PRAYAS

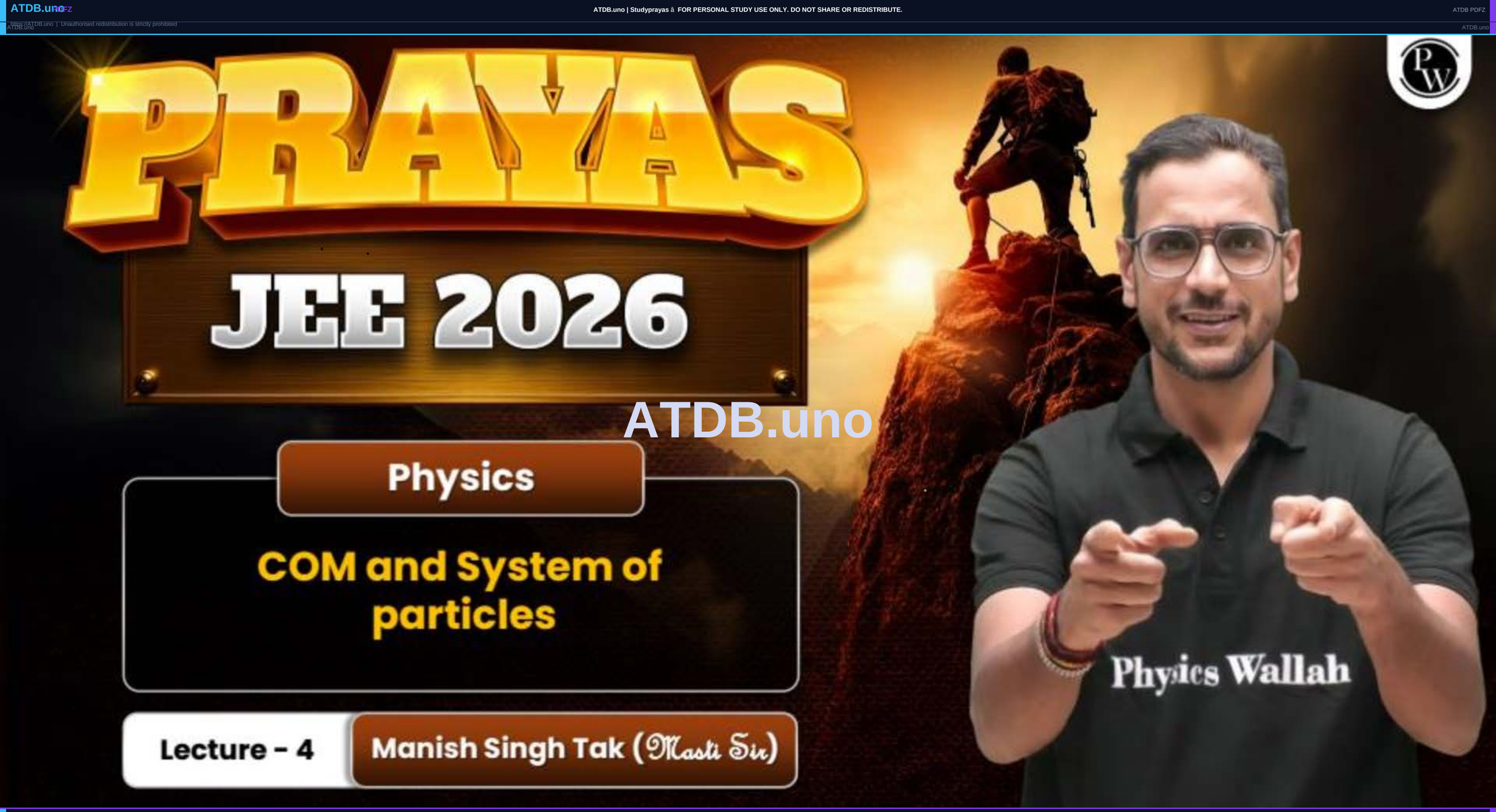
JEE 2026

Physics

COM and System of particles

Lecture – 4

Manish Singh Tak (Masti Sir)



