
MANZIL
FOR JEE ASPIRANTS
Mathematics
Determinants
In One Shot
By – Tarun Khandelwal Sir
B-Tech IIT Delhi, (A.I.R. 289)
Physics Wallah

Topics *to be covered*

1. Minor & Cofactors
2. Area of Triangle
3. Properties of Determinants
4. Some Important Determinants to Remember
5. Summation & Multiplication of Determinants
6. Differentiation of Determinants
7. Cramer's Rule

FOR NOTES & DPP CHECK DESCRIPTION

Determinant

$$A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix}_{(2\times 3)}$$

$$A = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}_{(2\times 2)}$$

$$|A| = \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 5 & 7 \end{vmatrix} = 7\times 1 - 5\times 3 = 7 - 15 = -8$$

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Minor of an Element

The **determinant obtained by deleting** the row and the column in which the given element lies.

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$M_a = d$

$M_b = c$

$M_c = b$

$M_d = a$

$i=1$, $j=1$

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\boxed{M_{a_1} = b_2c_3 - c_2b_3}$ = Cofactor

$M_{b_2} = a_1c_3 - a_3c_1$ = Cofactor

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Cofactor of an Element a_{ij}

C_{ij}

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$C_{21} = -M_{21} = -(a_{12}a_{33} - a_{32}a_{13})$

Cofactor of the element a_{ij} is denoted by C_{ij} and is given by

$\boxed{C_{ij} = (-1)^{i+j} M_{ij}}$

Where i and j denotes the row and column in which the particular element lies.

If $i+j =$ Even $\Rightarrow C_{ij} = M_{ij}$

If $i+j =$ odd $\Rightarrow C_{ij} = -M_{ij}$

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$A = (-4)(-3) + 3(27) + 2(11)$

QUESTION

If $A = \begin{vmatrix} 5 & 6 & 3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$, then cofactors of the elements of 2^{nd} row are

Minor $= 18 + 21 = 39$

Cofactor $= -39$

$5(-39) + 6 \times 27 + 3 \times 11$

$= -39 \times 5 + 162 + 33$

$-\cancel{195} + \cancel{195}$

$= 0$

(A) 39, −3, 11

(B) −39, 3, 11

Minor $= 15 + 12 = 27 =$ Cofactor

(C) −39, 27, 11

Minor $= -35 + 24 = -11$

Cofactor $= 11$

(D) −39, −3, 11

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Expansion of a Determinant

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\boxed{a_{11}C_{21} + a_{12}C_{22} + a_{13}C_{23} = 0}$$

$$a_{12}C_{13} + a_{22}C_{23} + a_{32}C_{33} = 0$$

Expansion along R_1

$$\Delta = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

Expansion along R_3 $\Delta = a_{31}C_{31} + a_{32}C_{32} + a_{33}C_{33}$

Exp. along C_1 : $\Delta = a_{11}C_{11} + a_{21}C_{21} + a_{31}C_{31}$

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Note

(i) The sum of the product of elements of any row (or column) with their corresponding cofactors is always equal to the value of the determinant.

(ii) The sum of the product of elements of any row (or column) with the cofactors of other row (or column) is always equal to zero.

QUESTION

If in the determinant $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$, A_i, B_i, C_i etc. be the co-factors of a_i, b_i, c_i etc., then which of the following relations is incorrect

(A) $a_1A_1 + b_1B_1 + c_1C_1 = \Delta$ True

(B) $a_2A_2 + b_2B_2 + c_2C_2 = \Delta$ → True

(C) $a_3A_3 + b_3B_3 + c_3C_3 = 0$ → False

(D) $a_1B_1 + a_2B_2 + a_3B_3 = 0$ → True.

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QUESTION [1981, 2]

[Ans. 0]

Let $\boxed{p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t} = \begin{vmatrix} \lambda^2+3\lambda & \lambda-1 & \lambda+3 \\ \lambda+1 & -2\lambda & \lambda-4 \\ \lambda-3 & \lambda+4 & 3\lambda \end{vmatrix}$

be an identity in λ, where p, q, r, s and t are constants. Then, the value of t is

$p+q+r+s+t = ?$

Put $\lambda = 1$

Put $\boxed{\lambda=0}$

$$t = \begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix}$$

→ direct 0

→ skew symm

$t = 0\times \;+1[0-12]+3[4]$

$t = -12+12 = 0$

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QUESTION [JEE Main 2023 (11 Apr Shift 2)]

[Ans. D]

If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix} = \frac{9}{8}(103x+81)$, for all $x \in R$

then $\lambda, \frac{\lambda}{3}$ are the roots of the equation

(A) $4x^2 + 24x - 27 = 0$

(B) $4x^2 - 24x - 27 = 0$

(C) $4x^2 + 24x + 27 = 0$

(D) $4x^2 - 24x + 27 = 0$

Put $\boxed{x = 0}$

$$\rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^2 \end{vmatrix} = \frac{9 \times 81}{8}$$

$$1[\lambda^3 - 0] = \frac{9 \times 81}{8}$$

$$\lambda^3 = \frac{9^3}{2^3} \Rightarrow \boxed{\lambda = 9/2}$$

$\lambda = 9/2 = \alpha$

$\lambda/3 = \frac{9/2}{3} = 3/2 = \beta$

$$\boxed{\begin{matrix} \alpha = 9/2 \\ \beta = 3/2 \end{matrix}}$$

$$\boxed{\begin{matrix} \alpha + \beta = 6 \\ \alpha\beta = 27/4 \end{matrix}}$$

$x^2 - (\text{sum})x + \alpha\beta = 0$

$x^2 - 6x + 27/4 = 0$

$4x^2 - 24x + 27 = 0$

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QUESTION

The cofactor of the element '4' in the determinant $\begin{vmatrix} 1 & 3 & 5 & 1 \\ 2 & 3 & 4 & 2 \\ 8 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \end{vmatrix}$ is

$i = 2$
$j = 3$
odd

$M_4 = \begin{vmatrix} 1 & 3 & 1 \\ 8 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$

$= 1 \times [0-2] - 3[8-0] + 1[16-0]$

$= -2 - 24 + 16.$

$-26 + 16$

$= -10$

$\boxed{C_4 = 10}$

(A) 4

(B) 10 ✓

(C) −10

(D) −4

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QUESTION

[Ans. 4]

If $x, y, z \in R$ & $\begin{vmatrix} x & y^2 & z^3 \\ x^4 & y^5 & z^6 \\ x^7 & y^8 & z^9 \end{vmatrix} = 2$, find the value of

$$\begin{array}{ccc} \to C_{11} & \to C_{12} & \\ \end{array}$$

$$\begin{vmatrix} y^5z^6(z^3-y^3) & x^4z^6(x^3-z^3) & x^4y^5(y^3-x^3) \\ y^2z^3(y^6-z^6) & xz^3(z^6-x^6) & xy^2(x^6-y^6) \\ y^2z^3(z^3-y^3) & xz^3(x^3-z^3) & xy^2(y^3-x^3) \end{vmatrix}$$

$$C_x = y^5z^9 - y^8z^6$$
$$= y^5z^6[z^3 - y^3]$$

$$C_{y^2} = -[x^4z^9 - x^7z^6]$$
$$= -x^4z^6[z^3 - x^3]$$
$$= x^4z^6[x^3 - z^3]$$

$$\begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$$

$$|\text{adj} A| = |A|^{n-1} = |A|^2$$
$$= 2^2 = 4$$

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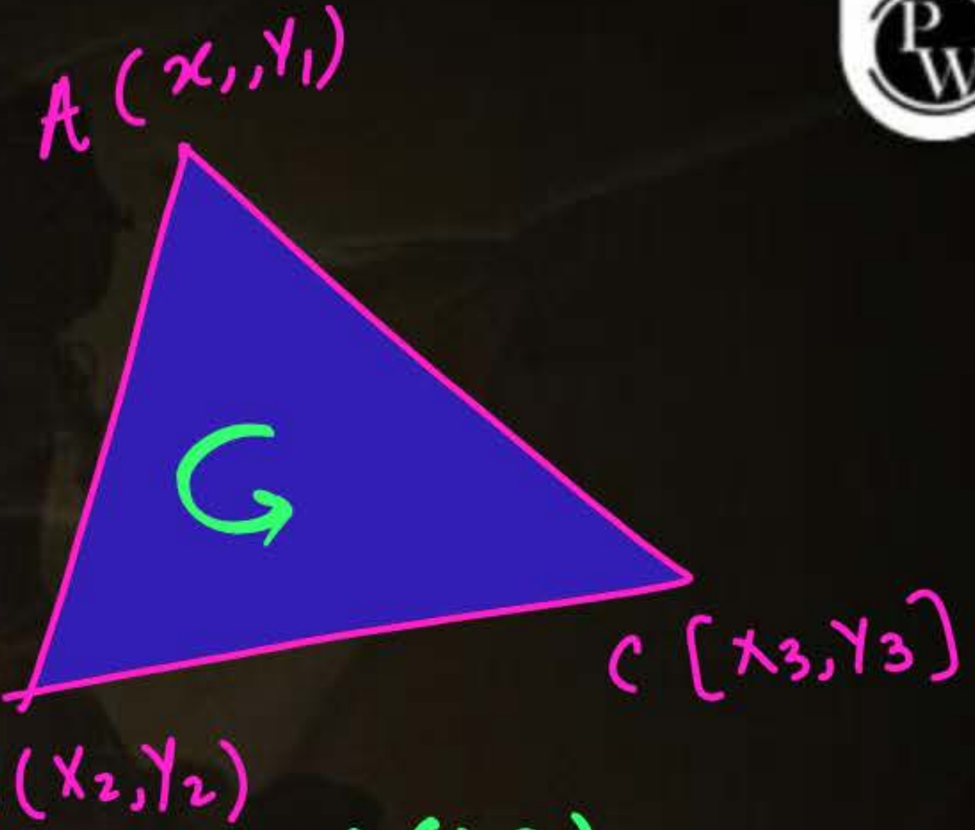
Area of a Triangle

$$\text{Area} = \left| \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \right|$$

