CBSE 2025-26

Class 12 - Mathematics

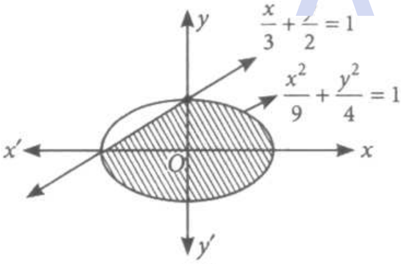
Application of Integrals

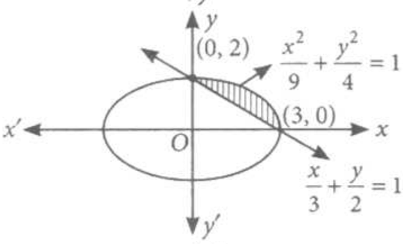
Important Question

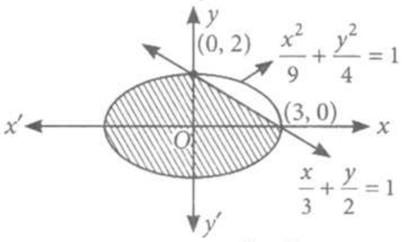
Question No. 1 to 5 are based on the given text. Read the text carefully and answer the questions:

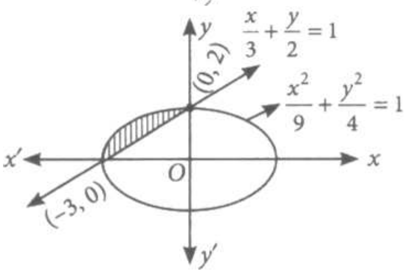
A mirror in the shape of an ellipse represented by $\frac{x^2}{9}+\frac{y^2}{4}=1$ was hanging on the wall. Arun and his sister 9, 4 were playing with the ball inside the house, even their mother refused to do so. All of sudden, the ball hit the mirror and x, y got a scratch in the shape of line represented by $\frac{x}{3}+\frac{y}{2}=1$.

1. Point(s) of the intersection of ellipse and scratch (straight line) is (are)
   a. (2, 0), (0, 3)
   b. (2, 3), (0, 0)
   c. (0, 2), (3, 0)
   d. (0, 3), (3, 0)
2. Area of smaller region bounded by the ellipse and line is represented by:

   a. 

   b. 

   c. 

   d. 

3. The value of $\frac{2}{3}\int_{0}^{3}\sqrt{9-x^2}\,dx$ is:
   a. $\frac{\pi}{4}$
   b. $\frac{3\pi}{2}$
   c. $\frac{\pi}{2}$
   d. π

4. The value of $2\int_{0}^{3}\left(1-\frac{x}{3}\right)$ dx is:
   a. 1
   b. 3
   c. 2
   d. 0

5. Area of the smaller region bounded by the mirror and scratch is:
   a. $3\left(\frac{\pi}{2}+1\right)$ sq. units
   b. $\left(\frac{\pi}{2}+1\right)$ sq. units
   c. $3\left(\frac{\pi}{2}-1\right)$ sq. units
   d. $\left(\frac{\pi}{2}-1\right)$ sq. units

6. The area of the region { (x , y) : $x^2+y^2 \leqslant 1 \leqslant x+y$ } is equal to
   a. $\frac{\pi-2}{4}$ sq. units
   b. $\frac{3\pi-2}{4}$ sq. units
   c. none of these
   d. $\frac{1}{2}$ sq. units

7. The area of the region between the curve y = 4 –x^2 , $0 \leqslant x \leqslant 3$ and the x –axis is equal to
   a. $\frac{7}{3}$
   b. $\frac{16}{3}$
   c. 3
   d. $\frac{23}{3}$

8. For which of the following values of m, is the area of the region bounded by the curve y = x - x^2 and the line y = mx equal to $\frac{9}{2}$?
   a. -4
   b. 2
   c. none of these
   d. -2

9. The area (in sq units) of the region $\left\{(x,y) : x \geq 0, x+y \leq 3, x^2 \leq 4y \text{ and } y \leq 1+\sqrt{x}\right\}$ is
   a. $\frac{3}{2}$
   b. $\frac{59}{12}$
   c. $\frac{5}{2}$
   d. $\frac{7}{3}$

