PRAYAS

JEE 2026

Physics

COM and System of particles

Lecture – 3

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Topics to be covered

A Calculation of COM

B

C

D

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Question - 11

A square of side $2R$ and a disc of radius R are kept in contact as shown. Find distance of COM of system from contact point. Given both object have same mass per unit area (made of same material and have same thickness)

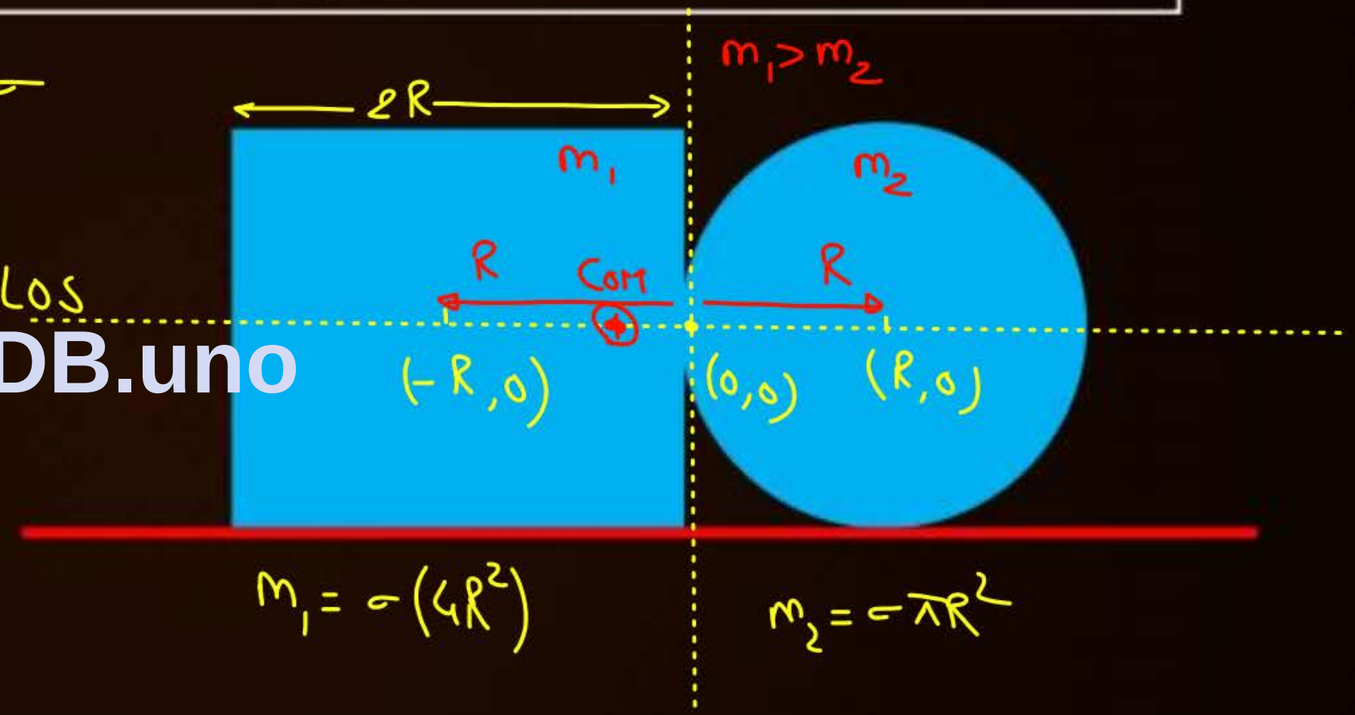
Let mass per unit area = σ

$$y_{cm} = 0$$

$$x_{cm} = \frac{\sigma 4R^2(-R) + \sigma \pi R^2(R)}{\sigma 4R^2 + \sigma \pi R^2}$$

$$x_{cm} = \frac{-4R + \pi R}{4 + \pi} = \left(\frac{\pi - 4}{\pi + 4}\right)R$$

$$\text{distance} = \left(\frac{4 - \pi}{4 + \pi}\right)R$$

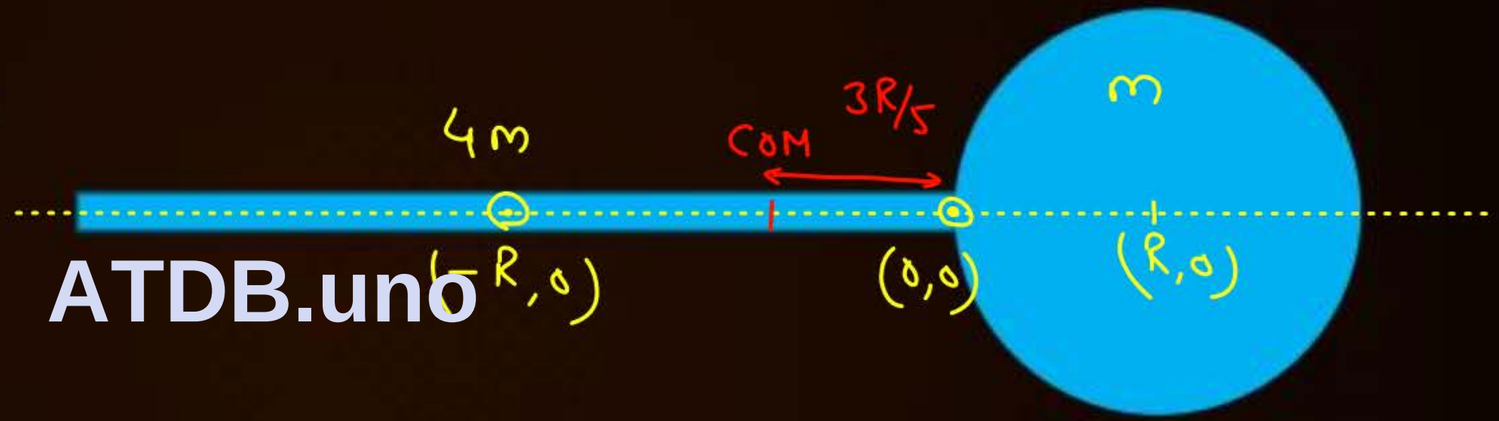


Question

A rod of mass 4m and length 2R and a disc of mass m and radius R are kept in contact as shown. Find distance of COM of system from contact point

$$y_{cm} = 0$$

$$x_{cm} = \frac{m(R) + 4m(-R)}{5m} = -\frac{3R}{5}$$



Question

A rod of length L have variable mass density $\lambda = aX+b$, where X is distancee from point A find (1) total mass of rod (2) COM from end A

Rod → Element dx at distance x (along the rod)

$$X_{cm} = \frac{\int_0^L x\,(ax+b)\,dx}{\int_0^L (ax+b)\,dx}$$

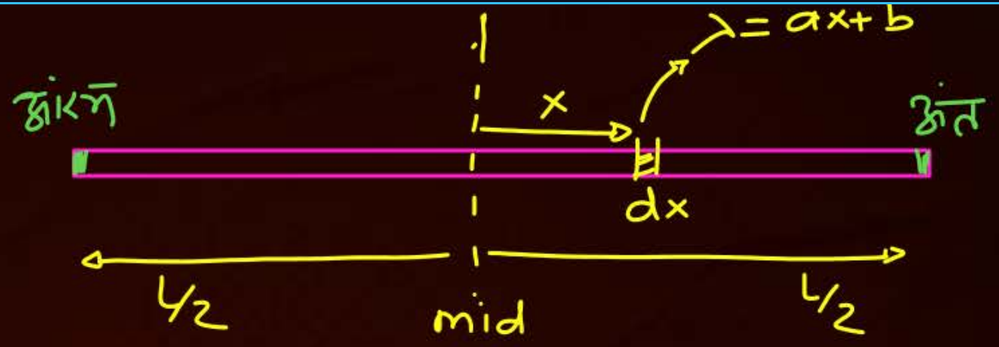
$$dm = \lambda dx = (ax+b)dx$$

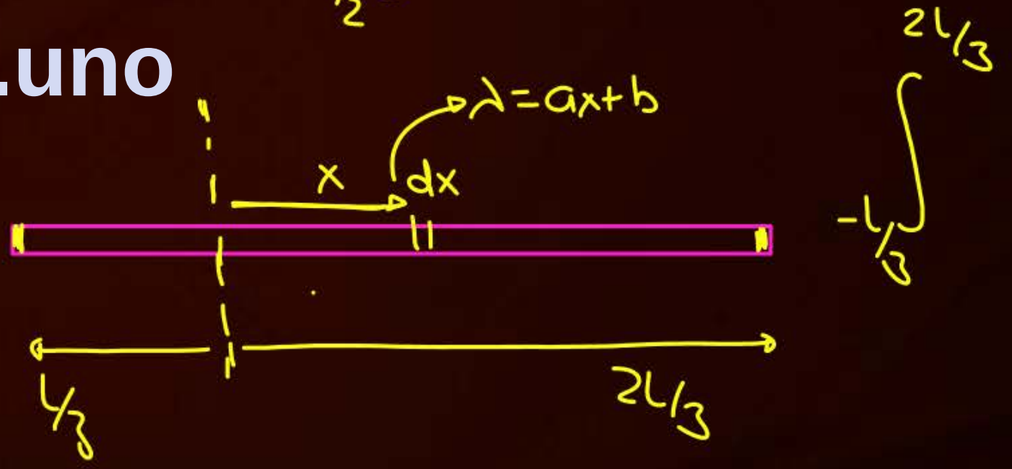
$$= \frac{\int_0^L (ax^2+bx)\,dx}{\int_0^L (ax+b)\,dx} = \frac{\frac{aL^3}{3}+\frac{bL^2}{2}}{\frac{aL^2}{2}+bL} =$$

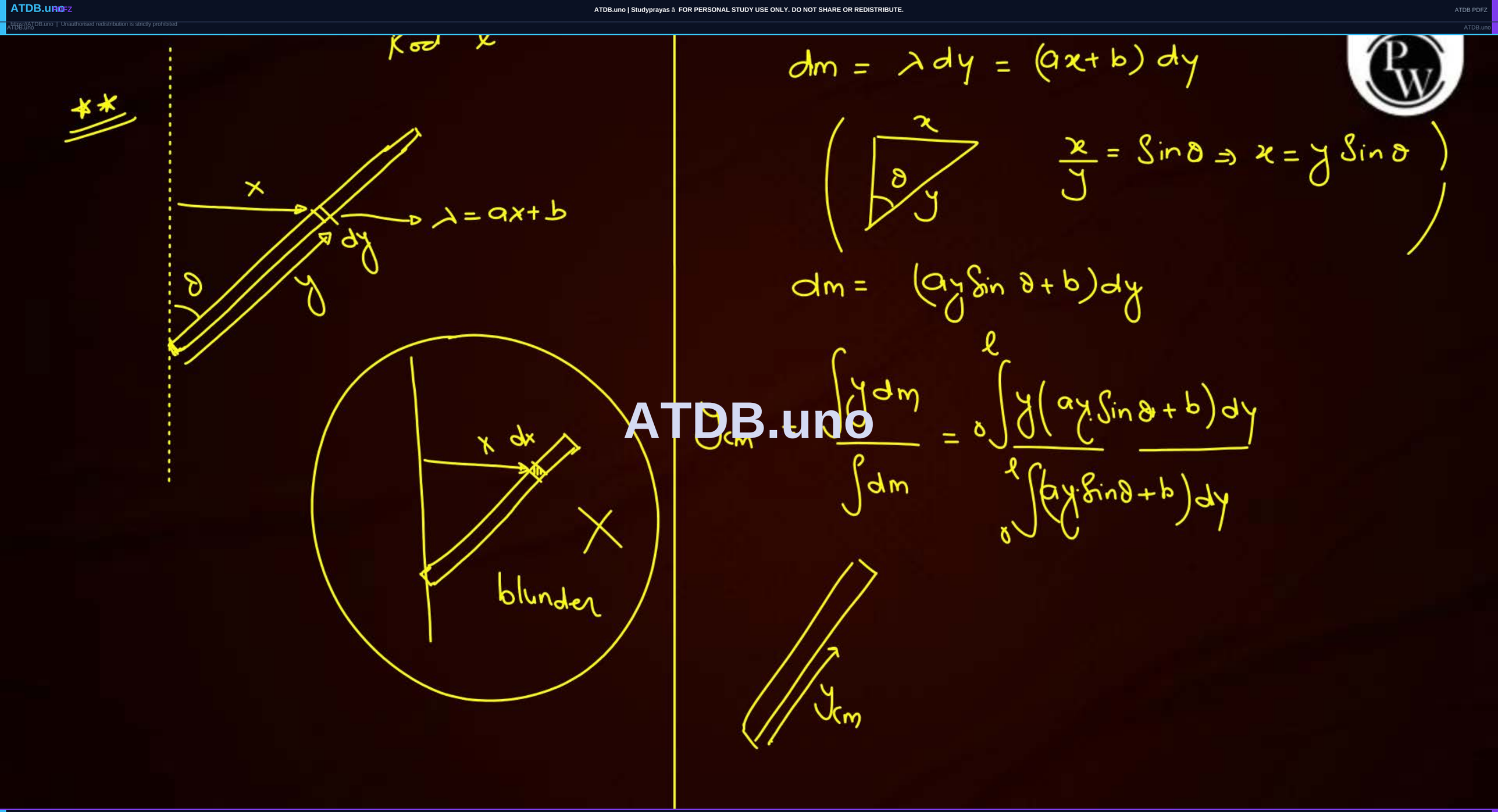
$$dm = \lambda dx = \sigma dA = \rho dV$$

