# CBSE 2025-26

# Class 12 - Mathematics

# Determinants

# Important Question

**Question No. 1 to 5 are based on the given text. Read the text carefully and answer the questions:**

Pankaj purchased 5 pens, 3 bags and 1 instrument box and pays ₹16. From the same shop, Dinesh purchased 2 pens, 1 bag and 3 instrument boxes and pays ₹19, while Ankit purchased 1 pen, 2 bags and 4 instrument boxes and pays ₹25.

1. The cost of one pen is
    a. ₹2
    b. ₹3
    c. ₹5
    d. ₹1
2. What is the cost of one pen and one bag?
    a. ₹5
    b. ₹8
    c. ₹3
    d. ₹7



3. What are the cost of one pen and one instrument box?
    a. ₹6
    b. ₹7
    c. ₹8
    d. ₹9
4. Which of the following is correct?
    a. A determinant is a number associated to a square matrix.
    b. A determinant is a number associated to a matrix.
    c. A determinant is a square matrix.
    d. All of these
5. From the matrix equation AB = AC, it can be concluded that B = C provided
    a. A is non-singular
    b. A is singular
    c. A is symmetric
    d. A is square
6. Let M and N be two $3 \times 3$ non-singular skew-symmetric matrices such that MN = NM. If $P^T$ denotes the transpose of P, then $M^2N^2(M^TN)^{-1}(MN^{-1})^T$ is equal to
    a. $-M^2$

b. MN

c. $-N^2$

d. $M^2$

7. Let a, b, c, d, u, v be integers. If the system of equations, ax + by = u, cx + dy = v, has a unique solution in integers, then
   a. ad - bc need not be equal to $\pm1$.
   b. ad – bc = - 1
   c. ad – bc = 1
   d. ad – bc = $\pm1$

8. The number of values of x for which the matrix A = $\begin{bmatrix} x-1 & x & x+1 \\ 2 & -1 & 3 \\ x+3 & x-2 & x+7 \end{bmatrix}$ has no inverse is
   a. 2
   b. 1
   c. 3
   d. infinitely many

9. Adj.(KA) = ______
   a. $K^{n-1}$ Adj. A
   b. None of these
   c. K Adj. A
   d. $K^n$ Adj.A

10. State True or False:
    i. If A = $\begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & x \\ 2 & 3 & 1 \end{bmatrix}$, $A^{-1}$ = $\begin{bmatrix} \frac{1}{2} & -4 & \frac{5}{2} \\ -\frac{1}{2} & 3 & -\frac{3}{2} \\ \frac{1}{2} & y & \frac{1}{2} \end{bmatrix}$ then x = 1, y = -1
       a. True
       b. False
    ii. For a square matrix A in matrix equation AX = B if |A| = 0 and (adj A)B $\neq$ 0, then there exists no solution.
       a. True
       b. False

11. Fill in the blanks:
    a. If A and B are square matrices each of order 3 and |A| = 5, |B| = 3, then the value of |3AB| is _________.
    b. If all the elements of a determinant above or below the main diagonal consists of zeros, then the value of the determinant is equal to the product of ________ elements.

12. Match the column:

| | |
|---|---|
| (a) The determinant $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 6 & 10 \\ 31 & 11 & 38 \end{vmatrix} =$ | (i) 1 |
| (b) The determinant $\begin{vmatrix} 5 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 3 \end{vmatrix} =$ | (ii) 18 |
| (c) The determinant $\begin{vmatrix} sin10^\circ & cos10^\circ \\ sin80^\circ & cos80^\circ \end{vmatrix} =$ | (iii) 0 |
| | (iv) 60 |

| | |
|---|---|
| (d) The determinant $\begin{vmatrix} 0 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix} =$ | |

13. Evaluate the determinant: $\begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$

14. Write $A^{-1}$ for A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$.

15. Using matrix method, solve the system of equations
    4x + 3y + 2z = 60;
    x + 2y + 3z = 45;
    6x + 2y + 3z = 70.

16. Use determinants to show that the following points are collinear. A (3, 8), B (-4,2) and C(10,14).

17. If the co-ordinates of the vertices of an equilateral triangle with sides of length ‘a’ are $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$, then
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3a^4}{4}.$$

18. Find adjoint of the matrix $\begin{vmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{vmatrix}$

# Solution

1. (d) ₹1

   **Explanation:** ₹1
2. (c) ₹3

   **Explanation:** ₹3
3. (a) ₹6

   **Explanation:** ₹6
4. (a) A determinant is a number associated to a square matrix.

   **Explanation:** A determinant is a number associated to a square matrix.
5. (a) A is non-singular

   **Explanation:** A is non-singular
6. (a) $-M^2$

   **Explanation:** Given, $M^T = -M$, $N^T = -N$ and $MN = NM$ ...(i)

   $\therefore M^2N^2 (M^TN)^{-1} (MN^{-1})^T$

   $\Rightarrow M^2N^2N^{-1} (M^T)^{-1} (N^{-1})^T .M^T$

   $\Rightarrow M^2N (NN^{-1}) (-M)^{-1} (N^T)^{-1} (-M)$

   $\Rightarrow M^2N I (-M^{-1}) (-N)^{-1} (-M)$

   $\Rightarrow -M^2NM^{-1} N^{-1} M$

   $\Rightarrow -M \cdot (MN) M^{-1} N^{-1} M = -M (NM) M^{-1} N^{-1} M$

   $\Rightarrow -MN (NM^{-1}) N^{-1} M = -M (NN^{-1}) M \Rightarrow -M^2$
7. (a) ad - bc need not be equal to $\pm 1$.