Mod det



If $\boxed{\text{Area} = 0}$

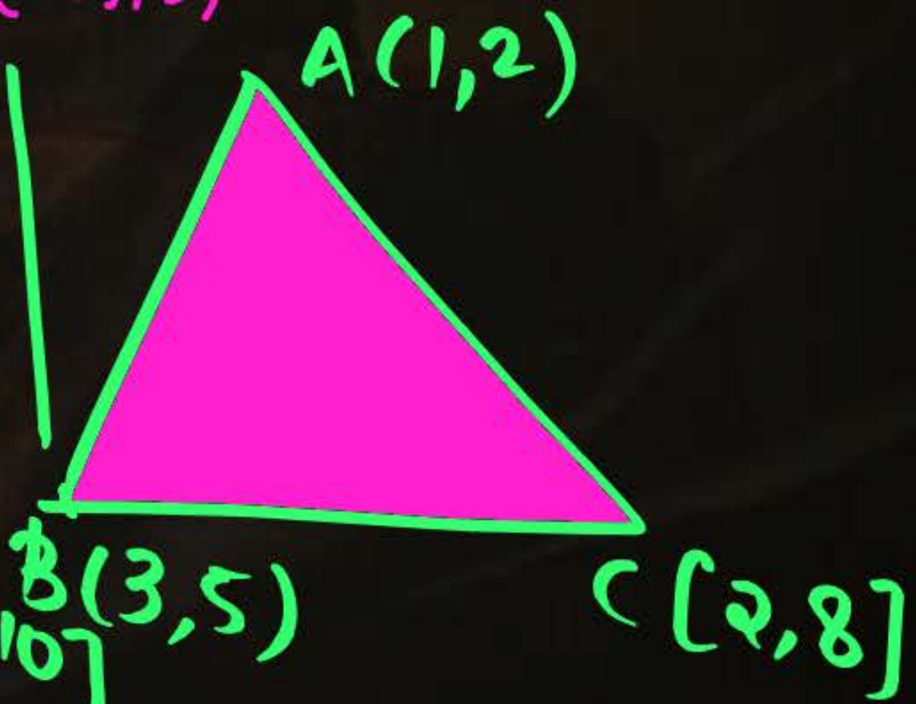
$\Rightarrow$ A, B, C are collinear points.

Ex →

$$A = \frac{1}{2} \begin{vmatrix} 1 & 2 & 1 \\ 3 & 5 & 1 \\ 2 & 8 & 1 \end{vmatrix}$$

$$\frac{1}{2}\left[(5-8) - 2(3-2) + 24 - 10\right]$$

$$\frac{1}{2}\left[-3-2+14\right] = 9/2$$



Properties of Determinants

$$= \begin{vmatrix} 0 & 0 & 0 \\ 2 & 3 & 5 \\ 8 & 7 & 10 \end{vmatrix} = 0 \checkmark$$

Property 1 If all the elements of a row (or column) are zero, then the value of the determinant is zero.

Property 2 If all the rows and columns of a determinant are inter-changed, the value of determinant remains unchanged.

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\boxed{|A| = |A^T|}$$

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Properties of Determinants

Property 3

If any two rows (or any two columns) of a determinant are **interchanged**, the value of determinant **changes sign** only.

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \Rightarrow R_1 \leftrightarrow R_2 \quad \begin{vmatrix} b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = -\Delta$$

$$R_1 \leftrightarrow R_3 \quad \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ a_1 & a_2 & a_3 \end{vmatrix} = -\Delta$$

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Properties of Determinants

Property 4

If any **two pair of rows** (or any two pair of columns) of a determinant are **interchanged**, the value of determinant remains same.

$$\Delta \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \quad R_1 \leftrightarrow R_2 \ \& \ R_2 \leftrightarrow R_3 \quad \begin{vmatrix} b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \end{vmatrix} = \Delta$$

4

Property 5

If any two rows (or any two columns) of a determinant are **identical**, then the value of determinant is always **zero**.

Ex →
$$\begin{vmatrix} 1 & 3 & 10 \\ 1 & 3 & 10 \\ 5 & 8 & 12 \end{vmatrix} = 0$$

$$\Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ a & b & c \end{vmatrix} \underset{R_1 \leftrightarrow R_3}{=} - \underbrace{\begin{vmatrix} a & b & c \\ d & e & f \\ a & b & c \end{vmatrix}}_{\Delta}$$

$\Delta = -\Delta$

$\Rightarrow 2\Delta = 0 \Rightarrow \Delta = 0$

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QUESTION

$$\begin{vmatrix} 3\sin 90^\circ & 3 & \pi \\ \ln e^3 & 3 & \sqrt{5} \\ \log_{10}1000 & 3 & e \end{vmatrix}$$ equals

$\ln e^3 = 3(\ln e) = 3\times 1 = 3$

$\log_{10}(10)^3 = 3\log_{10}10 = 3$

(A) 1

(B) e

(C) $\sqrt{\pi}$

(D) 0 ✓

$$\Delta = \begin{vmatrix} 3 & 3 & \pi \\ 3 & 3 & \sqrt{5} \\ 3 & 3 & e \end{vmatrix}$$

$C_1 = C_2$

$\Rightarrow \Delta = 0$

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QUESTION

Without expanding the determinants, prove that

$$\begin{vmatrix} 103 & 115 & 114 \\ 111 & 108 & 106 \\ 104 & 113 & 116 \end{vmatrix} + \begin{vmatrix} 113 & 116 & 104 \\ 108 & 106 & 111 \\ 115 & 114 & 103 \end{vmatrix} = 0$$

$R_1 \leftrightarrow R_3$

$$-\begin{vmatrix} 104 & 113 & 116 \\ 111 & 108 & 106 \\ 103 & 115 & 114 \end{vmatrix}$$

$C_1 \leftrightarrow C_2$

$$\begin{vmatrix} 113 & 104 & 116 \\ 108 & 111 & 106 \\ 115 & 103 & 114 \end{vmatrix} = -\begin{vmatrix} 113 & 116 & 104 \\ 108 & 106 & 111 \\ 115 & 114 & 103 \end{vmatrix}$$

Ans = 0 ✓

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$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} a+p & b+q \\ c+r & d+s \end{bmatrix} ✓$$

Properties of Determinants

Property 6

If each element of any row (or column) is expressed as a **sum of two** (or more) terms, then the determinant can be expressed as the sum of two (or more) determinants.

$$\begin{vmatrix} a_1+\alpha & b_1+\beta & c_1+\gamma \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha & \beta & \gamma \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Ex:

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 3 & 7 & 5 \end{vmatrix} + \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 2 & 3 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 10 & 10 \end{vmatrix} ✓$$

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$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} a & b \\ p & q \end{vmatrix} = \begin{vmatrix} a & b \\ c+p & d+q \end{vmatrix} \checkmark$$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} + \begin{vmatrix} p & q \\ r & s \end{vmatrix} = (ad-bc)+(ps-qr)$$

Ex→ $$\begin{vmatrix} a & c \\ b & d \end{vmatrix} + \begin{vmatrix} a & p \\ b & q \end{vmatrix} = \begin{vmatrix} a & c+p \\ b & d+q \end{vmatrix}$$

$\checkmark$ $$\begin{vmatrix} a & c \\ b & d \end{vmatrix} + \begin{vmatrix} a & c \\ b & d \end{vmatrix} = 2\begin{vmatrix} a & c \\ b & d \end{vmatrix}$$

Properties of Determinants

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$2A = \begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$

Property 7

If all the elements of any row (or column) are multiplied by the a scalar number "K", then the value of determinant is multiplied by that number "K".

Example:

If $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $D_1 = \begin{vmatrix} Ka_1 & Kb_1 & Kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$.

Then $D_1 = KD$.

$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$2\Delta = \begin{vmatrix} 2a & 2b \\ c & d \end{vmatrix}$

$K\Delta = \begin{vmatrix} Ka & b \\ Kc & d \end{vmatrix}$

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$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$3\Delta = \begin{vmatrix} a_1 & 3b_1 & c_1 \\ a_2 & 3b_2 & c_2 \\ a_3 & 3b_3 & c_3 \end{vmatrix}$$

$$\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} = 2\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

Ex $\rightarrow$ $\begin{vmatrix} 2a & b \\ 2c & d \end{vmatrix}$

$$2\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\begin{vmatrix} 3 & 5 & 8 \\ 6 & 4 & 10 \\ 9 & 2 & 12 \end{vmatrix}$$

$$2 \times 3 \begin{vmatrix} 1 & 5 & 4 \\ 2 & 4 & 5 \\ 3 & 2 & 6 \end{vmatrix}$$

QUESTION

The value of $\begin{vmatrix} 5^{200} & 5^{300} & 5^{400} \\ 5^{300} & 5^{400} & 5^{500} \\ 5^{400} & 5^{500} & 5^{700} \end{vmatrix}$ is

$$5^{400} \cdot 5^{300} \cdot 5^{200} \begin{vmatrix} 1 & 5^{100} & 5^{200} \\ 1 & 5^{100} & 5^{200} \\ 1 & 5^{100} & 5^{300} \end{vmatrix} = \underline{\underline{0}}$$

(A) 5^{200}

(B) 0 ✓

(C) 5^{1300}

(D) 5^{900}

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QUESTION

Prove that $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin(\alpha+\delta) \\ \sin\beta & \cos\beta & \sin(\beta+\delta) \\ \sin\gamma & \cos\gamma & \sin(\gamma+\delta) \end{vmatrix} = 0$

LHS $\begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha\cos\delta + \cos\alpha\sin\delta \\ \sin\beta & \cos\beta & \sin\beta\cos\delta + \cos\beta\sin\delta \\ \sin\gamma & \cos\gamma & \sin\gamma\cos\delta + \cos\gamma\sin\delta \end{vmatrix}$

$\cos\delta \begin{vmatrix} \sin\alpha & \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta & \sin\beta \\ \sin\gamma & \cos\gamma & \sin\gamma \end{vmatrix} + \sin\delta \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha \\ \sin\beta & \cos\beta & \cos\beta \\ \sin\gamma & \cos\gamma & \cos\gamma \end{vmatrix}$

$0 + 0 = 0$

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QUESTION

If x, y, z are respectively the p^{th} q^{th} r^{th} terms of an AP, then without expanding at any stage show that $\begin{vmatrix} x & y & z \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = 0$

$$\boxed{T_n = a+(n-1)d}$$

$x = a+(p-1)d$

$y = a+(q-1)d$

$z = a+(r-1)d$

$$\begin{vmatrix} a+(p-1)d & a+(q-1)d & a+(r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$\begin{vmatrix} a & a & a \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} + \begin{vmatrix} (p-1)d & (q-1)d & (r-1)d \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$$a\begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} + d\begin{vmatrix} p-1 & q-1 & r-1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} p & q & r \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 1 \\ p & q & r \\ 1 & 1 & 1 \end{vmatrix}$$

