Complete Book 2

Integrals

$\boxed{f(x) = \sin x}$

$\boxed{f'(x) = \underbrace{\cos x}_{\text{Derivative}}}$

Differentiation

$f(x) = \sin x + 4$ ✓

$\boxed{f'(x) = \cos x}$

$\int \cos x \, dx = \sin x$

$f(x) = \sin x + 100$ ✓

$\boxed{f'(x) = \cos x}$

$\int \cos x \, dx = \sin x$

$\boxed{\text{Definite Int.}}$

$\int f(x)\,dx = 4x^2 + 3$

✓ Anti-Differentiation

↓

$\boxed{\text{Intigration}}$

$\boxed{\int \cos x \, dx = \sin x}$

$\boxed{\int \cos x \, dx = \sin x + C}$

↓

Indefinite Integrals ✓

$\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

$\boxed{\int \frac{1}{1+x^2}\,dx = \tan^{-1}x + C}$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\int \sec^2 x \, dx = \boxed{\tan x + C}$

Integrals

	Derivatives	Integrals (Anti Derivatives)
(i)	$\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$; Particularly, we note that $\frac{d}{dx}(x) = 1$	$\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$ $\int dx = x + C$
(ii)	$\frac{d}{dx}(\sin x) = \cos x$;	$\int \cos x\, dx = \sin x + C$
(iii)	$\frac{d}{dx}(-\cos x) = \sin x$;	$\int \sin x\, dx = -\cos x + C$
(iv)	$\frac{d}{dx}(\tan x) = \sec^2 x$;	$\int \sec^2 x\, dx = \tan x + C$

$\frac{d}{dx}\left[\frac{1}{n+1}x^{n+1}\right]$

$\frac{1}{\cancel{n+1}}\cancel{(n+1)}\, x^{n+\cancel{1}-\cancel{1}}$

x^n

$$\boxed{\int x^n dx = \frac{x^{n+1}}{n+1} + C}$$

$\int x^4 dx = \frac{x^{4+1}}{4+1} + C$

$= \frac{x^5}{5} + C$

Integrals

	Derivatives	Integrals (Anti Derivatives)
(v)	$\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x$;	$\int \operatorname{cosec}^2 x\, dx = -\cot x + C$
(vi)	$\frac{d}{dx}(\sec x) = \sec x \tan x$;	$\int \sec x \tan x\, dx = \sec x + C$
(vii)	$\frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x \cot x$;	$\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$
(viii)	$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$;	$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$
(ix)	$\frac{d}{dx}(-\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$;	$\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$

Integrals

	Derivatives	Integrals (Anti Derivatives)
(x)	$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$;	$\int \frac{dx}{1+x^2} = \tan^{-1} x + C$
(xi)	$\frac{d}{dx}(e^x) = e^x$;	$\int e^x dx = e^x + C$
(xii)	$\frac{d}{dx}(\log\lvert x\rvert) = \frac{1}{x}$;	$\int \frac{1}{x} dx = \log\lvert x\rvert + C$
(xiii)	$\frac{d}{dx}\left(\frac{a^\tau}{\log a}\right) = a^x$;	$\int a^x dx = \frac{a^x}{\log a} + C$

Integrals

(i) $\int \tan x \, dx = \log|\sec x| + C$

(ii) $\int \cot x \, dx = \log|\sin x| + C$

(iii) $\int \sec x \, dx = \log|\sec x + \tan x| + C$

(iv) $\int \text{cosec}\, x \, dx = \log|\text{cosec}\, x - \cot x| + C$

Integrals

3. Methods of integration

(a) $\boxed{\text{Integration by substitution}}$

(b) Integration by trigonometric formula

(c) Method of partial fraction.

QUESTIONS

$$\boxed{\int \left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2 dx}$$

$$(\sqrt{x})^2+\left(\frac{1}{\sqrt{x}}\right)^2-2\sqrt{x}\times\frac{1}{\sqrt{x}}$$

$$\int x+\frac{1}{x}-2\;dx$$

$$\boxed{\frac{x^2}{2}+\log|x|-2x+C}=$$

$$\int x\,dx+\int\frac{1}{x}dx-2\int dx$$

QUESTIONS

$$\boxed{\int \frac{x^3+5x^2-4}{x^2}\,dx}$$

$$\int \frac{x^3}{x^2}+\frac{5x^2}{x^2}-\frac{4}{x^2}\,dx$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int x+5-4x^{-2}\,dx$$

$$\int x\,dx + 5\int dx - 4\int x^{-2}\,dx$$

$$\frac{x^2}{2}+5x-4\,\frac{x^{-2+1}}{-2+1}+C$$

$$\boxed{\frac{x^2}{2}+5x+\frac{4}{x}+C}$$

QUESTIONS

$$\boxed{\int \frac{2 - 3\sin x}{\cos^2 x}\,dx}$$

$$\frac{1 \times \sin x}{\cos x \times \cos x}$$

$$\int \frac{2}{\cos^2 x} - \frac{3\sin x}{\cos^2 x}\,dx$$

$$\boxed{\int 2\sec^2 x - 3\sec x \tan x\,dx}$$

$$2\int \sec^2 x\,dx - 3\int \sec x \tan x\,dx$$

$$\boxed{2\tan x - 3\sec x + C}$$

QUESTIONS

The anti derivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ equals

$\int \frac{1}{\sqrt{x}}dx = 2\sqrt{x}$

1. $\frac{1}{3}x^{\frac{1}{3}}+2x^{\frac{1}{2}}+C$
2. $\frac{2}{3}x^{\frac{2}{3}}+\frac{1}{2}x^{2}+C$
3. $\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C$
4. $\frac{3}{2}x^{\frac{3}{2}}+\frac{1}{2}x^{\frac{1}{2}}+C$

$\int \sqrt{x}+\frac{1}{\sqrt{x}}\,dx$

$\int x^{\frac{1}{2}}+\frac{1}{x^{\frac{1}{2}}}\,dx$

$\int x^{\frac{1}{2}}+x^{-\frac{1}{2}}\,dx$

$\int x^{\frac{1}{2}}dx+\int x^{-\frac{1}{2}}dx$

$\boxed{\frac{2}{3}x^{\frac{3}{2}}+2\sqrt{x}+C}$

$\frac{x^{-\frac{1}{2}+\frac{1}{1}}}{-\frac{1}{2}+\frac{1}{1}} = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} = 2x^{\frac{1}{2}} = 2\sqrt{x}$

QUESTIONS

If $\boxed{\frac{d}{dx}f(x) = 4x^3 - \frac{3}{x^4}}$ such that $\boxed{f(2) = 0}$. Then $\boxed{f(x)}$ is

1. $\boxed{x^4 + \frac{1}{x^3} - \frac{129}{8}}$
2. $x^3 + \frac{1}{x^4} + \frac{129}{8}$
3. $x^4 + \frac{1}{x^3} + \frac{129}{8}$
4. $x^3 + \frac{1}{x^4} - \frac{129}{8}$

$\int 4x^3 - \frac{3}{x^4}\, dx$

$\int 4x^3 - 3x^{-4}\, dx$

$4\int x^3 dx - 3\int x^{-4} dx$

$4\frac{x^{3+1}}{3+1} - 3\frac{x^{-4+1}}{-4+1} + C$

$\cancel{4}\frac{x^4}{\cancel{4}} - \frac{\cancel{3}x^{-3}}{-\cancel{3}} + C$

$\boxed{x^4 + \frac{1}{x^3} + C = f(x)}$

$2^4 + \frac{1}{2^3} + C = 0$

$\frac{16}{1} + \frac{1}{8} = -C$

$-\frac{128+1}{8} = C$

$\boxed{-\frac{129}{8} = C}$

$\boxed{x^4 + \frac{1}{x^3} - \frac{129}{8}}$

QUESTIONS

Integrate the following functions w.r.t. x :

(i) $\dfrac{\tan^4\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}$ (ii) $\boxed{\dfrac{\sin(\tan^{-1}x)}{1+x^2}}$

× Integration ×

→ Probability → 8

$$\boxed{\frac{d}{dx}(\tan x) = \sec^2 x}$$

(ii) $\displaystyle\int \frac{\sin(\tan^{-1}x)}{1+x^2}\,dx$

let $\tan^{-1}x = t$ ✓

$$\boxed{\frac{1}{1+x^2}\,dx = dt} ✓$$

$$\boxed{\int \sin t\,dt}$$

$$\boxed{-\cos t + C}$$

$$\boxed{-\cos\tan^{-1}x + C}$$

(i) $\displaystyle\int \frac{\tan^4\sqrt{x}\sec^2\sqrt{x}}{\sqrt{x}}\,dx$

$$\boxed{\frac{2}{5}\tan^5\sqrt{x} + C}$$

let $\tan\sqrt{x} = t$

$$\sec^2\sqrt{x} \times \frac{1}{2\sqrt{x}}\,dx = dt$$

$$\boxed{\frac{\sec^2\sqrt{x}}{\sqrt{x}}\,dx = 2\,dt}$$

$$2\int t^4\,dt = \boxed{\frac{2}{5}t^5 + C}$$

QUESTIONS

Find the following integrals:

(i) $\int \sin^3 x \cos^2 x \, dx$ (ii) $\int \frac{\sin x}{\sin(x+a)} dx$ (iii) $\int \frac{1}{1+\tan x} dx$

$$\boxed{\int \sin^3 x \cos^2 x \, dx}$$

$$\int \sin x \times \sin^2 x \cdot \cos^2 x \, dx$$

$$\int \sin x (1-\cos^2 x) \cos^2 x \, dx$$

$$\boxed{\int \sin x [\cos^2 x - \cos^4 x] \, dx}$$

let $\cos x = t$

$-\sin x \, dx = dt$

$$\boxed{\sin x \, dx = -dt}$$

$$-\int (t^2 - t^4) \, dt$$

$$-\left[\frac{t^3}{3} - \frac{t^5}{5}\right] + c$$

$$-\frac{t^3}{3} + \frac{t^5}{5} + c$$

$$\boxed{-\frac{1}{3}\cos^3 x + \frac{1}{5}\cos^5 x + c}$$

QUESTIONS

Find the following integrals:

3 marks

(i) $\boxed{\int \sin^3 x \cos^2 x \, dx}$ (ii) $\int \frac{\sin x}{\sin(x+a)} dx$ (iii) $\int \frac{1}{1+\tan x} dx$

(ii) $\int \frac{\sin x}{\sin(x+a)} dx$

let $x+a=t$ or $\boxed{x=t-a}$

$\boxed{dx=dt}$

$\int \frac{\sin(t-a)}{\sin t} dt$

$\int \frac{\sin t \cos a - \cos t \sin a}{\sin t} dt$

$\int \frac{\cancel{\sin t} \cos a}{\cancel{\sin t}} - \frac{\cos t \sin a}{\sin t} dt$

$\boxed{\cos a \int dt - \sin a \int \cot t \, dt}$

$t \cos a - \sin a \log \sin t + C$

$\boxed{(x+a)\cos a - \sin a \log \sin(x+a) + C}$

QUESTIONS

Find the following integrals:

(i) $\boxed{\int \sin^3 x \cos^2 x \, dx}$ (ii) $\int \frac{\sin x}{\sin(x+a)} dx$ (iii) $\int \frac{1}{1+\tan x} dx$

$\frac{1}{2}\int \frac{\cos x+\sin x}{\cos x+\sin x} + \frac{\cos x-\sin x}{\cos x+\sin x} dx$

$\frac{1}{2}\int 1 + \frac{\cos x-\sin x}{\cos x+\sin x} dx$

let $\cos x+\sin x = t$

$(-\sin x+\cos x)dx = dt$

$\boxed{(\cos x-\sin x)dx = dt}$

$\frac{1}{2}\int dx + \boxed{\frac{1}{2}\int \frac{(\cos x-\sin x)\,dx}{\cos x+\sin x}}$

$\frac{x}{2} + \frac{1}{2}\int \frac{1}{t} dt$

$\frac{x}{2} + \frac{1}{2}\log|t| + C = \boxed{\frac{x}{2} + \frac{1}{2}\log|\cos x+\sin x| + C}$

$\int \frac{1}{1+\frac{\sin x}{\cos x}}$

$\int \frac{1}{\frac{\cos x+\sin x}{\cos x}}$

$\boxed{\frac{1}{2}\int \frac{2\cos x}{\cos x+\sin x} dx}$

$\frac{1}{2}\int \frac{\cos x+\sin x+\cos x-\sin x}{\cos x+\sin x} dx$

QUESTIONS

$$\boxed{\frac{e^{2x}-1}{e^{2x}+1}}$$

$$\int \frac{\cancel{e^x}\left[e^x-\frac{1}{e^x}\right]}{\cancel{e^x}\left[e^x+\frac{1}{e^x}\right]}\,dx \qquad \int \frac{dt}{t}$$

$$\boxed{\log|t|+C}$$

$$\boxed{\int \frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx} \qquad \boxed{\log|e^x+e^{-x}|+C}$$

let $e^x+e^{-x}=t$

$e^x + e^{-x}\times-(1)\,dx = dt$

$$\boxed{(e^x-e^{-x})\,dx = dt}$$

QUESTIONS

$$\frac{e^{5\log x}-e^{4\log x}}{e^{3\log x}-e^{2\log x}}$$

$$\boxed{\log x^m = m\log x}$$

$$\boxed{e^{\log x} = x}$$

$$\# \int \frac{e^{\log x^5}-e^{\log x^4}}{e^{\log x^3}-e^{\log x^2}}\,dx$$

$$\int \frac{x^5-x^4}{x^3-x^2}\,dx$$

$$\int \frac{x^4\cancel{[x-1]}}{\cancel{x^2}\cancel{[x-1]}}\,dx$$

$$\int x^2\,dx = \boxed{\frac{x^3}{3}+C}$$

QUESTIONS

$$\boxed{\frac{\sqrt{\tan x}}{\sin x \cos x}}$$

3 marks

$\tan x$, $\sec^2 x$ }

$$\int \frac{\frac{\sqrt{\tan x}}{\cos^2 x}}{\frac{\sin x \cancel{\cos x}}{\cancel{\cos} x}} dx$$

$\sqrt{x} \times \frac{1}{x} = \frac{1}{\sqrt{x}}$

let $\tan x = t$

$\boxed{\sec^2 x\, dx = dt}$

$$\int \frac{\sqrt{\tan x}\sec^2 x}{\tan x} dx$$

$$\int \frac{dt}{\sqrt{t}}$$

$$\boxed{\int \frac{\sec^2 x}{\sqrt{\tan x}} dx}$$

$2\sqrt{t} + C$

$\boxed{2\sqrt{\tan x} + C}$

QUESTIONS

$$\boxed{\int \frac{1-\cos x}{1+\cos x}\,dx}$$

$$\int \frac{\cancel{2}\sin^2\frac{x}{2}}{\cancel{2}\cos^2\frac{x}{2}}\,dx$$

$$\boxed{\int \tan^2\frac{x}{2}\,dx}$$

$$\int \sec^2\frac{x}{2}-1\,dx$$

$$\int \sec^2\frac{x}{2}\,dx-1\int dx$$

$$\boxed{2\tan\frac{x}{2}-x+C}$$

$$\boxed{\int \sec^2x\,dx=\tan x+C}$$

$$1+\tan^2x=\sec^2x$$

$$\downarrow$$

$$\boxed{\tan^2x=\sec^2x-1}$$

$$\int \cos x=\sin x+C$$

$$\int \cos 4x\,dx=\boxed{\frac{\sin 4x}{4}+C}$$

$$\int \cos 5x\,dx=\frac{1}{5}\sin 5x+C$$

QUESTIONS

$$\boxed{\int \frac{\cos x}{1+\cos x}\,dx}$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\boxed{\cos x = 2\cos^2\frac{x}{2} - 1}$$

$$\int \frac{\cos x}{2\cos^2\frac{x}{2}}$$

$$\int \frac{2\cos^2\frac{x}{2} - 1}{2\cos^2\frac{x}{2}}$$

$$\int \frac{\cancel{2\cos^2\frac{x}{2}}}{\cancel{2\cos^2\frac{x}{2}}} - \frac{1}{2\cos^2\frac{x}{2}}$$

$$\int 1 - \frac{1}{2}\sec^2\frac{x}{2}\,dx$$

$$\int dx - \frac{1}{2}\int \sec^2\frac{x}{2}\,dx$$

$$x - \cancel{\frac{1}{2}} \times \cancel{2}\tan\frac{x}{2} + C$$

$$\boxed{x - \tan\frac{x}{2} + C}$$

QUESTIONS

$\int \frac{1\,dx}{\sin^2 x \cos^2 x}$ equals

$1 = \sin^2 x + \cos^2 x$

1. $\tan x + \cot x + C$
2. $\tan x - \cot x + C$
3. $\tan x \cot x + C$
4. $\tan x - \cot 2x + C$

$$\int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cdot \cos^2 x}\,dx$$

$$\int \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x} \cdot \cos^2 x} + \frac{\cancel{\cos^2 x}}{\sin^2 x \cdot \cancel{\cos^2 x}}\,dx$$

$$\int \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}\,dx$$

$$\int \sec^2 x\,dx + \int \csc^2 x\,dx$$

$$\boxed{\tan x - \cot x + C}$$

QUESTIONS

$$\boxed{\sin x\cdot\cos y=\frac{1}{2}\left[\sin(x+y)+\sin(x-y)\right]}$$

Find

(i) $\boxed{\int \cos^2 x\,dx}$ (ii) $\boxed{\int \sin 2x\cos 3x\,dx}$ $\boxed{\text{(iii) } \int \sin^3 x\,dx}$

$\cos 2x = 2\cos^2 x - 1$

$\cos 2x + 1 = 2\cos^2 x$

$$\boxed{\cos^2 x = \frac{\cos 2x + 1}{2}}$$

$\int \frac{\cos 2x+1}{2}\,dx$

$\frac{1}{2}\int \cos 2x + 1\,dx$

$\frac{1}{2}\left[\frac{\sin 2x}{2}+x\right]+C$

$$\boxed{\frac{1}{4}\sin 2x+\frac{x}{2}+C}$$

$\frac{1}{2}\int \sin 5x+\sin(-x)\,dx$

$$\boxed{\frac{1}{2}\int \sin 5x-\sin x\,dx}$$

$\frac{1}{2}\left[-\frac{\cos 5x}{5}-(-\cos x)\right]+C$

$\frac{1}{2}\left[-\frac{1}{5}\cos 5x+\cos x\right]+C$

$$\boxed{-\frac{1}{10}\cos 5x+\frac{1}{2}\cos x+C}$$

QUESTIONS

Find

(i) $\int \cos^2 x dx$ (ii) $\int \sin 2x \cos 3x \, dx$ (iii) $\boxed{\int \sin^3 x dx}$