$$X_{cm} = \frac{\int x\,dm}{\int dm}$$

$$\text{Total mass} = \int dm = \int_0^L (ax+b)\,dx$$



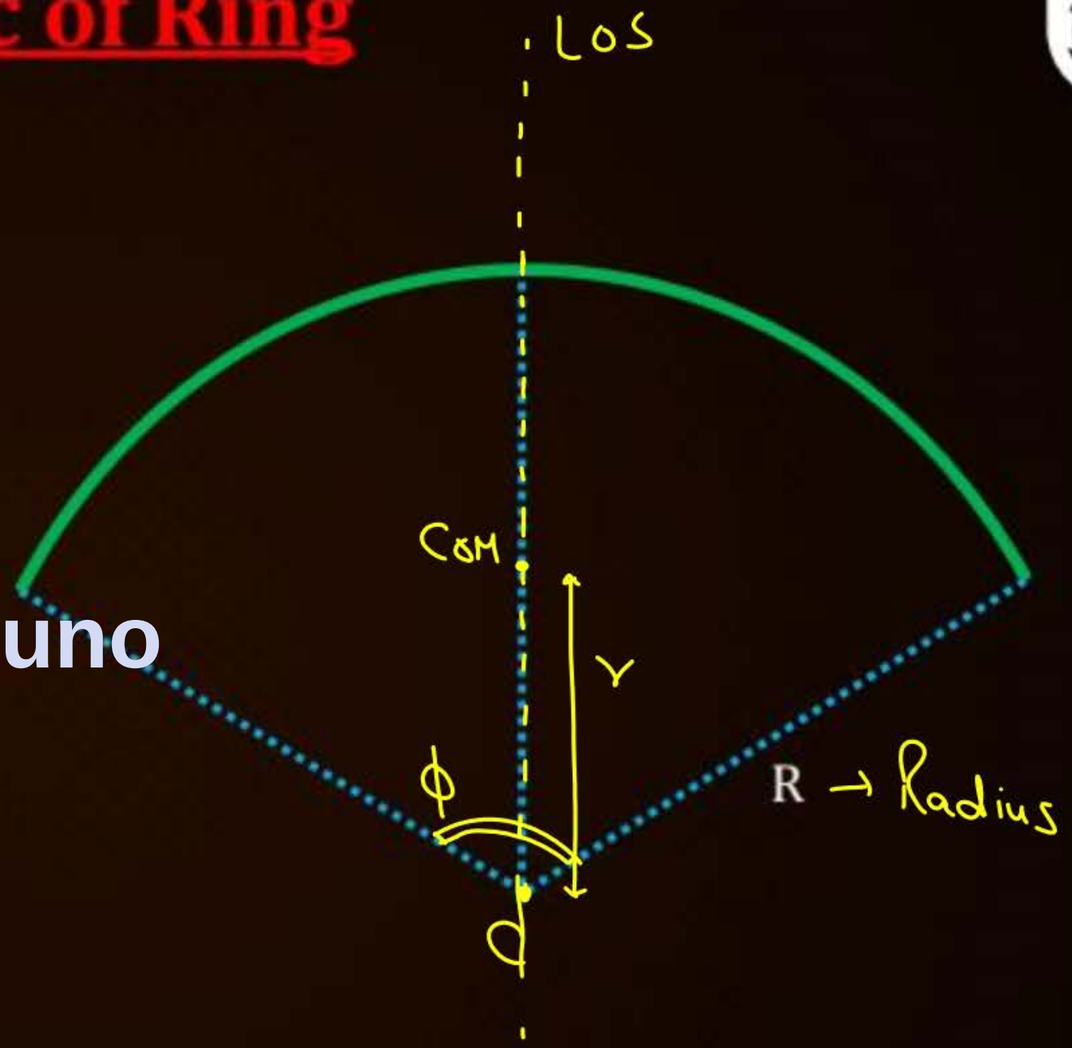




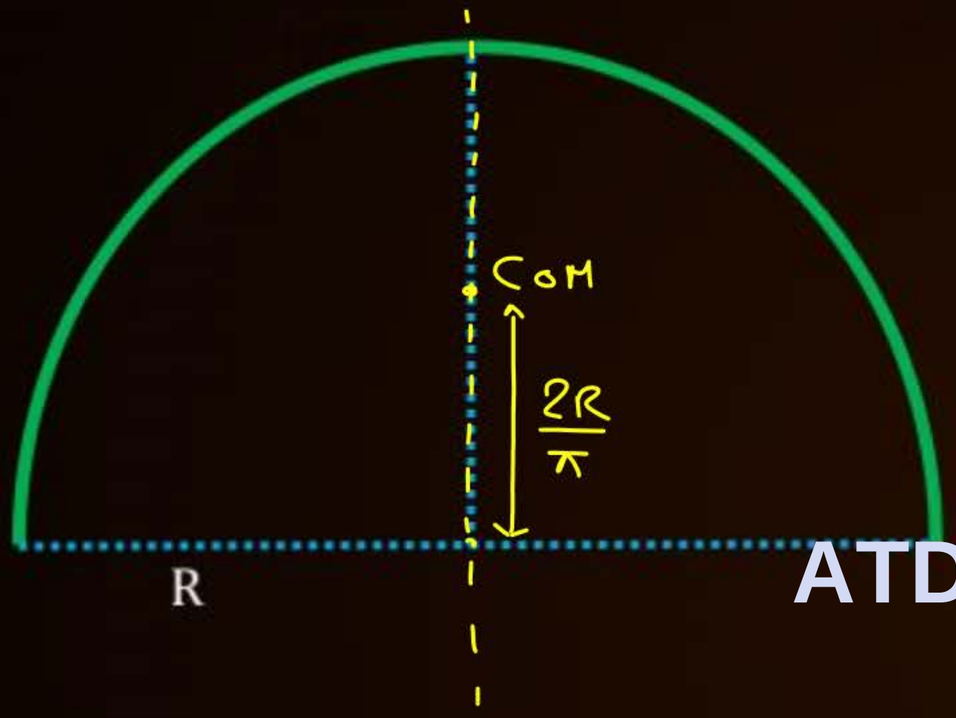
COM of Ring (Uniform)

COM of Arc of Ring

$$y = \frac{2R}{\phi} \sin\frac{\phi}{2}$$

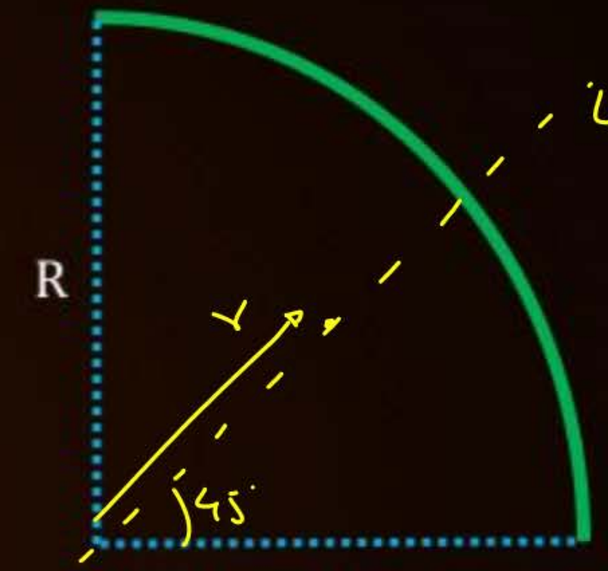


Semi-Circle :→ ($\phi = \pi$)



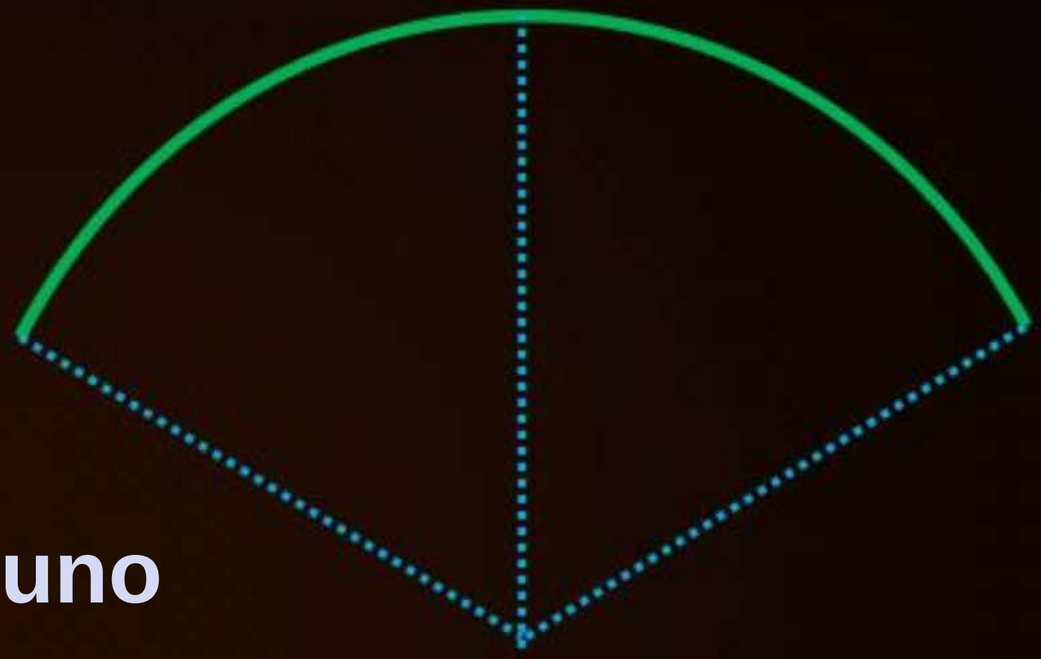
$$Y = \frac{2R}{\pi} \sin\frac{\pi}{2} = \frac{2R}{\pi}$$

quarter ($\phi = \pi/2$)



$$Y = \frac{2R}{\pi/2} \sin\frac{\pi/2}{2} = \frac{4R}{\pi} \cdot \frac{1}{\sqrt{2}} = \frac{2\sqrt{2}R}{\pi}$$

