CBSE 2025-26

Class 12 - Mathematics

Relations and Functions

Important Question

Question No. 1 to 5 are based on the given text. Read the text carefully and answer the questions:

Students of Grade 9, planned to plant saplings along straight lines, parallel to each other to one side of the playground ensuring that they had enough play area. Let us assume that they planted one of the rows of the saplings along the line y = x − 4. Let L be the set of all lines which are parallel on the ground and R be a relation on L.

1. Let relation R be defined by R = $\{(L_1, L_2) : L_1 \| L_2$ where $L_1, L_2 \in L\}$ then R is ________ relation.
   a. Only reflexive
   b. Symmetric but not transitive
   c. Not reflexive
   d. Equivalence
2. Let R = $\{(L_1, L_2) : L_1 \perp L_2$ where $L_1, L_2 \in L\}$ which of the following is true?
   a. R is an Equivalence relation
   b. R is Reflexive but neither symmetric nor transitive.
   c. R is Symmetric but neither reflexive nor transitive.
   d. R is Reflexive and transitive but not symmetric.

3. The function f : R $\rightarrow$ R defined by f(x) = x - 4 is ________.
   a. Neither Surjective nor Injective
   b. Surjective but not injective
   c. Bijective
   d. Injective but not Surjective
4. Let f : R $\rightarrow$ R be defined by f(x) = x − 4. Then the range of f(x) is ________.
   a. W
   b. R
   c. Z
   d. Q
5. Let R = $\{(L_1, L_2) : L_1$ is parallel to L_2 and L_1 : y = x - 4} then which of the following can be taken as L_2?
   a. 2x + 2y + 7 = 0
   b. 2x + y = 5
   c. 2x - 2y + 5 = 0
   d. x + y = 7
6. R is a relation on the set Z of integers and it is given by (x, y) $\in$ R $\Leftrightarrow$ |x – y| $\leq$ 1. Then, R is
   a. an equivalence relation
   b. symmetric and transitive

c. reflexive and symmetric

d. reflective and transitive

7. f : R $\rightarrow$ R is defined by $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$ is

a. neither one-one nor onto

b. many-one but onto

c. none of the above

d. one-one and onto

8. Let S be the set of all real numbers and let R be a relation on S, defined by a Rb $\Leftrightarrow$ |a - b| < 1. Then, R is

a. Reflexive and symmetric but not transitive

b. Reflexive and transitive but not symmetric

c. Symmetric and transitive but not reflexive

d. An equivalence relation

9. If the set Z of all integers, which of the following relation R is not an equivalence relation?

a. x R y : if x $\equiv$ y |3|

b. x R y : if x – y is an even integer

c. x R y : if x = y

d. x R y : if x $\leq$ y

10. Function f: X $\rightarrow$ Y is called many – one if

a. f has many elements

b. f is many – many

c. f is not well defined

d. f is not one – one

11. State True or False:

i. The relation R on the set A = {1, 2, 3} defined as R = {{1, 1), (1, 2), (2, 1), (3, 3)} is reflexive, symmetric and transitive.

a. True

b. False

ii. f is one-one iff $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in X$

a. True

b. False

12. Show that a one – one function f : {1, 2, 3} $\rightarrow$ {1, 2, 3} must be onto.

13. Define surjective function.

14. Find the domain and range of the real function, defined by f(x) = $\frac{x^2}{(1+x^2)}$. Show that f is many one.

15. Let f:R $\rightarrow$ R be define as f(x) = x^4 check whether the given function is one – one onto, or other.

16. If Z is the set of all integers and R is the relation on Z defined as R = {(a, b): a, $\in$ Z and a - b is divisible by 5}. Prove that R is an equivalence relation.

17. If R and S are relations on a set A, then prove the following:

i. R and S are symmetric $\Rightarrow$ R $\cap$ S and R $\cup$ S are symmetric.

ii. R is reflexive and S is any relation $\Rightarrow$ R $\cup$ S is reflexive.