Topics to be covered

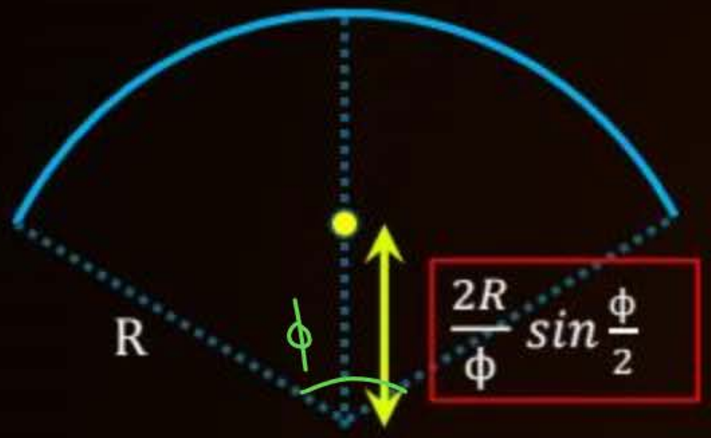
A Motion of COM

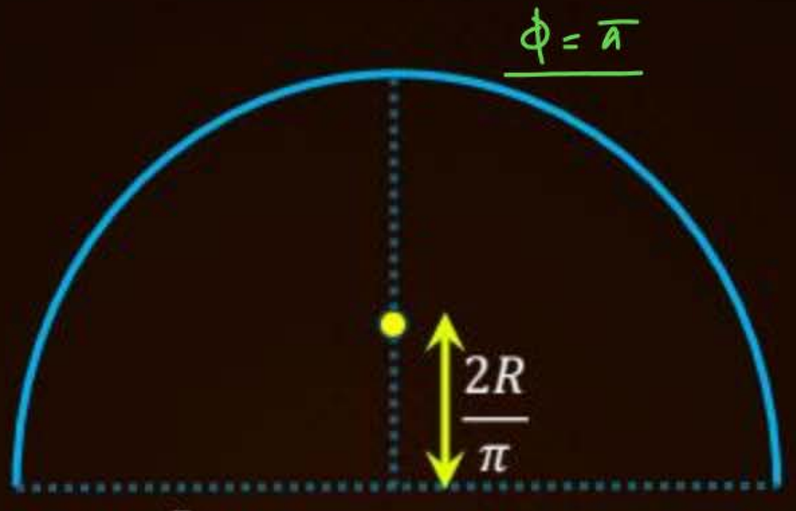
B

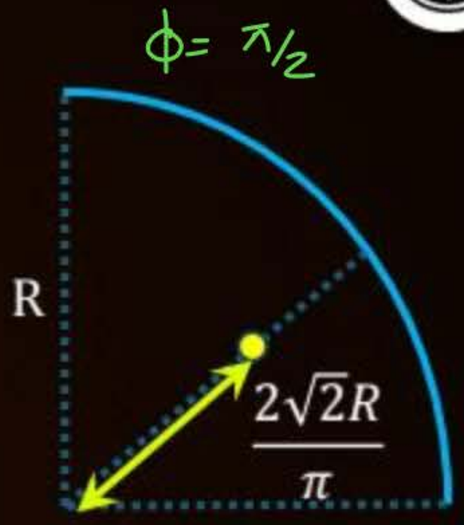
C

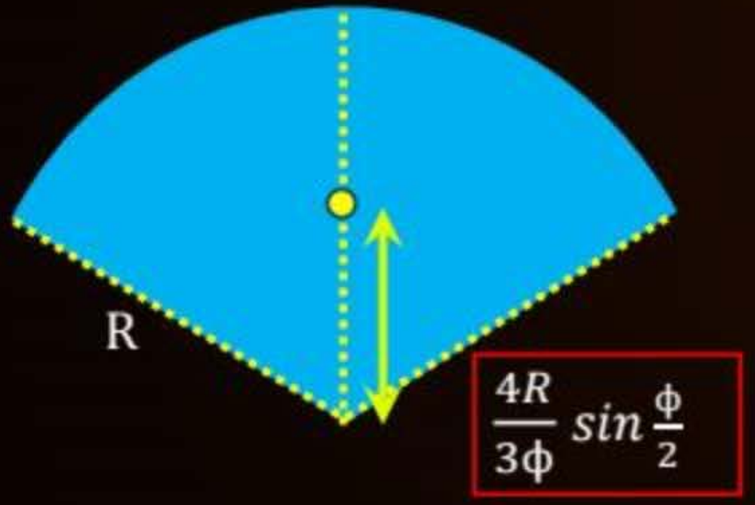
D

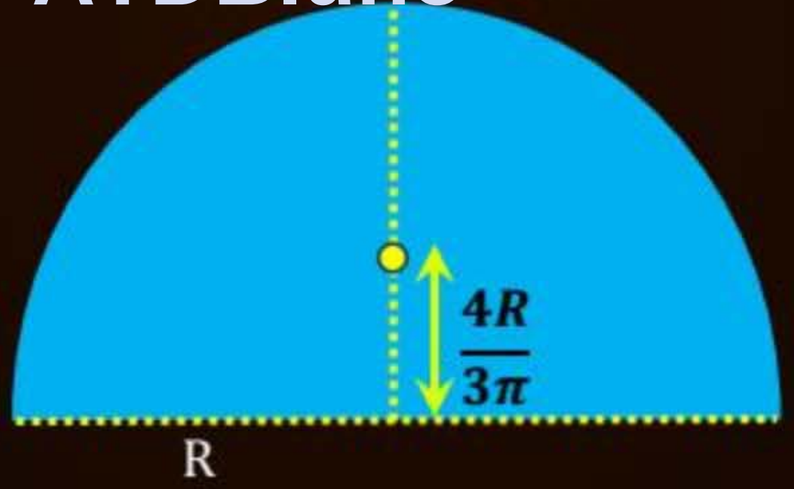
ATDB.uno

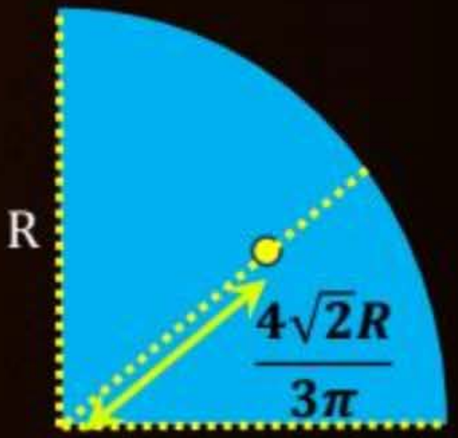


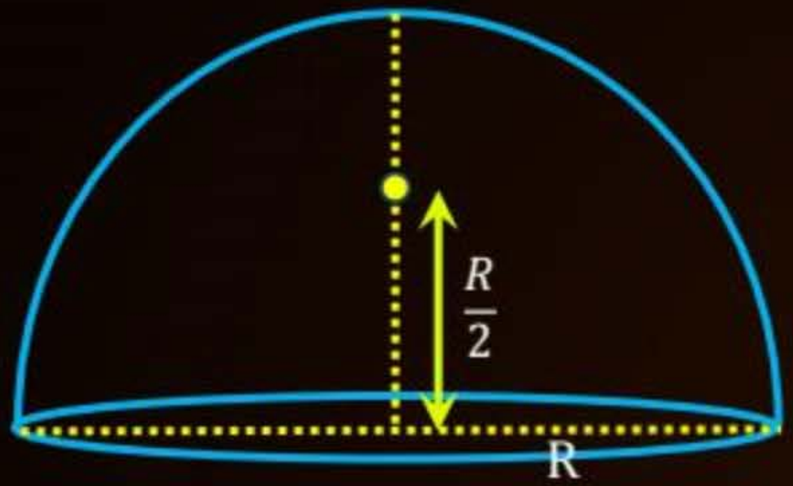


