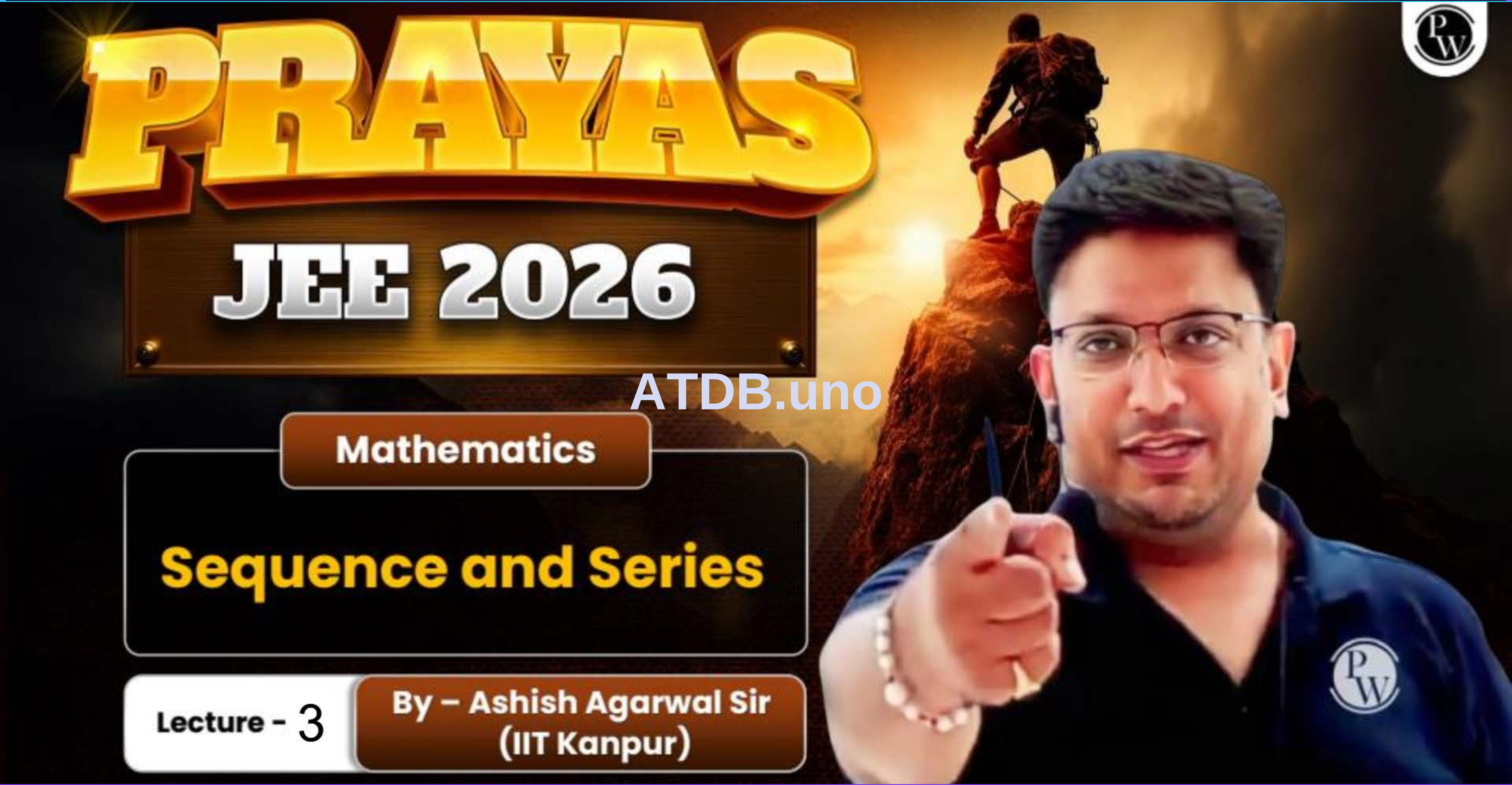
PRAYAS
JEE 2026
Mathematics
Sequence and Series
Lecture - 3
By – Ashish Agarwal Sir
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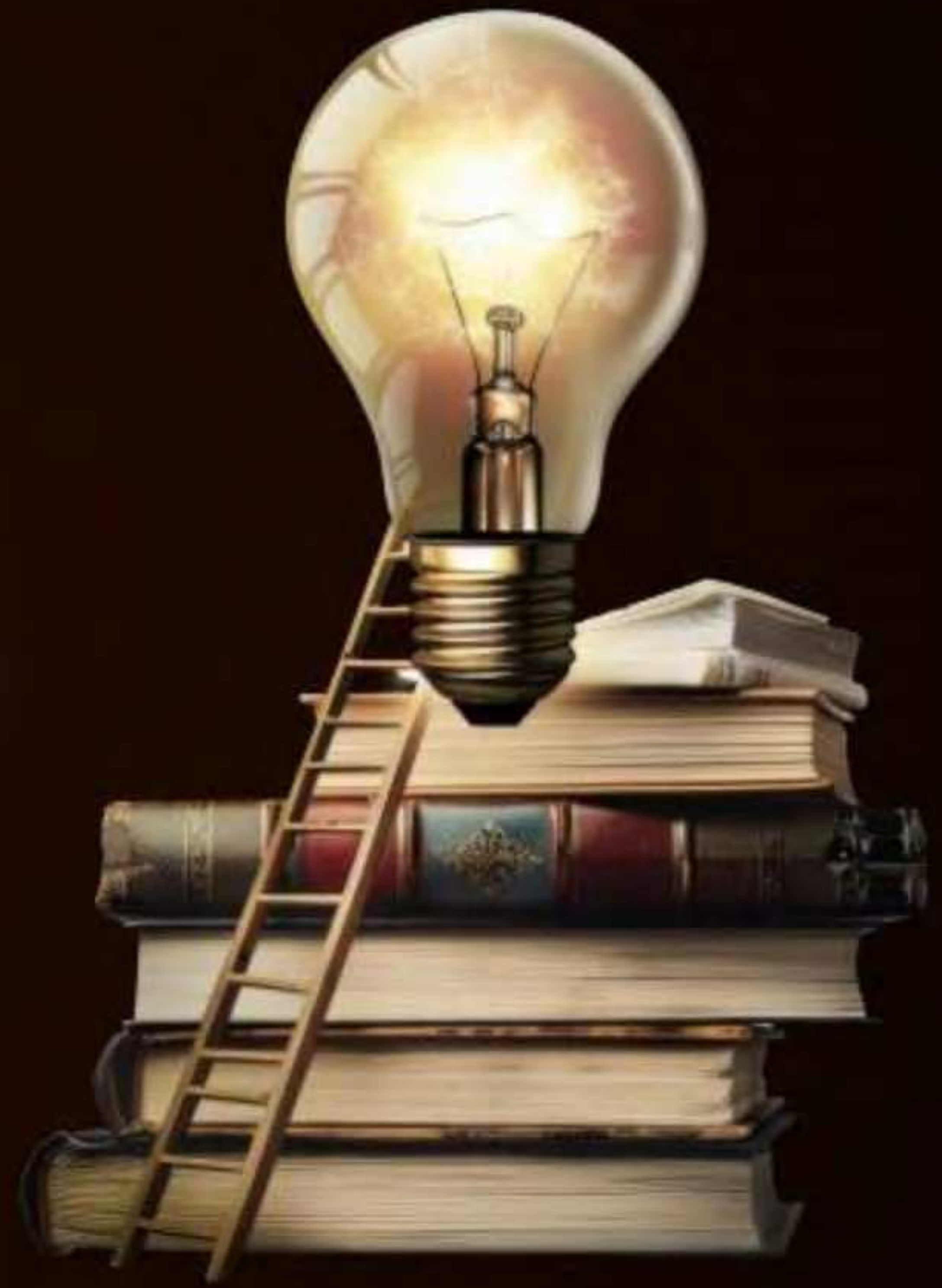
Topics *to be covered*

