PRAYAS

JEE 2026

Physics

COM and System of particles

Lecture - 1

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Topics to be covered

A Concept of COM

B

C

D

ATDB.uno

Question

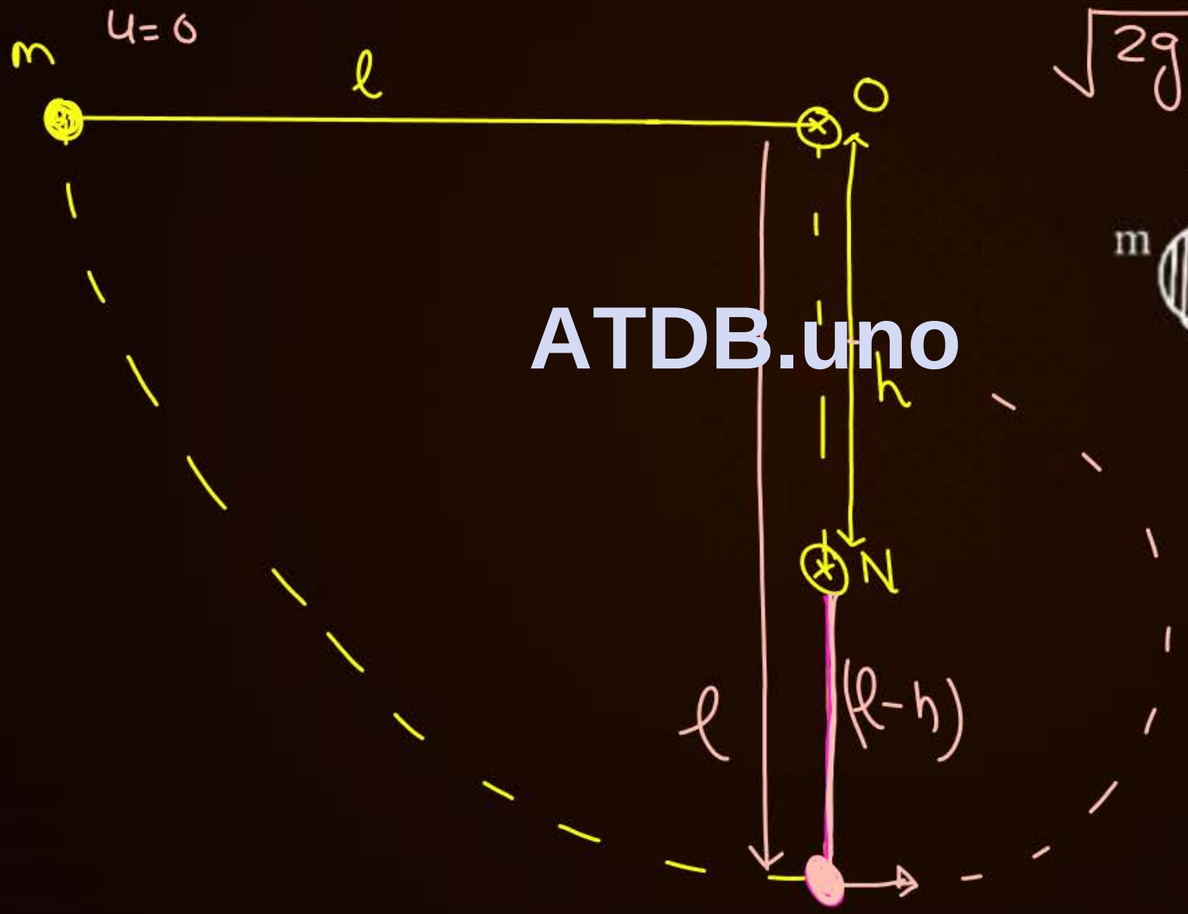
A particle of mass m attached to the end of a string of length l is released from the horizontal position. The particle rotates in a circle about O as shown. When it is vertically below O , the string makes contact with a nail N placed directly below O at a distance h and rotates around it. For the particle to swing completely around the nail in a circle.

