PRAYAS

JEE 2026

Physics

COM and System of particles

Lecture - 07

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Topics to be covered

A Momentum Conservation

B

C

D

ATDB.uno

Question

The center of mass of a thin rectangular plate (fig - x) with sides of length a and b, whose mass per unit area (σ) varies as $\sigma = \frac{\sigma_0 x}{ab}$ (where σ_0 is a constant), would be_____

(January 2025)/28-01-2025/Morning Shift)

1. $\left(\frac{2}{3}a, \frac{b}{2}\right)$
2. $\left(\frac{2}{3}a, \frac{2}{3}b\right)$
3. $\left(\frac{a}{2}, \frac{b}{2}\right)$
4. $\left(\frac{1}{3}a, \frac{b}{2}\right)$

$$X_{cm} = \frac{\int x\,dm}{\int dm}$$

$$dm = \sigma\,dA = \left(\frac{\sigma_0 x}{ab}\right)(b\,dx) = \frac{\sigma_0 x\,dx}{a}$$

$$= \frac{\int_0^a \frac{\sigma_0}{a} x^2\,dx}{\int \frac{\sigma_0}{a} x\,dx} = \frac{a^3/3}{a^2/2}$$

$$= \frac{2a}{3}$$

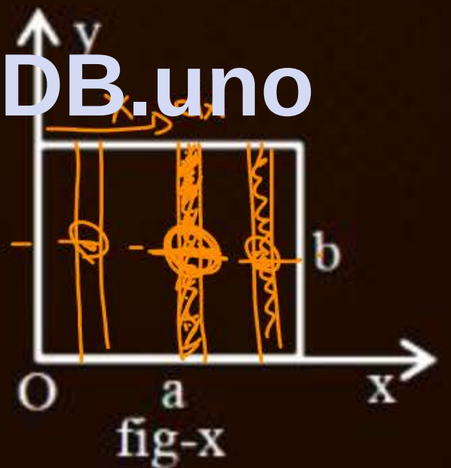


fig-x

$$y_{cm} = \frac{b}{2}$$

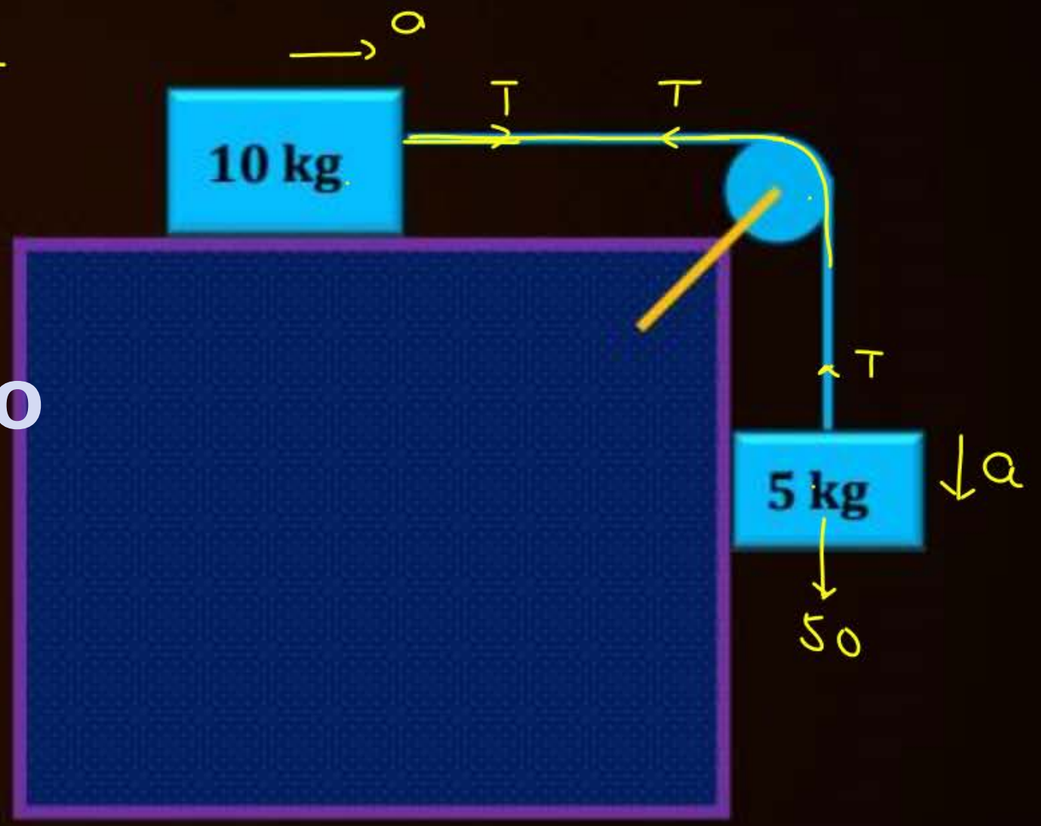
Question-6

System is release from rest. Find acceleration of each block.