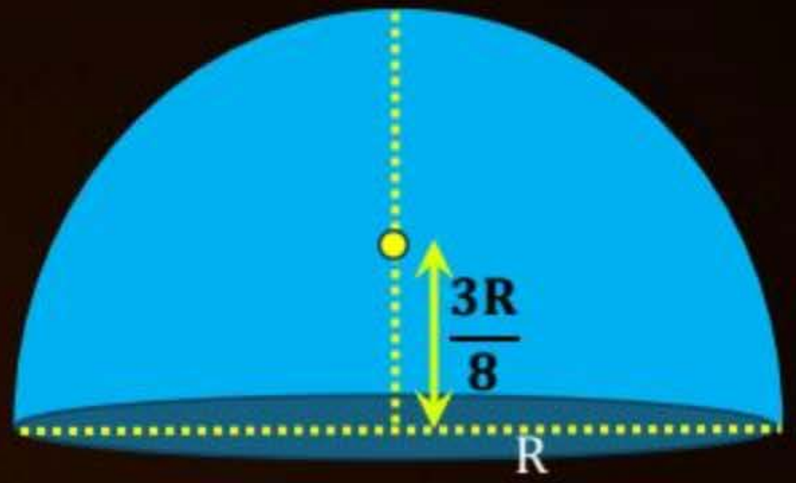


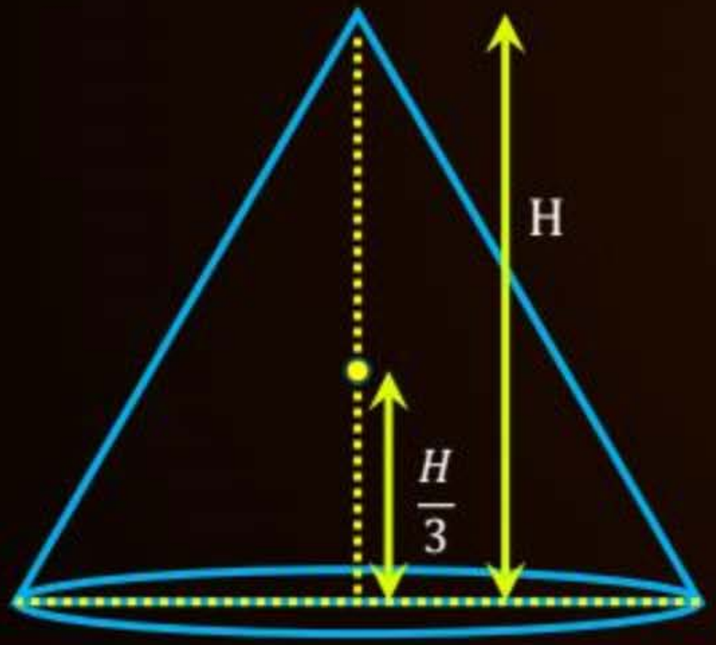


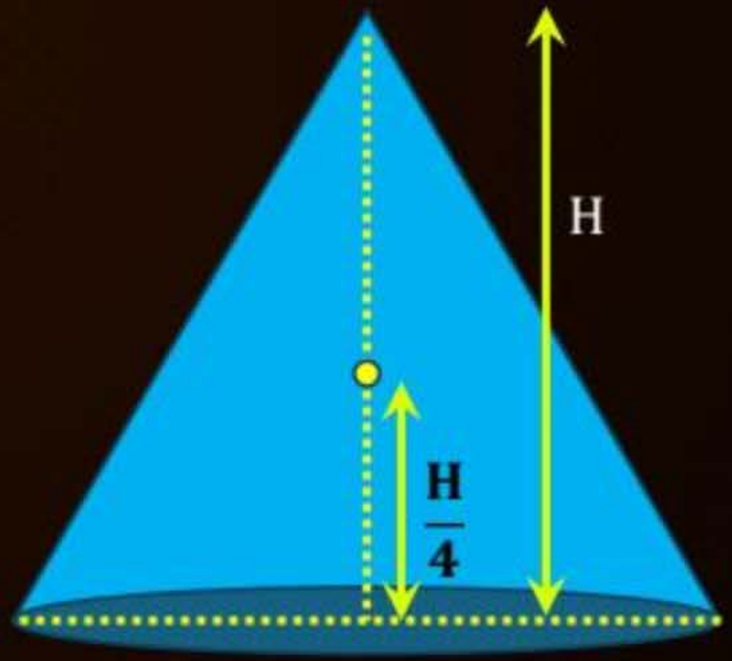






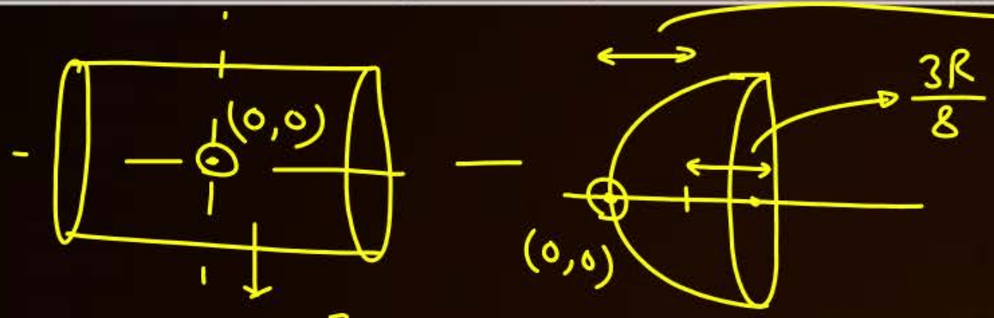






Question — 21

A spherical part is removed from a cylinder. Find Distance of com of remaining part from center of cylinder. Assume uniform density throughout. (R = 32cm)

