PRAYAS

JEE 2026

Physics

COM and System of particles

Lecture – 6

Manish Singh Tak (Masti Sir)

Topics to be covered

A Motion of COM

B

C

D

ATDB.uno

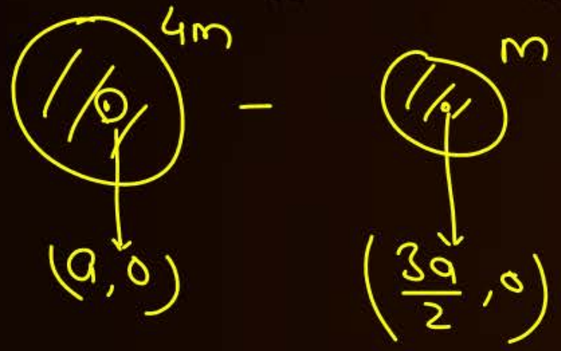
PYQ's

Question – 26

A circular hole of radius $\left(\frac{a}{2}\right)$ is cut of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be:

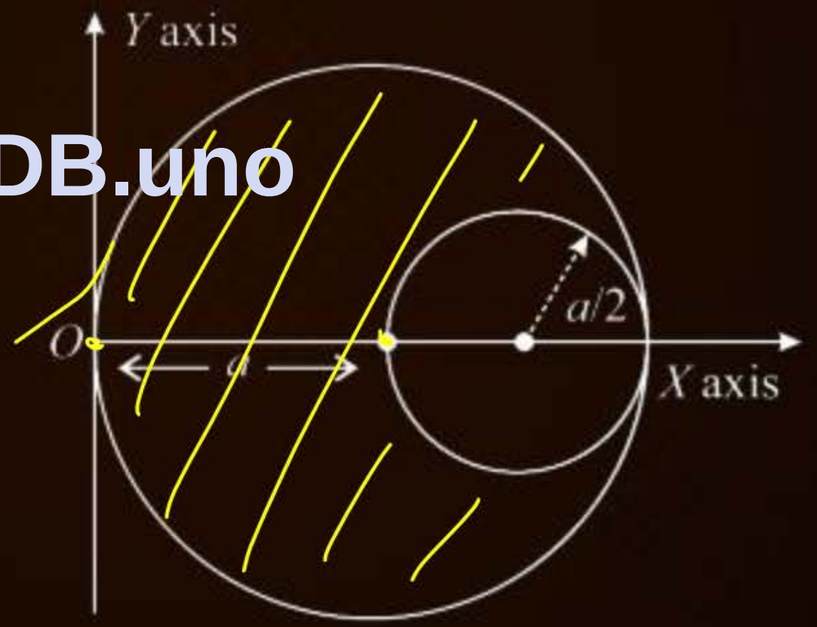
[24 Feb, 2021 (Shift-II)]

1. $\frac{5}{6}a$
2. $\frac{1}{6}a$
3. $\frac{10}{11}a$
4. $\frac{2}{3}a$



$$\frac{4m(a) - m\left(\frac{3a}{2}\right)}{4m - m} = \left(\frac{8-3}{6}\right)a$$

$$= \frac{5a}{6}$$

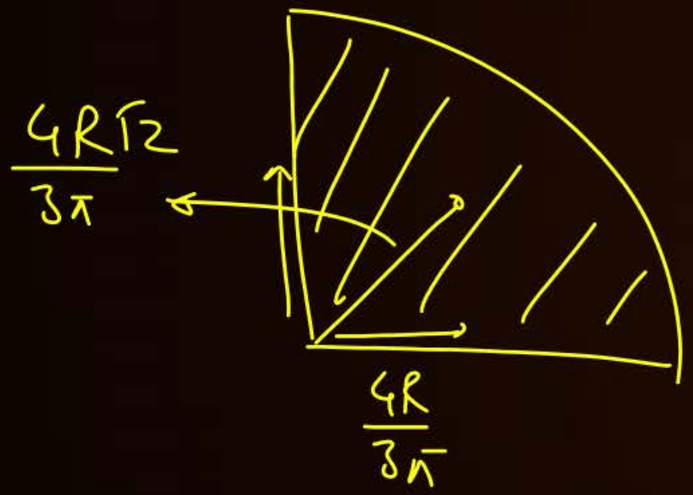


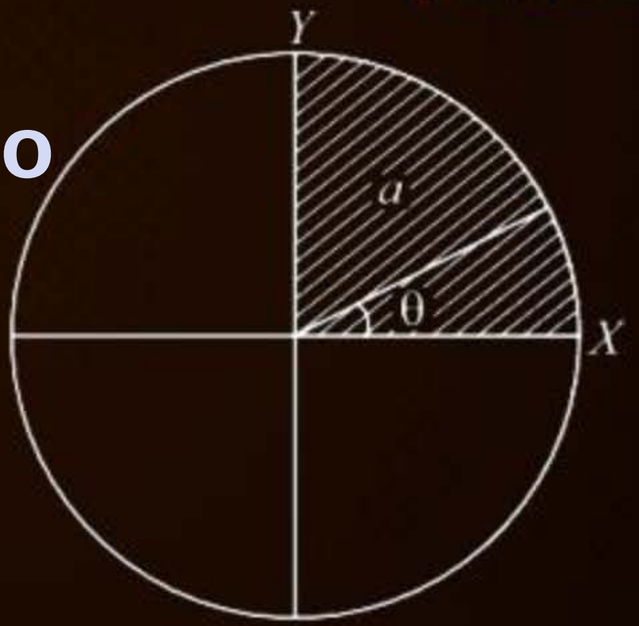
Question – 27

The disc of mass M with uniform surface mass density σ is shown in the figure. The centre of mass of the quarter disc (the shaded area) is at the position $\frac{x\,a}{3\,\pi}, \frac{x\,a}{3\,\pi}$ where x is _(Round off to the Nearest Integer) [a is an area as shown in the figure]

Radius

[17 March, 2021 (Shift-II)]