10. The area enclosed by the curve $y = 2\sqrt{1-x^2}, x \in [0,1]$ is
   a. none of these
   b. $\frac{\pi}{2}$
   c. π

d. $\frac{\pi}{4}$

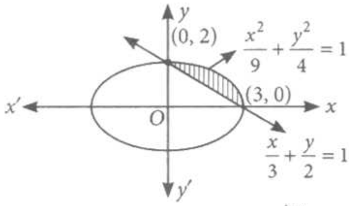
11. The area of the region bounded by the line 2y = -x + 8, x-axis and the lines x = 2 and x = 4 is 5 sq. units. (True/False)
12. The area of the bounded by the lines y = 2, x = 1, x = a and the curve y = f(x), which cuts the last two lines above the first line for all $a \geq 1$, is equal to $\frac{2}{3}\left[(2a)^{3/2} - 3a + 3 - 2\sqrt{2}\right]$. Find f(x)
13. Find the value of c for which the area of figure bounded by the curve y = 3, the straight lines x=1 and x=c and the x-axis is equal to $\frac{16}{3}$
14. Find the area of the region enclosed by the lines y=x, x=e, and the curve $y = \frac{1}{x}$ and the positive x-axis
15. Using integration, find the area of the region in the first quadrant enclosed by the x-axis, the line y = x and the circle $x^2 + y^2 = 18$.
16. Find the area of the region $\left\{(x, y) : x^2 \leqslant y \leqslant x\right\}$
17. Sketch the region lying in the first quadrant and bounded by $y = 9x^2$, x = 0, y = 1 and y = 4. Find the area of the region using integration.
18. Find the area bounded by the curve y -x|x|, x-axis and the ordinates x = - 3 and x = 3. Find the equation of the curve?

Solution

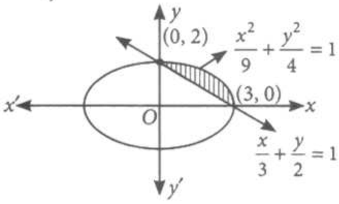
1. (c) (0, 2), (3, 0)

   Explanation: Points (0, 2) and (3, 0) pass through both line and ellipse.

2. (b)



   Explanation:



3. (b) $\frac{3\pi}{2}$

   Explanation: $\frac{2}{3}\int_0^3 \sqrt{9-x^2}\,dx = \frac{2}{3}\int_0^3 \sqrt{(3)^2-x^2}\,dx$

   $\frac{2}{3}\left[\frac{1}{2}x\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$

   $\frac{2}{3}\left[\frac{3}{2}\sqrt{0}+\frac{9}{2}\sin^{-1}(1)-\frac{1}{2}(0)-\frac{9}{2}\sin^{-1}(0)\right]$

   $\frac{2}{3}\left[\frac{9}{2}\cdot\frac{\pi}{2}\right]=\frac{3\pi}{2}$

4. (b) 3

   Explanation: $2\int_0^3\left(1-\frac{x}{3}\right)dx = 2\left[x-\frac{x^2}{6}\right]_0^3$

   2(3 - $\frac{9}{6}$ - 0 - 0) = 2 × $\frac{3}{2}$ = 3

5. (c) $3\left(\frac{\pi}{2}-1\right)$ sq. units

   Explanation: Area of smaller region bounded by the mirror and scratch

   = $\frac{2}{3}\cdot\int_0^3\sqrt{9-x^2}\,dx - 2\int_0^3\left(1-\frac{x}{3}\right)$ dx

   = $\frac{3\pi}{2}$ - 3 = $3\left(\frac{\pi}{2}-1\right)$ sq. units

6. (a) $\frac{\pi-2}{4}$ sq. units

   Explanation: $x^2+y^2=1, x+y=1$

   Meets when

   $x^2+(1-x)^2=1$

   $\Rightarrow x^2+1+x^2-2x=1$

   $\Rightarrow 2x^2-2x=0 \Rightarrow 2x(x-1)=0$

   i.e. points (1 ,0) ,(0 ,1). Therefore , required area is ;

$\int_0^1 \left(\sqrt{1-x^2} - (1-x)\right) dx$

$= \left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x - x + \frac{x^2}{2}\right]_0^1$

