CBSE 2025-26

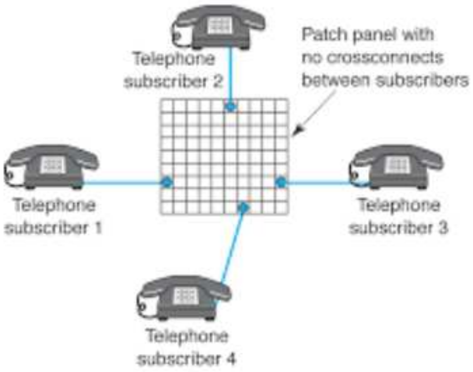
Class 12 - Mathematics

Application of Derivatives

Important Question

Question No. 1 to 5 are based on the given text. Read the text carefully and answer the questions:

A telephone company in a town has 500 subscribers on its list and collects fixed charges of 300 per subscriber per year. The company proposes to increases the annual subscription and it is believed that for every increase of 1 one subscriber will discontinue the service.



1. If x be the annual subscription then the total revenue of the company after increment will be:
   a. $R(x) = -x^2 + 200x + 150000$
   b. $R(x) = -x^2 + 100x + 100000$
   c. $R(x) = x^2 - 200x - 140000$
   d. $R(x) = 200x^2 + x + 150000$
2. To find maximum profit we put
   a. $R'(x) = 0$
   b. $R'(x) < 0$
   c. $R'(x) > 0$
   d. $R''(x) = 0$
3. How much fee the company should increase to have maximum profit?
   a. ₹ 250
   b. ₹ 150
   c. ₹ 200
   d. ₹ 100

4. Find the maximum profit that the company can make if the profit function is given by $P(x) = 41 + 24x - 18x^2$.
   a. 49
   b. 25
   c. 44
   d. 45
5. Find both the maximum and minimum value respectively of $3x^4 - 8x^3 + 48x + 1$ on the interval [1, 4].
   a. 257, -63
   b. -63, 257
   c. 258, -63

d. -63, -257

6. If the function f defined as f(x) = $2x^3 - 9ax^2 + 12a^2x + 1$, where a > 0, attains its maxima and minima at p and q respectively such that $p^2 = q$, then a equals:
   a. 3
   b. $\frac{1}{2}$
   c. 2
   d. 1
7. The global maximum value of f(x) = cot x - $\sqrt{2}$ cosec x in interval (0, π) is equal to:
   a. 0
   b. -1
   c. 1
   d. non-existent
8. Let f (x) = $x^3 - 6x^2 + 9x + 8$, then f (x) is decreasing in
   a. $(-\infty, 1)$
   b. [1, 3]
   c. $(-\infty, 1) \cup (3, \infty)$
   d. $[3, \infty]$
9. The function f given by f(x) = cos x + cos $\sqrt{3}$x
   a. attains its global maximum at x = 0
   b. is odd
   c. is periodic
   d. is not differentiable
10. For the function f defined as f(x) = x cos $\left(\frac{1}{x}\right)$, x $\geq$ 1, which one of the following is incorrect?
   a. For at least one x in the interval (1, ∞), f(x + 2) - f(x) < 2
   b. f(x) is decreasing in the interval (1, ∞)
   c. $\lim_{x \to \infty}$ f'(x) = 1
   d. for all x in the interval (1, ∞), f(x + 2) - f(x) > 2
11. State True or False:
   i. The f: R $\rightarrow$ R be defined by f(x) = 2x + cosx, then f is a decreasing function.
      a. True
      b. False
   ii. The function f(x) = tan - x always increases.
      a. True
      b. False

12. Fill in the blanks:
   a. y = $x(x - 3)^2$ decreases for the values of x given by ________.
   b. The interval on which the function f(x) = $2x^3 + 9x^2 + 12x - 1$ is decreasing is ________.
13. If h(x) = f(x) + f(-x), then show that h(x) has maximum value at the point where f'(x) is an even function.
14. Show that f(x) = x + cos x - a is an increasing function on R for all values of a.
15. Determine the values of $f(x) = x^x, x > 0$ for which f is increasing or decreasing.
16. Prove that the function f given by f(x) = $x^2 - x + 1$ is neither strictly increasing nor strictly decreasing on (-1, 1).