   **Explanation:** ax + by = u , cx +dy = v,

   $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc;$

   $\Delta_1 = \begin{vmatrix} u & b \\ v & d \end{vmatrix} = ud - bv;$

   $\Delta_2 = \begin{vmatrix} a & u \\ c & v \end{vmatrix} = av - cu;$

   $\Rightarrow x = \frac{\Delta_1}{\Delta} = \frac{ud-bv}{ad-bc}, y = \frac{\Delta_2}{\Delta} = \frac{av-cu}{ad-bc}$

   since the solution is unique in integers. $\Delta = \pm 1$ , $ad - bc = \pm 1$
8. (d) infinitely many

   **Explanation:** Given matrix has no inverse if |A| = 0

   $|A| = \begin{vmatrix} x-1 & x & x+1 \\ 2 & -1 & 3 \\ x+3 & x-2 & x+7 \end{vmatrix}$

   = (x - 1) {-1 (x + 7) - 3 (x - 2)} - x {2 (x + 7) - 3 (x + 3)} + (x + 1) {2 (x - 2) + (x + 3)}

   = (x - 1) (-4x - 1) - x (-x + 5) + (x + 1) (3x - 1)

= $-4x^2 - x + 4x + 1 + x^2 - 5x + 3x^2 - x + 3x - 1$

= 0

$\Rightarrow$ For infinitely many values of x, the given matrix has no inverse.

9. (a) $K^{n-1}$ Adj. A

**Explanation:** Adj. (KA) = $K^{n-1}$ Adj.A, where K is a scalar and A is a n $\times$ n matrix.

10. State True or False:
    i. (a) True
       **Explanation:** True
    ii. (a) True
       **Explanation:** True
11. Fill in the blanks:
    a. 405
    b. Diagonal
12. (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)
13. Let $\Delta = \begin{vmatrix} 1 & -3 & 2 \\ 4 & -1 & 2 \\ 3 & 5 & 2 \end{vmatrix}$

$\Rightarrow \Delta = 2\begin{vmatrix} 1 & -3 & 1 \\ 4 & -1 & 1 \\ 3 & 5 & 1 \end{vmatrix}$

Applying, $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$\Rightarrow \Delta = 2\begin{vmatrix} 1 & -3 & 1 \\ 3 & 2 & 0 \\ 2 & 8 & 0 \end{vmatrix}$



= 2[1(24 - 4)] = 40

So, $\Delta$ = 40.

14. We have, A = $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Clearly, adj A = $\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

and $|A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = 6 - 5 = 1$

$\therefore \quad A^{-1} = \frac{1}{|A|} \text{ adj } (A) = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

15. Given set of equations are:-

4x + 3y + 2z = 60

x + 2y + 3z = 45

6x + 2y + 3z = 70

Converting the following set of equations in matrix form, we have

AX = B

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & 2 & 3 \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 45 \\ 70 \end{bmatrix}$$

$R_2 \to 4R_2 - R_1$

$R_3 \to 2R_3 - 3R_1$

$$\begin{bmatrix} 4 & 3 & 2 \\ 0 & 5 & 10 \\ 0 & -5 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 60 \\ 120 \\ -40 \end{bmatrix}$$

Again converting into equations, we get

4x + 3y + 2z = 60

5y + 10z = 120

-5y = - 40

Y = 8

5 × 8 + 10z = 120

10z = 120 - 40

10z = 80

z = 8

4x + 3 × 8 + 2 × 8 = 60

4x = 60 - 24 - 16

4x = 20

x = 5

∴ x = 5, y = 8, z = 8

16. If area of triangle = 0, then points are collinear.

Now Area of a triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$



$= \frac{1}{2}\begin{vmatrix} 3 & 8 & 1 \\ -4 & 2 & 1 \\ 10 & 14 & 1 \end{vmatrix}$

Expanding with $C_3$

$= \frac{1}{2}[(-56 - 20) - (42 - 80) + (6 + 32)]$

$= \frac{1}{2}[-76 + 38 + 38]$

=0

Since the area between the 3 points is 0, the three points lie in a straight line, i.e. they are collinear.

17. Since, we know that area of a triangle with vertices $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$, is given by $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$\Rightarrow \Delta^2 = \frac{1}{4}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$ ...(i)

We know that, area of an equilateral triangle with side a,

$\Delta = \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)a^2 = \frac{\sqrt{3}}{4}a^2$

$\Rightarrow \Delta^2 = \frac{3}{16}a^4$ ...(ii)

from eq. (i) and (ii), $\frac{3}{16}a^4 = \frac{1}{4}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2$

$$= \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}^2 = \frac{3}{4}a^4$$

Hence proved.

18. Here $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

$$\Rightarrow |A| = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

$$A_{11} = +\begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$$

$$A_{12} = -\begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2+10) = -12$$

$$A_{13} = +\begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 6$$

$$A_{21} = -\begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} - (-1) = 1$$

$$A_{22} = +\begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 1 + 4 = 5$$

$$A_{23} = -\begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} - (-2) = 2$$

$$A_{31} = +\begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$$

$$A_{32} = +\begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -(5-4) = -1$$

$$A_{33} = +\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 3 + 2 = 5$$

$$\therefore Adj.\,A = \begin{bmatrix} 3 & -12 & 6 \\ 1 & 5 & 2 \\ -11 & -1 & 5 \end{bmatrix}'$$

$$= \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$$