CBSE 2025-26

Class 12 - Mathematics

Integrals

Important Question

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1. $\int \frac{\sin x}{\sin(x-\alpha)} dx = ?$
   a. $x \sin \alpha - (\sin \alpha) \log|\sin(x-\alpha)| + C$
   b. $x \cos \alpha - (\sin \alpha) \log|\sin(x-\alpha)| + C$
   c. $x \sin \alpha + (\sin \alpha) \log|\sin(x-\alpha)| + C$
   d. $x \cos \alpha + (\sin \alpha) \log|\sin(x-\alpha)| + C$
2. Let $\alpha \in (0, \pi/2)$ be fixed. If the integral $\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = A(x) \cos 2\alpha + B(x)$ sin 2 α + C, where C is a constant of integration, then the functions A(x) and B (x) are respectively
   a. $x + \alpha \ and \log_e |\sin(x-\alpha)|$
   b. $x + \alpha \ and \log_e |\sin(x+\alpha)|$
   c. $x - \alpha \ and \log_e |\cos(x-\alpha)|$
   d. $x - \alpha \ and \log_e |\sin(x-\alpha)|$

3. $\int \frac{\sqrt{x}}{\sqrt{x}+\sqrt[3]{x}} dx$ = f(x) + c. If f(1) = $\frac{m+n\log 2}{10}$, then |m - n| equals
   a. 97
   b. 34
   c. 91
   d. 38

4. $\int \sqrt{ax+b} dx = ?$
   a. $\frac{2(ax+b)^{3/2}}{3a} + C$
   b. $\frac{1}{2\sqrt{ax+b}} + c$
   c. None of these
   d. $\frac{3(ax+b)^{3/2}}{2a} + C$
5. The value of $\int_0^2$(|x - 2| + [x])dx, where [x] denotes the greatest integer less than or equal to x, is
   a. 4
   b. 1
   c. 2
   d. 3
6. State True or False:
   i. The value of integral P_7 : $\int_{-a}^{a} f(x)dx$ is $2\int_0^a f(x)dx$.
      a. True
      b. False
   ii. Two indefinite integrals with the same derivative lead to the same family of curves and so they are equivalent.
      a. True
      b. False
7. Fill in the blanks:
   a. The value of $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x dx$ is ________.

b. $\int_0^{\frac{\pi}{2}} \cos x e^{\sin x} dx$ is equal to ________

8. Evaluate $\int \frac{x^2}{1+x^3} dx$.

9. Write the value of $\int \frac{1-\sin x}{\cos^2 x} dx$.

10. Evaluate: $\int \frac{2x+1}{(x+1)(x-2)} dx$

11. Evaluate: $\int_0^{\pi/2} \frac{dx}{\left(a^2 \cos^2 x+b^2 \sin^2 x\right)}$

12. Evaluate $\int_0^{\pi/2} x^2 \sin x dx$.

13. Evaluate: $\int \frac{x^3-3x}{x^4+2x^2-4} dx$

Solution

1. (d) $x \cos \alpha + (\sin \alpha) \log |\sin (x - \alpha)| + C$

Explanation: Given:

$\int \frac{\sin x}{\sin(x-\alpha)} dx$

Let x - α = t

dx = dt

$I = \int \frac{\sin(t+\alpha)}{\sin t} dx$

$= \int \frac{\sin t \cos \alpha + \cos t \sin \alpha}{\sin t} dt$

$= \int \cos \alpha + \sin \alpha \cot t \, dt$

$= t \cos \alpha + \sin \alpha \ln |\sin t| + c$

= (x - a) cos α + (sin α) log |sin (x - α)| + c

=x cos α+ (sin α) log|sin(x - α) | + c

2. (d) $x - \alpha \; and \log_e |\sin (x - \alpha)|$

Explanation: Let $I = \int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx, \alpha \in \left(0, \frac{\pi}{2}\right)$

$= \int \frac{\frac{\sin x}{\cos x} + \frac{\sin \alpha}{\cos \alpha}}{\frac{\sin x}{\cos x} - \frac{\sin \alpha}{\cos \alpha}} dx$

$= \int \frac{\sin x \cos \alpha + \sin \alpha \cos x}{\sin x \cos \alpha - \sin \alpha \cos x} dx$

$= \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)} dx$

Now, put x - α = t $\Rightarrow$dx = dt, so

$I = \int \frac{\sin(t+2\alpha)}{\sin t} dt$

$= \int \frac{\sin t \cos 2\alpha + \sin 2\alpha \cos t}{\sin t} dt$

$= \int \left(\cos 2\alpha + \sin 2\alpha \frac{\cos t}{\sin t}\right) dt$

= t (cos 2α) + (sin 2α) $\log_e$ | sin t | + C

= (x - α) cos 2α + (sin 2α) $\log_e$ | sin (x - α) | + C

= A(x) cos 2α + B(x) sin α + C (given)