(for 60° Arc. $Y = \frac{2R}{\pi/3} \sin\frac{\pi/3}{2} = \frac{3R}{\pi}$)



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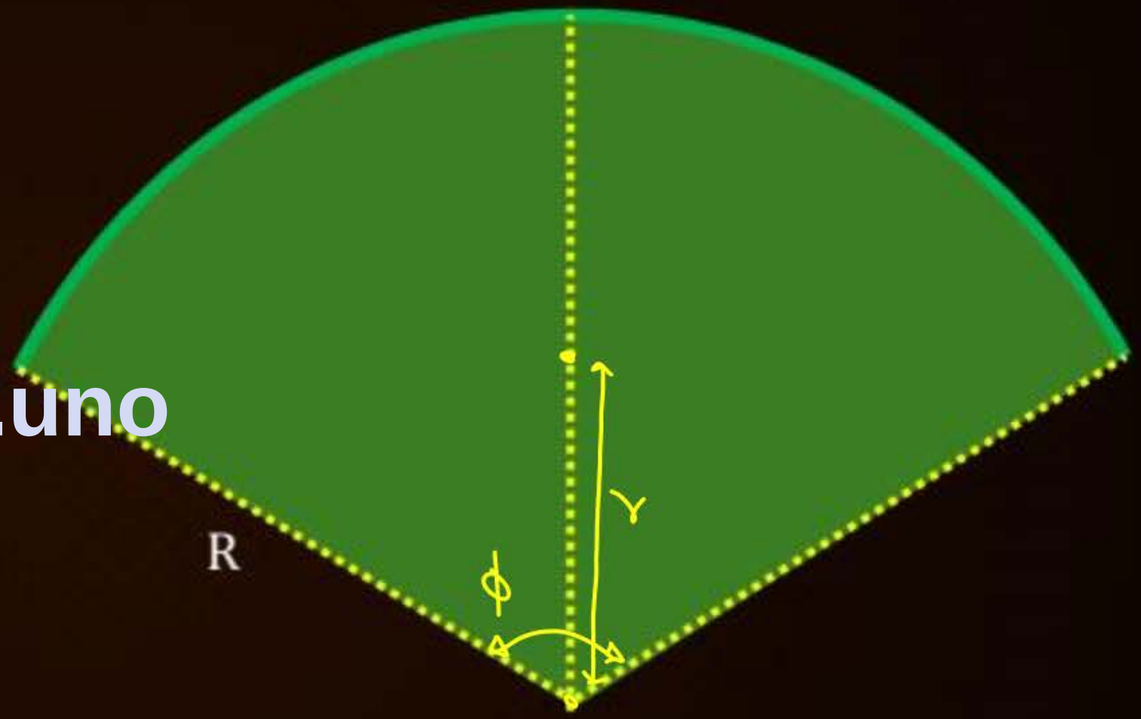
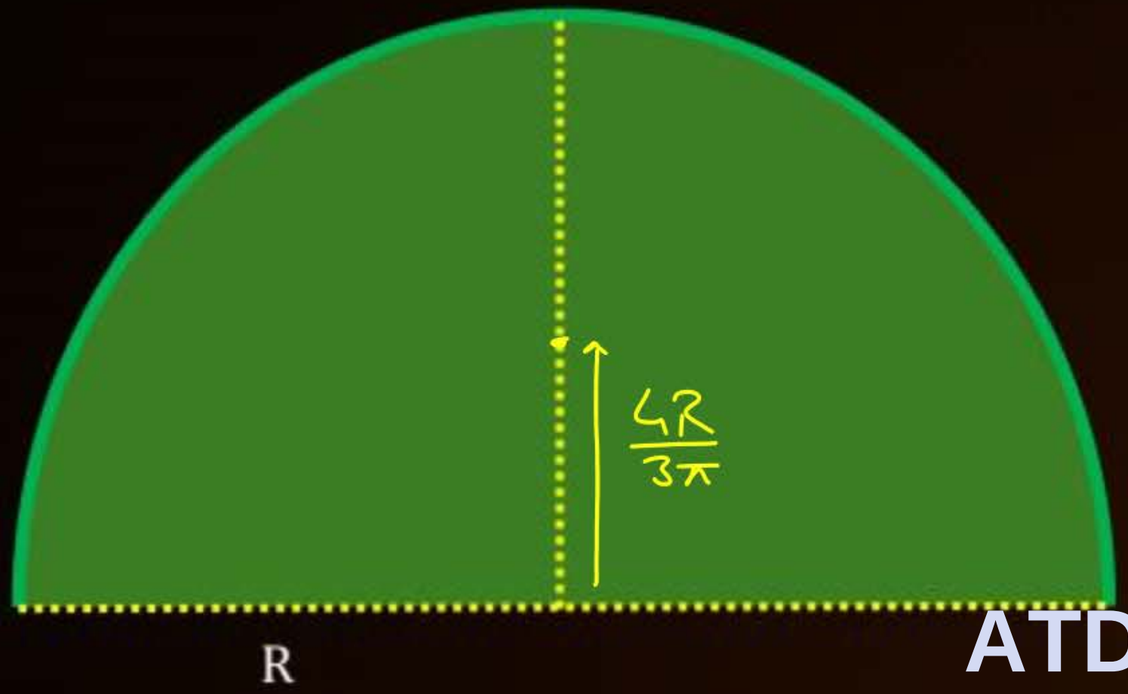
COM of Disc

COM of Sector (part of Disc)

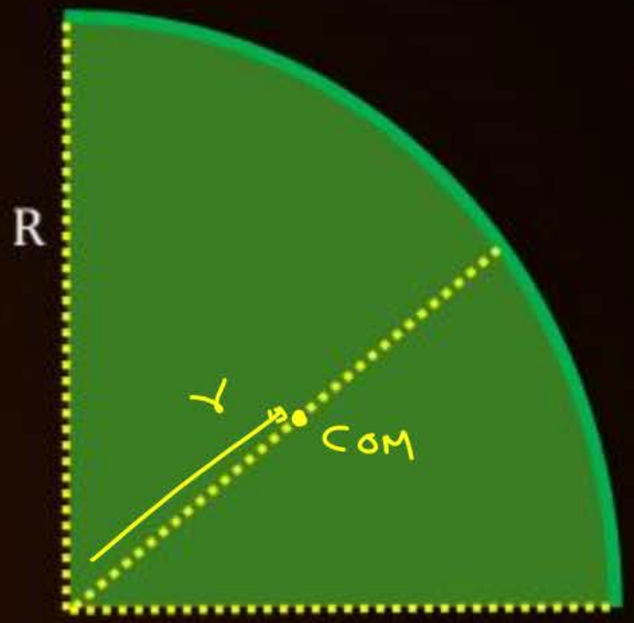
$$Y = \frac{4R}{3\phi} \sin\frac{\phi}{2}$$





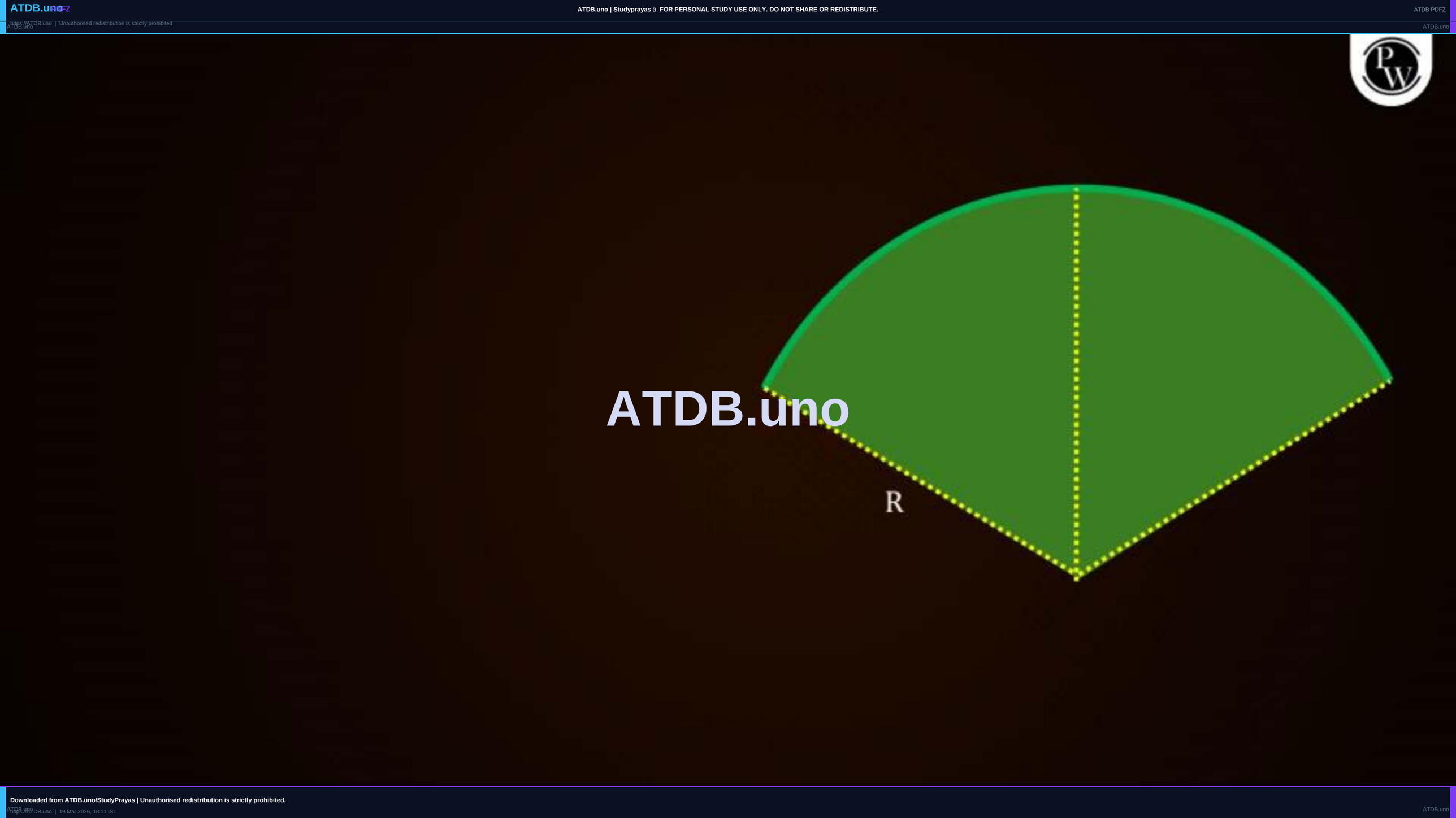
$\phi = \pi$

$$\gamma = \frac{4R}{3\pi} \sin \pi/2 = \frac{4R}{3\pi}$$



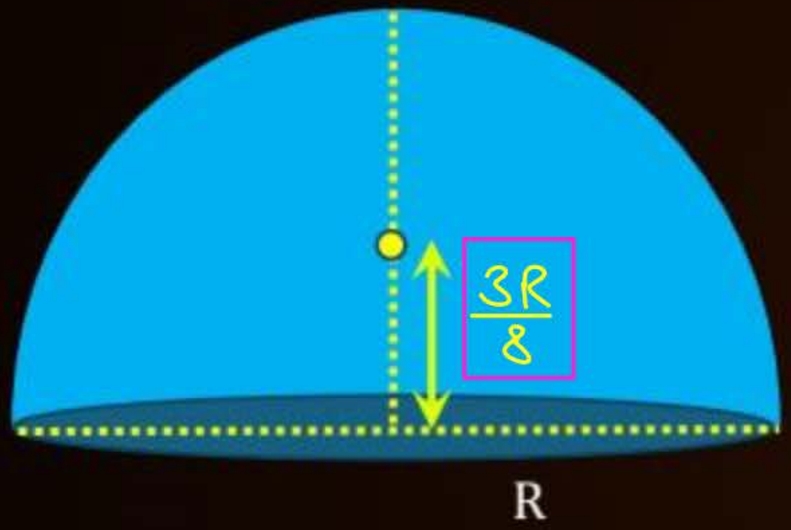
$\phi = \pi/2$

$$\gamma = \frac{4R}{3(\pi/2)} \sin \pi/4$$

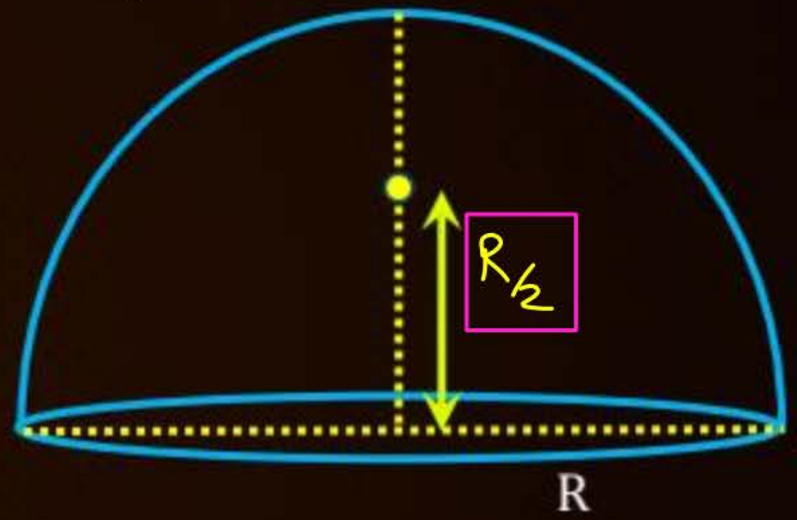


COM of Standard Objects (Half Sphere)

