CBSE 2025-26

Class 12 - Mathematics

Differential Equations

Important Question

Question No. 1 to 5 are based on the given text. Read the text carefully and answer the questions:

A rumour on whatsapp spreads in a population of 5000 people at a rate proportional to the product of the number of people who have heard it and the number of people who have not. Also, it is given that 100 people initiate the rumour and a total of 500 people know the rumour after 2 days.

1. If y(t) denote the number of people who know the rumour at an instant t, then maximum value of y(t) is
   a. None of these
   b. 100
   c. 5000
   d. 500
2. $\frac{dy}{dt}$ is proportional to
   a. y(500 - y)
   b. y(5000 - y)
   c. (y - 5000)
   d. y(y - 500)
3. The value of y(0) isTo practice more questions & prepare well for exams, download **myCBSEguide App**. It provides complete study material for CBSE, NCERT, JEE (main), NEET-UG and NDA exams. Teachers can use **Examin8 App** to create similar papers with their own name and logo.
   a. 200
   b. 600
   c. 100
   d. 500
4. The: value of y(2) is
   a. 600
   b. 200
   c. 500
   d. 100
5. The value of y at any time t is given by
   a. y = $\frac{5000}{1+e^{5000kt}}$
   b. y = $\frac{5000}{49e^{-5000kt}+1}$
   c. y = $\frac{5000}{e^{-5000kt}+1}$
   d. y = $\frac{5000}{49\left(1+e^{-5000kt}\right)}$

6. Find the equation of a curve passing through the point (0, –2) given that at any point (x, y) on the curve, the product of the slope of its tangent and y coordinate of the point is equal to the x coordinate of the point.
   a. $y^3 - x^2 = 4$
   b. $y^3 - x^3 = 4$
   c. $y^2 - x^3 = 4$
   d. $y^2 - x^2 = 4$
7. The slope of the tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is given by $\frac{y}{x}$ - $\cos^2(\frac{y}{x})$, then the equation of the curve is:
   a. y = x $\tan^{-1}[\log(\frac{x}{e})]$
   b. None of these
   c. y = $\tan^{-1}[\log(\frac{e}{x})]$
   d. y = x $\tan^{-1}[\log(\frac{e}{x})]$
8. The equation of a curve passing through the origin and satisfying the differential equation $\frac{dy}{dx}$ = $(x - y)^2$ is:
   a. $e^{2x}(1 - x + y) = 1 + x - y$
   b. $e^{2x}(1 + x + y) = 1 - x + y$
   c. $e^{2x}(1 + x - y) = 1 - x + y$
   d. $e^{2x}(1 - x + y) + (1 + x - y) = 0$
9. Differential equations are equations containing functions y = f(x), g(x) and
   a. derivatives of y
   b. tangent of y at zero
   c. minima of y
   d. maxima of y

10. Given a curve C. Suppose that the tangent line at P(x, y) on C is perpendicular to the line joining P and Q (1, 0). If the line 2x + 3y -1 5 = 0 is tangent to the curve C then the curve C denotes. :
    a. circle whose y-intercept is $4\sqrt{3}$
    b. a parabola with axis parallel to y-axis
    c. a circle touching the x-axis
    d. a circle touching the y-axis
11. State True or False:
    i. Order of the differential equation representing the family of ellipses having centre at origin and foci on x-axis is two.
       a. True
       b. False
    ii. The general solution of the differential equation $\frac{dy}{dx} + y\sec x = \tan x$ is y(secx - tanx) = secx - tanx + x + k.
       a. True
       b. False
12. Fill in the blanks:
    a. The degree of the differential equation $\frac{d^2y}{dx^2} + e^{\frac{dy}{dx}} = 0$ is ________.
    b. The order of the differential equation of all circles of given radius a is ________.
13. Solve differential equation: tan y dx + tan x dy = 0
14. Write order and degree (if defined) of the differential equation: $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^4$ + y^5 = 0.
15. Given that $\frac{dy}{dx} = e^{-2y}$ and y = 0 when x = 5. Find the value of x when y = 3.