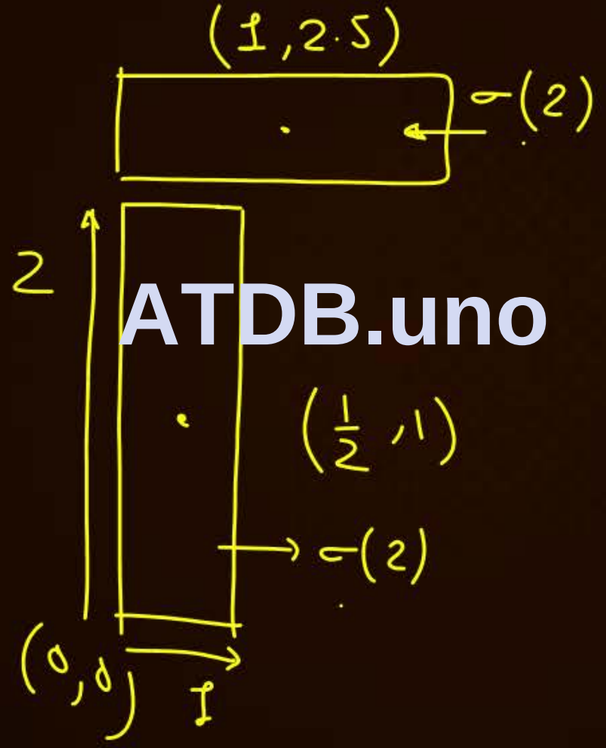


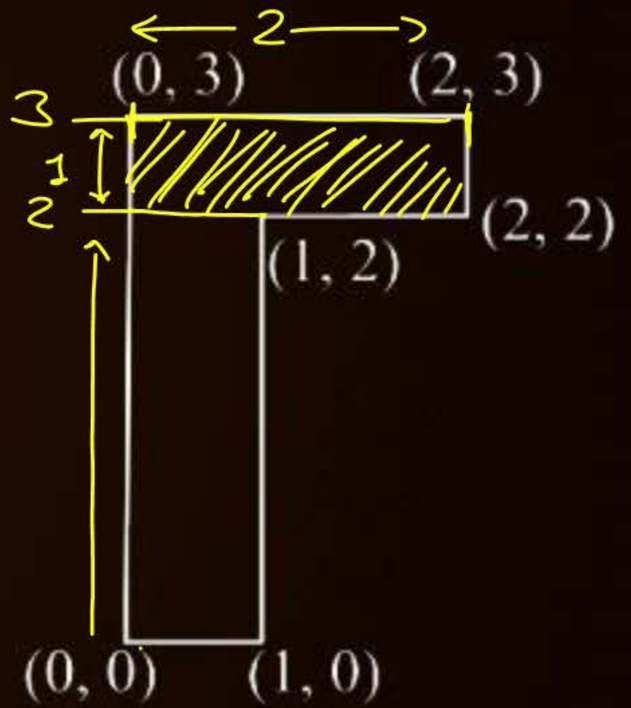
Question –28

$X_{cm} = \frac{m(2) + \cdots}{2m} = \frac{3}{4}$, $Y_{cm} = \frac{m(1) + m(2.5)}{2m} = \frac{3.5}{2} = \frac{7}{4}$

The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are: **[8 Jan, 2020 (Shift-I)]**

1. 1.25 m, 1.50 m)
2. (0.75 m, 0.75 m)
3. (0.75 m, 1.75 m)
4. (1 m, 1.75 m)





Question — 29

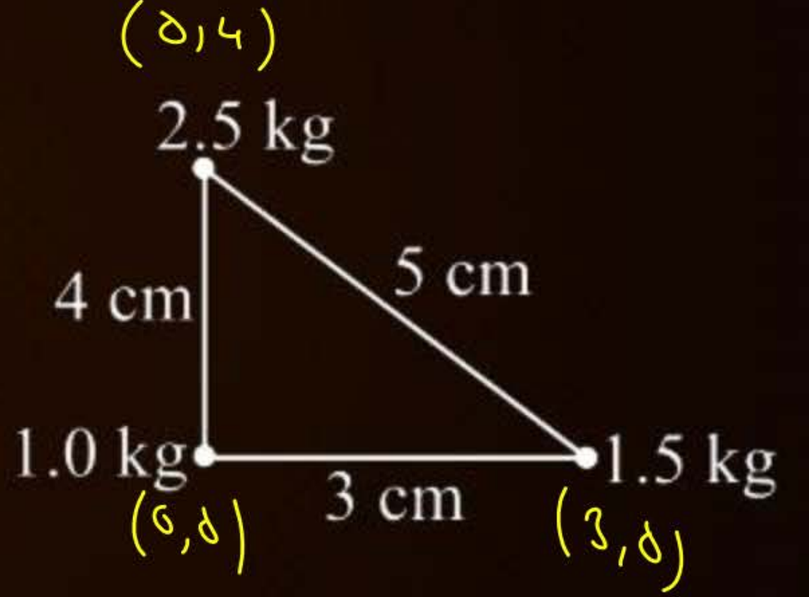
$x_{cm} = \frac{1(0) + 1.5(3) + 2.5(0)}{5} = 0.9$

$y_{cm} = \frac{1(0) + 1.5(0) + 2.5(4)}{5} = 2$

Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:

[7 Jan, 2020 (Shift-I)]

1. 0.6 cm right and 2.0 cm above 1kg mass
2. 2.0 cm right and 0.9 cm above 1 kg mass
3. 0.9 cm right and 2.0 cm above 1kg mass
4. 1.5 cm right and 1.2 cm above 1kg mass

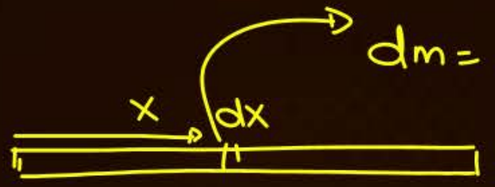


Question – 30

A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$ where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at:

[9 Jan, 2020 (Shift-II)]

1. $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$
2. $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$
3. $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$
4. $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$



$$x_{cm} = \frac{\int x\,dm}{\int dm} = \frac{\int x\left(a + \frac{bx^2}{L^2}\right)dx}{\int\left(a + \frac{bx^2}{L^2}\right)dx} = \frac{\frac{aL^2}{2} + \frac{bL^4}{4L^2}}{aL + \frac{bL^3}{3L^2}}$$

$$= \frac{3(2a+b)L}{4(3a+b)}$$

Question – 31

Three particles of masses 50 g, 100g and 150g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be:

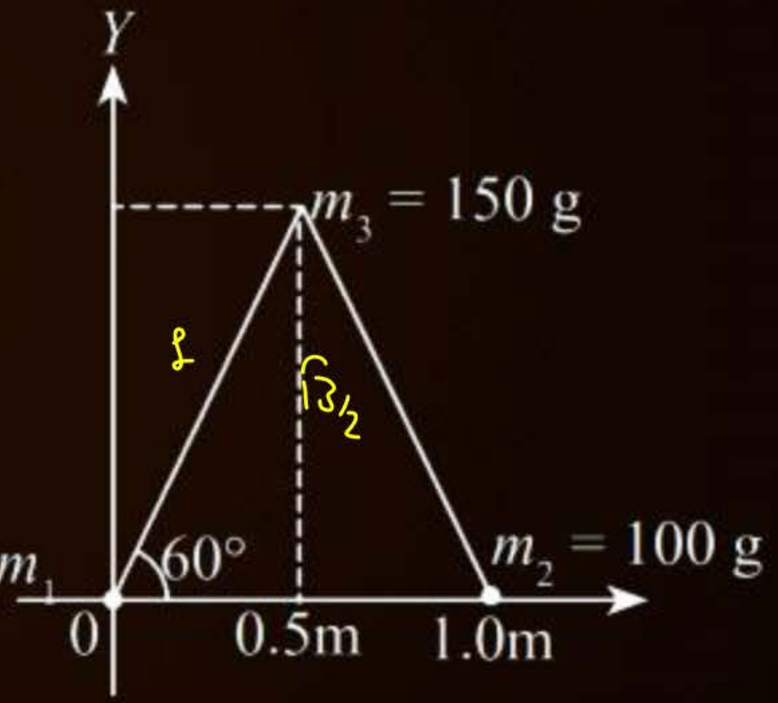
[12 April, 2019 (Shift-II)]

1. $\left(\frac{\sqrt{3}}{7}\text{m}, \frac{7}{12}\text{m}\right)$
2. $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{8}\text{m}\right)$
3. $\left(\frac{\sqrt{3}}{4}\text{m}, \frac{5}{12}\text{m}\right)$
4. $\left(\frac{7}{12}\text{m}, \frac{\sqrt{3}}{4}\text{m}\right)$

$$\frac{50(0)+100(1)+150(.5)}{300}$$

$$= \frac{50(0)+100(0)+150\left(\sqrt{3}/2\right)}{300}$$

$$= \frac{\sqrt{3}}{4}$$

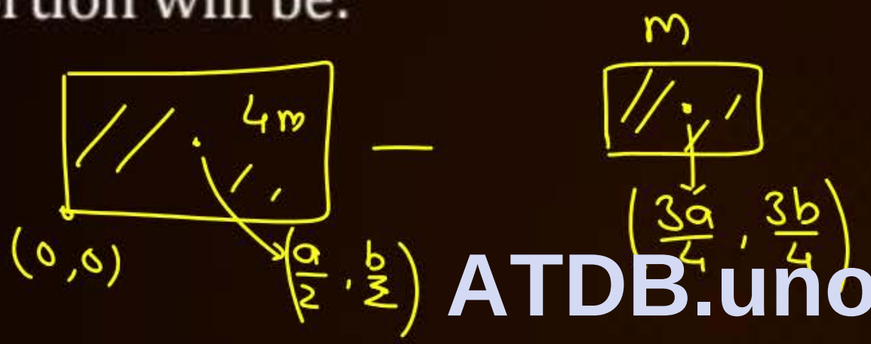


Question – 32

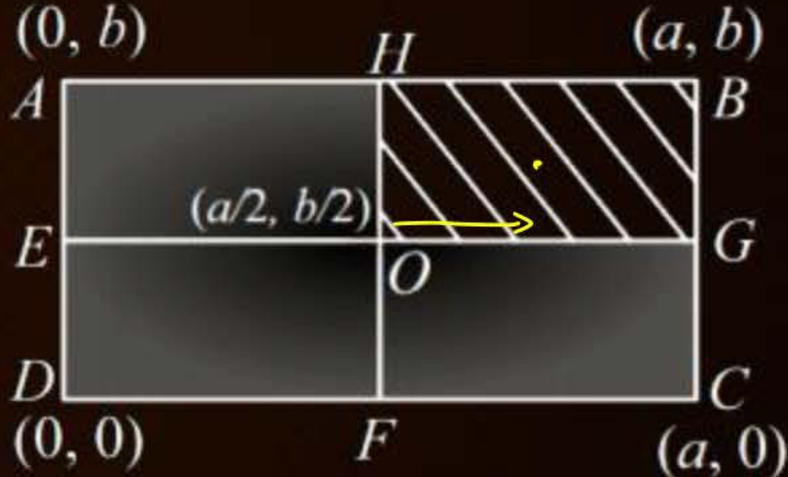
A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut off, the coordinates of the centre of mass of the remaining portion will be:

[8 April, 2019 (Shift-II)]

1. $\left(\frac{2a}{3}, \frac{2b}{3}\right)$
2. $\left(\frac{5a}{3}, \frac{5b}{3}\right)$
3. $\left(\frac{3a}{4}, \frac{3b}{4}\right)$
4. $\left(\frac{5a}{12}, \frac{5b}{12}\right)$ ✓



$$\frac{4m\left(\frac{a}{2}\right) - m\left(\frac{3a}{4}\right)}{8m} = \frac{(8-3)a}{12}$$

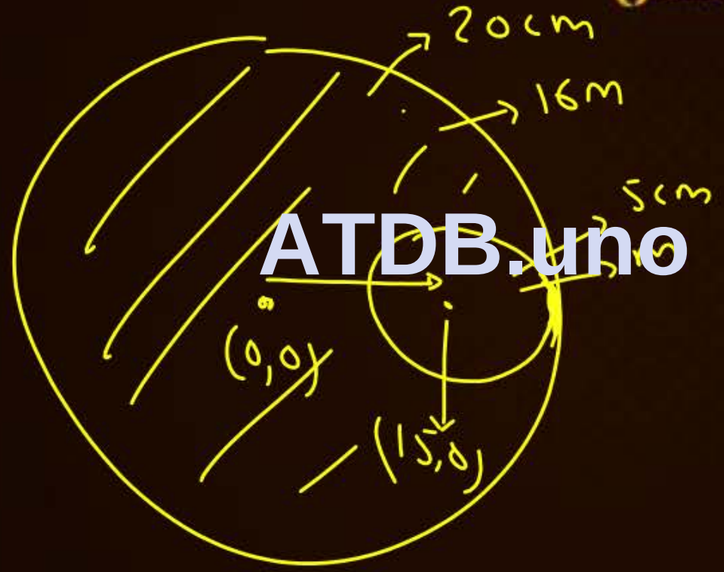


Question — 33

Consider a circular disc of radius 20 cm with center located at the origin. A circular hole of a radius 5 cm is cut from this disc in such a way that the edge of the hole touches the edge of the disc. The distance of center of mass of residual or remaining disc from the origin will be-