17. A tank with a rectangular base and rectangular sides open at the top is to be constructed so that its depth is 3 m and volume is 75 m^3. If building of tank costs Rs. 100 per square metre for the base and Rs. 50 per square metres for the sides, find the cost of least expensive tank.
18. A point on the hypotenuse of a triangle is at distance a and b from the sides of the triangle. Show that the minimum length of the hypotenuse is $\left(a^{\frac{2}{3}} + b^{\frac{3}{2}}\right)$.

Solution

1. (a) $R(x) = -x^2 + 200x + 150000$

   Explanation: $R(x) = -x^2 + 200x + 150000$
2. (a) R'(x) = 0

   Explanation: R'(x) = 0
3. (d) ₹ 100

   Explanation: ₹ 100
4. (a) 49

   Explanation: 49
5. (a) 257, -63

   Explanation: 257, -63

6. (c) 2

   Explanation: $f(x) = 2x^3 - 9ax^2 + 12a^2 x + 1$

   Differentiating with respect to x, we get

   $f'(x) = 6x^2 - 18ax + 12a^2$

   f'(x) = 0

   $\Leftrightarrow 6x^2 - 18ax + 12a^2 = 0$

   $\Leftrightarrow x^2 - 3ax + 2a^2 = \Leftrightarrow (x - a)(x - 2a) = 0$

   $\Rightarrow$ x = a or x = 2a

   x = a : a point of relative maxima

   x = 2a : a point of relative minima

   $\Rightarrow$ p = a, q = 2a

   $\Rightarrow a^2 = 2a \Rightarrow a = 2$ or a = 0

   $a > 0 \Rightarrow a = 2$
7. (b) -1

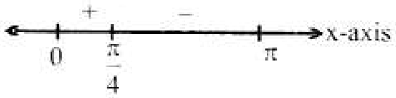
   Explanation: We have $f(x) = \cot x - \sqrt{2} \operatorname{cosec} x = \frac{\cos x - \sqrt{2}}{\sin x}$

   So, $f'(x) = \frac{\sin x(-\sin x) - (\cos x - \sqrt{2}) \cos x}{\sin^2 x}$

   $= \frac{-1 + \sqrt{2} \cos x}{\sin^2 x}$

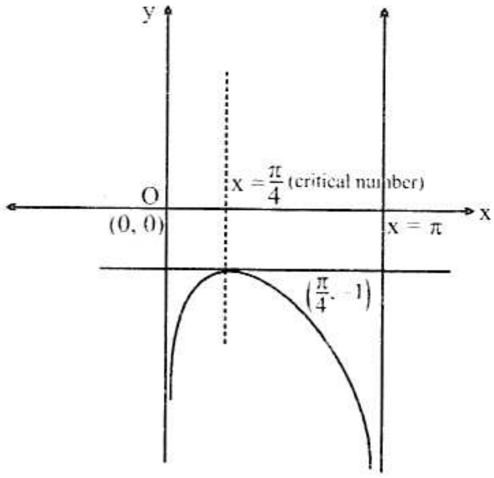
   $\therefore f'(x) = 0$

   $\Rightarrow \cos x = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4} \in (0, \pi)$

   (point of local maximum) Sign scheme of $f'(x)$

$\therefore$ f(x) $\uparrow$ on $\left(0, \frac{\pi}{4}\right)$ and f(x) $\downarrow$ on $\left(\frac{\pi}{4}, \pi\right)$



Graph of f(x) = cot x - $\sqrt{2}$ cosec x in (0, π)

Also, $\lim\limits_{x \to 0^+}$, f(x) = $\lim\limits_{x \to 0^+} \left(\frac{\cos x - \sqrt{2}}{\sin x}\right) \to -\infty$ and $\lim\limits_{x \to \pi^-}$ f(x) = $\lim\limits_{x \to \pi^-} \left(\frac{\cos x - \sqrt{2}}{\sin x}\right) \to -\infty$ and f(x) is also continuous on (0, π)

Clearly, f(x) is also continuous on (0, π)

$\therefore$ At x = $\frac{\pi}{4}$ (a local maximum point),

So, f(x) takes its absolute maximum value also at x = $\frac{\pi}{4}$

Hence, absolute maximum value of f(x = $\frac{\pi}{4}$) = 1 - $\sqrt{2}(\sqrt{2})$ = 1 - 2 = -1

8. (b) [1, 3]

   Explanation: Here ,$f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3) \leq 0 \Leftrightarrow x \in [1, 3]$

9. (a) attains its global maximum at x = 0

   Explanation: f (-x) = f(x) $\Rightarrow$ f is an even function.