A Arithmetic Mean

B Geometric Progression



Recap *of previous lecture*

1. K^{th} term from end in on A.P. = $(n-K+1)^{th}$ term from beginning.

2. If $T_n = 2n + 3$ then the sequence is an A·P

3. $30 + \frac{89}{3} + \frac{88}{3} + \cdots$ has largest possible sum then number of terms that should be taken is ____________

Recap *of previous lecture*

4. If a sequence has 40 terms then number of even numbered terms is 20

41 terms then no: of even numbered terms = 20
odd numbered terms = 21

5. If a sequence has 49 terms then number of even numbered terms is 24 & number of odd numbered terms is ______

Recap of previous lecture

FIRST TERM IS Not a

6. Three numbers in A.P. $a-d, a, a+d$ — $c.d = d$

Four numbers in A.P. $a-3d, a-d, a+d, a+3d$ — $c.d = 2d$

Five numbers in A.P. $a-2d, a-d, a, a+d, a+2d$ — $c.d = d$

Six numbers in A.P. $a-5d, a-3d, a-d, a+d, a+3d, a+5d$ — $c.d = 2d$

7. If we pick terms in an A.P. at an interval of 5 then the picked terms will form an A.P with common difference $5d$

Recap *of previous lecture*

8. If $a_1, a_2, a_3, a_4, a_5, \ldots\ldots$ be an A.P. with common difference 'd' then $a_1, a_5, a_9, a_{13}, \ldots\ldots$ will be in A.P with common difference 4d. Similarly $a_2, a_4, a_6, \ldots\ldots$ will also be in A.P with common difference 2d

9. If $m, n, p, q \in N$ & $m + n = p + q$ then in A.P. we have $T_m + T_n = T_p + T_q$.

10. If $a_1 + a_{12} = 10$ in an A.P. then value of $a_3 + a_7 + a_9 + a_4 + a_6 + a_{10}$ is $3(a_1 + a_{12}) = 30$.

Homework Discussion

QUESTION [JEE Mains 2022 (27 July)]

TAH 01
(ADBST)

Suppose $a_1, a_2, ..., a_n$ ..... be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is 5 : 17 and, $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to

(A) 290

(B) 380

(C) 460

(D) 510

Ans. B

QUESTION [JEE Mains 2025 (29 Jan)]

TAH 02

(ADB8T)

Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11^{th} term is :

(A) 108

(B) 90

(C) 122

(D) 84

Ans. B

QUESTION [JEE Advanced 2018]

Tah03

Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is

(ADBST)

QUESTION [JEE Mains 2024 (27 Jan)]

Jan 04

The number of common terms in the progressions

4, 9, 14, 19, ..., up to 25^{th} term and 3, 6, 9, 12, ..., up to 37^{th} term is:

(A) 9

(B) 8

(C) 5

(D) 7

Ans. D

QUESTION [JEE Mains 2023 (1 Feb)]

Tah 05

The sum of the common terms of the following three arithmetic progressions.

(ADBST)

3, 7, 11, 15, ..., 399,

2, 5, 8, 11, ..., 359 and

2, 7, 12, 17, ..., 197,

is equal to ______

Ans. 321

QUESTION [JEE Mains 2025 (4 April)]

Tah06

Let $A = \{1, 6, 11, 16, ...\}$ and $B = \{9, 16, 23, 30, ...\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

(A) 3814

(B) 4003

(C) 4027

(D) 3761

Ans. D

QUESTION [JEE Mains 2021 (31 Aug)]

Tah07

Let $a_1, a_2, a_3, ...$ be an A.P. If $\frac{a_1+a_2+\cdots+a_{10}}{a_1+a_2+\cdots+a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to :

(A) $\frac{19}{21}$

(B) $\frac{100}{121}$

(C) $\frac{21}{19}$

(D) $\frac{121}{100}$

$$\frac{\frac{10}{2}(2a+9d)}{\frac{p}{2}(2a+(p-1)d)} = \frac{100}{p^2}$$

$$\frac{2a+9d}{2a+(p-1)d} = \frac{10}{p}$$

$$2ap + 9dp = 20a + 10dp - 10d$$

$$2a(p-10) = dp - 10d$$

$$2a(p-10) = d(p-10)$$

$$2a = d$$

$$\frac{a+10d}{a+9d} = \frac{a+20a}{a+18a} = \frac{21}{19}$$

Ans. C

QUESTION

KTK 1

For a, b $\in$ R – {0}, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x+\frac{7}{4}\right) = f\left(\frac{7}{4}-x\right)\ \forall\ x \in R$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.
The value of (a + b) is equal to

(A) 4

(B) 5

(C) 6

(D) 7

$f\left(\frac{7}{4}+x\right)=f\left(\frac{7}{4}-x\right)$

f is symmt about $x=\frac{7}{4}$

$f(x)=ax^2+bx+a$

symmt abt. $x=-\frac{b}{2a}$

$-\frac{b}{2a}=\frac{7}{4}$

$-2b=7a$.

f(x) = 7x+a has only one real & distinct soln.

$ax^2+bx+\cancel{a}=7x+\cancel{a}$

$ax^2+(b-7)x=0$

$D=(b-7)^2-4a\cdot 0=0$

$b=7$

$-14=7a$

$a=-2$

$a+b=5$

Ans. B

QUESTION

KTK 2

For a, b $\in$ R – {0}, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x+\frac{7}{4}\right) = f\left(\frac{7}{4}-x\right)\ \forall\ x \in R$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.
The minimum value f(x) in $\left[0, \frac{3}{2}\right]$ is equal to

(A) $-\frac{33}{8}$

(B) 0

(C) 4

(D) –2

$$f(x) = -2x^2 + 7x - 2$$
$$= -2\left(x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}\right) - 2$$
$$= -2\left(x - \frac{7}{4}\right)^2 + \frac{49}{8} - 2$$
$$= \frac{33}{8} - 2(x - 7/4)^2$$

Ans. D

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

Arithmetic Mean

- If a, b, c are in A·P then 'b' is called single A·M b/w a & c

$$b-a=c-b$$

$$2b=a+c \longrightarrow b=\frac{a+c}{2}$$

a, b, c are in A·P $\Leftrightarrow 2b=a+c$

- If a, $A_1, A_2, A_3 - - - , A_n$, b are in A·P then $A_1, A_2 - - , A_n$ are called n A·Ms b/w a & b.

$$T_1=a$$

$$T_{n+2}=b \Rightarrow b=a+(n+1)d$$

$$d=\frac{b-a}{n+1}$$

$$A_1 = a + d$$

$$A_2 = a + 2d$$

$$A_3 = a + 3d$$

$$\vdots \qquad \vdots$$

$$A_n = a + nd$$

$$A_1 + A_2 + \cdots + A_n = na + d(1 + 2 + 3 + \cdots + n)$$

$$\sum_{i=1}^{n} A_i = na + d \cdot \frac{n(n+1)}{2} = na + \frac{b-a}{\cancel{n+1}} \cdot \frac{n\cancel{(n+1)}}{2}$$

$$\sum_{i=1}^{n} A_i = na + \frac{n(b-a)}{2} = n\left(\frac{2a+b-a}{2}\right) = n\left(\frac{a+b}{2}\right)$$

$a, A_1, A_2, \ldots, A_n, b$ are in A.P

$\underbrace{A_1, A_2, \ldots, A_n}$

n A.Ms b/w a & b.

$$\sum_{i=1}^{n} A_i = n\left(\frac{a+b}{2}\right)$$

$\boxed{\text{Sum of } n \text{ A.Ms b/w 2 No.s} = n \cdot (\text{single A.M b/w the two no:s})}$

Ex: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

8 A.Ms b/w 2 & 20

Sum of 8 A.Ms = 4+6+8+10+12+14+16+18 = 88 = $8 \cdot \left(\frac{2+20}{2}\right)$

QUESTION

Insert 20 AM's between 4 and 67.

$4, A_1, A_2, ---, A_{20}, 67$ are in A.P

$T_{22} = 67 = 4 + 21d$

$d = \frac{63}{21} = 3$

20 A.Ms : 7, 10, 13, ------61, 64

QUESTION

If p arithmetic mean are inserted between 5 and 41 so that ratio $\frac{A_3}{A_{p-1}} = \frac{2}{5}$, then find the value of p.