$0 \qquad 0$

QUESTION

[Ans. 3]

Let $\Delta_1 = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} 6a_1 & 2a_2 & 2a_3 \\ 3b_1 & b_2 & b_3 \\ 12c_1 & 4c_2 & 4c_3 \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} 3a_1+b_1 & 3a_2+b_2 & 3a_3+b_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix}$ & $\Delta_3 - \Delta_2 = k\Delta_1$, find k.

$$\Delta_2 = 24\begin{vmatrix} 3a_1 & a_2 & a_3 \\ 3b_1 & b_2 & b_3 \\ 3c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Delta_2 = 8 \cdot 3 \underbrace{\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}}_{\Delta_1}$$

$$\boxed{\Delta_2 = 24\Delta_1}$$

$$\Delta_3 = \begin{vmatrix} 3a_1 & 3a_2 & 3a_3 \\ 3b_1 & 3b_2 & 3b_3 \\ 3c_1 & 3c_2 & 3c_3 \end{vmatrix} + 3\begin{vmatrix} b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \\ \end{vmatrix}$$

$$\boxed{\Delta_3 = 27\Delta_1}$$

$$\boxed{\Delta_3 - \Delta_2 = 3\Delta_1}$$

$K = 3$

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QUESTION [JEE MAIN 2020 (7 Jan. II)]

HW ✓

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be two 3×3 real matrices such that $b_{ij} = (3)^{(i+j-2)} a_{ij}$, If the determinant of B is 81, then $|A|$ is:

$$\begin{matrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{matrix}$$

(A) 1/3

(B) 3

(C) 1/81

(D) 1/9

FOR NOTES & DPP CHECK DESCRIPTION

Properties of Determinants

PW

Property 8

Row-column operation: If any row (or any one column) of a determinant is added to or subtracted with equi-multiples of another Row (or another column), the value of determinant remains unchanged.

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$\boxed{R_1 \to R_1 + KR_2}$, $\boxed{K \in R}$

$R_1 \to R_1 + 2R_2$

$$\Delta' = \begin{vmatrix} a_1+2b_1 & a_2+2b_2 & a_3+2b_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \Delta$$

$$\Delta' = \begin{vmatrix} a_1+Kb_1 & a_2+Kb_2 & a_3+Kb_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$\Delta' = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} Kb_1 & Kb_2 & Kb_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

0

$\Delta' = \Delta + 0$

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$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$\boxed{C_1 \rightarrow C_1 + 3C_3}$$

$$\Delta = \begin{vmatrix} a_1+3c_1 & b_1 & c_1 \\ a_2+3c_2 & b_2 & c_2 \\ a_3+3c_3 & b_3 & c_3 \end{vmatrix} \checkmark$$

$$\Delta = \begin{vmatrix} 3 & 5 & 7 \\ 2 & 1 & 3 \\ 8 & 1 & 0 \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$

$$\Delta = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 3 \\ 8 & 1 & 0 \end{vmatrix} \checkmark$$

Valid operations :→

1) $R_1 \rightarrow R_1 \pm KR_2$ ✓

2) $\boxed{R_1 \rightarrow R_2 + KR_3}$ (X)

3) $\boxed{R_1 \rightarrow \underbrace{R_1 + 3R_1}}$ (X)

$R_1 \rightarrow 4R_1$

(4) $R_1 \rightarrow \underbrace{R_1 + K_1R_2} + K_2R_3$ ✓

(5) $C_1 \rightarrow C_1 + 5R_2$ (x)

(6) $C_1 \rightarrow C_1 + C_2 + C_3$ ✓

Some Important Determinants to Remember

1.

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$$

Proof:

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$= \begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^2-b^2 & b^2-c^2 & c^2 \end{vmatrix}$$

$$(b-c)(a-b)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a+b & b+c & c^2 \end{vmatrix}$$

$$(b-c)(a-b)\left[(\cancel{b}+c)-(a+\cancel{b})\right]$$

$$(b-c)(a-b)(c-a)$$

$= \underline{\underline{RHS}}$

Some Important Determinants to Remember

2.

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)(a+b+c)}$$

Proof:

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ a^3-b^3 & b^3-c^3 & c^3 \end{vmatrix}$$

$$\begin{vmatrix} 0 & 0 & 1 \\ a-b & b-c & c \\ (a-b)(a^2+b^2+ab) & (b-c)(b^2+c^2+bc) & c^3 \end{vmatrix}$$

$$(a-b)(b-c)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ a^2+b^2+ab & b^2+c^2+bc & c^3 \end{vmatrix}$$

$$(a-b)(b-c)\left[(\cancel{b^2}+c^2+bc)-(a^2+\cancel{b^2}+ab)\right]$$

$$(a-b)(b-c)\left[(\underbrace{c^2-a^2})+b(c-a)\right]$$

$$(a-b)(b-c)(c-a)\left[c+a+b\right]$$

FOR NOTES & DPP CHECK DESCRIPTION

Recap Some Important Determinants to Remember

$bc^2 \to 3^\circ$ term

1. $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)}$$

$bc^3 \to 4^\circ$ term

2. $$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)(a+b+c)}$$

5° term

3. $$\begin{vmatrix} 1 & 1 & 1 \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = \boxed{(a-b)(b-c)(c-a)(ab+bc+ca)}$$

2°

4. *Cyclic or Circulant Determinant* $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3+b^3+c^3-3abc)$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If $x \neq y \neq z$ and $\begin{vmatrix} x & x^3 & x^4-1 \\ y & y^3 & y^4-1 \\ z & z^3 & z^4-1 \end{vmatrix} = 0$, then the value of $xy + yz + zx$ is

(A) $x+y+z$

(B) xyz

(C) $\dfrac{xyz}{x+y+z}$

(D) $\dfrac{x+y+z}{xyz}$ ✓

$$\begin{vmatrix} x & x^3 & x^4 \\ y & y^3 & y^4 \\ z & z^3 & z^4 \end{vmatrix} - \begin{vmatrix} x & x^3 & 1 \\ y & y^3 & 1 \\ z & z^3 & 1 \end{vmatrix}$$

$C_1 \leftrightarrow C_3$

$C_2 \leftrightarrow C_3$

$$xyz \underbrace{\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}} - \begin{vmatrix} 1 & x & x^3 \\ 1 & y & y^3 \\ 1 & z & z^3 \end{vmatrix}$$

$$xyz \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} - \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^3 & y^3 & z^3 \end{vmatrix} = 0$$

$$xyz\,\cancel{(x-y)}\,\cancel{(y-z)}\,\cancel{(z-x)}\,\underbrace{(xy+yz+zx)} = \cancel{(x-y)}\,\cancel{(y-z)}\,\cancel{(z-x)}\,(x+y+z)$$

$$xy+yz+zx = \frac{x+y+z}{xyz}$$

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Some Important Determinants to Remember

4. Cyclic or Circulant Determinant

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = -(a^3+b^3+c^3-3abc)$$

Proof:

$R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} a+b+c & b+c+a & c+a+b \\ b & c & a \\ c & a & b \end{vmatrix}$$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix}$$

$$(a+b+c)\left[(a-b)(b-c)-(c-a)^2\right]$$

$$(a+b+c)\left[ab-\cancel{ac}+bc-b^2-c^2-a^2+\cancel{2}ac\right]$$

$$(a+b+c)\left[ab+bc+ca-a^2-b^2-c^2\right]$$

$$-(a+b+c)\left[a^2+b^2+c^2-ab-bc-ca\right]$$

$$-(a^3+b^3+c^3-3abc)$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If α, β and γ are the roots of the equations $\boxed{x^3 + px + q = 0}$ then value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is

α, β, γ

$\boxed{\alpha + \beta + \gamma = 0}$

$\Rightarrow \alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$

$-(\cancel{\alpha^3 + \beta^3 + \gamma^3} - \cancel{3\alpha\beta\gamma})$

$= 0$ ✓

(A) p

(B) q

(C) $p^2 - 2q$

(D) 0 ✓

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

If ω is the cube root of unity, then $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} =$

$R_1 \to R_1 + R_2 + R_3$

$\boxed{1+\omega+\omega^2=0}$ ✓

$\boxed{\omega^3=1}$ ✓

$\begin{vmatrix} 0 & 0 & 0 \\ - & - & - \\ - & - & - \end{vmatrix}$

(A) 1

(B) 0 ✓

(C) ω

(D) ω^2

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2019 (Apr.)]

HW

[Ans. A]

Let α and β be the roots of the equation $\boxed{x^2+x+1=0}$.

Then for $y \neq 0$ in R, $\begin{vmatrix} y+1 & \alpha & \beta \\ \alpha & y+\beta & 1 \\ \beta & 1 & y+\alpha \end{vmatrix}$ is equal to

(A) y^3

(B) y^3-1

(C) $y(y^2-1)$

(D) $y(y^2-3)$

ω, ω^2

$\alpha = \omega$
$\beta = \omega^2$

$1+\omega+\omega^2=0$
$\omega^3=1$

$$\begin{vmatrix} y+1 & \omega & \omega^2 \\ \omega & \omega^2+y & \\ \omega^2 & 1 & y+\omega \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

HW

If ω, ω^2 are imaginary cube roots of unity and

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} \text{ and } \Delta_2 = \begin{vmatrix} 1 & 1 & \omega \\ 1 & 1 & \omega^2 \\ \omega^2 & \omega & 1 \end{vmatrix}, \text{ then } \frac{\Delta_1}{\Delta_2} \text{ is equal to}$$

(A) $\sqrt{3}$

(B) $\sqrt{3}i$

(C) 1

(D) -1

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [JEE MAIN 2019 (Jan.)]

[Ans. D]

$x^2 = 2$

If $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)(x+a+b+c)^2$,

$\boxed{x \neq 0}$ and $a+b+c \neq 0$, then x is equal to.