\# $\boxed{\sin 3x = 3\sin x - 4\sin^3 x}$

$4\sin^3 x = 3\sin x - \sin 3x$

$$\boxed{\sin^3 x = \frac{1}{4}[3\sin x - \sin 3x]}$$

\# $\frac{1}{4}\int 3\sin x - \sin 3x \, dx$

$$\frac{1}{4}\left[3(-\cos x) - \left[\frac{-\cos 3x}{3}\right]\right] + C$$

$$\frac{1}{4}\left[-3\cos x + \frac{1}{3}\cos 3x\right] + C$$

$$\boxed{-\frac{3}{4}\cos x + \frac{1}{12}\cos 3x + C}$$

QUESTIONS

$$\boxed{\int \frac{dx}{\cos(x-a)\cos(x-b)}}$$

v.v.v.imp

$\int \tan x = \log|\sec x| + C$

$\sin(a-b) = \boxed{\sin[(x-b)-(x-a)]}$

$$\boxed{\sin(A-B) = \sin A\cos B - \cos A\sin B}$$

$$\frac{1}{\sin(a-b)}\int \frac{\sin(a-b)}{\cos(x-a)\cos(x-b)}\,dx$$

$$\frac{1}{\sin(a-b)}\int \frac{\sin[(x-b)-(x-a)]}{\cos(x-a)\cos(x-b)}\,dx$$

$$\frac{1}{\sin(a-b)}\int\left[\frac{\sin(x-b)\cos(x-a)-\cos(x-b)\sin(x-a)}{\cos(x-a)\cos(x-b)}\right]$$

$$\frac{1}{\sin(a-b)}\int\left[\tan(x-b)-\tan(x-a)\right]dx$$

$$\frac{1}{\sin(a-b)}\left[\log|\sec(x-b)| - \log|\sec(x-a)|\right]+C$$

$$\boxed{\frac{1}{\sin(a-b)}\left[\log\left|\frac{\sec(x-b)}{\sec(x-a)}\right|\right]+C}\ ✓$$

$$\frac{1}{\sin(a-b)}\left[\log\left|\frac{\cos(x-a)}{\cos(x-b)}\right|\right]+C\ ✓$$

Integrals

(1) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$

(2) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

(3) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

(4) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log \left| x + \sqrt{x^2 - a^2} \right| + C$

(5) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$

(6) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log \left| x + \sqrt{x^2 + a^2} \right| + C$

QUESTIONS

Find the following integrals :

$$\boxed{\int \frac{dx}{3x^2 + 13x - 10}}$$

Completing the square method

$3x^2 + 13x - 10$

$3\left[x^2 + \frac{13x}{3} - \frac{10}{3}\right]$

$3\left[x^2 + \frac{13}{3}x + \left(\frac{13}{6}\right)^2 - \left(\frac{13}{6}\right)^2 - \frac{10}{3}\right]$

$3\left[\left(x + \frac{13}{6}\right)^2 - \left(\frac{17}{6}\right)^2\right]$

$$\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + C$$

$$\boxed{\frac{1}{3}\int \frac{dx}{\left(x+\frac{13}{6}\right)^2 - \left(\frac{17}{6}\right)^2}}$$

$$\frac{1}{3} \times \frac{6}{2 \times 17} \log\left|\frac{x + \frac{13}{6} - \frac{17}{6}}{x + \frac{13}{6} + \frac{17}{6}}\right| + C$$

$$\frac{1}{17}\log\left|\frac{\frac{6x-4}{6}}{\frac{6x+13+17}{6}}\right| + C$$

$$\boxed{\frac{1}{17}\log\left|\frac{6x-4}{6x+30}\right| + C}$$

QUESTIONS

$$\frac{1}{\sqrt{7-6x-x^2}}$$

QUESTIONS

Imp

$\frac{1}{2}\times 3$

Find the following integrals:

$$\boxed{\int \frac{x+2}{2x^2+6x+5}\,dx}$$

$\int \frac{x+2}{2x^2+6x+5}dx = \int \frac{1}{4}\left[\frac{4x+6}{2x^2+6x+5}\right] + \frac{1}{2}\times\left[\frac{1}{2x^2+6x+5}\right]dx$

I_1 $\qquad$ I_2

$I_1 = \frac{1}{4}\int \frac{4x+6}{2x^2+6x+5}dx$

$\boxed{I_2 = \frac{1}{2}\int \frac{1}{2x^2+6x+5}} = \frac{1}{4}\int \frac{dx}{\left(x+\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2}$

$\boxed{x+2 = A\frac{d}{dx}(2x^2+6x+5)+B}$

$x+2 = A[4x+6]+B$

$x+2 = 4Ax+6A+B$

$4Ax = x$

$\boxed{A = \frac{1}{4}}$

$6A+B = 2$

$6\left[\frac{1}{4}\right]+B = 2$

$\boxed{B = \frac{2}{1}-\frac{3}{2} = \frac{1}{2}}$

let $2x^2+6x+5 = t$

$\boxed{(4x+6)\,dx = dt}$

$I_1 = \frac{1}{4}\int \frac{dt}{t}$

$\boxed{I_1 = \frac{1}{4}\log|2x^2+6x+5|}$

$2x^2+6x+5$

$2\left[x^2+\frac{6}{2}x+\frac{5}{2}\right]$

$2\left[x^2+3x+\frac{5}{2}\right]$

$2\left[x^2+3x+\left(\frac{3}{2}\right)^2-\left(\frac{3}{2}\right)^2+\frac{5}{2}\right]$

$2\left[\left(x+\frac{3}{2}\right)^2+\left(\frac{1}{2}\right)^2\right]$

Comment

Integrals

S. No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
	where x^2+bx+c cannot be factorized further	

QUESTIONS

Find $\boxed{\int \frac{dx}{(x+1)(x+2)}} = \int \frac{1}{x+1} - \frac{1}{x+2}\, dx = \boxed{\log|x+1| - \log|x+2| + C}$

$\boxed{\log\left|\frac{x+1}{x+2}\right| + C}$

$$I = \int \frac{dx}{(x+1)(x+2)}$$

$$\boxed{\int \frac{1}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}}$$

$$\boxed{1 = A(x+2) + B(x+1)}$$

put $x = -2$

$1 = A(-2+2) + B(-2+1)$

$1 = -B$

$\boxed{B = -1}$

$x = -1$

$1 = A(-1+2)$

$\boxed{1 = A}$

QUESTIONS

Find $\boxed{\int \frac{3x-2}{(x+1)^2(x+3)}\,dx}$ ② ①

$$\boxed{\frac{3x-2}{(x+1)^2(x+3)} = \frac{A}{x+1} + \frac{B}{(x+1)^2} + \frac{C}{x+3}}$$

$$\boxed{3x-2 = A(x+1)(x+3) + B(x+3) + C(x+1)^2}$$

$x=-1$

$3(-1)-2 = B(-1+3)$

$-3-2 = B(2)$

$\boxed{-\frac{5}{2} = B}$

$\boxed{A = \frac{11}{4}}$

\# $x=-3$

$3(-3)-2 = C[-3+1]^2$

$-9-2 = C[-2]^2$

$-11 = 4C$

$\boxed{C = -\frac{11}{4}}$

\# $\frac{11}{4}\int \frac{1}{x+1}dx - \frac{5}{2}\int \frac{1}{(x+1)^2}dx - \frac{11}{4}\int \frac{1}{x+3}dx$