$50 - T = 5a$
$T = 10a$
$\Rightarrow a = \frac{10}{3}\ m/s^2$

$\vec{a}_{cm} = \frac{10\left(\frac{10}{3}\hat{i}\right) + 5\left(-\frac{10}{3}\hat{j}\right)}{15} \neq 0$

$=$



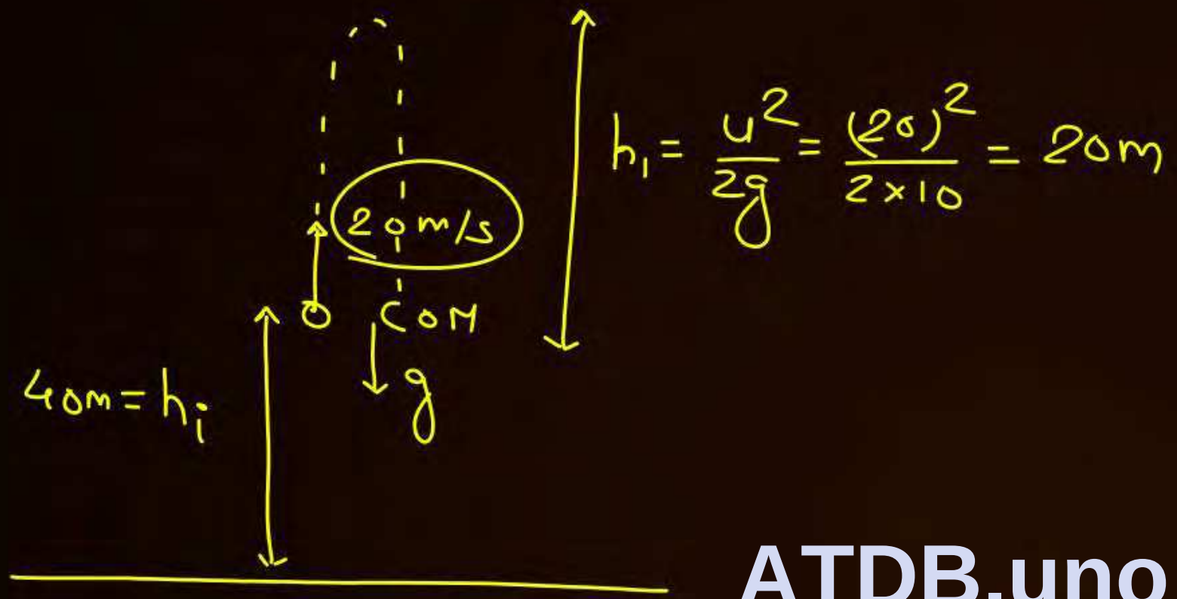
Question 35

Two balls A and B of mass m and 2m are projected up as shown.
Find (1) initial velocity and height of COM
(2) find maximum height of COM

(1) $u_{cm} = \frac{m(10) + 2m(25)}{3m} = 20\ m/s$

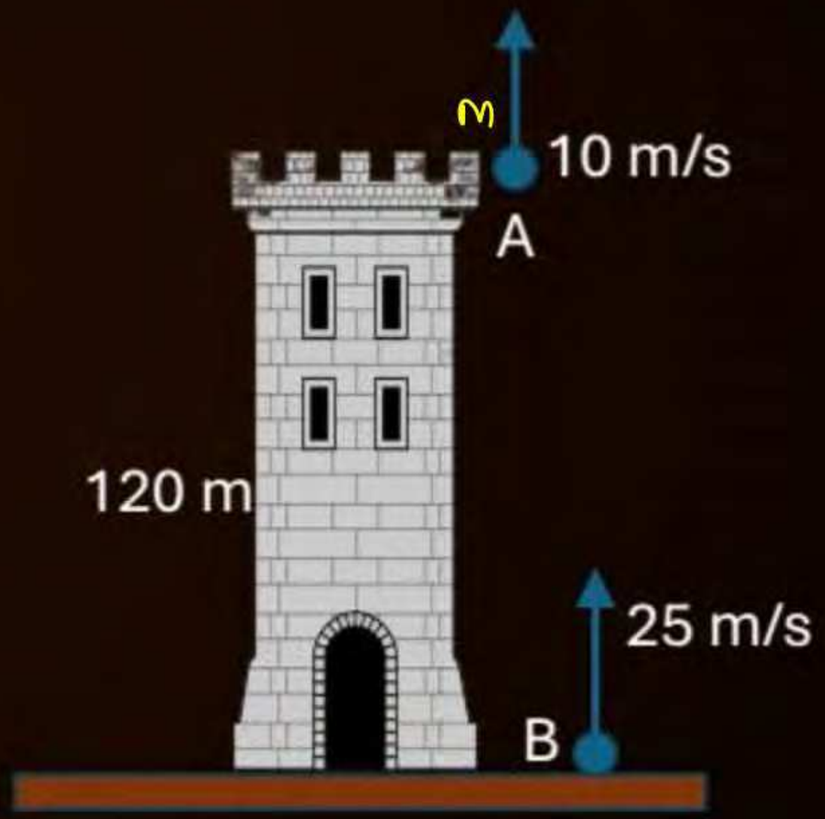
$h_{i)_{cm}} = \frac{m(120) + 2m(0)}{3m} = 40\ m$