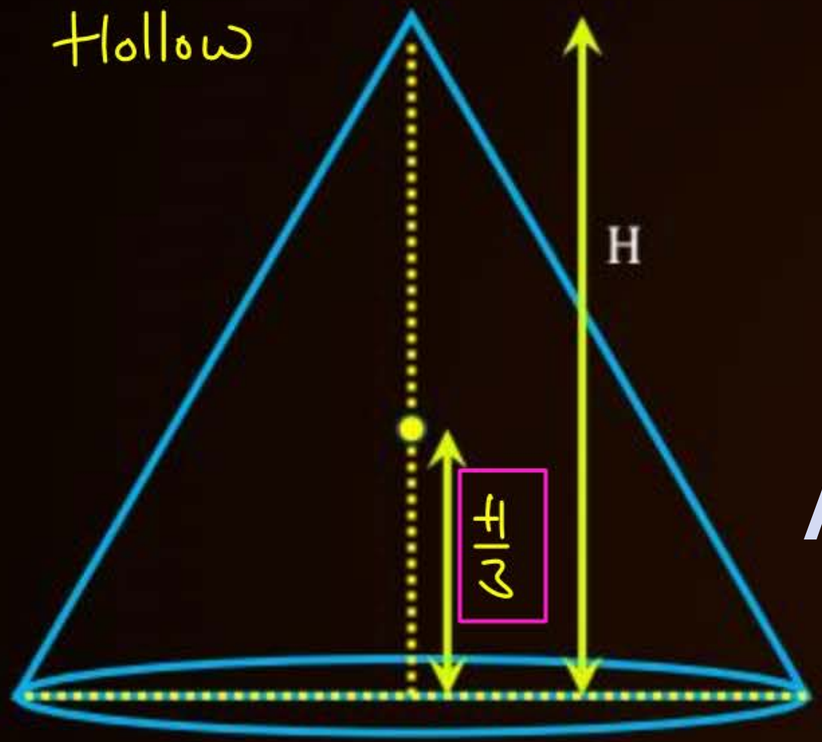
Solid

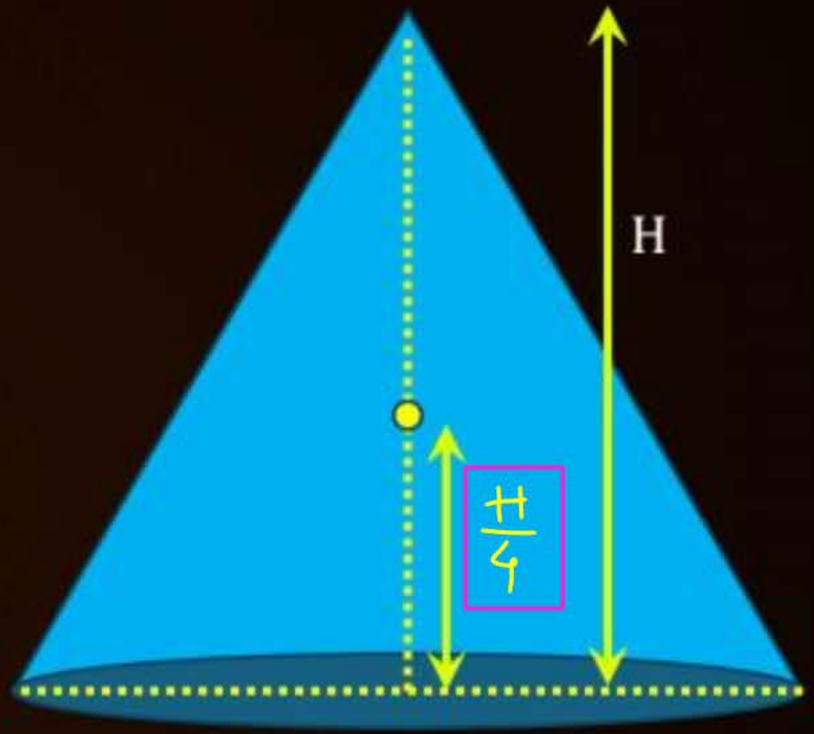


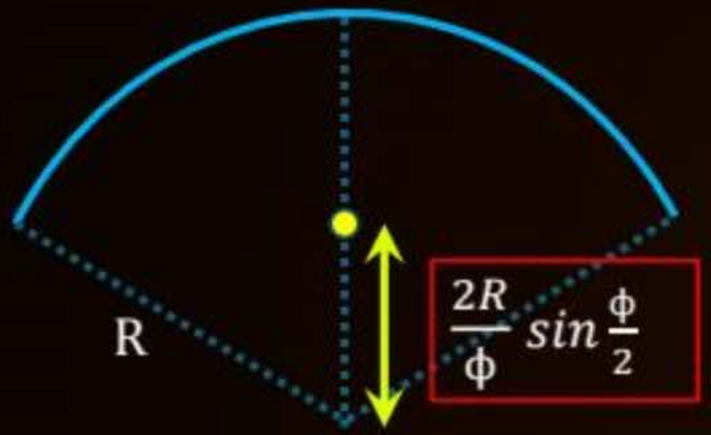
Hollow

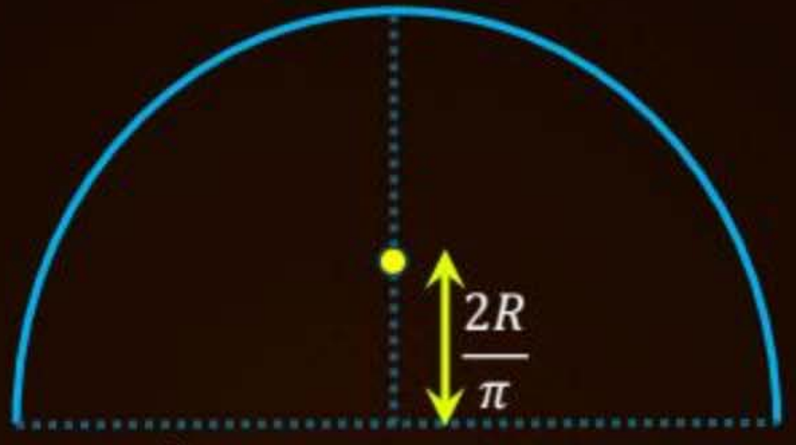


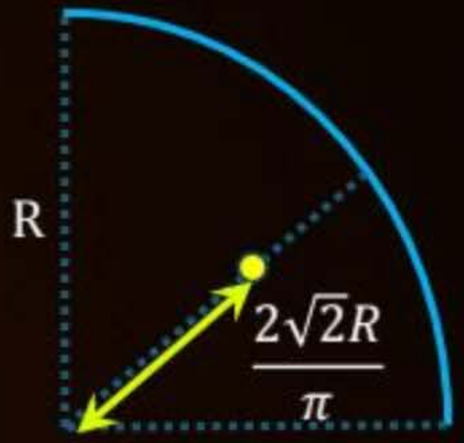
COM of Standard Objects (Cone)