16. Solve the differential equation: $(x + 2)\frac{dy}{dx} = x^2 + 3x + 7$

17. Solve the differential equation: $xy\ log\frac{x}{y}dx + [y^2 - x^2 log(\frac{x}{y})]dy = 0$

18. Solve differential equation: $\frac{dy}{dx} = \sin^3 x \cos^4 x + x\sqrt{x+1}$

Solution

1. (c) 5000

   Explanation: Since, size of the population is 5000

   ∴ Maximum value of y(t) is 5000
2. (b) y(5000 - y)

   Explanation: Clearly, according to given information, $\frac{dy}{dt}$ = ky(5000 - y), where k is the constant of proportionality.
3. (c) 100

   Explanation: Since, rumour is initiated with 100 people.

   ∴ When t = 0, then y = 100

   Thus y(0) = 100

4. (c) 500

   Explanation: Since, rumour is spread in 500 people, after 2 days.

   ∴ When t = 2, then y = 500.

   Thus, y(2) = 500
5. (b) y = $\frac{5000}{49e^{-5000kt}+1}$

   Explanation: We know that, when t = 0, then y = 100

   This condition is satisfied by this option only.
6. (d) $y^2 - x^2 = 4$

   Explanation: Given that $y\frac{dy}{dx} = x$

   $ydy = xdx$

   $\int ydy = \int xdx$

   $\frac{y^2}{2} = \frac{x^2}{2} + c$

   When x = 0 and y = 2, we get

   $\frac{-2^2}{2} = \frac{0^2}{2} + c$

   c = 2

   $\frac{y^2}{2} = \frac{x^2}{2} + 2$

   $y^2 - x^2 = 4$
7. (d) y = x $\tan^{-1}[\log(\frac{e}{x})]$

   Explanation: $\frac{dy}{dx} = \frac{y}{x}$ - $\cos^2 \frac{y}{x}$ ...(i)

   Let y = vx ...(ii)

   ⇒ $\frac{dy}{dx}$ = v + x $\frac{dv}{dx}$ ...(iii)

   Substituting (ii) and (iii) in (i), we get

   v + x $\frac{dv}{dx}$ = v - $\cos^2$v ⇒ x $\frac{dv}{dx}$ = -$\cos^2$ v

   Integrating on both sides, we get

   $\int \sec^2$ vd v = -$\int \frac{dx}{x}$ + c

   ⇒ tan v = -log x + c

$\Rightarrow$ tan $\frac{y}{x}$ = -log x + c ...(iv)

The required curve passes through $\left(1, \frac{\pi}{4}\right)$.

$\Rightarrow$ tan $\frac{\pi}{4}$ = -log 1 + c $\Rightarrow$ c = 1

(iv) $\Rightarrow$ tan $\frac{y}{x}$ = -log x + 1

$\Leftrightarrow$ tan $\frac{y}{x}$ = -log x + log e

$\Leftrightarrow$ y = x $\tan^{-1}$[log($\frac{e}{x}$)]

8. (a) e^{2x}(1 - x + y) = 1 + x - y

Explanation: $\frac{dy}{dx}$ = $(x - y)^2$ ...(i)

Let x - y = v ...(ii)

$\Rightarrow$ 1 - $\frac{dy}{dx} = \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = 1 - \frac{dv}{dx}$ ...(iii)

Substituting (ii) and (iii) in (i), we get

1 - $\frac{dv}{dx}$ = v^2

$\Rightarrow$ dx = $\frac{dv}{1-v^2}$ (Variables separable)

Integrating on both sides, we get

$2\int dx = 2\int \frac{dv}{1-v^2}$ + log c

$\Leftrightarrow$ 2x = $2\left(\frac{1}{2}\right)\log\left(\frac{1+v}{1-v}\right)$ + log c

$\Leftrightarrow$ 2x = $\log\left(\frac{1+v}{1-v}\right) \cdot c \Leftrightarrow e^{2x} = \left(\frac{1+v}{1-v}\right) c$