a. $h < \frac{3}{5}l$

b. $h \geq \frac{3}{5}l$

c. $h < \frac{2}{5}l$

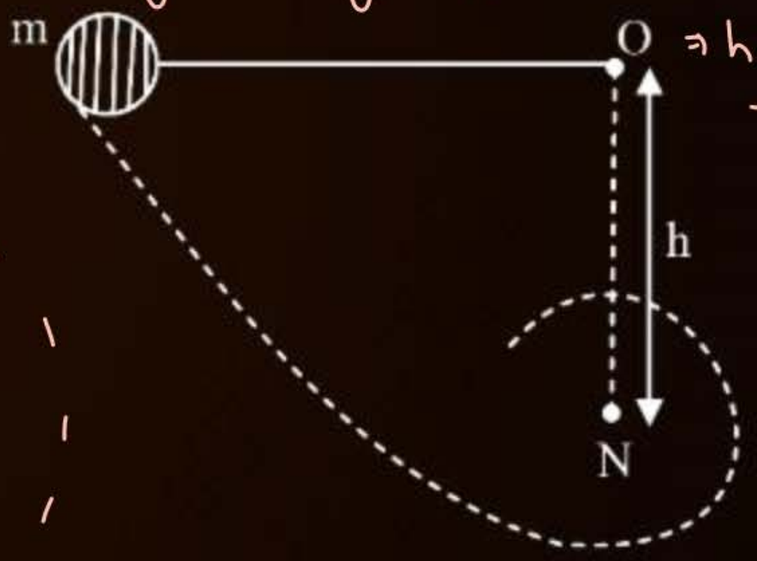
d. $h \geq \frac{2}{5}l$



$\sqrt{2gl} \geq \sqrt{5g(l-h)}$

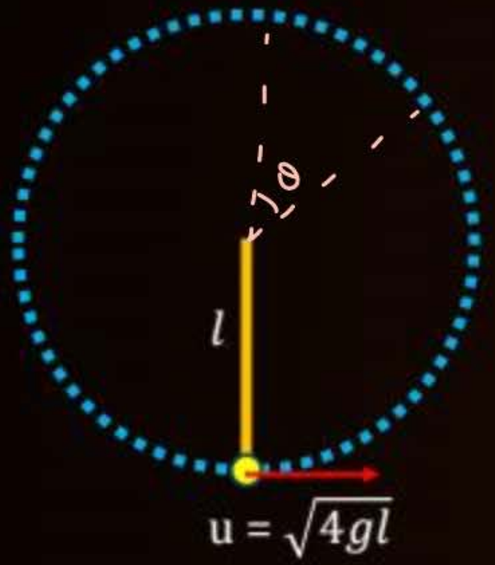
$2gl \geq 5g(l-h) \Rightarrow 2l \geq 5l - 5h$

$\Rightarrow h \geq \frac{3l}{5}$



Question

A particle of mass m attached to the end of a string of length l is projected with speed $u = \sqrt{4gl}$. Find angle with vertical line at which string is about to slack also find maximum height attend by the particle



$\cos\theta$

(A) $1/3$

(B) $2/3$

(C) $\sqrt{3}/2$

(D) $1/2$

Sol^n $\sqrt{2gl} < u < \sqrt{5gl}$ ⇒ particle leave circular motion in upper half

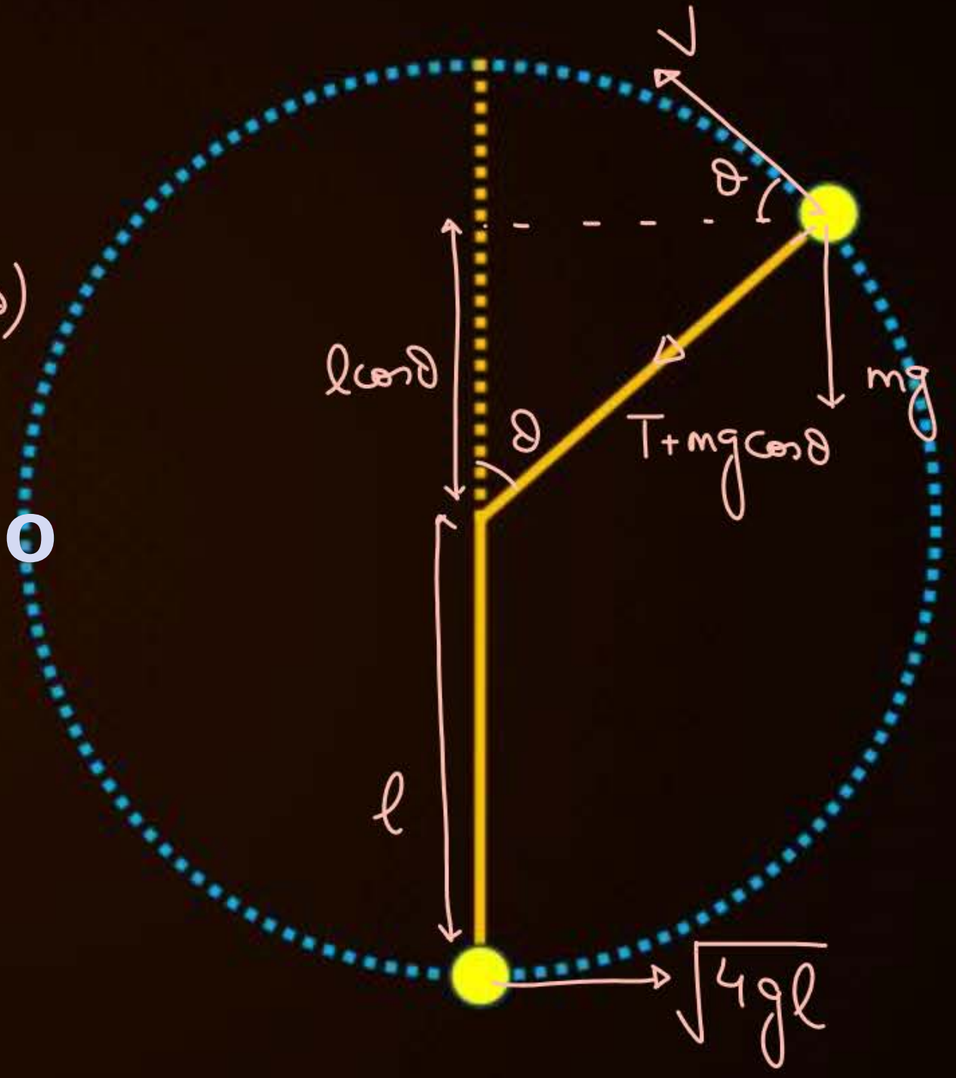
$\longrightarrow$ $T + mg\cos\theta = \frac{mv^2}{l} \Rightarrow v^2 = gl\cos\theta$

