



PRAAYAS

JEE 2026

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Physics

COM and System of
particles

Lecture - 13

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Physics Wallah



Topics to be covered

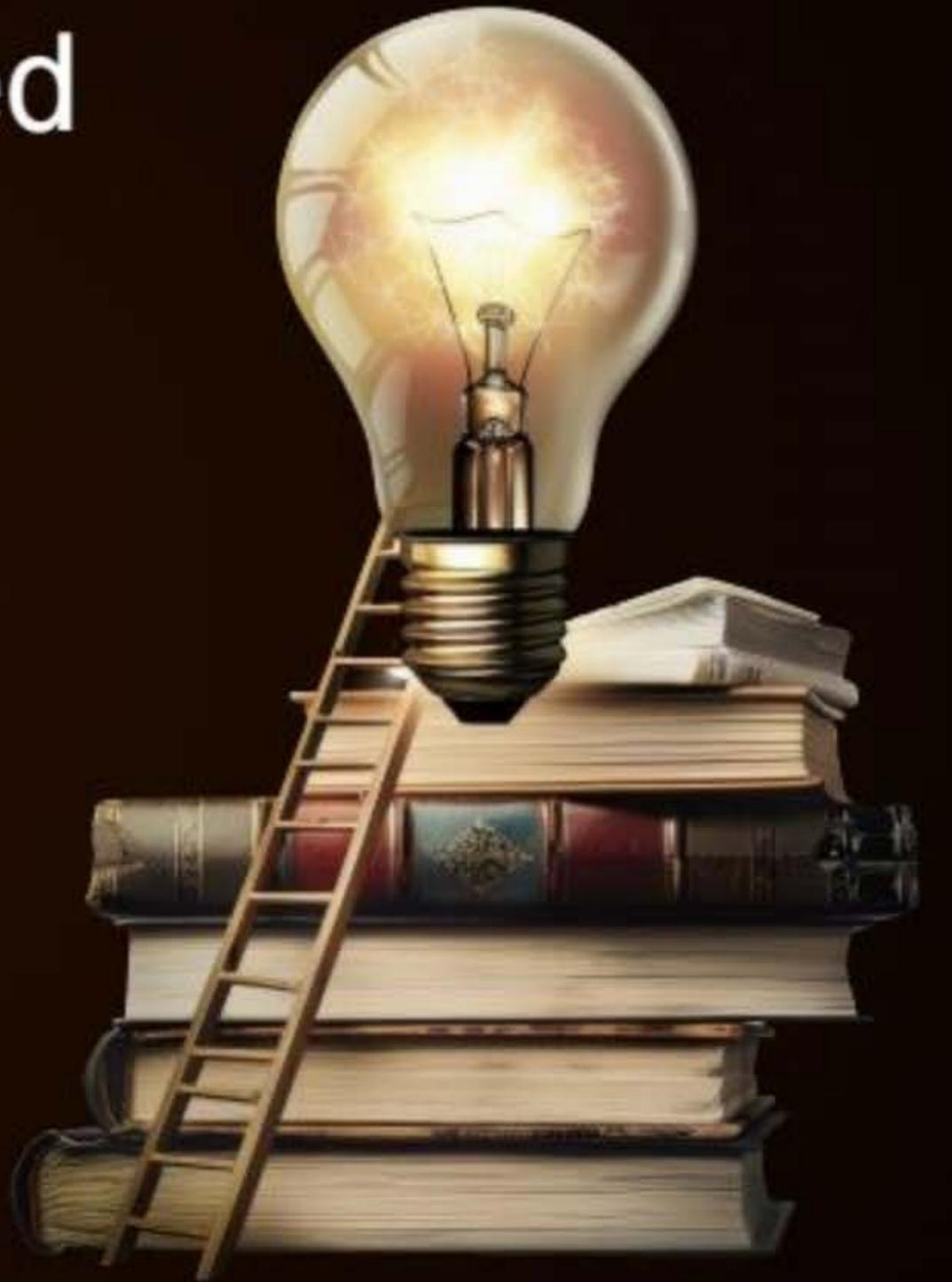
A Variable mass system

B

C

D

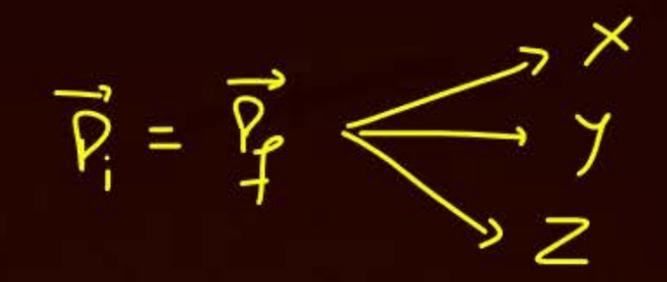
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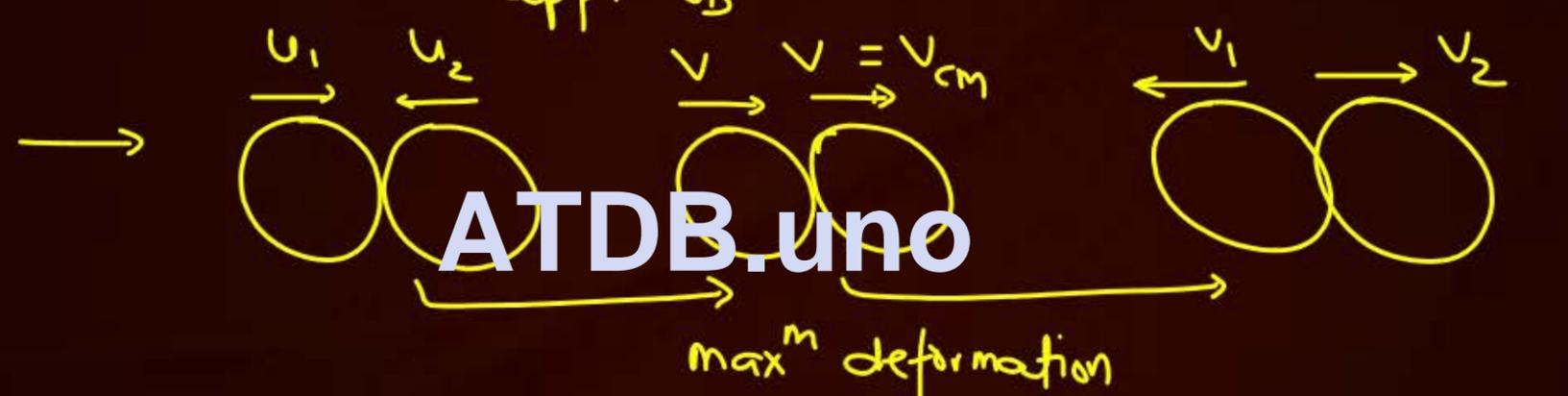
Tools