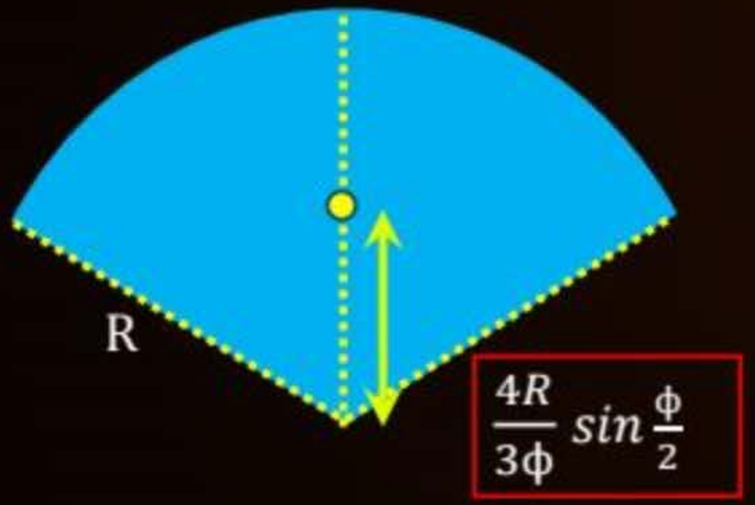


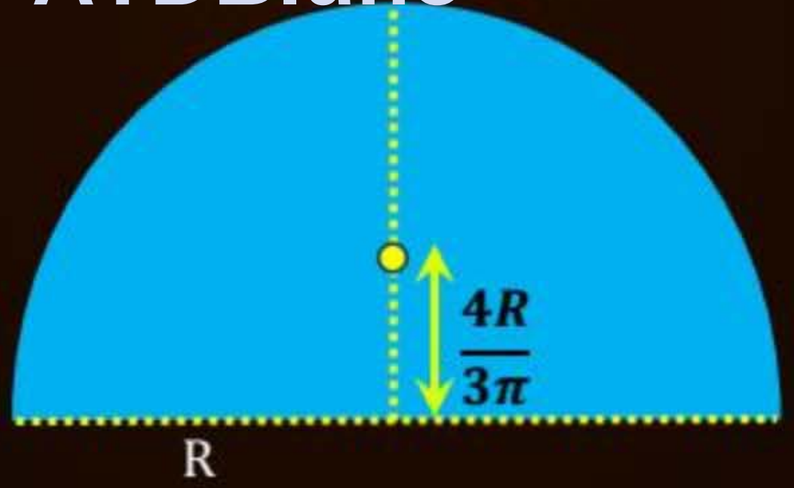


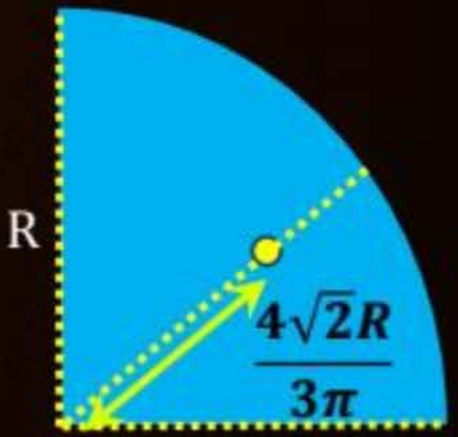


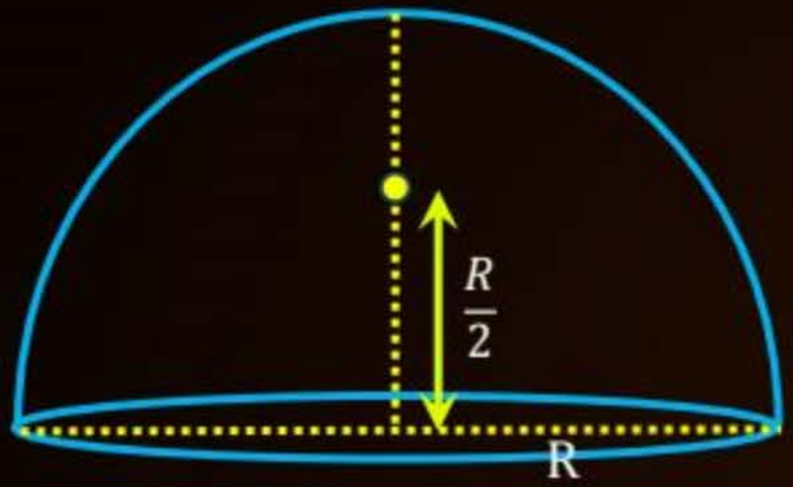


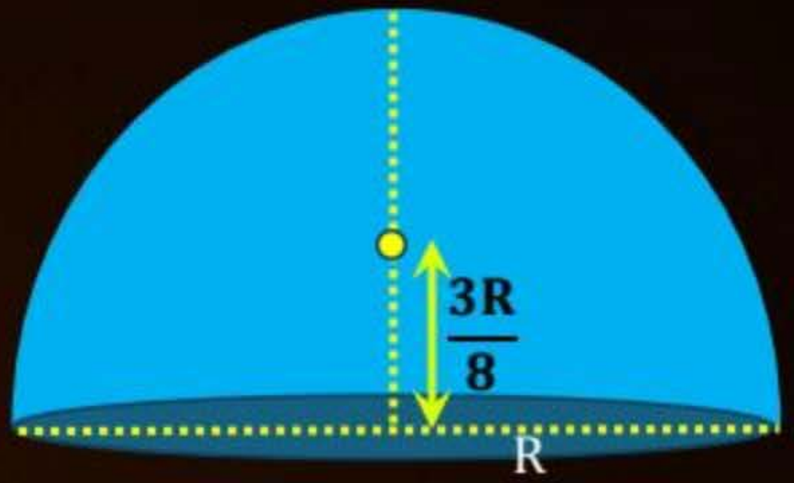


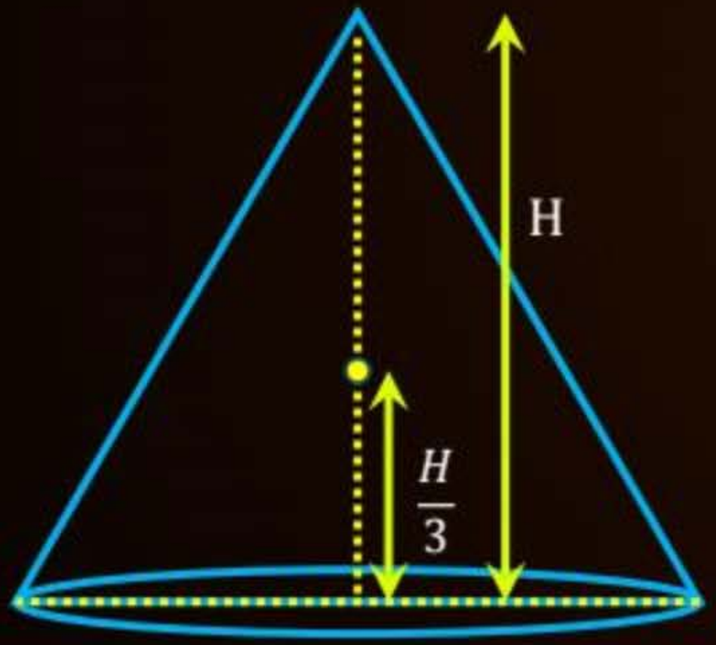


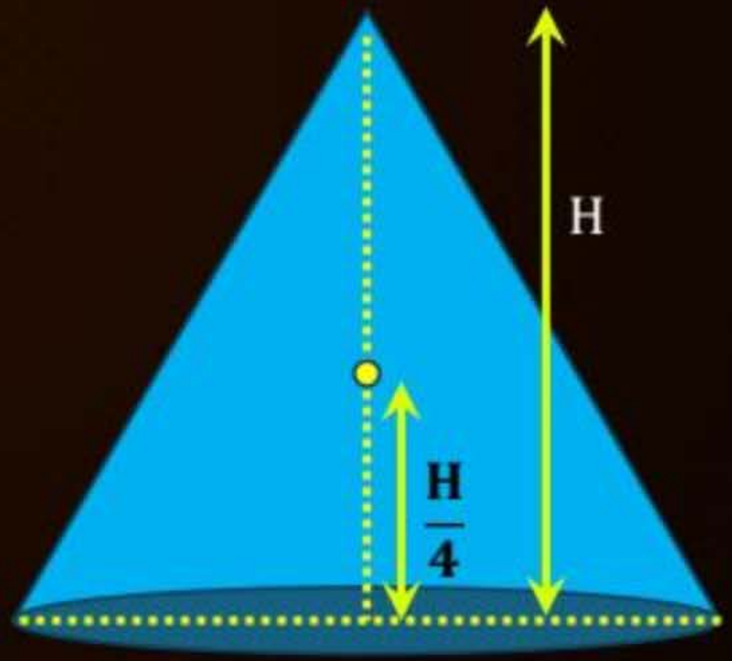












Question 14

A solid cone of radius R and Height 4R and solid hemisphere of radius R are joined as shown find com of system from point O. both objects have same mass density

$R = 120\,cm$

Solid hemisphere $\left(\frac{3R}{8} \text{ from base}\right)$

$X_{cm} = 0$

$$y_{cm} = \frac{2m(R) + m\left(\frac{-3R}{8}\right)}{3m} = \left(\frac{16-3}{3\times 8}\right)R$$

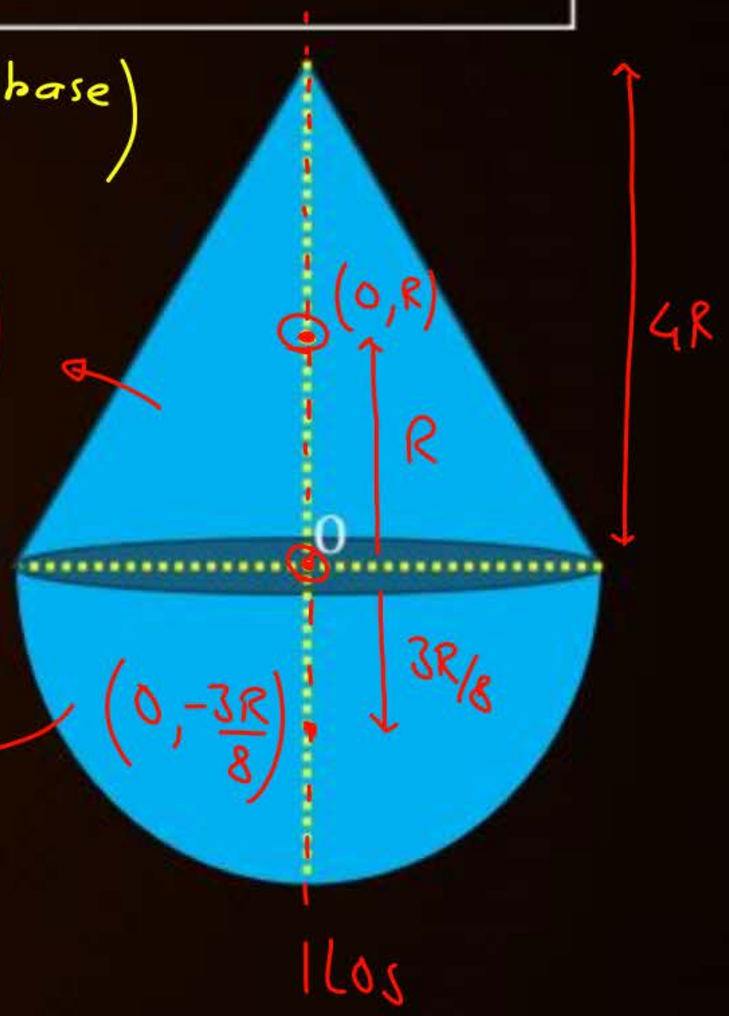
$$= \frac{13R}{24} = \frac{13}{24} \times 120\,cm = \underline{65\,cm}$$

$m_1 = \rho\left(\frac{1}{3}\pi R^2 \cdot 4R\right)$

$= 2m$

$m_2 = \rho\left(\frac{2}{3}\pi R^3\right)$

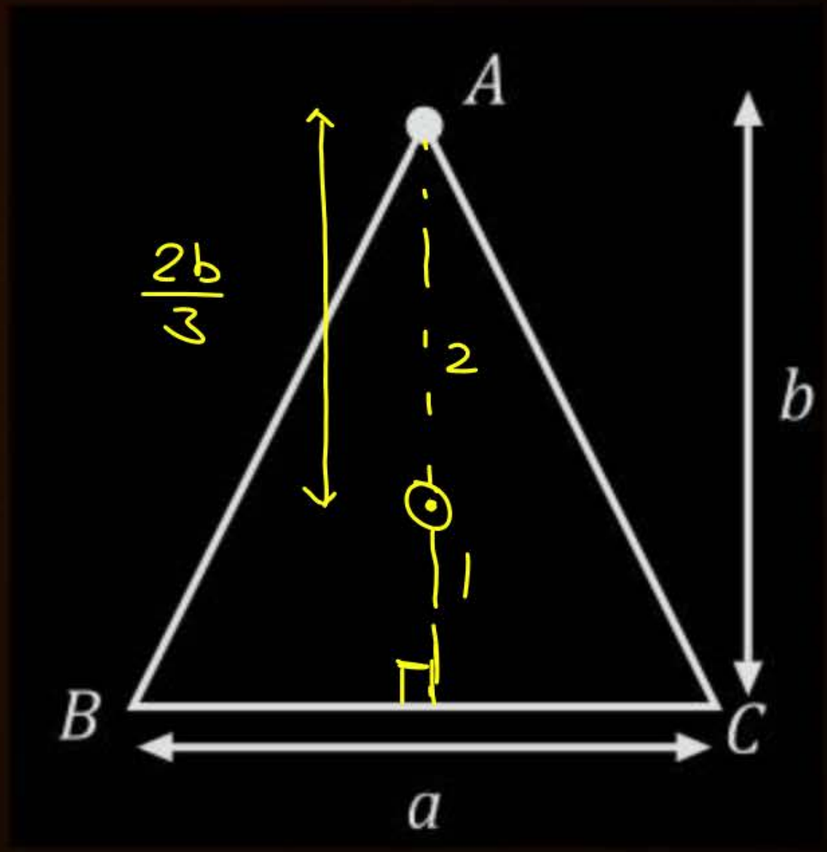
$= m\,(\text{let})$

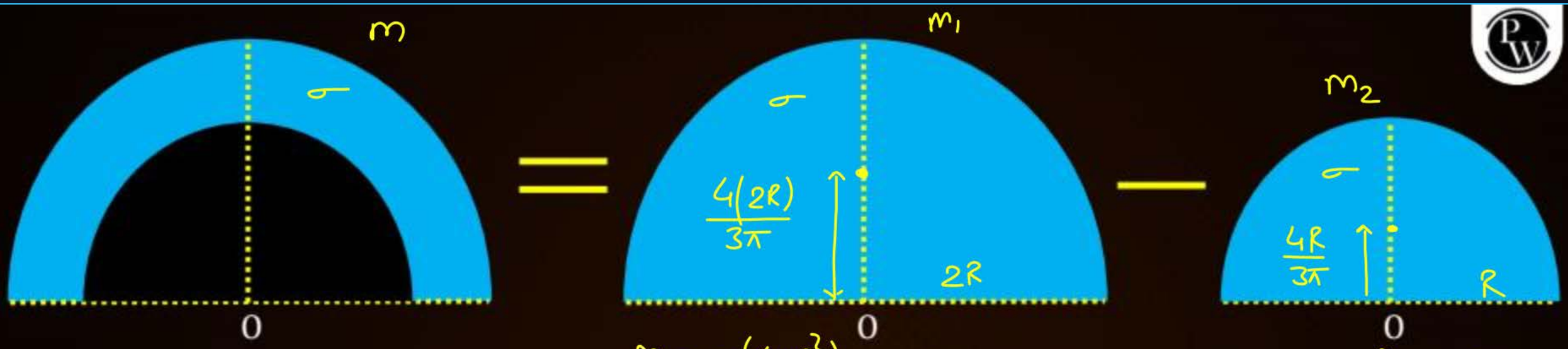


Question 13

Find out the distance of center of mass from vertex A of an isosceles triangular sheet of base length a and altitude b. Assume that the mass of the triangle is uniformly distributed over its area. (b = 12cm)

