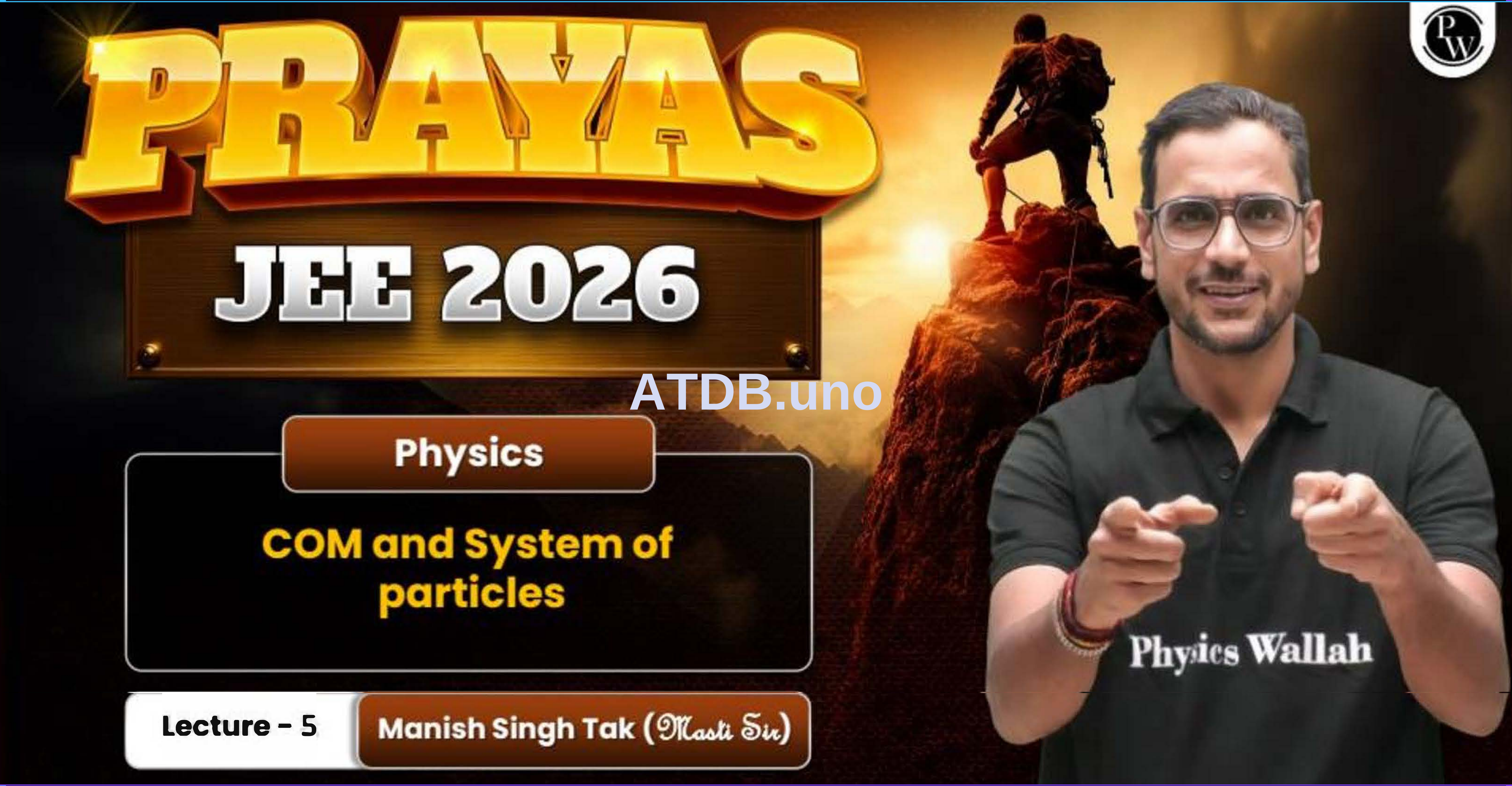

PRAYAS
JEE 2026
Physics
COM and System of particles
Lecture - 5
Manish Singh Tak (Masti Sir)
Physics Wallah

Topics to be covered

A. Derivation of COM of Standard Objects

B.

C.

D.

$$x_{cm} = \frac{\int x\,dm}{\int dm}$$

$$y_{cm} = \frac{\int y\,dm}{\int dm}$$

$$\vec{r}_{cm} = \frac{\int \vec{r}\,dm}{\int dm}$$

COM of Arc of Ring

$$r_{cm} = \frac{2R}{\phi} \sin\frac{\phi}{2}$$

Ring :→ (Rdθ at angle θ)