→ $f_{ext.} = 0$
(Impulsive)



→ $e = \frac{v_{sep.}}{v_{app.}} = \frac{J_R}{J_D}$ along LOI

$e = 1$



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→ Collision से just पहले and just बादका diagram

Question

63



A ball of mass m is moving towards another stationary ball of mass $2m$ as shown. Find velocity of each ball and total loss in kinetic energy after collision for

1. Elastic collision
2. Inelastic collision ($e = 0.5$)
3. Perfectly inelastic collision



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Smooth

$$\rightarrow \vec{P}_i = \vec{P}_f \Rightarrow mu + 0 = mv_1 + 2mv_2 \Rightarrow 2v_2 + v_1 = u \quad \text{---(i)}$$

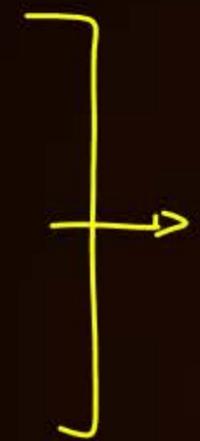
$$\rightarrow e = \frac{v_{sep.}}{v_{app.}} \Rightarrow \frac{v_2 - v_1}{u} = e \Rightarrow v_2 - v_1 = eu \quad \text{---(ii)}$$

$$\Rightarrow 3v_2 = (e+1)u \Rightarrow v_2 = \frac{(e+1)u}{3}, \quad v_1 = \frac{u(1-2e)}{3}$$



$$V_2 = \frac{u(1+e)}{3}$$

$$V_1 = \frac{u(1-2e)}{3}$$

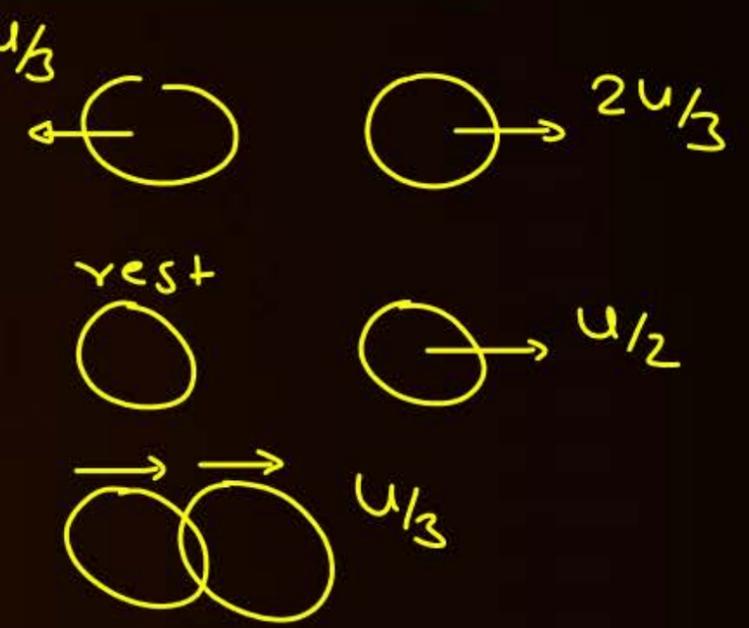


$$e = 1 \quad V_2 = \frac{2u}{3}, \quad V_1 = -\frac{u}{3}$$

$$e = \frac{1}{2} \quad V_2 = \frac{u}{2}, \quad V_1 = 0$$

$$e = 0 \quad V_2 = \frac{u}{3}, \quad V_1 = \frac{u}{3}$$

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Question



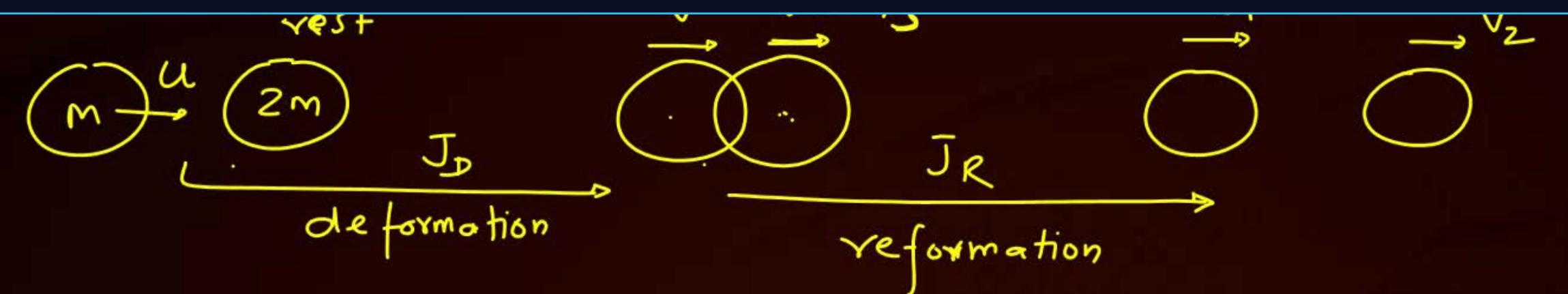
A ball of mass m is moving towards another stationary ball of mass $2m$ as shown. Find impulse of deformation and reformation for $\left(\text{max}^m \text{ deformation energy, loss in k.E.} \right)$

1. Elastic collision
2. Inelastic collision ($e = 0.5$)
3. Perfectly inelastic collision



$$V_{cm} = \frac{mu + 2m(0)}{3m} = \frac{u}{3}$$

Constant



→ Max^m deformation energy and J_D are same for every value of e $0 \leq e \leq 1$
 (M.D.E.)

$$\text{MDE} = k \cdot \xi_1 - k \cdot \xi_2 = \frac{1}{2} m u^2 - \left(\frac{1}{2} 3m \left(\frac{u}{3} \right)^2 \right) = \frac{1}{2} m u^2 \left(1 - \frac{1}{3} \right) = \frac{1}{3} m u^2$$