   Let g (x) = cos x, h (x) = cos $\sqrt{3}$x.

   The function h and g are periodic with periods

   2π (= a, say) and $\frac{2\pi}{\sqrt{3}}$ (= b, say).

   Since $\frac{a}{b}$ is not rational.

   Therefore f is not periodic.

   f is differentiable everywhere,

   cos x $\leq$ 1 and cos $\sqrt{3}$ x $\leq$ 1 ...(i)

   $\Rightarrow$ cos x + cos $\sqrt{3}$ x $\leq$ 2

   $\Leftrightarrow$ f(x) $\leq$ 2

   The global maximum value of f(x) = 2 if equality occurs in (i).

   (Equality in (i) occurs when x = 0).

   $\Rightarrow$ Global maximum of f occurs at x = 0

10. (a) For at least one x in the interval (1, ∞), f(x + 2) - f(x) < 2

    Explanation: f(x) = x cos ($\frac{1}{x}$), x $\geq$ 1

    f differentiable.

    $\Rightarrow$ f'(x) = $\cos\left(\frac{1}{x}\right) + \frac{1}{x}\sin\left(\frac{1}{x}\right)$ and f''(x) = $-\frac{1}{x^3}\cos\left(\frac{1}{x}\right)$

    $\Rightarrow \lim\limits_{x \to \infty}$ f'(x) = 1

    Also, f''(x) < 0 for all x > 1

    $\Rightarrow$ f' is decreasing in the interval (1, ∞).

    Let g(x) = f(x + 2) - f(x). Then,

    g'(x) = f'(x + 2) - f'(x) < 0 ...[as f' is strictly decreasing]

$\Rightarrow$ g is strictly decreasing on $(1, \infty)$

Now, $\lim\limits_{x \to \infty} g(x) = \lim\limits_{x \to \infty} [f(x + 2) - f(x)]$

$= \lim\limits_{x \to \infty} [(x + 2) \cos\left(\frac{1}{x+2}\right) - x \cos\left(\frac{1}{x}\right)]$

$= \lim\limits_{x \to \infty} [x\{\cos\left(\frac{1}{x+2}\right) - \cos\left(\frac{1}{x}\right)\} + 2\cos\left(\frac{1}{x+2}\right)]$

$= \lim\limits_{x \to \infty} [2x \sin\left(\frac{x+1}{x^2+2x}\right) \sin\left(\frac{1}{x^2+2x}\right) + 2 \cos\left(\frac{1}{x+2}\right)]$

$= 2 \lim\limits_{x \to \infty} \left[\frac{\sin\left(\frac{x+1}{x^2+2x}\right)}{\left(\frac{x+1}{x^2+2x}\right)} \times \frac{\sin\left(\frac{1}{x^2+2x}\right)}{\left(\frac{1}{x^2+2x}\right)} \times \frac{x(x+1)}{(x^2+2x)^2} + 2\cos\left(\frac{1}{x+2}\right)\right]$

$= 2(1)(1)(0) + 2 = 2$

As function g is decreasing on $(1, \infty)$ and $\lim\limits_{x \to \infty} g(x) = 2$

$\Rightarrow g(x) > 2$ for all $x \in (1, \infty)$

$\Rightarrow f(x + 2) - f(x) > 2$ for all $x \in (1, \infty)$

11. State True or False:
    i. (b) False

       Explanation: False
    ii. (a) True

       Explanation: True
12. Fill in the blanks:
    a. $1 < x < 3$
    b. [-2, -1]

13. we have, h'(x)=f'(x)-f'(-x)

    For extreme values of h(x),h'(x)=0

    f'(x)-f'(-x)=0

    f'(x)=f'(-x)

    f'(x) is an even function.