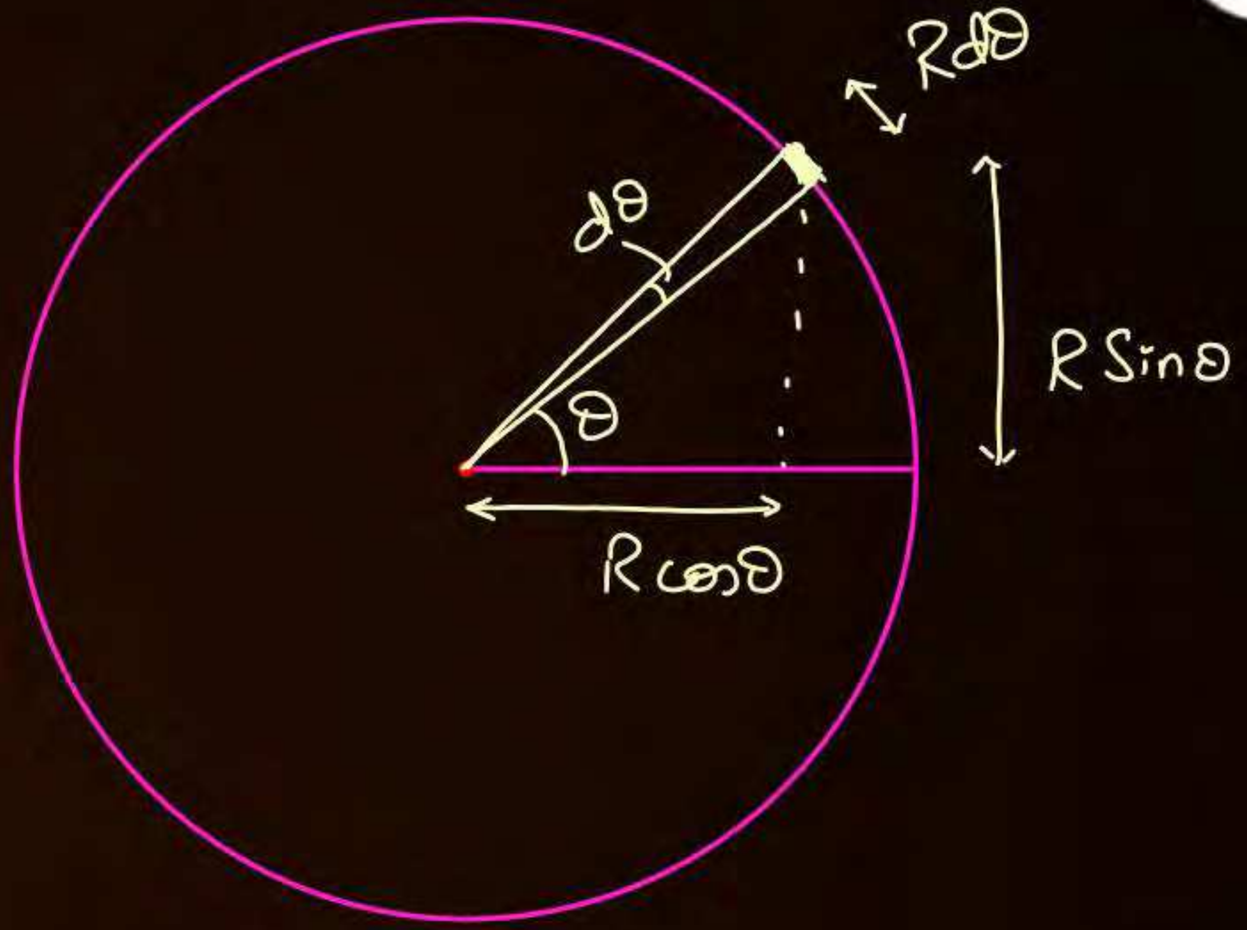
Element → point mass / charge

length → $Rd\theta$

mass/charge = $\lambda(Rd\theta)$

λ → (mass/charge) per unit length

$dm = \lambda R d\theta$



COM of Arc of Ring

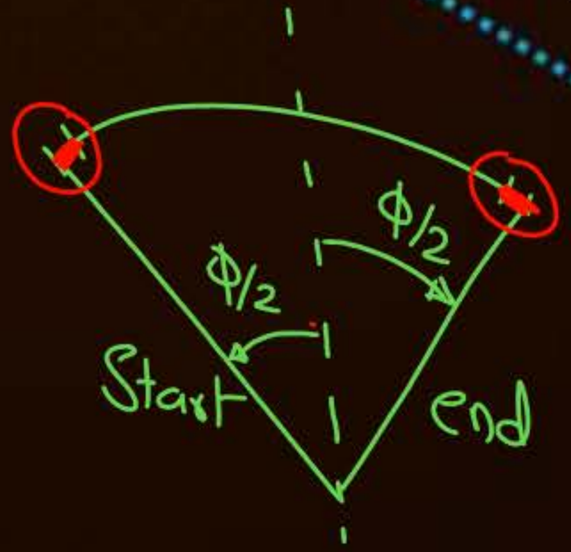
$$x_{cm} = 0$$

$$\boxed{dm = \lambda R\, d\theta}$$

$$y_{cm} = \frac{\int y\, dm}{\int dm} = \frac{\int_{-\phi/2}^{\phi/2} (R\cos\theta)(\lambda R\, d\theta)}{\int_{-\phi/2}^{\phi/2} \lambda R\, d\theta}$$

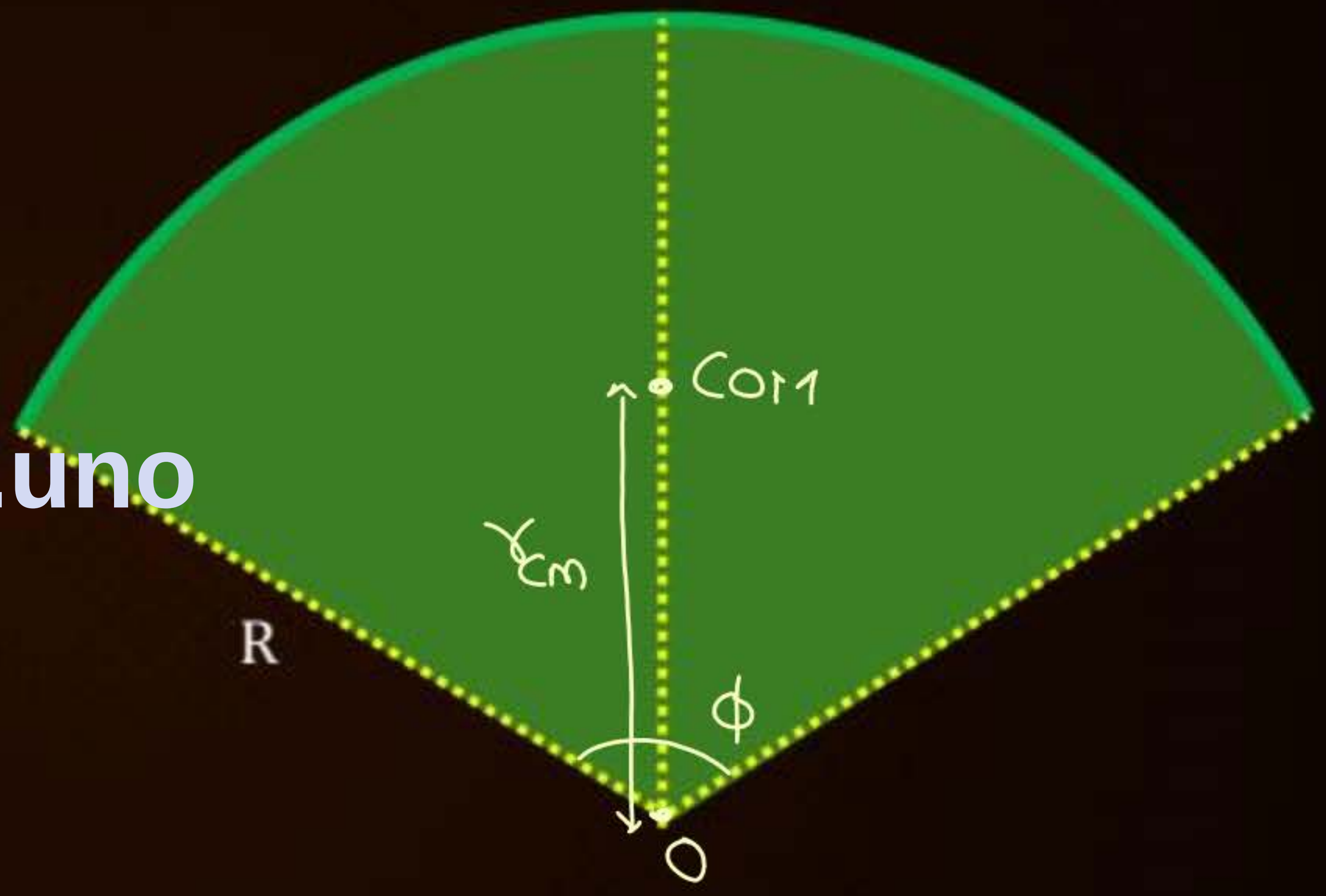
$$y_{cm} = \frac{R\left[\sin\theta\right]_{-\phi/2}^{\phi/2}}{\left[\theta\right]_{-\phi/2}^{\phi/2}} = \frac{R\left(\sin\phi/2 + \sin\frac{\phi}{2}\right)}{\left(\frac{\phi}{2} + \frac{\phi}{2}\right)}$$

$$= \frac{2R\sin\phi/2}{\phi}$$



COM of Sector (part of Disc)

