CBSE 2025-26

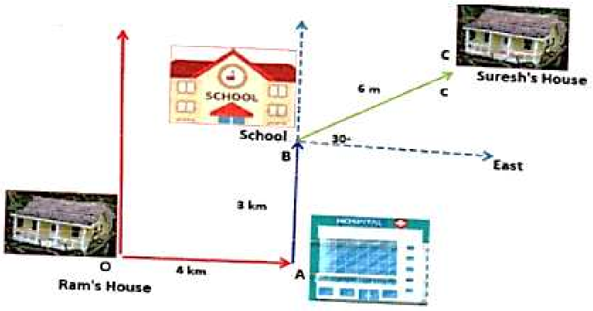
Class 12 - Mathematics

Vector Algebra

Important Question

Question No. 1 to 5 are based on the given text. Read the text carefully and answer the questions:

Ram's house is situated at Gandhi Nagar at Point O, for going to school he first travels by bus in the east. Here at Point A, a hospital is situated. From Hospital Ram takes an auto and goes 3 km in the north direction, here at point B school is situated. Suresh's house is at 30^{o} east, 6 km from point B. **(Refer image for information)**



1. What is vector distance between Ram's house and school?
   a. 8 km
   b. 4 km
   c. 5 km
   d. 7 km
2. How many km Ram travels to reach school?
   a. 5 km
   b. 7 km
   c. 8 km
   d. 4 km
3. What is the vector distance from school to Suresh's home?
   a. $6\hat{i}$
   b. $6\hat{j}$
   c. $\sqrt{3}\hat{i}+\hat{j}$
   d. $3\sqrt{3}\hat{i}+3\hat{j}$
4. What is the displacement from Ram's house to Suresh house?
   a. $(4+3\sqrt{3})\hat{i}+6\hat{j}$
   b. $13\hat{j}$
   c. $4\hat{i}+6\hat{j}$
   d. $13\hat{i}$
5. What is the total distance from Ram's house to Suresh's home?
   a. 13 km
   b. 5 km
   c. 11 km
   d. 9 km

6. If $\begin{vmatrix} |\vec{a}|^2 & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & |\vec{b}|^2 & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & |\vec{c}|^2 \end{vmatrix} = \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}^n$ where n ∈ N, then n is
   a. 1
   b. none of these
   c. 2
   d. 3
7. Let A(3, 0, -1), B(2, 10, 6) and C(1, 2,1) be the vertices of a triangle and M be the mid-point of AC. If G divides BM in the ratio 2 : 1, then cos (∠GOA) (O being the origin) is equal to
   a. $\frac{1}{6\sqrt{10}}$
   b. $\frac{1}{\sqrt{30}}$
   c. $\frac{1}{2\sqrt{15}}$
   d. $\frac{1}{\sqrt{15}}$
8. If the vectors $4\hat{i} + 11\hat{j} + m\hat{k}$, $7\hat{i} + 2\hat{j} + 6\hat{k}$ and $\hat{i} + 5\hat{j} + 4\hat{k}$ are coplanar, then m =
   a. 38
   b. -10
   c. 0
   d. None of these
9. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ and $\vec{d}$ be such that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = \vec{0}$. If P_1 and P_2 are planes determined by the pairs of vectors $\vec{a}, \vec{b}$, and $\vec{c}, \vec{d}$ respectively, then the angle between P_1 and P_2 is
   a. 0
   b. $\frac{\pi}{3}$
   c. $\frac{\pi}{2}$
   d. $\frac{\pi}{4}$

10. The vector $(\cos\alpha\cos\beta)\hat{i} + (\cos\alpha\sin\beta)\hat{j} + (\sin\alpha)\hat{k}$ is a
   a. none of these
   b. constant vector
   c. null vector
   d. unit vector
11. State True or False:
   i. The length of the vector $\overrightarrow{AB}$ is called a magnitude of $\overrightarrow{AB}$.
      a. True
      b. False
   ii. If $\overrightarrow{a}, \overrightarrow{b}$ are two vectors, then write the truth value of the statement: $\overrightarrow{a} = -\overrightarrow{b} \Rightarrow |\overrightarrow{a}| = |\overrightarrow{b}|$.
      a. True
      b. False
12. Fill in the blanks:
   a. A quantity that has magnitude as well as direction, is called a ________.
   b. The number of vectors of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is ________.
13. Find the components along the coordinate axes of the position vector of the point: S(4, -3).
14. Write two different vectors having same magnitude.