(January 2025)/ 23-01-2025/Morning Shift

1. 2.0 cm
2. 0.5 cm
3. 1.5 cm
4. 1.0 cm ✓



$$\frac{16m(0) - m(15)}{15m}$$

Question – 34

The center of mass of a thin rectangular plate (fig - x) with sides of length a and b, whose mass per unit area (σ) varies as $\sigma = \frac{\sigma_0 x}{ab}$ (where σ_0 is a constant), would be_____

(January 2025)/28-01-2025/Morning Shift)

1. $\left(\frac{2}{3}a, \frac{b}{2}\right)$
2. $\left(\frac{2}{3}a, \frac{2}{3}b\right)$
3. $\left(\frac{a}{2}, \frac{b}{2}\right)$
4. $\left(\frac{1}{3}a, \frac{b}{2}\right)$

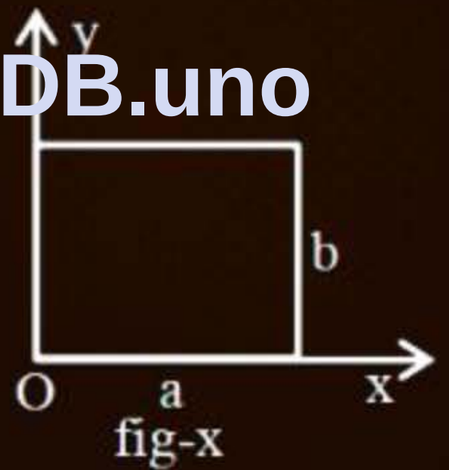


fig-x

$$\vec{F}_{ext.} = M\vec{a}_{cm}$$

$$P = M\vec{V}_{cm}$$

$$\vec{F}_{ext} = \frac{d\vec{P}}{dt}$$

Impulse

$$\vec{J} = \vec{P}_f - \vec{P}_i = \int \vec{F}\, dt$$

→ internal forces

(action + Reaction within System)

→ Internal forces Can't change momentum of System, a_{cm}, V_{cm}, motion of COM

→ internal forces Can change K.E., P.E., T.M.E

Momentum Conservation

if $\vec{F}_{ext} = 0$ then $\vec{P}_f = \vec{P}_i$ (System)

(inertial frame)

Mechanical Energy Conservation

if $W_{NC} = 0$ then $K_1 + U_1 = K_2 + U_2$

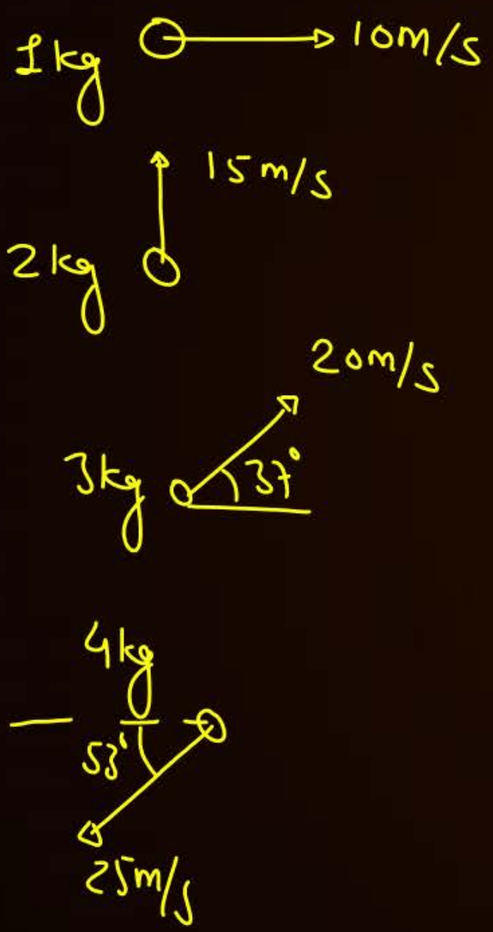
Work-Energy theorem $W_{net} = \Delta K.E$ (from inertial frame)

Bomb Explosion + Gun-Bullet + Cannon Shell

- due to internal forces
- Very Short time event
- Energy of explosion = $k \cdot \mathcal{E}_f - k \cdot \mathcal{E}_i$
- Explosion Can't change motion of COM

Question – 33

Find Velocity of center of mass.



$$V_{cm} = \frac{1(10) + 2(15) + 3(20) + 4(25)}{10}$$

$$\vec{V}_{cm} = \frac{1(10\hat{i}) + 2(15\hat{j}) + 3(16\hat{i} + 12\hat{j}) + 4(-15\hat{i} - 20\hat{j})}{10}$$

$$\vec{V}_{cm} = \frac{-2\hat{i} - 2\hat{j}}{10}$$

V, a, P → vector

K.E, P.E → Scalar

Question 38

External forces

Find acceleration of center of mass.