$$r_{cm} = \frac{4R}{3\phi} \sin\frac{\phi}{2}$$



Disc →

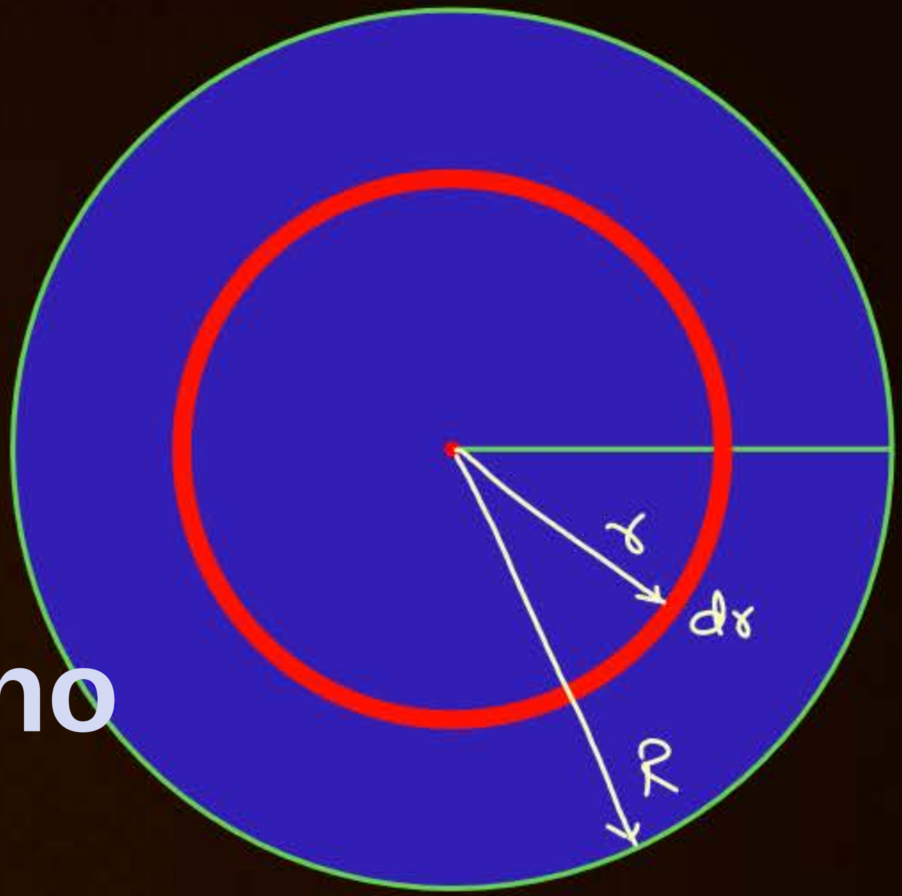
Ring element radius r and thickness dr

(onion Ring)

area → (length of Ring)(thickness)

$= (2\pi r)(dr)$

$dA = \pi r\, dr$



mass $dm = \sigma(dA) = \sigma(2\pi r\, dr)$

Charge dq

COM of Sector (part of Disc)

Element → Arc of Ring

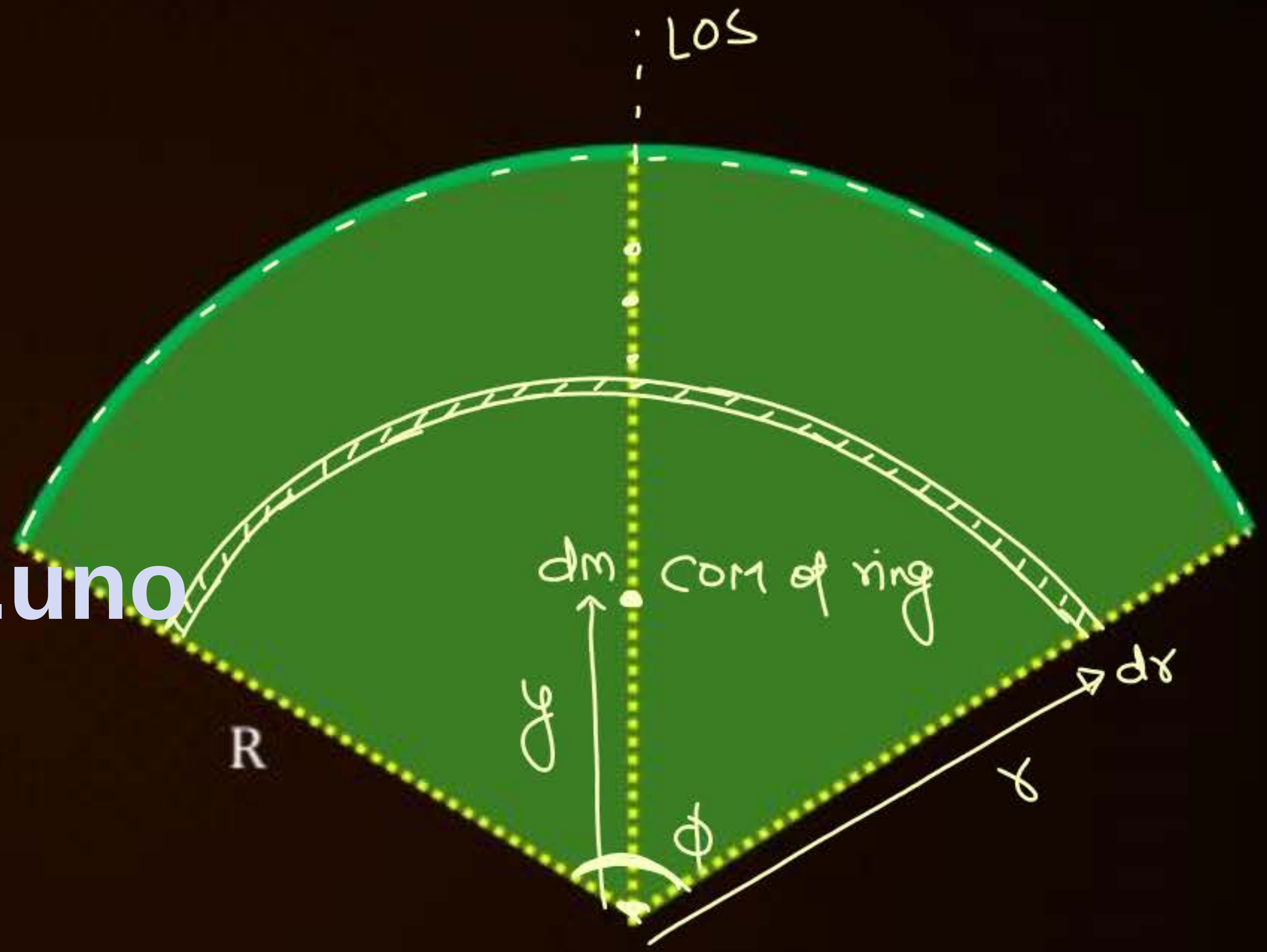
$$dA = (r\phi)\,dr$$

$$dm = \sigma r\phi\, dr$$

$$y = \frac{2r}{\phi}\sin\frac{\phi}{2}$$

$$y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int_0^R \left(\frac{2r}{\phi}\sin\frac{\phi}{2}\right)(\sigma r\phi\, dr)}{\int_0^R \sigma r\phi\, dr}$$

$$y_{cm} = \frac{2}{\phi}\sin\frac{\phi}{2}\,\frac{\int r^2 dr}{\int r\,dr} = \frac{2}{\phi}\sin\frac{\phi}{2}\left[\frac{R^3/3}{R^2/2}\right] = \frac{4R}{3\phi}\sin\frac{\phi}{2}$$



COM of Sector (part of Disc)