$\Rightarrow e^{2x} = \left(\frac{1+x-y}{1-x+y}\right) c$ ...(iv)

This curve passes through the origin (0, 0)

$\Rightarrow$ e^0 = (1) c $\Rightarrow$ c = 1

(iv) $\Rightarrow e^{2x} = \frac{1+x-y}{1-x+y}$

$\Rightarrow$ e^{2x} (1 - x + y) = 1 + x - y

9. (a) derivatives of y

Explanation: Differential equations are equations containing functions y = f(x), g(x) and derivatives of y with respect to x.

10. (a) circle whose y-intercept is $4\sqrt{3}$

Explanation: circle whose y-intercept is $4\sqrt{3}$

11. State True or False:
    i. (a) True

       Explanation: True
    ii. (b) False

       Explanation: False
12. Fill in the blanks:
    a. not defined
    b. 2
13. The given differential equation is

tan y dx+tan x dy=0

$\Rightarrow \tan x \frac{dy}{dx} = -\tan y$

$\Rightarrow \cot y dy = -\cot x dx$

Integrating both sides, we get

$\int \cot y dy = -\int \cot x dx$

$\Rightarrow$ log |sin y|=-log |sin x|+log C

⇒log |sin y|+log |sin x|=log C
⇒log |(sin y)(sin x)|=log C
⇒(sin y)(sin x)=C
⇒sin x sin y=C

14. In the given equation, the highest-order derivative is $\frac{d^2y}{dx^2}$ and its power is 2
∴ its order = 3 and degree = 2.

15. Given that, $\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{dy}{e^{-2y}} = dx$
$\Rightarrow \int e^{2y} dy = \int dx \Rightarrow \frac{e^{2y}}{2} = x + c$ ..(i)
When x = 5 and y = 0, then substituting these values in Eq. (i), we get
$\frac{e^0}{2} = 5 + C$
$\Rightarrow \frac{1}{2} = 5 + C \Rightarrow C = \frac{1}{2} - 5 = -\frac{9}{2}$
Eq. (i) becomes e^{2y} = 2x - 9
when y = 3, then $e^6 = 2x - 9 \Rightarrow 2x = e^6 + 9$
$\therefore x = \frac{(e^6+9)}{2}$

16. We have, (x + 2) $\frac{dy}{dx}$ = x^2 + 3x + 7
$\Rightarrow \frac{dy}{dx} = \frac{x^2+3x+7}{x+2}$
$\Rightarrow dy = \left(\frac{x^2+3x+7}{x+2}\right) dx$
Integrating both sides, we get
$\int dy = \int \left(\frac{x^2+3x+7}{x+2}\right) dx$
$\Rightarrow \int dy = \int \left(\frac{x^2+3x+2+5}{x+2}\right) dx$
$\Rightarrow \int dy = \int \left[\frac{(x+2)(x+1)+5}{x+2}\right] dx$
$\Rightarrow \int dy = \int \left(x + 1 + \frac{5}{x+2}\right) dx$
⇒ y = $\frac{x^2}{2}$ + x + 5 log |x + 2| + C
So, y = $\frac{x^2}{2}$ + x + 5 |log|x + 2| + C
Hence, y = $\frac{x^2}{2}$ + x + 5 log |x + 2| + C, it is the solution to the given differential equation.

