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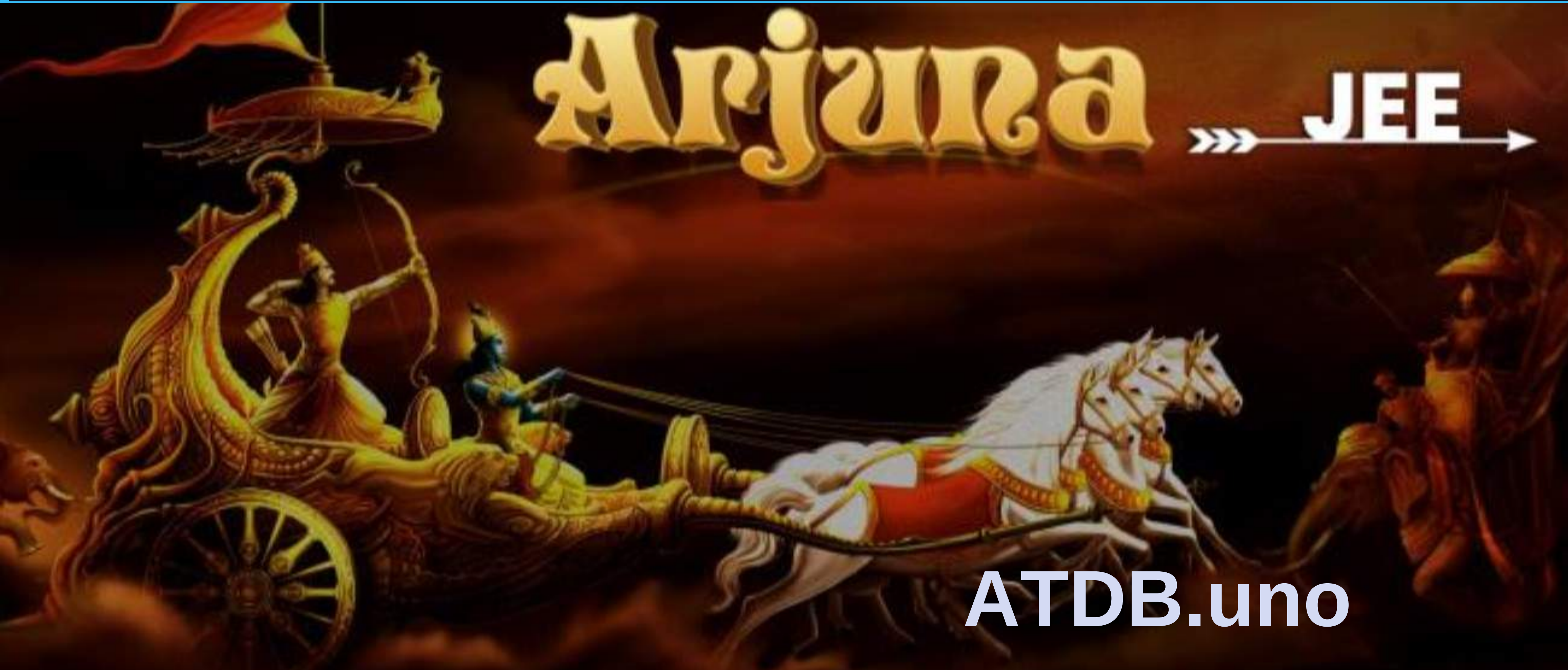
Arjuna JEE



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☐ MATHEMATICS

☐ Sequence & Series

One Shot

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Today's Targets

1. AP , GP , HP
2. AGP
3. Insertion of Means
4. Special Series
5. Series with Difference in AP/ GP
6. Telescoping Series
7. AM GM HM Inequality
8. PYQs

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Introduction

Sequence

1, 3, 5, 7, 9, - - ..

OR Progressions

→ 1) A.P.

2) G.P.

3) H.P.

4) A.G.P.

Series

1 + 3 + 5 + 7 + 9 + - . .

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ARITHMETIC PROGRESSION (A.P.)

$\underbrace{a}_{T_1}, \underbrace{a+d}_{T_2}, \underbrace{a+2d}_{T_3}, \underbrace{a+3d}_{T_4}, - - .$

General term of AP

✓ $\boxed{T_n = a + (n-1)\,d}$

Sum of n terms of A.P.

✓✓ $\boxed{S_n = \frac{n}{2}\left[2a + (n-1)d\right]} = \boxed{\frac{n}{2}[a + L]}$

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$T_{10} = a + 9d$

$T_{21} = a + 20d.$

$\Rightarrow T_n = a + (n-1)\,d$

$\underline{\underline{Ex}} \rightarrow \cdot 1, 3, 5, 7, 9, - - . \cdot \bigcirc\!\!\!\!\!\!101$

$101 = a + (n-1)\,d.$

$101 = 1 + (n-1)\,2.$ $\Rightarrow 2n = 102 \Rightarrow n = 51$

$101 = 1 + 2n - 2$

$S = \frac{n}{2}[I + L]$

$= \frac{51}{2}[102] = (51)^2$

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1, 3, 5, 7, 9, - - - - n terms

$a = 1,\ d = 2$

no. of terms = n

$$S = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2\times 1 + (n-1)\cdot 2\right]$$

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$$\frac{n}{2}\left[\cancel{2} + 2n - \cancel{2}\right]$$

$$\frac{n}{\cancel{2}}\cdot\cancel{2}n$$

$$= \boxed{n^2}$$

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Q JEE Main-2023 (Jan.-I)

$$\underbrace{a_1\ a_2\ a_3\ a_6}\ a_7\ \underbrace{a_8.\ \ \cdot\ a_{14}}$$

[Ans. 754] ✓

Let $a_1 = 8$, a_2, a_3, ... a_n be an A.P. If the sum of its first four terms is 50 and the sum of its last four terms is 170, then the product of its middle two terms is ....

$a = 8$ $S_4 = 50$

$$\frac{4}{2}\left[2a + (4-1)d\right] = 50$$

$$2(16 + 3d) = 50.$$

$$16 + 3d = 25$$

$$\boxed{d = 3}$$

$$a_n = a + (n-1)d$$

$$a_{n-1} = a + (n-2)d$$

$$a_{n-2} = a + (n-3)d$$

$$a_7 a_8 = (a+6d)(a+7d)$$

$$= (8+18)(8+21) = 26 \times 29$$

$$\underbrace{a_1, a_2, a_3, a_4}, - \underbrace{a_{n-3}, a_{n-2}, a_{n-1}, a_n}$$

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$$170 = 4a + d\left[n-1+n-2+n-3+n-4\right]$$

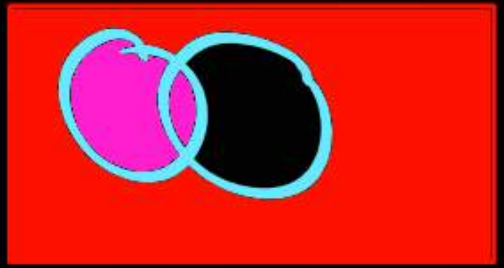
$$170 = 32 + 3\left[4n - 10\right]$$

$$138 = 3\left[4n-10\right]$$

$$46 = 4n - 10$$

$$56 = 4n$$

$$\boxed{n = 14}$$



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Q JEE Main-2023 (Feb.-I)

$a = 11, d = 20$

[Ans. 151]

The 8th common term of the series

$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots,$

$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots$

is ....

$d_1 = 4$

$d_2 = 5$

Common diff of common AP is LCM of d_1 & d_2

$T_8 = a + 7d$

$= 11 + 7 \times 20$

$= 11 + 140$

$= 151$

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Q **JEE Main-2022**

[Ans. 2223]

Let 3, 6, 9, 12, ... upto 78 terms and 5, 9, 13, 17, ... upto 59 terms be two series. Then, the sum of terms common to both the series is equal to __________.

$a_1 = 3$

$d_1 = 3$

$T_{78} = 3 + (78-1).3.$

$= \cancel{3} + 78 \times 3 - \cancel{3}$

$T_{78} = 234$

$3, 6, 9, 12, - - . \boxed{234}$

$a_2 = 5$

$d_2 = 4$

$L = 5 + 58 \times 4.$

$= 5 + 232$

$= 237$

$n = 19, \quad a = 9, \quad d = 12$

$S = \frac{19}{\cancel{2}}\left[\cancel{2} \times 9 + \overset{9}{\cancel{18}} \times 12\right]$

$S = 19\,[9 + 108]$

$S = 19 \times 117 = 2223$

Common AP

$d = 12$

$a = 9$

$L \le 234$

$a + (n-1)d \le 234$

$9 + (n-1)12 \le 234$

$12n - 3 \le 234.$

$12n \le 237$

$n \le \frac{79}{4}$

$n_{max} = 19$

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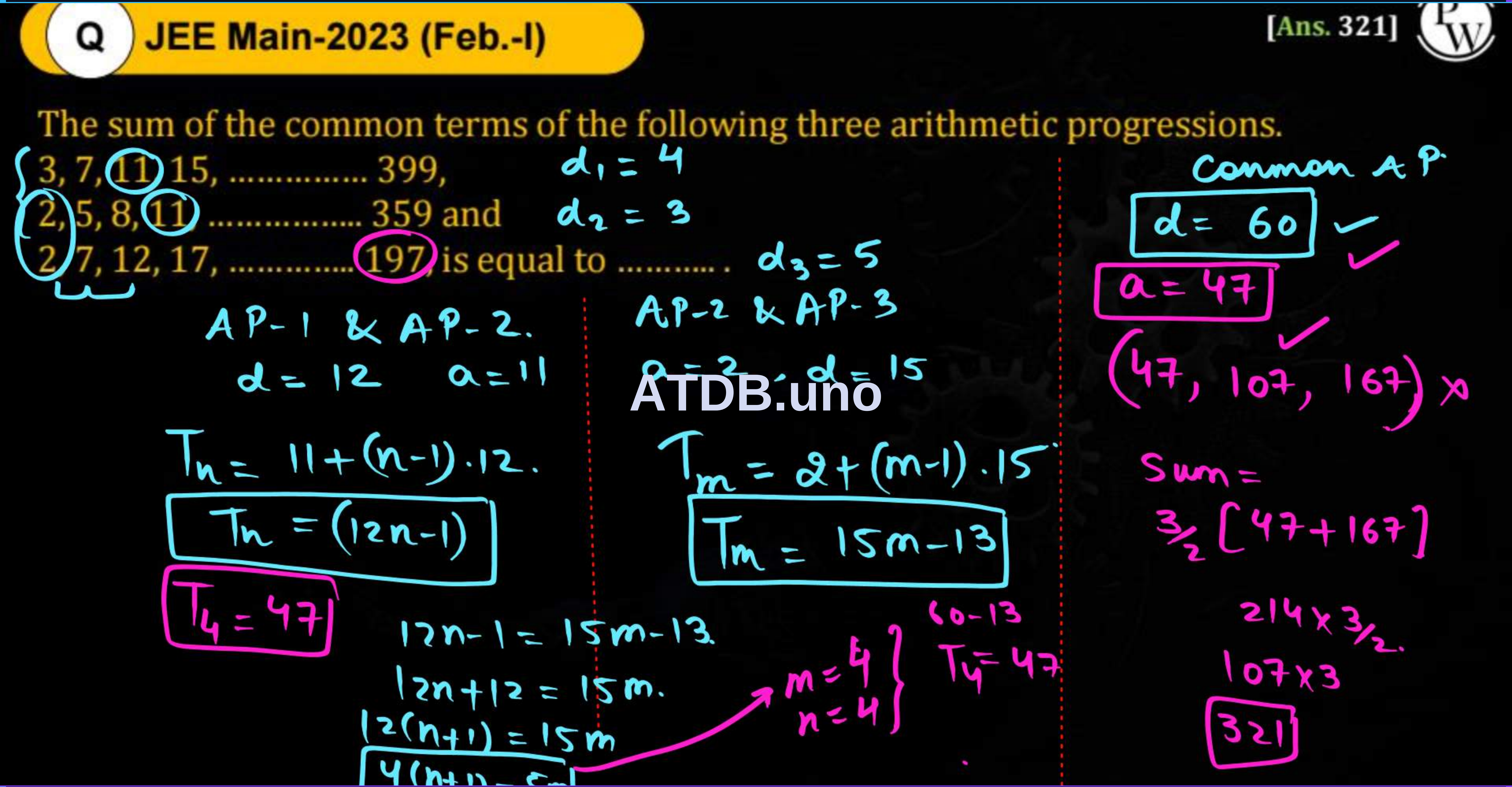

Q JEE Main-2023 (Feb.-I)

[Ans. 321]

The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, ... 399,
2, 5, 8, 11 .... 359 and
2, 7, 12, 17, ... 197, is equal to ....