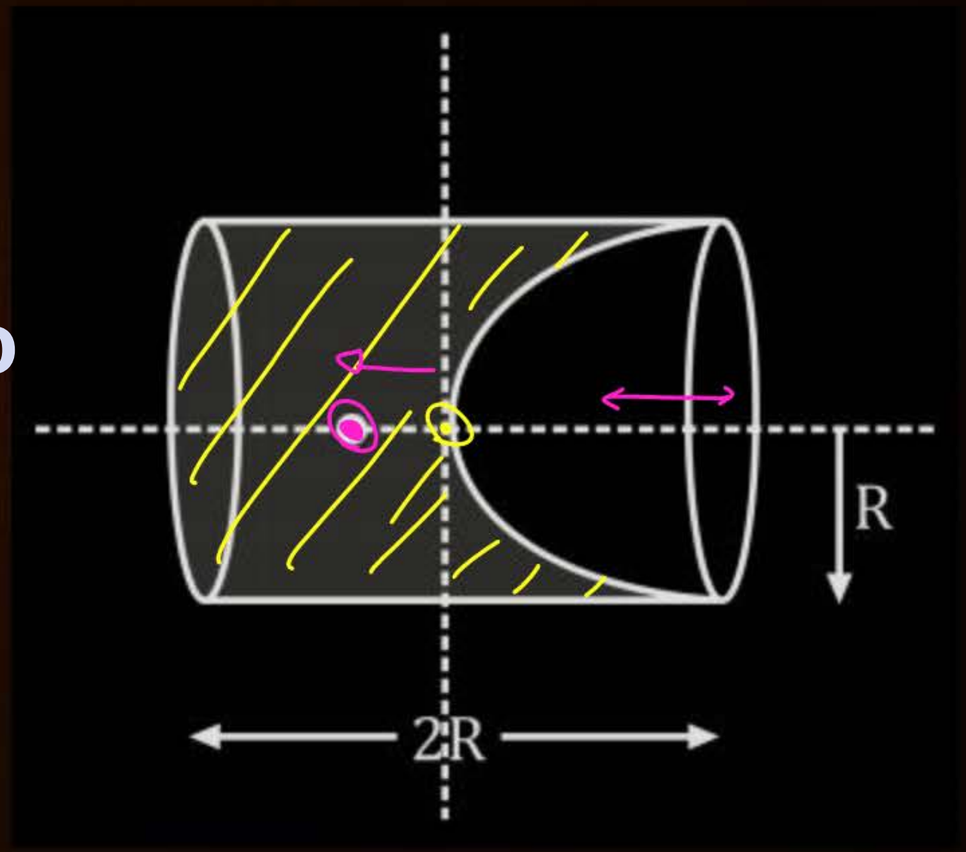


$m_1 = \rho \cdot \pi R^2 (2R) = 3m$

$m_2 = \rho \cdot \frac{2}{3}\pi R^3 = m$

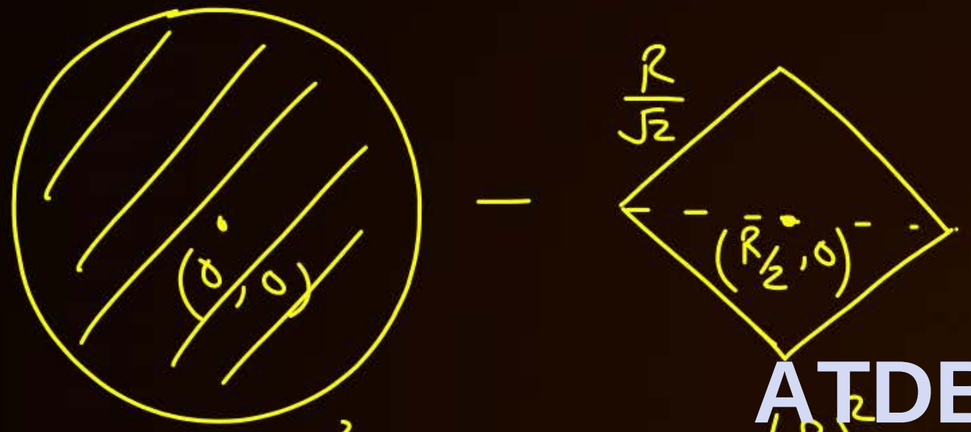
$$X_{cm} = \frac{3m(0) - m\left(\frac{5R}{8}\right)}{3m - m} = -\frac{5R}{16}$$

$$\text{distance} = \frac{5R}{16} = \frac{5 \times 32}{16} = 10\,\text{cm}$$



Question — 22

Find the center of mass of the disc of radius R (R = 12cm) as shown in figure. A square is removed from the disc as shown. Distance of COM from point C is $\frac{N}{(2\pi-1)}$.find Value of N



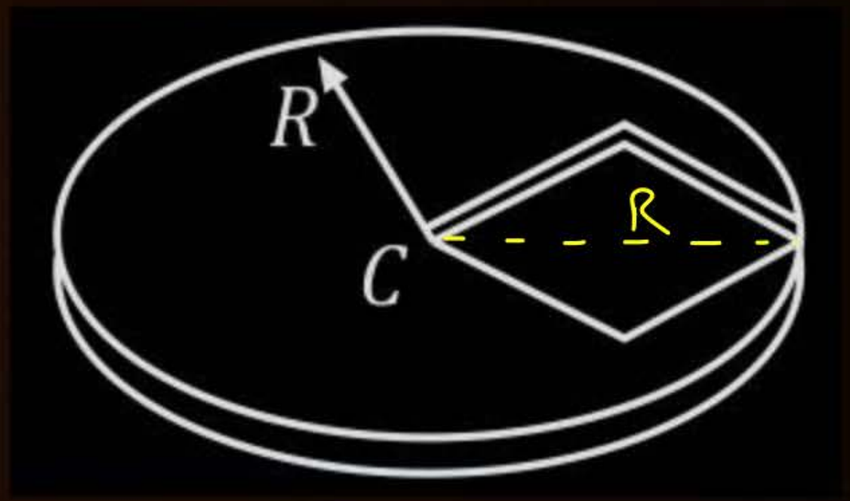
diagonal = $R = \sqrt{2}a$

$a = \frac{R}{\sqrt{2}}$

$m_1 = \sigma\pi R^2$

$= \pi m$

$m_2 = \sigma\left(\frac{R}{\sqrt{2}}\right)^2 = \frac{m}{2}$

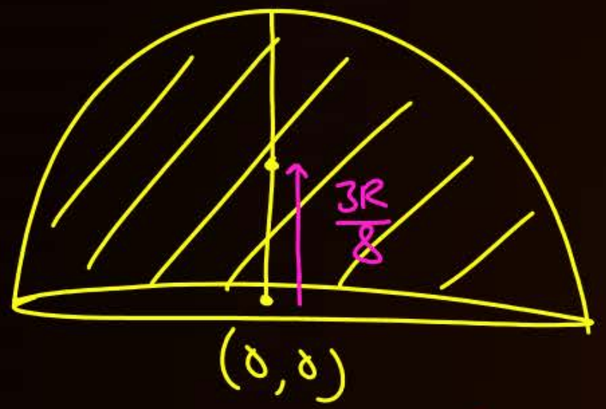


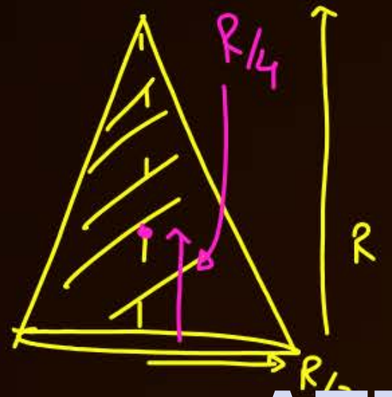
$$X_{cm} = \frac{\pi m(0) - \frac{m}{2}\left(\frac{R}{2}\right)}{\pi m - \frac{m}{2}} = \frac{-R/4}{\frac{(2\pi-1)}{2}} = \frac{-R}{2(2\pi-1)}$$