(A) abc

(B) $-(a+b+c)$

(C) $2(a+b+c)$

(D) $-2(a+b+c)$ ✓

$R_1 \to R_1 + R_2 + R_3$

$$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b+c+a & -(b+c+a) & 2b \\ 0 & c+a+b & c-a-b \end{vmatrix}$$

$(a+b+c)\left[(a+b+c)^2 - 0\right]$

$= (a+b+c)^3$

$(a+b+c)^3 = (x+a+b+c)^2 \cancel{(a+b+c)}$

$(a+b+c)^2 = (x+a+b+c)^2$

$x+a+b+c = \pm(a+b+c)$

$(+)$ $\boxed{x=0}$ ✗

$(-)$ $x+a+b+c = -a-b-c$

$\boxed{x = -2a-2b-2c}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Show that $\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$

$$\frac{1}{\cancel{a}bc}\begin{vmatrix} \cancel{a}(a^2+1) & a^2\cancel{b} & a^2\cancel{c} \\ \cancel{a}b^2 & \cancel{b}(b^2+1) & b^2\cancel{c} \\ \cancel{a}c^2 & \cancel{b}c^2 & \cancel{c}(c^2+1) \end{vmatrix}$$

$$\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

$$(a^2+b^2+c^2+1)\begin{vmatrix} 1 & 1 & 1 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

$C_2 \to C_2 - C_3$

$$(a^2+b^2+c^2+1)\begin{vmatrix} 0 & 0 & 1 \\ -1 & 1 & b^2 \\ 0 & -1 & c^2+1 \end{vmatrix}$$

$$(a^2+b^2+c^2+1)\{1-0\}$$

BRAIN TEASER

If $\boxed{a^2+b^2+c^2=1}$ then show that the value of the determinant

$$\begin{vmatrix} a^2+(b^2+c^2)\cos\theta & ba(1-\cos\theta) & ca(1-\cos\theta) \\ ab(1-\cos\theta) & b^2+(c^2+a^2)\cos\theta & cb(1-\cos\theta) \\ ac(1-\cos\theta) & bc(1-\cos\theta) & c^2+(a^2+b^2)\cos\theta \end{vmatrix}$$

simplifies to $\boxed{\cos^2\theta}$

$$\frac{1}{\cancel{a}\cancel{b}\cancel{c}}$$

$$\begin{vmatrix} a^2+(b^2+c^2)\cos\theta & a^2(1-\cos\theta) & a^2(1-\cos\theta) \\ b^2(1-\cos\theta) & b^2+(c^2+a^2)\cos\theta & b^2(1-\cos\theta) \\ c^2(1-\cos\theta) & c^2(1-\cos\theta) & c^2+(a^2+b^2)\cos\theta \end{vmatrix}$$

$R_1 \to R_1+R_2+R_3$

$$\begin{vmatrix} 1 & 1 & 1 \\ b^2(1-\cos\theta) & b^2+(c^2+a^2)\cos\theta & b^2(1-\cos\theta) \\ c^2(1-\cos\theta) & c^2(1-\cos\theta) & c^2+(a^2+b^2)\cos\theta \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ -\cos\theta & \cos\theta & b^2(1-\cos\theta) \\ 0 & -\cos\theta & \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$(\cos^2\theta - 0) = \cos^2\theta$ ✓

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

Prove that

$$\begin{vmatrix} (b+c)^2 & ba & ac \\ ba & (c+a)^2 & cb \\ ca & cb & (a+b)^2 \end{vmatrix} = \underbrace{\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}}_{} = 2abc(a+b+c)^3.$$

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\begin{vmatrix} (b+c)^2 - a^2 & 0 & a^2 \\ b^2 - (c+a)^2 & (c+a)^2 - b^2 & b^2 \\ 0 & c^2 - (a+b)^2 & (a+b)^2 \end{vmatrix}$$

$$(a+b+c)^2 \begin{vmatrix} b+c-a & 0 & a^2 \\ b-c-a & c+a-b & b^2 \\ 0 & c-a-b & (a+b)^2 \end{vmatrix}$$

HW

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QUESTION

If in a triangle, s denotes the semi-perimeter and a, b, c denote the lengths of sides, then prove that

$$\begin{vmatrix} a^2 & (s-a)^2 & (s-a)^2 \\ (s-b)^2 & b^2 & (s-b)^2 \\ (s-c)^2 & (s-c)^2 & c^2 \end{vmatrix} = 2s^3(s-a)(s-b)(s-c)$$

$$S = \frac{a+b+c}{2}$$

Let $s-a=\alpha$, $s-b=\beta$, $s-c=\gamma$

Add

$$3s-(a+b+c) = \alpha+\beta+\gamma$$

$$3s-2s = \alpha+\beta+\gamma$$

$$\boxed{\alpha+\beta+\gamma = s}$$

$$a = s-\alpha = \beta+\gamma$$

$$b = s-\beta = \alpha+\gamma$$

$$c = s-\gamma = \alpha+\beta$$

$$\begin{vmatrix} a^2 & \alpha^2 & \alpha^2 \\ \beta^2 & b^2 & \beta^2 \\ \gamma^2 & \gamma^2 & c^2 \end{vmatrix}$$

$$\begin{vmatrix} (\beta+\gamma)^2 & \alpha^2 & \alpha^2 \\ \beta^2 & (\alpha+\gamma)^2 & \beta^2 \\ \gamma^2 & \gamma^2 & (\alpha+\beta)^2 \end{vmatrix}$$

$$= 2\alpha\beta\gamma(\alpha+\beta+\gamma)^3 = 2(s-a)(s-b)(s-c)(s)^3$$

Use from previous Ques.

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

$$\begin{vmatrix} \frac{a^2+b^2}{c} & c & c \\ a & \frac{b^2+c^2}{a} & a \\ b & b & \frac{c^2+a^2}{b} \end{vmatrix}$$ is equal to

(A) abc

(B) $2\,abc$

(C) $4\,abc$

(D) 0

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QUESTION [JEE MAIN 2020 (Jan.)]

Let $\boxed{a-2b+c=1}$. If $f(x)=\begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then:

(A) $f(-50)=-1$

(B) $f(50)=1$

(C) $f(50)=-501$

(D) $f(-50)=501$

$R_1 \to R_1+R_3-2R_2$

$a_{11} = x+a+x+c-2(x+b)$

$2\cancel{x}+a+c-\cancel{2x}-2b$

$a+c-2b$
$=1$

a_{12}

$(x+2)+(x+4)-2(x+3)$

$\cancel{2x}+\cancel{6}-\cancel{2x}-\cancel{6}$

$=0$

$$\begin{vmatrix} 1 & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

$(x+3)^2-(x+4)(x+2)$

$\cancel{x^2}+\cancel{6x}+9-(\cancel{x^2}+\cancel{6x}+8)$

$\boxed{f(x)=1}$

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QUESTION [JEE MAIN 2024]

[Ans. B]

The values of α, for which $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$, lie in the interval

(A) $(-2, 1)$

(B) $(-3, 0)$

(C) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

(D) $(0, 3)$

$R_1 \to R_1 - R_2$

$$\begin{vmatrix} 0 & \left(\frac{3}{2}-\frac{1}{3}\right) & \left(\frac{3}{2}-\frac{1}{3}\right) \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

$$\cancel{\left(\frac{3}{2}-\frac{1}{3}\right)}\begin{vmatrix} 0 & 1 & 1 \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

$C_2 \to C_2 - C_3$

$$\begin{vmatrix} 0 & 0 & 1 \\ 1 & -\alpha & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$$

$(3\alpha+1) + \alpha(2\alpha+3) = 0$

$3\alpha + 1 + 2\alpha^2 + 3\alpha = 0$

$\boxed{2\alpha^2 + 6\alpha + 1 = 0}$

$\alpha = \dfrac{-6 \pm \sqrt{36-8}}{2\times 2}$

$= \dfrac{-6 \pm \sqrt{28}}{2\cdot 2} = \dfrac{-6 \pm 2\sqrt{7}}{2\times 2}$

$\boxed{\alpha = \dfrac{-3 \pm \sqrt{7}}{2}}$

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QUESTION [JEE Main 2024 (08 Apr Shift 2)]

[Ans. B]

If $\alpha \neq a, \beta \neq b, \gamma \neq c$ and $\begin{vmatrix} \alpha & b & c \\ a & \beta & c \\ a & b & \gamma \end{vmatrix} = 0$, then $\frac{a}{\alpha-a} + \frac{b}{\beta-b} + \frac{\gamma}{\gamma-c}$ is equal to:

(A) 3

(B) 0

(C) 1

(D) 2

$\frac{\alpha}{\alpha-a} + \frac{b}{\beta-b} + \frac{\gamma}{\gamma-c}$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$$\begin{vmatrix} \alpha-a & b-\beta & 0 \\ 0 & \beta-b & c-\gamma \\ a & b & \gamma \end{vmatrix} = 0$$

$$(\alpha-a)\left[(\beta-b)\gamma - b(c-\gamma)\right] + (b-\beta)\left[a(c-\gamma)\right] = 0$$

$$(\alpha-a)(\beta-b)\gamma - b(c-\gamma)(\alpha-a) + a(b-\beta)(c-\gamma) = 0$$

$$(\alpha-a)(\beta-b)\gamma + b(\gamma-c)(\alpha-a) + a(\beta-b)(\gamma-c) = 0$$

$$(\alpha-a)(\beta-b)(\gamma-c)$$

$$\frac{\gamma-c+c}{\gamma-c} + \frac{b}{\beta-b} + \frac{a}{\alpha-a} = 0$$

$$1 + \frac{c}{\gamma-c} + \frac{b}{\beta-b} + \frac{a}{\alpha-a} = 0$$

$$\boxed{\frac{a}{\alpha-a} + \frac{b}{\beta-b} + \frac{c}{\gamma-c} = -1}$$

ATDB.uno

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QUESTION [JEE MAIN 2023]

HW

[Ans. 495]

Let A_1, A_2, A_3 be the three A.P. with the same common difference d and having their first terms as $A, A+1, A+2$, respectively. Let $\boldsymbol{a, b, c}$ be the $7^{th}, 9^{th}, 17^{th}$ terms of A_1, A_2, A_3, respectively such that $\begin{vmatrix} a & 7 & 1 \\ 2b & 17 & 1 \\ c & 17 & 1 \end{vmatrix} + 70 = 0$. If a = 29, then the sum of first 20 terms of an AP whose first term is $c - a - b$ and common difference is $\frac{d}{12}$, is equal

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QUESTION [JEE MAIN 2019 (Jan.)]