$d_1 = 4$
$d_2 = 3$
$d_3 = 5$

AP-1 & AP-2.

$d = 12 \quad a = 11$

$T_n = 11 + (n-1)\cdot 12.$

$\boxed{T_n = (12n-1)}$

$\boxed{T_4 = 47}$

$12n - 1 = 15m - 13.$

$12n + 12 = 15m.$

$12(n+1) = 15m$

$\boxed{4(n+1) = 5m}$

AP-2 & AP-3

$a = 2, \ d = 15$

$T_m = 2 + (m-1)\cdot 15$

$\boxed{T_m = 15m - 13}$

$\left.\begin{matrix} m = 4 \\ n = 4 \end{matrix}\right\}$ 60-13 $T_4 = 47$

Common A.P.

$\boxed{d = 60}$ ✓

$\boxed{a = 47}$ ✓

(47, 107, 167) ✓ ×

Sum =

$\frac{3}{2}[47 + 167]$

$214 \times 3/2.$

107×3

$\boxed{321}$

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Properties of an A.P.

1, 4, 7, 10, 13, - ..

-1, 2, 5, 8, 11, - - ..

(i) If a fixed number is added (or subtracted) to each term of a given A.P. then the resulting sequence is also an A.P. with the same common difference as that of the given A.P.

2, 8, 14, 20, 26

$d = 6$

(ii) If each term of an A.P. is multiplied by a fixed number (say k) (or divided by a non-zero fixed number), the resulting sequence is also an A.P. with the common difference multiplied by k. $(k \neq 0)$

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(iii) In an A.P. the sum of terms equidistant from the beginning and end is constant and equal to sum of first and last terms.

1, 4, 7, 10, 13, 16, 19

For example, If a_1, a_2, a_3, ..., an are in A.P.,

then $\underbrace{a_1 + a_n} = \underbrace{a_2 + a_{n-1}} = \underbrace{a_3 + a_{n-2}} =$ .... and so on.

1, 4, 7, 10, 13, 16, 19, 22

✓

(iv) If a, b, c are three numbers in A.P. then $\boxed{2b = a + c}$.

$a, b, c \rightarrow AP \quad b - a = c - b$

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Q JEE Main-2019

If $a_1, a_2, a_3, ..., a_n$ are in A.P and $\overset{x}{a_1 + a_4} + a_7 + ... + a_{16} = 114$, then $\overbrace{a_1 + \underbrace{a_6 + a_{11}}_{x} + a_{16}}^{x}$ is equal to:

$= 2x$

$= 2\times38$

$= 76$

$$\underbrace{a_1 + \overbrace{a_4 + \underbrace{a_7 + a_{10}}_{} + a_{13}}^{x} + a_{16}}_{x} = 114$$

$3x = 114.$

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$a_1 + a_{16} = \frac{114}{3}$

$a_1 + a_{16} = 38.$

$\boxed{M\text{-}2}$

$a_1 = a \quad , 3d.$

$n = 6$

$\frac{6}{2}\left[2a + 5\times3d\right] = 114$

$2a + 15d = 38$

$a \quad , 5d$

$n = 4$

$\frac{4}{2}\left[2a + 3\times5d\right]$

$2\left[2a + 15d\right]$

$2\times38 = 76$

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Important Note

If sum of numbers in A.P. is given then always assume the numbers to be-

3 numbers in A.P. : $a - d, a, a + d$

4 numbers in A.P. : $a - 3d, a - d, a + d, a + 3d$

5 numbers in A.P. : $a - 2d, a - d, a, a + d, a + 2d$

6 numbers in A.P. : $a - 5d, a - 3d, a - d, a + d, a + 3d\ a + 5d$

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Q JEE ADV 2022

[Ans. 18900]

Let $l_1, l_2, \ldots, l_{100}$ be consecutive terms of an arithmetic progression with common difference d_1 and let $w_1, w_2, \ldots, w_{100}$ be consecutive terms of another arithmetic progression with common difference d_2, where $d_1d_2 = 10$. For each $i = 1, 2, \ldots, 100$, let R_i be a rectangle with length l_i, width w_i and area A_i. If $A_{51} - A_{50} = 1000$, then the value of $A_{100} - A_{90}$ is ________.

$$\boxed{d_1 d_2 = 10}$$

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$$A_{51} = l_{51} w_{51} = (l_1 + 50d_1)(w_1 + 50d_2) = \cancel{l_1 w_1} + 50 l_1 d_2 + 50 d_1 w_1 + (50)^2 d_1 d_2$$

$$A_{50} = l_{50} w_{50} = (l_1 + 49d_1)(w_1 + 49d_2) = \cancel{l_1 w_1} + 49 l_1 d_2 + 49 d_1 w_1 + (49)^2 d_1 d_2$$

$$1000 = l_1 d_2 + d_1 w_1 + \underbrace{d_1 d_2}_{} [50^2 - 49^2]$$

$$1000 = l_1 d_2 + d_1 w_1 + 990 \qquad 10 \times 99$$

$$10 = l_1 d_2 + d_1$$

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$$A_{100} = l_{100} w_{100} = (l_1 + 99d_1)(w_1 + 99d_2)$$

$$A_{90} = l_{90} w_{90} = (l_1 + 89d_1)(w_1 + 89d_2)$$

$$A_{100} - A_{90} = 10\underline{l_1 d_2 + 10 d_1} w_1 + [(99)^2 - (89)^2]\underline{d_1 d_2}$$

$$= 10 \times 10 + (99+89) \times 10 \cdot \times 10$$

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$$= 100 + 100 \times [188]$$

$$= 100[1 + 188]$$

$$= \underline{18900}$$

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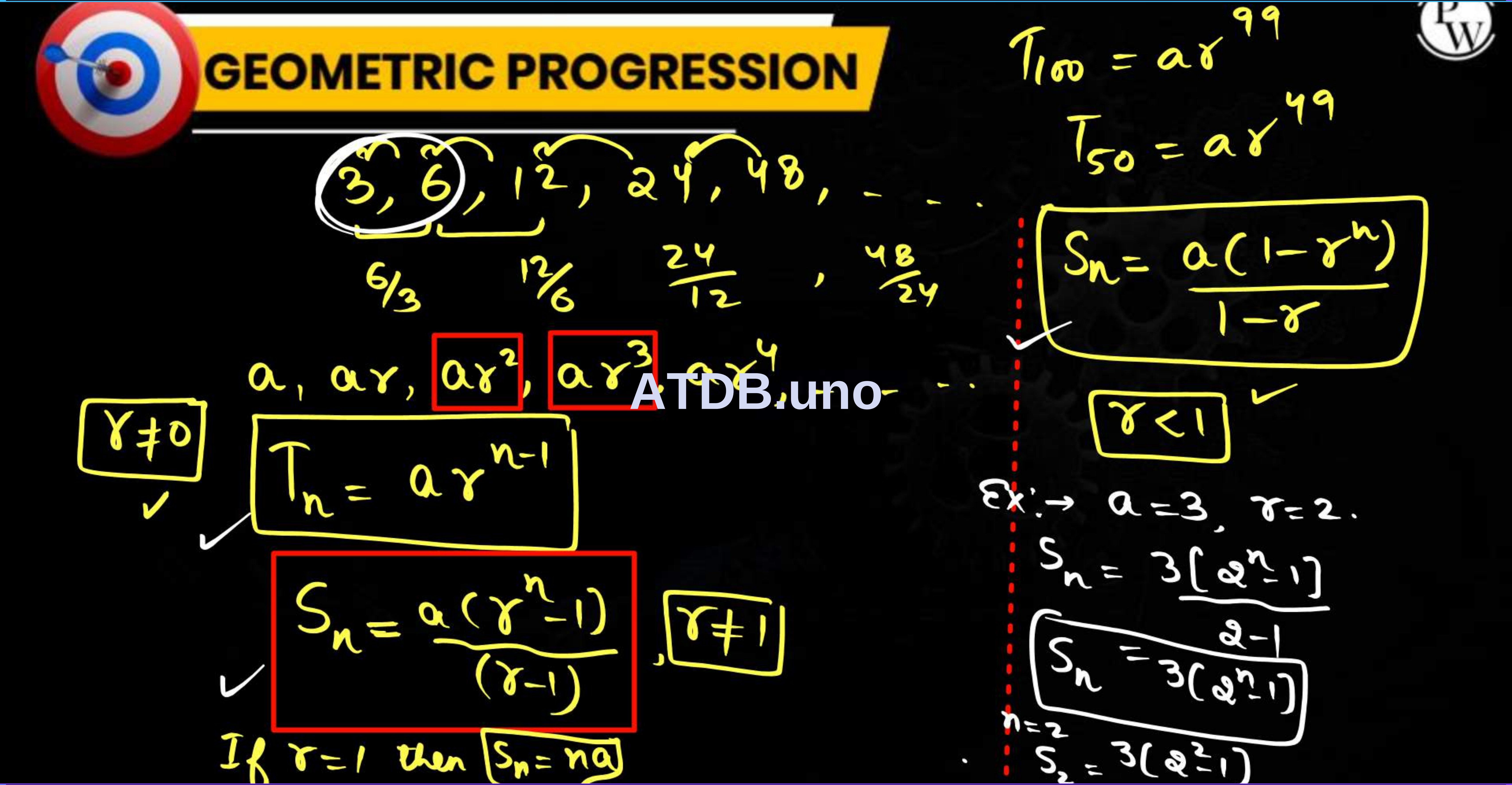