$$a = \frac{150-100}{25} = 2\,m/s^2$$

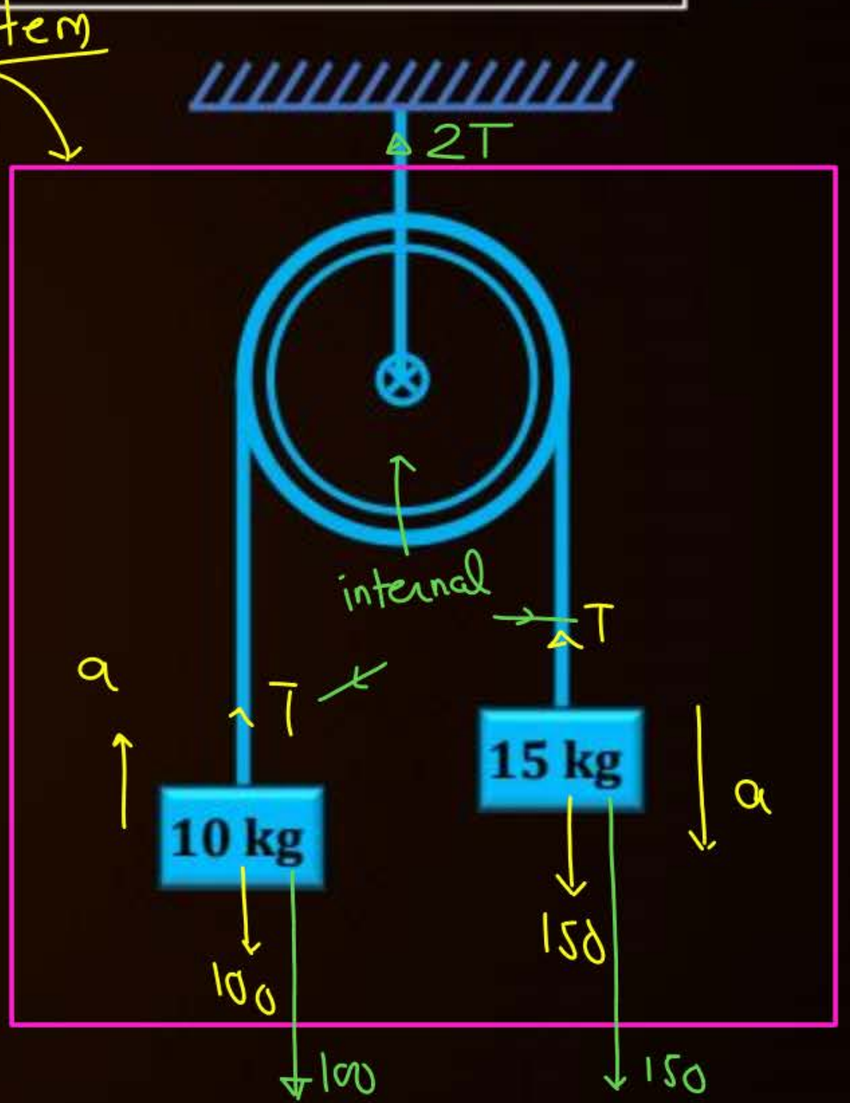
$$a_{cm} = \frac{15(2)+10(2)}{25} = 2\,m/s^2$$

$$\vec{a}_{cm} = \frac{15(-2\hat{j})+10(2\hat{j})}{25} = -\frac{2}{5}\hat{j}$$

$$f = ma \rightarrow T-100 = 10a = 10\times 2 \Rightarrow T = 120\,N$$

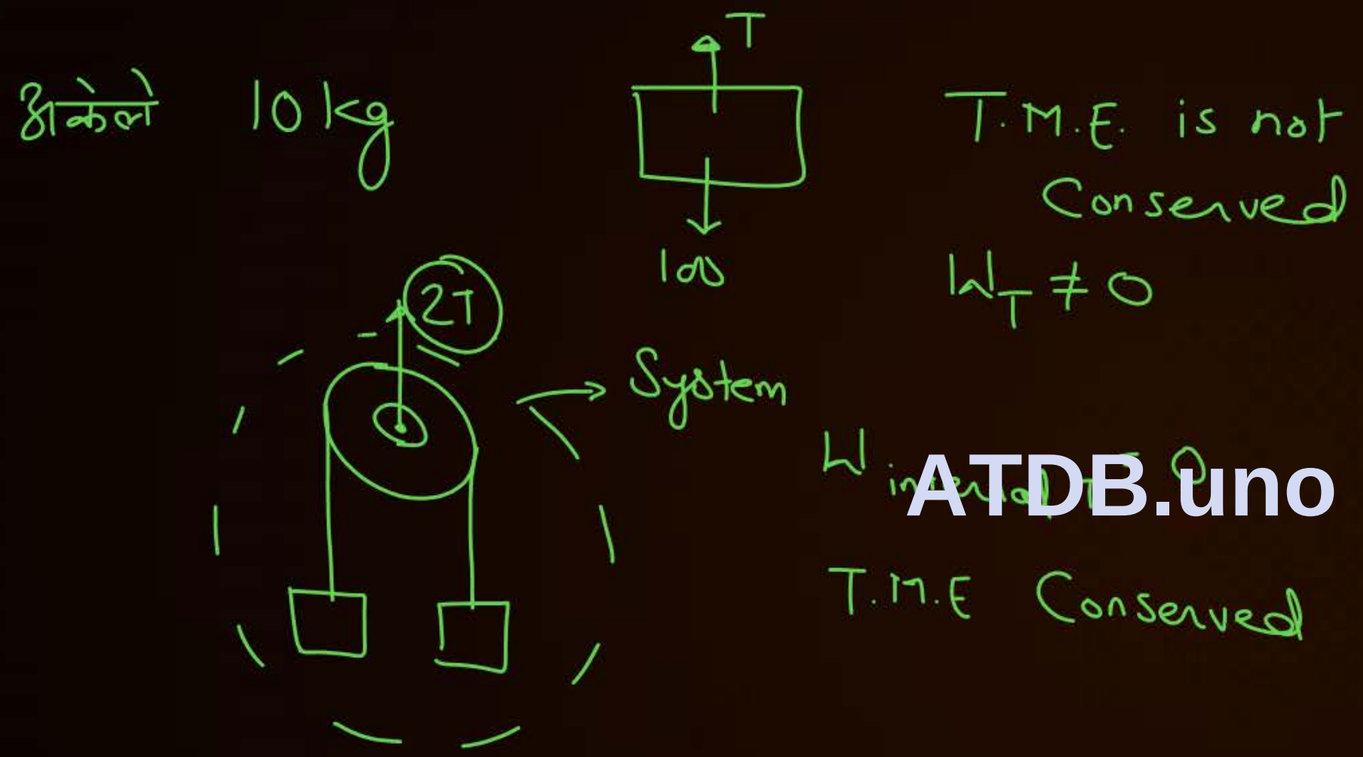
$$\vec{F}_{ext} = M\vec{a}_{cm} \Rightarrow 2T\hat{j} - (100+150)\hat{j} = 25\vec{a}_{cm}$$