HW

[Ans. A]

Let $A = \begin{bmatrix} 2 & b & 1 \\ b & b^2+1 & b \\ 1 & b & 2 \end{bmatrix}$ where $b > 0$. Then the minimum value of $\frac{\det(A)}{b}$ is

(A) $2\sqrt{3}$

(B) $-2\sqrt{3}$

(C) $-\sqrt{3}$

(D) $\sqrt{3}$

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Summation of Determinants

$\sum \rightarrow$ add

$$\Delta_r = \begin{vmatrix} r & 1 & 7 \\ 2r & 3 & 8 \\ 3r & 5 & 9 \end{vmatrix} \quad 1,3,5 \text{ and } 7,8,9 \rightarrow \text{const}$$

$$\sum_{r=1}^{5} \Delta r = \Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 + \Delta_5 = ?$$

$$\Sigma \Delta r = \begin{vmatrix} \Sigma r & 1 & 7 \\ \Sigma 2r & 3 & 8 \\ \Sigma 3r & 5 & 9 \end{vmatrix}$$

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Special Series_Remember

1. Sum of the first n natural numbers $\sum_{r=1}^{n} r = \boxed{\frac{n(n+1)}{2}}$

2. Sum of the squares of the first n natural numbers $\boxed{\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}}$

3. Sum of the cubes of the first n natural numbers $\boxed{\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4} = \left[\sum_{r=1}^{n} r\right]^2}$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION

The value of $\sum_{n=1}^{N} U_n$ if $U_n = \begin{vmatrix} n & 1 & 5 \\ n^2 & 2N+1 & 2N+1 \\ n^3 & 3N^2 & 3N \end{vmatrix}$ is

(A) 0

(B) 1

(C) −1

(D) 2

$n \to$ Variable

$N \to$ const

$$\sum U_n = \begin{vmatrix} \sum n & - & - \\ \sum n^2 & - & - \\ \sum n^3 & - & - \end{vmatrix} \quad \text{Const}$$

$$= \begin{vmatrix} \frac{N(N+1)}{2} & 1 & 5 \\ \frac{N(N+1)(2N+1)}{6} & 2N+1 & 2N+1 \\ \left(\frac{N(N+1)}{2}\right)^2 & 3N^2 & 3N \end{vmatrix}$$

$$\frac{N(N+1)}{2}\begin{vmatrix} 1 & 1 & 5 \\ \frac{2N+1}{3} & 2N+1 & 2N+1 \\ \frac{N(N+1)}{2} & 3N^2 & 3N \end{vmatrix}$$

$$\frac{N^2(N+1)}{2}(2N+1)\begin{vmatrix} 1 & 1 & 5 \\ \frac{1}{3} & 1 & 1 \\ \frac{N+1}{2} & 3N & 3 \end{vmatrix}$$

R_3

$$\frac{N+1}{2}(-4) - 3N\left(1-\frac{5}{3}\right) + 3\cdot\frac{2}{3}$$

$$-2(N+1) - 3N\cdot\left(-\frac{2}{3}\right) + 2$$

$$-\cancel{2N} - \cancel{2} + \cancel{2N} + \cancel{2} = 0$$

FOR NOTES & DPP CHECK DESCRIPTION

QUESTION [1993]

[Ans. A]

If $\Delta_r = \begin{vmatrix} 2^r & 2.3^r & 4.5^r \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$, then the value of $\sum_{r=0}^{n-1} \Delta_r$ is

(A) 0

(B) $\alpha\beta\gamma$

(C) $\alpha+\beta+\gamma$

(D) $\alpha \cdot \alpha^n + \beta \cdot 3^n + \gamma \cdot 4^n$

$S_n = \frac{a(r^n-1)}{r-1}$

$\begin{vmatrix} \sum 2^r & \sum 2.3^r & \sum 4.5^r \\ - & - & - \\ - & - & \end{vmatrix}$

$\begin{vmatrix} 2^n-1 & 3^n-1 & 5^n-1 \\ \alpha & \beta & \gamma \\ 2^n-1 & 3^n-1 & 5^n-1 \end{vmatrix}$

0

$\sum_{r=0}^{n-1} 2^r = 2^0+2^1+2^2+\dots+2^{n-1}$

$= 1\frac{[2^n-1]}{2-1} = (2^n-1)$

$\sum_{r=0}^{n-1} 2.3^r = 2[3^0+3^1+3^2+\dots+3^{n-1}]$

$= \frac{\cancel{2}[3^n-1]}{\cancel{3-1}} = 3^n-1$

$\sum_{r=0}^{n-1} 4.5^r = 4[5^0+5^1+5^2+\dots]$

$= \frac{\cancel{4}(5^n-1)}{\cancel{(5-1)}} = (5^n-1)$

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QUESTION [JEE MAIN 2023]

[Ans. 6]

Let $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$. If $\sum_{k=1}^{n} D_k = 96$, then n is equal to

Const

$\Sigma 1 = 1+1+1+\dots 1 = n$

$\Sigma 2K = 2[1+2+\dots+n]$

$= \cancel{2}\dfrac{(n)(n+1)}{\cancel{2}}$

$= n^2+n$

$\sum_{k=1}^{n}(2K-1) = \Sigma 2K - \Sigma 1$

$= n^2 + \cancel{n} - \cancel{n}$

$= n^2$

$$\sum D_K = \begin{vmatrix} \sum_{K=1}^{n} 1 & \sum_{K=1}^{n} 2K & \sum_{K=1}^{n} 2K-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$\sum_{K=1}^{n} 2K-1 \to n^2$

$$= \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = \begin{vmatrix} 0 & -2 & 0 \\ 0 & 2 & -n-2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$$

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$2[n(n+2)] = 96$

$\boxed{n(n+2) = 48}$

6×8

$\boxed{n = 6}$

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QUESTION [JEE Main 2024 (06 Apr Shift 1)]

[Ans. B]

For $\alpha, \beta \in \mathbb{R}$ and $n \in N$, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $2A_{10} - A_8 =$

(A) 0

(B) $4\alpha + 2\beta$

(C) $2\alpha + 4\beta$

(D) $2n$

$$2A_{10} = \begin{vmatrix} 2\times 10 & - & - \\ 4\times 10 & - & - \\ 2(28) & - & - \end{vmatrix}$$

$$A_8 = \begin{vmatrix} 8 & - & - \\ 16 & - & - \\ 22 & - & - \end{vmatrix}$$

$$\Rightarrow 2A_{10} - A_8 = \begin{vmatrix} 12 & 1 & \frac{n^2}{2}+\alpha \\ 24 & 2 & n^2-\beta \\ 34 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$C_1 \to C_1 - 12C_2$

$$\begin{vmatrix} 0 & 1 & \frac{n^2}{2}+\alpha \\ 0 & 2 & n^2-\beta \\ -2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$

$$-2\left[n^2-\beta-2\left(\frac{n^2}{2}+\alpha\right)\right] = -2[-\beta-2\alpha] = 2(\beta+2\alpha)$$

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HW_QUESTION

HW

If 'm' is a positive integer $\Delta_r = \begin{vmatrix} 2r-1 & {}^mC_r & 1 \\ m^2-1 & 2^m & m+1 \\ \sin^2(2m) & \sin^2(m) & \sin^2(m+1) \end{vmatrix}$ then value of $\sum_{r=0}^{m} \Delta_r$ is:

(A) 1

(B) 0

(C) 2

(D) −1

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HW_QUESTION

HW

Ans = 0

If $f(x) = \begin{vmatrix} \sin^5 x & \ln \sin x & \dfrac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} \\ n & \sum_{k=1}^{n} k & \prod_{k=1}^{n} k \\ \dfrac{8}{15} & \dfrac{\pi}{2}\ln\left(\dfrac{1}{2}\right) & \dfrac{\pi}{4} \end{vmatrix}$ then find the value of $\int_0^{\pi/2} f(x)\,dx$

Const.

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Differentiation of Determinant

$$\frac{d}{dx}(fg) = fg' + gf'$$

$$\Delta(x) = \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix}$$

$$\Delta'(x) = \begin{vmatrix} f_1' & f_2' & f_3' \\ g_1 & g_2 & g_3 \\ h_1 & h_2 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1' & g_2' & g_3' \\ h_1 & h_2 & h_3 \end{vmatrix} + \begin{vmatrix} f_1 & f_2 & f_3 \\ g_1 & g_2 & g_3 \\ h_1' & h_2' & h_3' \end{vmatrix}$$

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QUESTION

If $\begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} \equiv px^5+qx^4+rx^3+sx^2+tx+w$

then find the value of t & w.

Identity.

for w

put $\boxed{x=0}$

$$\begin{vmatrix} 1 & 1 & 2x \\ x & 1+x & x^2 \\ x^2 & x & 1+x \end{vmatrix} + \begin{vmatrix} 1+x & x & x^2 \\ 1 & 1 & 2x \\ x^2 & x & 1+x \end{vmatrix} + \begin{vmatrix} 1+x & x & x^2 \\ x & 1+x & x^2 \\ 2x & 1 & 1 \end{vmatrix} = \cancel{5px^4} + \cancel{4qx^3} + \cancel{3rx^2} + \cancel{2sx} + t$$

Put $x=0$

$$\underbrace{\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}_{1} + \underbrace{\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}}_{1} + \underbrace{\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}}_{1} = t = \begin{vmatrix} 3 & 0 & 0 \\ 1 & 3 & 0 \\ 0 & 1 & 3 \end{vmatrix}$$

$\Rightarrow \boxed{t=3}$

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QUESTION [JEE MAIN 2024] HW **[Ans. C]**

If $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in \mathbb{R}$, then $2f(0)+f'(0)$ is equal to

(A) 48

(B) 24

(C) 42

(D) 18

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QUESTION [JEE MAIN 2024]

[Ans. A]

$\sin^4 x + \cos^4 x = 1 - 2\sin^2 x \ldots$

$\frac{d}{dx}(\cos^4 x) = 4\cos^3 x(-\sin x) = 0$

$\frac{d}{dx}(\sin^2 2x) = 2\sin 2x \cos 2x \cdot 2$

If $f(x) = \begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)$ is equal to ________

(A) 0

(B) 1

(C) 2

(D) 6

$R_1 \to R_1 - R_2$ & $R_2 \to R_2 - R_3$

$f(x) = \begin{vmatrix} -3 & 0 & 3 \\ 3 & -3 & 0 \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$

$f(x) = 9\begin{vmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$

$= 9\left[\sin^2 2x + 3 + \underbrace{2\sin^4 x + 2\cos^4 x}\right]$

$f(x) = 9\left[\sin^2 2x + 3 + 2(1 - 2\sin^2 x\cos^2 x)\right]$

$9\left[4\sin^2 x\cancel{\cos^2 x} + 5 - 4\sin^2 \cancel{x}\cos^2 x\right]$

$\boxed{f(x) = 45}$

$\Rightarrow f'(x) = 0$

M-2

$\underbrace{\begin{vmatrix} 0 & - & - \\ 0 & - & - \\ 0 & - & - \end{vmatrix}}_{0} + \underbrace{\begin{vmatrix} - & 0 & - \\ - & 0 & - \\ - & 0 & - \end{vmatrix}}_{0} + \underbrace{\begin{vmatrix} - & - & 0 \\ - & - & 0 \\ - & - & 0 \end{vmatrix}}_{0}$