GEOMETRIC PROGRESSION

$3, 6, 12, 24, 48, \ldots$

$6/3 \quad 12/6 \quad \frac{24}{12}, \quad \frac{48}{24}$

$a, ar, \boxed{ar^2}, \boxed{ar^3}, ar^4 \ldots$

$\boxed{r \neq 0}$ ✓

✓ $\boxed{T_n = ar^{n-1}}$

✓ $\boxed{S_n = \frac{a(r^n - 1)}{(r-1)}}$, $\boxed{r \neq 1}$

If $r=1$ then $\boxed{S_n = na}$

$T_{100} = ar^{99}$

$T_{50} = ar^{49}$

✓ $\boxed{S_n = \frac{a(1-r^n)}{1-r}}$

$\boxed{r<1}$ ✓

Ex:→ $a=3, r=2.$

$S_n = \frac{3[2^n - 1]}{2-1}$

$\boxed{S_n = 3(2^n - 1)}$

$n=2$

$S_2 = 3(2^2 - 1)$

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Q JEE Main 2023

[Ans. D]

$\boxed{a, r > 0}$

Let $a_1, a_2, a_3, \ldots\ldots$ be a G.P. of increasing positive numbers. Let the sum of its 6th and 8th terms be 2 and the product of its 3rd and 5th terms be $\frac{1}{9}$. Then $6(a_2 + a_4)(a_4 + a_6)$ is equal to

(A) $2\sqrt{2}$

(B) $3\sqrt{3}$

(C) 2

(D) 3 ✓

$T_6 + T_8 = 2$

$ar^5 + ar^7 = 2$

$\underbrace{ar^5}[1+r^2] = 2 \sim ①$

$\boxed{ar^3} r^2[1+r^2] = 2$

$r^2(1+r^2) = 6.$

$\boxed{r^2 = z}$

$z(1+z) = 6.$

$z^2 + z - 6 = 0$

$(z+3)(z$

$T_3 \cdot T_5 = 1/9$

$ar^2 \cdot ar^4 = 1/9.$

$a^2r^6 = \frac{1}{9} \Rightarrow (ar^3)^2 = \frac{1}{9}$

$\boxed{ar^3 = +\frac{1}{3}} \rightarrow ②$

$a \times 2\sqrt{2} = \frac{1}{3}$

$z = 2 \Rightarrow r^2 = 2 \Rightarrow \boxed{r = \sqrt{2}}$

$\boxed{a = \frac{1}{6\sqrt{2}}}$

$a^2 = \frac{1}{72}$

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$$6(a_2+a_4)(a_4+a_6).$$

$$6(ar+ar^3)(ar^3+ar^5)$$

$$6\,\underbrace{ar(1+r^2]\,ar^3}(1+r^2)$$

$$6\,a^2r^4(1+r^2)^2$$

$$6\times\frac{1}{72}\times(2)^2\cancel{(3)}^{\cancel{2}}.$$

$$\frac{6\times(2)^2}{8}=③$$

For any sequence:

$$\boxed{S_n-S_{n-1}=T_n}$$

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Question

The sum of 10 terms of the series .7 + .77 + .777 + ... is

A $\frac{7}{9}\left(89+\frac{1}{10^{10}}\right)$

B $\frac{7}{81}\left(89+\frac{1}{10^{10}}\right)$ ✓

C $\frac{7}{9}\left(89+\frac{1}{10^{9}}\right)$

D None of these

$$S = 0.7 + 0.77 + 0.777 + \text{------}$$

$$S = 7\,[0.1 + 0.11 + 0.111 + \text{----}]$$

$$S = \frac{7}{9}\,[0.9 + \underbrace{0.99} + 0.\underbrace{999} + \text{----}]$$

$$S = \frac{7}{9}\left[1-(0.1)^1 + 1-(0.1)^2 + 1-(0.1)^3 + \text{----} + 1-(0.1)^{10}\right]$$

$$= \frac{7}{9}\left[10 - \boxed{\{(0.1)^1 + (0.1)^2 + (0.1)^3 + \text{---}\,(0.1)^{10}\}}\right]$$

$$= \frac{7}{9}\left[10 - 0.1\,\frac{[1-(0.1)^{10}]}{1-0.1}\right]$$

$$\left.\begin{array}{l} a = 0.1 \\ r = 0.1 \\ n = 10 \end{array}\right\}$$

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$$= \frac{7}{9}\left[10 - \frac{1}{9}\left(1-(0.1)^{10}\right)\right]$$

$$= \frac{7}{9}\;\frac{90 - 1 + (0.1)^{10}.}{9}$$

$$= \frac{7}{81}.\left(89 + (0.1)^{10}\right)$$

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Q JEE Main-2023 (Jan.-I)

$S_6 = T_1 + T_2 + T_3 + \ldots + T_6$ [Ans. 12]

$S_5 = T_1 + T_2 \ldots + T_5$

The 4th term of GP is 500 and its common ratio is $\frac{1}{m}$, $\underbrace{m \in N}$. Let S_n denote the sum of the first n terms of this GP. If $\underbrace{S_6 > S_5 + 1}$ and $\underbrace{S_7 < S_6} + \frac{1}{2}$, then the number of possible values of m is

$\boxed{ar^3 = 500}$ $r = 1/m$, $m \in N$

$S_6 > S_5 + 1$

$\boxed{S_6 - S_5 > 1}$

$\Rightarrow \boxed{T_6 > 1}$

$\Rightarrow ar^5 > 1$

$ar^3 r^2 > 1$

$r^2 > \frac{1}{\ldots}$

$\frac{1}{m^2} > \frac{1}{500}$

$\Rightarrow \boxed{m^2 < 500} \rightarrow ①$

$\boxed{m^3 > 1000}$

$\Rightarrow \boxed{m > 10} \rightarrow ②$

$m = 11, 12, 13, \ldots\ldots 22$

$S_7 - S_6 < 1/2$

$T_7 < 1/2$

$ar^6 < 1/2$

$ar^3 r^3 < 1/2$

$500 r^3 < 1/2$

$\boxed{r^3 < \frac{1}{1000}}$

$\frac{1}{m^3} < \frac{1}{1000}$

22
22
44
44
484

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Important Note

If product of some numbers in G.P. is given then also assume the numbers to be:

3 numbers in G.P. : $\boxed{a/r, a, ar}$

4 numbers in G.P. : $\underbrace{a/r^3, a/r, ar, ar^3}$

5 numbers in G.P. : $a/r^2, a/r, a, ar, ar^2$

6 numbers in G.P. : $a/r^5, a/r^3, a/r, ar, ar^3, ar^5$

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INFINITE GP

$\boxed{r=-2}$

$\{3,-6,12,-24,48,\dots\}$

when no of terms (n) → ∞

$r=2 \rightarrow \{3,6,12,24,48,\dots\} \rightarrow S_\infty \rightarrow \infty$

$\rightarrow r=\frac{1}{2} \rightarrow \boxed{\{3, 3/2, 3/4, 3/8, 3/16, \dots\}}$

$\boxed{T_\infty \rightarrow 0}$

$\boxed{S_\infty \rightarrow \text{Finite}}$

$S_\infty=\frac{a}{1-r}$

$=\frac{3}{1-1/2}$

$=\frac{3}{1/2}=6$

For Infinite GP to be defined $\boxed{-1<r<1}$

$\boxed{r\neq 0}$

$S_n \rightarrow$ sum of n terms

$S_\infty \rightarrow$ sum of Infinite terms

$$\boxed{S_\infty = \frac{a}{1-r}}$$

$$S_n=\frac{a(1-r^n)}{1-r}$$

$n\rightarrow\infty$

$$S_\infty=\frac{a(1-\cancel{r^\infty}\,0)}{1-r}$$

$r^\infty \rightarrow (1/2)^\infty \rightarrow 0$

$$S_\infty=\frac{a(1-0)}{1-r}$$

$(-\frac{1}{2})^\infty \rightarrow 0$

$$\boxed{S_\infty=\frac{a}{1-r}}$$ ✓

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INFINITE GP

Note 1 The sum of Infinite GP is finite only when $\boxed{|r| < 1 \text{ or } -1 < r < 1}$ and is given by $\boxed{S_\infty = \frac{a}{1-r}}$

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Question

A ball falls from a height of 100 mts. on a floor. If in each rebound, it describes $(4/5)^{th}$ height of the previous falling height, then the total distance travelled by the ball before it comes to rest is ?

✓

$$d = 100 + 2\cdot(4/5)100 + 2(4/5)^2 100 + 2(4/5)^3\cdot 100 + \dots\infty$$

$$d = \boxed{\left(\underbrace{2\cdot\times 100} + 2(4/5)100 + 2(4/5)^2 100 + \dots\infty\right)} - 100$$

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$a = 200$

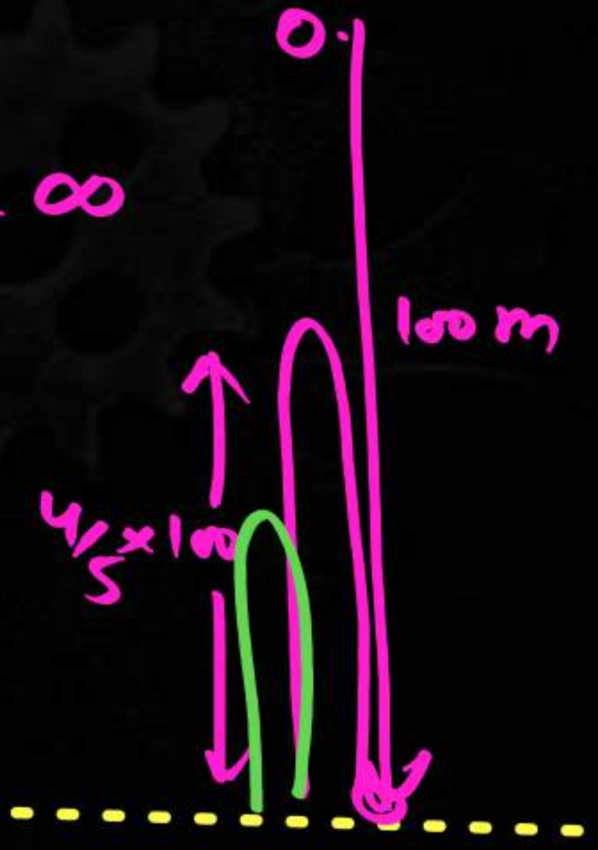
$r = 4/5$

$$d = \frac{a}{1-r}$$

$$= \frac{200}{1-4/5} = \frac{200}{1/5} = 1000$$

$$d = 1000 - 100$$

$$= 900\,m$$



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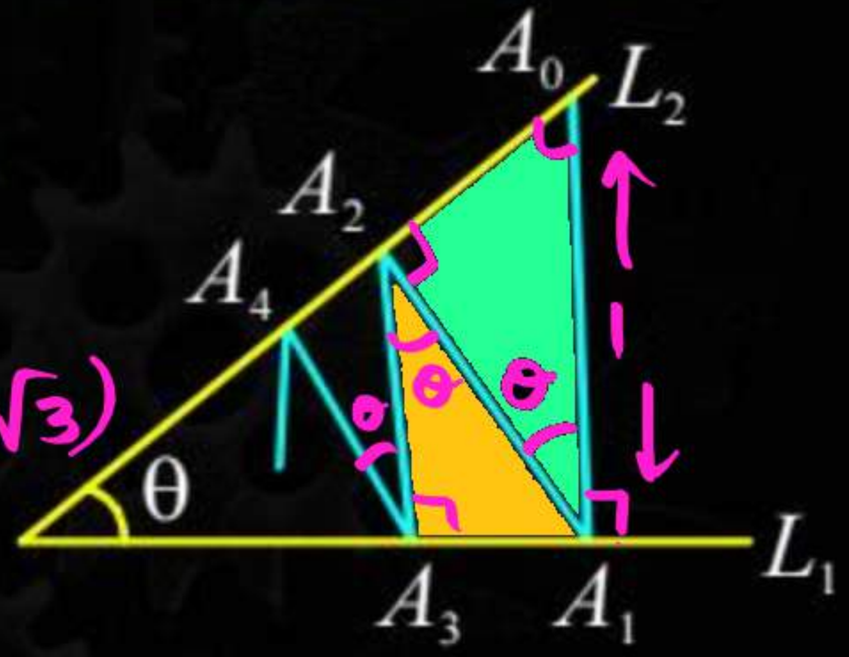
Question

In the figure, $A_0 A_1, A_2 A_3, A_4 A_5$, ... are all perpendicular to L_1; $A_1 A_2, A_3 A_4, A_5 A_6$, ... are all perpendicular to L_2.