$\frac{11}{4}\log|x+1| + \frac{5}{2}\frac{1}{(x+1)} - \frac{11}{4}\log|x+3| + C$

$\frac{11}{4}\log|x+1| - \frac{11}{4}\log|x+3| + \frac{5}{2(x+1)} + C$

$$\boxed{\frac{11}{4}\log\left|\frac{x+1}{x+3}\right| + \frac{5}{2(x+1)} + C}$$

QUESTIONS

Find $\int \frac{x^2+1}{x^2-5x+6}dx$

$x^2-3x-2x+6$

$x(x-3)-2(x-3)$

$(x-3)(x-2)$

$x^2-5x+6)\ \cancel{x^2}+1\ (\ 1$

$\cancel{x^2}-5x+6$

$-\quad+\quad-$

$5x-5$

$$\boxed{\int \frac{x^2+1}{x^2-5x+6} = \int 1 + \frac{5x-5}{x^2-5x+6}}$$

$$\boxed{1\int dx + 5\int \frac{x-1}{(x-2)(x-3)}dx}$$

$x+5I_1$

$I_1 = \int \frac{x-1}{(x-2)(x-3)}dx$

$x-1 = \frac{A}{x-2} + \frac{B}{x-3}$

$x-1 = A(x-3) + B(x-2)$

$x=3$

$3-1 = B(3-2)$

$\boxed{2 = B}$

$x=2$

$2-1 = A(2-3)$

$1 = -A$

$\boxed{A=-1}$

$I_1 = -\int \frac{1}{x-2}dx + 2\int \frac{1}{x-3}dx$

$= \boxed{-\log|x-2| + 2\log|x-3|}$

$\boxed{I = x - 5\log|x-2| + 10\log|x-3| + C}$

QUESTIONS

Find $\boxed{\int \frac{x^2}{(x^2+1)(x^2+4)}\,dx}$

2025|2024

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$$

let $x^2 = t$

$$I = \int \frac{t}{(t+1)(t+4)}\,dx$$

$$\frac{t}{(t+1)(t+4)} = \frac{A}{t+1} + \frac{B}{t+4}$$

$$t = A(t+4) + B(t+1)$$

$t = -4$

$-4 = B(-4+1)$

$-4 = -3B$

$\boxed{B = \frac{4}{3}}$

$t = -1$

$-1 = A(-1+4)$

$\boxed{A = \frac{-1}{3}}$

$$\boxed{-\frac{1}{3}\int\frac{dx}{x^2+1} + \frac{4}{3}\int\frac{dx}{x^2+2^2}}$$

$$\boxed{-\frac{1}{3}\tan^{-1}x + \frac{4}{6}\tan^{-1}\frac{x}{2} + C}$$

$$-\frac{1}{3}\tan^{-1}x + \frac{2}{3}\tan^{-1}\frac{x}{2} + C$$

QUESTIONS

Find $\boxed{\int \frac{x^2 + x + 1\, dx}{(x+2)(x^2+1)}}$ V.V.imp

$$Bx + C = \frac{2x}{5} + \frac{1}{5} = \frac{1}{5}(2x+1)$$

3 5

$$\boxed{\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}}$$

$$x^2+x+1 = A(x^2+1) + (x+2)(Bx+C)$$

$$x^2+x+1 = Ax^2+A+Bx^2+Cx+2Bx+2C$$

$$x^2+x+1 = Ax^2+Bx^2+2Bx+Cx+A+2C$$

$$1x^2+1x+1 = x^2[A+B]+x[2B+C]+[A+2C]$$

$A+B=1$ — (i) $B=1-A$

$2B+C=1$ — (ii) $2(1-A)+C=1$

$A+2C=1$ — (iii) $2-2A+C=1$

$-2A+C=-1$

$\boxed{C=2A-1}$

$A+2(2A-1)=1$

$A+4A-2=1$

$5A=1+2$

$A=\frac{3}{5}$

$C=2\left(\frac{3}{5}\right)-1$

$C=\frac{6}{5}-\frac{1}{1}$

$C=\frac{6-5}{5}$

$\boxed{C=\frac{1}{5}}$

$B=\frac{1}{1}-\frac{3}{5}=\boxed{\frac{2}{5}}$

$$\# \frac{3}{5}\int \frac{dx}{x+2} + \frac{1}{5}\int \frac{2x+1}{x^2+1}\,dx$$

$$\frac{3}{5}\log|x+2| + \frac{1}{5}\int \frac{2x}{x^2+1} + \frac{1}{x^2+1}\,dx$$

$$\frac{3}{5}\log|x+2| + \frac{1}{5}\left[\log(x^2+1) + \tan^{-1}x\right] + C$$

$$\frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + C$$

QUESTIONS

Find $\int x \cos x \, dx$

Integration by parts

ILATE

$$\boxed{I = \int \underset{\text{I}}{x} \, \underset{\text{II}}{\cos x} \, dx}$$

$$x \int \cos x \, dx - \int \left[\frac{d}{dx}(x) \int \cos x \, dx \right] dx$$

$$x \sin x - \int \sin x \, dx$$

$$\boxed{x \sin x + \cos x + C}$$

QUESTIONS

$$\boxed{\text{Find } \int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx}$$

$\int \frac{x \sin^{-1} x}{\sqrt{1-x^2}}\, dx$

let $\sin^{-1} x = t$ $\quad x = \sin t$

$$\boxed{\frac{1}{\sqrt{1-x^2}}\, dx = dt}$$

$$\int \underset{\text{II}}{\sin t} \cdot \underset{\text{I}}{t}\; dt$$

$$t \int \sin t\, dt - \int \left[\frac{d}{dt}(t) \int \sin t\, dt\right] dt$$

$$-t \cos t + \int \cos t\, dt$$

$$\boxed{-t \cos t + \sin t + C}$$

$$-\sin^{-1} x \sqrt{1-x^2} + x + C$$

$$\boxed{x - \sin^{-1} x \sqrt{1-x^2} + C}$$

$\cos t = ?$

$\sin t = \frac{x}{1} = \frac{P}{h}$

$b = \sqrt{1-x^2}$

$$\boxed{\cos t = \sqrt{1-x^2}}$$

QUESTIONS

Find $\int e^x \sin x \, dx$

ILATE

$$I = \int \underset{\text{II}}{e^x} \underset{\text{I}}{\sin x} \, dx$$

$$I = \sin x \int e^x dx - \int \left[\frac{d}{dx}(\sin x) \int e^x dx\right] dx$$

$$I = e^x \sin x - \int \cos x \, e^x \, dx$$

$$I = e^x \sin x - \left[\cos x \int e^x dx - \int \left[\frac{d}{dx}(\cos x) \int e^x dx\right] dx\right]$$

$$I = e^x \sin x - e^x \cos x - \boxed{\int e^x \sin x \, dx}$$

$$2I = e^x[\sin x - \cos x]$$

$$\boxed{I = \frac{e^x[\sin x - \cos x]}{2}} + C$$

QUESTIONS

$$\boxed{e^x\left(\frac{1+\sin x}{1+\cos x}\right)}$$

$$\boxed{\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$\int e^x\left[\frac{1+\sin x}{2\cos^2\frac{x}{2}}\right]dx$$

$$\int e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\frac{\cancel{2}\sin\frac{x}{2}\cancel{\cos\frac{x}{2}}}{\cancel{2}\cos^{\cancel{2}}\frac{x}{2}}\right]dx$$

$$\boxed{\int e^x\left[\frac{1}{2}\sec^2\frac{x}{2}+\tan\frac{x}{2}\right]} dx$$

$$\int e^x\left[\underbrace{\tan\frac{x}{2}}_{f(x)}+\underbrace{\frac{1}{2}\sec^2\frac{x}{2}}_{f'(x)}\right]dx = \boxed{e^x\tan\frac{x}{2}+C}$$

$$\# \int e^x[f(x)+f'(x)]\,dx$$

$$\# \boxed{e^x f(x)+C}$$

$$\int e^x[\underbrace{\sin x}_{f(x)}+\underbrace{\cos x}_{f'(x)}]\,dx$$

$$\Rightarrow \boxed{e^x\sin x+C}$$

QUESTIONS

$$\boxed{\frac{(x-3)e^x}{(x-1)^3}}$$

$$\boxed{e^x[f(x)+f'(x)]}$$

$$e^x\left[\frac{x-1-2}{(x-1)^3}\right]$$

$$e^x\left[\frac{\cancel{x-1}}{(x-1)^{\cancel{3}}}-\frac{2}{(x-1)^3}\right]$$

$$\int e^x\left[\underbrace{\frac{1}{(x-1)^2}}_{f(x)}-\underbrace{\frac{2}{(x-1)^3}}_{f'(x)}\right]dx$$

$$e^x\left[\frac{1}{(x-1)^2}\right]+C=\boxed{\frac{e^x}{(x-1)^2}+C}$$

QUESTIONS

$$\boxed{\sin^{-1}\left(\frac{2x}{1+x^2}\right)}$$

Imp

$$\int \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$$

Putting $x = \tan\theta$

$$\boxed{\theta = \tan^{-1}x}$$

$$\int \sin^{-1}\left[\frac{2\tan\theta}{1+\tan^2\theta}\right] dx$$

$$\int \cancel{\sin^{-1}}\, \sin 2\theta \, dx$$

$$2\int \theta \, dx$$

$$\boxed{2\int \tan^{-1}x \, dx}$$

$$\boxed{\int \tan^{-1}x \cdot 1 \, dx}$$

$$\tan^{-1}x \int dx - \int \left[\frac{d}{dx}(\tan^{-1}x) \int dx\right] dx$$

$$x \tan^{-1}x - \int \left[\frac{1}{1+x^2} \times x\right] dx$$

$$x \tan^{-1}x - \int \frac{x}{1+x^2}\, dx = x \tan^{-1}x - \frac{1}{2}\int \frac{1}{t}\, dt$$

let $1+x^2 = t$

$2x\,dx = dt$

$$\boxed{x\,dx = \frac{1}{2}\, dt}$$

$$x \tan^{-1}x - \frac{1}{2}\log|1+x^2|$$

$$\boxed{2x \tan^{-1}x - \log|1+x^2| + C}$$

Integrals

3:30 Break 4:00 9-10

(i) $$\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + C$$

(ii) $$\boxed{\int \sqrt{x^2 + a^2}\,dx = \frac{1}{2}x\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + C}$$

(iii) $$\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

QUESTIONS

Find $\boxed{\int \sqrt{x^2 + 2x + 5}\, dx}$

x^2+2x+5

$x^2+2x+1+4$

$x^2+2x+1^2+2^2$

$\boxed{(x+1)^2+2^2}$

$\int \sqrt{(x+1)^2+2^2}\, dx$

$\frac{1}{2}(x+1)\sqrt{x^2+2x+5} \ldots$

Comment

QUESTIONS

$$\boxed{\text{Evaluate } \int_{-1}^{1} 5x^4\sqrt{x^5+1}\,dx}$$

Definite
- ✓ Indefinite + ✓ lim
- properties.