M 1 $\quad 5, A_1, A_2 - - -, A_p, 41$ are in A.P

$T_4 = A_3 = 5 + 3d \qquad T_{p+2} = 41$

$T_p = A_{p-1} = 5 + (p-1)d \qquad 5 + (p+1)d = 41$

$(p+1)d = 36.$

$$\frac{A_3}{A_{p-1}} = \frac{5+3d}{5+(p-1)d} = \frac{2}{5}.$$

$25 + 15d = 10 + 2(p-1)d$

$\cancel{25} + 15 \cdot \frac{36}{p+1} = \cancel{10} + 2(p-1) \cdot \frac{36}{p+1}$

$15 + \frac{15 \cdot 36}{p+1} = \frac{72(p-1)}{p+1} \Rightarrow 15p + 15 + 540 = 72p - 72$

$57p = 627.$

$p = 11$

QUESTION

If p arithmetic mean are inserted between 5 and 41 so that ratio $\frac{A_3}{A_{p-1}} = \frac{2}{5}$, then find the value of p.

M(2)

$\overset{d}{\longrightarrow} \qquad \overset{-d}{\longleftarrow}$

$5, A_1, A_2 - - -, A_p, 41$ are in A.P

$$\frac{A_3}{A_{p-1}} = \frac{2}{5}$$

$$\frac{5+3d}{41-2d} = \frac{2}{5}.$$

$$25+15d = 82-4d$$

$$19d = 57$$

$$d = 3$$

$$T_{p+2} = 41$$

$$5+(p+1)d = 41$$

$$(p+1)d = 36.$$

$$p+1 = 12$$

$$p = 11$$

QUESTION

If $A_1, A_2, A_3, \ldots\ldots A_{51}$ are arithmetic means inserted between the numbers a and b, then find the value of $\left(\frac{b+A_{51}}{b-A_{51}}\right)-\left(\frac{A_1+a}{A_1-a}\right)$.

$$\overbrace{a, A_1, A_2, \ldots A_{51}, b}. \quad T_{53} = b = a + 52d$$

$$b - a = 52d$$

$$\text{let } E = \frac{b+A_{51}}{b-A_{51}} - \frac{(A_1+a)}{A_1-a}$$

$$= \frac{b+A_{51}}{d} - \frac{A_1+a}{d}$$

$$= \frac{b+A_{51}-A_1-a}{d}$$

$$= \frac{b+b-d-(a+d)-a}{d}$$

$$= \frac{2b-2a-2d}{d} = \frac{104d-2d}{d} = 102 \text{ Ans}$$

QUESTION

Tah 01

If $a_1 a_2 a_3$ ..... a_n are n arithmetic means inserted between 7 and 2015 whose sum is 56616 then

$7, a_1, a_2, \dots a_n, 2015.$

$a_1 + a_2 + \dots + a_n = 56616.$

(A) n is 56

(B) n is 28

(C) $a_{19} = 2029/3$

(D) $a_{19} = 36543/57$

QUESTION

If $S_1, S_2, S_3, \ldots\ldots, S_p$ are the sums of n terms of 'p' arithmetic series whose first term and 1, 2, 3, 4, …… and whose common difference are 1, 3, 5, 7, ……

Prove that $S_1 + S_2 + S_3 + \ldots\ldots + S_p = \frac{np}{2}(np+1)$

$1, 3, 5, 7 - - - (2n-1)$ ← T_n

$3, 5, 7, 9 - - - (2n+1)$ ← T_n

$A \cdot P_1 : \quad T_1 = 1, \; d = 1 \longrightarrow S_1 = \frac{n}{2} \cdot (2 \cdot 1 + (n-1) \cdot 1)$

$A \cdot P_2 : \quad T_1 = 2, \; d = 3 \longrightarrow S_2 = \frac{n}{2}(2 \cdot 2 + (n-1)3)$

$A \cdot P_3 : \quad T_1 = 3, \; d = 5 \longrightarrow S_3 = \frac{n}{2}(2 \cdot 3 + (n-1)5)$

$A \cdot P_p : \quad T_1 = p, \; d = 2p-1 \longrightarrow S_p = \frac{n}{2}(2p + (n-1)(2p-1))$

$S_1 + S_2 + S_3 + - - + S_p = \frac{n}{2}\left(2(1+2+3+ - - + p) + (n-1)(1+3+5+ - - + (2p-1))\right)$

$= \frac{n}{2}\left(\cancel{2}\frac{p(p+1)}{\cancel{2}} + (n-1)p^2\right)$

$$S_1 + S_2 + \cdots + S_p = \frac{np}{2}\left(\cancel{p} + 1 + p(n - \cancel{1})\right)$$

$$S_1 + S_2 + \cdots + S_p = \frac{np}{2}(np + 1)$$

QUESTION [JEE Mains 2023 (Jan)]

Number of 4 digit integers less than 2800 which are either divisible by 3 or by 11 is equal to

$$n(D_3 \text{ or } D_{11}) = n(D_3) + n(D_{11}) - n(D_3 \cap D_{11})$$

$$D_3 = \{1002,\ 1005,\ ---,\ 2799\} \quad 2799 = 1002 + (n-1)\cdot 3 \longrightarrow n = 600$$

$$D_{11} = \{1001,\ 1012,\ 1023,\ ---,\ 2794\} \quad 2794 = 1001 + (n-1)11 \quad n = 164$$

$$D_{33} = \{1023,\ 1056\ ---\ 2772\} \quad 2772 = 1023 + (n-1)33 \quad n = 54$$

$$n(D_3 \text{ or } D_{11}) = 600 + 164 - 54$$
$$= 600 + 110 = 710 \ \underline{\text{Ans}}$$

Ans. 710

QUESTION [JEE Advanced 2018]

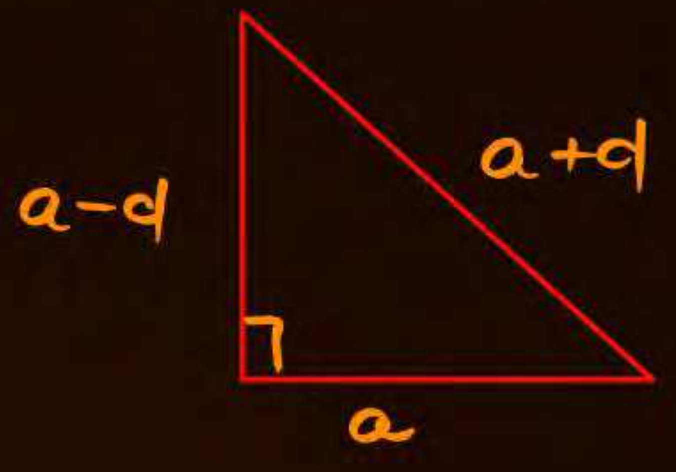
Tah02

The sides of a right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side?

$$\frac{1}{2}\cdot(a-d)\cdot a = 24$$

$$a(a-d) = 48$$

$$a^2 - ad = 48$$



Ans. 6

QUESTION [JEE Mains 2025 (4 April)]

Tan03

Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of A.P.'s in A and B respectively such that $D = d + 3, d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$, then $p - q$ is equal to

(A) 540

(B) 450

(C) 600

(D) 630

Ans. A

QUESTION [JEE Mains 2023 (31 Jan)]