7. (d) $\frac{23}{3}$

Explanation: y = $4 - x^2 > 0 \Rightarrow -2 < x < 2$ And $y = 4 - x^2 < 0 \Rightarrow x < -2$ or x > 2

Required area:

$= \int_0^2 (4-x^2)dx + \left|\int_2^3 (4-x^2)dx\right| = \frac{16}{3} + \left|-\frac{7}{3}\right| = \frac{23}{3}$sq.units

8. (d) -2

Explanation: Required area:

$\int_0^{1-m} (x - x^2 - mx)dx$

$= \left[(1-m)\frac{x^2}{2} - \frac{x^3}{3}\right]_0^{1-m}$

$= \frac{(1-m)^3}{2} - \frac{(1-m)^3}{3} = \frac{9}{2}$

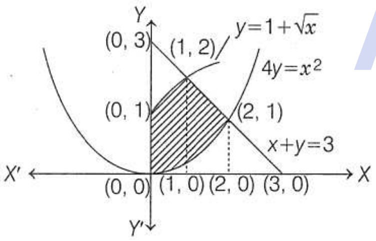
$\Rightarrow \frac{(1-m)^3}{6} = \frac{9}{2}$

$\Rightarrow$ m = - 2

9. (c) $\frac{5}{2}$

Explanation: Required area

$= \int_0^1 (1+\sqrt{x})dx + \int_1^2 (3-x)dx - \int_0^2 \frac{x^2}{4}dx$



$= \left[x + \frac{x^{3/2}}{3/2}\right]_0^1 + \left[3x - \frac{x^2}{2}\right]_1^2 - \left[\frac{x^3}{12}\right]_0^2$

$= \left(1 + \frac{2}{3}\right) + \left(6 - 2 - 3 + \frac{1}{2}\right) - \left(\frac{8}{12}\right)$

$= \frac{5}{3} + \frac{3}{2} - \frac{2}{3} = 1 + \frac{3}{2} = \frac{5}{2}$ sq units

10. (b) $\frac{\pi}{2}$

Explanation: Required area :

$= \int_0^1 y dx = \int_0^1 2\sqrt{1-x^2}\, dx = 2\left[\frac{x\sqrt{1-x^2}}{2} + \frac{1}{2}\sin^{-1}x\right]_0^1 = 2\left(\frac{1}{2}\sin^{-1}1\right) = \frac{\pi}{2}$

11. True

Explanation: True

12. we are given,

$\int_a^1 [f(x) - 2]dx = \frac{2}{3}\left[(2a)^{3/2} - 3a + 3 - 2\sqrt{2}\right]$

Differentiating w.r.t a, we get

f(a) - 2== $\frac{2}{3}\left[\frac{3}{2}\sqrt{2a}.2 - 3\right]$

f(a)=$2\sqrt{2a}, a \geq 1$

$\therefore\ f(x) = 2\sqrt{2x}, x \geq 1$

13. we have, $\int_0^c 3dx = \frac{16}{3}$

$3(x)_0^c = \frac{16}{3}$

$3c = \frac{16}{3}$

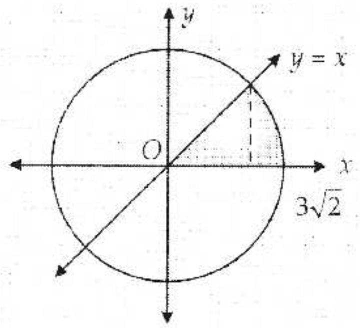
$c = \frac{16}{9}$

14. Required area = the area of the region enclosed by the lines y=x, x=e, and the curve $y = \frac{1}{x}$ and the positive x-axis

$=\int_0^1 xdx + \int_1^e \frac{1}{x}dx$

$= \frac{1}{2} + 1$

$= \frac{3}{2} sq\ units$

15. 

Point of intersection of Circle and Line is (3, 3).