15. If $A(-2,1,2)$ and $B(2,-1,6)$ are two given points, find a unit vector in the direction of $\overrightarrow{AB}$.
16. Find a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$, where $\overrightarrow{a} = 4\hat{i} - \hat{j} + 8\hat{k}, \vec{b} = -\hat{j} + \hat{k}$.
17. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} + \vec{b} + \vec{c} = \overrightarrow{0}$, then prove that $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$
18. If O is the circumcentre and O' the orthocentre of a triangle ABC, prove that $\overrightarrow{AO'} + \overrightarrow{O'B} + \overrightarrow{O'C} = \overrightarrow{AP}$, where $\overrightarrow{AP}$ is the diameter of the circumcircle.

Solution

1. (c) 5 km
   Explanation: 5 km
2. (b) 7 km
   Explanation: 7 km
3. (d) $3\sqrt{3}\hat{i} + 3\hat{j}$
   Explanation: $3\sqrt{3}\hat{i} + 3\hat{j}$
4. (a) $(4 + 3\sqrt{3})\hat{i} + 6\hat{j}$
   Explanation: $(4 + 3\sqrt{3})\hat{i} + 6\hat{j}$
5. (a) 13 km
   Explanation: 13 km
6. (c) 2
   Explanation: Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}, \vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$
   Let $\begin{vmatrix} |\vec{a}|^2 & \vec{a}\cdot\vec{b} & \vec{a}\cdot\vec{c} \\ \vec{b}\cdot\vec{a} & |\vec{b}|^2 & \vec{b}\cdot\vec{c} \\ \vec{c}\cdot\vec{a} & \vec{c}\cdot\vec{b} & |\vec{c}|^2 \end{vmatrix} = \Delta$. Then
   $\Delta = \begin{vmatrix} a_1^2 + a_2^2 + a_3^2 & a_1b_1 + a_2b_2 + a_3b_3 & a_1c_1 + a_2c_2 + a_3c_3 \\ a_1b_1 + a_2b_2 + a_3b_3 & b_1^2 + b_2^2 + b_3^2 & b_1c_1 + b_2c_2 + b_3c_3 \\ a_1c_1 + a_2c_2 + a_3c_3 & b_1c_1 + b_2c_2 + b_3c_3 & c_1^2 + c_2^2 + c_3^2 \end{vmatrix}$
   $= \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$
   $\Delta = \begin{bmatrix}\vec{a} & \vec{b} & \vec{c}\end{bmatrix}^2 \Rightarrow$ n = 2
7. (d) $\frac{1}{\sqrt{15}}$
   Explanation: Given vertices of a $\triangle$ABC are A(3,0, -1), B(2,10,6) and C (1, 2, 1) and a point M is mid-point of AC. An another point G divides BM in ratio 2 : 1, so G is the centroid of $\triangle$ABC.
   $\therefore G\left(\frac{3+2+1}{3}, \frac{0+10+2}{3}, \frac{-1+6+1}{3}\right)$ = (2, 4, 2)
   Now, cos ($\angle$GOA) = $\frac{\vec{OG}\cdot\vec{OA}}{|\vec{OG}||\vec{OA}|}$, where O is the origin.
   $\because \overrightarrow{OG} = 2\hat{i} + 4\hat{j} + 2\hat{k} \Rightarrow |\overrightarrow{OG}| = \sqrt{4 + 16 + 4} = \sqrt{24}$
   and $|\overrightarrow{OA}| = 3\hat{i} - \hat{k} \Rightarrow |\overrightarrow{OA}| = \sqrt{9 + 1} = \sqrt{10}$
   and $\overrightarrow{OG}.\overrightarrow{OA}$ = 6 - 2 = 4
   $\therefore \cos(\angle GOA) = \frac{4}{\sqrt{24}\sqrt{10}} = \frac{1}{\sqrt{15}}$
8. (d) None of these
   Explanation: Given vectors $4\vec{i} + 11\vec{j} + m\vec{k}.\ 7\vec{i} + 2\vec{j} + 6\vec{k}\ and\ \vec{i} + 5\vec{j} + 4\vec{k}$ are coplanar then

$$\begin{vmatrix} 4 & 11 & m \\ 7 & 2 & 6 \\ 1 & 5 & 4 \end{vmatrix} = 0$$

$\implies$ 4(8-30) - 11(28 - 6) + m(35 - 2) = 0

- 88 - 242 + 33 m = 0

- 330 + 33 m = 0

m = 10

9. (a) 0

Explanation: If θ is the angle between P_1 and P_2, then normal to the planes are

$$\left.\begin{aligned} N_1 &= \vec{a} \times \vec{b} \\ N_2 &= \vec{c} \times \vec{d} \end{aligned}\right\}$$