→ $J_D = \vec{p}_f - \vec{p}_i = 2m \left(\frac{u}{3} \right) - 2m(0) = \frac{2mu}{3}$ Same in all cases

$$e = \frac{J_R}{J_D} \Rightarrow J_R = e J_D = \frac{2mu e}{3}$$

- $e=1$ $2mu/3$
- $e=0.5$ $mu/3$
- $e=0$ 0

$$\text{Loss in k.e.} = k \cdot \xi_1 - k \cdot \xi_3 = \frac{1}{2} m(u)^2 - \left(\frac{1}{2} m v_1^2 + \frac{1}{2} 2m v_2^2 \right)$$



$$e=1 \quad \text{Loss} = 0 = \left(\frac{1}{2} m u^2 - \left(\frac{1}{2} m \left(\frac{u}{3} \right)^2 + \frac{1}{2} 2m \left(\frac{2u}{3} \right)^2 \right) \right)$$



$$e = \frac{1}{2} \quad \text{Loss} = \frac{1}{2} m u^2 - \left(\frac{1}{2} m (0)^2 + \frac{1}{2} 2m \left(\frac{u}{2} \right)^2 \right) = \frac{1}{2} m u^2 - \frac{1}{2} m u^2 \left(\frac{1}{2} \right) = \frac{m u^2}{4}$$

$$e=0 \quad \text{loss} = \frac{1}{2} m u^2 - \left(\frac{1}{2} m \left(\frac{u}{3} \right)^2 + \frac{1}{2} 2m \left(\frac{u}{3} \right)^2 \right) = \frac{1}{2} m u^2 - \frac{m u^2}{6} = \frac{m u^2}{3}$$

\uparrow
max^m loss

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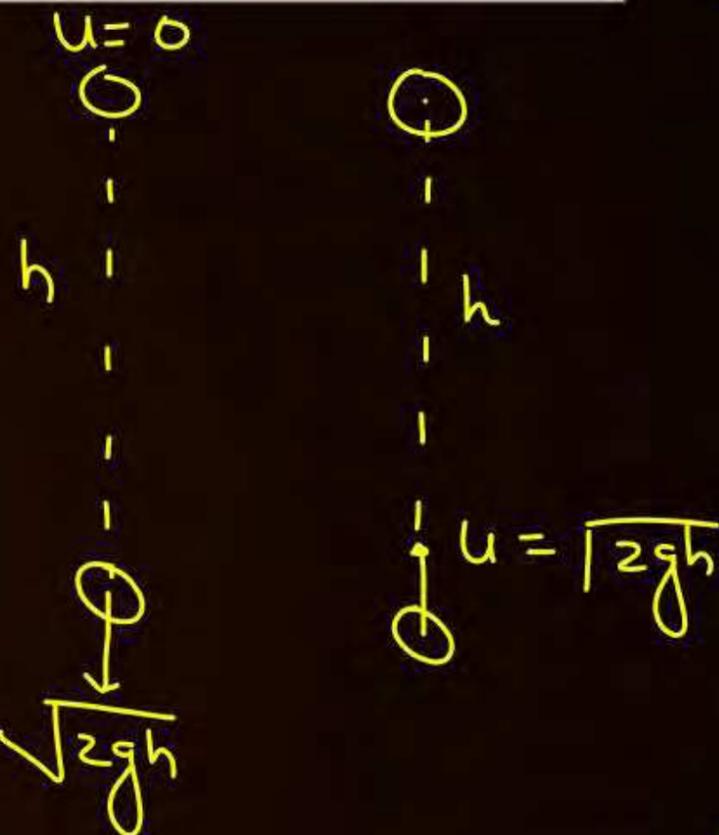
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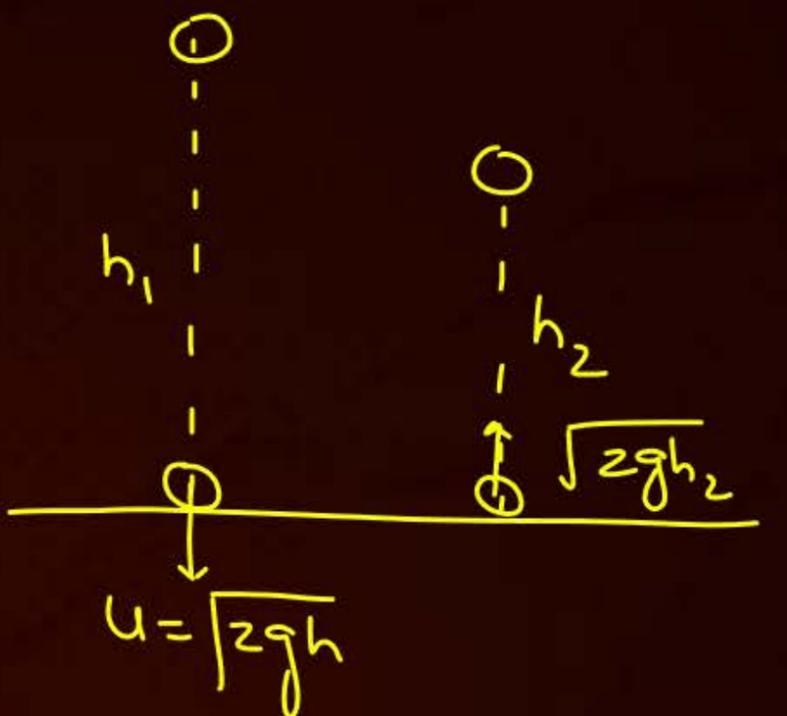
Question 57

A ball of mass m is released from rest from height h . After its collision with ground it rebound up to height $0.64h$. Find value of e



$$e = \frac{v_{\text{sep.}}}{v_{\text{app.}}} = \frac{0.8\sqrt{2gh}}{\sqrt{2gh}} = \underline{0.8}$$





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$$e = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}}$$