Element → Arc of Ring

$$dA = (r\phi)\,dr$$

$$dm = \sigma r\phi\,dr$$

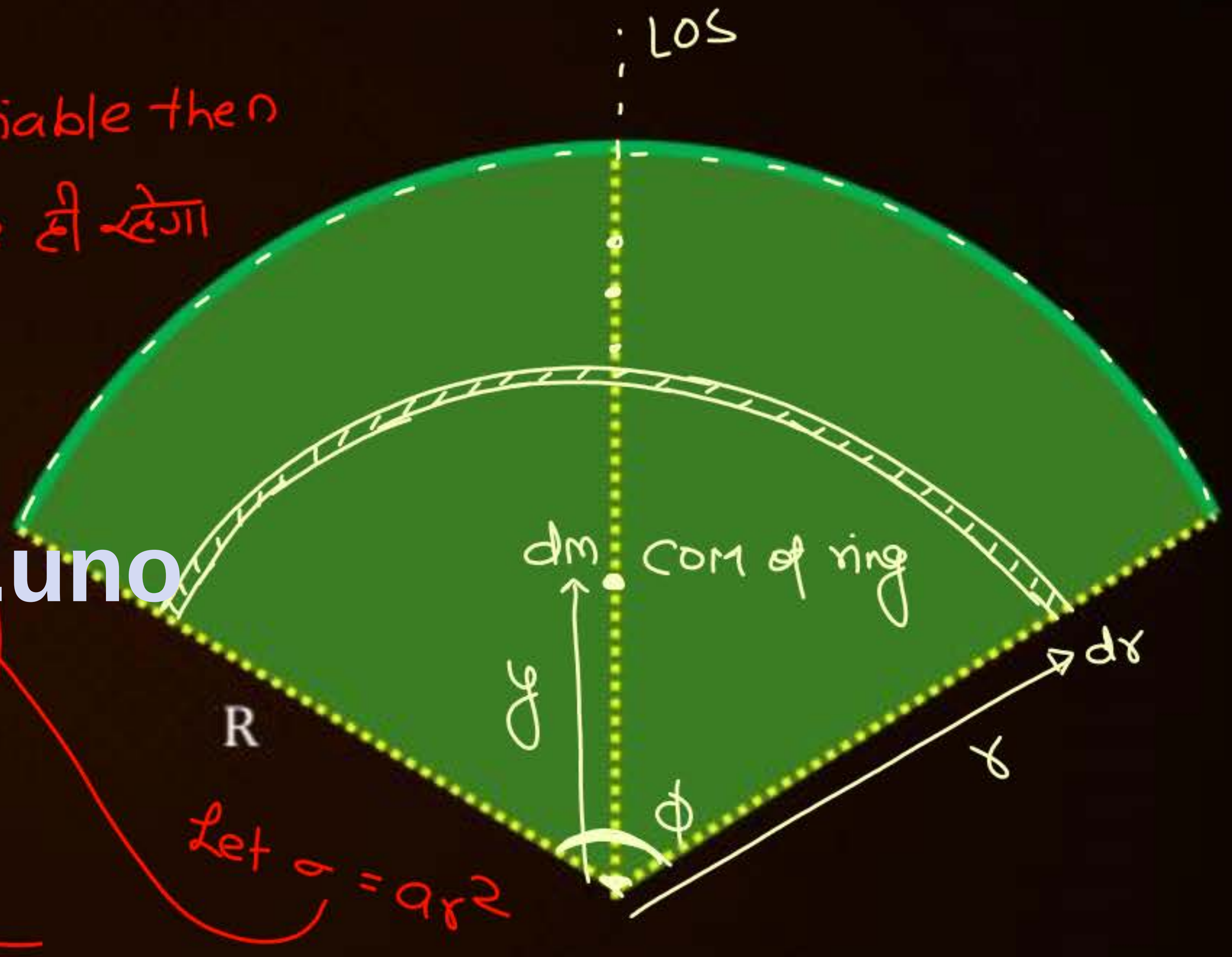
$$y = \frac{2r}{\phi}\sin\frac{\phi}{2}$$

$$y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int_0^R \left(\frac{2r}{\phi}\sin\frac{\phi}{2}\right)(\sigma r\phi\,dr)}{\int_0^R \sigma r\phi\,dr}$$

if σ is variable then
यहा तक तो Same ही रहेगा

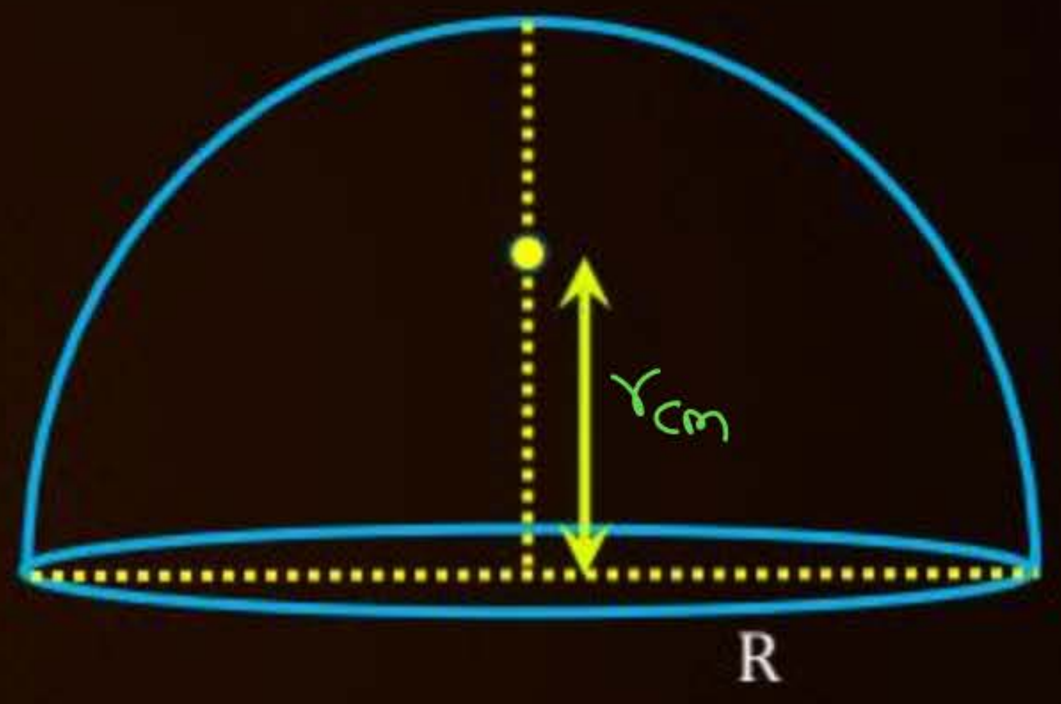
Let $\sigma = ar^2$

$$y_{cm} = \frac{2}{\phi}\sin\frac{\phi}{2}\,\frac{\int r^2 dr}{\int r\,dr} = \frac{2}{\phi}\sin\frac{\phi}{2}\left[\frac{R^3/3}{R^2/2}\right] = \frac{4R}{3\phi}\sin\frac{\phi}{2}$$