$a_{cm} = \frac{m(-g) + 2m(-g)}{3m} = -g$ (till both balls are in air)



$$h_1 = \frac{u^2}{2g} = \frac{(20)^2}{2\times 10} = 20\,m$$

$$\text{max}^m\ H = 40 + 20 = 60\,m$$



Question

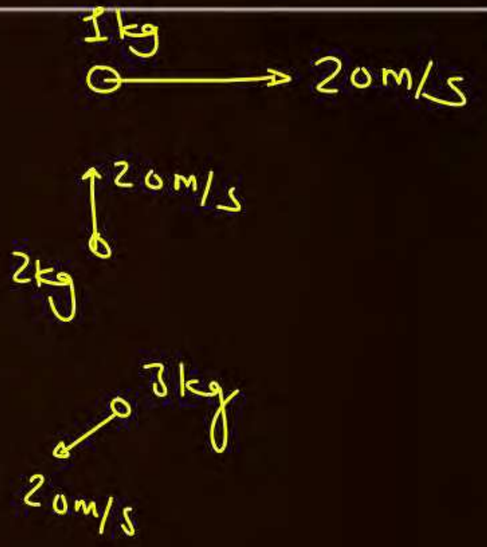
Three interacting particles of masses 1 kg, 2kg and 3kg each have a velocity of 20 m/s magnitude along the positive direction of x-axis, y-axis and z-axis. Due to force of interaction the third particle stops moving. The velocity of the second particle is $20\hat{i} + 5\hat{j} - 10\hat{k}$. What is the velocity of the first particle?

internal forces

No ext. force

$$\vec{P}_f = \vec{P}_i$$

$$3(0) + 2(20\hat{i} + 5\hat{j} - 10\hat{k}) + 1\vec{V} = 1(20\hat{i}) + 2(20\hat{j}) + 3(20\hat{k})$$



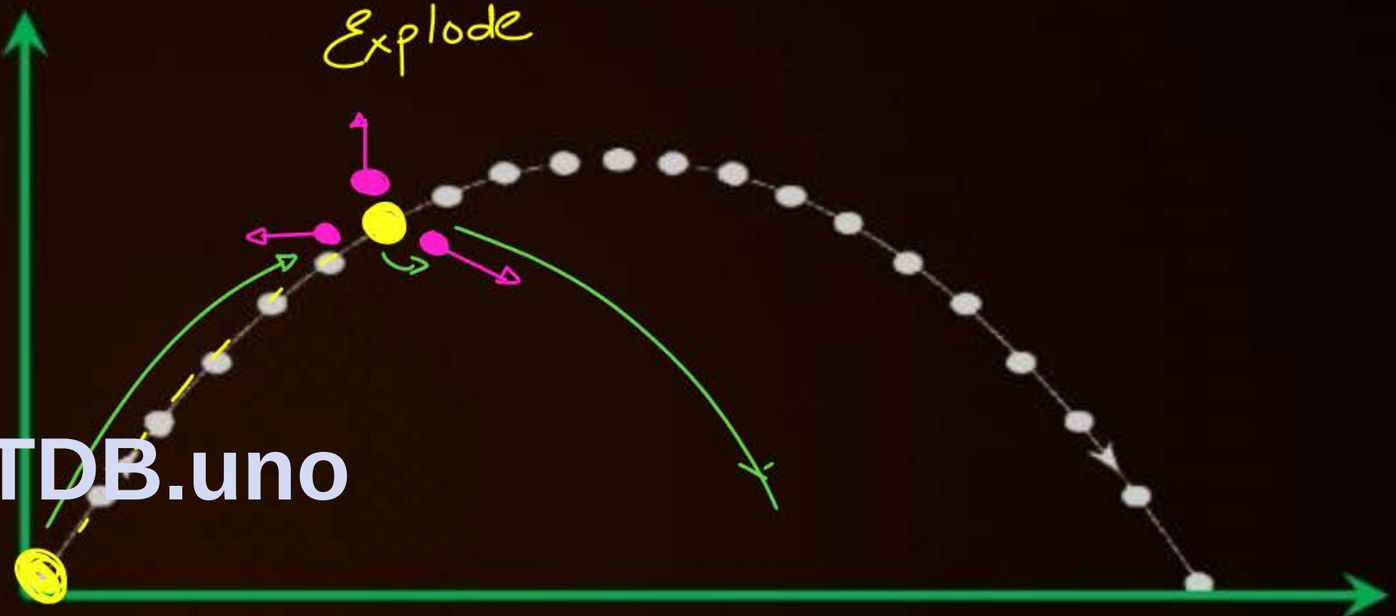
→ just before and just after Collision/explosion position of bomb and it's parts are Same