Now on comparing, we get

A(x) = x - α and B(x) = $\log_e$ | sin (x - α)|

3. (a) 97

Explanation: Let I = $\int \frac{\sqrt{x}}{\sqrt{x} + \sqrt[3]{x}} dx$

Let x = t^6 $\Rightarrow$ dx = $6t^5$ dt

I = $\int \frac{t^3}{t^2 + t^3}$ ($6t^5$)dt

= $\int \frac{6t^6}{1+t} dt$

= 6 $\int \frac{t^6 - 1 + 1}{1+t} dt$

= 6 $\int \left(\frac{(t+1)(t^5 - t^4 + t^3 - t^2 + t - 1)}{t+1} + \frac{1}{t+1} \right) dt$

= $6\int\left(t^5-t^4+t^3-t^2+t-1+\frac{1}{t+1}\right)$

= $6\left[\frac{t^6}{6}-\frac{t^5}{5}+\frac{t^4}{4}-\frac{t^3}{3}+\frac{t^2}{2}-t+\log|1+t|\right]$ + c,

where t = $x^{\frac{1}{6}}$

$\Rightarrow$ f(1) = $6\left[\frac{1}{6}-\frac{1}{5}+\frac{1}{4}-\frac{1}{3}+\frac{1}{2}-1+\log 2\right]$

= $\frac{-37+60\log 2}{10}$

$\Rightarrow$ m = -37, n = 60

$\Rightarrow$ |m - n| = |-37 - 60| = 97

4. (a) $\frac{2(ax+b)^{3/2}}{3a}+C$

Explanation: Given integral is $\int\sqrt{ax+b}$

Let, ax + b = z^2

$\Rightarrow$ adx = 2zdz

So,

$\int\sqrt{ax+b}dx$

$=\int z\frac{2zdz}{a}$

$=\frac{2}{a}\int z^2dz$ where, c is the integrating constant.

$=\frac{2}{a}\frac{z^3}{3}+c$

$=\frac{2}{3a}z^3+c$

$=\frac{2(ax+b)^{3/2}}{3a}+c$

Hence, $\int\sqrt{ax+b}==\frac{2(ax+b)^{3/2}}{3a}+c.$

Which is the required solution.

5. (d) 3

Explanation: $\int_0^2(|x-2|+[x])dx=\int_0^2|x-2|dx+\int_0^2[x]dx$

$=-\int_0^2(x-2)dx+\int_0^1[x]dx+\int_1^2[x]dx$

$=\int_0^2(2-x)dx+\int_0^1 0\cdot dx+\int_1^2 1\cdot dx$

$=\left[2x-\frac{x^2}{2}\right]_0^2+0+[x]_1^2=3$

6. State True or False:

i. (a) True

Explanation: We have,

$I=\int_{-a}^{a}f(x)dx$

$I=\int_{-a}^{0}f(x)dx+\int_0^a f(x)dx$

Let z = -x $\Rightarrow$ dz = -dx

When x = -a, z = a and x = 0, z = 0 and x = -z

$I=-\int_a^0 f(-z)dz+\int_0^a f(x)dx$

Since f(x) is even, f(-x) = f(x)

Since, $\int_a^b f(z)dz=\int_a^b f(x)dx$

$I=2\int_0^a f(x)dx$

which is the required solution.

ii. (a) True

Explanation: True

7. Fill in the blanks:

a. 0

b. e - 1

8. Let $I = \int \frac{x^2}{1+x^3} dx$

Put $1 + x^3 = t \Rightarrow 3x^2 dx = dt \Rightarrow x^2 dx = \frac{dt}{3}$

$\therefore \quad I = \int \frac{x^2}{1+x^3} dx = \frac{1}{3} \int \frac{dt}{t}$

$= \frac{1}{3} \log|t| + C \quad \left[\because \int \frac{dx}{x} = \log|x| + C\right]$

$= \frac{1}{3} \log\left|1 + x^3\right| + C \quad \left[\text{put } t = 1 + x^3\right].$

9. Let I = $\int \frac{1-\sin x}{\cos^2 x} dx$

$= \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right) dx$

$= \int \sec^2 x dx - \int \sec x \tan x dx$

= tan x - sec x + C

10. Let, I =$\int \frac{2x+1}{(x+1)(x-2)}$

Using partial fractions,

$\frac{2x+1}{(x+1)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x-2)}$

$\Rightarrow$ 2x + 1 = A (x - 2) + B(x + 1)

Put x = 2

$\Rightarrow$ 5 = 3B $\Rightarrow$ B $= \frac{5}{3}$

Put x = - 1

$\Rightarrow$ - 1 = - 3A $\Rightarrow$ A = $\frac{1}{3}$

So, we have

$I = \int \frac{2x+1}{(x+1)(x-2)} dx = \frac{1}{3} \int \frac{dx}{x+1} + \frac{5}{3} \int \frac{dx}{x-2}$

$\Rightarrow$ I = $\frac{1}{3} \log|x+1| + \frac{5}{3} \log|x-2|$ + c

Thus,

I $= \frac{1}{3} \log|x+1| + \frac{5}{3} \log|x-2|$ + c.