If $\boxed{A_0 A_1 = 1}$ and $\boxed{A_0 A_1 + A_1 A_2 + A_2 A_3 + \ldots \infty = 2(2+\sqrt{3})}$, find θ.



$$1 + \cos\theta + (\cos^2\theta) + \cos^3\theta + \ldots \infty = 2(2+\sqrt{3})$$

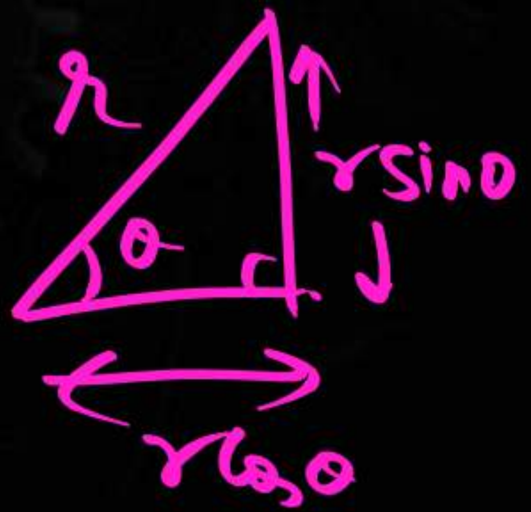
$$a = 1, \quad r = \cos\theta$$

$$\frac{1}{1-\cos\theta} = \frac{2(2+\sqrt{3})(2-\sqrt{3})}{2-\sqrt{3}}$$

$$\frac{1}{1-\cos\theta} = \frac{2}{2-\sqrt{3}}$$

$$1-\cos\theta = \frac{2-\sqrt{3}}{2}$$

$$\cancel{1} - \cos\theta = \cancel{1} - \frac{\sqrt{3}}{2}$$



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[Ans. 9]

Let $\{a_k\}$ and $\{b_k\}$, $k \in \mathbb{N}$, be two G.P.s with common ratio $\boxed{r_1 \text{ and } r_2}$ respectively such that $\boxed{a_1 = b_1 = 4}$ and $\boxed{r_1 < r_2}$. Let $\boxed{c_k = a_k + b_k}$, $k \in \mathbb{N}$. If $\underbrace{c_2}$ $= 5$ and $c_3 = \frac{13}{4}$ then $\underbrace{\sum_{k=1}^{\infty} c_k}_{24} - (\underbrace{12a_6 + 8b_4}_{15})$ is equal to $24 - 15 = 9$

$a_1 = 4 \qquad b_1 = 4$

$a_2 = 4r_1 \qquad b_2 = 4r_2$

$a_3 = 4r_1^2 \qquad b_3 = 4r_2^2$

$c_1 = a_1 + b_1$

$c_2 = a_2 + b_2$

$5 = 4r_1 + 4r_2$

$\Rightarrow \boxed{r_1 + r_2 = 5/4} \rightarrow ①$

$c_3 = a_3 + b_3$

$\frac{13}{4} = 4r_1^2 + 4r_2^2$

$\Rightarrow \boxed{\frac{13}{16} = r_1^2 + r_2^2} \rightarrow ②$

$\frac{13}{16} = (r_1 + r_2)^2 - 2r_1 r_2$

$c_1 + c_2 + c_3 + \dots \infty$

$(a_1 + a_2 + a_3 \dots \infty) + (b_1 + b_2 + b_3 \dots \infty)$

$\frac{a}{1-r_1} + \frac{a}{1-r_2} = \frac{4}{1-\frac{1}{2}} + \frac{4}{1-3/4} = 8 + 16 = 24$

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$$\frac{13}{16} = (5/4)^2 - 2r_1 r_2$$

$$2r_1 r_2 = \frac{25}{16} - \frac{13}{16}.$$

$$2r_1 r_2 = \frac{12}{16}$$

$$2r_1 r_2 = 3/4$$

$$\boxed{r_1 r_2 = 3/8}$$

Formation of Quad

$$x^2 - 5/4\, x + 3/8 = 0$$

$$8x^2 - 10x + 3 = 0$$

$$8x^2 - 6x - 4x + 3 = 0$$

$$(2x-1)(4x-3) = 0.$$

$$x = 1/2 \text{ or } 3/4$$

$r_1 = 1/2$ $r_2 = 3/4$

$$12a_6 + 8b_4$$

$$12ar_1^5 + 8ar_2^3$$

$$12\times 4\times\left(\frac{1}{2}\right)^5 + 8\times 4\times(3/4)^3.$$

$$\frac{3}{2} + \frac{\cancel{32}\times 27}{\cancel{64}\ 2}$$

$$3/2 + \frac{27}{2} = ⑮$$

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HARMONIC PROGRESSION (H.P.)

$1, 2, 3 = 6$

$1 + \frac{1}{2} + \frac{1}{3}$

A sequence whose reciprocals form an AP is called **HP**.

Ex → $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, - - -$

Ex → $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \frac{1}{a+3d}, - - - \frac{1}{a+(n-1)d}$

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$$a_n = \frac{1}{a+(n-1)d}$$

$S_n \to$ no formula

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HARMONIC PROGRESSION (H.P.)

Note 1

(i) The general form of a harmonic progression is

$$\boxed{\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots \frac{1}{a+(n-1)d}}$$

(ii) No term of any H.P. can be zero.

(iii) If a, b, c are in HP, then $b = \frac{2ac}{a+c}$ ✓

Note 2

Note that we do not have general formula for the sum of the n terms of an HP.

Proof →

$a, b, c \rightarrow HP$

$\frac{1}{a}, \frac{1}{b}, \frac{1}{c} \rightarrow A.P.$

$$\frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

$$\frac{2}{b} = \frac{a+c}{ac}$$

$$\boxed{b = \frac{2ac}{a+c}}$$

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Question

$\frac{c}{b} = \frac{d}{c}$

If $a, b, c \to$ AP, $b, c, d \to$ GP and $c, d, e \to$ HP. Show that $a, c, e \to$ GP.

$a, \overbrace{b, c, d}, e$

$\underbrace{a, b, c}_{AP}, \underbrace{d, e}_{HP}$

$\boxed{2b = a + c}$ ~ ①

$\boxed{c^2 = bd}$ ~→ ②

$d = \frac{2ce}{c+e}$ ~→ ③

To Prove → $\boxed{c^2 = ae}$

✓

$c^2 = bd$

$c^2 = \frac{a+c}{\cancel{2}} \cdot \frac{\cancel{2}ce}{c+e}$

$c = \frac{(a+c)e}{c+e}$

$c(c+e) = ae + ce$

$c^2 + \cancel{ce} = ae + \cancel{ce}$

$\boxed{c^2 = ae}$

$\frac{c}{a} = \frac{e}{c} \Rightarrow a, c, e \to GP$

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ARITHMETIC GEOMETRIC PROGRESSION (AGP)

A.G.P →

Ex → $a, (a+d)r, (a+2d)r^2, (a+3d)r^3, \ldots$

$$\boxed{T_n = [a+(n-1)d]r^{n-1}}$$

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ARITHMETIC GEOMETRIC PROGRESSION (AGP)

$$T_n = [a + (n - 1)d]r^{n-1}$$

$$S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]}{1-r}.r^n$$

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Note 1

Steps to find S_n for A.G.P

Step I : Write the given series upto T_n.

Step II : Multiply the whole series with common ratio (r) and write the series again by shifting on term to right.

Step III : Subtract the two equations to obtain S_n term.

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[Ans. B]

The sum $1 + 2.3 + 3.3^2 + 4.3^3 + 5.3^4 + \ldots\ldots\ldots + 10.3^9$

(A) $\dfrac{2.3^{12} + 10}{4}$

(B) $\dfrac{19.3^{10} + 1}{4}$ ✓

(C) $5.3^{10} - 2$

(D) $\dfrac{9.3^{10} + 1}{2}$

$$S_{10} = 1 + 2.3 + 3.3^2 + 4.3^3 + --- + 10.3^9$$

$$3S_{10} = 1.3 + 2.3^2 + 3.3^3 + ---- + 10.3^{10}$$

$$S_{10} - 3S_{10} = \boxed{1.3^0 + 1.3^1 + 1.3^2 + 1.3^3 + - - .1.3^9} - 10.3^{10}$$

$\boxed{n=10}$, $a = 1$, $r = 3$

$$-2S_{10} = 1\cdot\frac{[3^{10}-1]}{3-1} - 10.3^{10} \qquad \frac{a(r^n-1)}{r-1}$$

$$-2S_{10} = \frac{3^{10}-1}{2} - 10.3^{10}$$

$$S_{10} = \frac{10.3^{10}}{2} - \frac{(3^{10}-1)}{4}$$

$$\frac{2\times10.3^{10} - 3^{10} + 1}{4}$$

$$= 19.3^{10} + 1 \quad 4$$

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Question

$$1-\frac{4}{2}+\frac{7}{2^2}-\frac{10}{2^3}+\frac{13}{2^4}........\infty$$

$1, 4, 7, 10, 13, - - .$

$r = -\frac{1}{2}$

$$S = 1 - \frac{4}{2} + \frac{7}{2^2} - \frac{10}{2^3} + \frac{13}{2^4} - - - - - \infty$$

$$\left(-\frac{1}{2}\right)S = \downarrow \quad -\frac{1}{2} + \frac{4}{2^2} - \frac{7}{2^3} + \frac{10}{2^4} + - - - - \infty$$

$+ \quad + \quad + \quad -$

$$S + \frac{S}{2} = \textcircled{1} - \frac{3}{2} + \frac{3}{2^2} - \frac{3}{2^3} + \frac{3}{2^4} - - - - \infty$$

$$\frac{3S}{2} = 1 + \boxed{\left[-\frac{3}{2} + \frac{3}{2^2} - \frac{3}{2^3} + \frac{3}{2^4} - - - \infty\right]}$$

$$\frac{3S}{2} = \cancel{1} + (\cancel{-1})$$

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{-3/2}{1-(-1/2)}$$

$$= \frac{-3/2}{3/2}$$

$$= \textcircled{-1}$$

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Q JEE Main 2023

[Ans. 2175]

Let $\boxed{S = 109 + \frac{108}{5} + \frac{107}{5^2} + \ldots + \frac{2}{5^{107}} + \frac{1}{5^{108}}}$. Then the value of $\underbrace{(16S - (25)^{-54})}$ is equal to ______.