$$e = \frac{h_2}{h_1}$$

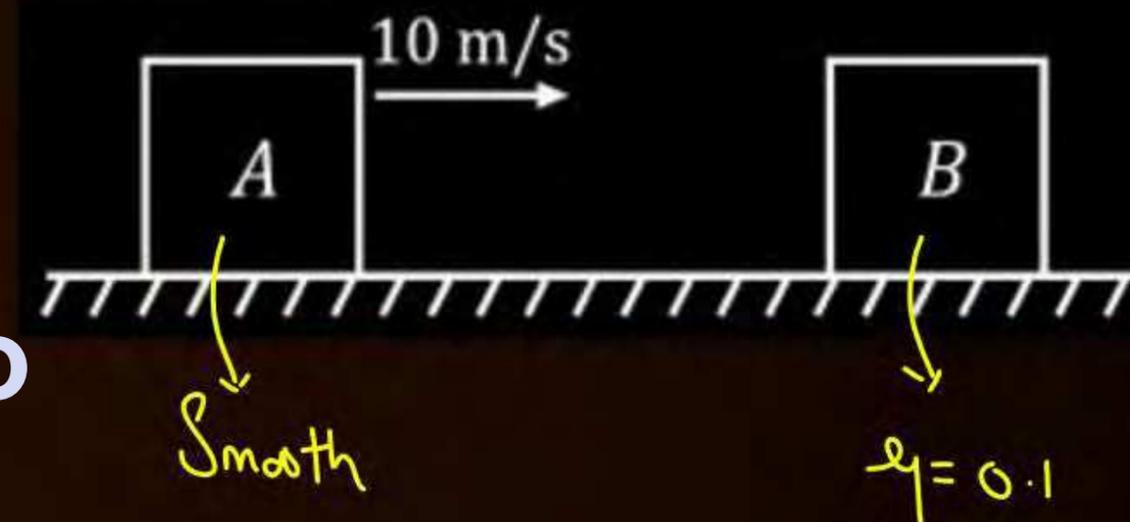
Question — 66

11W



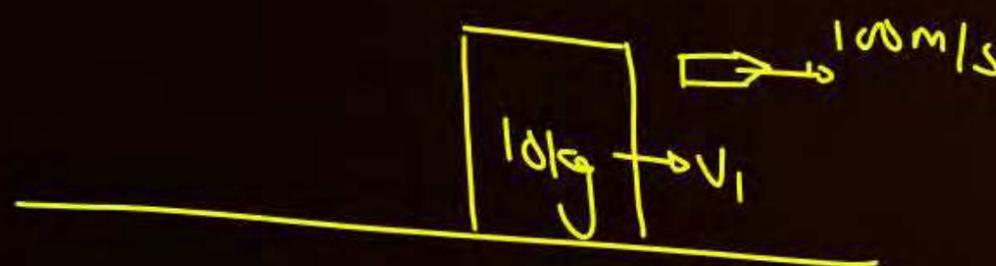
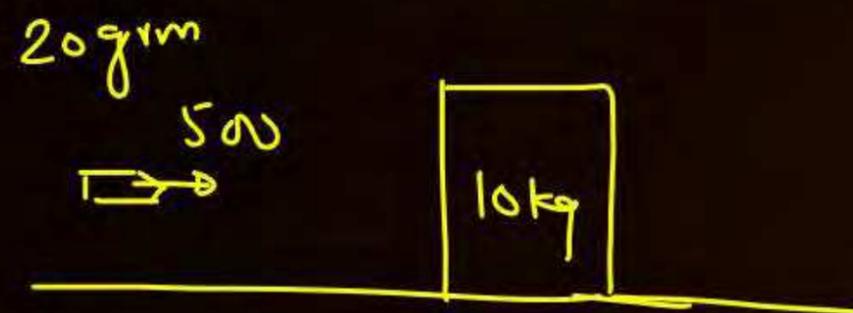
The blocks shown in figure have equal masses. The surface of A is smooth but that of B has a friction coefficient of 0.10 with the floor. Block A is moving at a speed of 10 m/s towards B which is kept at rest. Find the distance travelled by B if (a) the collision is perfectly elastic and (b) the collision is perfectly inelastic. Take $g = 10 \text{ m/s}^2$.

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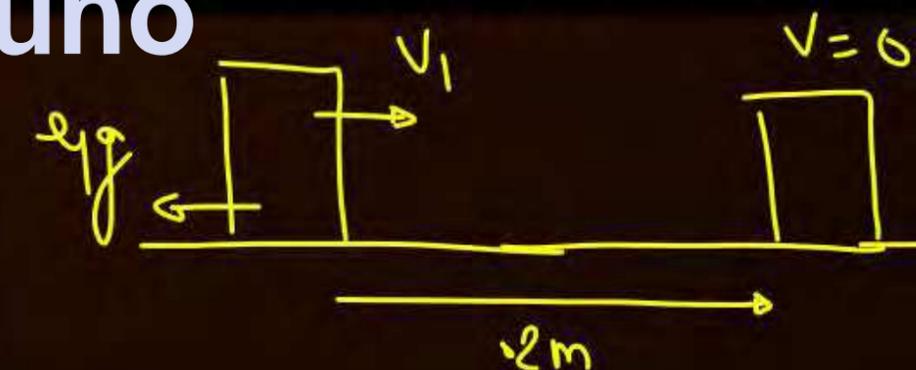
Question 62

A bullet of mass 20 g travelling horizontally with a speed of 500 m/s passes through a wooden block of mass 10.0 kg initially at rest on a level surface. The bullet emerges with a speed of 100 m/s and the block slides 20 cm on the surface before coming to rest. Find the friction coefficient between the block and the surface.



$$P_i = P_f = 20 \times 10^{-3} \times 500 + 0 = 10v_1 + 20 \times 10^{-3} \times 100$$

$$\Rightarrow \underline{v_1 = 0.8 \text{ m/s}}$$



$$v^2 = u^2 + 2as \Rightarrow 0 = (.8)^2 - 2(10a)(.2)$$

$$\Rightarrow \frac{0.64}{4} = a \Rightarrow \underline{a = 0.16}$$



Question 10

A ball of mass m strike the ground with speed u at an angle θ with horizontal. Find speed of ball just after collision if coefficient of restitution for collision with ground is e