17. The given differential equation is,
$xy\, log\frac{x}{y} dx + [y^2 - x^2 log(\frac{x}{y})]dy$
⇒ xy log $\left(\frac{x}{y}\right)$ dx = - {y^2 - x^2 log $\left(\frac{x}{y}\right)$} dy
$\Rightarrow \frac{dx}{dy} = \frac{-\left\{y^2-x^2 \log\left(\frac{x}{y}\right)\right\}}{xy \log\left(\frac{x}{y}\right)} = \frac{x^2 \log\left(\frac{x}{y}\right)-y^2}{xy \log\left(\frac{x}{y}\right)}$
It is a homogeneous equation
We put x = vy and $\frac{dx}{dy} = v + y\frac{dv}{dy}$
So, $v + y\frac{dv}{dy} = \frac{v^2y^2 \log(v)-y^2}{vy^2 \log(v)}$
$v + y\frac{dv}{dy} = \frac{v^2 \log(v)-1}{v \log(v)}$
$\Rightarrow y\frac{dv}{dy} = \frac{v^2 \log(v)-1}{\text{v}\log(v)} - v$
$\Rightarrow y\frac{dv}{dy} = \frac{v^2 \log(v)-1-v^2 \log(v)}{\text{v}\log(v)}$

$\Rightarrow y\frac{dv}{dy} = \frac{-1}{v\log(v)}$

$\Rightarrow$ v log (v) dv = $\frac{-1}{y}$ dy

On integrating both sides we get,

$\int$ v log (v) dv = $-\int \frac{1}{y} dy$

$\Rightarrow \frac{v^2}{2}$ log (v) $- \int \frac{v}{2}$ dv = -log y + C

$\Rightarrow \frac{v^2}{2}$ log(v) $- \frac{v^2}{4}$ = -log y + C

$\Rightarrow \frac{v^2}{2}\left[\log(v) - \frac{1}{2}\right]$ = -log y + C

$\Rightarrow$ v^2 [log (v) - $\frac{1}{2}$] = -2 log y + C

Now, putting back the values of v as $\frac{x}{y}$ we get,

$\frac{x^2}{y^2}\left[\log(v) - \frac{1}{2}\right]$ + log y^2 = C

18. The given differential equation is,

$\frac{dy}{dx} = \sin^3 x\cos^4 x + x\sqrt{x+1}$

$\Rightarrow dy = \left(\sin^3 x\cos^4 x + x\sqrt{x+1}\right) dx$

Integrating both sides, we have,

$\int dy = \int \left(\sin^3 x\cos^4 x + x\sqrt{x+1}\right) dx$

$\Rightarrow y = \int \sin^3 x\cos^4 x dx + \int x\sqrt{x+1} dx$

$\Rightarrow$ y=I_1+I_2 ......(1)

Here,

$I_1 = \int \sin^3 x\cos^4 x dx$

$I_1 = \int x\sqrt{x+1} dx$

Now,we have,

$I_1 = \int \sin^3 x\cos^4 x dx$

$= \int \left(1-\cos^2 x\right)\cos^4 x\sin x dx$

Putting t=cos x, we have,

dt=-sin x dx

$\therefore I_1 = -\int t^4\left(1-t^2\right) dt$

$= \int \left(t^6 - t^4\right) dt$

$= \frac{t^7}{7} - \frac{t^5}{5} + C_1$

$= \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C_1$

$I_2 = \int x\sqrt{x+1} dx$

Putting t^2=x+1, we have,

2t dt=dx

$\therefore I_2 = 2\int \left(t^2-1\right) t^2 dt$

$= 2\int \left(t^4 - t^2\right) dt$

$= \frac{2t^5}{5} - \frac{2t^3}{3} + C_2$

$= \frac{2(x+1)^{\frac{5}{2}}}{5} - \frac{2(x+1)^{\frac{3}{2}}}{3} + C_2$

Putting the value of I_1 and I_2 in (1), we have,

$y = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + C_1 + \frac{2(x+1)^{\frac{5}{2}}}{5} - \frac{2(x+1)^{\frac{3}{2}}}{3} + C_2$

$y = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + \frac{2(x+1)^{\frac{5}{2}}}{5} - \frac{2(x+1)^{\frac{3}{2}}}{3} + C \ \left[\because C = C_1 + C_2\right]$

Hence, $y = \frac{\cos^7 x}{7} - \frac{\cos^5 x}{5} + \frac{2(x+1)^{\frac{5}{2}}}{5} - \frac{2(x+1)^{\frac{3}{2}}}{3} + C$ is the solution of the given differential equation.