    Hence the maximum value of the function occurs when the derivative of f(x) is an even function.
14. We are given that,

    f(x) = x + cos x – a

    $f'(x) = 1 - \sin x = \frac{2\cos^2 x}{2}$

    Now

    $x \in R$

    $\Rightarrow \frac{\cos^2 x}{2} > 0$

    $\Rightarrow \frac{2\cos^2 x}{2} > 0$

    $\Rightarrow f'(x) > 0$

    Hence, f(x) is an increasing function for $x \in R$
15. Given: $f(x) = x^x$

    $\Rightarrow \quad f(x) = e^{x \log x}$

    $\Rightarrow \quad f'(x) = e^{x \log x} \frac{d}{dx}(x \log_e x)$

    $\Rightarrow \quad f'(x) = x^x (1 + \log_e x)$

    For f(x) to be increasing, we must have

    $f'(x) > 0$

$\Rightarrow \quad x^x(1+\log_e x) > 0$

$\Rightarrow \quad 1+\log_e x > 0$

$\Rightarrow \quad \log_e x > -1$

$\Rightarrow \quad x > e^{-1}$

Thus, f(x) is increasing on $(1/e, \infty)$

for f(x) to be decreasing, we must have

$f'(x) < 0$

$\Rightarrow \quad x^x(1+\log_e x) < 0$

$1+\log_e x < 0$

$x < e^{-1}$

$\Rightarrow \quad x \in (0, 1/e)$

Thus, F(x) increasing on $(1/e, \infty)$ and decreasing on $(0, 1/e)$

16. Given: $f(x) = x^2 - x + 1$ f(x) = x^2 - x + 1

$\Rightarrow$ f'(x) = 2x - 1

f(x) is strictly increasing if f'(x) < 0

$\Rightarrow$2x - 1 > 0

$\Rightarrow x > \frac{1}{2}$

i.e., increasing on the interval $\left(\frac{1}{2}, 1\right)$

f(x) is strictly decreasing if f'(x) < 0

$\Rightarrow$ 2x - 1 < 0

$\Rightarrow x < \frac{1}{2}$

i.e., decreasing on the interval $\left(-1, \frac{1}{2}\right)$

hence, f(x) is neither strictly increasing nor decreasing on the interval (-1, 1).

17. The given tank is in the form of cuboid.

Volume of cuboid = lbh = 75 m^3

$\Rightarrow$ lb(3) = 75

$\Rightarrow$ lb = 25 ...(1)

Cost = 100.l.b + 50(lh + bh + lh + bh)

=100(l)(b) + 100(l + b)h

= 2500 + 300(l + b)

Minimize 2500 + 300(l + b)

Cost = 2500 + 300($\frac{25}{b}$ + b)

Cost is minimum when $\frac{d}{db}$(Cost) = 0

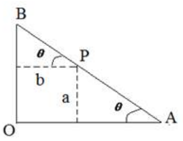
$0 + 300\left(25\left(-\frac{1}{b^2}\right) + 1\right) = 0$

$300\left(1 - \frac{25}{b^2}\right) = 0$

$1 - \frac{25}{b^2} = 0$

$b^2 = 25 \Rightarrow b = 5$

Therefore Minimum Cost = 2500 + 300(5 + 5) = 2500 + 3000 = 5500

18. 

$AP = a\,\mathrm{cosec}\,\theta$

$BP = b\sec\theta$

L = AP + BP

$l = \cos ec\theta + b\sec\theta$

$\frac{dl}{d\theta} = -a\cos ec\theta\cot\theta + b\sec\theta.\tan\theta$

$\frac{d^2l}{d\theta^2} = a\cos ec^3\theta + a\cos ec\theta\cot^2\theta + b\sec^3\theta + b\sec\theta.\tan^2\theta$

For maximum/minimum

$\frac{dl}{d\theta} = 0$

$\tan^3\theta = \frac{a}{b}$

$\tan\theta = \left(\frac{a}{b}\right)^{\frac{1}{3}}$

$\sin\theta = \frac{a^{1/3}}{\sqrt{a^{2/3}+b^{2/3}}}, \cos\theta = \frac{b^{1/3}}{\sqrt{a^{2/3}+b^{2/3}}}$

$\frac{d^2l}{d\theta^2} > 0$ for $\tan\theta = \left(\frac{a}{b}\right)^{\frac{1}{3}}$

L is minimum

$l = a\cos ec\theta + b\sec\theta$

$l = (a^{\frac{2}{3}} + b^{\frac{3}{2}})$