N = 6

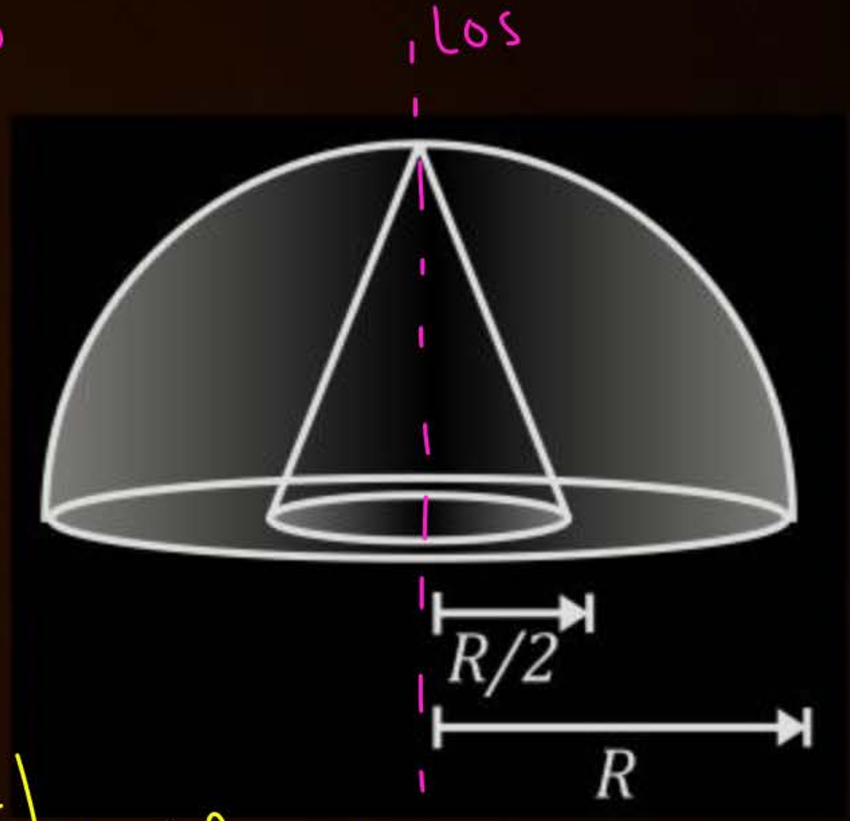
Question

From a hemisphere of radius R a cone of base radius $R/2$ and height R is cut as shown in figure. Find the height of center of mass of the remaining object. (R = 56cm)





$X_{cm} = 0$



$m_1 = \rho \cdot \frac{2}{3}\pi R^3$

$= 2m$

$m_2 = \rho \cdot \frac{1}{3}\pi\left(\frac{R}{2}\right)^2 \cdot R$

$= \frac{m}{4}$

$$y_{cm} = \frac{(2m)\left(\frac{3R}{8}\right) - \frac{m}{4}\left(\frac{R}{4}\right)}{2m - \frac{m}{4}} = \left(\frac{\frac{3}{4} - \frac{1}{16}}{\frac{8-1}{4}}\right)R = \left(\frac{\frac{11}{16}}{\frac{7}{4}}\right)R = \frac{11R}{28} = \frac{11}{28} \times 56 = 22\,\text{cm}$$

Question — 21

From the circular disc of radius 4R two small disc of radius R are cut off. The center of mass of the new structure will be : (distance of Center of mass from Center) R = 28√2 cm

$m_1 = \sigma 16\pi R^2 = 16m$

$m_2 = m_3 = \sigma\pi R = m$

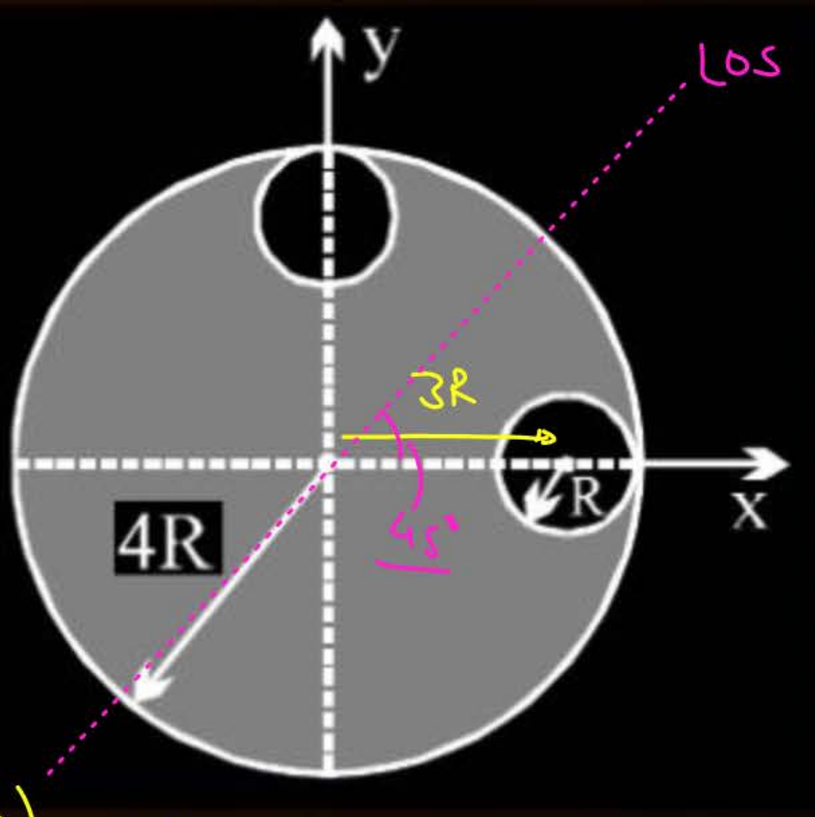
$$X_{cm} = \frac{16m(0) - m(0) - m(3R)}{14m} = -\frac{3R}{14}$$

$$Y_{cm} = \frac{16m(0) - m(3R) - m(0)}{14m} = -\frac{3R}{14}$$

distance

$$\frac{3\sqrt{2}R}{14}$$

$$= \frac{3\sqrt{2}(28\sqrt{2})}{14} = 12$$



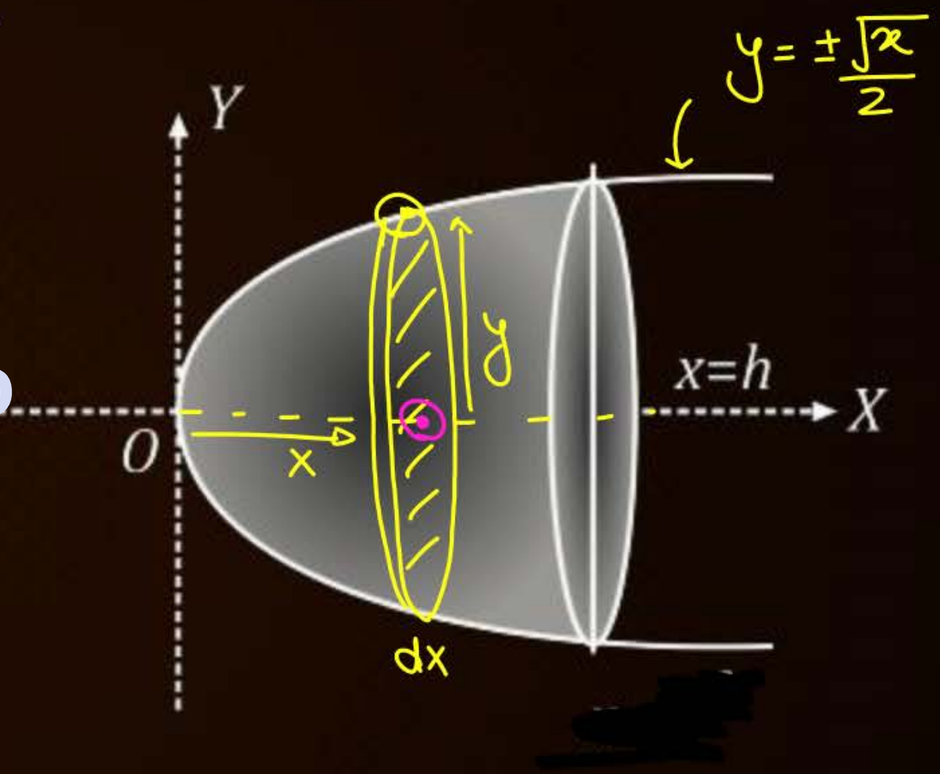
Question

Find center of mass of an object (Paraboloid) which is formed by rotating a parabola $x = 4y^2$ about x-axis as shown in figure (From x = 0 to x = h). Assume the object is of uniform density. (h = 15cm)