$$\frac{2b}{3} = \frac{2 \times 12}{3} = 8\text{cm}$$





$m_1 = \sigma\left(\frac{4\pi R^2}{2}\right) = 4m$

$m_2 = \sigma\frac{\pi R^2}{2} = m\,(\text{Let})$

⟶ density is Same (mass, charge, Current different)

add $\quad X_{cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$

Subtract $\quad X_{cm} = \dfrac{m_1x_1 - m_2x_2}{m_1 - m_2} = \dfrac{4m\left(\frac{8R}{3\pi}\right) - m\left(\frac{4R}{3\pi}\right)}{4m - m} = \dfrac{28R}{9\pi}$

Question

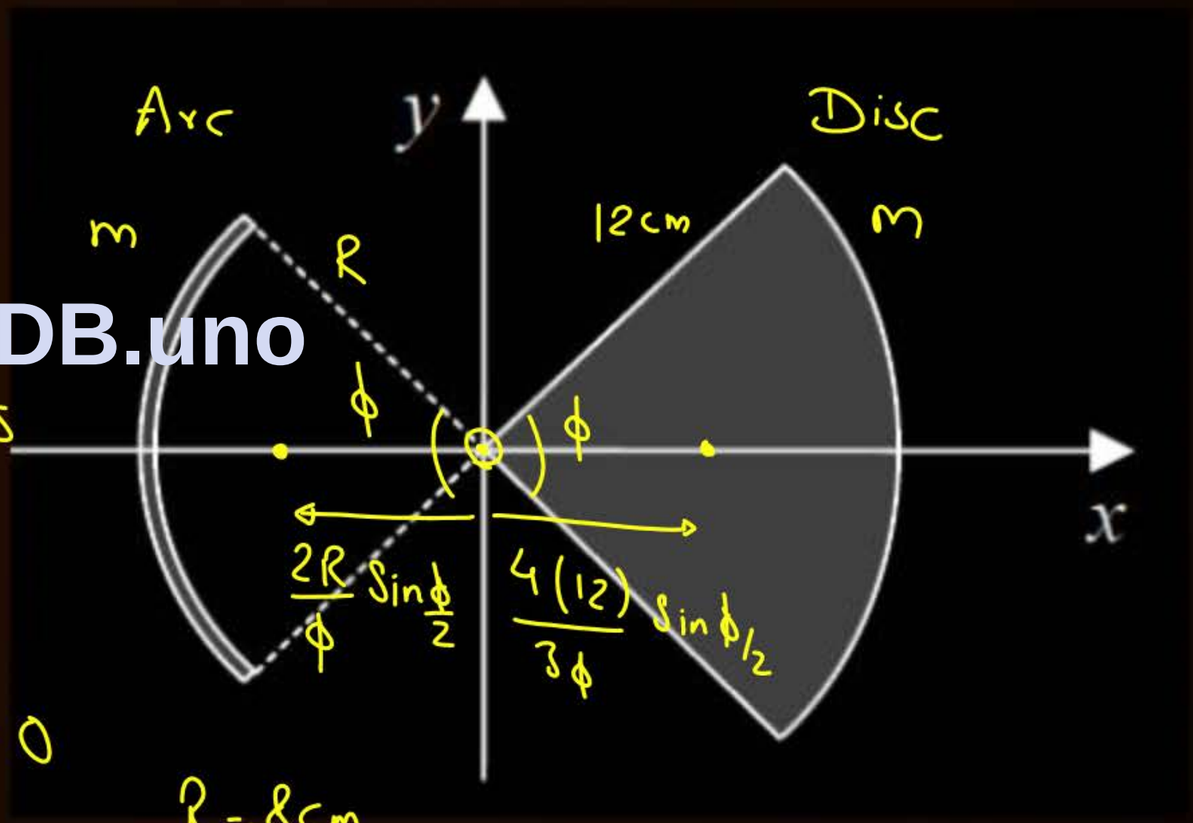
A sector cut from a uniform disk of radius 12 cm and a uniform rod of the same mass bent into shape of an arc are arranged facing each other as shown in the figure. If center of mass of the combination is at the origin, what is the radius of the arc?

Arc → $\frac{2R}{\phi}\left(\sin\frac{\phi}{2}\right)$

Sector → $\frac{4R}{3\phi}\left(\sin\frac{\phi}{2}\right)$

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = 0$$

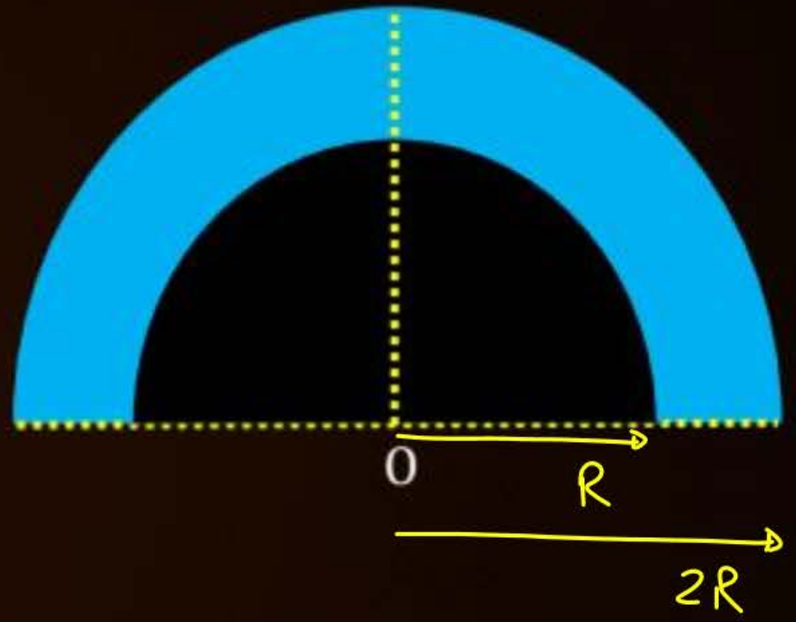
$$\Rightarrow \frac{m\left(-\frac{2R}{\phi}\sin\frac{\phi}{2}\right) + m\left(\frac{16}{\phi}\sin\frac{\phi}{2}\right)}{2m} = 0$$



$R = 8\,cm$

Question

Find COM of half disc of inner radius R and outer Radius 2R



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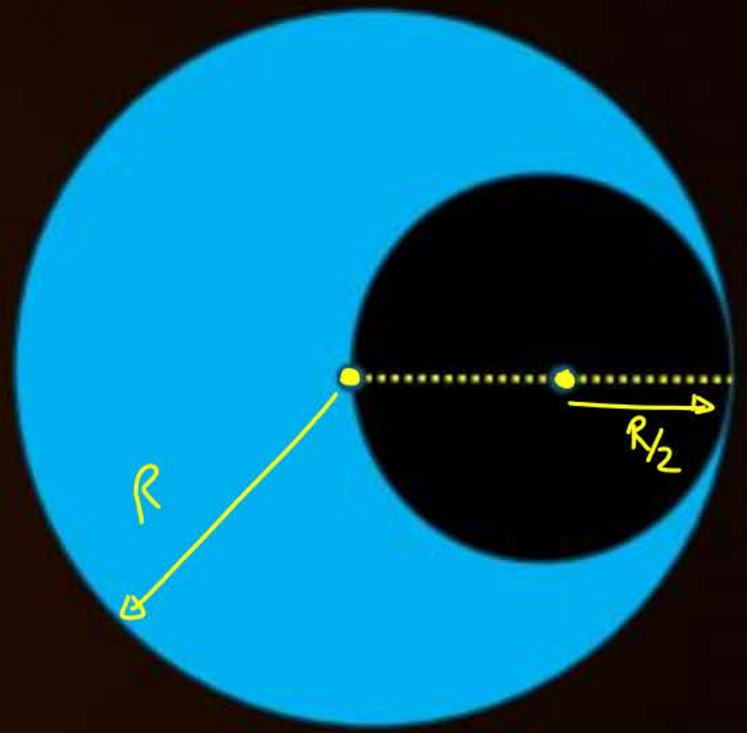
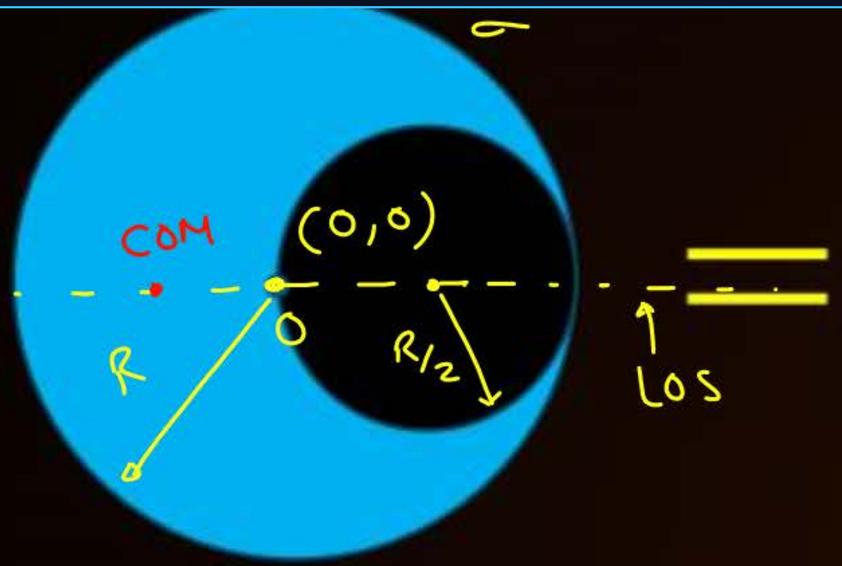
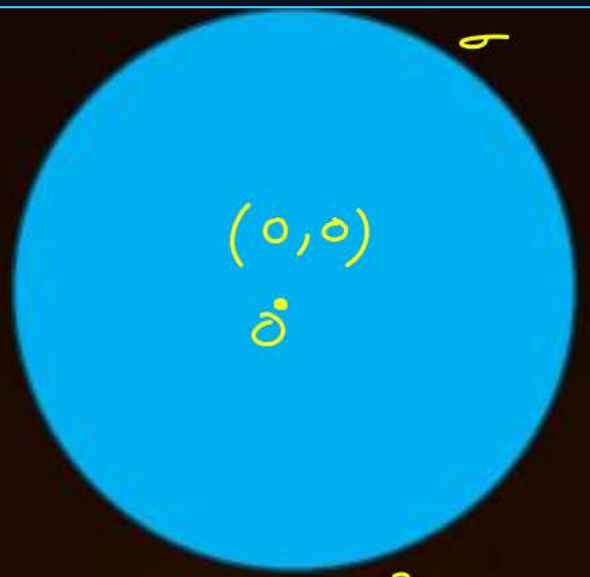
Question 18