Let $a_1, a_2, \ldots, a_n$ be in A.P. If $a_5 = 2a_7$ and $a_{11} = 18$, then

$12\left(\frac{1}{\sqrt{a_{10}}+\sqrt{a_{11}}} + \frac{1}{\sqrt{a_{11}}+\sqrt{a_{12}}} + \cdots + \frac{1}{\sqrt{a_{17}}+\sqrt{a_{18}}}\right)$ is equal to

$$a_{11} - 6d = 2(a_{11} - 4d)$$
$$18 - 6d = 2(18 - 4d)$$
$$9 - 3d = 18 - 4d$$
$$d = 9$$

$$12\left(\frac{\sqrt{a_{11}}-\sqrt{a_{10}}}{a_{11}-a_{10}} + \frac{\sqrt{a_{12}}-\sqrt{a_{11}}}{a_{12}-a_{11}} + \cdots + \frac{\sqrt{a_{18}}-\sqrt{a_{17}}}{a_{18}-a_{17}}\right)$$

$$\frac{12}{d}\left((\sqrt{a_{11}}-\sqrt{a_{10}}) + (\sqrt{a_{12}}-\sqrt{a_{11}}) + (\sqrt{a_{13}}-\sqrt{a_{12}}) + \cdots + (\sqrt{a_{18}}-\sqrt{a_{17}})\right)$$

$$\frac{12}{d}\left(\sqrt{a_{18}}-\sqrt{a_{10}}\right)$$

$$\frac{12}{9}\left(\sqrt{a_{11}+7d}-\sqrt{a_{11}-d}\right)$$

$$\frac{4}{3}(9-3) = 4 \cdot 2 = 8 \text{ Ans}$$

Ans. 8

QUESTION

$m+n = p+q$

$a_m + a_n = a_p + a_q$

$\underbrace{a_1+a_n}_{n+1} = \underbrace{a_2+a_{n-1}}_{n+1}$

Given $a_1, a_2, a_3, \dots . a_n$ in A.P. prove that

$$\frac{1}{a_1 a_n}+\frac{1}{a_2 a_{n-1}}+\frac{1}{a_3 a_{n-2}}+\cdots+\frac{1}{a_n a_1}=\frac{2}{a_1+a_n}\left[\frac{1}{a_1}+\frac{1}{a_2}+\ \dots+\frac{1}{a_n}\right]$$

$\underbrace{\text{LHS}}$

$$\frac{a_1+a_n}{a_1+a_n}\left(\frac{1}{a_1a_n}+\frac{1}{a_2a_{n-1}}+\frac{1}{a_3a_{n-2}}+---+\frac{1}{a_na_1}\right)=\frac{1}{a_1+a_n}\left(\frac{a_1+a_n}{a_1a_n}+\frac{a_1+a_n}{a_2a_{n-1}}+\frac{a_1+a_n}{a_3a_{n-2}}+--+\frac{a_n+a_1}{a_na_1}\right)$$

$$\frac{1}{a_1+a_n}\left(\frac{a_1+a_n}{a_1a_n}+\frac{a_2+a_{n-1}}{a_2a_{n-1}}+\frac{a_3+a_{n-2}}{a_3\cdot a_{n-2}}+--+\frac{a_1+a_n}{a_1a_n}\right)$$

$$\frac{1}{a_1+a_n}\left(\left(\frac{1}{a_n}+\frac{1}{a_1}\right)+\left(\frac{1}{a_{n-1}}+\frac{1}{a_2}\right)+\left(\frac{1}{a_{n-2}}+\frac{1}{a_3}\right)+--+\left(\frac{1}{a_1}+\frac{1}{a_n}\right)\right)$$

$$\frac{1}{a_1+a_n}\,2\cdot\left(\frac{1}{a_1}+\frac{1}{a_2}+\frac{1}{a_3}+--+\frac{1}{a_n}\right)$$

QUESTION [JEE Mains 2021 (August)]

Tah 04

Let $a_1, a_2, \ldots\ldots a_{21}$ be an A.P. such that $\sum_{n=1}^{20} \frac{1}{a_n a_{n+1}} = \frac{4}{9}$.

If the sum of this A.P. is 189, then $a_6 \cdot a_{16}$ is equal to:

(A) 57

(B) 72

(C) 48

(D) 36

$$S = \frac{1}{a_1 \cdot a_2} + \frac{1}{a_2 a_3} + \frac{1}{a_3 a_4} + \cdots + \frac{1}{a_{20} a_{21}} = \frac{4}{9}$$

QUESTION [JEE Mains 2024 (29 Jan)]

If $\log_e a, \log_e b, \log_e c$ are in an A.P. and $\log_e a - \log_e 2b, \log_e 2b - \log_e 3c, \log_e 3c - \log_e a$ are also in an A.P., then $a : b : c$ is equal to

(A) $6 : 3 : 2$

(B) $9 : 6 : 4$

(C) $25 : 10 : 4$

(D) $16 : 4 : 1$

$2\log_e b = \log_e a + \log_e c$

$\log_e b^2 = \log_e ac$

$b^2 = ac$

$2\log_e \frac{2b}{3c} = \log_e \frac{a}{2b} + \log_e \frac{3c}{a}$

$\left(\frac{2b}{3c}\right)^2 = \frac{\cancel{a}}{\cancel{2b}} \cdot \frac{3c}{\cancel{a}}$

$8b^3 = 27c^3$

$2b = 3c = 6\lambda$

$b = 3\lambda$

$c = 2\lambda$

$9\lambda^2 = a \cdot 2\lambda$

$a = \frac{9}{2}\lambda.$

$a : b : c = \frac{9}{2}\lambda : 3\lambda : 2\lambda = 9 : 6 : 4$

Ans. B

QUESTION [JEE Mains 2021 (27 July)]

Jah 05

If $\log_3 2, \log_3(2^x - 5), \log_3\left(2^x - \frac{7}{2}\right)$ are in an arithmetic progression, then the value of x is equal to

Ans. 3

QUESTION

How many terms of the series $20 + 19\frac{1}{3} + 18\frac{2}{3} + \ldots$ must be taken so that sum is 300. Explain the reason of double answer.

$d = -\frac{2}{3}$

$S_n = 300$

$\frac{n}{2}\left(40 + (n-1)\left(-\frac{2}{3}\right)\right) = 300$

$n\left(20 - \frac{n-1}{3}\right) = 300$

$n(61 - n) = 900$

$-n^2 + 61n = 900$

$n^2 - 61n + 900$

$n = 25, 36$

$T_n \leq 0$

$20 + (n-1)(-2/3) \leq 0$

$10 \leq \frac{n-1}{3}$

$n \geq 31$

$\underbrace{T_1, T_2, \ldots, T_{25}}_{S_{25} = 300} \; \underbrace{T_{26}, T_{27} \ldots}_{+ve} \; \overset{0}{T_{31}}, \underbrace{T_{32}, T_{33}}_{-ve}$

$S_{36} = 300$

(b) $S_{MAX} = S_{30} = S_{31}$

Sum inc till terms are +ve

$$S_{30} = \frac{30}{2}\left(40 + 29\cdot\left(-\frac{2}{3}\right)\right)$$

$$= \frac{15(120-58)}{3}$$

$$= 5\cdot 62$$

$$S_{MAX} = 310$$

G.P

आरंभ

G.P

Geometric Progression

Definition :

G.P. is the collection of non-zero terms in which each bears the same constant ratio with its immediately preceding term the series is called a G.P. and the constant ratio is called the common ratio.

Standard appearance of a G.P. where a is first term and r is common ratio is

agli Term ÷ Pechli Term = constant = common ratio

G.P: $a, ar, ar^2, ar^3, ar^4, ----$

$T_1 = a$ common ratio $= r$

$T_n = ar^{n-1}$

None of the terms of G.P can be zero

Sum of n terms of G.P

$$S_n = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$$

$$rS_n = ar + ar^2 + ar3 + \cdots + ar^{n-1} + ar^n.$$

$$(1-r)S_n = a - ar^n$$

$$S_n = \frac{a(1-r^n)}{1-r} = \frac{a(r^n-1)}{r-1}, \quad r \neq 1$$

If $r=1$

$$S = \underbrace{a + a + \cdots + a}_{n\text{ terms.}} = na$$

$$S_n = \begin{cases} \dfrac{a(r^n-1)}{r-1} & r \neq 1 \\ na & r = 1. \end{cases}$$

- A G·P of +ve terms is inc if $r > 1$
- A G·P of +ve terms is dec if $0 < r < 1$
- A G·P of −ve terms is inc if $0 < r < 1$
- A G·P of −ve terms is dec if $r > 1$

- If $r < 0$ then G·P is alternatingly composed of +ve & −ve terms Ex: $1, -2, 4, -8, 16, -32 - - -$
- If $r = 1$ then it is a constant G·P.