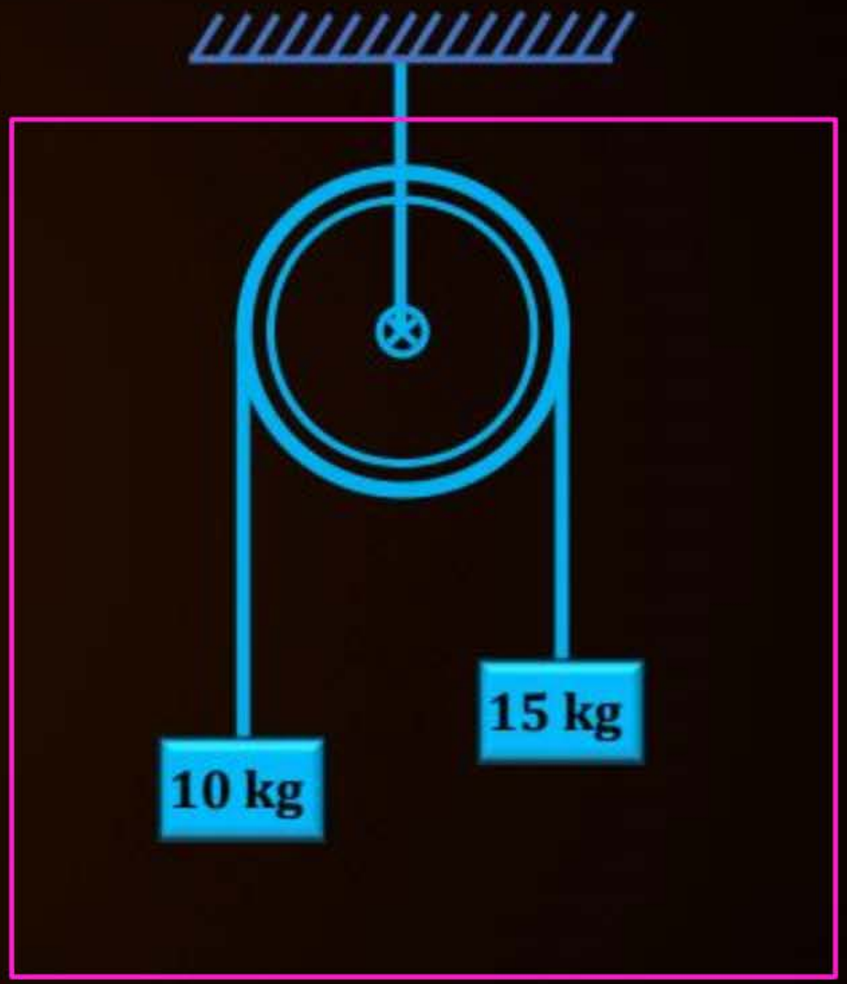
$$\vec{a}_{cm} = \frac{-10}{25}\hat{j}$$



Question 38

Find acceleration of center of mass.





internal force ⟶ Can change K.E.

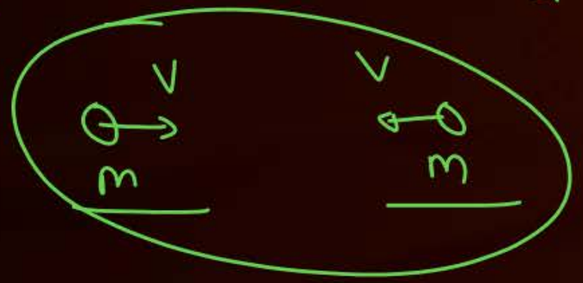
⟶ Can't change momentum

$\left(K.E. = \frac{p^2}{2m} \right)$ ⟶ for Particle

⟶ not for System

+q
m 0
u=0

−q
0 m
u=0

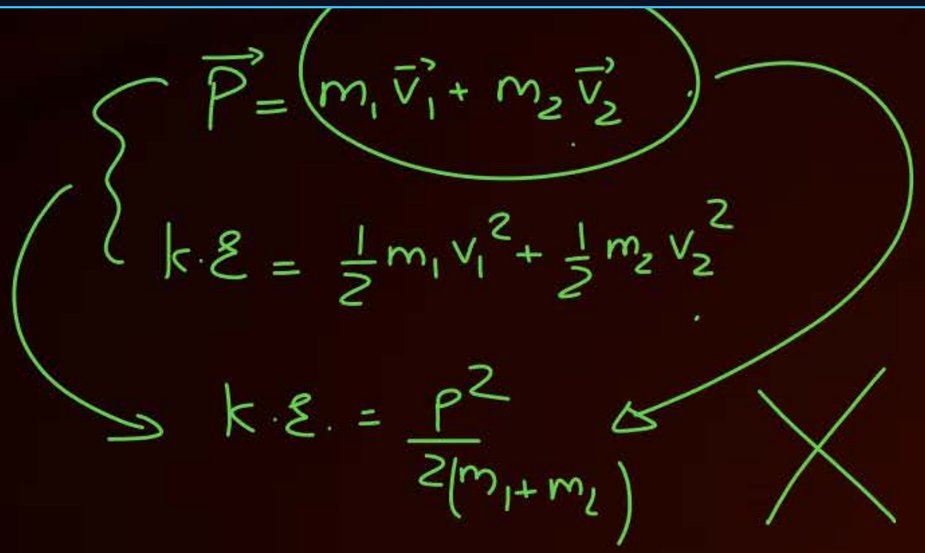


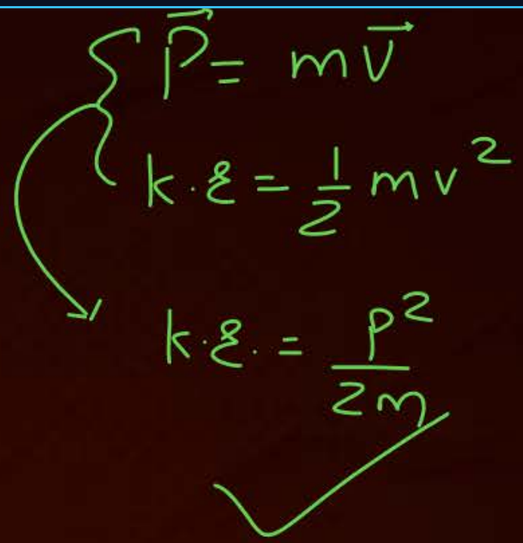
$\vec{P}_{total} = mv\hat{i} + mv(-\hat{i})$

$K.E_{total} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$

(is $(a+b) = a+b$

X $(a+b)^2 = a^2 + b^2$)