$\longrightarrow$ $v^2 = u^2 - 2gh \Rightarrow gl\cos\theta = 4gl - 2gl(1+\cos\theta)$

$\Rightarrow \cos\theta = 2/3$

$\left(\frac{1}{2}mv^2 + mgl(1+\cos\theta) = \frac{1}{2}mu^2\right)$

$v = \sqrt{\frac{2}{3}gl}$ Circular motion end
Projectile motion Start



$V = \sqrt{\frac{2}{3}gl}$ $\quad \cos\theta = 4/3$

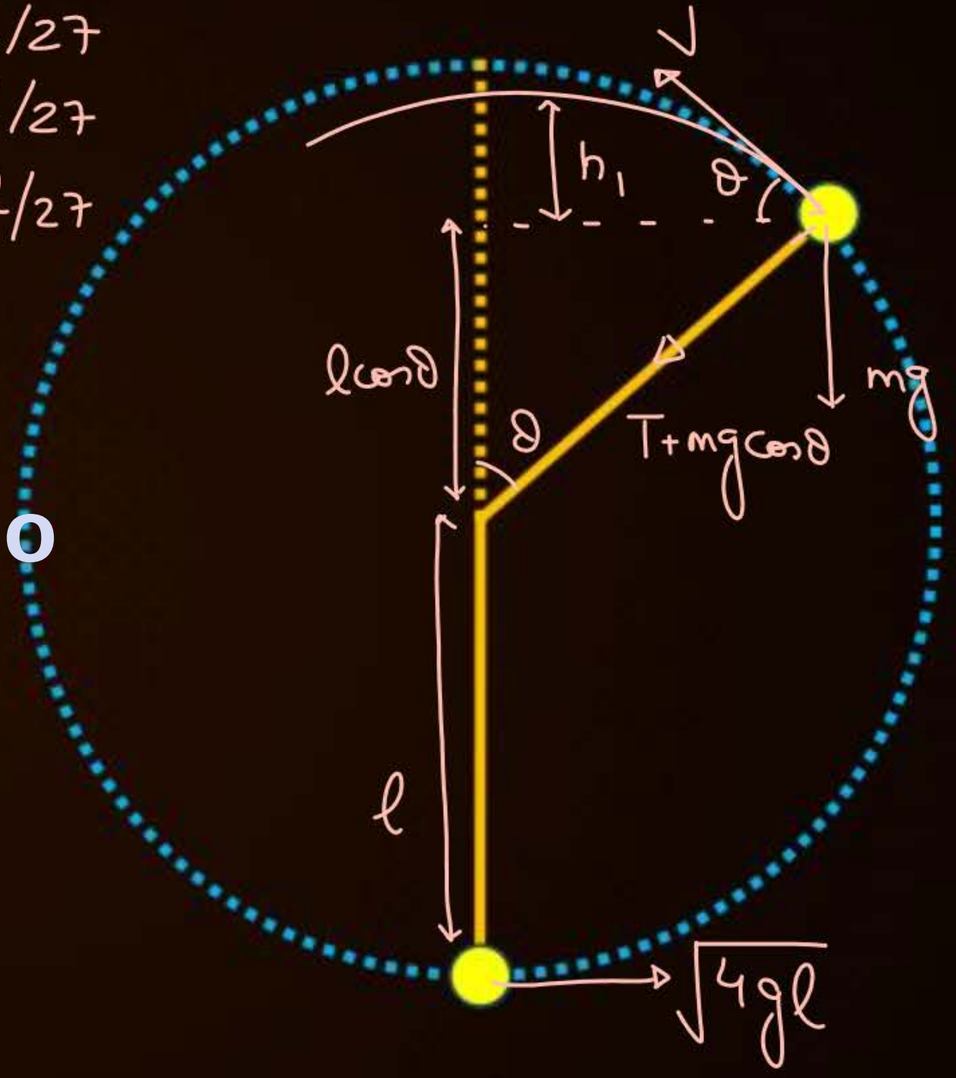
$$h_1 = \frac{v^2 \sin^2\theta}{2g}$$

$$h_1 = \frac{\left(\frac{2}{3}gl\right)\left(1 - \frac{4}{9}\right)}{2g} = \frac{5l}{27}$$