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Multiplication of 2 Determinants

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} \begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix} = \begin{vmatrix} a_1\alpha_1 + a_2\beta_1 + a_3\gamma_1 & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{vmatrix}$$

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QUESTION

For all real values of A, B, C and P, Q, R show that

$$\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-R) \end{vmatrix} = 0$$

$$\begin{vmatrix} \cos A\cos P + \sin A\sin P & \cos A\cos Q + \sin A\sin Q & \cos A\cos R + \sin A\sin R \\ \cos B\cos P + \sin B\sin P & \cos B\cos Q + \sin B\sin Q & — \\ \cos C\cos P + \sin C\sin P & \cos C\cos Q + \sin C\sin Q & — \end{vmatrix}$$

$$= \begin{vmatrix} \cos A & \sin A & 0 \\ \cos B & \sin B & 0 \\ \cos C & \sin C & 0 \end{vmatrix} \cdot \begin{vmatrix} \cos P & \cos Q & \cos R \\ \sin P & \sin Q & \sin R \\ 0 & 0 & 0 \end{vmatrix}$$

$$0 \times 0 = 0$$

3

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QUESTION

$(c+d)(a+b)+(a+b)(c+d)+0$

$2(a+b)(c+d)$

Without expanding at any stage show that:

$$\begin{vmatrix} 2 & a+b+c+d & ab+cd \\ a+b+c+d & 2(a+b)(c+d) & ab(c+d)+cd(a+b) \\ ab+cd & ab(c+d)+cd(a+b) & 2abcd \end{vmatrix} = 0$$

$$\begin{vmatrix} 1 & 1 & 0 \\ c+d & a+b & 0 \\ cd & ab & 0 \end{vmatrix} \begin{vmatrix} 1 & a+b & ab \\ 1 & c+d & cd \\ 0 & 0 & 0 \end{vmatrix}$$

$0 \times 0 = 0$ ✓

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QUESTION

If α, β, γ are real numbers, then without expanding at any stage, show that

$$\begin{vmatrix} 1 & \cos(\beta-\alpha) & \cos(\gamma-\alpha) \\ \cos(\alpha-\beta) & 1 & \cos(\gamma-\beta) \\ \cos(\alpha-\gamma) & \cos(\beta-\gamma) & 1 \end{vmatrix} = 0$$

QUESTION

Prove that

$$\begin{vmatrix}(a-x)^2 & (a-y)^2 & (a-z)^2\\(b-x)^2 & (b-y)^2 & (b-z)^2\\(c-x)^2 & (c-y)^2 & (c-z)^2\end{vmatrix} = \begin{vmatrix}(1+ax)^2 & (1+bx)^2 & (1+cx)^2\\(1+ay)^2 & (1+by)^2 & (1+cy)^2\\(1+az)^2 & (1+bz)^2 & (1+cz)^2\end{vmatrix}$$

$$= 2(b-c)(c-a)(a-b)\times(y-z)(z-x)(x-y)$$

LHS $\begin{vmatrix}a^2+x^2-2ax & a^2+y^2-2ay & a^2+z^2-2az\\ b^2+x^2-2bx & b^2+y^2-2by & b^2+z^2-2bz\end{vmatrix} = \begin{vmatrix}a^2 & 1 & -2a\\ b^2 & 1 & -2b\\ c^2 & 1 & -2c\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ x^2 & y^2 & z^2\\ x & y & z\end{vmatrix}$

$C_1 \leftrightarrow C_2$ & $C_2 \leftrightarrow C_3$

$2\begin{vmatrix}a^2 & 1 & a\\ b^2 & 1 & b\\ c^2 & 1 & c\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ x & y & z\\ x^2 & y^2 & z^2\end{vmatrix}$

$2\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\ x & y & z\\ x^2 & y^2 & z^2\end{vmatrix}$

Rhs: $2(a-b)(b-c)(c-a)(x-y)(y-z)(z-x)$

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QUESTION

Prove that

$$\begin{vmatrix}(a-x)^2 & (a-y)^2 & (a-z)^2\\(b-x)^2 & (b-y)^2 & (b-z)^2\\(c-x)^2 & (c-y)^2 & (c-z)^2\end{vmatrix} = \boxed{\begin{vmatrix}(1+ax)^2 & (1+bx)^2 & (1+cx)^2\\(1+ay)^2 & (1+by)^2 & (1+cy)^2\\(1+az)^2 & (1+bz)^2 & (1+cz)^2\end{vmatrix}}$$

$$= 2(b-c)(c-a)(a-b)\times(y-z)(z-x)(x-y)$$

$$\begin{vmatrix}1+a^2x^2+2ax & 1+b^2x^2+2bx & 1+c^2x^2+2cx\\1+a^2y^2+2ay & & \\ & & \end{vmatrix}$$

$$= \begin{vmatrix}1 & x^2 & x\\1 & y^2 & y\\1 & z^2 & z\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\a^2 & b^2 & c^2\\2a & 2b & 2c\end{vmatrix}$$

$$2\begin{vmatrix}1 & x & x^2\\1 & y & y^2\\1 & z & z^2\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\a & b & c\\a^2 & b^2 & c^2\end{vmatrix}$$

$$= 2(x-y)(y-z)(z-x)(a-b)(b-c)(c-a)$$

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QUESTION [JEE Adv.-2015]

[Ans. B,C]

Which of the following values of α satisfy the equation $\begin{vmatrix}(1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2\\(2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2\\(3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2\end{vmatrix} = -648\alpha$

(A) -4

(B) 9

(C) -9

(D) 4

$$\begin{vmatrix}1+\alpha^2+2\alpha & 1+4\alpha^2+4\alpha & 1+9\alpha^2+6\alpha\\4+\alpha^2+4\alpha & 4+4\alpha^2+8\alpha & \\9+\alpha^2+6\alpha & & \end{vmatrix} = \begin{vmatrix}1 & \alpha^2 & \alpha\\4 & \alpha^2 & 2\alpha\\9 & \alpha^2 & 3\alpha\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\1 & 4 & 9\\2 & 4 & 6\end{vmatrix}$$

$$= \alpha^3\begin{vmatrix}1 & 1 & 1\\4 & 1 & 2\\9 & 1 & 3\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\1 & 4 & 9\\2 & 4 & 6\end{vmatrix}$$

$$= 2\alpha^3\begin{vmatrix}1 & 1 & 1\\4 & 1 & 2\\9 & 1 & 3\end{vmatrix}\begin{vmatrix}1 & 1 & 1\\1 & 4 & 9\\1 & 2 & 3\end{vmatrix}$$

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HW_QUESTION

[Ans. C]

If $\alpha, \beta \neq 0$ and $\boxed{f(n) = \alpha^n + \beta^n}$ and $\begin{vmatrix} 3 & 1+f(1) & 1+f(2) \\ 1+f(1) & 1+f(2) & 1+f(3) \\ 1+f(2) & 1+f(3) & 1+f(4) \end{vmatrix} = k(1-\alpha)^2(1-\beta)^2(\alpha-\beta)^2$

then 'k' is equal to:

(A) $\alpha\beta$

(B) $1/\alpha\beta$

(C) 1

(D) -1

$$\begin{vmatrix} 3 & 1+\alpha+\beta & 1+\alpha^2+\beta^2 \\ 1+\alpha+\beta & 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 \\ 1+\alpha^2+\beta^2 & 1+\alpha^3+\beta^3 & 1+\alpha^4+\beta^4 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \beta \\ 1 & \alpha^2 & \beta^2 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \end{vmatrix}$$

$C_1 \to C_1 - C_2$

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Solving System of Linear Equation in 3 Variable

Case 1: Non Homogenous System of Equation

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Case 2: Homogenous System of Equation

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

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Cramer's Rule for Non Homogeneous System of Equations

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$\boxed{\begin{aligned} x &= D_1/D \\ y &= D_2/D \\ z &= D_3/D \end{aligned}}$$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$D_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$D_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

Proof →

$$D_1 = \begin{vmatrix} a_1x + b_1y + c_1z & b_1 & c_1 \\ a_2x + b_2y + c_2z & b_2 & c_2 \\ a_3x + b_3y + c_3z & b_3 & c_3 \end{vmatrix}$$

$$C_1 \rightarrow C_1 - yC_2 - zC_3$$

$$D_1 = \begin{vmatrix} a_1x & b_1 & c_1 \\ a_2x & b_2 & c_2 \\ a_3x & b_3 & c_3 \end{vmatrix}$$

$$D_1 = x\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D_1 = xD \Rightarrow \boxed{x = D_1/D}$$

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Nature of Solutions

Case → 1 If $\boxed{D_1 = D_2 = D_3 = 0 \ \& \ D \neq 0}$ ⇒ $\boxed{\text{Unique}}$ $\underbrace{\text{trivial}}_{\text{all "zero"}}$ sol^n $(x = y = z = 0)$

Case → 2 If atleast one of D_1, D_2, D_3 is non zero & $\boxed{D \neq 0}$

Ex → $\left[\underbrace{D_1 = 10, \ D_2 = 20, \ D_3 = 5}_{x = \frac{10}{7}, \ y = \frac{20}{7}, \ z = 5/7}, \ D = 7 \right]$ ⇒ $\boxed{\text{Unique}}$ non trivial sol^n

Case → 3 If $\boxed{D_1 = D_2 = D_3 = 0 \ \& \ D = 0}$ ⇒ x, y & z will have infinite sol^n

Case → 4 If atleast one of D_1, D_2, D_3 is non zero but $D = 0$ ⇒ no sol^n → system is inconsistent

$\boxed{\text{☆☆ Unique sol}^n}$ ⇒ $\boxed{\text{Case → 1 \& 2 Union}}$ ⇒ $\boxed{D \neq 0}$ ☆☆

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QUESTION

Investigate for what values of λ, μ the simultaneous equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ have;

(a) A unique solution.

(b) An infinite number of solutions.