A circular cavity of radius R/2 is made inside uniform Disc of radius R as shown. Find distance of COM of object from center of disc







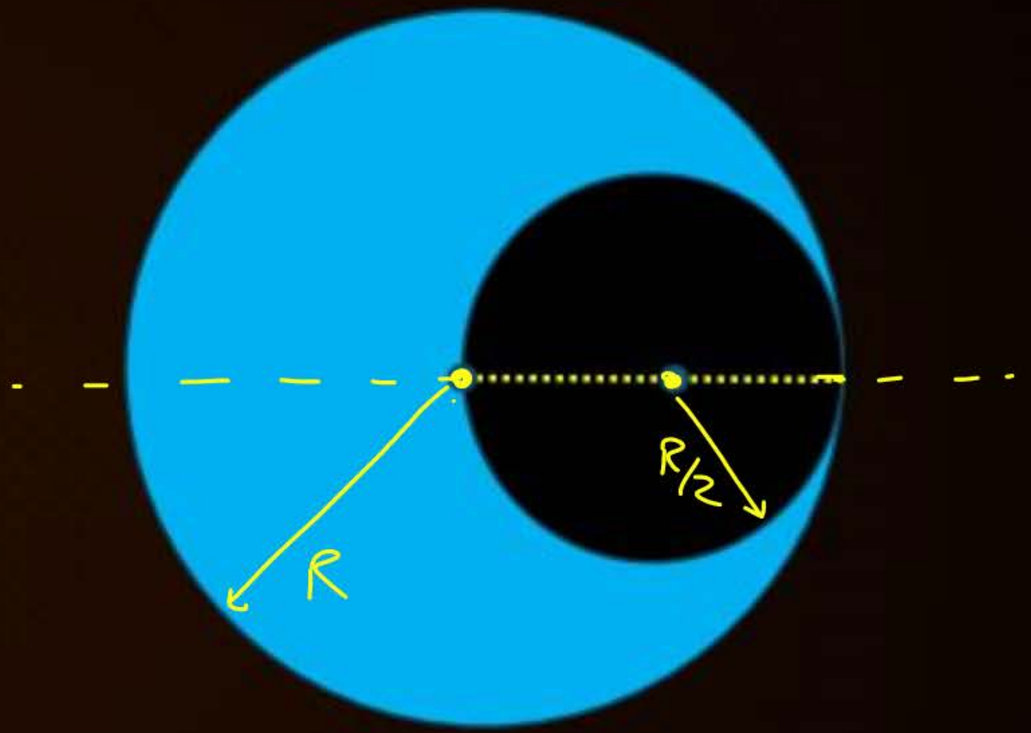
$m_1 = \sigma \pi R^2 = 4m$

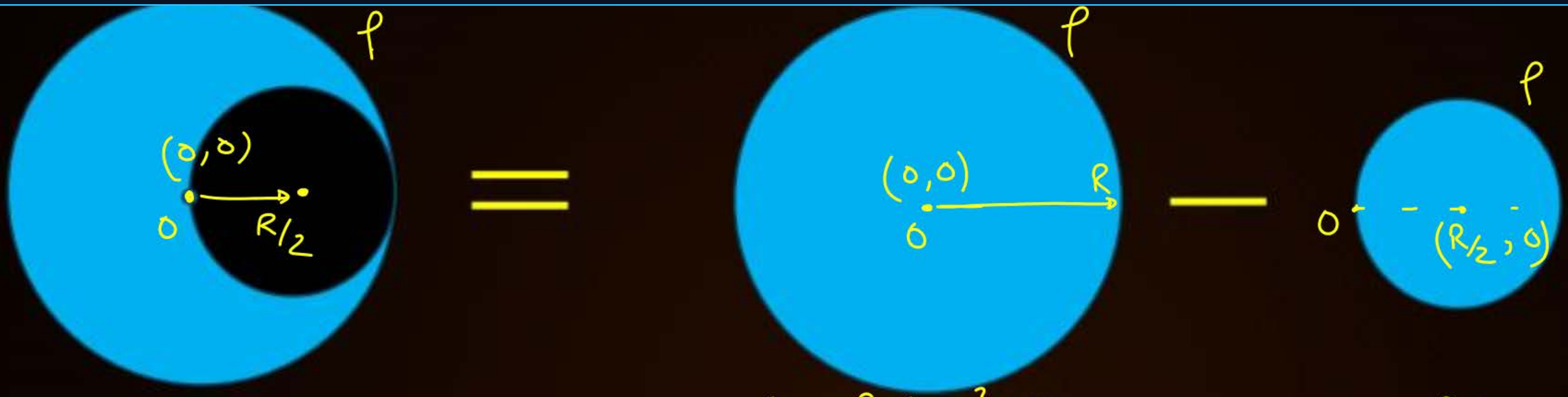
$m_2 = \sigma \pi \left(\frac{R}{2}\right)^2 = m$

$$X_{CM} = \frac{4m(0) - m\left(\frac{R}{2}\right)}{4m - m} = -\frac{R}{6}$$

Question

A spherical cavity of radius R/2 is made inside uniform sphere of radius R as shown. Find distance of COM of object from center of sphere





$m_1 = \rho \cdot \frac{4}{3}\pi R^3 = 8m$

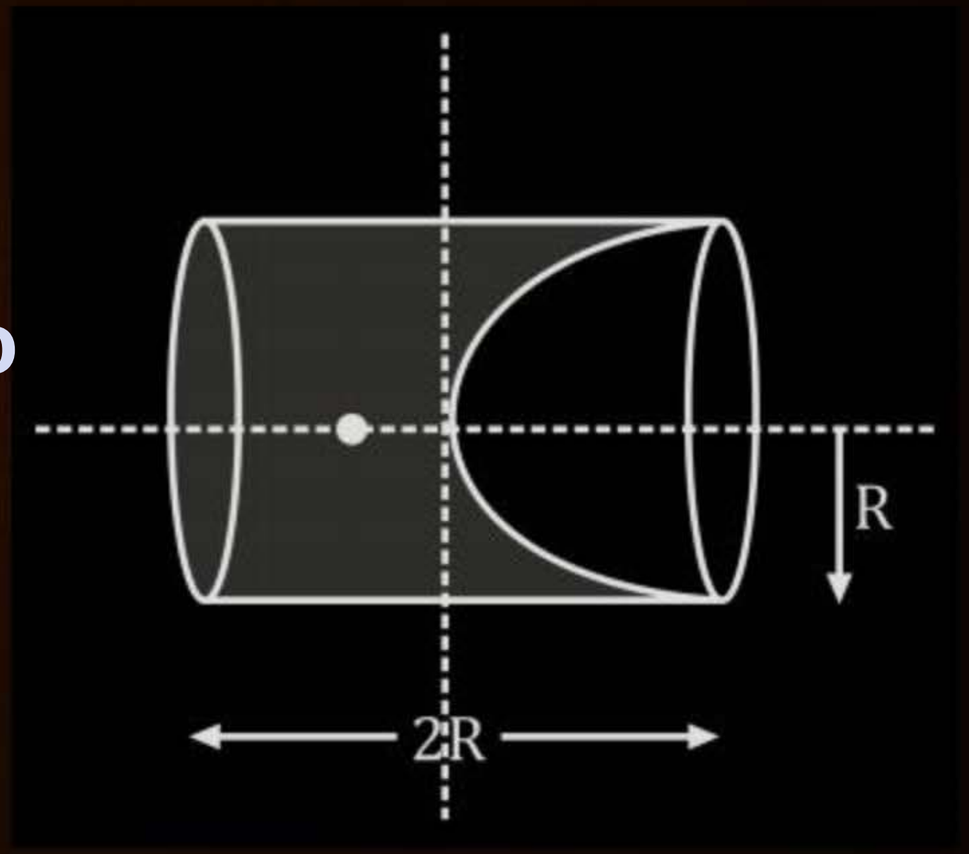
$m_2 = \rho \cdot \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = m$ (let)

$y_{cm} = 0$

$$X_{cm} = \frac{m_1 x_1 - m_2 x_2}{m_1 - m_2} = \frac{8m(0) - m\left(\frac{R}{2}\right)}{8m - m} = -\frac{R}{14}$$

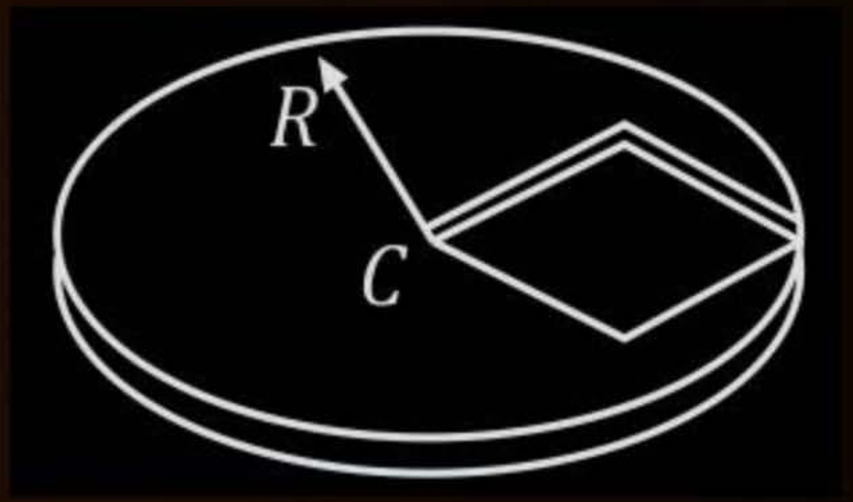
Question

A spherical part is removed from a cylinder. Find Distance of com of remaining part from center of cylinder. Assume uniform density throughout. (R = 32cm)



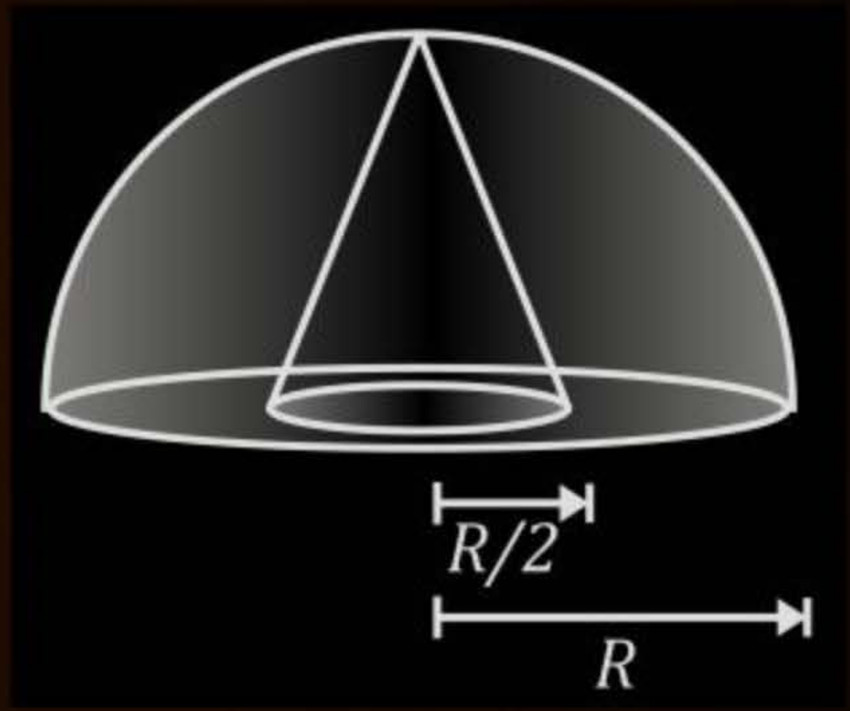
Question

Find the center of mass of the disc of radius R (R = 12cm) as shown in figure. A square is removed from the disc as shown. Distance of COM from point C is $\frac{N}{(2\pi-1)}$.find Value of N