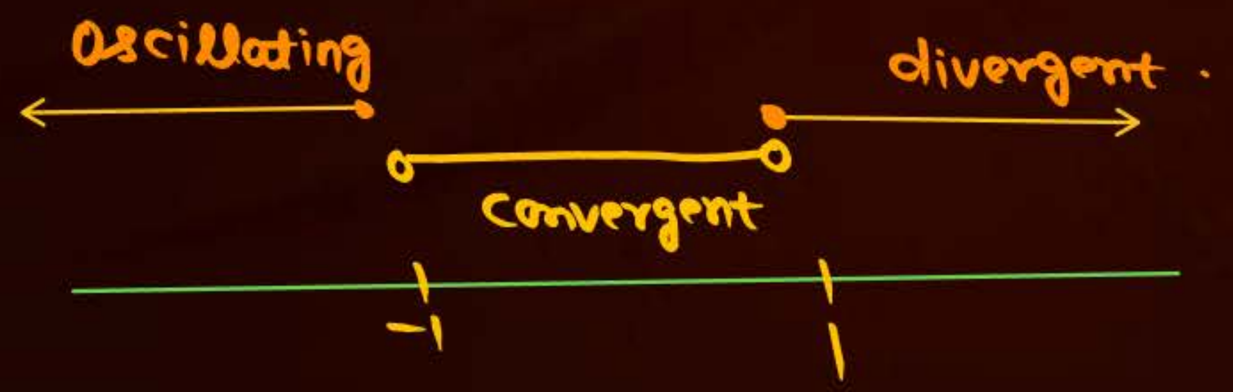
Sum of infinite Terms

$$S_\infty = a + ar + ar^2 + ar^3 + \text{----} \infty \qquad 0 < |r| < 1$$

$$-1 < r < 1,\ r \neq 0$$

$$S_\infty = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a(1-\overset{\to 0}{r^n})}{1-r} = \frac{a}{1-r}$$

$$\boxed{S_\infty = \frac{a}{1-r} \quad -1 < r < 1,\ r \neq 0}$$



* K^{th} from End in G.P of n terms $= (n-K+1)^{th}$ term from Begining

* T_K from End $= \ell \cdot \left(\frac{1}{r}\right)^{K-1}$

$$\xrightarrow{r} \qquad \xleftarrow{1/r}$$

$$a, ar, ar^2, ar^3, \ldots, \ell$$

Properties of G.P.

Property 1:

In an G.P. product of k^{th} term from beginning and k^{th} term from the last is always constant which equal to product of first term and last term.

$$T_k \cdot T_{n-k+1} = \text{constant} = a \cdot \ell$$

$T_K = a \cdot r^{K-1}$

T_K from End $= \ell \cdot (1/r)^{K-1}$

$T_K \cdot (T_K \text{ from End}) = a\ell.$

1 3 9 27 81 243

243

243

243

If $m, n, p, q \in N$ & $m+n = p+q$ then $T_m \cdot T_n = T_p \cdot T_q$ for a G.P

(prove yourself)

Property 2:

If product of first few terms is given

Three numbers in G.P. : $a/r, a, ar$ — $CR = r$

Five numbers in G.P. : $a/r^2, a/r, a, ar, ar^2$ — $CR = r$

first term is not a.

Four numbers in G.P. : $a/r^3, a/r, ar, ar^3$ — $CR = r^2$

Six numbers in G.P. : $a/r^5, a/r^3, a/r, ar, ar^3, ar^5$ — $CR = r^2$

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Solution to Previous TAH

QUESTION [JEE Advanced 2015]

Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is $6 : 11$ and the seventh term lies in between 130 and 140, then the common difference of this A.P. is

TAH-01

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Q) Suppose that all the terms of an A.P are natural numbers. If the ratio of the sum of the first seven terms to the first eleven terms is 6:11 and the seventh term lies between 130 and 140. then the common difference of the AP is:

Sol^n:

all terms of AP are natural given

$$\frac{\frac{7}{2}(2a+6d)}{\frac{11}{2}(2a+10d)} = \frac{6}{11}$$

$$\frac{2a+6d}{2a+10d} = \frac{6}{7} \qquad 130 < T_7 = a+6d < 140$$

$$\frac{a+3d}{a+5d} = \frac{6}{7}$$

$7a + 21d = 6a + 30d$

$a = 9d$

$$T_7 = a+6d = \underline{15d}$$

factor 15's number lying between 130 and 140 is 135
so the value of 15d is 135 $\Rightarrow d = 9$

$a = 9d \Rightarrow a = 81$ and $d = 9$

Lecture-02 (Sequence & Series) JEE Adv. - 2013

Tah-01] Suppose that all the terms of an arithmetic progression (A.P) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first 11 terms is 6:11 and the seventh term lies in b/w 130 and 140, then the common difference of this A.P is :-

$$\Rightarrow \frac{\frac{7}{2}(2a+6d)}{\frac{11}{2}(2a+10d)} = \frac{6}{11} \quad \& \quad \boxed{130 < a+6d < 140}$$

$$\Rightarrow \frac{7(2a+6d)}{2a+10d} = 6$$

$$\Rightarrow \frac{7(a+3d)}{a+5d} = 6 \Rightarrow \boxed{7a+21d = 6a+30d}$$

$a = 9d$

as, $130 < a+6d < 140$

$130 < 15d < 140$

$$\frac{\cancel{130}^{26}}{\cancel{15}_{3}} < d < \frac{\cancel{140}^{28}}{\cancel{15}_{3}}$$

$$\frac{26}{3} < d < \frac{28}{3}$$ as the terms of the A.P are natural numbers

$$\boxed{d = \frac{27}{3} = 9} \text{ (Ans)}$$

Kritisha

QUESTION [JEE Mains 2022 (27 July)]

TAH

Suppose $a_1, a_2, ..., a_n$ ..... be an arithmetic progression of natural numbers. If the ratio of the sum of first five terms to the sum of first nine terms of the progression is 5 : 17 and, $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to

(A) 290

(B) 380

(C) 460

(D) 510

Ans. B

Tah2

$a_1, a_2, \ldots a_n \rightarrow A.P.$

$\frac{S_5}{S_9} = \frac{5}{17}$

$\frac{5a+10d}{9a+36d} = \frac{5}{17}$

$\frac{a+2d}{a+4d} = \frac{9}{17}$

$17a - 9a = 36d - 34d$

$\boxed{4a = d}$

$110 < a_{15} < 120$

$110 < a + 14d < 120$

$110 < a + 56d < 120$

$\frac{110}{57} < a < \frac{120}{57}$

$1.9 < a < 2.1$

$\boxed{a = 2}$ $\boxed{d = 8}$

SAKSHI

Now we find Sum of first ten terms $= S_{10} = a_1 + a_2 + a_3 \ldots + a_{10}$

$S_{10} = \frac{10}{2}[2\times2 + 9\times8]$

$S_{10} = 5(4+72) = 380$ Ans (B)

of natural numbers. If the ratio of the sum of first five numbers terms to the sum of first nine terms of the progression is 5 : 17 and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to:

$\Rightarrow \frac{S_5}{S_9} = \frac{\frac{5}{2}(2a+4d)}{\frac{9}{2}(2a+8d)} = \frac{5}{17}$

$\Rightarrow \frac{a+2d}{9(a+4d)} = \frac{1}{17}$

$\Rightarrow 17a + 34d = 9a + 36d$

$\Rightarrow \boxed{8a = 2d}$

$a_{15} = (a+14d)$ acc. to the question,

$110 < (a+14d) < 120$

$110 < 57a < 120$

$\frac{110}{57} < a < \frac{120}{57}$

Kritisha

as $a \in N$

$a = \frac{114}{57} = 2$; $d = 8$

$S_{10} = 5(4 + 9(8)) = 5(4+72) = 20 + 360 = 380$ (B) Ans.