(c) No solution.

a) Unique solⁿ

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} \neq 0 \qquad \boxed{\lambda \neq 3}$$

$$D_2 = \begin{vmatrix} 1 & 6 & 1 \\ 1 & 10 & 3 \\ 1 & \mu & 3 \end{vmatrix} = 0 \Rightarrow \mu = 10$$

(b) $$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{vmatrix} = 0 \Rightarrow \boxed{\lambda = 3}$$

&

$$D_3 = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 10 \\ 1 & 2 & \mu \end{vmatrix} = 0 \qquad \boxed{\mu = 10}$$

$$D_1 = \begin{vmatrix} 6 & 1 & 1 \\ 10 & 2 & 3 \\ \mu & 2 & 3 \end{vmatrix} = 0 \Rightarrow \mu = 10$$

(b) $\lambda = 3$ & $\mu = 10$

(c) No solⁿ: $\lambda = 3,\ \mu \in R - \{10\}$

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QUESTION [JEE Main 2024 (08 Apr Shift 2)]

[Ans. B]

If the system of equations $x + 4y - z = \lambda, 7x + 9y + \mu z = -3, 5x + y + 2z = -1$ has infinitely many solutions, then $(2\mu + 3\lambda)$ is equal to :

(A) 3

(B) −3

(C) −2

(D) 2

$$D = \begin{vmatrix} 1 & 4 & -1 \\ 7 & 9 & \mu \\ 5 & 1 & 2 \end{vmatrix} = 0$$

$$1[18-\mu] - 4[14-5\mu] - 1[7-45] = 0$$

$$\cancel{18} - \mu - \cancel{56} + 20\mu + \cancel{38} = 0$$

$$19\mu = 0$$

$$\Rightarrow \boxed{\mu = 0}$$

$$D_3 = \begin{vmatrix} 1 & 4 & \lambda \\ 7 & 9 & -3 \\ 5 & 1 & -1 \end{vmatrix} = 0$$

$$1[-9+3] - 4[-7+15] + \lambda[7-45] = 0$$

$$-6 - 32 - 38\lambda = 0$$

$$\cancel{38} = -\cancel{38}\lambda$$

$$\boxed{\lambda = -1}$$

Check $D_2 = D_1 = 0$ for $\boxed{\lambda = -1}$

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QUESTION [JEE Main 2024 (Feb.-I)]

[Ans. B]

If the system of equations

$2x + 3y - z = 5$

$x + \alpha y + 3z = -4$

$3x - y + \beta z = 7$

has infinitely many solutions, then $13\alpha\beta =$

(A) 1110

(B) 1120

(C) 1210

(D) 1220

$$D = \begin{vmatrix} 2 & 3 & -1 \\ 1 & \alpha & 3 \\ 3 & -1 & \beta \end{vmatrix} = 0$$

$$D_3 = \begin{vmatrix} 2 & 3 & 5 \\ 1 & \alpha & -4 \\ 3 & -1 & 7 \end{vmatrix} = 0$$

$$= 2[7\alpha - 4] - 3[7+12] + 5[-1-3\alpha] = 0$$

$$= 14\alpha - 8 - 57 - 5 - 15\alpha = 0$$

$$\boxed{\alpha = -70}$$

$$D_2 = \begin{vmatrix} 2 & 5 & -1 \\ 1 & -4 & 3 \\ 3 & 7 & \beta \end{vmatrix} = 0$$

$$= 2[-4\beta - 21] - 5[\beta - 9] - 1[7+12] = 0$$

$$-8\beta - 42 - 5\beta + 45 - 19 = 0$$

$$-13\beta + 3 - 19 = 0$$

$$13\beta = -16$$

$$\boxed{\beta = \frac{-16}{13}}$$

$$13\alpha\beta = \cancel{13} \cdot 70 \times \frac{16}{\cancel{13}} = \underline{1120}$$

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QUESTION [JEE Main 2023]

[Ans. C]

For $\alpha, \beta \in \mathbb{R}$, suppose the system of linear equations

$$x - y + z = 5$$
$$2x + 2y + \alpha z = 8$$
$$3x - y + 4z = \beta$$

has infinitely many solutions. Then $\boxed{\alpha \text{ and } \beta \text{ are the roots of}}$

(A) $x^2 - 10x + 16 = 0$

(B) $x^2 + 18x + 56 = 0$

(C) $x^2 - 18x + 56 = 0$

(D) $x^2 - 14x + 24 = 0$

$$D = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0$$

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QUESTION [JEE Main 2024 (05 Apr Shift 1)]

[Ans. C]

If the system of equations

$11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to :

(A) 51

(B) 45

(C) 47

(D) 49

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QUESTION [JEE Main 2024 (Jan.-II)]

HW

[Ans. 113]

Let for any three distinct consecutive terms a, b, c of an A.P., the lines $ax + by + c = 0$ be concurrent at the point P and $Q(\alpha, \beta)$ be a point such that the system of equations

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$ and

$x + 2y + 3z = 4$,

has infinitely many solutions. Then $(PQ)^2$ is equal to ?

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QUESTION [JEE Main 2023 (01 Feb. Shift-I)]

[Ans. D]

Let S denote the set of all real values of λ such that the system of equations

$$\left.\begin{aligned}\lambda x + y + z &= 1\\ x + \lambda y + z &= 1\\ x + y + \lambda z &= 1\end{aligned}\right\}$$

Put $\lambda = 1 \Rightarrow \left.\begin{aligned}x+y+z&=1\\x+y+z&=1\\x+y+z&=1\end{aligned}\right\}$ Infinite sol^n

is $\boxed{\text{inconsistent,}}$ then $\sum_{\lambda \in S} (|\lambda|^2 + |\lambda|)$ is equal to

$\Rightarrow \lambda = -2$ gives no sol^n

$S = \{-2\}$

$(-2)^2 + |-2| = 4 + 2 = 6$

no sol^n

$\Rightarrow D = 0$

$\boxed{D_1, D_2, D_3 \neq 0}$

(A) 2

(B) 12

(C) 4

(D) 6 ✓

$$D = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 0$$

$D = \lambda[\lambda^2 - 1] - 1[\lambda - 1] + 1[1 - \lambda] = 0$

$\lambda^3 - \lambda - \lambda + 1 + 1 - \lambda = 0$

$$\boxed{\lambda^3 - 3\lambda + 2 = 0}$$

$\lambda^3 - 3\lambda + 3 - 1 = 0$

$(\lambda^3 - 1) - 3(\lambda - 1)$

$(\lambda - 1)(\lambda^2 + 1 + \lambda) - 3(\lambda - 1) = 0$

$(\lambda - 1)[\lambda^2 + 1 + \lambda - 3] = 0$

$(\lambda - 1)(\lambda^2 + \lambda - 2) = 0$

$(\lambda - 1)(\lambda + 2)(\lambda - 1) = 0 \Rightarrow \lambda = 1, -2$

Infinite

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QUESTION [JEE Main 2024 (30 Jan Shift 2)]

[Ans. D]

Consider the system of linear equations $x + y + z = 5, x + 2y + \lambda^2 z = 9, x + 3y + \lambda z = \mu$, where $\lambda, \mu \in R$. Then, which of the following statement is NOT correct?

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \lambda^2 \\ 1 & 3 & \lambda \end{vmatrix} = 0 \Rightarrow 1[2\lambda - 3\lambda^2] - 1[\lambda - \lambda^2] + 1(3-2) = 0$$

$$2\lambda - 3\lambda^2 - \lambda + \lambda^2 + 1 = 0 \Rightarrow -2\lambda^2 + \lambda + 1 = 0$$

$$2\lambda^2 - \lambda - 1 = 0$$

$$2\lambda^2 - 2\lambda + \lambda - 1 = 0$$

$$(2\lambda + 1)(\lambda - 1) = 0$$

$$\lambda = -\tfrac{1}{2}, \lambda = 1$$

(A) System has infinite number of solution if $\lambda = 1$ and $\mu = 13$ → True

(B) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$
True   no soln

(C) System is consistent if $\lambda \neq 1$ and $\mu = 13$

(D) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$
False

$\lambda = 1$

$$D_2 = \begin{vmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

$$D_1 = \begin{vmatrix} 5 & 1 & 1 \\ 9 & 2 & 1 \\ \mu & 3 & 1 \end{vmatrix} = 0$$

at $\lambda = 1 \Rightarrow \boxed{D = 0}$

$$D_3 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & 9 \\ 1 & 3 & \mu \end{vmatrix}$$

$$= 1[2\mu - 27] - 1[\mu - 9] + 5(3-2) = 0$$

$$= 2\mu - 27 - \mu + 9 + 5 = 0$$

$$\boxed{\mu = 13}$$

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Cramer's Rule for Homogeneous System of Equations

$$a_1x + b_1y + c_1z = 0$$
$$a_2x + b_2y + c_2z = 0$$
$$a_3x + b_3y + c_3z = 0$$

here $\boxed{D_1 = D_2 = D_3 = 0}$

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

Nature of solⁿ: →

1) If $D \neq 0 \Rightarrow$ Unique trivial solⁿ

2) If $D = 0 \Rightarrow$ Infinite solⁿ.

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QUESTION [JEE Main 2023 (08 Apr Shift 2)]

[Ans. A]

Let S be the set of all values of $\theta \in [-\pi, \pi]$ for which the system of linear equations

$$\boxed{\begin{aligned} x + y + \sqrt{3}z &= 0 \\ -x + (\tan\theta)y + \sqrt{7}z &= 0 \\ x + y + (\tan\theta)z &= 0 \end{aligned}}$$

→ Homogenous system.