$$dm = \rho\, dV = \rho(\pi y^2)dx = \rho\frac{\pi \cdot x}{4}dx$$

$$x_{cm} = \frac{\int_0^h x\, dm}{\int_0^h dm} = \frac{\int_0^h x \cdot \rho\frac{\pi x dx}{4}}{\int_0^h \rho\frac{\pi x dx}{4}}$$

$$= \frac{h^3/3}{h^2/2} = \frac{2h}{3} = \frac{2(15)}{3} = 10\,cm$$



$$dm = \lambda dx$$

$$dm = \sigma dA = -(\text{length of element})(dx)$$

$$dm = \rho dv = \rho(\text{Surface area})(dx)$$

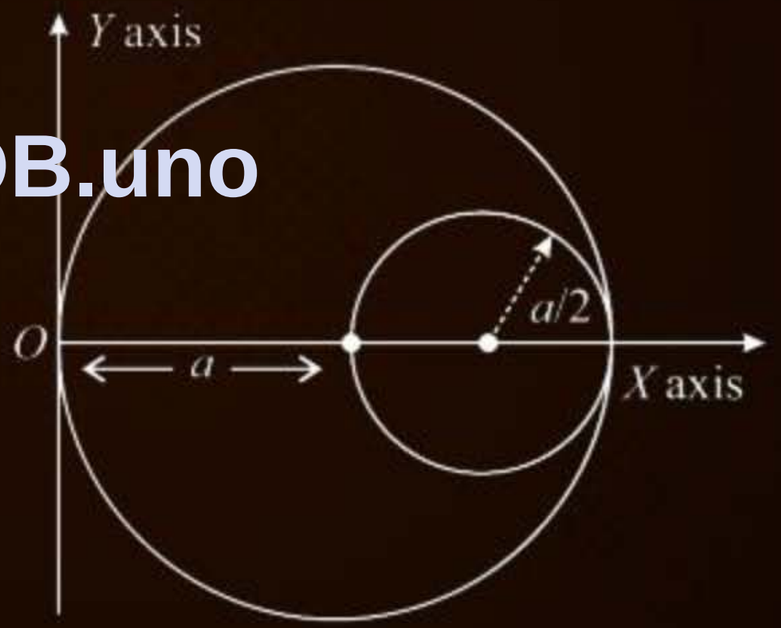
ATDB.uno

Question 26

A circular hole of radius $\left(\frac{a}{2}\right)$ is cut of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be:

[24 Feb, 2021 (Shift-II)]

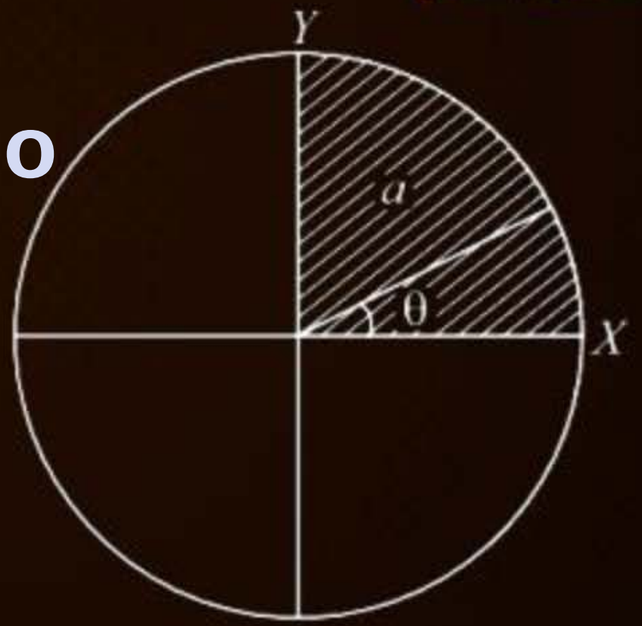
1. $\frac{5}{6}a$
2. $\frac{1}{6}a$
3. $\frac{10}{11}a$
4. $\frac{2}{3}a$



Question — 27

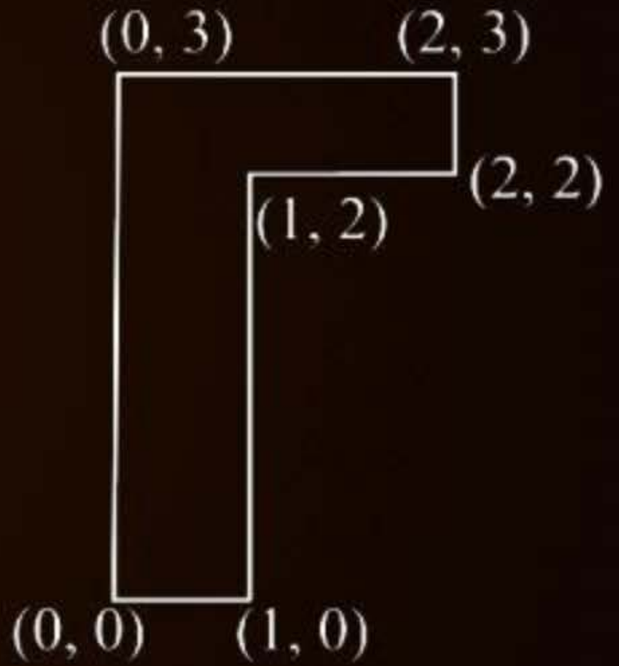
The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x}{3\pi}a, \frac{x}{3\pi}a$ where x is _(Round off to the Nearest Integer) [a is an area as shown in the figure]

[17 March, 2021 (Shift-II)]



Question 28

The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are: **[8 Jan, 2020 (Shift-I)]**