Smooth

m

u

θ

$u \cos \theta$

$u \sin \theta$

LOI

Impulsive

$v_1 = e u \sin \theta$

ϕ

$u \cos \theta = v_2$

$\tan \phi = \frac{e u \sin \theta}{u \cos \theta}$

$\tan \phi = e \tan \theta$

$f_{ext})_y \neq 0$ in y -dirⁿ

$f_{ext})_x = 0$ in x -dirⁿ $\Rightarrow \vec{p}_i)_x = \vec{p}_f)_x$

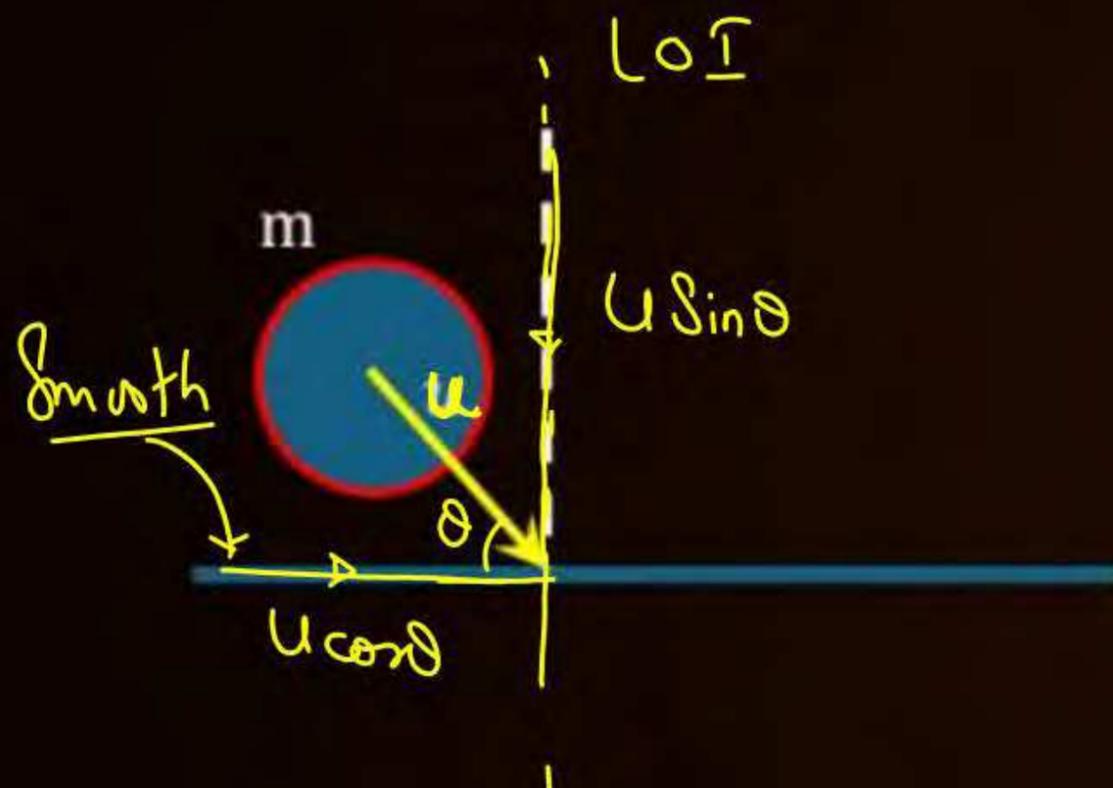
$m u \cos \theta = m v_2$

$e = \frac{v_{sep.}}{v_{app}} = \frac{v_1}{u \sin \theta}$

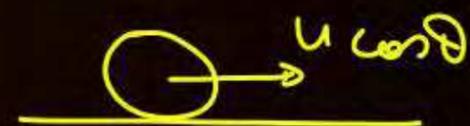
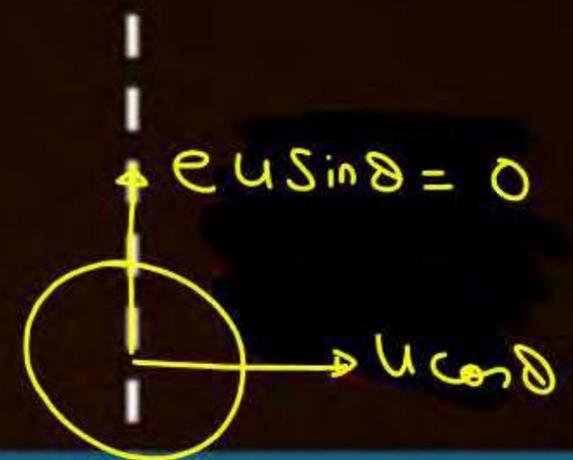
$v_1 = e u \sin \theta$

Question

A ball of mass m strike the ground with speed u at an angle θ with horizontal. Find speed of ball just after collision if coefficient of restitution for collision with ground is $e = 0$



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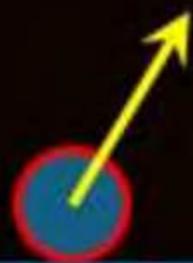


Question — 12



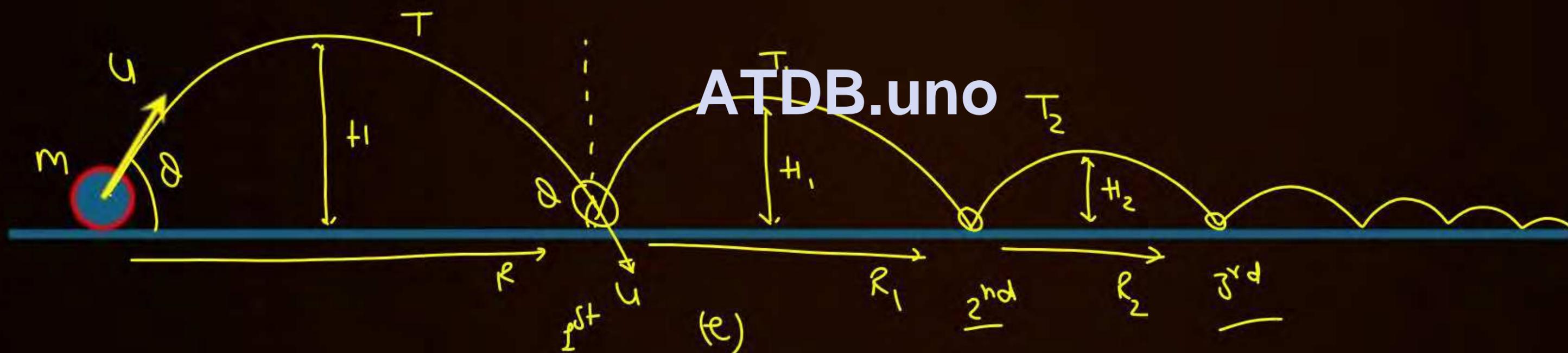
A ball of mass m is projected from ground with speed u at an angle θ with horizontal. coefficient of restitution for collision with ground is e . find time of flight, maximum height and range between n th and $(n+1)$ th collision. Also find total time of flight and total range $(e < 1)$

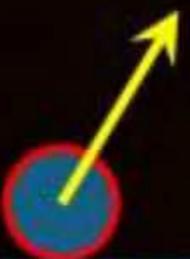
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Question

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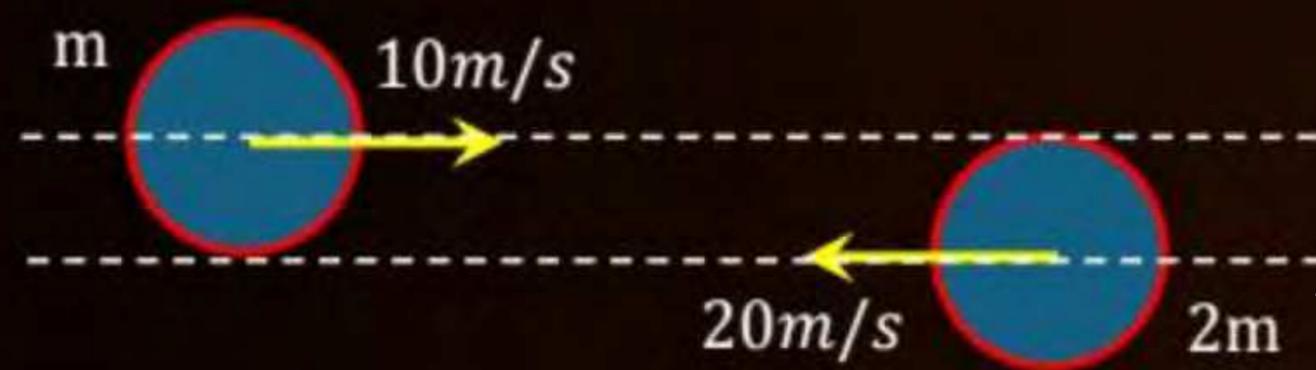