from Ground $\quad H = l + l\cos\theta + h_1$

$$= l + \frac{2}{3}l + \frac{5l}{27} = \frac{50l}{27} < 2l$$

(i) $50l/27$
(ii) $55l/27$
(iii) $52l/27$
(iv) $54l/27$

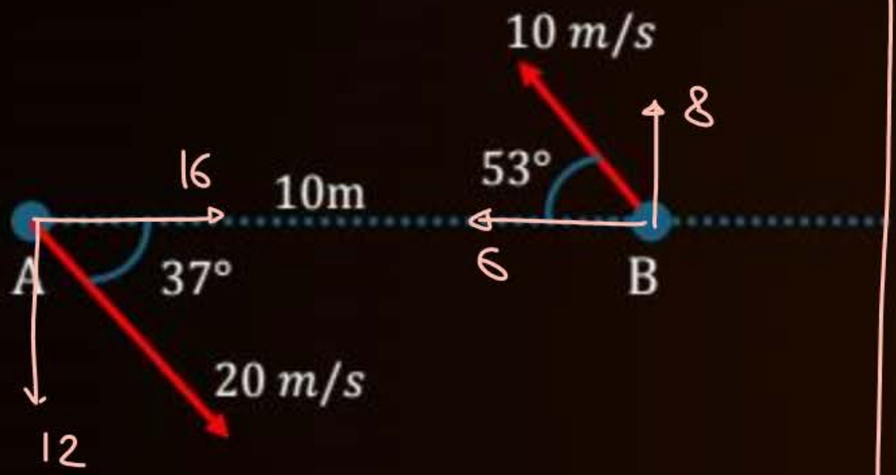


Question

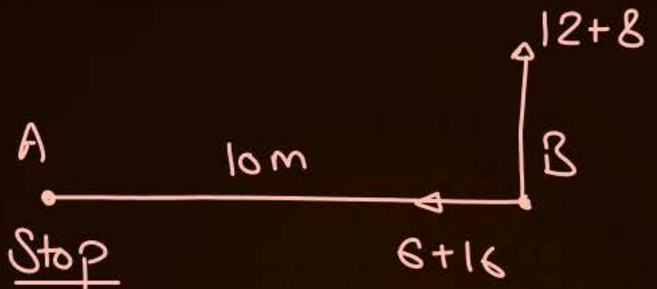
Two particle A and B are moving as shown. At given instant find

1 Velocity of separation or approach

2 Angular velocity of particle B w.r.t. particle A



w.r.t A



$V_{app.} = 22\ m/s$

$\omega = \frac{12+8}{10} = 2\ rad/s$

Question

A particle is projected with speed 50m/s at an angle 53° with horizontal. Find radius of curvature