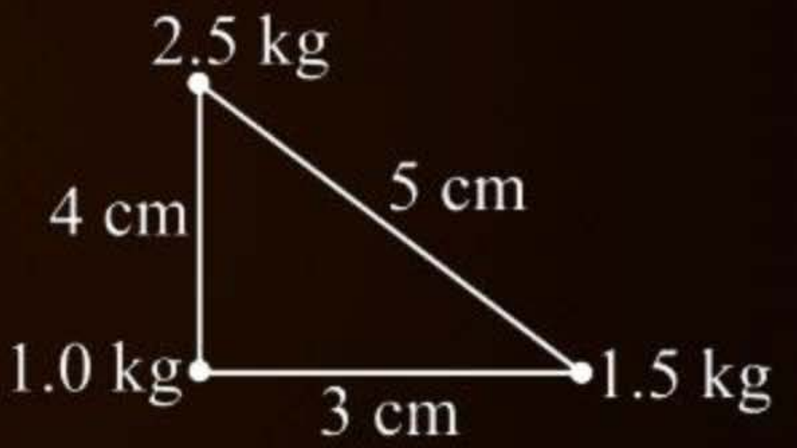
1. 1.25 m, 1.50 m)
2. (0.75 m, 0.75 m)
3. (0.75 m, 1.75 m)
4. (1 m, 1.75 m)

Question 29

Three point particles of masses 10 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:

[7 Jan, 2020 (Shift-I)]

1. 0.6 cm right and 2.0 cm above 1kg mass
2. 2.0 cm right and 0.9 cm above 1 kg mass
3. 0.9 cm right and 2.0 cm above 1kg mass
4. 1.5 cm right and 1.2 cm above 1kg mass



Question – 30

A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$ where a and b are constants and $0 \leq x \leq L$. The value of x for the centre of mass of the rod is at:

[9 Jan, 2020 (Shift-II)]

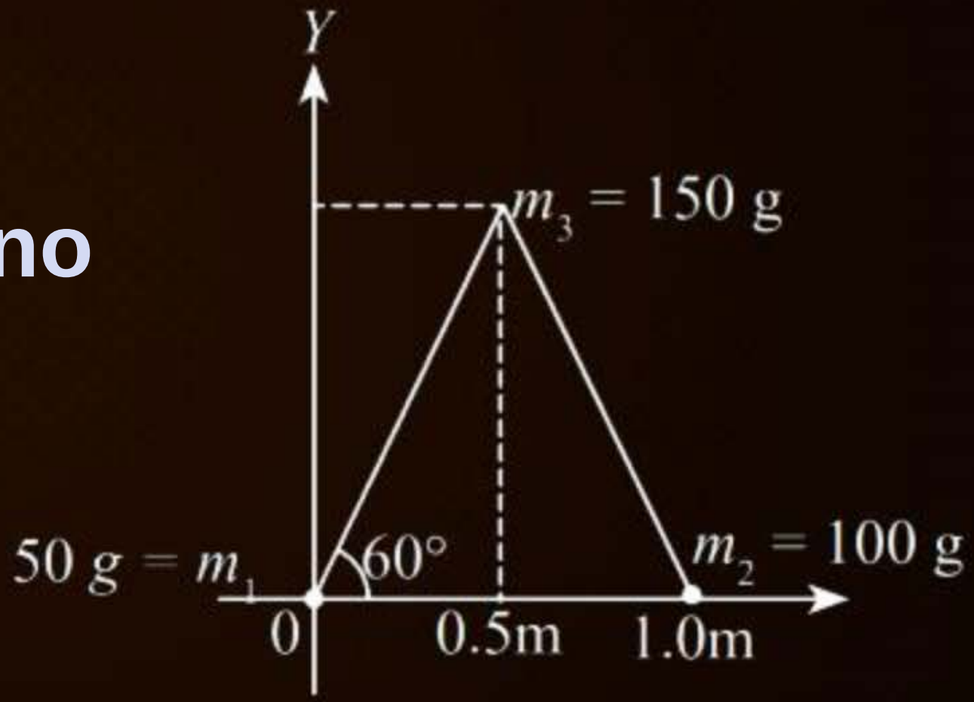
1. $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$
2. $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$
3. $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$
4. $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$

Question — 31

Three particles of masses 50 g, 100g and 150g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be:

[12 April, 2019 (Shift-II)]

1. $\left(\frac{\sqrt{3}}{7}\text{m}, \frac{7}{12}\text{m}\right)$
2. $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{8}\text{m}\right)$
3. $\left(\frac{\sqrt{3}}{4}\text{m}, \frac{5}{12}\text{m}\right)$
4. $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{4}\text{m}\right)$

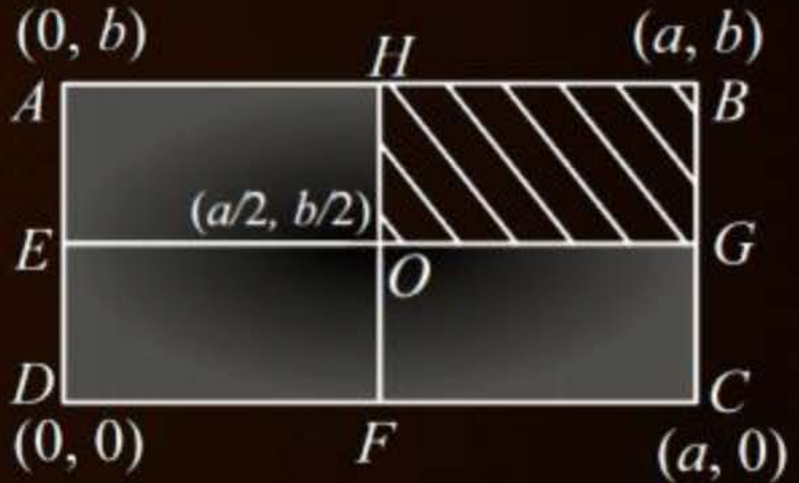


Question – 32

A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut off, the coordinates of the centre of mass of the remaining portion will be:

[8 April, 2019 (Shift-II)]

1. $\left(\frac{2a}{3}, \frac{2b}{3}\right)$
2. $\left(\frac{5a}{3}, \frac{5b}{3}\right)$
3. $\left(\frac{3a}{4}, \frac{3b}{4}\right)$
4. $\left(\frac{5a}{12}, \frac{5b}{12}\right)$



Question 33

Consider a circular disc of radius 20 cm with center located at the origin. A circular hole of a radius 5 cm is cut from this disc in such a way that the edge of the hole touches the edge of the disc. The distance of center of mass of residual or remaining disc from the origin will be-

(January 2025)/ 23-01-2025/Morning Shift

1. 2.0 cm
2. 0.5 cm
3. 1.5 cm
4. 1.0 cm

Question — 34

The center of mass of a thin rectangular plate (fig - x) with sides of length a and b, whose mass per unit area (σ) varies as $\sigma = \frac{\sigma_0 x}{ab}$ (where σ_0 is a constant), would be_____