COM of half Hollow Sphere

$$r_{cm} = \frac{R}{2}$$



COM of half Hollow Sphere

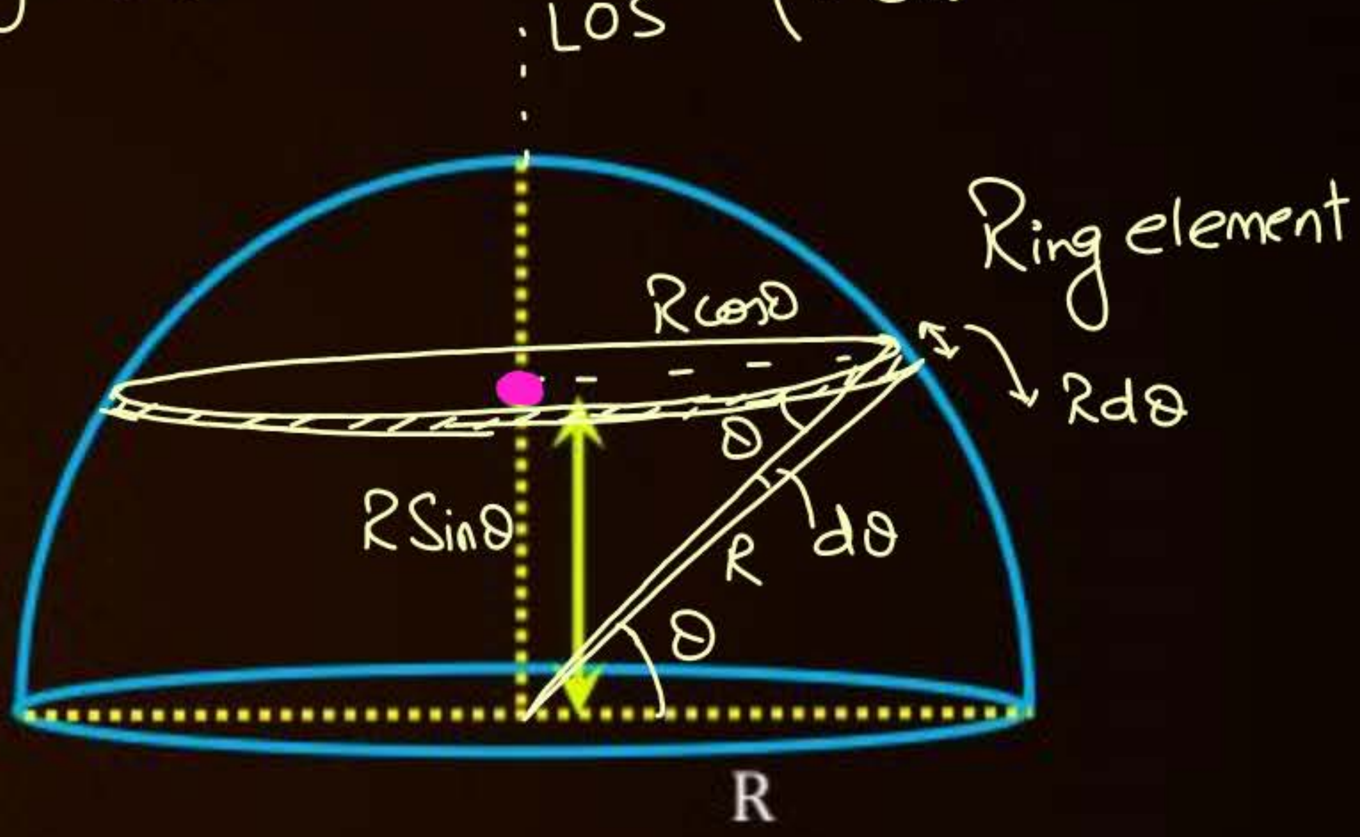
Element → at angle θ, a Ring making an angle dθ $(X_{cm} = 0)$

$dm = \sigma dA$

$dA = (\text{length of Ring})(\text{thickness})$

$= (2\pi R\cos\theta)(R d\theta)$

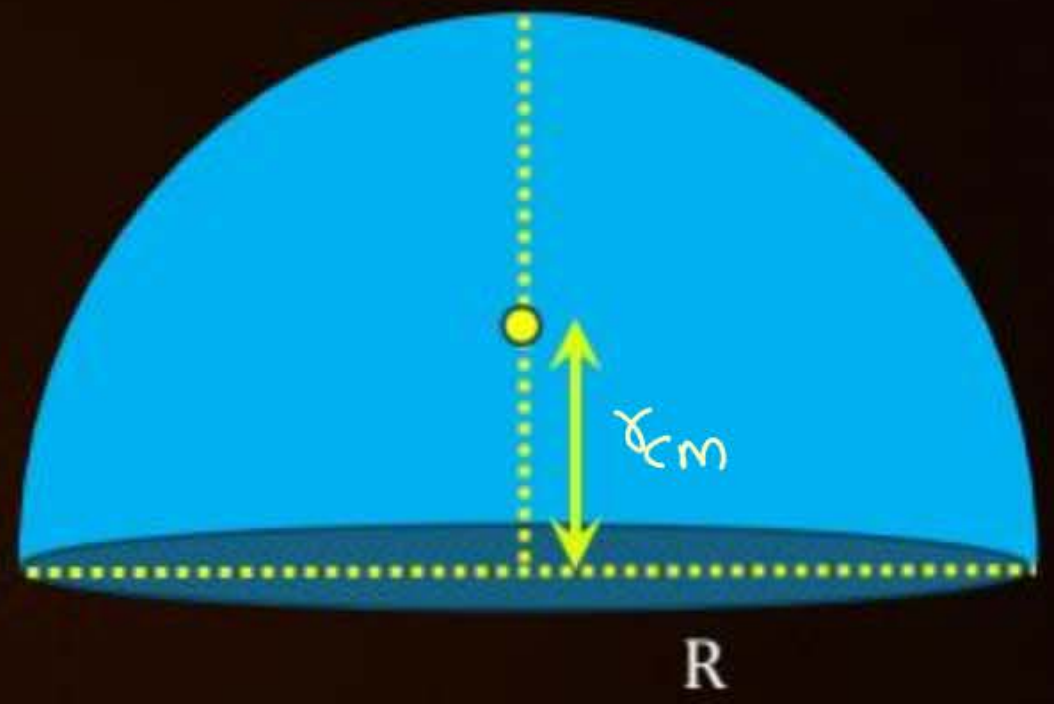
$dm = \sigma(2\pi R^2 \cos\theta\, d\theta)$



$$y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int (R\sin\theta)(\sigma 2\pi R^2 \cos\theta\, d\theta)}{\int \sigma(2\pi R^2 \cos\theta\, d\theta)} = \frac{R}{2}\frac{\int_0^{\pi/2} \sin 2\theta\, d\theta}{\int_0^{\pi/2} \cos\theta\, d\theta} = -\frac{R}{4}\frac{[\cos\pi - \cos 0]}{[\sin\pi/2 - \sin 0]} = \frac{R}{2}$$