Ans 2175 ✓

$$S = 109 + \frac{108}{5^1} + \frac{107}{5^2} + \text{-----} + \frac{1}{5^{108}}$$

$$S/5 = \frac{109}{5^1} + \frac{108}{5^2} + \frac{107}{5^3} + \text{----} + \frac{1}{5^{109}}$$

$-$

$$\frac{4S}{5} = 109 + \boxed{\frac{(-1)}{5^1} + \frac{(-1)}{5^2} + \frac{(-1)}{5^3} + \cdots \frac{(-1)}{5^{108}}} - \frac{1}{5^{109}}$$

$$\frac{a(1-r^n)}{1-r} \quad n = 108, \; a = -1/5, \; r = 1/5$$

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$$\frac{4S}{5} = 109 + \frac{-\frac{1}{5}\left[1-\left(\frac{1}{5}\right)\right]}{\left(1-\frac{1}{5}\right)} - \frac{1}{5^{109}}$$

$$\frac{4S}{5} = 109 + \frac{-\left[1-\left(\frac{1}{5}\right)^{108}\right]}{4} - \frac{1}{5^{109}}$$

multiply with 5×4

$$16S = 109\times5\times4 - 5\left[\text{[illegible]}\,5^{108}\right] - \frac{4}{5^{108}}$$

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$$16S = 109\times20 - 5 + \left(\frac{5}{5^{108}} - \frac{4}{5^{108}}\right)$$

$$16S = 2180 - 5 + \boxed{\frac{1}{5^{108}}}.$$

$$16S = 2175 + (25)^{-54}$$

$$16S - (25)^{-54} = 2175$$

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Q **JEE Main 2023**

[Ans. 16]

Suppose $a_1, a_2, 2, a_3, a_4$ be in an arithmetic-geometric progression. If the common ratio of the corresponding geometric progression is 2 and the sum of all 5 terms of the Arithmetic-geometric progression is $\boxed{\frac{49}{2}}$, then a_4 is equal to _____.

$$a,\ (a+\underbrace{d}_{})r,\ \boxed{(a+\underbrace{2d}_{})r^2},\ (\underbrace{a+3d}_{})r^3,\ (\underbrace{a+4d}_{})r^4$$

$$\boxed{r=2}$$

$$(a+2d)2^2 = 2$$

$$\boxed{a+2d = \frac{1}{2}} \rightarrow ①$$

$$2+\underbrace{8+24}_{}+64$$

$$2+32+64$$

$$\boxed{98}$$

$$S_5 = a(1\underbrace{+r+r^2+r^3+}_{}r^4) + dr + 2dr^2 + 3dr^3 + 4dr^4$$

$$= a\left[\frac{r^5-1}{r-1}\right] + d[\underbrace{2+2.2^2+3.2^3+4.2^4}_{}]$$

$$\boxed{\frac{49}{2} = 31a + 98d} \rightarrow ②$$

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$$49\left(a+2d=\tfrac{1}{2}\right) \Rightarrow \quad 49a + 9\cancel{8d} = \cancel{49/2}$$

$$31a+98d=\frac{49}{2} \Rightarrow \quad \underline{-}\,31a \ \underline{+}\ 9\cancel{8d} = \underline{-}\,\cancel{\frac{49}{2}}$$

$$18a=0$$

$$\boxed{a=0}$$

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$$\boxed{d = \frac{\text{[illegible]}}{4}}$$

$$a_4 = (a+4d)r^4$$

$$= (0+1).(2)^4.$$

$$= 2^4 = 16$$

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INSERTION OF MEANS– A.M.S

$2 \quad , 9 \quad , \quad 16$

Case-1 Inserting Single AM between 2 Given numbers a & b.

$a, A, \quad b \rightsquigarrow A.P.$

$2A = a+b \Rightarrow \boxed{A = \frac{a+b}{2}}$

Case-2 Inserting "n" AMs between 2 Given numbers a & b.

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$\boxed{a, A_1, A_2, A_3, A_4, \ldots A_n, b} \Rightarrow A.P.$

$b = T_{n+2}$

$b = a+(n+1)d.$

$\boxed{d = \frac{b-a}{n+1}}$

$$\left\{\begin{aligned} A_1 &= a+d \\ A_2 &= a+2d \\ &\vdots \\ A_n &= a+nd \end{aligned}\right\} \rightarrow \text{add}: A_1+A_2+A_3+\cdots+A_n =$$

$= na+d\,(1+2+3+4+\cdots+n]$

$= na+\text{ⓓ}\,\frac{n(n+1)}{2}$

$= na+\frac{(b-a)}{(n+1)}\cdot\frac{n(\cancel{n+1})}{2}$

$=$

$= na+\frac{(b-a)n}{2}$

$= \frac{2na+bn-\cancel{an}}{2}$

$= \frac{na+bn}{2}$

$= n\left(\frac{a+b}{2}\right)$

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Question

Insert 12 AMs between the numbers 1 and 99.

$$A_1 + A_2 + \cdots + A_{12} = 12 \cdot \left[\frac{1+99}{2}\right]$$

$$= \underline{\underline{600}}$$ ✓

$$1,\ A_1,\ \underbrace{A_2},\ \underbrace{A_3},\ \ldots\ldots A_{12},\ 99$$

$\boxed{a=1}$ ✓

$\boxed{n=14}$

$$T_{14} = 99$$

$$a + 13d = 99$$

$$1 + 13d = 99.$$

$$13d = 98.$$

$$d = \frac{98}{13}.$$

$A_1 = a+d$ ✓

$A_2 = a+2d.$ ✓

$A_3 = a+3d$

$\vdots$

$A_{12} = a+12d$

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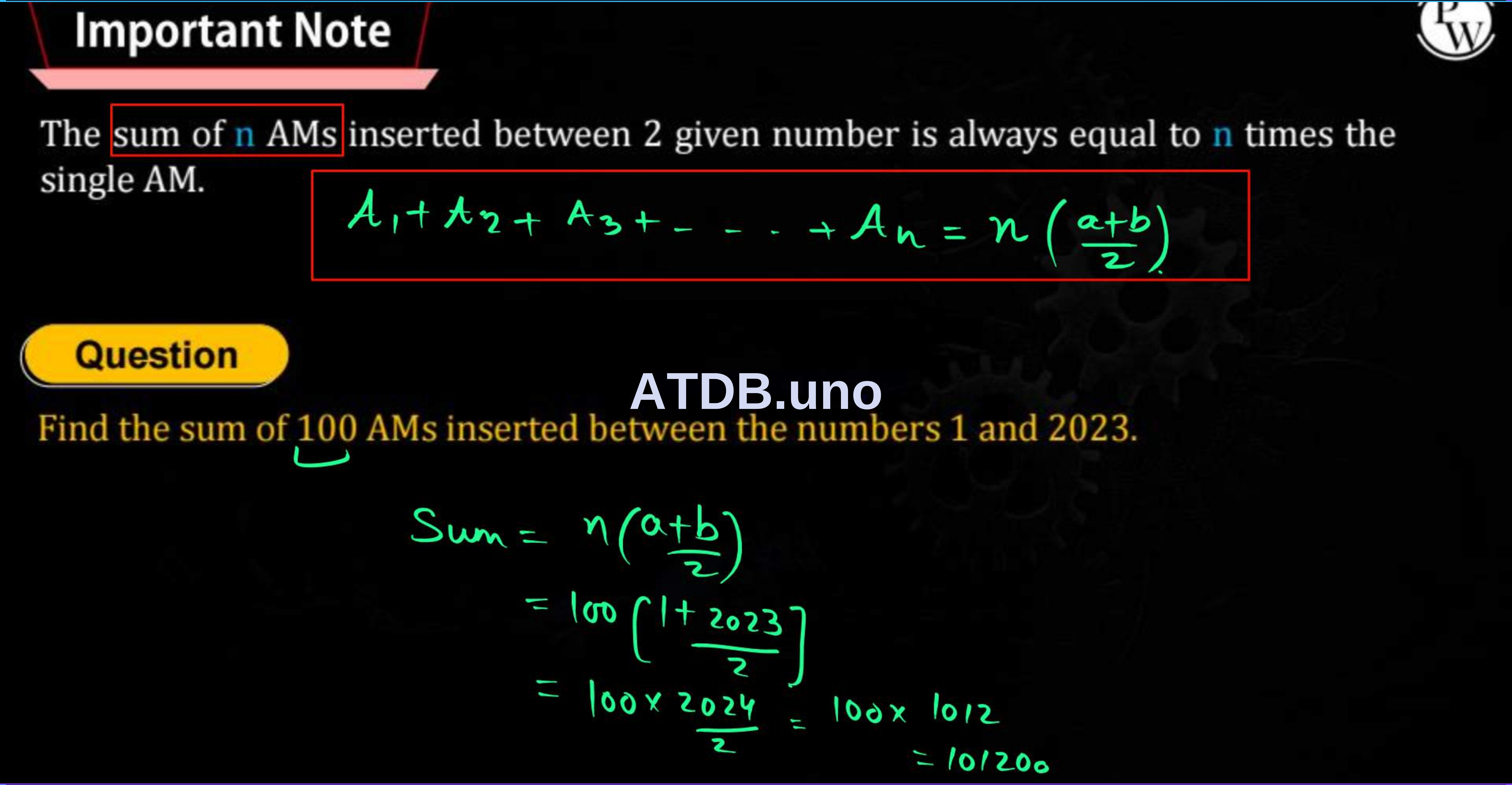

Important Note

The sum of n AMs inserted between 2 given number is always equal to n times the single AM.

$$\boxed{A_1 + A_2 + A_3 + - - - + A_n = n\left(\frac{a+b}{2}\right)}$$

Question

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Find the sum of 100 AMs inserted between the numbers 1 and 2023.

$$\text{Sum} = n\left(\frac{a+b}{2}\right)$$

$$= 100\left[\frac{1+2023}{2}\right]$$

$$= 100 \times \frac{2024}{2} = 100 \times 1012$$

$$= 101200$$

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INSERTION OF MEANS_G.M.S

Case-1 Inserting Single GM between 2 Given numbers a & b.

$a, G, b \rightarrow G.P.$

$G^2 = ab \Rightarrow \boxed{G = \sqrt{ab}}$

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Case-2 Inserting "n" GMs between 2 Given numbers a & b.

$a, G_1, G_2\ G_3 . - - - G_n, b \rightarrow G.P.$

$$\left.\begin{aligned} G_1 &= ar \\ G_2 &= ar^2 \\ G_3 &= ar^3 \\ &\vdots \\ G_n &= ar^n \end{aligned}\right\} \Rightarrow \text{multiply}$$

$b = T_{n+2}$

$b = ar^{n+1}$

$(b/a) = r^{n+1}$

$r = (b/a)^{\frac{1}{n+1}} \uparrow$

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Question

Insert 7 GMs between the numbers 2 and 32. Also Find their Product.

$2,\ G_1\ G_2\ G_3 - - - - G_7,\ 32 \longrightarrow$ G.P.

$32 = T_9$

$32 = ar^8$

$32 = 2r^8$

$16 = r^8.$

$r^2 = 2$

$\boxed{r = \sqrt{2}}$

$G_1 = ar^1 = 2\sqrt{2}$

$G_2 = ar^2 = 4$

$G_3 = ar^3 = 4\sqrt{2}$

$= 8$

$= 8\sqrt{2}$

$G_7 = ar^7 = 16.$

$\searrow 16\sqrt{2}$

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Important Note

The Product of n GMs inserted between 2 given number is always equal to n th power of the single GM.

$$\boxed{G_1 G_2 G_3 G_4 \ldots G_n = (\sqrt{ab})^n}$$

Question

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Find the product of 100 GMs inserted between the numbers 3 and 243.

$$G_1 G_2 G_3 \ldots G_{100} = (\sqrt{ab})^{100}$$

81×3

$$= (\sqrt{3 \times 243})^{100}$$

$$= (3 \times 9)^{100}$$

$$= (27)^{100}$$

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Q JEE Main-2020

If m arithmetic mean (*A*Ms) and three geometric means (*G*Ms) are inserted between 3 and 243 such that 4th *A*.M. is equal to 2nd *G*.M. Find "m".