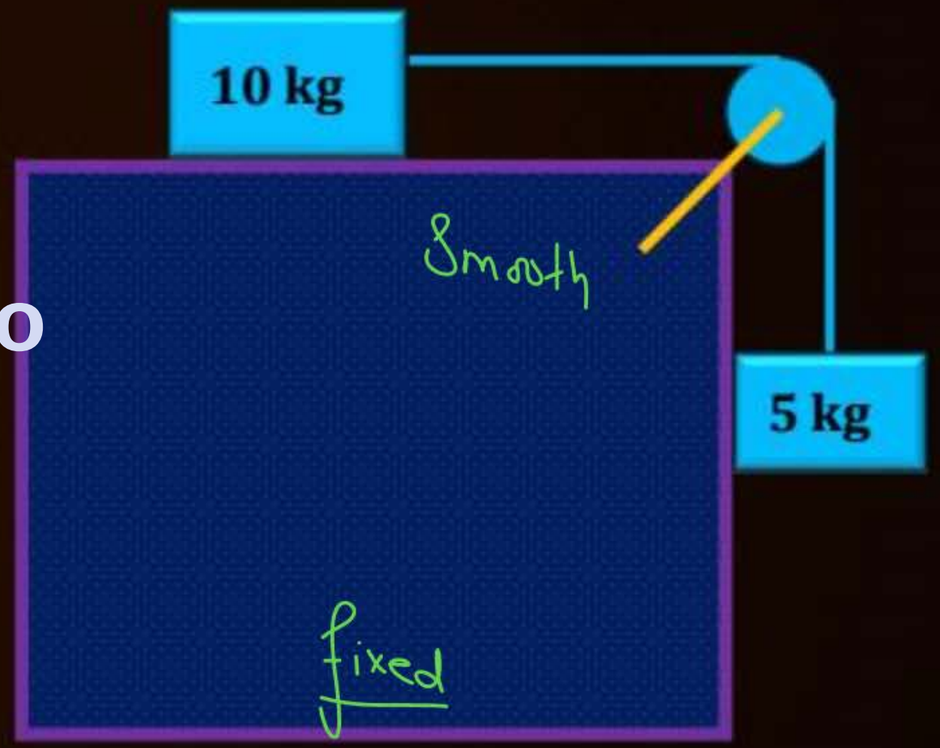




ATDB.uno

Question-

System is release from rest. Find acceleration of each block.



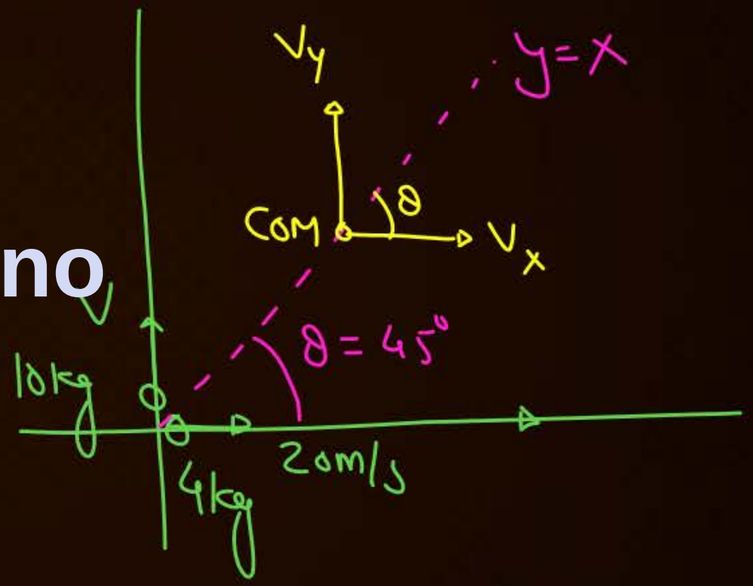
Question 58

A body of mass 4 kg moves along $+X$ direction at a speed of 20 m/s. Find the speed with which a 10 kg mass should move along $+Y$ direction such that their center of mass travels along line $x = y$. Assume both particle start at the same time from origin.

$$\vec{V}_{cm} = \frac{4(20\hat{i}) + 10(v\hat{j})}{14}$$

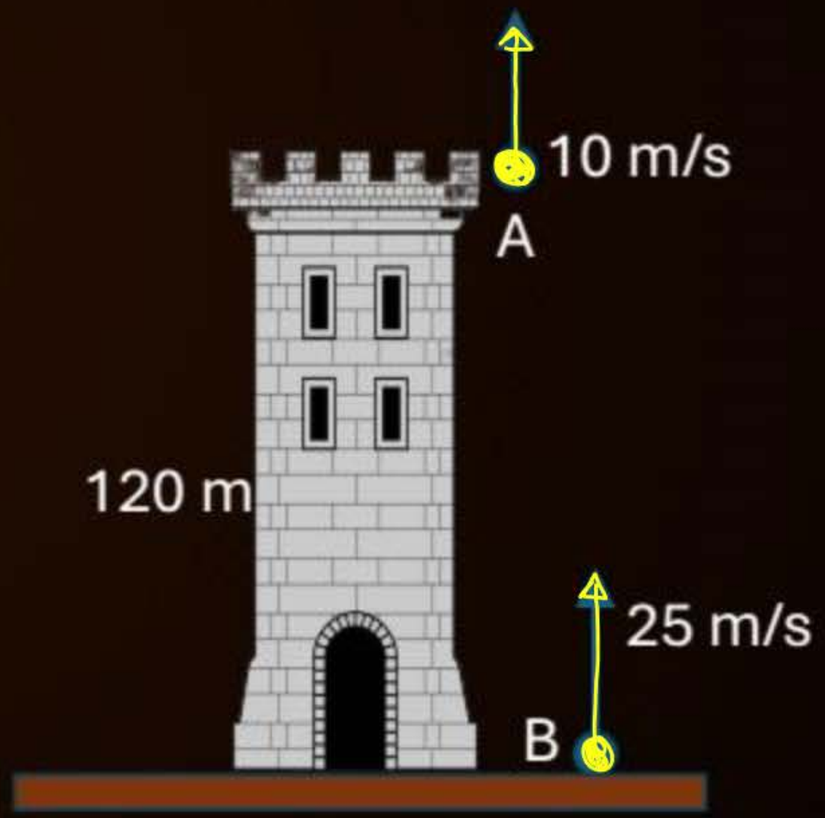
$$\tan 45^\circ = \frac{V_y}{V_x} = \frac{10v}{80} = 1$$

$$v = 8 \text{ m/s}$$



Question 39

Two balls A and B of mass m and 2m are projected up as shown.
Find (1) initial velocity and height of COM
(2) find maximum height of COM