COM of half Solid Sphere

$$r_{cm} = \frac{3R}{8}$$



Solid Sphere :→

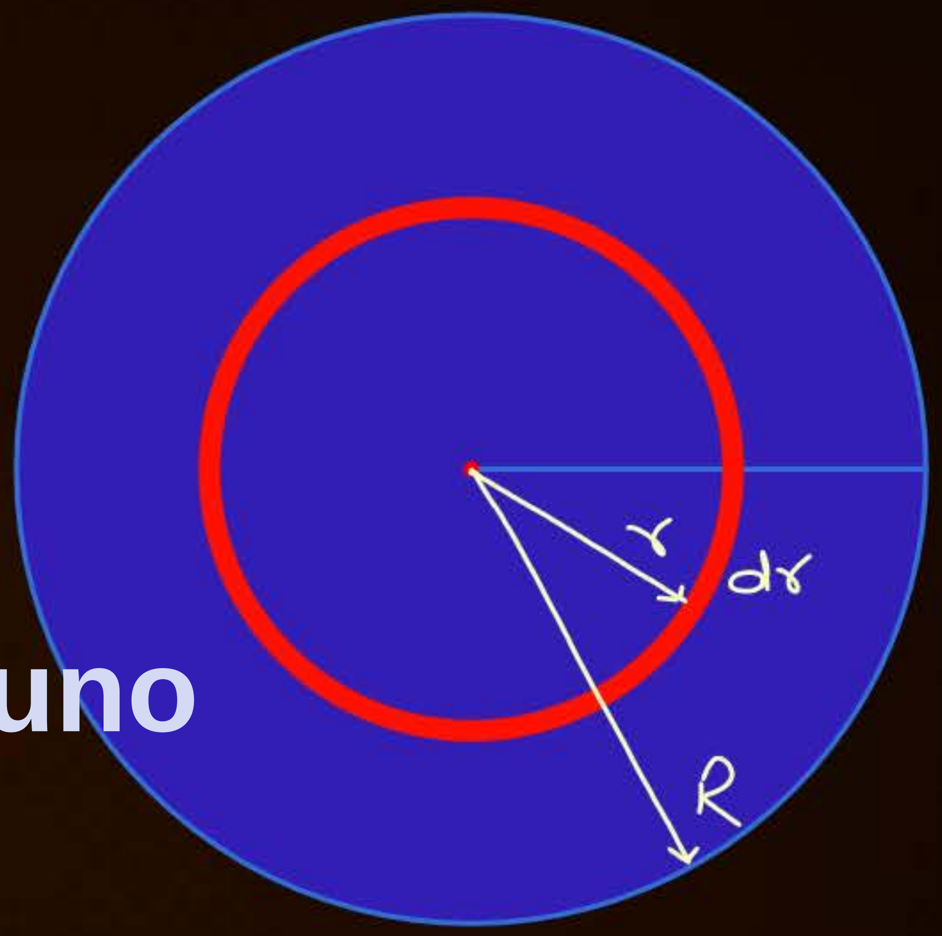
Spherical Shell (Hollow Sphere) of radius r and thickness dr

Volume = (Surface area)(thickness)