→ $F_{ext.} \longrightarrow mg$

$$\text{Impulse} = \vec{P}_f - \vec{P}_i = \int F\,dt$$

$$= mg(t) \rightarrow \text{very small}$$

We ignore effect of mg during explosion



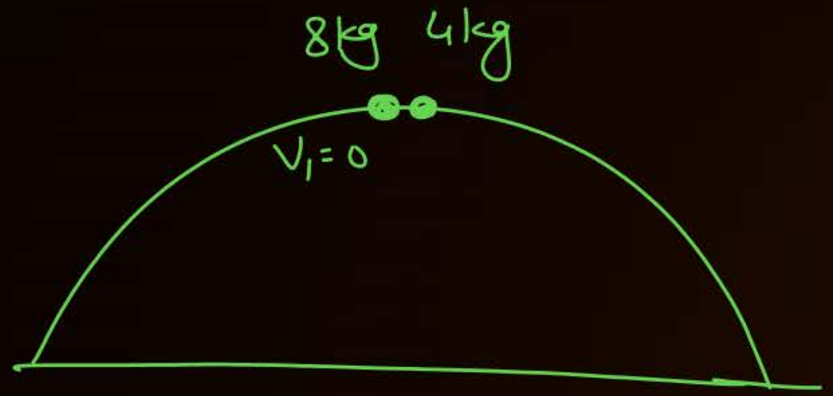
Question 93

A 12 kg projectile is fired at an angle of 37° above the horizontal with an initial speed of 50 m/s. At the highest point in its trajectory, the projectile explodes into two fragments of 8kg and 4kg mass, the first of which falls vertically with zero initial speed.

(i) How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)

(ii) How much energy was released during the explosion?