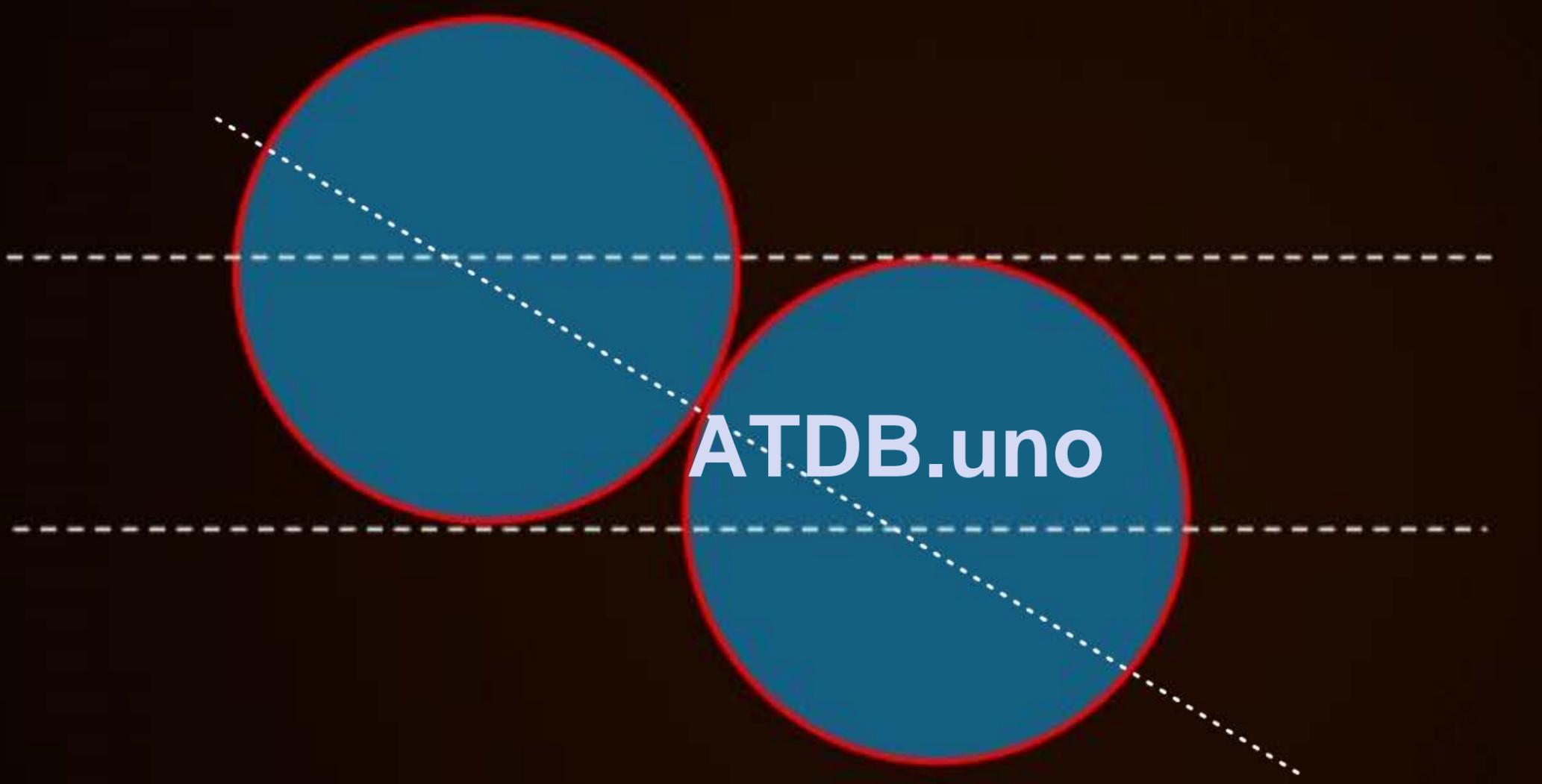
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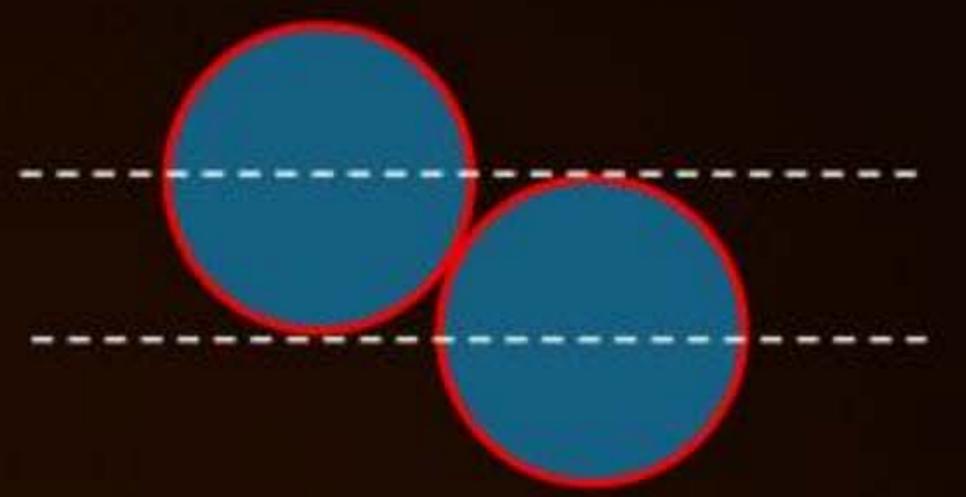
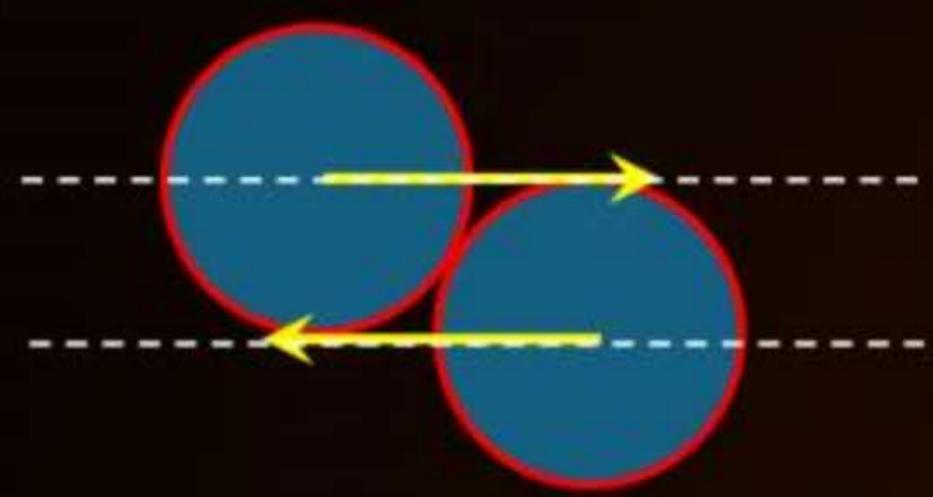
Question