$$dV = (4\pi r^2)(dr)$$

$$dm = \rho(4\pi r^2 dr)$$

or dq

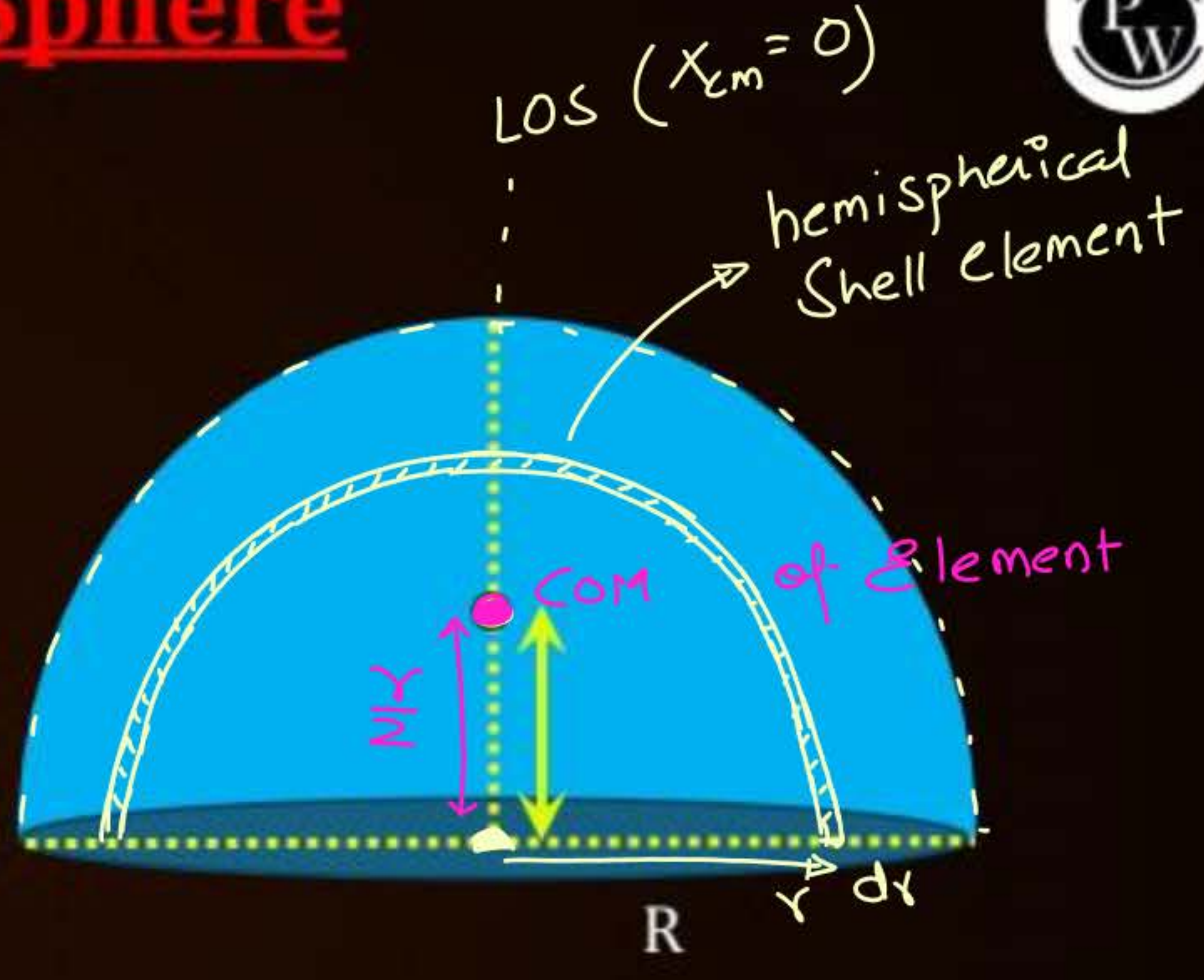


COM of half Solid Sphere

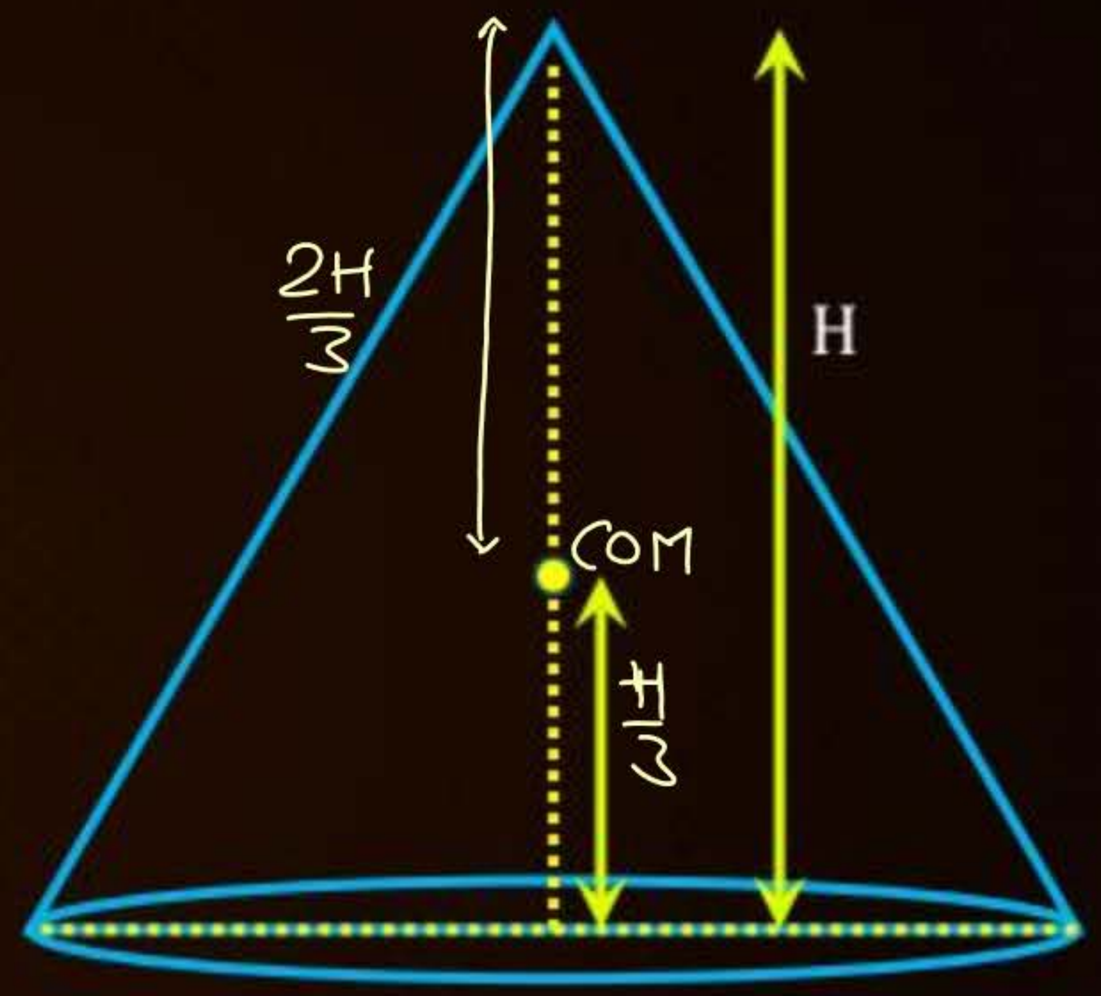
$$dm = \rho dV = \rho(2\pi r^2 dr)$$

$$y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int \frac{r}{2}\cdot \rho \cdot 2\pi r^2 dr}{\int \rho \cdot 2\pi r^2 dr}$$

$$y_{cm} = \frac{\int_0^R \frac{r^3}{2} dr}{\int_0^R r^2 dr} = \frac{1}{2}\left[\frac{\frac{R^4}{4}}{\frac{R^3}{3}}\right] = \frac{3R}{8}$$



COM of Hollow Cone



COM of Hollow Cone

Ring element

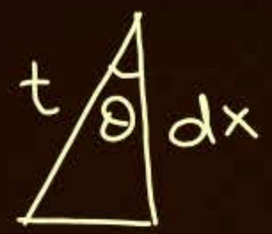
$$dm = \sigma\, dA = \sigma(\text{length})(\text{thickness})$$

$$dm = \sigma(2\pi r)(t) = 2\pi(x\tan\theta)(dx\sec\theta)$$

$$= (2\pi\tan\theta\sec\theta)(x\,dx)\sigma$$

$$y_{cm} = \frac{\int y\,dm}{\int dm} = \frac{\int x(2\pi\tan\theta\sec\theta)(x\,dx)}{\int \sigma(2\pi\tan\theta\sec\theta)\,x\,dx}$$

$$y_{cm} = \frac{\int_0^H x^2\,dx}{\int_0^H x\,dx} = \frac{H^3/3}{H^2/2} = \frac{2H}{3}$$