QUESTION [JEE Mains 2025 (29 Jan)]

TAH

Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11^{th} term is :

(A) 108

(B) 90

(C) 122

(D) 84

Ans. B

TAH-03

$S_3 = 54$

$\frac{3}{2}(2a+2d) = 54$

$3(a+d) = 54$

$a+d = 18$

$\boxed{a = 18-d}$

$1600 < S_{20} < 1800$ — (1)

$\therefore S_{20} = \frac{20}{2}(2a+19d)$

$10(2(18-d)+19d)$

$10(36-2d+19d)$

$360+170d$

Put in (1)

$1600 < 360+170d < 1800$

$1240 < 170d < 1440$

$7.2 < d < 8.4$

$\boxed{d = 8}$

$\boxed{a = 10}$

$T_{11} = a+10d$

$= 10+10(8)$

$\boxed{\text{Ans} = 90}$

Tah3 $a_1+a_2+a_3 = 54$

$a+a+d+a+2d = 54$

$3a+3d = 54$

$a+d = 18$

$a = 18-d$

$\boxed{a = 10}$

$1600 < S_{20} < 1800$

$1600 < \frac{20}{2}(2(18-d)+19d) < 1800$

$1600 < 360+170d < 1800$

$\frac{124\cancel{0}}{17\cancel{0}} < d < \frac{144\cancel{0}}{17\cancel{0}}$

$7.2 < d < 8.4$

$\boxed{d = 8}$

$a_{11} = ?$

Now, $a_{11} = a+10d$

$a_{11} = 10+80$

$a_{11} = 90$ Ans

SAKSHI

tah-03 Consider an A.P. of positive integers, whose sum of the first three terms is 54 and the sum of the first twenty terms lies between 1600 and 1800. Then its 11^{th} term is :-

$\longrightarrow$ $S_3 = \frac{3}{2}(2a+2d) = 54$ 18 ; $a+d=18$

$S_{20} = \frac{20}{2}(2a+19d)$

$1600 < 10(2a+19d) < 1800$

$160 < (2a+19d) < 180$

$160 < 2(18-d)+19d < 180$

$160 < 36+17d < 180$

$\frac{124}{17} < d < \frac{144}{17}$

$d=8$ [as terms of A.P are +ve integers.
$a=10$

$t_{11} = a+10d$
$= 10+80 = 90$ (B) (Ans)

Kritisha

let A.P of positive integers

given. $S_3 = \frac{3}{2}(2a+2d) = 54$ 18

$\underline{a+d=18}$

and. $1600 < S_{20} < 1800$

Hint

$2a+19d$
$(a+d)+(a+d)+17d$
$18+18+17d$
$36+17d$

$1600 < \frac{20}{2}(2a+19d) < 1800$

$1600 < 10(36+17d) < 1800$

$160-36 < 17d < 180-36$

$\frac{124}{17} < d < \frac{144}{17}$

$7.2 < d < 8.9$

$\boxed{d = 8 \in I}$, $a = 18-8 = 10$

Now. $t_{11} = a + 10d$
$= 10 + 10\times 8$
$= 90$ Ans

tah-3 by vandana from bihar

QUESTION [JEE Advanced 2018]

Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ..., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ... Then, the number of elements in the set $X \cup Y$ is upto 2018 terms

$X = 1, 6, 11, 16, \ldots$ $\quad d_1 = 5$
$Y = 9, 16, 23, \ldots$ $\quad d_2 = 7$ $\Big\}\ 35 = D$

$A = 16$
$D = 35$

$a_{n_1} = a_1 + (n-1)d_1$
$1 + 2017(5)$
10086

$a_{n_2} = a_2 + (n-1)d_2$
$= 9 + 2017 \times 7$
$= 14128$

$16 + (n-1)35 \leq 10086$
$(n-1)35 \leq 10070$
$(n-1) \leq 287.7$
$n \leq 288.7$

$n \leq 288$
$\hookrightarrow (X \cap Y)$

$n(X) = 2018$
$n(Y) = 2018$
$n(X \cap Y) = 288$
$n(X \cup Y) = 2018 + 2018 - 288$

$\boxed{n(X \cup Y) = 3748}$

Ans

Sol^n:

$X: 1, 6, 11, 16, 17, \ldots$ upto 2018 terms
$Y: 9, 16, 23, \ldots$ upto 2018 terms

$d_X = 5$
$d_Y = 7$ $> LCM = 35$

$\therefore (X \cap Y) = 16, 54, \ldots$

$X_{2018} = 1 + (2017 \times 5) = 10086$
$Y_{2018} = 9 + (2017 \times 7) = 14128$

let $(X \cap Y)$ has n terms

$(X \cap Y)_{n^{th}} = a + (n-1)d \leq 10086$
$16 + (n-1)(35) \leq 10086$
$(n-1) \leq \frac{10070}{35}$
$n - 1 \leq 287.7\ldots$
$n - 1 = 287$
$n = 288$

$\therefore n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$
$= 2018 + 2018 - 288$
$= 3748$

Ans 3748

RASIDUL

Jan 04 :- Let X be the set consisting of the first 2018 terms of the A.P 1, 6, 11 ..... and Y be the set consisting of the first 2018 terms of the A.P, 9, 16, 23 .... Then the no. of elements in the set X∪Y is

⇒ We have to find the common terms in X & Y

$X = \{1, 6, 11, 16 \ldots \ldots 2018^{th} \text{ term}\}$

$Y = \{9, 16, 23 \ldots \ldots 2018^{th} \text{ term}\}$

LCM of Common diff ⇒ 35

hence, A.P, consisting of the common terms of X & Y would have common diff = 35 & 1st term = 16

Now, 2018^{th} term of $X \longrightarrow 1 + (2017)(5) = 10086$

" " " $Y \longrightarrow 9 + 2017(7) = 14128$

(Last term of new AP) < 10086

$10086 > 16 + 35(n-1)$

$\frac{10070}{35} > n-1$

$\boxed{n < 288.71}$

putting $n = 288$

$16 + 35(287) = 10061$

$(X \cap Y) = 16, 51, 86, \ldots, 10061$ & $n(X \cap Y) = 288$ ✓

Thus, $n(X \cup Y) = n(X) + n(Y) - 288$

$= 2018 + 2018 - 288$

$n(X \cup Y) = 3748$ (Ans)

Kritisha

Jan 4

$n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$ —①

$n(X) = 2018$

$n(Y) = 2018$

1, 6, 11, 16 .... $T_n = 10086$ and 9, 16, 23, 30 .... $T_n = 14128$

d = 5   d = 7

LCM (35)

Common A.P. ⇒ 16, 51, 86 ...... d = 35

$T_n \le 10086$ and $T_n \le 14128$

$T_n \le 10086$

$a + (n-1)d \le 10086$

$16 + 35n - 35 \le 10086$

$35n \le 10105$

$n \le \frac{10105}{35}$

$n \le 288.71$ $n \approx 288$

from eqn ①

$n(X \cup Y) = 2 \times 2018 - 288$

$= 4036 - 288$

$= 3748$ Ans

SAKSHI

DATE
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Gyan

QUESTION [JEE Mains 2024 (27 Jan)]

The number of common terms in the progressions

4, 9, 14, 19, ..., up to 25^{th} term and 3, 6, 9, 12, ..., up to 37^{th} term is:

(A) 9

(B) 8

(C) 5

(D) 7

Ans. D

Tab 5

4, 9, 14, 19 .... $T_n = 124$ and 3, 6, 9, 12 .... $T_n = 111$

↳ upto 25^{th} $d = 5$ ↳ upto 37^{th} $d = 3$

common diff. of comm term A.P. = 15

common A.P. → 9, 24, 39 ..... $d = 15$

$T_n \le 111$ & $T_n < 124$

↓

$9 + (n-1)15 \le 111$

$9 + 15n - 15 \le 111$

$15n \le 117$

$n \le 7.8$

almost $n = 7$ An

$T_n = 371$

Tah-05 The no. of common terms in the
4, 9, 14, 19 ..... upto 25^{th} term,
3, 6, 9, 12 ...... " 37^{th} term is