$$I = \int_{-1}^{1} 5x^4\sqrt{x^5+1}\;dx$$

let $x^5+1 = t$

$$\boxed{5x^4\,dx = dt}$$

if $x = 1$: $\boxed{t = 1+1 = 2}$

if $x = -1$: $(-1)^5+1$, $\boxed{t = 0}$

$$I = \int_0^2 \sqrt{t}\,dt = \int_0^2 t^{\frac{1}{2}}\,dt$$

$$\left[\frac{2}{3}t^{\frac{3}{2}}\right]_0^2$$

$$\frac{2}{3}\left[2^{\frac{3}{2}} - 0^{\frac{3}{2}}\right]$$

$$\frac{2}{3}\times 2^{3/2}$$

$$\frac{2}{3}\times 2\times 2^{\frac{1}{2}}$$

$$\boxed{\frac{4\sqrt{2}}{3}}$$

QUESTIONS

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\,dx$$

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\,dx$$

let $\boxed{\cos x = t}$

$-\sin x\,dx = dt$

$\boxed{\sin dx = -dt}$

if $x = \frac{\pi}{2}$ if $x = 0$

$\cos\frac{\pi}{2} = t$ $\cos 0 = t$

$\boxed{t=0}$ $\boxed{t=1}$

$$\boxed{-\int_1^0 \frac{dt}{t^2+1}}$$

$$-1\left[\tan^{-1} t\right]_1^0$$

$$-\left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$-\left(\cancel{\tan^{-1}\tan} 0 - \cancel{\tan^{-1}\tan}\frac{\pi}{4}\right)$$

$$-\left[-\frac{\pi}{4}\right] = \boxed{\frac{\pi}{4}}$$

Integrals

Some Properties of Definite Integrals

$$\mathbf{P}_0 : \int_a^b f(x)dx = \int_a^b f(t)dt$$ rare

$$\mathbf{P}_1 : \boxed{\int_a^b f(x)dx} = \boxed{-\int_b^a f(x)dx.} \quad \boxed{\int_a^a f(x)dx = 0}$$

$$\boxed{\mathbf{P}_2 : \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx}$$

Integrals

$\boxed{\mathbf{P_3} : \int_a^b f(x)dx = \int_a^b f(a+b-x)dx}$ King property

$\mathbf{P_4} : \int_0^a f(x)dx = \int_0^a f(a-x)dx$

$\boxed{\text{(Note that } P_4 \text{ is a particular case of } P_3\text{)}}$

$\mathbf{P_5} : \int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

$\mathbf{P_6} : \boxed{\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx}$, if $f(2a-x) = f(x)$ and 0 $f(2a-x) = -f(x)$

$\mathbf{P_7}$: (i) $\int_{-a}^{a} f(x)dx = 2\int_0^a f(x)dx$, if f is an even function, i.e., if $f(-x) = f(x)$.

(ii) $\int_{-a}^{a} f(x)dx = 0$, if f is an odd function, i.e., if $f(-x) = -f(x)$.

Function

Odd: $f(-x)$ → $-f(x)$

Even: $f(-x)$ → $f(x)$

$\boxed{f(x) = \sin^3 x}$

$f(-x) = \sin^3(-x)$

$\Rightarrow -\sin^3 x$

$f(-x) = -f(x)$

QUESTIONS

Evaluate $\boxed{\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx}$

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$2\sin^2 x = 1 - \cos 2x$$

$$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$$

$$\boxed{I = 2\int_0^{\frac{\pi}{4}} \sin^2 x\,dx} = 2\int_0^{\frac{\pi}{4}} \frac{1-\cos 2x}{\cancel{2}}\,dx$$

↳ Comment

$$= \int_0^{\frac{\pi}{4}} (1-\cos 2x)\,dx$$

$$\Rightarrow \left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{4}}$$

$$\boxed{\frac{\pi}{4} - \frac{1}{2}}$$

QUESTIONS

Evaluate $\boxed{\int_{-1}^{1} \sin^5 x \ \cos^4 x \, dx} = 0$

$$\boxed{f(x) = (\sin x)^5 \cdot (\cos x)^4}$$

\# $f(-x) = (\sin(-x))^5 \cdot (\cos(-x))^4$

$\Rightarrow (-\sin x)^5 \cdot (\cos x)^4$

$\Rightarrow -\sin^5 x \cdot \cos^4 x$

$$\boxed{f(-x) = -f(x)}$$

It is an odd function

QUESTIONS

Evaluate $\boxed{\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx} = I \quad \text{— (i)}$

$$\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^4\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)} dx$$

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^4 x}{\cos^4 x + \sin^4 x} dx \quad \text{— (ii)}$$

adding (i) and (ii)

$$2I = \int_0^{\pi/2} \frac{\cos^4 x + \sin^4 x}{\cos^4 x + \sin^4 x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} dx$$

$$2I = [x]_0^{\pi/2}$$

$$2I = \frac{\pi}{2}$$

$$\boxed{I = \frac{\pi}{4}}$$

QUESTIONS

$$\boxed{\int_0^{\frac{\pi}{2}} (2\log \sin x - \log \sin 2x)dx}$$

$$I = \int_0^{\frac{\pi}{2}} 2\log\sin x - \log[\overset{a}{2}\overset{b}{\sin x}\overset{c}{\cos x}]\,dx$$

$$I = \int_0^{\frac{\pi}{2}} 2\log\sin x - \log 2 - \log\sin x - \log\cos x\,dx$$

$$I = \int_0^{\pi/2} \log\sin x - \log\cos x - \log 2\,dx \quad -\text{(i)}$$

$$I = \int_0^{\frac{\pi}{2}} \log\sin\left(\frac{\pi}{2}-x\right) - \log\cos\left(\frac{\pi}{2}-x\right) - \log 2\,dx$$

$$I = \int_0^{\pi/2} \log\cos x - \log\sin x - \log 2\,dx \quad -\text{(ii)}$$

$\boxed{\text{adding (i) and (ii)}}$

$$2I = \int_0^{\pi/2} -2\log 2\,dx$$

$$\cancel{2}I = -\cancel{2}\log 2\int_0^{\pi/2} dx$$

$$I = -\log 2\,[x]_0^{\pi/2}$$

$$I = -\log 2 \times \frac{\pi}{2}$$

$$\boxed{I = -\frac{\pi}{2}\log 2}$$

QUESTIONS

Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

QUESTIONS

Evaluate $\boxed{\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}}$ → Imp

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

$$\frac{\pi}{3}+\frac{\pi}{6}=\frac{\pi}{2}$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\frac{1}{1}+\frac{\sqrt{\sin x}}{\sqrt{\cos x}}}$$

$$\boxed{I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx} \quad -\text{(i)}$$

$$I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}{\sqrt{\cos\left(\frac{\pi}{2}-x\right)}+\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}\,dx$$

$$I = \int_{\frac{\pi}{6}}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad -\text{(ii)}$$

adding (i) and (ii)

$$2I = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cancel{\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}}\,dx$$

$$2I = [x]_{\frac{\pi}{6}}^{\pi/3}$$

$$2I = \left[\frac{\pi}{3}-\frac{\pi}{6}\right]$$

$$2I = \frac{\pi}{6}$$

$$\boxed{I = \frac{\pi}{12}}$$

QUESTIONS

$$\boxed{\int_0^{\frac{\pi}{4}} \log(1+\tan x)\,dx}$$ Imp

$$\boxed{\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A\tan B}}$$

$$I = \int_0^{\pi/4} \log(1+\tan x)\,dx$$

$$I = \int_0^{\pi/4} \log\left[1+\tan\left[\frac{\pi}{4}-x\right]\right]dx$$

$$I = \int_0^{\pi/4} \log\left(1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\cdot\tan x}\right]$$

$$I = \int_0^{\pi/4} \log\left[\frac{1}{1}+\frac{1-\tan x}{1+\tan x}\right]dx$$

$$I = \int_0^{\pi/4} \log\left[\frac{1+\cancel{\tan x}+1-\cancel{\tan x}}{1+\tan x}\right]dx$$

$$I = \int_0^{\pi/4} \log\left[\frac{2}{1+\tan x}\right]dx$$

$$I = \int_0^{\pi/4} \log 2 - \log(1+\tan x)\,dx$$

$$I = \int_0^{\pi/4} \log 2\,dx - \int_0^{\pi/4} \log(1+\tan x)\,dx$$

$$2I = \log 2\,[x]_0^{\pi/4}$$

$$2I = \frac{\pi}{4}\log 2$$

$$\boxed{I = \frac{\pi}{8}\log 2}$$

QUESTIONS

$$\boxed{\int_0^a \frac{\sqrt{x}}{\sqrt{x}+\sqrt{a-x}}\,dx}$$

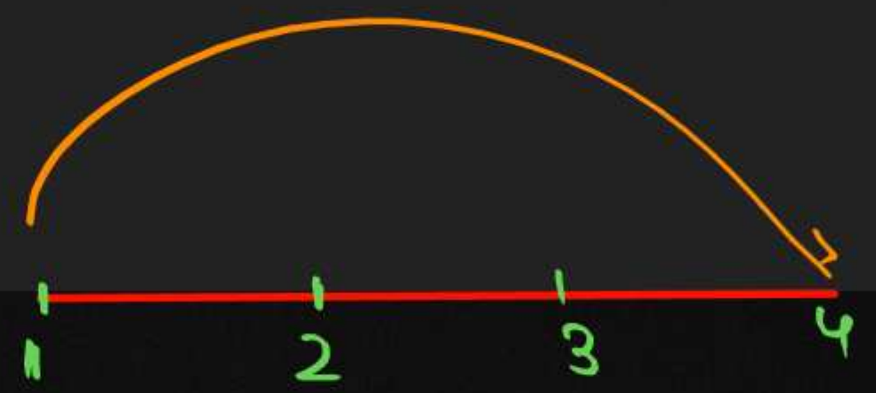
Home work

QUESTIONS

v.v. imp

$$\boxed{\int_1^4 [|x-1|+|x-2|+|x-3|]dx}$$

$x=1,\ x=2,\ x=3$



$$\int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx + \int_3^4 f(x)\,dx$$

$$\boxed{\int_1^2 -x+4\,dx + \int_2^3 x\,dx + \int_3^4 3x-6\,dx}$$

$$\left[-\frac{x^2}{2}+4x\right]_1^2 + \left[\frac{x^2}{2}\right]_2^3 + \left[\frac{3x^2}{2}-6x\right]_3^4$$

$$F(x) = x-1-(x-2)-(x-3)$$
$$= \cancel{x}-1-\cancel{x}+2-x+3$$
$$= -x+5-1$$
$$= -x+4$$