$\therefore N_1 \times N_2 = 0$

Then, $|N_1| \times |N_2| \sin\theta = 0$

$\Rightarrow \sin\theta = 0 \Rightarrow \theta = 0$

10. (d) unit vector

Explanation: Given vector $(cos\alpha cos\beta)\hat{i} + (cos\alpha sin\beta)\hat{j} + sin\alpha\hat{k}$

The magnitude of the vector = $\sqrt{cos^2\alpha cos^2\beta + cos^2\alpha sin^2\beta + sin^2\alpha}$

= $\sqrt{cos^2\alpha(cos^2\beta + sin^2\beta) + sin^2\alpha}$

= $\sqrt{cos^2\alpha + sin^2\alpha}$

= 1

$\therefore$ It is unit vector

11. State True or False:

   i. (a) True

   Explanation: True

   ii. (a) True

   Explanation: True

   Let $\overrightarrow{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$

   $\overrightarrow{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$

   Given that, a = -b

   $a_1\hat{i} + b_1\hat{j} + c_1\hat{k} = -a_2\hat{i} - b_2\hat{j} - c_2\hat{k}$

   Comparing the coefficients of i, j, k in LHS and RHS,

   $a_1 = -a_2$ ... (i)

   $b1 = -b_2$ ... (ii)

   $c1 = -c_2$ .... (iii)

   $|\vec{a}| = \sqrt{a_1^2 + b_1^2 + c_1^2}$

   Using (i), (ii) and (iii),

   $|\overrightarrow{a}| = \sqrt{(-a_2)^2 + (-b_2)^2 + (-c_2)^2}$

   $|\overrightarrow{a}| = \sqrt{a_2^2 + b_2^2 + c_2^2}$

   $\therefore |\overrightarrow{a}| = |\overrightarrow{b}|$

12. Fill in the blanks:

   a. vector

   b. two

13. The position vector of the given point S(4, -3) is given by ;

$\overrightarrow{OS} = 4\hat{i} - 3\hat{j}$

Component of $\overrightarrow{OS}$ along x- axis = a vector of magnitude 4 having its direction along the positive direction of x-axis.

Component of $\overrightarrow{OS}$ along y-axis = a vector of magnitude 3 having its direction along the negative direction of y-axis.

14. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$|\vec{a}| = \sqrt{1+4+9} = \sqrt{14}$

$\vec{b} = 3\hat{i} + 2\hat{j} + 1\hat{k}$

$\left|\vec{b}\right| = \sqrt{9+4+1} = \sqrt{14}$

15. We have, A = (-2, 1, 2), and B = (2, -1, 6)

$\therefore \overrightarrow{AB}$ = position vector of B - position vector of A = $\{2-(-2)\}\hat{\imath} + \{(-1)-1\}\hat{\jmath} + \{6-2\}\hat{k}$

= $4\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

We know that for any vector $\overrightarrow{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ the unit vector is represented as $\widehat{a} = \frac{a_x\hat{i}+a_y\hat{j}+a_z\hat{k}}{\sqrt{a_x^2+a_y^2+a_z^2}}$

$\therefore \widehat{AB} = \frac{4\hat{\imath}-2\hat{\jmath}+4\hat{k}}{\sqrt{4^2+2^2+4^2}}$

= $\frac{4}{6}\hat{\imath} - \frac{2}{6}\hat{\jmath} + \frac{4}{6}\hat{k}$

= $\frac{2}{3}\hat{\imath} - \frac{1}{3}\hat{\jmath} + \frac{2}{3}\hat{k}$

16. Here we have $\overrightarrow{a} = 4\hat{i} - \hat{j} + 8\hat{k}, \vec{b} = -\hat{j} + \hat{k}$

$\overrightarrow{a} \times \overrightarrow{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 8 \\ 0 & -1 & 1 \end{vmatrix} = 7\hat{i} - 4\hat{j} - 4\hat{k}$