[Ans. 39]

$3, A, \enclose{circle}{A_2}, A_3 - - - A_m, \boxed{243}$

$\boxed{A_4 = G_2} = 27$

$a + 4d = 27$

$3 + 4d = 27$

$4d = 24$

$\boxed{d = 6}$

T_{m+2} $243 = a + (m+1)d$

$243 = 3 + (m+1)6$

$\boxed{m = 39}$

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$3, G, G_2\ G_3, 243$

$243 = T_5 = ar^4$

$243 = 3r^4.$

$r^4 = 81$

$\boxed{r = 3}$

$G_1 = 3 \times 3 = 9$

$\boxed{G_2 = 27}$

$G_3 = 81$

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Q JEE Main-2022

29
132

$161, 29$

[Ans. 23]

If n AMs are inserted between 2 numbers a and 100, Such that the ratio of first mean to last mean is 1: 7 & a + n = 33, then find the value of n ?

$$a, A_1, A_2, A_3 \ldots\ldots A_n, 100$$

$$\frac{A_1}{A_n} = \frac{1}{7}$$

$$\frac{a+d}{a+nd} = \frac{1}{7}$$

$$7a + 7d = a + nd$$

$$\boxed{6a = (n-7)d} \rightarrow ①$$

$$6(33-n) = (n-7)d$$

$$d =$$

$$100 = a + (n+1)d$$

$$100 = (33-n) + (n+1)\cdot\frac{6(33-n)}{(n-7)}$$

$$100 = (33-n)\left[1 + \frac{6(n+1)}{n-7}\right]$$

$$100 = (33-n)\left[\frac{n-7+6n+6}{n-7}\right]$$

$$100(n-7) = (33-n)(7n-1)$$

$$100n - 700 = 231n - 33 - 7n^2 + n$$

$$\boxed{7n^2 - 132n - 667 = 0}$$

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$$7n^2 - 161n + 29n - 667$$

$$(7n+29)(n-23) = 0$$

$$\boxed{n = 23}$$ ✓

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INSERTION OF MEANS_(HARMONIC MEANS)

Case-1 Inserting Single HM between 2 Given numbers a & b.

$a, H, b \rightsquigarrow \boxed{HP}$

$\frac{1}{a}, \frac{1}{H}, \frac{1}{b} \rightarrow AP$

$\frac{2}{H} = \frac{1}{a} + \frac{1}{b}$

$\boxed{H = \frac{2ab}{a+b}}$

Case-2 Inserting "n" HMs between 2 Given numbers a & b.

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$a, H, H_2\ H_3 . - - - H_n, b \rightarrow HP$

$\frac{1}{a}, \boxed{\frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3} . - - - \frac{1}{H_n}}, \frac{1}{b} \rightarrow AP$

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Important Note

The Sum of Reciprocals of n HMs inserted between 2 given number is always equal to n times the reciprocal of the single HM.

$$\boxed{\frac{1}{H_1}+\frac{1}{H_2}+\frac{1}{H_3}+----+\frac{1}{H_n}=\frac{n}{H}}$$

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(a, b) →

$A=\frac{a+b}{2}$

$G=\sqrt{ab}$

$H=\frac{2ab}{a+b}$

→ always form. GP

To Prove: $\boxed{G^2=AH}$

$\boxed{AH=G^2}$

$AH=\frac{\cancel{a+b}}{\cancel{2}}\cdot\frac{\cancel{2}ab}{\cancel{(a+b)}}=$

2, 32

$A=17$

$G=8$

$H=\frac{2\times2\times32}{}$

$H=\frac{\cancel{64}}{17}$ 34

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Homework HW

If m is the A.M. of two distinct real number l and n (l, n > 1) and G_1, G_2 and G_3 are three geometric means between l an n then $G_1^4 + 2G_2^4 + G_3^4$ equals.

(JEE MAIN 2015)

[Ans. B]

A $4l^2mn$

B $4lm^2n$

C $4lmn^2$

D $4l^2mn^2$

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SERIES-1 (SPECIAL SERIES)

Note 1 Sum of the first n natural numbers $\boxed{\sum_{r=1}^{n} r = \frac{n(n+1)}{2}}$

$$\sum_{r=1}^{n} r = 1+2+3+4+ \dots + n = \frac{n(n+1)}{2}$$

Note 2 Sum of the squares of the first n natural numbers $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$

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$$\sum r^2 = 1^2+2^2+3^2+4^2+ \dots + n^2 =$$

Note 3 Sum of the cubes of the first n natural numbers $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} = \left[\sum_{r=1}^{n} r\right]^2$

$$\sum r^3 = 1^3+2^3+3^3+ \dots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

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SERIES-1 (SPECIAL SERIES)

Properties of Sigma Notations (Σ)

1) $$\sum_{r=1}^{n}(a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$$

2) $$\sum_{r=1}^{n}(k a_r) = k \sum_{r=1}^{n} a_r$$

3) $$\sum_{r=1}^{n} 1 = 1+1+1+1+..+1 = \textcircled{n}$$

4) $$\sum_{r=1}^{n} k = k \sum 1 = \boxed{kn}$$

$$\int 2x^2 dx$$

$$\int (f(x)+g(x))dx$$

Ex→ $\sum (r^2 + r)$

$$= \sum r^2 + \sum r$$

✓ ✓

Ex→ $\sum (2r^2 - 3r)$

$$\sum 2r^2 - \sum 3r$$

$$2\underbrace{\sum r^2} - 3\underbrace{\sum r}$$

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Q→ If $\boxed{T_r = 2r^3 - 4r^2 + r + 7}$ find $\boxed{S_n}$

$$\boxed{S_n = \sum_{r=1}^{n} T_r} = T_1 + T_2 + T_3 + - - + T_n$$

$$S_n = \sum_{r=1}^{n} \left(2r^3 - 4r^2 + r + 7\right)$$

$$= 2\boxed{\sum r^3} - 4\boxed{\sum r^2} + \boxed{\sum r} + 7\boxed{\sum 1}$$

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$$S_n = 2\left(\frac{n(n+1)}{2}\right)^2 - 4\,\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} + 7n$$

Q→ $T_r = 3r^2 + 5$

$S_{10} = ?$

$$S_{10} = \sum_{r=1}^{10} T_r$$

$$\sum \left(3r^2 + 5\right)$$

$$= 3\sum r^2 + \sum 5$$

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Question

Let $S_k = \frac{1+2+3+\ldots+k}{k}$. If $S_1^2 + S_2^2 + \ldots + S_{10}^2 = \frac{5}{12}A$, then A is equal to: (JEE MAIN 2019 (Jan.))

[Ans. C]

A 283

B 301

C 303 ✓

D 156

22
46
46

$S_K = \frac{\cancel{K}(K+1)}{2\cancel{K}}$

$\boxed{S_K = \frac{K+1}{2}}$

$(S_K)^2 = \frac{(K+1)^2}{4}$

$(S_1)^2 = \frac{2^2}{4}$

$(S_2)^2 = 3^2/4$

$(S_3)^2 = 4^2/4$

$(S_{10})^2 = \frac{(11)^2}{4}$

$S_1^2 + S_2^2 + S_3^2 + - - + (S_{10})^2 = \frac{1}{4}\left(2^2+3^2+4^2+\ldots 11^2\right)$

$\frac{1}{4}\left[1^2+2^2+3^2+ - - - + 11^2 - 1\right]$

$\frac{1}{4}\left[\frac{11\times \cancel{12}\times 23}{\cancel{6}} - 1\right] = \frac{5}{12}A$

$\frac{1}{4}\left[22\times 23 - 1\right] = \frac{5}{12}A$

$\frac{(505)}{4} = \frac{5A}{12}$

$\boxed{A = 303}$

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Question #

Find the sum to the first n terms of the series:

$$\boxed{\frac{1^3}{1}}+\boxed{\frac{1^3+2^3}{1+2}}+\boxed{\frac{1^3+2^3+3^3}{1+2+3}}+\dots$$

$$T_r = \frac{1^3+2^3+3^3+\dots+r^3}{1+2+3+4+\dots+r}$$

$$T_r = \frac{\left[\frac{r(r+1)}{2}\right]^2}{\frac{r(r+1)}{2}}$$

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$$T_r = \frac{r(r+1)}{2}$$

$$\boxed{S_n = \sum_{r=1}^{n} T_r}$$

$$= \sum_{r=1}^{n} \frac{r(r+1)}{2} = \frac{1}{2}\sum(r^2+r) = \frac{1}{2}\left(\sum r^2+\sum r\right)$$

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Q JEE Main-2023 (Jan.-II)

[Ans. 5]

If $\dfrac{1^3+2^3+3^3+......\text{upto } n \text{ terms}}{1.3+2.5+3.7+......\text{upto } n \text{ terms}} = \dfrac{9}{5}$, then the value of n is

$\rightarrow S_n$

$\rightarrow S_n'$

$$\boxed{\frac{S_n}{S_n'} = 9/5}$$

$$S_n = \left[\frac{n(n+1)}{2}\right]^2 \rightsquigarrow ①$$

$$S_n' = ①\cdot 3 + ②\cdot 5 + ③\cdot 7 + + - - , - -$$

$$\boxed{T_r = (r)(2r+1)}$$

$$T_r = (2r^2+r)$$

$$S_n' = \sum_{r=1}^{n} T_r$$

$$\sum_{r=1}^{n} (2r^2+r)$$

$$= 2\sum r^2 + \sum r$$

$$= \cancel{2}\,\frac{n(n+1)(2n+1)}{\cancel{6}\;3} + \frac{n(n+1)}{2}$$

$$S_n' = n(n+1)\left[\frac{(2n+1)}{3} + \frac{1}{2}\right]$$

$$S_n' = \frac{n(n+1)}{6}(4n+5) \qquad n(n+1)\left[\frac{2(2n+1)+3}{6}\right]$$

$$\frac{\left(\frac{n(n+1)}{2}\right)^2}{\frac{\cancel{n(n+1)}}{\cancel{2}}\left(\frac{4n+5}{3}\right)} = 9/5$$

$$\frac{n(n+1)\cancel{3}}{2(4n+5)} = \overset{3}{9}/5$$

$$5n(n+1) = 6(4n+5)$$

$$5n^2+5n = 24n+30$$

$$\boxed{5n^2-19n-30=0}$$

$$5n^2 - 25n + 6n - 30 = 0$$

$$(5n+6)(n-5) = 0$$

$$\boxed{n=5}$$

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SERIES-2

Series in which the Difference of Consecutive Terms are in A.P or G.P.

Note

Steps to get the sum

Step I Write the given series.

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Step II Write the series again by shifting one term to right.

Step III Subtract the two equations to obtain n^{th} term (T_n) of the series.