Two balls of mass m and $2m$ of same size are moving toward each other and collide as shown. coefficient of restitution e is 0.5 . find speed of each ball after collision and total energy loss



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Variable mass system



$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

$$\vec{F} = m\vec{a} + \vec{v}\frac{dm}{dt} \Rightarrow \vec{F} - \vec{v}\frac{dm}{dt} = m\vec{a}$$

$$\vec{F} + \vec{F}_{Th} = m\vec{a}$$

$\vec{F}_{Th} \rightarrow$ Thrust force

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$$\vec{F}_{Th} = -v\frac{dm}{dt} \Rightarrow \vec{F}_{Th} = v_{rel.}\frac{dm}{dt}$$

$v_{rel.} \rightarrow$ relative velocity of dm w.r.t object



Variable mass system

$$\vec{F}_{Th} = \vec{V}_{rel} \cdot \frac{dm}{dt}$$

\vec{V}_{rel} → velocity of dm
w.r.t object

mass increase → $\frac{dm}{dt} = +ive$

mass decrease → $\frac{dm}{dt} = -ive$

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$$\vec{F}_{Th} = (-v_r \hat{j}) \left(\frac{dm}{dt} \right)$$

-ive
↓

$$\vec{F}_{Th} \rightarrow +\hat{j}$$



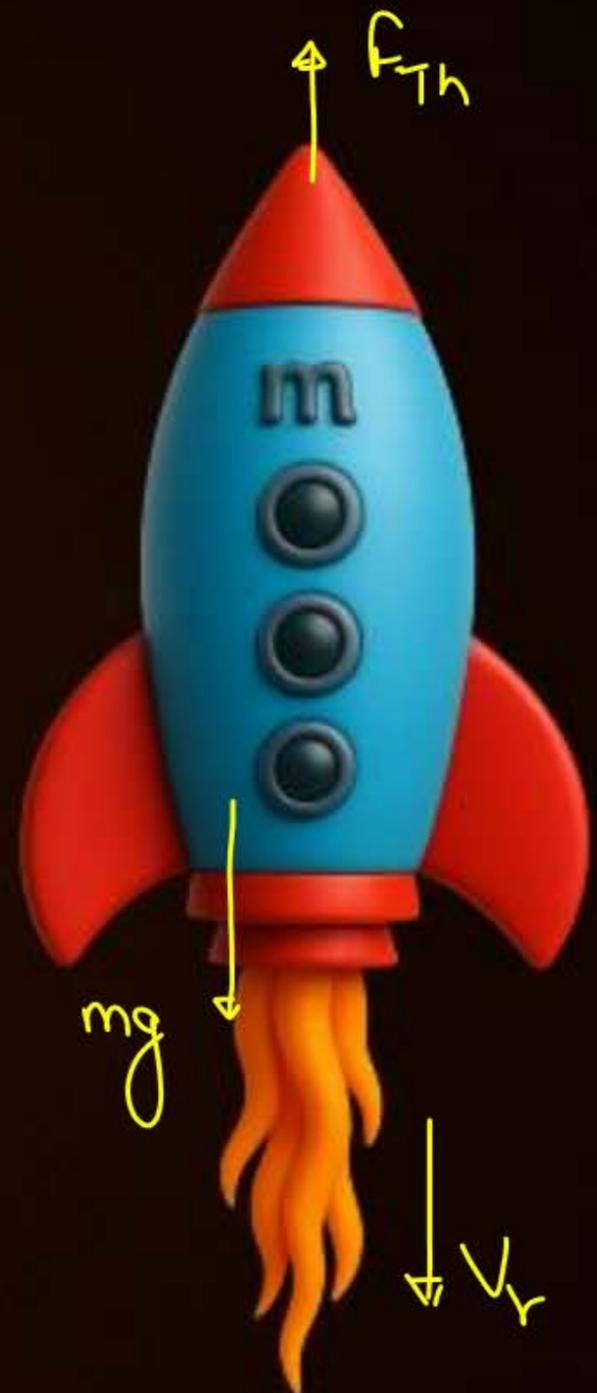
Rocket propulsion

Let gases are ejected out with constant rate of λ kg/s and with relative speed V_r and initially mass of rocket is m_0 and velocity u_0

$$F_{Th} - mg = ma \Rightarrow -V_r \frac{dm}{dt} - mg = m \frac{dv}{dt}$$

$$\int_{m_0}^m -V_r \frac{dm}{m} - \int_0^t g dt = \int_{u_0}^v dv \Rightarrow -V_r \ln\left(\frac{m}{m_0}\right) - gt = v - u_0$$