Required unit vector = $\frac{(\vec{a}\times\vec{b})}{|\vec{a}\times\vec{b}|}$

= $\frac{1}{9}(7\hat{i} - 4\hat{j} - 4\hat{k})$

17. Given that ,

$\vec{a} + \vec{b} + \vec{c} = \overrightarrow{0}$

$\Rightarrow \vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times \overrightarrow{0}$ [Taking cross-product on left with $\vec{a}$]

$\Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \overrightarrow{0}$ [Using distributive law]

$\Rightarrow \vec{a} \times \vec{b} - \vec{c} \times \vec{a} = \overrightarrow{0}$ $[\because \vec{a} \times \vec{a} = \overrightarrow{0}$ and $\vec{a} \times \vec{c} = -\vec{c} \times \vec{a}]$

$\Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$ ...(i)

Again, $\vec{a} + \vec{b} + \vec{c} = \overrightarrow{0}$

$\Rightarrow \vec{b} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{b} \times \overrightarrow{0}$ [Taking cross-product on left with $\vec{b}$]

$\Rightarrow \vec{b} \times \vec{a} + \vec{b} \times \vec{b} + \vec{b} \times \vec{c} = \overrightarrow{0}$

$\Rightarrow -\vec{a} \times \vec{b} + \overrightarrow{0} + \vec{b} \times \vec{c} = \overrightarrow{0}$ $[\because \vec{b} \times \vec{b} = \overrightarrow{0}$ and $\vec{b} \times \vec{a} = -\vec{a} \times \vec{b}]$

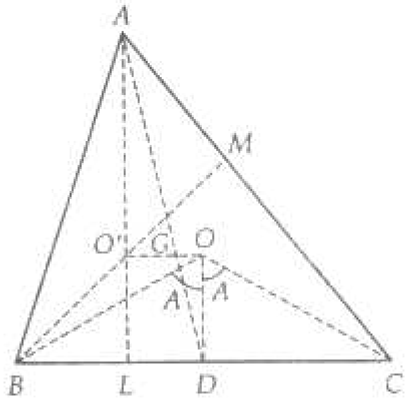
$\Rightarrow \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ ...(ii)

Form (i) and (ii), we obtain

$\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$. Hence proved.

18. Suppose that G be the centroid of triangle ABC. First we will show that the circumcentre O, orthocentre O' and centroid G are collinear and O'G = 2OG

Suppose AL and BM be perpendiculars on the sides BC and CA respectively. Suppose ADAD be the median and OD be the perpendicular from O on side BC. If R is the circum radius of circumcircle of $\triangle ABC$, then we get OB = OC = R.



In $\triangle OBD$, we have

OD = R cos A .... (i)

In $\triangle ABM$, we have

AM = AB cosA = c cosA .... (ii)

In $\Delta AO'M$, we have

AO' = Am sec $\angle O'AM$

$\Rightarrow$ AO' = c cos A sec (90^o - C) [Using (ii)]

$\Rightarrow$ AO' = c cos A cosec C

$\Rightarrow \quad AO' = \frac{c}{\sin C}\cos A = 2R\cos A \left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{C}{\sin C} = 2R\right]$

$\triangle$ AO' = 2 OD .... (iii) [Using (i)]

Triangle AGO' and OGD are similar

$\therefore \quad \frac{OG}{O'G} = \frac{GD}{GA} = \frac{OD}{AO'} = \frac{1}{2}$ [Using (iii)

$\Rightarrow \quad 2 \cdot OG = O'G$ .... (iv)

We have,

$\overrightarrow{AO'} + \overrightarrow{O'B} + \overrightarrow{O'C} = 2\overrightarrow{AO}' + (\overrightarrow{O'A} + \overrightarrow{O'B} + \overrightarrow{O'C})$

$\Rightarrow \overrightarrow{AO'} + \overrightarrow{O'B} + \overrightarrow{O'C} = 2\overrightarrow{AO'} + 2\overrightarrow{O'O}$ [using (iii)]

$\Rightarrow \overrightarrow{AO'} + \overrightarrow{O'B} + \overrightarrow{O'C} = 2\left(\overrightarrow{AO'} + \overrightarrow{O'}O\right)$ [using (iii)]

$\Rightarrow \overrightarrow{AO'} + \overrightarrow{O'B} + \overrightarrow{O'C} = 2\overrightarrow{AO} = \overrightarrow{AP}$ [$\because$ AO is the circum-radius of $\triangle ABC$]