$f(x) = x-1+x-2+x-3$
$$= 3x-3-3$$
$$= 3x-6$$

QUESTIONS

u-s fixed

equation of circle: centre → origin

Area lying in the first quadrant and bounded by the circle $\boxed{x^2 + y^2 = 4}$ and the lines $x = 0$ and $x = 2$ is

$\boxed{y = \sqrt{2^2 - x^2}}$

1. $\boxed{\pi}$
2. $\frac{\pi}{2}$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{4}$

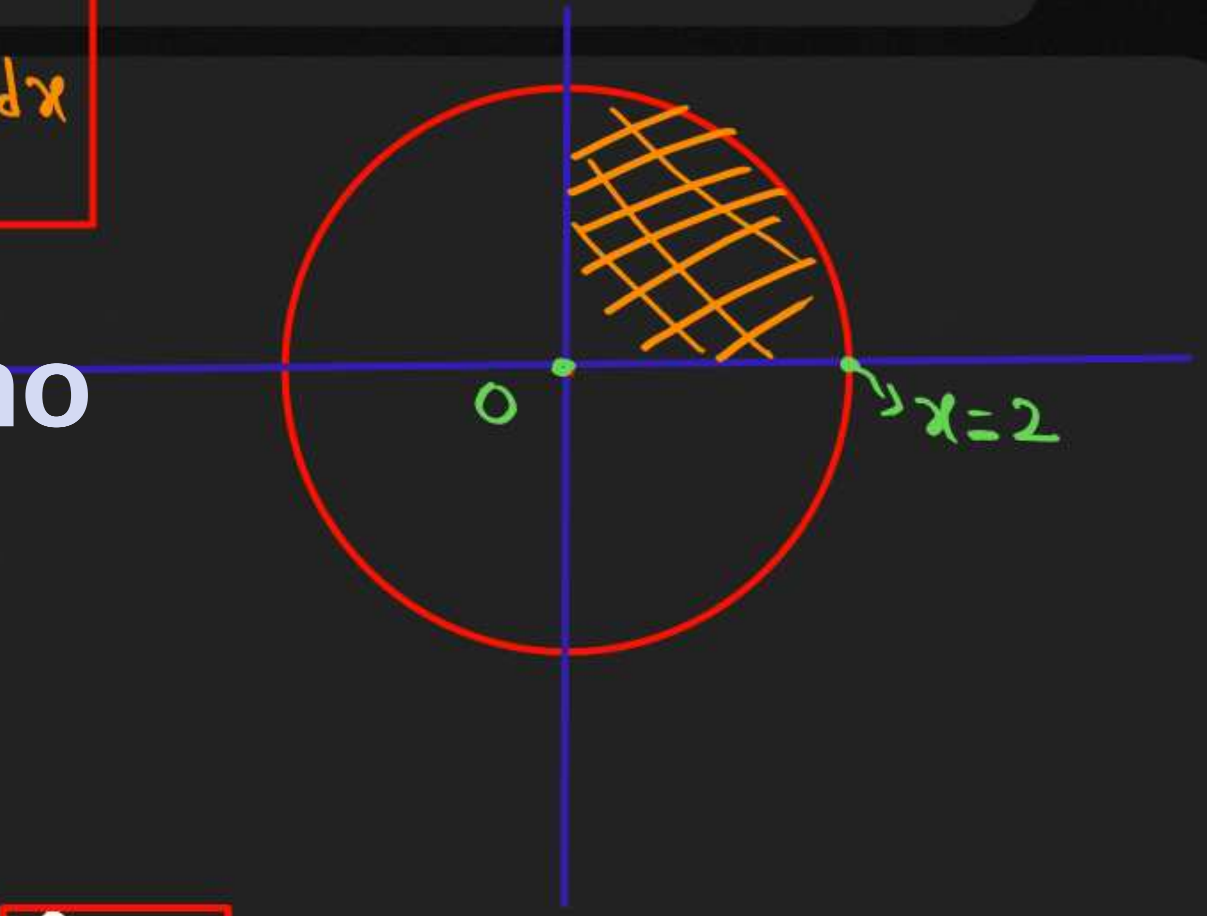
$\boxed{\text{Area} = \int_0^2 y\,dx}$

$\boxed{\text{Area} = \int_a^b f(x)\,dx}$

$\boxed{A = \int_0^2 \sqrt{2^2 - x^2}\,dx}$

$A = \left[\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2}\right]_0^2$

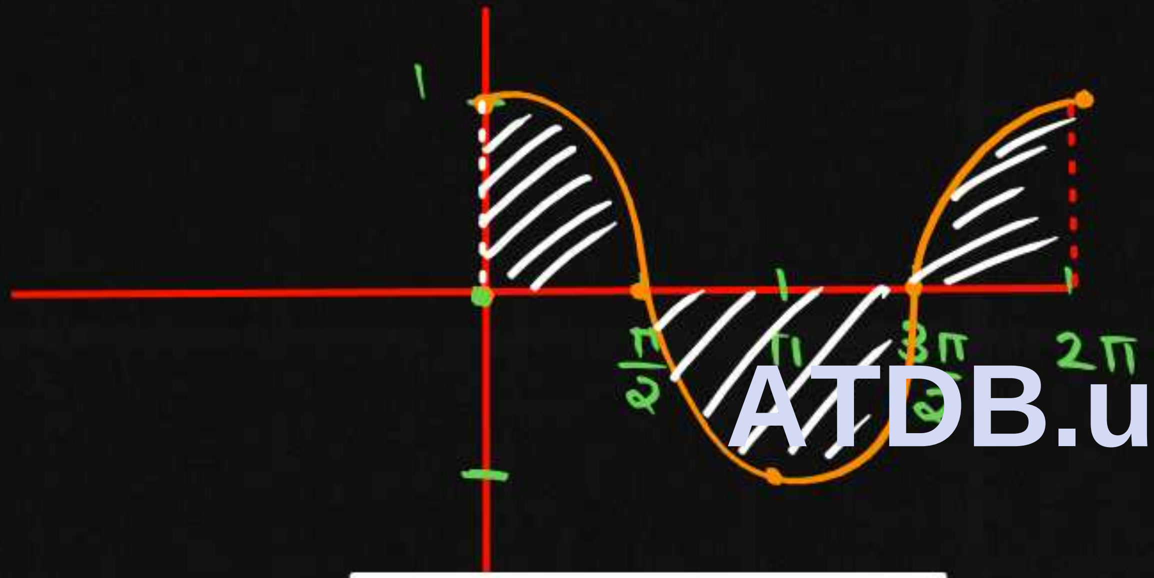
$A = \left[2\sin^{-1}\frac{2}{2}\right] = A = 2\sin^{-1}\sin\frac{\pi}{2}$ $\boxed{A = \pi}$



QUESTIONS

Find the area bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$.