Just after :-

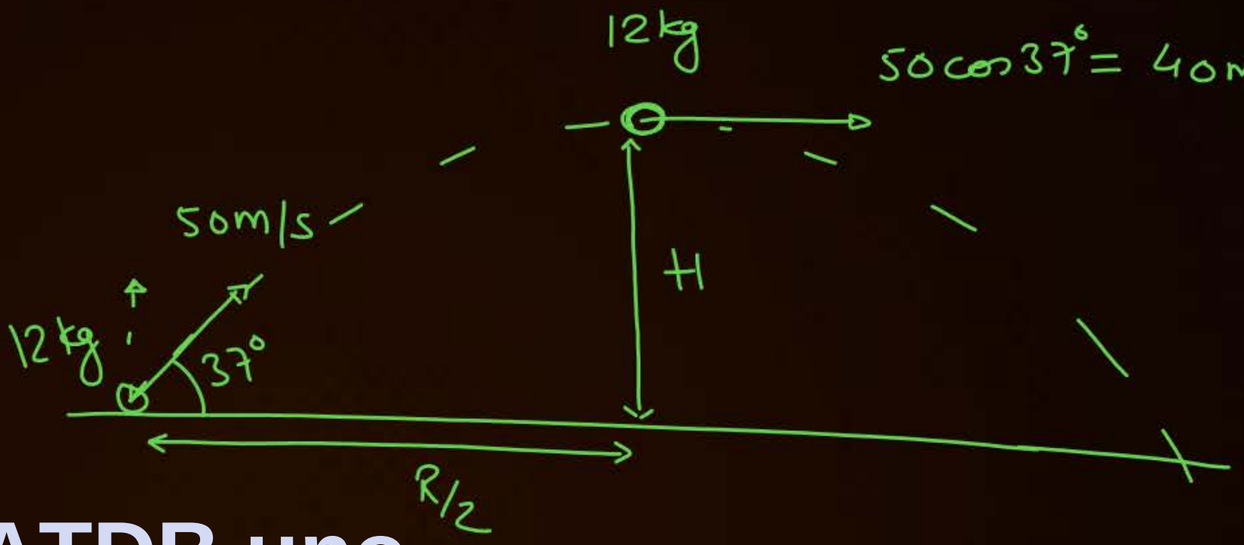


$F_{ext.} = 0$ during explosion

$$\vec{P}_i = \vec{P}_f \Rightarrow (12)(40\hat{i}) = 8(0) + 4(\vec{V}_2)$$

$$V_2 = 120\hat{i}$$

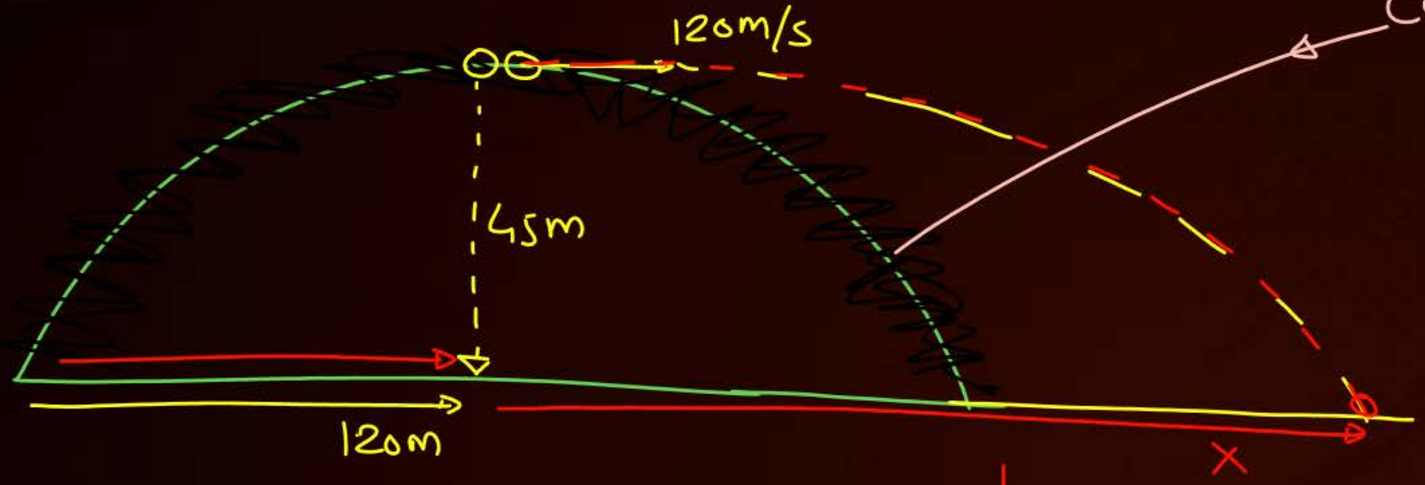
Just before



$$H = \frac{(30)^2}{2g} = 45\text{m}$$

$$\frac{R}{2} = \frac{2(40)(30)}{2(10)} = 120\text{m}$$

Horizontal projectile



Path of COM

(internal force can't change motion of COM)

4kg :→ $S = ut + \frac{1}{2}at^2$

$-45 = \frac{1}{2}(-10)t^2$

$t = 3\,\text{sec}$

$t = \sqrt{\frac{2h}{g}}$

$X = 120 \times 3$

$= 360$

from initial position

$120 + 360 = 480\text{m}$

Energy of explosion

$= k \cdot \mathcal{E}_f - k \cdot \mathcal{E}_i$ (just after and before)

$= \frac{1}{2}8(0)^2 + \frac{1}{2}4(120)^2 - \frac{1}{2} \times 12 \times (40)^2$

$=$

Ex. at top most point.