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta \in S}\theta$ is equal to

↓ Infinite sol^n

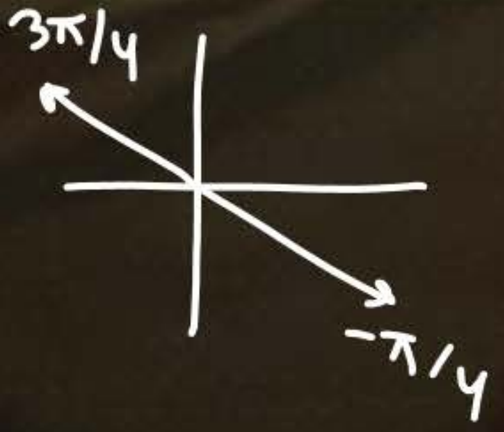
$\boxed{D = 0}$

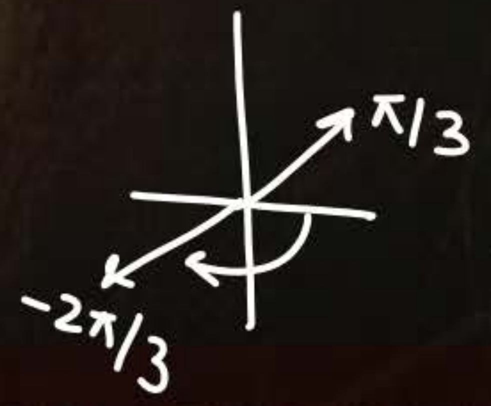
A 20 ✓

B 40

C 30

D 10





$D = 0$

$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_3$

$$\begin{vmatrix} 0 & 0 & \sqrt{3} - \tan\theta \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix}$$

$(\sqrt{3} - \tan\theta)[-1 - \tan\theta] = 0$

$(\sqrt{3} - \tan\theta)(1 + \tan\theta) = 0$

$\tan\theta = \sqrt{3}$ or $\tan\theta = -1$

$\theta = \pi/3$ or $-2\pi/3$ $\qquad$ $\theta = -\pi/4, \frac{3\pi}{4}$

$\frac{120}{\pi}\left[-\pi/3 + \pi/2\right] = \frac{120}{\pi} \cdot \pi/6 = 20$

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Note

If the homogenous system of equations

$$\begin{cases} a_1x + b_1y + c_1 = 0; \\ a_2x + b_2y + c_2 = 0; \\ a_3x + b_3y + c_3 = 0 \end{cases}$$

have some Non trivial Solution then this implies that the given system must have Infinite Solutions

$$\Rightarrow D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

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QUESTION [JEE Main 2024 (04 Apr Shift 1)]

[Ans. C]

If the system of equations

$x + (\sqrt{2}\sin\alpha)y + (\sqrt{2}\cos\alpha)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$x + (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is

$D = 0$

(A) $\frac{11\pi}{24}$

(B) $\frac{7\pi}{24}$

(C) $\frac{5\pi}{24}$

(D) $\frac{3\pi}{4}$

$$\begin{vmatrix} 1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \\ 1 & \sin\alpha & -\cos\alpha \end{vmatrix} = 0$$

$$1[-\cos^2\alpha - \sin^2\alpha] - \sqrt{2}\sin\alpha[-\cos\alpha - \sin\alpha] + \sqrt{2}\cos\alpha[\sin\alpha - \cos\alpha]$$

$$= -1 + \sqrt{2}\sin\alpha\cos\alpha + \sqrt{2}\sin^2\alpha + \sqrt{2}\sin\alpha\cos\alpha - \sqrt{2}\cos^2\alpha = 0$$

$$= -1 + \sqrt{2}\sin 2\alpha - \sqrt{2}\cos 2\alpha = 0$$

$$\sqrt{2}(\sin 2\alpha - \cos 2\alpha) = 1$$

$$\boxed{\sin 2\alpha - \cos 2\alpha = \frac{1}{\sqrt{2}}}$$

dont S.B.S

divide by $\sqrt{2}$

$$\frac{\sin 2\alpha}{\sqrt{2}} - \frac{\cos 2\alpha}{\sqrt{2}} = \frac{1}{2}$$

$$\sin(2\alpha - \pi/4) = 1/2$$

$$2\alpha - \pi/4 = \pi/6$$

$$2\alpha = \pi/4 + \pi/6$$

$$2\alpha = \frac{3\pi + 2\pi}{12}$$

$$2\alpha = 5\pi/12$$

$$\alpha = 5\pi/24$$

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QUESTION [JEE Main 2019 (April)]

[Ans. C]

If the system of equations $2x + 3y - z = 0$, $x + ky - 2z = 0$ and $2x - y + z = 0$ has a non-trivial solution (x, y, z), then $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + k$ is equal to

(A) 3/4

(B) −4

(C) 1/2

(D) −1/4

QUESTION [JEE Main 2018]

[Ans. A]

If the system of linear

$x + ky + 3z = 0;$

$3x + ky - 2z = 0;$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z) then $\dfrac{xz}{y^2} = ?$

(A) 10

(B) −30

(C) 30

(D) −10

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QUESTION [JEE Adv. 2023]

$$D = \begin{vmatrix} 1 & 0 & \alpha \\ 2 & -3 & \beta \end{vmatrix} = 0$$

Let α, β and γ be real numbers. Consider the following system of linear equations

$$\boxed{\begin{aligned} x + 2y + z &= 7 \\ x + \alpha z &= 11 \\ 2x - 3y + \beta z &= \gamma \end{aligned}}$$

$1[3\alpha] - 2[\beta - 2\alpha] + 1(-3) = 0$

$3\alpha - 2\beta + 4\alpha - 3 = 0$

$7\alpha = 2\beta + 3$

$\boxed{\beta = \frac{7\alpha - 3}{2}}$

Match each entry in List-I to the correct entries in List-II.

List – I

(P) If $\boxed{\beta = \frac{1}{2}(7\alpha - 3)}$ and $\boxed{\gamma = 28}$, then the system has

(Q) If $\beta = \frac{1}{2}(7\alpha - 3)$ and $\gamma \neq 28$, then the system has

(R) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\underline{\alpha = 1}$ and $\underline{\gamma \neq 28}$, then the system has

$\boxed{D_3 \neq 0}$

$\Rightarrow z = D_3/D \neq 0$

(S) If $\beta \neq \frac{1}{2}(7\alpha - 3)$ where $\alpha = 1$ and $\gamma = 28$, then the system has

List-II

(1) A unique solution

(2) No solution

(3) Infinitely many solution

(4) $x = 11, y = -2$ and $z = 0$ as a solution

(5) $x = -15, y = 4$ and $\boxed{z = 0}$ as a solution

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$$D_3 = \begin{vmatrix} 1 & 2 & 7 \\ 1 & 0 & 11 \\ 2 & -3 & \gamma \end{vmatrix} = 0$$

$$D_3 = 1[33] - 2[\gamma - 22] + 7(-3) = 0$$

$$33 - 2\gamma + 44 - 21 = 0$$

$$2\gamma = 77 - 21$$

$$2\gamma = 56$$

$$\boxed{\gamma = 28}$$

Check D_1 & D_2 also

QUESTION [JEE Adv. 2023]

[Ans. A]

The correct option is:

(A) $(P) \rightarrow (3)$, $(Q) \rightarrow (2)$, $(R) \rightarrow (1)$, $(S) \rightarrow (4)$

(B) $(P) \rightarrow (3)$, $(Q) \rightarrow (2)$, $(R) \rightarrow (5)$, $(S) \rightarrow (4)$

(C) $(P) \rightarrow (2)$, $(Q) \rightarrow (1)$, $(R) \rightarrow (4)$, $(S) \rightarrow (5)$

(D) $(P) \rightarrow (2)$, $(Q) \rightarrow (1)$, $(R) \rightarrow (1)$, $(S) \rightarrow (5)$

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QUESTION [JEE Main 2019 (Jan.)]

[Ans. B]

The number of values of $\theta \in (0, \pi)$ for which the system of linear equations

$x + 3y + 7z = 0;$

$-x + 4y + 7z = 0;$

$x \sin 3\theta + y \cos 2\theta + 2z = 0$

has non-trivial solutions, is

(A) three

(B) two

(C) four

(D) one

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QUESTION [JEE Main 2023 (01 Feb. Shift-II)]

[Ans. A]

For the system of linear equations

$\alpha x + y + z = 1, x + \alpha y + z = 1, x + y + \alpha z = \beta,$

which one of the following statements is NOT correct?

(A) It has infinitely many solutions if $\alpha = 2$ and $\beta = -1$

(B) It has no solution if $\alpha = -2$ and $\beta = 1$

(C) $x + y + z = \frac{3}{4}$ if $\alpha = 2$ and $\beta = 1$

(D) It has infinitely many solutions if $\alpha = 1$ and $\beta = 1$

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QUESTION [JEE Main 2019 (Jan.)]

[Ans. D]

The system of linear equation

$x + y + z = 2,$

$2x + 3y + 2z = 5,$

$2x + 3y + (a^2 - 1)z = a + 1$

(A) is inconsistent when $a = 4$

(B) has a unique solution for $|a| = \sqrt{3}$

(C) has infinitely many solutions for $a = 4$

(D) inconsistent when $|a| = \sqrt{3}$

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BRAIN TEASER

$Y_1 \to 2, -2$ $\quad Y_2 \to \pm 2$ $\quad Y_3 \to \pm 2$

If every element of a **(3 × 3)** determinant is from the set **{1, –1}** then show that the value of such determinant can only be **0, –4 or 4**.

$$\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$= a_1[b_2c_3 - c_2b_3] - a_2[b_1c_3 - c_1b_3] + a_3[b_1c_2 - c_1b_2]$$

$$\Delta = (a_1b_2c_3 - a_1b_3c_2) + (a_2b_3c_1 - a_2b_1c_3) + (a_3b_1c_2 - a_3b_2c_1)$$

$$\Delta = Y_1 + Y_2 + Y_3$$

$\Delta \to \pm 6, \pm 4, \pm 2, 0$

$\pm 6, \pm 2 \to$ Reject ?

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Consider $\boxed{\Delta = 6}$

$\Rightarrow \boxed{y_1 = 2 = y_2 = y_3}$

$$\Rightarrow \left\{ \begin{array}{lcl} a_1b_2c_3 = 1 & \& & a_1b_3c_2 = -1 \\ a_2b_3c_1 = 1 & \& & a_2b_1c_3 = -1 \\ a_3b_1c_2 = 1 & \& & a_3b_2c_1 = -1 \end{array} \right\}$$ not possible

multiply

$\boxed{a_1a_2a_3b_1b_2b_3c_1c_2c_3 = 1}$

$a_1a_2a_3b_1b_2b_3c_1c_2c_3 = -1$

Contradict

$\boxed{\Delta = -6} \rightarrow$ Rejected

$\Delta = 2$

$y_1 = 2$, $y_2 = y_3 = 0$

HW

$y_1 = 2$, $y_2 = 2$, $y_3 = -2$

BRAIN TEASER

[Ans 0, 1, -1, 2, -2]

If every element of a (3×3) determinant is from the set $\{0, 1\}$ then find the possible number of values of such determinant.

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QUESTION [JEE Adv. 2024]

HW

[Ans. 16]

Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0, 1\} \text{ and } |A| \in \{-1, 1\} \right\}$,

where $|A|$ denotes the determinant of A.

Then the number of elements in S is __________ .

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Homework

REDO ALL QUES
+ H.W.

FOR NOTES & DPP CHECK DESCRIPTION


Thank
YOU
Keep Hustling!