(January 2025)/28-01-2025/Morning Shift)

1. $\left(\frac{2}{3}a, \frac{b}{2}\right)$
2. $\left(\frac{2}{3}a, \frac{2}{3}b\right)$
3. $\left(\frac{a}{2}, \frac{b}{2}\right)$
4. $\left(\frac{1}{3}a, \frac{b}{2}\right)$

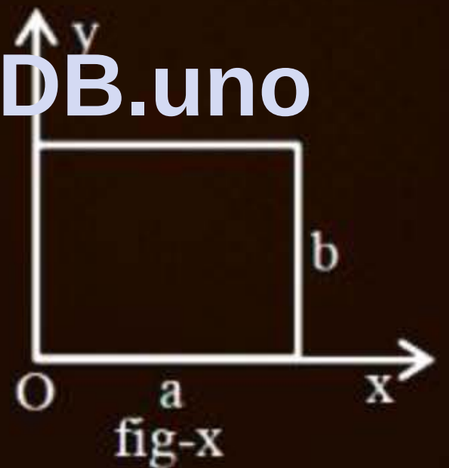


fig-x

Motion of Center of Mass

→ $\vec{V}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2 + - -}{M}$

→ $\vec{a}_{cm} = \dfrac{m_1\vec{a}_1 + m_2\vec{a}_2 + - - -}{M}$

→ $\vec{S}_{cm} = \dfrac{m_1\vec{s}_1 + m_2\vec{s}_2 + - - -}{M}$

→ We treat COM as a Particle to study it's motion (kinematics)

Ex. $u_{cm} = 10\ m/s$

$a_{cm} = 2\ m/s^2$ Constant

find V_{cm} (at $t=2$) and S_{cm} in 2 Sec.

Soln $V_{cm} = u_{cm} + a_{cm}t = 10 + 2\times 2 = 14\ m/s$

$S_{cm} = u_{cm}t + \frac{1}{2}a_{cm}t^2 = 10(2) + \frac{1}{2}\times 2\times 2^2 = 24\ m$

** $\vec{a}_{cm} = \dfrac{d\vec{V}_{cm}}{dt}$, $\vec{V}_{cm} = \dfrac{d\vec{r}_{cm}}{dt}$

$a_{cm} = \dfrac{dV_{cm}}{dt} = V_{cm}\dfrac{dV_{cm}}{dx}$, $V_{cm} = \dfrac{dx_{cm}}{dt}$

$$\vec{F}_{ext.} = M\vec{a}_{cm}$$

$$\vec{P} = M\vec{V}_{cm}$$

$$\vec{F}_{ext.} = \frac{d\vec{P}}{dt}$$

Impulse ($\vec{J}$)

$$\vec{J} = \Delta\vec{P} = \int \vec{F}\, dt$$

ATDB.uno

Effect of internal Forces

→ internal forces Can not change acc. and Velocity of COM

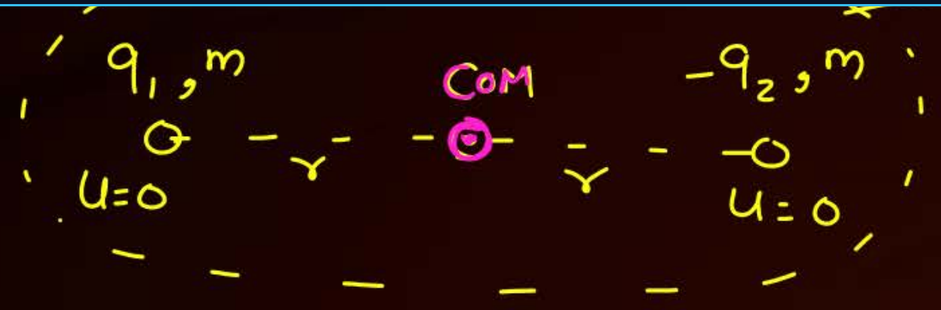
($\vec{F}_{ext.} = M\vec{a}_{cm}$, $\vec{a}_{cm}$ only depend on $\vec{f}_{ext.}$)

→ net Sum of internal forces is zero (if action and reaction of force is in boundary of System then it is internal)

→ internal forces Can not change motion of COM

→ internal forces Can change kinetic energy / Potential energy / Mechanical energy of System

Ex:



electric force b/w charges → internal / External ×

थोड़ी देर पास

→ momentum $P_i = 0 + 0 = 0$ $\left(F_{ext} = 0 \Rightarrow \vec{P}_f = \vec{P}_i\right)$

$P_f = mv_1 - mv_2 = P_i$

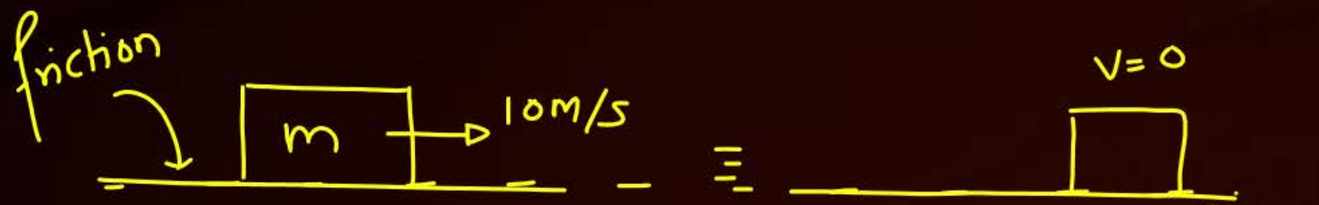
$\Rightarrow v_1 = v_2$

→ $a_{cm} = 0$, $U_{cm} = 0 \Rightarrow V_{cm} = 0 \Rightarrow \mathcal{E}_{cm} = 0$

→ kinetic energy $K.E_i = 0 + 0$ → change

$K.E_f = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \neq 0$

→ Potential energy $\left(\frac{kq_1q_2}{r}\right)$ → change



Mechanical energy loss

System → block + Earth

System
both block

ATDB.uno

Mechanical Energy Conservation

if Work done by non-Conservative forces is Zero then total mechanical energy is Conserved

if $W_{NC} = 0$ then $K_1 + U_1 = K_2 + U_2$

(यहा internal, external Concept की कोई जरूरत नहीं है यहा Conservative and Non-Conservative देखने है)

momentum Conservation

if $f_{ext} = 0$ then $\vec{P}_i = \vec{P}_f$

(इसको Conservative and non-Cons. से कोई मतलब नहीं इसको internal and external से मतलब है)

Momentum Conservation

from inertial frame

ATDB.uno

Bomb Explosion + Gun-Bullet + Cannon Shell

→ due to internal forces

→ very short time event

→ kinetic energy generally change

→ energy of explosion = final − initial