$12\,kg \longrightarrow 8\,kg + 4\,kg$

$8\,kg \rightarrow$ initial velocity

$= 20\hat{i} + 10\hat{j}$

find initial velocity of 4kg

---

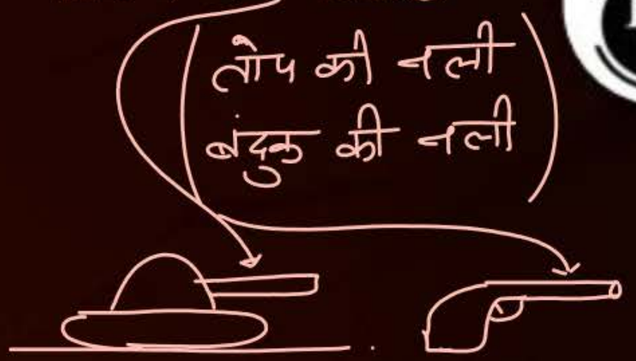
during explosion $f_{ext} = 0$

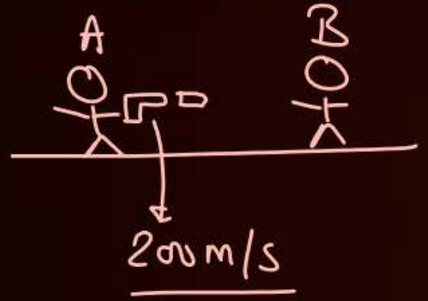
$$\vec{P}_f = \vec{P}_i$$

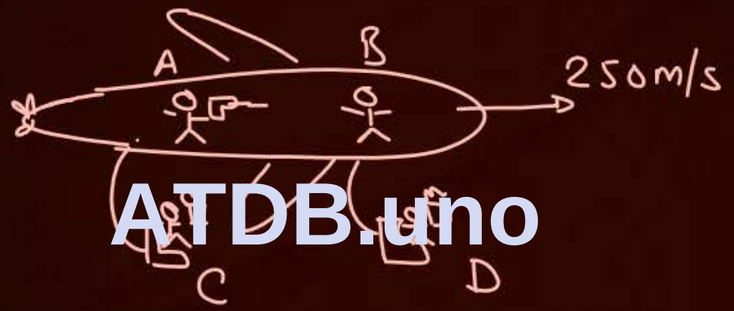
$$8(20\hat{i} + 10\hat{j}) + 4\vec{v}_2 = 12(40\hat{i})$$

muzzle Velocity :→ velocity of bullet/Shell w.r.t muzzle

$$\vec{V}_{muzzle} = \vec{V}_{Shell/Cannon} \quad \text{bullet/Gun}$$







250

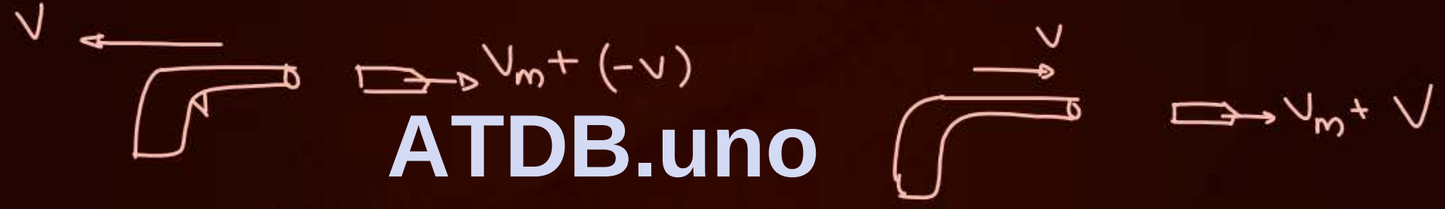
250+200

250

$$\vec{V}_{muzzle} = \vec{V}_{Shell/Cannon}$$

$$= \vec{V}_{Shell} - \vec{V}_{Cannon}$$

$$\vec{V}_{Shell} = \vec{V}_{muzzle} + \vec{V}_{Cannon}$$



Just after firing

→ Velocity of bullet = muzzle Velocity + Vectorially Velocity of Gun

Question

A tank a total mass of 500kg horizontal fire shell of mass 25kg with muzzle velocity 100m/s. Tank is initially at rest and is on smooth horizontal surface. Find speed of shell and tank after firing

$f_{ext.} = 0$

$\vec{P}_f = \vec{P}_i \Rightarrow 25(100-v) - 475(v) = 0$

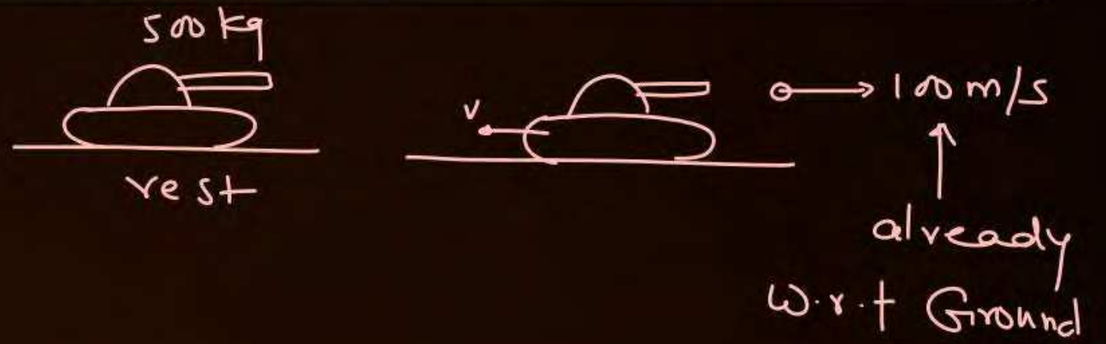
$V = \frac{2500}{500} = 5\ m/s$

$\vec{V}_{tank} = -5\hat{i}$ $\quad V_{Shell} = (100-5)\hat{i}$

Sir ने कहा था momentum Coservation ground frame से लगायेंगे मतलब सारी Velocity Ground frame में लाने है।