1 At point of projection
2 At highest point
3 After 1 sec. of projection

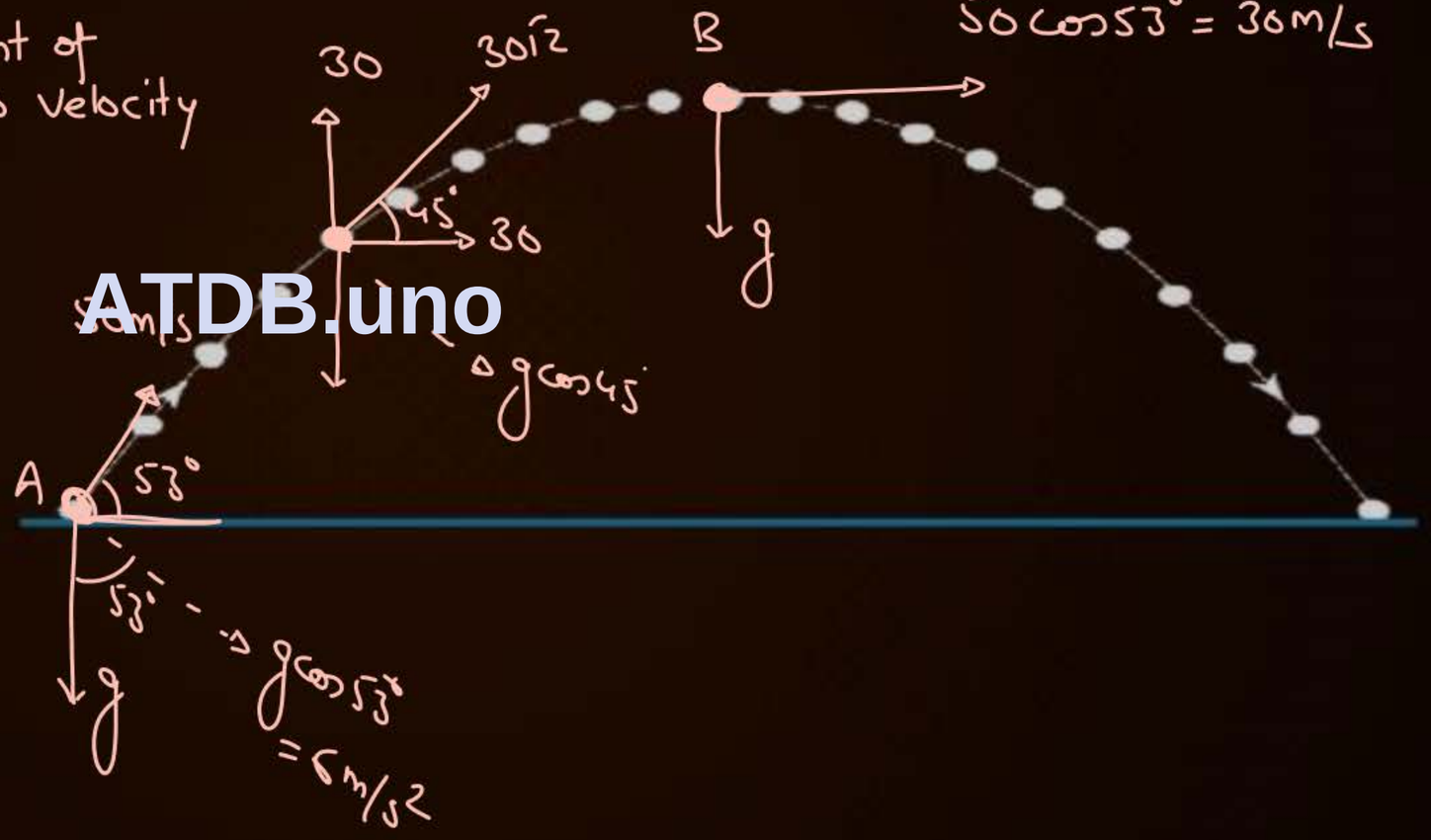
$$\star\star \quad ROC = \frac{v^2}{a_c} = \frac{v^2}{a_\perp}$$

→ Component of acc. ⊥ to Velocity

1] $ROC_A = \frac{(50)^2}{60}$

2] $ROC_B = \frac{(30)^2}{10}$

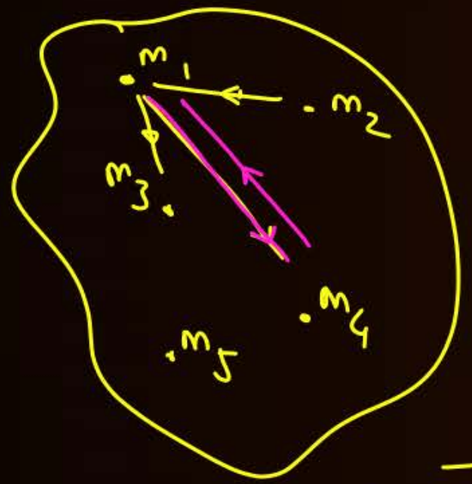
3] $ROC_C = \frac{(30\sqrt{2})^2}{10 \cdot \frac{1}{\sqrt{2}}}$



Concept of COM

$$\vec{F} = \frac{d\vec{P}}{dt} = m\vec{a}$$

for particle



$$\vec{f}_1 + \vec{f}_2 + \vec{f}_3 + \dots = m_1\vec{a}_1$$

$$\vec{f}_1' + \vec{f}_2' + \vec{f}_3' + \dots = m_2\vec{a}_2$$

$$\vec{f}_1'' + \vec{f}_2'' + \vec{f}_3'' + \dots = m_3\vec{a}_3$$

$$\left(\vec{f}_1 + \vec{f}_1' + \vec{f}_1'' + \dots\right) + \left(\vec{f}_2 + \vec{f}_2' + \vec{f}_2'' + \dots\right) = m_1\vec{a}_1 + m_2\vec{a}_2 + \dots$$

$$\sum \vec{F} = m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots$$

$$\sum \vec{F}_{internal} + \sum \vec{F}_{external}$$

→ Sum of all internal forces is zero

(internal force → action + Reaction both are in System)

$$0 + \sum \vec{F}_{external} = \text{net } \vec{F}_{ext.}$$

→ Let $m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots = M\vec{a}_{cm}$