inverse



$$\boxed{\text{Area} = 4\int_0^{\frac{\pi}{2}} y\,dx}$$

$$= 4\int_0^{\frac{\pi}{2}} \cos x\,dx$$

$$A = 4\left[\sin x\right]_0^{\frac{\pi}{2}}$$

$$A = 4\left[\sin\frac{\pi}{2}\right]$$

$$\boxed{A = 4\text{ sq unit}}$$

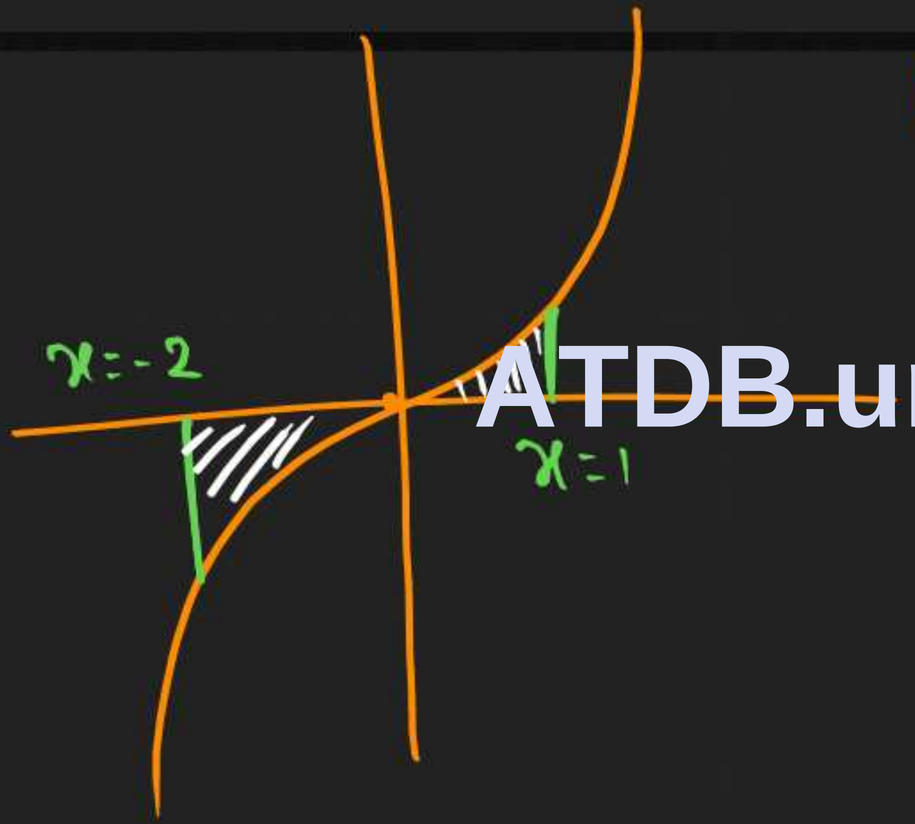
QUESTIONS

Area cannot be negative

-16

Area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2$ and $x = 1$ is

1. -9
2. $\frac{-15}{4}$
3. $\frac{15}{4}$
4. $\frac{17}{4}$



$\int_{-2}^{0} x^3\,dx \quad \int_{0}^{1} x^3\,dx$

$$\left[\frac{x^4}{4}\right]_{-2}^{0} + \left[\frac{x^4}{4}\right]_{0}^{1}$$

$$\left[0 - \frac{(-2)^4}{4}\right] + \left[\frac{1}{4}\right]$$

$$|-4| + \frac{1}{4}$$

$$4 + \frac{1}{4} = \frac{16+1}{4} = \boxed{\frac{17}{4}}$$

QUESTIONS

Find the area under the given curves and given lines :

(i) $y = x^2, x = 1, x = 2$ and x-axis

(ii) $y = x^4, x = 1, x = 5$ and x-axis

Trick

$A = \int_1^2 y\,dx$

$A = \int_1^2 x^2\,dx$

$A = \left[\frac{x^3}{3}\right]_1^2$

$A = \frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}}$

(ii) $A = \int_1^5 y\,dx$

$A = \int_1^5 x^4\,dx$

$A = \left[\frac{x^5}{5}\right]_1^5$

$\boxed{A = \frac{5^5}{5} - \frac{1}{5}} = \boxed{\frac{3124}{5}}$

Comment

QUESTIONS

→ Imp

Find the area of the region bounded by the line $y = 3x + 2$, the x-axis and the ordinates $x = -1$ and $x = 1$.

$$\boxed{y = 3x + 2}$$

x	0	1
y	2	5

$0 = 3x + 2$

$-2 = 3x$

$x = -\frac{2}{3}$

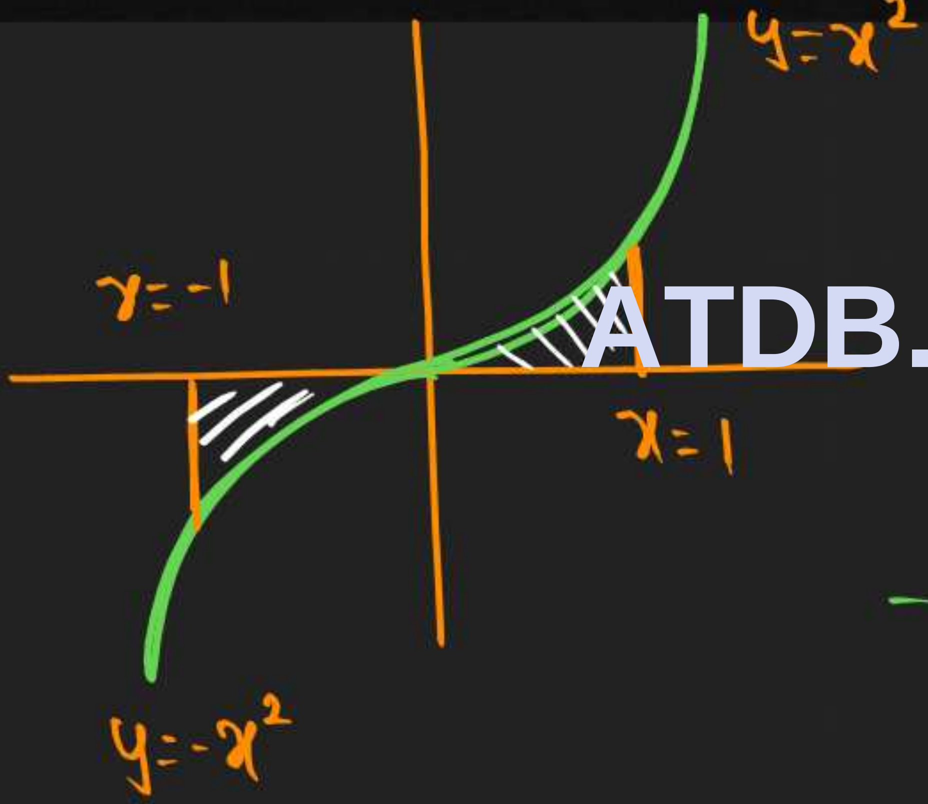
$$\int_{-1}^{-\frac{2}{3}} 3x+2\,dx + \int_{-\frac{2}{3}}^{1} 3x+2\,dx$$

$$\boxed{\left[\frac{3x^2}{2}+2x\right]_{-1}^{-2/3} + \left[\frac{3x^2}{2}+2x\right]_{-2/3}^{1} = ?} = \boxed{\frac{13}{3}}$$

QUESTIONS

The area bounded by the curve $y = x|x|$, x-axis and the ordinates $x = -1$ and $x = 1$ is given by

1. 0
2. $\frac{1}{3}$
3. $\frac{2}{3}$
4. $\frac{4}{3}$



$x<0$ $\quad$ $x>0$

$y=x(-x)$ $\quad$ $y=x\cdot x$

$y=-x^2$ $\quad$ $y=x^2$

$$-\int_{-1}^{0} -x^2\,dx + \int_{0}^{1} x^2\,dx$$

$$-\left[-\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^3}{3}\right]_{0}^{1}$$

$$-\left[-\left(-\frac{1}{3}\right)\right] + \frac{1}{3}$$

$$\left|-\frac{1}{3}\right| + \frac{1}{3} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

$\boxed{\text{Differential equations}}$

$\left(\frac{dy}{dx}\right)^{\text{whole no}}$

Equation having derivative

$$\boxed{\left(\frac{d^2y}{dx^2}\right)^1 + 4\frac{dy}{dx} + \sin x = 0}$$

$$\boxed{\sqrt{\frac{dy}{dx}} - y = 0}$$

Order = 1

$$\sqrt{\frac{dy}{dx}} = y$$

$$\frac{dy}{dx} = y^2$$

$$\boxed{\frac{dy}{dx} - y^2 = 0}$$

degree = 1

Degree X

$\sin\left(\frac{dy}{dx}\right)$ $e^{\frac{dy}{dx}}$

$\log\left(\frac{dy}{dx}\right)$ $\sqrt{\frac{dy}{dx}}$

(i) order → Highest order → 2

(ii) degree → power of highest order derivative.

↓

1

$\left(\frac{d^2y}{dx^2}\right)^2 + \sin x = 0$

Degree = 2

Order = 2

QUESTIONS

The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$

Degree = not

1. 3
2. 2
3. 1
4. Not defined

QUESTIONS

The order of the differential equation $\boxed{2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0}$ is

(1) $\boxed{2}$

(2) 1

(3) 0

(4) Not defined

Solution

- (i) Variable Seperable Method
- (ii) Homogenous D.E
- (iii) linear differentral equation

QUESTIONS

Find the general solution of the differential equation $\boxed{\frac{dy}{dx} = \frac{1+y^2}{1+x^2}}$.

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

$$\boxed{\int \frac{dy}{y^2+1} = \int \frac{dx}{x^2+1}}$$

$$\boxed{\tan^{-1} y = \tan^{-1} x + C}$$

$$\boxed{\tan^{-1} y - \tan^{-1} x = C}$$

General solution

Solution

- $\boxed{\text{General}}$ → Arbitary constant → Depends on order
- $\boxed{\text{Particular}}$ → no arbitory constant, # = 0

QUESTIONS

Find the equation of a curve passing through the point (-2, 3), given that the slope of the tangent of the curve at any point (x, y) is $\frac{2x}{y^2}$.

$x \quad y$

$$\boxed{\frac{dy}{dx} = \frac{2x}{y^2}}$$

$$y^2 dy = 2x\,dx$$

$$\int y^2 dy = 2\int x\,dx$$

$$\boxed{\frac{y^3}{3} = \cancel{2}\frac{x^2}{\cancel{2}} + C}$$

If $x = -2$ then $y = 3$

$$\frac{3^3}{3} = (-2)^2 + C$$

$$9 = 4 + C$$

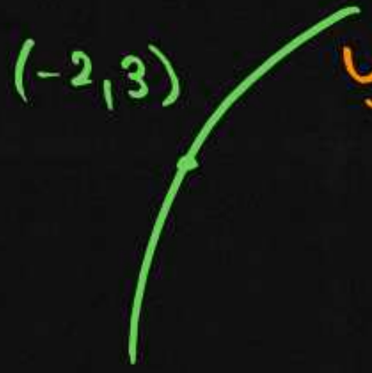
$$9 - 4 = C$$

$$\boxed{C = 5}$$

$$\frac{y^3}{3} = x^2 + 5$$

$$y^3 = 3x^2 + 15$$

$$\boxed{y = (3x^2 + 15)^{1/3}}$$



$y = \boxed{}$

QUESTIONS

Show that the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ is homogeneous and solve it.