Question #45

A tank a total mass of 500kg horizontal fire shell of mass 25kg with [redacted] velocity 100m/s. Tank is initially at rest and is on smooth horizontal surface. Find speed of shell and tank after firing



$$P_f = P_i$$

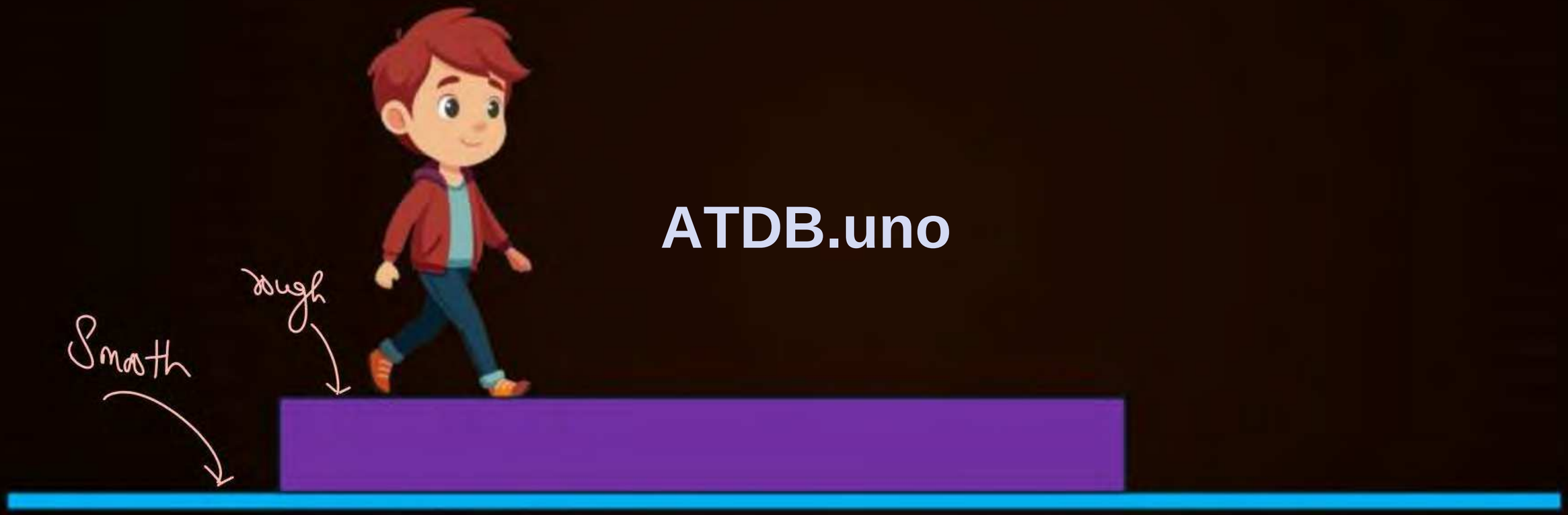
$$25(100) - 475V = 0$$

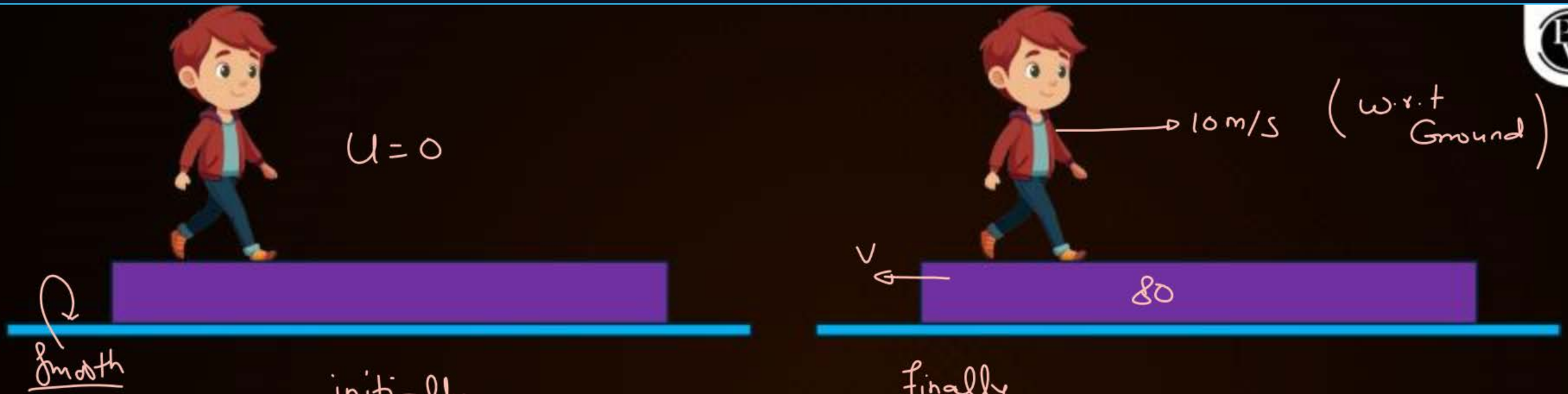
Question

A tank a total mass of 500kg horizontal fire 2 shells of each of mass 25kg with muzzle velocity 100m/s. Tank is initially at rest and is on smooth horizontal surface. Find speed of tank after firing both shells

Question 97

A boy of mass 20 kg is standing on a plank of mass 80kg. Plank is kept on a smooth horizontal surface. Initially system was at rest. Now boy start moving on plank with speed 10m/s w.r.t. ground. Find speed of plank





initially

finally

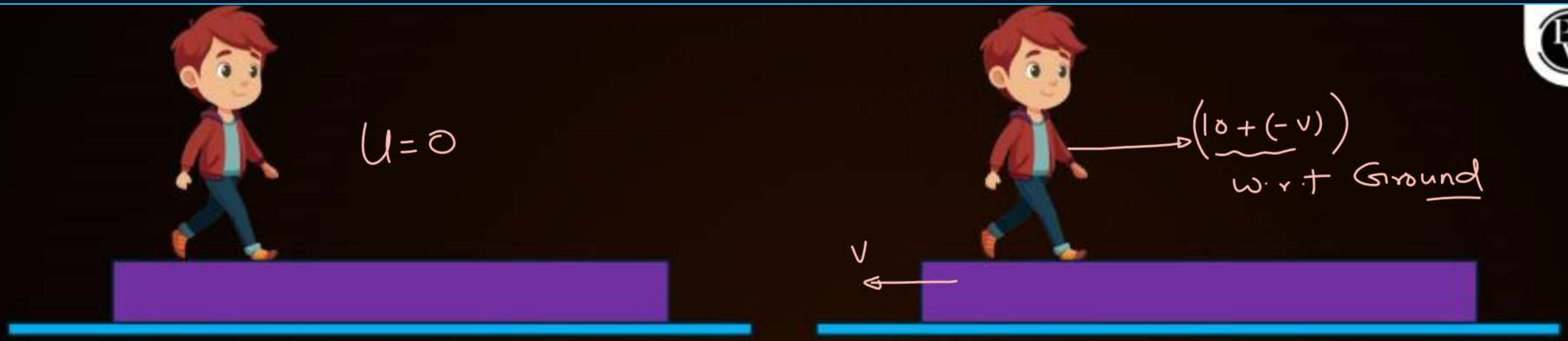
System (boy + plank) $\longrightarrow$ $F_{ext})_x = 0 \Rightarrow \vec{P}_f)_x = \vec{P}_i)_x$

$20(10) - 80V = 0 \Rightarrow V = \frac{200}{80} = \frac{5}{2}$ m/s

Question # 98

A boy of mass 20 kg is standing on a plank of mass 80kg. Plank is kept on a smooth horizontal surface. Initially system was at rest. Now boy start moving on plank with speed 10m/s w.r.t. plank. Find speed of plank

bhaiya Game हो गया



initially $(P_i)_x = 0$

$$\vec{P}_f = \vec{P}_i$$

$$20(10-v) - 80v = 0$$

$$v = 2\ m/s$$

$$V_{boy} = 10 - 2 = 8\ m/s$$

Question

A uniform thin rod of mass M and length L is standing vertically along the Y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at $t = 0$ causes the lower end to slip on the smooth surface along the positive X-axis, and the rod starts falling.

(A) What is the path followed by the center of mass of the rod during its fall?

(B) Find the equation of the trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point? **(1993)**

Question

A shell is fired from a cannon with a speed of 100 m/s at an angle 30° with the vertical (y-direction). At the highest point of its trajectory, the shell explodes into two fragments of masses in the ratio 1 : 2. The lighter fragment moves vertically upwards with an initial speed of 200 m/s. What is the speed of the heavier fragment at the time of explosion?