M → Total mass

$M = m_1 + m_2 + \cdots$

$$\text{net } \vec{F}_{ext.} = M\vec{a}_{cm} \quad \star\star$$

$\vec{a}_{cm}$ → acceleration of COM

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots}{M}$$

$$\frac{d\vec{V}_{cm}}{dt} = \frac{m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} + \cdots}{M}$$

$$\int d\vec{V}_{cm} = \frac{\int m_1 d\vec{v}_1 + \int m_2 d\vec{v}_2 + \int \cdots}{M}$$

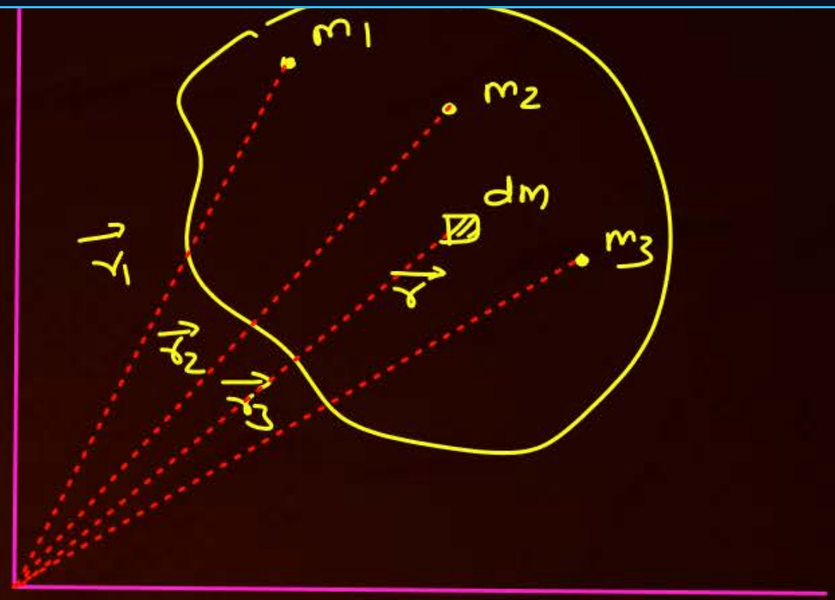
$$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{M}$$

$$\frac{d\vec{r}_{cm}}{dt} = \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + \cdots}{M}$$

$$\int d\vec{r}_{cm} = \frac{\int m_1 d\vec{r}_1 + \int m_2 d\vec{r}_2 + \cdots}{M}$$

$$\boxed{\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots}{M}}$$

Position of Center of mass



for object

$$\vec{r}_{cm} = \frac{\sum m_i\vec{r}_i}{M} = \frac{\int \vec{r}\,dm}{\int dm}$$

→ Position of COM

$$\vec{r}_{cm} = \frac{\sum m_i \vec{r}_i}{M} = \frac{\int \vec{r}\, dm}{\int dm}$$

→ Velocity of COM

$$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots}{M} = \frac{\int \vec{v}\, dm}{\int dm}$$

→ acceleration of COM

$$\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + \cdots}{M} = \frac{\int \vec{a}\, dm}{\int dm}$$

displacement of COM

$$\vec{S}_{cm} = \vec{r}_{cm})_f - \vec{r}_{cm})_i$$

$$\vec{S}_{cm} = \frac{m_1\vec{S}_1 + m_2\vec{S}_2 + \cdots}{M}$$

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + \cdots}{M} = \frac{\int x\, dm}{\int dm}$$

→ $\vec{F}_{ext.} = M\vec{a}_{cm}$ from inertial frame (Ground frame)

momentum of System

→ $\vec{P} = M\vec{V}_{cm}$

$$\vec{P}_{System} = \vec{P}_1 + \vec{P}_2 + \cdots$$

$$= m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots = M\vec{V}_{cm}$$

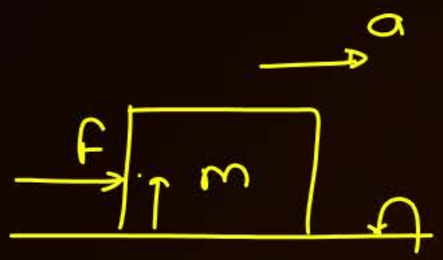
→ $\vec{F}_{ext.} = \frac{d\vec{P}}{dt}$