Solution

1. (d) Equivalence
   Explanation: Equivalence
2. (c) R is Symmetric but neither reflexive nor transitive.
   Explanation: R is Symmetric but neither reflexive nor transitive.
3. (c) Bijective
   Explanation: Bijective

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4. (b) R
   Explanation: R
5. (c) 2x - 2y + 5 = 0
   Explanation: 2x - 2y + 5 = 0
6. (c) reflexive and symmetric
   Explanation: According to the condition,
   $(x,y) \in R \implies |x - y| \leq 1$
   Reflexive: let $(x,x) \in R \implies |x-x| = 0 < 1$
   $\Rightarrow$ R is Reflexive
   Symmetric:
   If $(x,y) \in R \implies |x-y| \leq 1$
   and $(y,x) \in R \implies |y-x| \leq 1$ [Since $|x-y| = |y-x|$]
   $\Rightarrow$ R is Symmetric
   Transitive:
   If $(x,y) \in R \Rightarrow |x-y| \leq 1$
   and $(y,z) \in R \Rightarrow |y-z| \leq 1$
   $\Rightarrow |x-y| = |x-y+y-z|$
   $\leq |x-y| + |y-z| \leq 1+1 = 2$
   $\Rightarrow |x-z| \leq 2$
   $\therefore$ R is not transitive
7. (a) neither one-one nor onto
   Explanation: Given that f : R $\rightarrow$ R where
   $f(x) = \frac{e^{x^2} - e^{-x^2}}{e^{x^2} + e^{-x^2}}$
   Here, we can see that for negative as well as positive x we will get same value.
   So, it is not one-one.
   f(x) = y
   By definition of onto, each element of R is not mapped to at least one element of R.
   So, it is not onto.
8. (a) Reflexive and symmetric but not transitive
   Explanation: Reflexivity: clearly for every a $\in$ S (real number)
   $|a - a| = 0 < 1$

$\Rightarrow$ R is Reflecxive

Symmetric: (a, b) $\in$ R

$\Rightarrow \quad |a-b|<1$

$\Rightarrow \quad |b-1|<1$

$\Rightarrow (b, a) \in R$

$\therefore$ R is Symmetric

Transitive: (a, b), (b, c) in R

$\Rightarrow |a-b|<1$ and $|b-c|<1$

Now, $|a-c|=|a-b+b-c| \leq |a-b|+|b-c|$

Which need not be less than 1

So, R is not Transitive

$\therefore$ R is Reflexive and symmetric but not transitive.

9. (d) x R y : if x $\leq$ y

   Explanation: x R y : if x $\leq$ y

   2 ,3 are in Z with $2 \leq 3$ but $3 \nleq 2$

   Therefore,R is not symmetric,hence it is not an equivalnce relation.

10. (d) f is not one – one

    Explanation: A function f: X $\rightarrow$ Y is called many – one iff it is not one-one, i.e. if there exist at least two elements x_1, x_2 in A such that $x_1 \neq x_2$ but $f(x_1) = f(x_2)$.

11. State True or False:

    i. (b) False

       Explanation: False

       Given that, A = {1, 2, 3} and R = {(1, 1), (1, 2), (2, 1), (3, 3)}

       Now,

       R is not reflexive $\because$ (2,2) $\notin$ R

    ii. (a) True

       Explanation: True

12. Since f is one – one three element of {1, 2, 3} must be taken to 3 different element of the co – domain {1, 2, 3} under f.

    Hence, f has to be onto.

    {by definition ONTO FUNCTION- every element of co-domain have a pre-image in domain}

13. If the function f: A$\rightarrow$B is such that each element in B (co-domain) is the ‘f’ image of at least one element in A, then we say that f is a function of A ‘onto’ B.

    Thus f: A$\rightarrow$B is surjective if, for all b$\in$B, there are some a$\in$A such that f(a) = b.