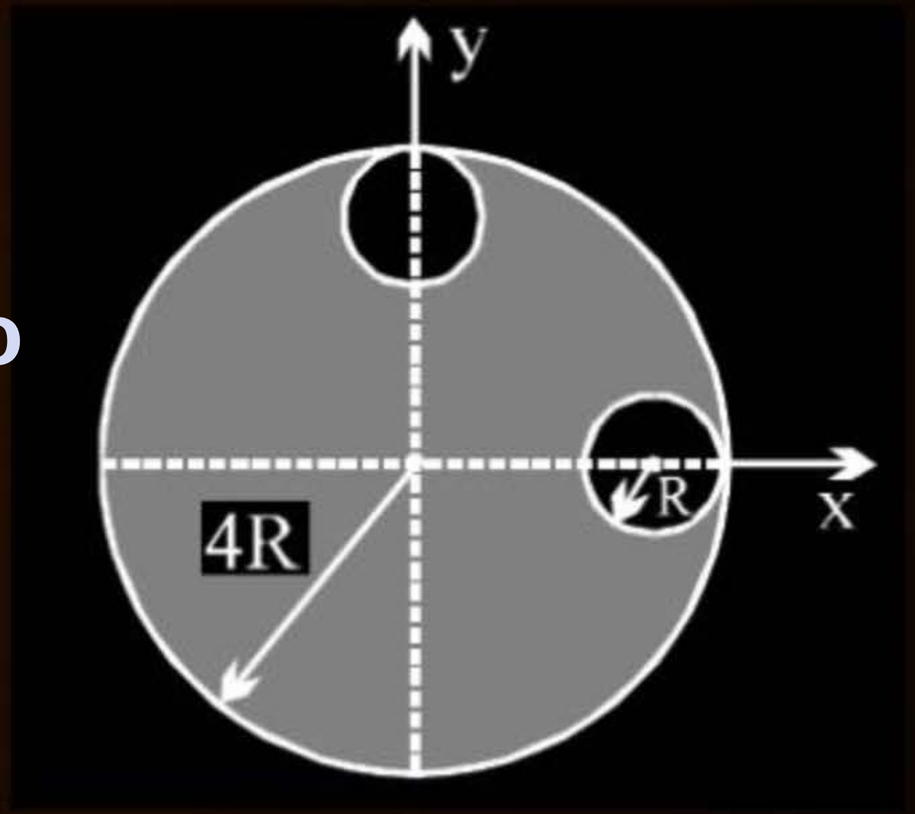
Question

From a hemisphere of radius R a cone of base radius $R/2$ and height R is cut as shown in figure. Find the height of center of mass of the remaining object. (R = 56cm)



Question

From the circular disc of radius 4R two small disc of radius R are cut off. The center of mass of the new structure will be :



Question

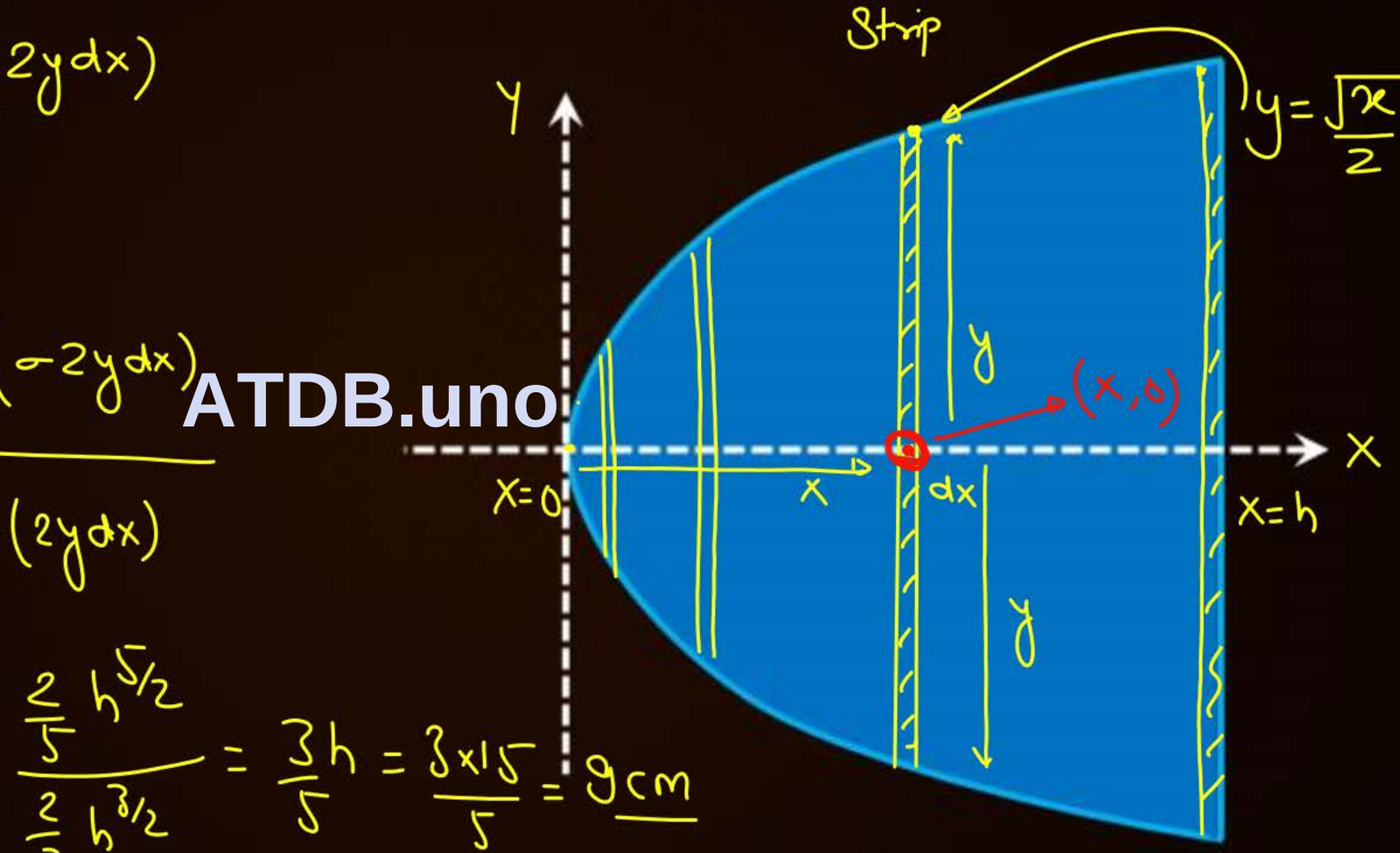
Find center of mass of an object (Parabola) which is formed by a parabola $x = 4y^2$ as shown in figure (From x = 0 to x = h). Assume the object is of uniform density. (h = 15cm)

$$dm = \sigma dA = \sigma(2y\,dx)$$

$$y_{cm} = 0$$

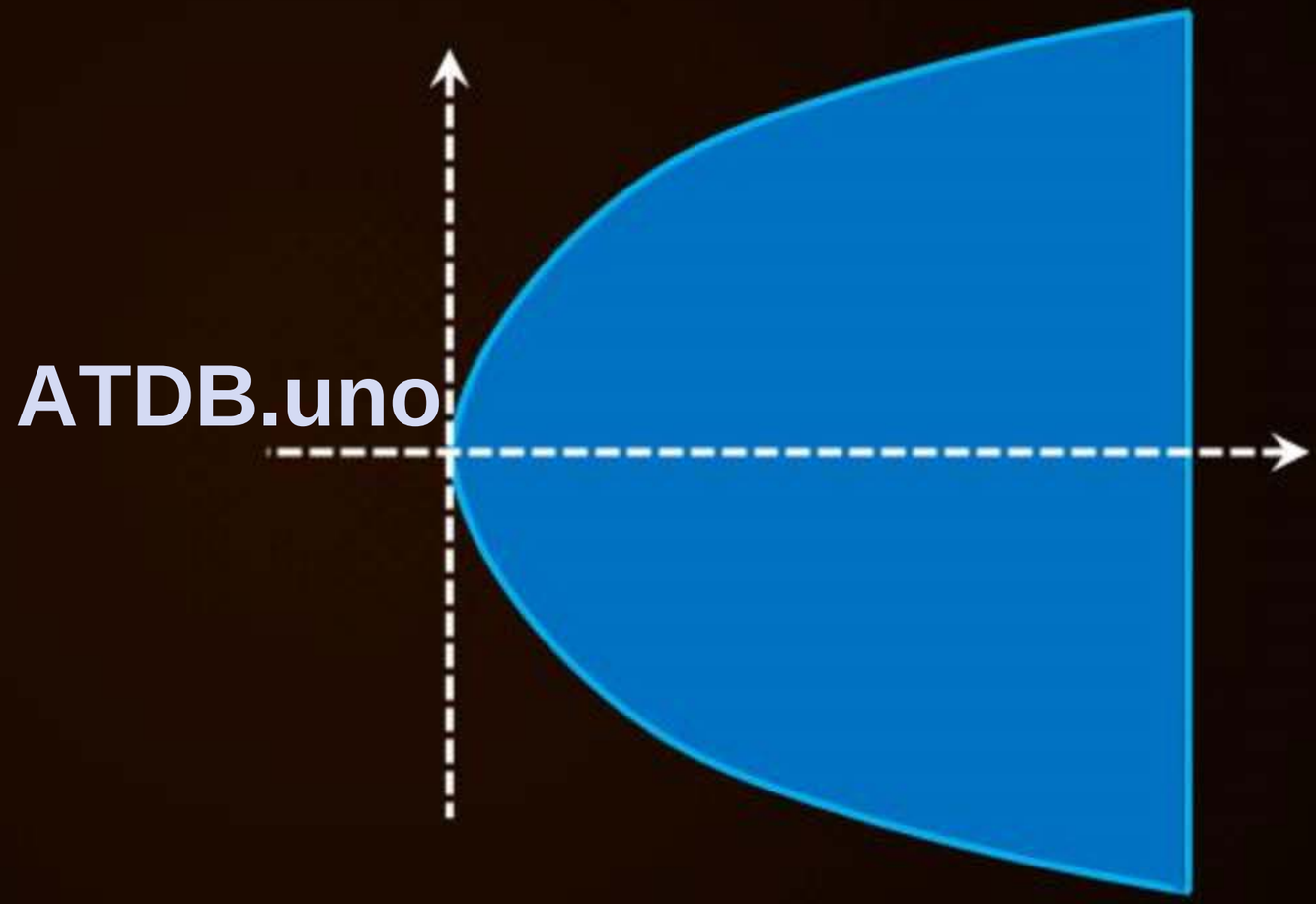
$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int_0^h x(\sigma 2y\,dx)}{\int_0^h \sigma(2y\,dx)}$$

$$x_{cm} = \frac{\int_0^h x \cdot \frac{x^{1/2}}{2}\,dx}{\int_0^h \frac{x^{1/2}}{2}\,dx} = \frac{\frac{2}{5}h^{5/2}}{\frac{2}{3}h^{3/2}} = \frac{3}{5}h = \frac{3\times 15}{5} = 9\,cm$$



Question

Find center of mass of an object (Parabola) which is formed by a parabola $x = 4y^2$ as shown in figure (From x = 0 to x = h). Assume the object is of uniform density. (h = 15cm)



Question

Find center of mass of an object (Paraboloid) which is formed by rotating a parabola $x = 4y^2$ about x-axis as shown in figure (From x = 0 to x = h). Assume the object is of uniform density. (h = 15cm)