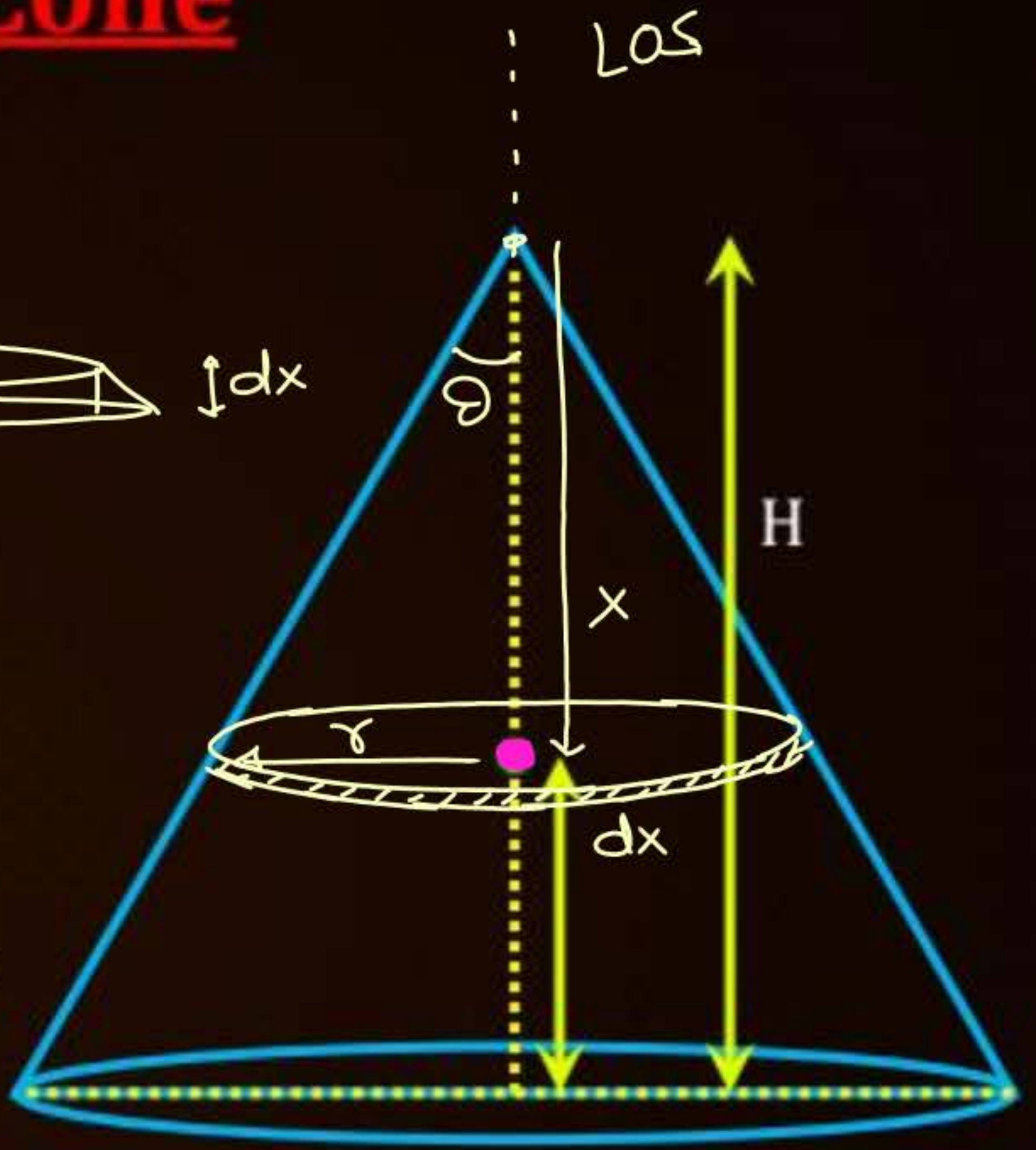


t

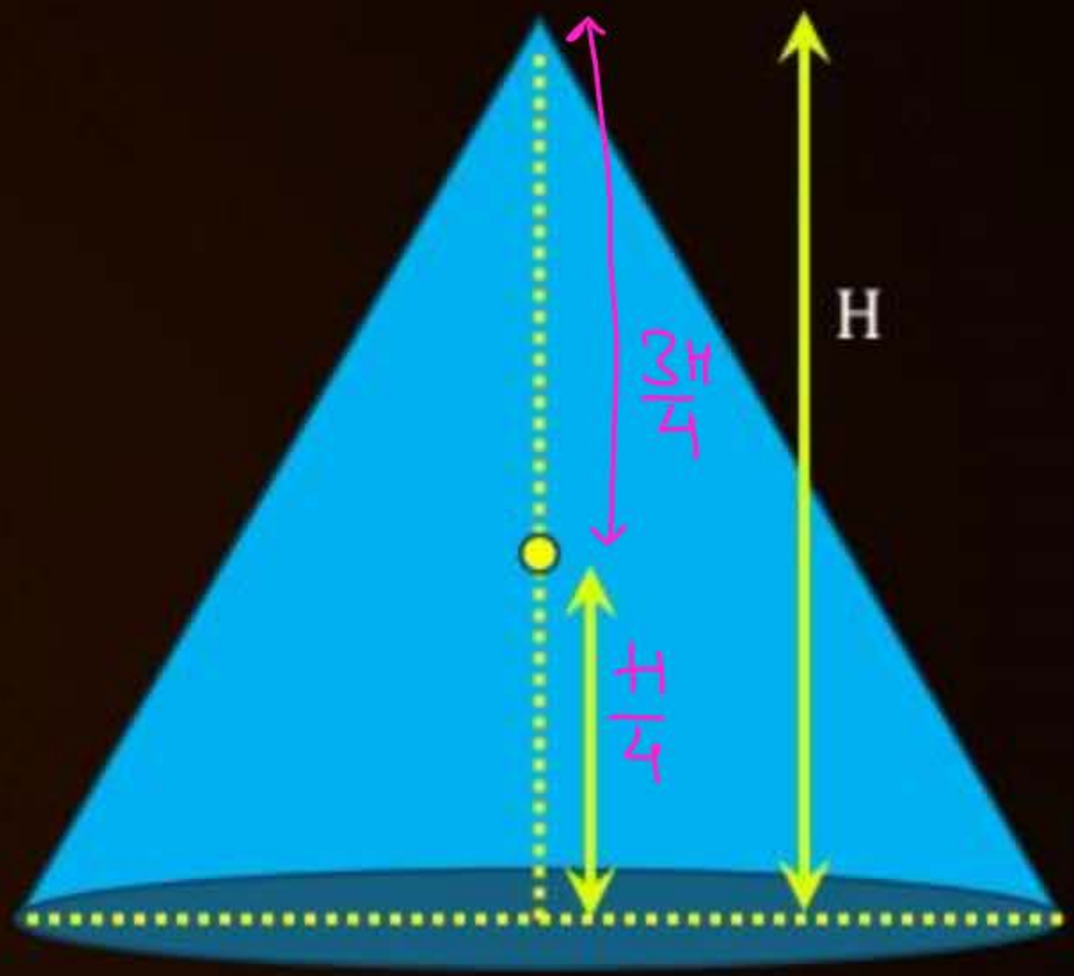
$$t = (dx)\sec\theta$$

$$\frac{r}{x} = \tan\theta$$

$$r = x\tan\theta$$



COM of Solid Cone



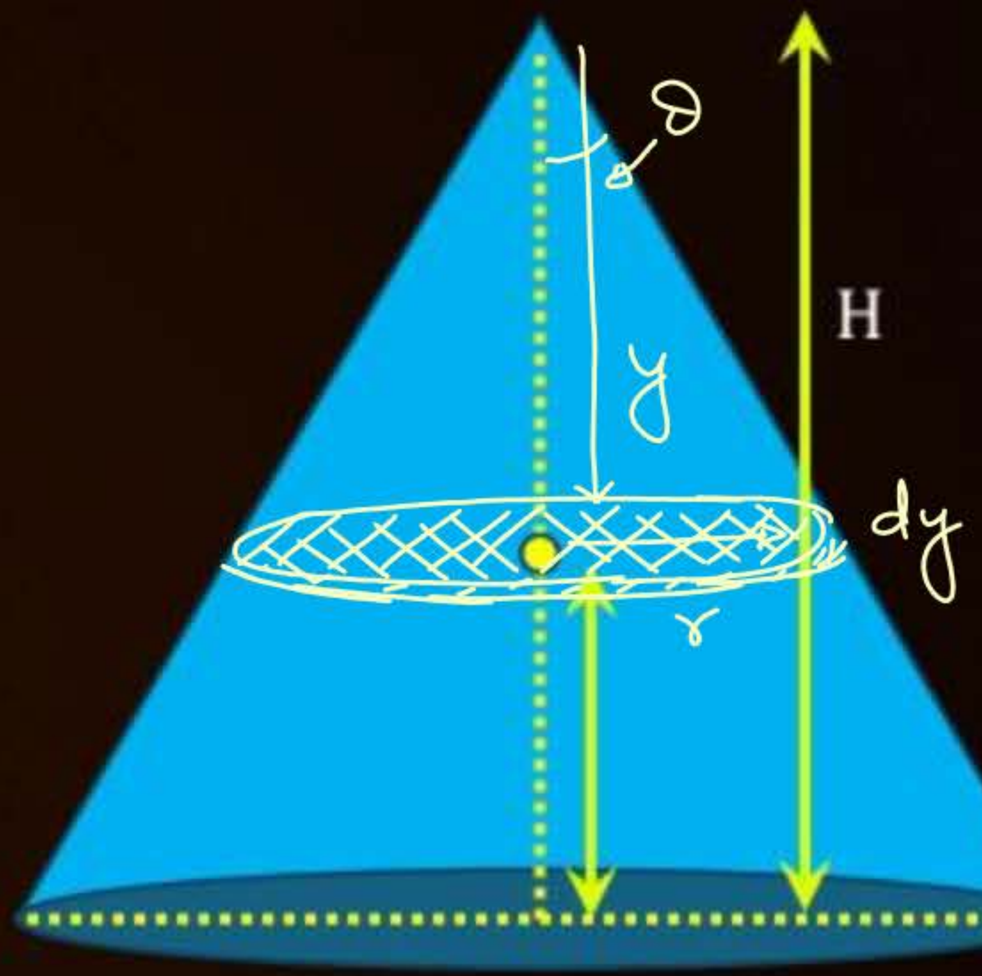
COM of Solid Cone

disc element

$$dm = \rho dV = \rho(\text{Surface area})(\text{thickness})$$

$$dm = \rho(\pi r^2)(dy) = (\rho \pi \tan^2\theta) y^2 dy$$

$$y_{cm} = \frac{\int_0^H y\, dm}{\int_0^H dm} = \frac{\int_0^H (\rho \pi \tan^2\theta)(y^3 dy)}{\int_0^H (\rho \pi \tan^2\theta) y^2 dy}$$



$$y_{cm} = \frac{H^4/4}{H^3/3} = \frac{3H}{4}$$

$$\tan\theta = \frac{r}{y}$$

$$r = y\tan\theta$$


THANK YOU
BAWAL
BACCHA
PARTY
Physics Wallah