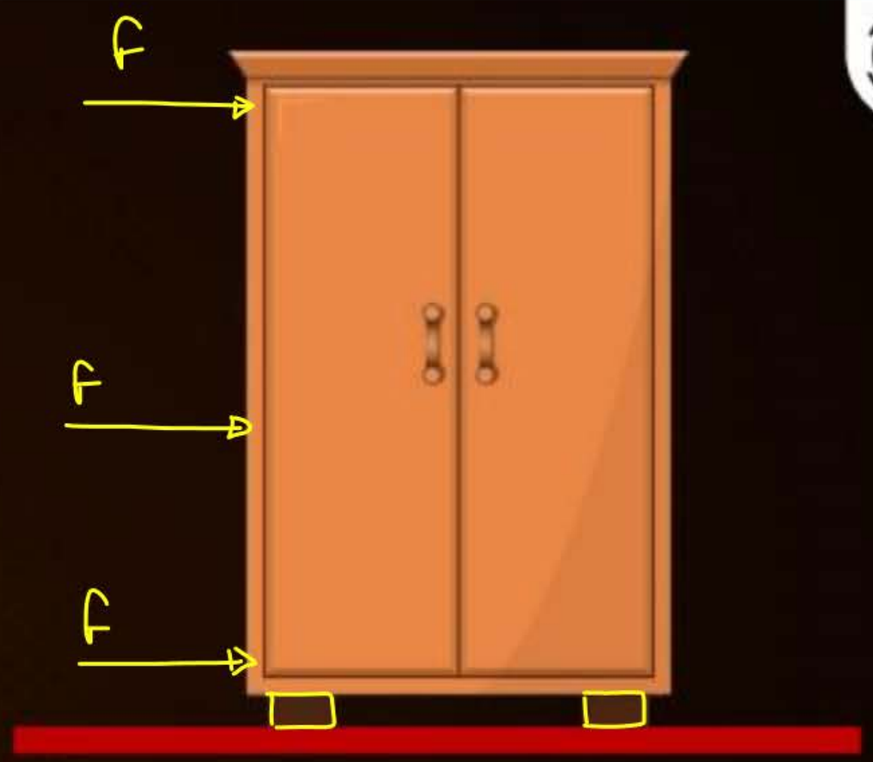
→ if $\vec{F}_{ext.} = 0$ (net external force on System is zero) then momentum of system is Conserved this is known as momentum Conservation (from inertial frame)

$\vec{F}_{ext.} = M\vec{a}_{cm} = 0 \Rightarrow \vec{a}_{cm} = 0$

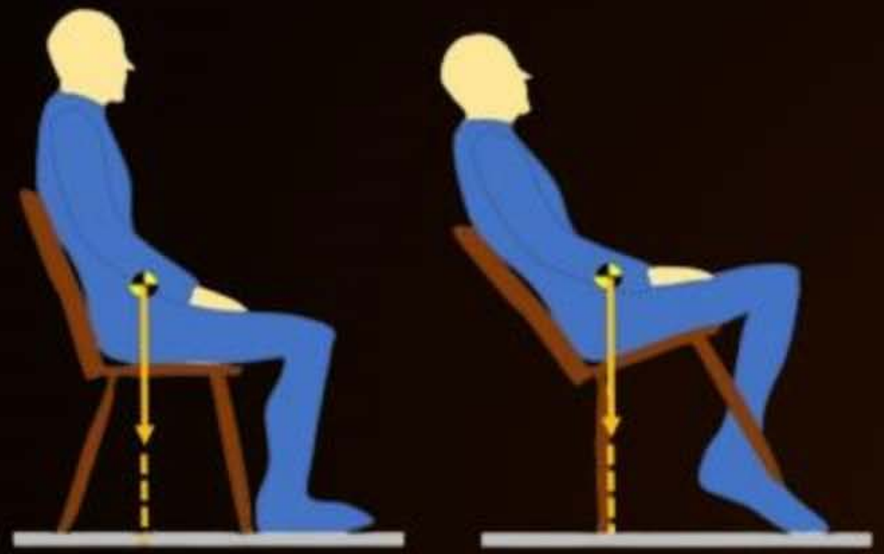
$\Rightarrow \vec{V}_{cm}$ = Constant $\Rightarrow \vec{P}$ = Constant

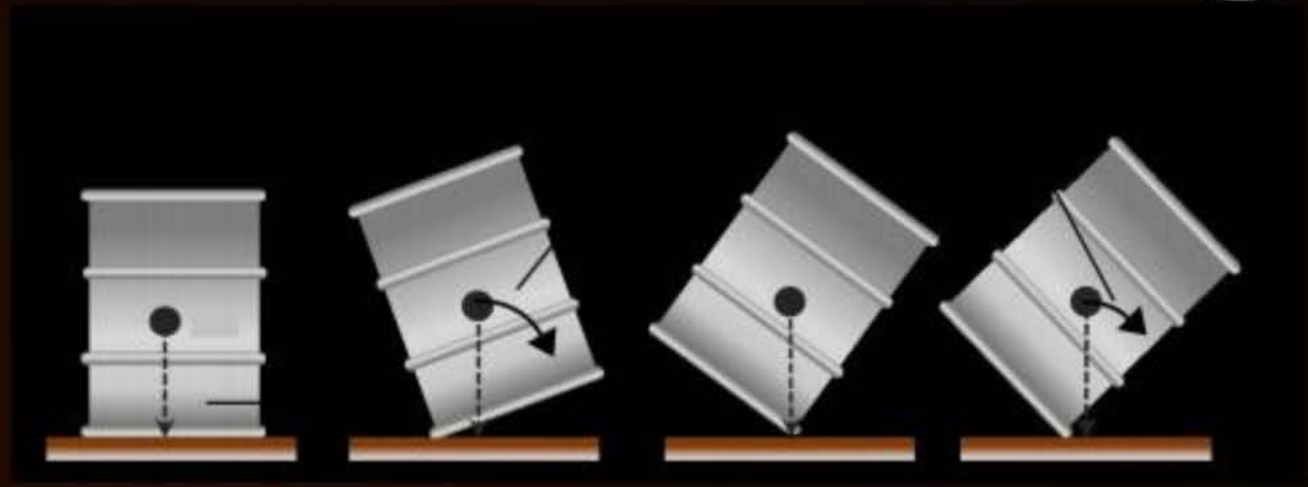
Impulse $\vec{J} = \vec{P}_f - \vec{P}_i = \int \vec{F}\, dt$ (ext.)