Step IV Find the sum of the n terms of the sequence as

$$S_n = \sum_{n=1}^{n} T_n.$$

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Question

Method of difference T_r

Find S_n for the series 3 + 7 + 14 + 24 + 37 + ... up to *n* terms.

$$\boxed{T_n = \frac{3n^2 - n + 4}{2}}$$

check: $T_2 = \frac{3\times4 - 2 + 4}{2} = 7$

$$S = 3 + 7 + 14 + 24 + 37 + - - - - + T_n$$

$$S = \quad 3 + 7 + 14 + 24 + - - - + T_n$$

$-$

$$0 = 3 + \boxed{(4 + 7 + 10 + 13 - - - ..)} - T_n$$

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$$T_n = 3 + (4 + 7 + 10 + 13 + - - ..)$$

$a = 4$, $d = 3$, no of terms# $= (n-1)$

$$T_n = 3 + \frac{n-1}{2}\left[2\times4 + (n-2).3\right]$$

$$= 3 + \frac{(n-1)}{2}(8 + 3n - 6) = 3 + \frac{(n-1)(3n+2)}{2} = \frac{6 + 3n^2 + 2n - 3n - 2}{2}$$

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$$T_r = \frac{3r^2 - r + 4}{2}$$

$$S_n = \sum T_r$$

$$= \sum \frac{3r^2 - r + 4}{2}$$

$$= \frac{1}{2}\left(\underbrace{3\sum r^2}_{\text{Cubic}} - \sum r + \sum 4\right)$$

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Shortcut ✓

$$\boxed{T_n = an^2 + bn + c}$$ if diff in A.P.

$$\boxed{S_n = An^3 + Bn^2 + Cn + D}$$

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Question

Find the sum of *n*-terms of the series
$1 + 4 + 10 + 22 + 46 + \ldots\ldots\ldots\ldots$

$$S = 1 + 4 + 10 + 22 + 46 + - - - + T_n$$

$$S = \quad 1 + 4 + 10 + 22 + - - + T_n$$

$-$

$$0 = 1 + (3 + 6 + 12 + 24 + - -) - T_n$$

$$T_n = 1 + (3 + 6 + 12 + 24 + - -)$$

$r = 2,\ a = 3,$ no of terms $- n-1$

$$T_n = 1 + 3\underbrace{[2^{n-1} - 1]}$$

$$T_n = 1 + 3[2^{n-1} - 1] \quad 2-1$$

$$= 3\cdot 2^{n-1}$$

$$\boxed{T_r = 3\cdot 2^{r-1} - 2}$$

check $r = 3$

$$T_3 = 3\cdot 2^2 - 2$$

$$= 12 - 2 = 10 \checkmark$$

$$S_n = \sum_{r=1}^{n} T_r$$

$$= \sum(3\cdot 2^{r-1} - 2)$$

$$= \sum 3\cdot 2^{r-1} - \sum 2$$

$$= 3\sum_{r=1}^{n} 2^{r-1} - 2n \checkmark$$

$$= 3\underbrace{(2^0 + 2^1 + 2^2 + - - + 2^{n-1})}$$

$$S_n = 3\times 1\,(2^n - 1] - 2n \qquad -2n$$

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Q JEE Main 2023

[Ans. A]

Let $S_n = 4 + 11 + 21 + \underbrace{34}_{} + 50 + \ldots\ldots$ then $\frac{1}{60}(S_{\underbrace{29}_{}} - S_{\underbrace{9}_{}})$ is equal to

(A) 223 ✓

(B) 226

(C) 220

(D) 227

$$\boxed{T_n = an^2 + bn + c}$$ ✓

$T_1 = a + b + c = 4.$

$T_2 = \boxed{4a + 2b + c = 11} \Rightarrow 3a + b + 4 = 11$

$\rightarrow \boxed{3a + b = 7}$

$T_3 = \underbrace{9a + 3b}_{} + c = 21$

$9a + 3b = 21$

$21 + c = 21$

$\boxed{c = 0}$

$\boxed{a + b = 4}$

$3a + b = 7$

$2a + 4 = 7$

$\frac{3}{2} + b = 4$

$b = 4 - \frac{3}{2}$

$b = \frac{5}{2}$

$$\boxed{T_n = \frac{3}{2}n^2 + \frac{5}{2}n}$$

$T_4 = \frac{\cancel{3}}{\cancel{2}} \times \cancel{16} + \frac{5}{2} \times 4$

$= 24 + 10 = \underline{34}$

$S_n = \Sigma T_r$

$= \Sigma \frac{3}{2}r^2 + \frac{5}{2}r.$

$= \frac{3}{2}\Sigma r^2 + \frac{5}{2}\Sigma r$

$S_n = \frac{3}{2} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{5}{2} \cdot \underline{n(n+1)}$

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$$S_n = \frac{n(n+1)}{2}\left[\frac{3}{6}(2n+1) + 5/2\right].$$

$$= \frac{n(n+1)}{2\times 2}\left[2n+1+5\right].$$

$$\begin{array}{r} {}^{7}29 \\ 8 \\ \hline 232 \end{array}$$

$$S_n = \frac{n(n+1)(n+3)}{2}$$

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$$S_{29} = \frac{29\times 30\times 32}{2}. = 29\times 30\times 16$$

$$S_9 = \frac{9\times 10\times 12}{2} = 9\times 30\times 2$$

$$S_{29} - S_9 = \underbrace{30}\times 2\left[29\times 8 - 9\right].$$

$$\frac{S_{29}-S_9}{60} = \left[29\times 8 - 9\right] = 232 - 9 = \boxed{223}$$

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Q JEE Main 2023

HW

[Ans. B]

Let a_n be the n^{th} term of the series $5 + 8 + 14 + 23 + 35 + 50 + \ldots$ and

$3 \quad 6 \quad 9 \quad 12 \quad 15$

$S_n = \sum_{k=1}^{n} a_k$. Then $S_{30} - a_{40}$ is equal to

$$\boxed{a_n = an^2 + bn + c}$$

$$\left\{\begin{aligned} a+b+c &= 5 \\ 4a+2b+c &= 8 \\ 9a+3b+c &= 14 \end{aligned}\right\}$$

A 11260

B 11290

C 11310

D 11280

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TELESCOPING SERIES

A series in which the general Term can be written as a difference of two symmetric terms is called Telescoping Series.

$$\boxed{T_r = a_r \ominus a_{r-1}}$$

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Question

$$\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots\cdots\cdots\cdots$$

$$T_r = \frac{1}{r(r+1)}$$

$$= \frac{(r+1)-r}{r(r+1)}$$

$$= \frac{\cancel{r+1}}{r(\cancel{r+1})} - \frac{\cancel{r}}{\cancel{r}(r+1)}$$

$$\boxed{T_r = \frac{1}{r} - \frac{1}{r+1}}$$

$r=1$ $$T_1 = \frac{1}{1} - \cancel{\frac{1}{2}}$$

$r=2$, $$T_2 = \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}}$$

$r=3$, $$T_3 = \cancel{\frac{1}{3}} - \cancel{\frac{1}{4}}$$

⋮

$$T_n = \cancel{\frac{1}{n}} - \frac{1}{n+1}$$

Add

$$S_n = 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1}$$

$$\boxed{S_n = \frac{n}{n+1}}$$

$$\boxed{S_\infty = 1}$$

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Question

$S = \frac{1}{1.2.3} + \frac{1}{2.3.4} + \frac{1}{3.4.5} + \cdots$ upto n terms.

$$T_r = \frac{1}{r(r+1)(r+2)}$$

$$\frac{1}{2}\left[\frac{(r+2) - r}{r(r+1)(r+2)}\right]$$

$$T_r = \frac{1}{2}\left[\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)}\right]$$

$$T_1 = \frac{1}{2}\left[\frac{1}{1\cdot 2} - \cancel{\frac{1}{2\cdot 3}}\right]$$

$$T_2 = \frac{1}{2}\left[\cancel{\frac{1}{2\cdot 3}} - \cancel{\frac{1}{3\cdot 4}}\right]$$

$$\vdots$$

$$T_n = \frac{1}{2}\left[\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right]$$

$$S_n = \frac{1}{2}\left[\frac{1}{2} - \frac{1}{(n+1)(n+2)}\right]$$

$$S_\infty = \frac{1}{2}\left[\frac{1}{2} - 0\right]$$

$$\boxed{S_\infty = \frac{1}{4}}$$

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Question

Find the sum to n terms and S_∞ for the series

$$\frac{1}{1.2.3.4}+\frac{1}{2.3.4.5}+\frac{1}{3.4.5.6}\text{-----------------}$$

$$T_r = \frac{1}{r(r+1)(r+2)(r+3)}$$

$$\frac{1}{3}\,\frac{(r+3)-r}{r(r+1)(r+2)(r+3)}$$

$$T_r = \frac{1}{3}\left[\frac{1}{r(r+1)(r+2)} - \frac{1}{(r+1)(r+2)(r+3)}\right]$$

$$\frac{1}{3}\left[\frac{1}{1\cdot2\cdot3} - \sim\right]$$

$$\boxed{S_\infty = \frac{1}{18}}\ \checkmark$$

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Question

$S_\infty = \frac{1}{2}$

$\boxed{r^4+r^2+1 = (r^2-r+1)(r^2+r+1)}$

Find the sum to *n* terms and S_∞ for the series

$$\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}\text{----------------}$$

$\underbrace{r^4+2r^2+1}-r^2$

$(r^2+1)^2-r^2$

$$T_r = \frac{r}{\boxed{1+r^2+r^4}}$$

$$= \frac{r}{(1+r+r^2)(1-r+r^2)}$$

$$= \frac{1}{2}\,\frac{(r^2+r+1)-(r^2-r+1)}{(r^2+r+1)(r^2-r+1)}$$

$$T_r = \frac{1}{2}\left[\frac{1}{r^2-r+1} - \frac{1}{r^2+r+1}\right]$$

$$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$$

$$T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{7}\right]$$

$$T_3 = \frac{1}{2}\left[\frac{1}{7} - \frac{1}{}\right.$$

$$T_n = \frac{1}{2}\left(- \frac{1}{n^2+n+1}\right]$$

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Q JEE Main-2023 (Feb.-I)

[Ans. B]

The sum to 10 terms of the series $\frac{1}{1+1^2+1^4}+\frac{2}{1+2^2+2^4}+\frac{3}{1+3^2+3^4}+\ldots$ is

HW

(A) $\frac{59}{111}$

(B) $\frac{55}{111}$

(C) $\frac{56}{111}$

(D) $\frac{58}{111}$

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Q JEE Main-2023 (Jan.-I)

[Ans. C]

If $a_n = \frac{-2}{4n^2-16n+15}$, then $a_1 + a_2 + \ldots + a_{25}$ is equal to : ✓

- (A) $\frac{51}{144}$
- (B) $\frac{49}{138}$
- [✓ (C) $\frac{50}{141}$]
- (D) $\frac{52}{147}$

$$a_n = \frac{-2}{(2n-3)(2n-5)}$$

$$= \frac{(2n-5)-(2n-3)}{(2n-3)(2n-5)}$$

$$\boxed{a_n = \frac{1}{2n-3} - \frac{1}{2n-5}}$$

$$a_1 = \frac{1}{-1} - \frac{1}{-3}$$

$$a_2 = \frac{1}{1} - \frac{1}{-1}$$

$$a_3 = \frac{1}{3} - \frac{1}{1}$$

$$a_{25} = \frac{1}{47} -$$

$$S_{25} = \frac{1}{47} + \frac{1}{3}$$

$$\frac{3+47}{47 \times 3} = \frac{50}{141}$$

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Q JEE Main-2023 (Jan.-I)