14. For domain $(1 + x^2) \neq 0$

    or $x^2 \neq -1$

    So, dom(f) = R

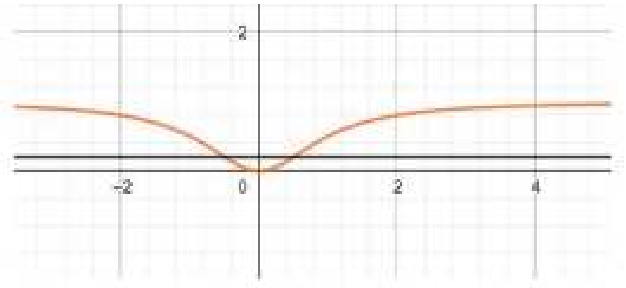
    For the range of x:

    $y=\frac{x^2+1-1}{x^2+1}=1-\frac{1}{x^2+1}$

    $y_{min} = 0$ (when x = 0)

    $y_{max} = 1$ (when x = ∞)

    $\therefore$ range of f(x) = [0,1)



For many one the lines cut the curve in 2 equal valued points of y therefore the function f(x) $= \frac{x^2}{x^2+1}$ is many - one.

dom(f) = R

range(f) = [0,1)

function f(x) $= \frac{x^2}{x^2+1}$ is many - one

15. Let $x_1, x_2 \in$R

Since f(1)=1 and f(-1)=1 so, two distinct elements have same image

Therefore function is not one one

Now,

For y = x^4 ,

Negative numbers have no pre-image

So, f(x) is not onto function

16. The given relation is R = $\{(a, b) : a, b \in$ Z and $a - b$ is divisible by 5$\}$.

To prove:- R is an equivalence relation, we have to prove R is reflexive, symmetric and transitive.

Reflexive :

As for any x∈Z, we have $x - x = 0$, which is divisible by 5

$\Rightarrow$ $(x - x)$ is divisible by 5

$\Rightarrow (x, x) \in$ R, ∀ x∈Z

Therefore, R is reflexive.

Symmetric:

Let (x, y)∈R where x, y∈Z

$\Rightarrow$ $(x - y)$ is divisible by 5. [by definition of R]

$\Rightarrow$ $x - y = 5A$ for some A∈Z

$\Rightarrow$ $y - x = 5(-A)$

$\Rightarrow$ $(y - x)$ is also divisible by 5

$\Rightarrow$ (y, x)∈R

∴ R is symmetric.

Transitive:

Let$(x, y)\in$R ,where x, y∈Z $\Rightarrow$ $(x - y)$ is divisible by 5

$\Rightarrow$ x - y = 5A for some A∈Z

Again, let$(y, z)\in$R where y, z∈Z

$\Rightarrow$ $(y - z)$ is divisible by 5

$\Rightarrow$ $y - z = 5B$ for some B∈Z.

Now, $(x - y) + (y - z) = 5A + 5B$

$\Rightarrow$ $x - z = 5(A + B)$

$\Rightarrow (x - z)$ is divisible by 5 for some

$(A + B) \in$ Z

$\Rightarrow (x, z) \in$ R

∴ R is transitive.

Thus, R is reflexive, symmetric and transitive

Hence, it is an equivalence relation.

17. R and S are two symmetric relations on set A.

i. To prove: $R \cap S$ is symmetric

Let $(a, b) \in R \cap S$

$\Rightarrow (a, b) \in R$ and $(a, b) \in S$

$\Rightarrow (b, a) \in R$ and $(b, a) \in S$ [$\because$ R and S are symmetric]

$\Rightarrow (b, a) \in R \cap S$

$\Rightarrow R \cap S$ is symmetric

To prove: $R \cup S$ is symmetric.

Let $(a, b) \in R$ or $(b, a) \in S$ [$\because$ R and S are symmetric]

$\Rightarrow (a, b) \in R$ or $(a, b) \in S$

$\Rightarrow (b, a) \in R$ or $(b, a) \in S$ [$\because$ R and S are symmetric]

$\Rightarrow (b, a) \in R \cup S$

$\Rightarrow R \cup S$ is symmetric

ii. R and S are two relations on A such that R is reflexive.

To prove: $R \cup S$ is reflexive

Suppose $R \cup S$ is not reflexive.

This means that there is an $a \in R \cup S$ such that $(a, a) \notin R \cup S$

Since $a \in R \cup S$,

$\therefore a \in R$ or $a \in S$

If $a \in R$, then $(a, a) \in R$ [$\because$ R is reflexive]

$\Rightarrow (a, a) \in R \cup S$ which contradicts our supposition.

Hence, $R \cup S$ is reflexive.