$\Rightarrow (A.P)_1 = 4, 9, 14, 19 \ldots . 25^{th}$ term
$(A.P)_2 = 3, 6, 9, 12 \ldots . 37^{th}$ term

Last term of $(A.P)_1 \Rightarrow \cancel{4(24\times5}$
$= 4 + 24(5) = 124$

Last term of $(A.P)_2 = 3 + 3(36) = 111$

common diff. of new A.P $\Rightarrow 15$
1st term " " " $\Rightarrow 9$
Last " " " " $\Rightarrow < 111$

Now,
$111 > 9 + 15(n-1)$

$\frac{102+15}{15} > n$

$n < 7.8 \Rightarrow \boxed{n = 7}$ (Am)
(D)

Kritisha

Sol For 1st AP : $T_n = 4 + (25-1)5$
$= 4 + 24\times5$
$= 4 + 120$
$= 124$

For 2nd AP : $T_n = 3 + (37-1)3$
$= 3 + 36\times3$
$= 3 + 108$
$= 111$

AP_1 : 4, 9, 14, 19, ... 124, $d_1 = 5$
AP_2 : 3, 6, 9, 12, ..., 111, $d_2 = 3$
LCM = 15

New AP : 9, 24, 39, ... T_n

$\Rightarrow T_n \le 124$ & $T_n \le 111$
$\Rightarrow 9 + (n-1)15 \le 111$
$\Rightarrow (n-1)15 \le 102$
$\Rightarrow (n-1) \le \frac{102}{15}$
$\Rightarrow n-1 \le 6.8$
$\Rightarrow n \le 7.8$

$\boxed{n = 7}$ Ans

Aniket raj
From patna

Galaxy F15 5G

Question :- [JEE Mains 2024]

$4, 9, 14, 19 \ldots$

$n = 25$
$a = 4$
$d = 5$

$a_n = a + (n-1)d$
$= 4 + 24 \times 5$
$= 124$

$3, 6, 9, 12 \ldots$

$n = 37$
$a = 3$
$d = 3$

$a_n = 3 + 36 \times 3$
$= 111$

$4, 9, 14, 19 \ldots 124$ $\quad d = 5$
$3, 6, 9, 12 \ldots 111$ $\quad d = 3$

L.C.M = 15

$a = 9$, $d = 15$ and $T_n \leq 111$

$a + (n-1)d \leq 111$

$9 + 15(n-1) \leq 111$

$n \leq \frac{111 - 9 + 15}{15}$

$n \leq \frac{117}{15}$

$n \leq 7$ ———— Option D

QUESTION [JEE Mains 2023 (1 Feb)]

The sum of the common terms of the following three arithmetic progressions.

3, 7, 11, 15, ..., 399,

2, 5, 8, 11, ..., 359 and

2, 7, 12, 17, ..., 197,

is equal to ______

Ans. 321

the sum of the common terms of the following three arithmetic progressions

3, 7, 11, 15 399

2, 5, 8, 11 ...... 359 and

2, 7, 12, 17, ... 197 is equal to

→

3, 7, 11, 15, ... 399
2, 5, 8, 11 359
2, 7, 12, 17 197

LCM(4, 3) = 12
com. terms → 11, 23, 35, 47 .....

LCM(3, 5) = 15
Com. terms → 2, 17, 32, 47 ....

common terms in all three A.P
⇒ LCM(12, 15)
⇒ 60

New A.P ⇒ 47, 107, 167 [LAST TERM CAN'T EXCEED 197

Thus, sum of all the terms → 47 + 107 + 167
= 321 (Ans)

Kritisha

(d=4) ← P = 3, 7, 11, 15, ... 399
(d=3) ← Q = 2, 5, 8, 11 359
(d=5) ← R = 2, 7, 12, 17 ... 197

P or Q से
common term = 11
common difference = L.C.M = 4 × 3 = 12

(d=12) ← PQ ⇒ 11, 23, 35, 47, 59, 71 359

Q or R से,
common term = 2
" difference = L.C.M = 3 × 5 = 15

(d=15) ← QR ⇒ 2, 17, 32, 47, 62, ... 197

(a) common terms = 47
(d) " diffrence = L.CM = |12, 15 = 60

$T_n \le 197$

$47 + (n-1)60 \le 197$

$n \le \frac{197 - 47 + 60}{60}$

$n \le \frac{210}{60}$

manik bihar

$n \le 3.$

Sum of common term $S_3 = \frac{3}{2}(2a + (n-1)d)$

$= \frac{3}{2}(2 \times 47 + 2 \times 60)$

$= 3(47 + 60)$

QUESTION [JEE Mains 2025 (4 April)]

Let $A = \{1, 6, 11, 16, ...\}$ and $B = \{9, 16, 23, 30, ...\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

(A) 3814

(B) 4003

(C) 4027

(D) 3761

Ans. D

lah-07 Let $A = \{1, 6, 11, 16 \ldots\}$ and $B = \{9, 16, 23, 30 \ldots\}$ be the sets consisting of the first 2025 terms of two arithmetic progressions. Then $n(A \cup B)$ is

$\Rightarrow \quad A = \{1, 6, 11, 16 \ldots 2025^{th}\text{ term}\}$

$B = \{9, 16, 23, 30 \ldots 2025^{th}\text{ term}\}$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Last term of $A \Rightarrow 1 + 2024(5) = 10121$

" " " $B \Rightarrow 9 + 2024(7) = 14177$

common diff of $(A \cap B) \Rightarrow 35$

1st term " $(A \cap B) \Rightarrow 16$

Last " " $(A \cap B) < 10121$

~~35~~ $16 + 35(n-1) < 10121$

$35(n-1) < 10105$

$(n-1) < 288.71$

$n < 289.71$

$n = 289$

$n(A \cap B) = 289$

hence, $n(A \cup B) = (2025) \times 2 - 289$

$n(A \cup B) = 3761$

Kritisha

IAH → 07

Question [JEE Mains 2025 (4 April)]

$A = 1, 6, 11, 16 \ldots$

$a = 1, d = 5, n = 2025$

$a_n = 1 + (2024) \times 5$

$= 1 + 10110$

$= 10111$

Now

$1, 6, 11, 16, \ldots 10111$

$B = 9, 16, 23, 30 \ldots$

$a = 9, d = 7, n = 2025$

$a_n = 9 + (2024) \times 7$

$= 9 + 14168$

$= 14177$

Now

$9, 16, 23, 30 \ldots 14177$

Common term $(a) = 16$

Common difference $(d) = L.CM = 5 \times 7 = 35$

$T_n \leq 10111$

$a + (n-1)d \leq 10111$

$16 + (n-1)35 \leq 10111$

$n \leq \dfrac{10111 - 16 + 35}{35}$

$n \leq \dfrac{10130}{35}$

$35)10130(289$
70
313
280
$\times 330$

$(A \cap B) \rightarrow n \leq 289$

$(A \cup B) = n(A) + n(B) - (A \cap B)$

$= 2025 + 2025 - 289$

$= 4050 - 289$

$= 3761$

[JEE-Mains-2025]

Tah-07 let $A = \{1, 6, 11, 16 \ldots\}$ and $B = \{9, 16, 23, 30 \ldots\}$ be the sets consisting of the first 2025 terms of two A.P. Then $n(A \cup B)$ is

A. 3814 C. 4027

B. 4003 ~~D.~~ 3761

$A = \{1, 6, 11, 16 \ldots\} \longrightarrow d_1 = 5$

$B = 9, 16, 23, 30 \longrightarrow d_2 = 7$

$A \cap B = 16, 51, 86 \ldots \quad d = 35$

$a = 16$ $\quad$ 2025^{th} term of A

$d = 35$ $\quad$ $T_{2025} = 1 + (2024)5$

2025^{th} term of B $\quad$ T_{2025} 10121

$T_{2025} = 9 + (2024)7$

$T_{2025} = 9 + 14168 \rightarrow 14177$

Let n^{th} term of $A \cap B = 10191$

$$a_n = a + (n-1)d$$

$$10191 = 16 + (n-1)35$$

$$n-1 = \frac{10191-16}{35}$$

$$n-1 = \frac{10105}{35}$$

$$\boxed{n = 289}$$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B) = 2025 + 2025 - 289$$

$$n(A \cup B) = 4050 - 289$$

$$\boxed{n(A \cup B) = 3761} \text{ (D)}$$

[Mains-2021]