Point of application of force all matters





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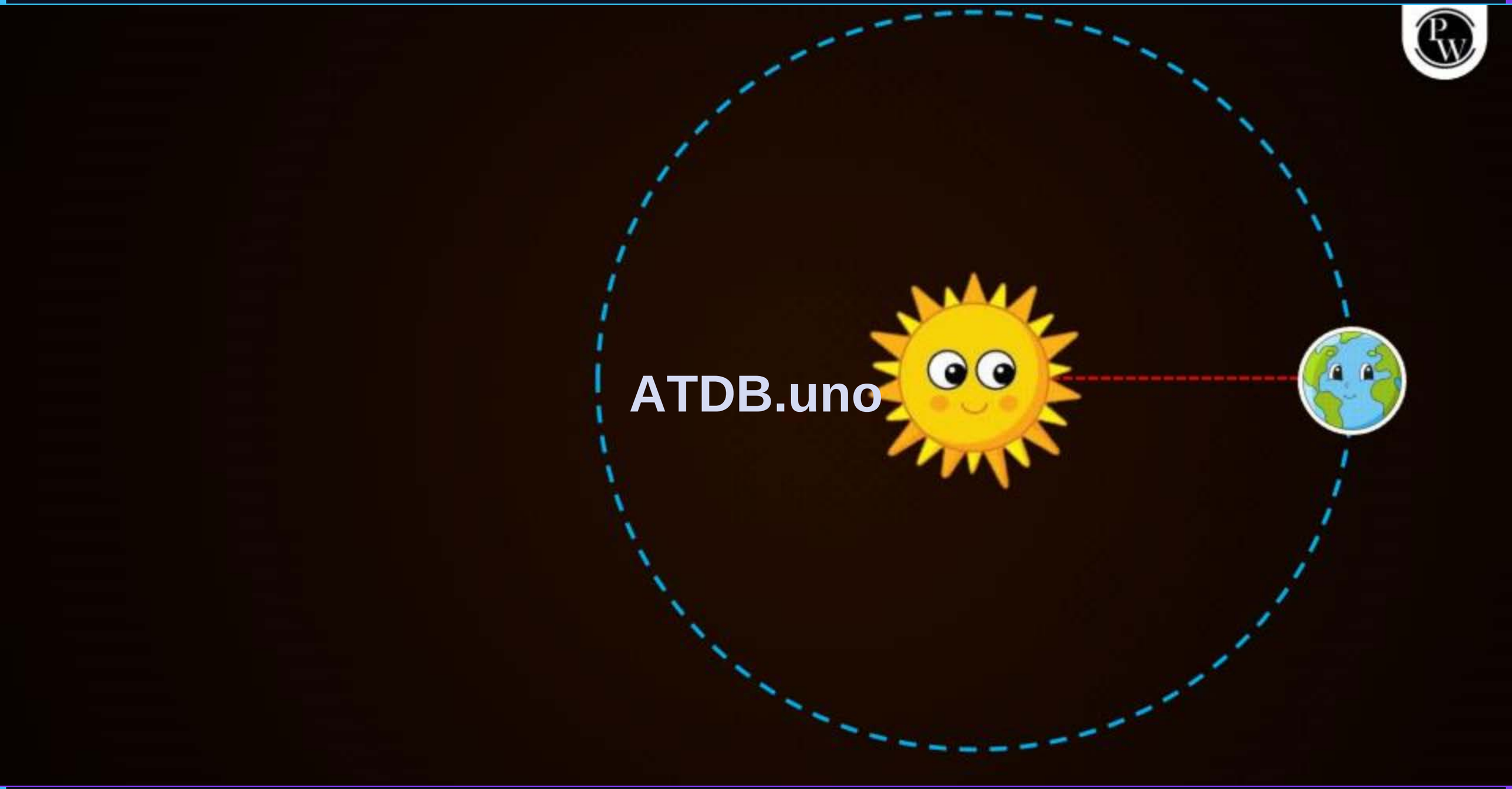

→ both revolve around COM of System