[Ans. 8]

$\frac{\ }{9} = \frac{\ }{3}$

$= ⑧$

Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \ldots \frac{1}{\underbrace{\sqrt{a_{17}}+\sqrt{a_{18}}}}\right)$ is equal to ....

$$\left(\frac{\sqrt{a_{10}}-\sqrt{a_{11}}}{a_{10}-a_{11}}\right)$$

$-d$

$$\frac{\sqrt{a_{10}}-\sqrt{a_{11}}}{-d}$$

$$= \left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d}\right)$$

$$\frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d}$$

$$\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{d}$$

$$\frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{d}$$

$$\frac{\sqrt{a_{13}}-\sqrt{a_{12}}}{d}$$

$$\frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{d}$$

$$\text{Sum} = \frac{\sqrt{a_{18}}-\sqrt{a_{10}}}{d} = \frac{9-3}{9} = \frac{6}{9}$$

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$$a_5 = 2a_7$$

$$a+4d = 2(a+6d)$$

$$a+4d = 2a+12d$$

$$\boxed{a+8d=0} \rightarrow ①$$

$$\boxed{a_9 = 0}$$

$$a_9 + d + d = a_{11}$$

$$2d = 18$$

$$\boxed{d = 9}$$

$$\boxed{a = -72}$$

$$\boxed{a_{10} = 9}$$

$$a_{18} = a + 17d$$

$$= -72 + 17 \times 9$$

$$= 9[-8+17]$$

$$= 9 \times 9$$

$$a_{18} = \boxed{81}$$

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AM-GM- HM INEQUALITY

$\boxed{2, 8}$

$AM = 5$

$GM = \sqrt{16} = 4$

$HM = \frac{2 \times 2 \times 8}{10} = 3.2$

1. FOR 2 positive Numbers

Consider two positive numbers *a* and *b*, A.M. = $\frac{a+b}{2}$, G.M. = $\sqrt{ab}$, H.M. = $\frac{2ab}{a+b}$ $\rightarrow \frac{2}{\frac{1}{a}+\frac{1}{b}}$

According to A.M. G.M. H.M. Inequality A.M. ≥ G.M. ≥ H.M.

The Equality holds (AM = GM = HM) When the given numbers are equal.

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2. FOR 3 positive Numbers

a, b, c

$AM = \frac{a+b+c}{3}$, $GM = \sqrt[3]{abc}$

$HM = \frac{3}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$

$\boxed{AM \geqslant GM \geqslant HM}$

$x + \frac{1}{x} \geqslant 2$ for $\boxed{x > 0}$

$x, \frac{1}{x} \rightarrow 2$ +ve nos.

$x + \frac{1}{x} \leq -2$ for $x < 0$

$\frac{x + \frac{1}{x}}{2} \geqslant \sqrt{x \cdot \frac{1}{x}} \Rightarrow \boxed{x + \frac{1}{x} \geq 2}$

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$$\boxed{-x, \ -\frac{1}{x}} \rightarrow +ve$$

AM ≥ GM

$$\frac{-x-\frac{1}{x}}{2} \geq \sqrt{-\cancel{x}\cdot\left(-\frac{1}{\cancel{x}}\right)}$$

$$\Rightarrow -\left[x+\frac{1}{x}\right] \geq 2$$

$$\boxed{x+\frac{1}{x} \leq -2}$$ ✓

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AM–GM– HM INEQUALITY

FOR n positive Numbers

Let a_1, a_2, ..., a_n be n $\underbrace{\text{positive}}$ real numbers, then we define their arithmetic mean (A), geometric mean (G) and harmonic mean (H) as $\boxed{A = \dfrac{a_1 + a_2 + \cdots + a_n}{n}}$,

$\boxed{G = (a_1\, a_2 \ldots a_n)^{1/n}}$ and $\boxed{H = \dfrac{n}{\left(\dfrac{1}{a_1} + \dfrac{1}{a_2} + \dfrac{1}{a_3} + \cdots \dfrac{1}{a_n}\right)}}$.

$\boxed{A \geqslant G \geqslant H}$

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$\log_3(3 + 3 + 3) \geq 4$

$\Rightarrow m = 4$

Q JEE ADV 2020

Let $\boxed{m}$ be the minimum possible value of $\log_3(3^{y_1} + 3^{y_2} + 3^{y_3})$, where y_1, y_2, y_3 are real numbers for which $y_1 + y_2 + y_3 = 9$. Let M be the maximum possible value of $(\log_3 x_1 + \log_3 x_2 + \log_3 x_3)$, where x_1, x_2, x_3 are positive real numbers for which $x_1 + x_2 + x_3 = 9$. Then the value of $\log_2(m^3) + \log_3(M^2)$ is

Consider $3^{y_1}, 3^{y_2}, 3^{y_3}$

AM ≥ GM

$$\frac{3^{y_1} + 3^{y_2} + 3^{y_3}}{3} \geq \left[3^{y_1} 3^{y_2} 3^{y_3}\right]^{1/3}$$

$$\frac{3^{y_1} + 3^{y_2} + 3^{y_3}}{3} \geq \left[3^{9}\right]^{1/3}$$

$$\boxed{3^{y_1} + 3^{y_2} + 3^{y_3} \geq 81}$$

x_1, x_2, x_3

$$\frac{x_1 + x_2 + x_3}{3} \geq \left[x_1 x_2 x_3\right]^{1/3}$$

$$3 \geq \left[x_1 x_2 x_3\right]^{1/3}$$

$$27 \geq x_1 x_2 x_3$$

$$\log_3 27 \geq \log_3 x_1 + \log_3 x_2 + \log_3 x_3$$

$$3 \geq \log_3 x_1 + \log_3 x_2 + \log_3 x_3$$

$$\boxed{M = 3}$$

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$$\log_2(m^3) + \log_3(M^2)$$

$$\underbrace{\log_2 (4)^3} + \log_3 3^2$$

$$6 + 2$$

$\underline{\text{Ans}} = \underline{⑧}$

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EXPONENTIAL & LOGARITHMIC SERIES

$e = 2.71$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots \infty$$

Put $x = 1 \Rightarrow e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots \infty$

$x = -1 \Rightarrow e^{-1} = 1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \dots \infty$

$$a^x = 1 + \frac{x \ln a}{1!} + \frac{x^2 (\ln a)^2}{2!} + \frac{x^3 (\ln a)^3}{3!} + \dots \infty$$

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$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \dots \infty$$

$x = 1$

$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots \infty$

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Question

Simplify: $S = \frac{2}{1!} + \frac{4}{3!} + \frac{6}{5!} + \frac{8}{7!} + \ldots\ldots\infty$

$\frac{1}{0!} + \frac{1}{2!} + \frac{1}{4!} \text{----}$

$\frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \text{---}$

$$\frac{(1+1)}{1!} + \frac{(1+3)}{3!} + \frac{(1+5)}{5!} + \left(\frac{1+7}{7!}\right) \text{----}\infty$$

$$\boxed{\left(\frac{1}{} + \frac{1}{1!} + \frac{1}{3!} + \frac{1}{2!} + \frac{1}{5!} + \frac{1}{4!} + \frac{1}{7!} + \frac{1}{6!}\right) \text{---}\infty}$$

$\downarrow$ e

$\boxed{M-2}$ By General Term →

$$S_n = \sum_{n=1}^{\infty} T_n \qquad T_n = \frac{(2n-1)+1}{(2n-1)!}$$

$$= \frac{(2n-1)}{(2n-1)!} + \frac{1}{(2n-1)!} = \boxed{\frac{1}{(2n-2)!} + \frac{1}{(2n-1)!}} \rightarrow = \boxed{e}$$

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Question

If α and β are the roots of the equation $\boxed{3x^2 - 6x + 2 = 0}$ then prove that:

$$\left(1+\frac{\alpha}{2}+\frac{\alpha^2}{\boxed{2!}\,4}+\frac{\alpha^3}{\boxed{3!}\,8}+\dots\right)\boxed{\left(1+\frac{\beta}{2}+\frac{\beta^2}{2!\,4}+\frac{\beta^3}{3!\,8}+\dots\right)}=e$$

$\nearrow \alpha$

$\searrow \beta$

$\alpha+\beta = 6/3 = 2$

$$\boxed{\left(1+\frac{\alpha/2}{1!}+\frac{(\alpha/2)^2}{2!}+\frac{(\alpha/2)^3}{3!}+\dots\infty\right)} \rightarrow e^{\alpha/2}$$

$$\downarrow e^{\beta/2}$$

$$e^{\alpha/2}\cdot e^{\beta/2} = e^{\frac{\alpha+\beta}{2}}$$

$$= e^{2/2} = e^{1}$$

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Question

$$S = \boxed{1} + \boxed{\frac{1+2}{2!}} + \boxed{\frac{1+2+2^2}{3!}} + \boxed{\frac{1+2+2^2+2^3}{4!}} + \ldots\ldots\infty$$

$\boxed{Ans = e^2 - e}$ ✓

$$T_n = \frac{1+2+2^2+2^3+\ldots+2^{n-1}}{n!} = \left(\frac{2^n-1}{n!}\right)$$

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$$S_n = \sum_{n=1}^{\infty}\frac{2^n}{n!} - \sum_{n=1}^{\infty}\frac{1}{n!}$$

↓

$$\underbrace{\left(\frac{2}{1!}+\frac{2^2}{2!}+\frac{2^3}{3!}+\ldots\infty\right)} - \left(\boxed{\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\ldots\infty}\right) \rightarrow (e-1)$$

Ans: $\rightarrow (e^2-1)-(e-1) = e^2-e.$

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Q JEE Adv 2022

[Ans. B, C]

Let $a_1, a_2, a_3, \ldots$ be an arithmetic progression with $\boxed{a_1 = 7}$ and common difference $\boxed{8}$. Let $T_1, T_2, T_3, \ldots$ be such that $\boxed{T_1 = 3}$ and $\boxed{T_{n+1} - T_n = a_n}$ for $n \geq 1$. Then, which of the following is/are TRUE?

(A) $T_{20} = 1604$

(B) $\boxed{\sum_{k=1}^{20} T_k} \rightarrow S_{20}$ $= 10510$

(C) $T_{30} = 3454$

(D) $\boxed{\sum_{k=1}^{30} T_k} \rightarrow S_{30}$ $= 35610$

$$T_{n+1} = 3 + n(4n+3)$$

$$n \rightarrow n-1$$

$$T_n = 3+(n-1)[4(n-1)+3]$$

$$T_n = 3+(n-1)(4n-1)$$

$$T_n = 3+4n^2-5n+1$$

$$\boxed{T_n = 4n^2-5n+4}$$

$n=20 \Rightarrow T_{20},$

$$\boxed{\begin{aligned} T_2 - T_1 &= a_1 \\ T_3 - T_2 &= a_2 \\ T_4 - T_3 &= a_3 \\ &\vdots \\ T_{n+1} - T_n &= a_n \end{aligned}}$$

$$T_{n+1} - T_1 = \frac{n}{2}[2a+(n-1)d]$$

$$T_{n+1} - 3 = \frac{n}{2}[14+(n-1)8]$$

$$T_{n+1} - 3 = n[7+4(n-1)]$$

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