11. Let I = $\int_0^{\frac{\pi}{2}} \frac{1}{a^2 \cos^2 x + b^2 \sin^2 x} dx$

Dividing by $\cos^2$x in numerator and denominator, we have

I = $\int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{a^2 + b^2 \tan^2 x} dx$

Let tan x = t then $\sec^2$xdx = dt

I = $\int_0^{\frac{\pi}{2}} \frac{1}{a^2 + b^2 t^2} dt = \frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{1}{\frac{a^2}{b^2} + t^2} dt$

Let t = $\frac{a}{b} \tan \theta$ = tanx

I = $\frac{1}{b^2} \int_0^{\frac{\pi}{2}} \frac{\frac{a}{b} \sec^2 \theta}{\frac{a^2}{b^2} + \frac{a^2}{b^2} \tan^2 \theta} d\theta$

$= \frac{1}{ab} \theta$

$\frac{1}{ab} \tan^{-1} \left(\frac{b}{a} \tan x \right) \Big|_0^{\frac{\pi}{2}}$

$= \frac{\pi}{2ab}$

12. Given, $I = \int_0^{\pi/2} x^2 \sin x dx$

$I = \int_0^{\pi/2} \underset{I}{x^2} \underset{II}{\sin x} \, dx$

By using integration by parts we get ,

$I = [-x^2\cos x]_0^{\pi/2} + 2\int_0^{\pi/2} \underset{I}{x}\cos \underset{II}{x}\, dx$

$= [-x^2\cos x]_0^{\pi/2} + 2\left[[x(\sin x)]_0^{\pi/2} - \int_0^{\pi/2} 1\cdot(\sin x)dx\right]$

$= [-x^2\cos x]_0^{\pi/2} + 2\left[[x(\sin x)]_0^{\pi/2} + [cosx]_0^{\pi/2}\right]$

$\therefore I = \int_0^{\pi/2} x^2\sin x dx = \left[-x^2\cos x + 2(x\sin x + \cos x)\right]_0^{\pi/2}$

$= \left[-\left(\frac{\pi}{2}\right)^2\cos\left(\frac{\pi}{2}\right) + 2\left(\frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\right) - 2(0+\cos 0)\right]$

$= -\frac{\pi^2}{4}\times 0 + 2\left(\frac{\pi}{2} + 0\right) - 2(0+1)$

$\therefore I = \pi - 2$

13. Let the given integral be,

I = $\int \frac{x^3-3x}{x^4+2x^2-4}dx$

$= \int \frac{x(x^2-3)}{x^4+2x^2-4}dx$

Let x^2 = t or,

2x dx = dt

$\Rightarrow I = \frac{1}{2}\int \frac{(t-3)}{t^2+2t-4}dt$

$= \frac{1}{4}\int \frac{2t-6}{t^2+2t-4}dt$

$= \frac{1}{4}\int \frac{2t+2-8}{t^2+2t-4}dt$

$= \frac{1}{4}\int \left(\frac{2t+2}{t^2+2t-4} - \frac{8}{t^2+2t-4}\right)dt$

$= \frac{1}{4}\left(\int \frac{2t+2}{t^2+2t-4}dt - \int \frac{8}{t^2+2t-4}dt\right)$

$\Rightarrow I = \frac{1}{4}$ $(I_1 + I_2)$ ...(i)

Now,

$I_1 = \int \frac{2t+2}{t^2+2t-4}dt$

$t^2 + 2t - 4 = u$

or, (2t + 2) dt = du

$\Rightarrow I_1 = \int \frac{1}{u}$ du = In |u| + c_1

$\Rightarrow$ I_1 = In |t + 2t - 4| + c_1

$\therefore$ I_1 = In $|x^4 + 2x^2 - 4| + c_1$

Now,

$I_2 = \int \frac{-8}{(t+1)^2-5}dt$

$\Rightarrow I_2 = \int \frac{8}{(\sqrt{5})^2-(t+1)^2}dt$

$\therefore I_2 = \frac{8}{2\sqrt{5}}\ln\left|\frac{\sqrt{5}+x^2+1}{\sqrt{5}-x^2-1}\right| + c_2$

So, from (i), we get

I = $\frac{1}{4}\left[\ln|x^4+2x^2-4| + \frac{4}{\sqrt{5}}\ln\left|\frac{\sqrt{5}+x^2+1}{\sqrt{5}-x^2-1}\right|\right] + C$

$\therefore I = \frac{1}{4}$ In $|x^2 + 2x^2 - 4|$ + $\frac{1}{\sqrt{5}}\ln\left|\frac{\sqrt{5}+x^2+1}{\sqrt{5}-x^2-1}\right| + C$