Question – 90

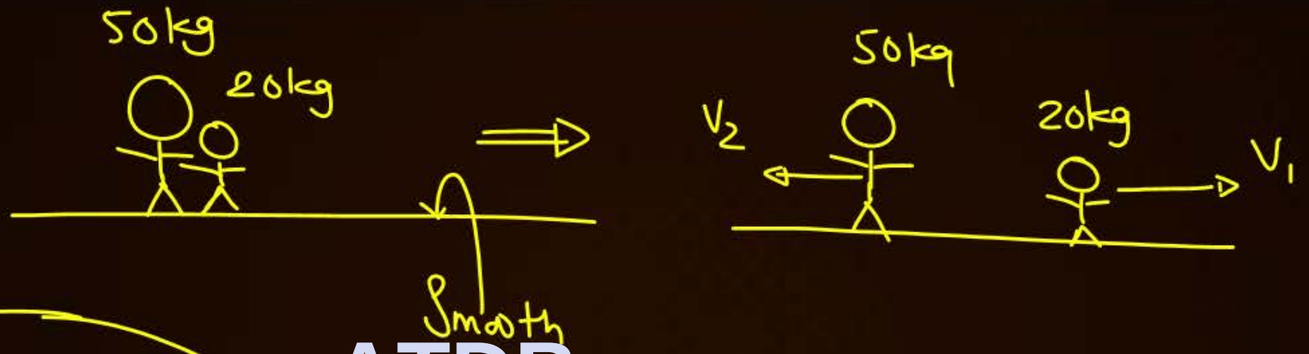
A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son, so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is **(2019)**

a. 0.28 m s^{-1}

b. 0.20 m s^{-1}

c. 0.47 m s^{-1}

d. 0.14 m s^{-1}



$\vec{V}_{b/m} = \vec{V}_b - \vec{V}_m$

$= V_1\hat{i} - (-V_2\hat{i})$

$= V_1 + V_2 = 0.7$ ---(i)

$(F_{ext})_x = 0$ (System → boy + man)

$\vec{P}_f = \vec{P}_i \Rightarrow 20V_1 - 50V_2 = 0 \Rightarrow 2V_1 = 5V_2$

$\Rightarrow$ eq (i) $\quad \frac{5V_2}{2} + V_2 = 0.7 \Rightarrow \frac{7V_2}{2} = 0.7 \Rightarrow V_2 = 0.2\ m/s$

Ans: (b)

Question – 90

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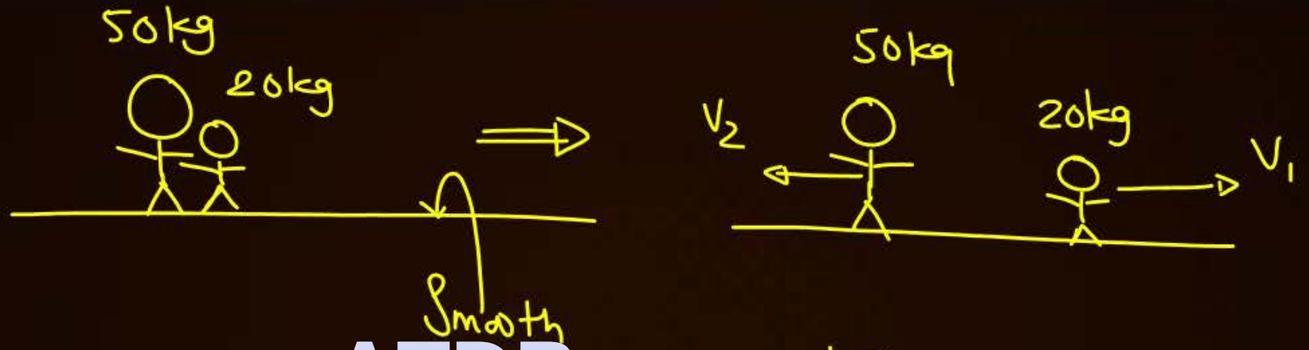


Mechanical energy is not Conserved

System (boy + man)

$M.E_i = (0+0)+(0+0)$ $\quad M.E_f = \frac{1}{2}20(v_1)^2+\frac{1}{2}(50)v_2^2+(0+0)$

(internal energy of man spend) (muscular force)

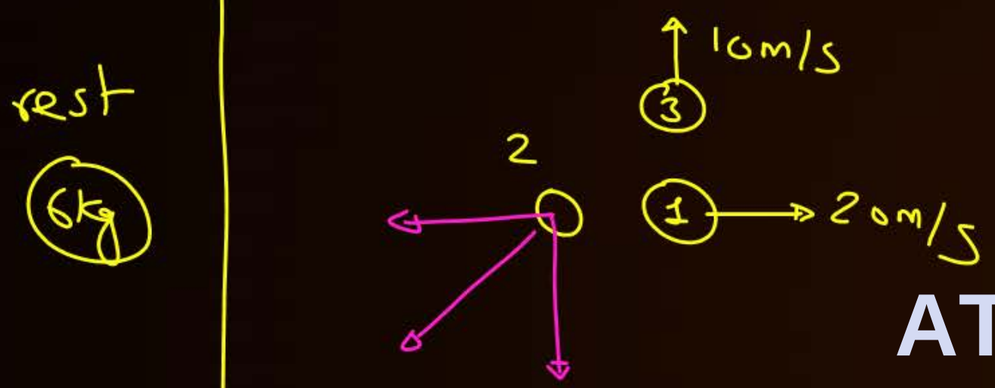
Ans: (b)

Question 91

Three interacting particles of masses 1 kg, 2kg and 3kg each have a velocity of 20 m/s magnitude along the positive direction of x-axis, y-axis and z-axis. Due to force of interaction the third particle stops moving. The velocity of the second particle is $20\hat{\imath} + 5\hat{\jmath} - 10\hat{k}$. What is the velocity of the first particle?

Question 92 ... of mass ... g

A bomb is placed at origin explodes into three fragments in mass ratio 1 : 2 : 3. If lightest particle move along $+X$ with 20 m/s, heaviest particle move along $+Y$ with 10 m/s. Find velocity of remaining part. and energy of explosion



$F_{ext.} = 0$

$\vec{P}_f = \vec{P}_i$

$1(20\hat{i}) + 3(10\hat{j}) + 2(\vec{v}) = 0$

$\vec{v} = -10\hat{i} - 15\hat{j}$

$V = \sqrt{100 + 225}$

Δ energy of explosion $= K.E_f - K.E_i = \frac{1}{2}1(20)^2 + \frac{1}{2}3(10)^2 + \frac{1}{2}2(325) - 0$