$v = \frac{y}{x}$

$\boxed{x = \lambda x,\ y = \lambda y}$

Step 1 $\boxed{\frac{dy}{dx} = \frac{y\cos\left(\frac{y}{x}\right) + x}{x\cos\left(\frac{y}{x}\right)}}$

Step-2 putting $\boxed{y = vx}$

$\boxed{\frac{dy}{dx} = x\frac{dv}{dx} + v}$

$x\frac{dv}{dx} + v = \frac{vx\cos\left[\frac{v\cancel{x}}{\cancel{x}}\right] + x}{x\cos\left[\frac{v\cancel{x}}{\cancel{x}}\right]}$

$x \cdot \frac{dv}{dx} + v = \frac{\cancel{x}[v\cos v + 1]}{\cancel{x}\cos v}$

$x \cdot \frac{dv}{dx} + v = \frac{v\cos v + 1}{\cos v}$

$x\frac{dv}{dx} = \frac{v\cos v + 1}{\cos v} - \frac{v}{1}$

$x\frac{dv}{dx} = \frac{v\cancel{\cos v} + 1 - v\cancel{\cos v}}{\cos v}$

$\boxed{x\frac{dv}{dx} = \frac{1}{\cos v}}$

$dv\cos v = \frac{dx}{x}$

$\int \cos v\, dv = \int \frac{1}{x} dx$

$\boxed{\sin v = \log x + C}$

$\boxed{\sin\frac{y}{x} = \log x + C}$

QUESTIONS

Show that the differential equation $\boxed{2y\, e^{\frac{x}{y}}\, dx + \left(y - 2x\, e^{\frac{x}{y}}\right) dy = 0}$ is homogenous and find its solution, given that, $x = 0$ when $y = 1.$

$$2y e^{\frac{x}{y}} dx = -\left[y - 2x e^{\frac{x}{y}}\right] dy$$

$$\boxed{\frac{dx}{dy} = \frac{2x e^{\frac{x}{y}} - y}{2y e^{\frac{x}{y}}}}$$

$$x = vy \qquad v = \frac{x}{y}$$

$$\boxed{\frac{dx}{dy} = y \cdot \frac{dv}{dy} + v}$$

$$y \cdot \frac{dv}{dy} + v = \frac{2vy e^{\cancel{\frac{vy}{y}}} - y}{2y e^{\cancel{\frac{vy}{y}}}}$$

$$y \cdot \frac{dv}{dy} + v = \frac{\cancel{y}\left[2ve^{v} - 1\right]}{2\cancel{y} e^{v}}$$

$$y \cdot \frac{dv}{dy} + v = \frac{2ve^{v} - 1}{2e^{v}}$$

$$y \cdot \frac{dv}{dy} = \frac{2ve^{v} - 1}{2e^{v}} - \frac{v}{1}$$

$$y \cdot \frac{dv}{dy} = \frac{\cancel{2ve^{v}} - 1 - \cancel{2e^{v}v}}{2e^{v}}$$

$$\boxed{y \cdot \frac{dv}{dy} = \frac{-1}{2e^{v}}}$$

$$e^{v} dv = -\frac{1}{2y} dy$$

$$\int e^{v} dv = -\frac{1}{2} \int \frac{dy}{y}$$

$$\boxed{e^{v} = -\frac{1}{2} \log|y| + C}$$

$$e^{\frac{x}{y}} = -\frac{1}{2} \log|y| + C$$

$$e^{0} = -\frac{1}{2} \log|1| + C$$

$$1 = C \qquad \neq C = 2$$

$$\boxed{e^{x/y} = -\frac{1}{2} \log y + 1}$$

$$2e^{\frac{x}{y}} + \log y = 2$$

QUESTIONS

$m \log x = \log x^m$

Find the general solution of the differential equation $\boxed{ydx - (x + 2y^2)dy = 0.}$

$ydx = (x+2y^2)dy$

$\frac{dx}{dy} = \frac{x+2y^2}{y}$

$\frac{dx}{dy} = \frac{x}{y} + \frac{2y^2}{y}$

$\boxed{\frac{dx}{dy} - \frac{1}{y}x = 2y}$

$\# \boxed{\frac{dx}{dy} + P_1 x = Q_1}$

$\boxed{\frac{dx}{dy} - \frac{1}{y}x = 2y}$

$\boxed{P_1 = -\frac{1}{y}, \; Q_1 = 2y}$

$I.f = e^{\int P_1 dy}$

$= e^{\int -\frac{1}{y} dy}$

$\Rightarrow e^{-\int \frac{1}{y} dy}$

$= e^{-\log y}$

$= e^{\log y^{-1}} = e^{\log \frac{1}{y}} = \boxed{\frac{1}{y}}$

$G.S = x \times I.f = \int Q_1 \times I.f \, dy$

$x \times \frac{1}{y} = \int 2\cancel{y} \times \frac{1}{\cancel{y}} dy$

$\frac{x}{y} = \int 2 dy$

$\frac{x}{y} = 2\int dy$

$\boxed{\frac{x}{y} = 2y + C}$

QUESTIONS

Home work

Find the general solution of the differential equation $\boxed{x\frac{dy}{dx} + 2y = x^2 (x \neq 0)}$

QUESTIONS

Find the particular solution of the differential equation $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x\,(x \neq 0)$ given that $y = 0$ when $x = \frac{\pi}{2}$.

v.v.imp

$\sin x \times \frac{\cos x}{\sin x}$

$\boxed{\frac{dy}{dx} + y\cot x = 2x + x^2\cot x}$

$\frac{dy}{dx} + Py = Q$

$P = \cot x$ $\quad \boxed{Q = 2x + x^2\cot x}$

$I.f = e^{\int \cot x\, dx}$

$I.f = e^{\log \sin x}$

$\boxed{I.f = \sin x}$

G.S $\quad y \times I.f = \int I.f \times Q\, dx$

$y \times \sin x = \int \sin x\,[2x + x^2\cot x]\, dx$

$y\sin x = \int 2x\sin x + x^2\sin x\cot x\, dx$

$y\sin x = \int \underbrace{2x\sin x}_{I_1} + \int x^2\cos x\, dx \Rightarrow x^2\sin x - \cancel{\int x^2\cos x\, dx} + \cancel{\int x^2\cos x\, dx}$

$I_1 = \int \underbrace{2x}_{II}\,\underbrace{\sin x}_{I}\, dx$

$I_1 = \sin x \int 2x\, dx - \int \left[\frac{d}{dx}(\sin x) \int 2x\, dx\right] dx$

$I_1 = \sin x \times \cancel{2} \times \frac{x^2}{\cancel{2}} - \int \left[\cos x \times \cancel{2}\frac{x^2}{\cancel{2}}\right]$

$\boxed{y\sin x = x^2\sin x + C}$

$y\sin x = x^2\sin x - \frac{\pi^2}{4}$

$\boxed{C = -\frac{\pi^2}{4}}$

QUESTIONS

V.V.Imp

The Integrating Factor of the differential equation $\boxed{x\frac{dy}{dx} - y = 2x^2}$ is

1. e^{-x}
2. e^{-y}
3. $\boxed{\frac{1}{x}}$
4. x

$$\frac{dy}{dx} - \frac{y}{x} = \frac{2x^2}{x}$$

$$\boxed{\frac{dy}{dx} - \frac{1}{x}y = 2x}$$

$$I.f = e^{\int P\,dx}$$

$$= e^{\int -\frac{1}{x}dx}$$

$$= e^{-\int \frac{1}{x}dx}$$

$$= e^{-\log x}$$

$$e^{\log x^{-1}}$$

$$x^{-1} = \frac{1}{x}$$

QUESTIONS

1 2025

The Integrating factor of the differential equation

$(1-y^2)\frac{dx}{dy}+yx=ay\,(-1<y<1)$ is

1. $\frac{1}{y^2-1}$
2. $\frac{1}{\sqrt{y^2-1}}$
3. $\frac{1}{1-y^2}$
4. $\frac{1}{\sqrt{1-y^2}}$

$\frac{dx}{dy}+\frac{yx}{1-y^2}=\frac{ay}{1-y^2}$

$P_1=\frac{y}{1-y^2}$

$\boxed{I.F=e^{\int P_1 dy}}$

$e^{\log\frac{1}{\sqrt{1-y^2}}}$

$\boxed{\frac{1}{\sqrt{1-y^2}}}$

$\int P_1 dy=\int\frac{y}{1-y^2}dy$

let $1-y^2=t$

$-2y\,dy=dt$

$\boxed{y\,dy=-\frac{1}{2}dt}$

$\int P_1 dy=-\frac{1}{2}\int\frac{dt}{t}=\boxed{-\frac{1}{2}\log(1-y^2)}$

$\log(1-y^2)^{-\frac{1}{2}}$

$\boxed{\log\frac{1}{\sqrt{1-y^2}}}$

HOMEWORK

Thank You