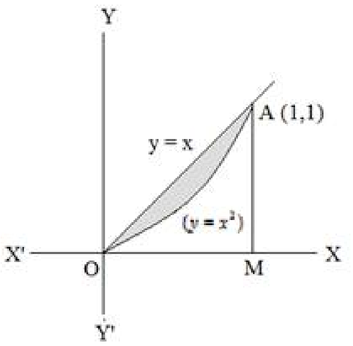
$\therefore$ Area = $\int_0^3 xdx + \int_3^{3\sqrt{2}} \sqrt{18 - x^2}\, dx$

$= \left[\frac{x^2}{2}\right]_0^3 + \left[\frac{x}{2}\sqrt{18 - x^2} + 9\sin^{-1}\frac{x}{3\sqrt{2}}\right]_3^{3\sqrt{2}}$

$= \frac{9}{2} + \frac{9\pi}{2} - \frac{9}{2} - \frac{9\pi}{4}$

$= \frac{9}{2} + \frac{9\pi}{2} - \frac{9}{2} - \frac{9\pi}{4}$

$= \frac{9\pi}{4}$ sq. units.

16. 

y = x^2

y = x

$\Rightarrow x = 0, y = 0$

x = 1, y = 1

Area $= \int_0^1 xdx - \int_0^1 x^2 dx.$

$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$ sq. units

17. The equation y = $9x^2$ represents an upward opening parabola with axis as y-axis and vertex at the origin.

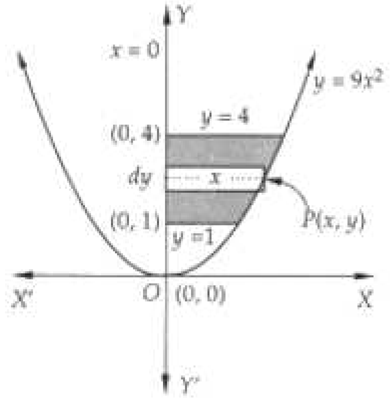
From the below it is Clear that the shaded region is the region lying in the first quadrant and bounded by y = $9x^2$, x = 0, y = 1 and y = 4.

Let us slice this region into horizontal rectangular strips.

The approximating rectangle shown in Fig. has length = $|x|$ and width = dy thus we have area = $|x|$ dy.

Clearly, it can move vertically between y = 1 and y = 4.

So, the required area denoted by A is given by



$A=\int_1^4|x|dy=\int_1^4 xdy\ [\because x\geq 0\quad \therefore |x|=x]$

$\Rightarrow\quad A=\int_1^4\sqrt{\frac{y}{9}}dy$ [∵ P(x, y) lies on y = 9x^2 ∴ $x=\sqrt{\frac{y}{9}}$]

$\Rightarrow\quad A=\frac{1}{3}\int_1^4\sqrt{y}dy$

$\Rightarrow\quad A=\frac{1}{3}\times\frac{2}{3}\left[y^{3/2}\right]_1^4$

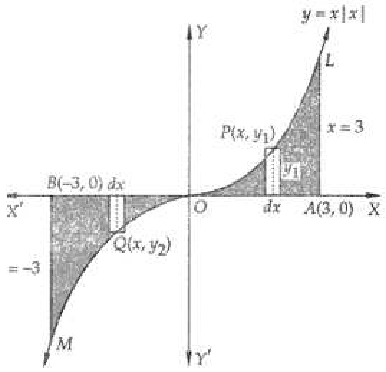
$=\frac{2}{9}(8-1)$

$=\frac{14}{9}$ sq. units

18. The equation of the curve is

y = $x|x|$ = $\begin{cases}x^2, x\geq 0\\ -x^2, x<0\end{cases}$

The graph of y = x$|x|$ is shown in Fig. and the region bounded by y = x$|x|$, x-axis and the ordinates x = - 3 and x = 3 is shaded in the given Fig.



Clearly, y = x|x|, being an odd function is symmetric in opposite quadrants. Therefore, the required area is twice the area of the shaded region in the first quadrant.

Let us slice the region in first quadrant into vertical strips. The approximating rectangle shown in Fig. has length = $|y_1|$dx. As it can move between x = 0 and x = 3, therefore, area of the shaded region in first quadrant

A = $\int_0^3|y_1|\,dx$ = $\int_0^3 y_1 dx$ ....[∵ $y_1\geq 0$ ∴ $|y_1|$ = y_1]

⇒ A = $\int_0^3 x^2dx$ ....[∵ P(x_1, y_1) lies on y = x^2 ∴ y_1 = x^2]

⇒ A = $\left[\frac{x^3}{3}\right]_0^3$ = 9 sq. units.

Hence required area = 2A = 2 × 9 = 18 sq. units.