QUESTION [JEE Mains 2021 (31 Aug)]

Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\frac{a_1+a_2+\cdots+a_{10}}{a_1+a_2+\cdots+a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ is equal to :

(A) $\frac{19}{21}$

(B) $\frac{100}{121}$

(C) $\frac{21}{19}$

(D) $\frac{121}{100}$

Ans. C

Jah-08

Let $a_1, a_2, a_3 \ldots$ be an A.P. If $\frac{a_1+a_2+\cdots+a_{10}}{a_1+a_2+\cdots+a_p} = \frac{100}{p^2}$

$p \neq 10$ then $\frac{a_{11}}{a_{10}}$ is equal to:—

$$\Rightarrow \frac{a_1+a_2+a_3+\cdots a_{10}}{a_1+a_2+\cdots+a_p} = \frac{100}{p^2} \quad (p \neq 10)$$

$$\Rightarrow \frac{\frac{10}{2}(2a_1+9d)}{\frac{p}{2}(2a_1+(p-1)d)} = \frac{100}{p^2}$$

$$\Rightarrow \frac{2a_1+9d}{2a_1+(p-1)d} = \frac{10}{p}$$

$$2a_1p + 9dp = 20a_1 + 10dp - 10d$$

$$\Rightarrow 2a_1(10-p) + dp - 10d = 0$$

$$\Rightarrow 2a_1(10-p) - d(10-p) = 0$$

$$\Rightarrow (2a_1-d)(10-p) = 0 \rightarrow \cancel{a_1 = d/2}$$

$p \neq 10$ hence, $2a_1 = d$

$$\boxed{a_{10} = a_1 + 9d}$$

$$\boxed{a_{11} = a_1 + 10d}$$

$$\frac{a_{11}}{a_{10}} = \frac{a_1+10d}{a_1+9d} = \frac{21a_1}{19a_1} = \frac{21}{19} \text{ (Ans)}$$

Kritisha

Jah-08 Let $a_1, a_2, a_3 \ldots$ be an AP. If $\frac{a_1+a_2+\ldots a_{10}}{a_2+a_1+\ldots a_p} = \frac{100}{p^2}$, $p \neq 10$, then $\frac{a_{11}}{a_{10}}$ equal to

(A) $19/21$

(C) $\frac{21}{19}$

(B) $\frac{100}{121}$

(D) $\frac{121}{100}$

$$\frac{a_1+a_2+\cdots+a_{10}}{a_1+a_2+\cdots+a_p} = \frac{100}{p^2}$$

$$\frac{\frac{10}{2}[2a_1+9d]}{\frac{p}{2}[2a_1+(p-1)d]} = \frac{100}{p^2}$$

$$\Rightarrow 2a_1p + 9dp = 20a_1 + 10dp - 10d$$

$$2a_1(10-p) + dp - 10d = 0$$

$$2a_1(10-p) - d(p-10) = 0$$

$$(2a_1+d)(10-p) = 0$$

$a_1 = \frac{d}{2}$; $p = 10$ not possible $p \neq 10$

$$\frac{a_1}{d} = \frac{1}{2}$$

$$\boxed{\frac{d}{a_1} = 2}$$

$$\frac{a_{11}}{a_{10}} \Rightarrow \frac{a_1+10d}{a_1+9d} = \frac{1+10\frac{d}{a_1}}{1+9\frac{d}{a_1}}$$

$$\frac{1+10\times2}{1+9\times2} \Rightarrow \frac{21}{19} \text{ (C)}$$

$$\boxed{\frac{a_{11}}{a_{10}} = \frac{21}{19}}$$

Solution to Previous KTKs

QUESTION

KTK 1

For $a, b \in R - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right) \forall x \in R$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.
The value of $(a + b)$ is equal to

(A) 4

(B) 5

(C) 6

(D) 7

Ans. B

QUESTION

KTK 2

For a, b $\in$ R − {0}, let $f(x) = ax^2 + bx + a$ satisfies $f\left(x + \frac{7}{4}\right) = f\left(\frac{7}{4} - x\right)\ \forall\ x \in R$. Also the equation $f(x) = 7x + a$ has only one real and distinct solution.

The minimum value f(x) in $\left[0, \frac{3}{2}\right]$ is equal to

(A) $-\frac{33}{8}$

(B) 0

(C) 4

(D) −2

Ans. D

$f\left(\frac{7}{4}-x\right) \forall x \in R$. Also the eq^n $f(x) = 7x + a$ has only one real ~~root~~ and distinct sol^n. Then value of $(a+b) = ?$

(A) 4
(B) 5
(C) 6
(D) 7

$$f\left(x+\frac{7}{4}\right) = f\left(\frac{7}{4}-x\right)$$

$\frac{7}{4} \rightarrow$ Symmetry

$f(x) = ax^2 + bx + a$ $\qquad$ $f(x) = 7x + a$

$$\frac{-b}{2a} = \frac{7}{4}$$

$$b = \frac{-7}{2}a$$

$$ax^2 + bx + \cancel{a} = 7x + \cancel{a}$$

$$ax^2 + x(b-7) = 0$$

$$(b-7)^2 - 4(a)(0) = 0$$

$$(b-7)^2 = 0$$

$$b = 7$$

$$b = \frac{-7 \times a}{2}$$

$$7 = \frac{-7}{2}a$$

$$\boxed{a = -2 \quad b = 7}$$

$a + b \Rightarrow -2 + 7$

$$\boxed{a+b=5}$$

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KTK-2 For $a, b \in R - \{0\}$ - Same question
The minimum value $f(x)$ in $\left[0, \frac{3}{2}\right]$ is equal to

(A) $-33/8$
(B) 0
(C) 4
(D) -2

$a = -2$ means downward parabola

$$f(x) = -2x^2 + 7x - 2$$

$x = \frac{7}{4} \rightarrow$ vertex

$f(0) = -2$ $\qquad$ $f(1) = -2 + 7 - 2 \Rightarrow 3$

min value $= -2$

KTK-01

Q. For $a, b \in R - \{0\}$, let $f(x) = ax^2 + bx + a$ satisfies; $f\left(x+\frac{7}{4}\right) = f\left(\frac{7}{4}-x\right) \forall x \in R$, Also the equation $f(x) = 7x + a$ has only one real and distinct sol^n; The value of $(a+b)$ is equal to:

$\Rightarrow$ $f(x) = ax^2 + bx + a$, is satisfies: $f\left(x+\frac{7}{4}\right) = f\left(-x+\frac{7}{4}\right)$

$\Rightarrow$ To see this eq^n:

Clearly graph is sym about $\boxed{x = \frac{7}{4}}$

$\Rightarrow \frac{-b}{2a} = \frac{7}{4}$

$\Rightarrow \boxed{7a = -2b}$

$\Rightarrow f(x) = ax^2 + bx + a$ & $f(x) = 7x + a$

$\Rightarrow 7x + \cancel{a} = ax^2 + bx + \cancel{a}$

$\Rightarrow ax^2 + (b-7)x = 0$ (only one real & Distinct sol^n)

$\Rightarrow (b-7)^2 - 4 \cdot a \cdot 0 = 0$ $\qquad \boxed{D = 0}$

$\Rightarrow b - 7 = 0$

$\Rightarrow \boxed{b = 7}$ $\qquad$ $7a = (-2)(7)$

$7a = -14$

$\boxed{a = -2}$

krish

$\Rightarrow$ find value of $(a+b)$.

$a + b = -2 + 7$

$= 5$ Ans.


THANK
YOU