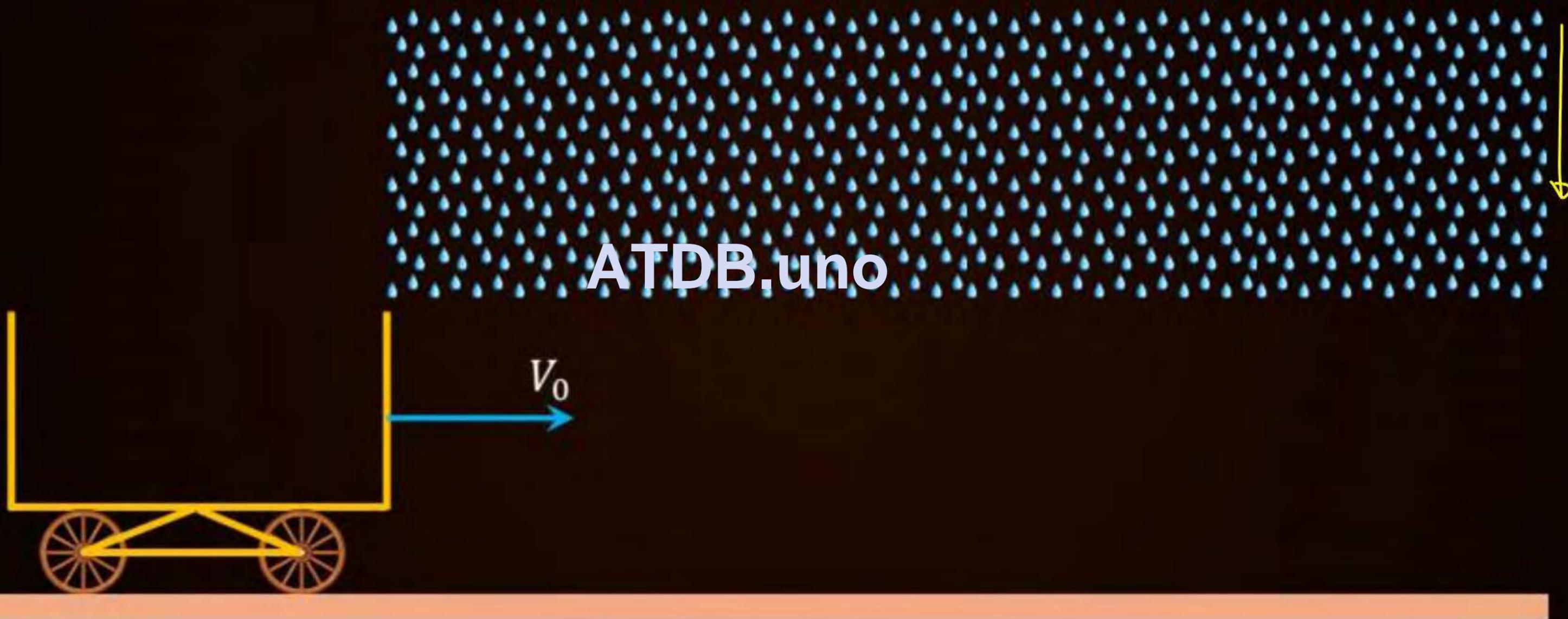
$$** \quad v = u_0 + V_r \ln\left(\frac{m_0}{m}\right) - gt$$



Question 79



An open trolley of mass M is moving on smooth horizontal surface. Rain is falling vertically downwards and due to rain, water start filling in trolley at constant rate of λ kg/s. find speed of trolley at time t . Initial velocity of trolley is V_0 and trolley has no water in it,

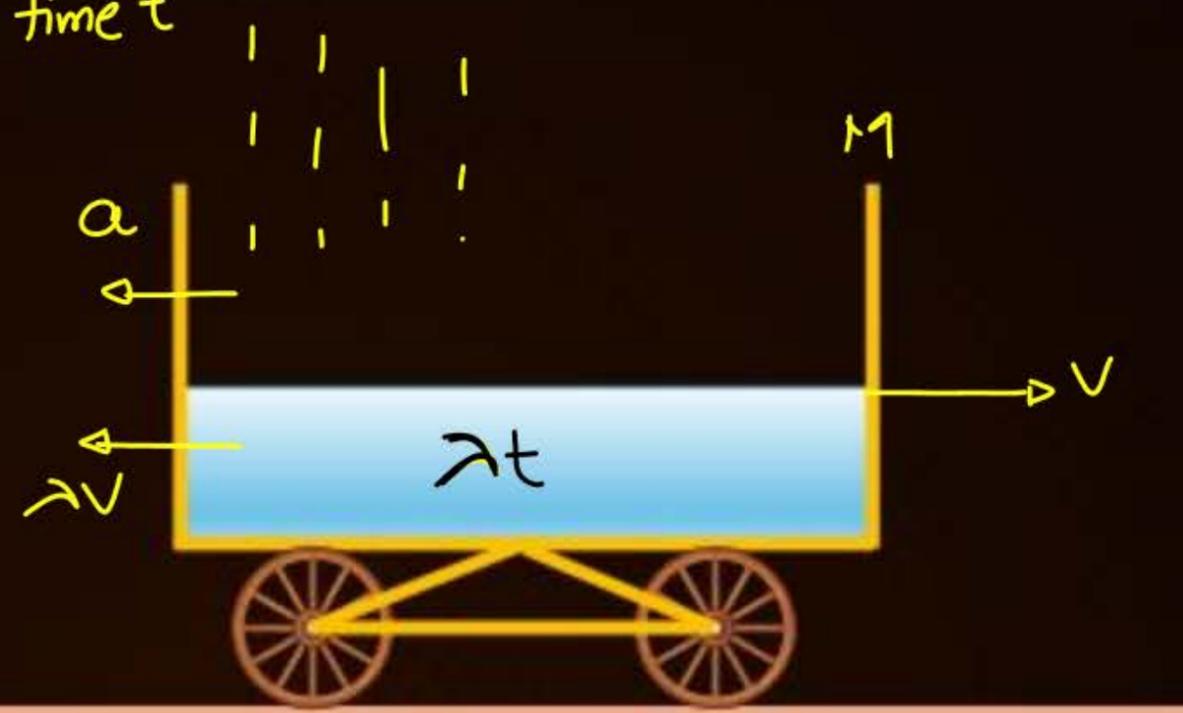




At t = 0



At time t

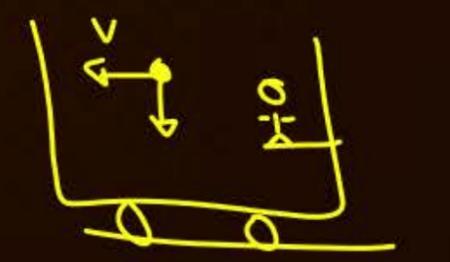


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$$f_{Th} = \vec{V}_{rel} \frac{dm}{dt} = (-v\hat{i})(\lambda) = -\lambda v\hat{i}$$

acc. => f = ma

$$\lambda v = (M + \lambda t)a = -(M + \lambda t) \frac{dv}{dt}$$



$$\frac{v}{v_0} = + \left(\frac{M}{M + \lambda t} \right)$$

$$\int_{v_0}^v \frac{dv}{v} = \int_0^t \frac{-\lambda dt}{M + \lambda t} \Rightarrow \ln\left(\frac{v}{v_0}\right) = -\frac{\lambda}{\lambda} \ln\left(\frac{M + \lambda t}{M}\right)$$

Question — 75

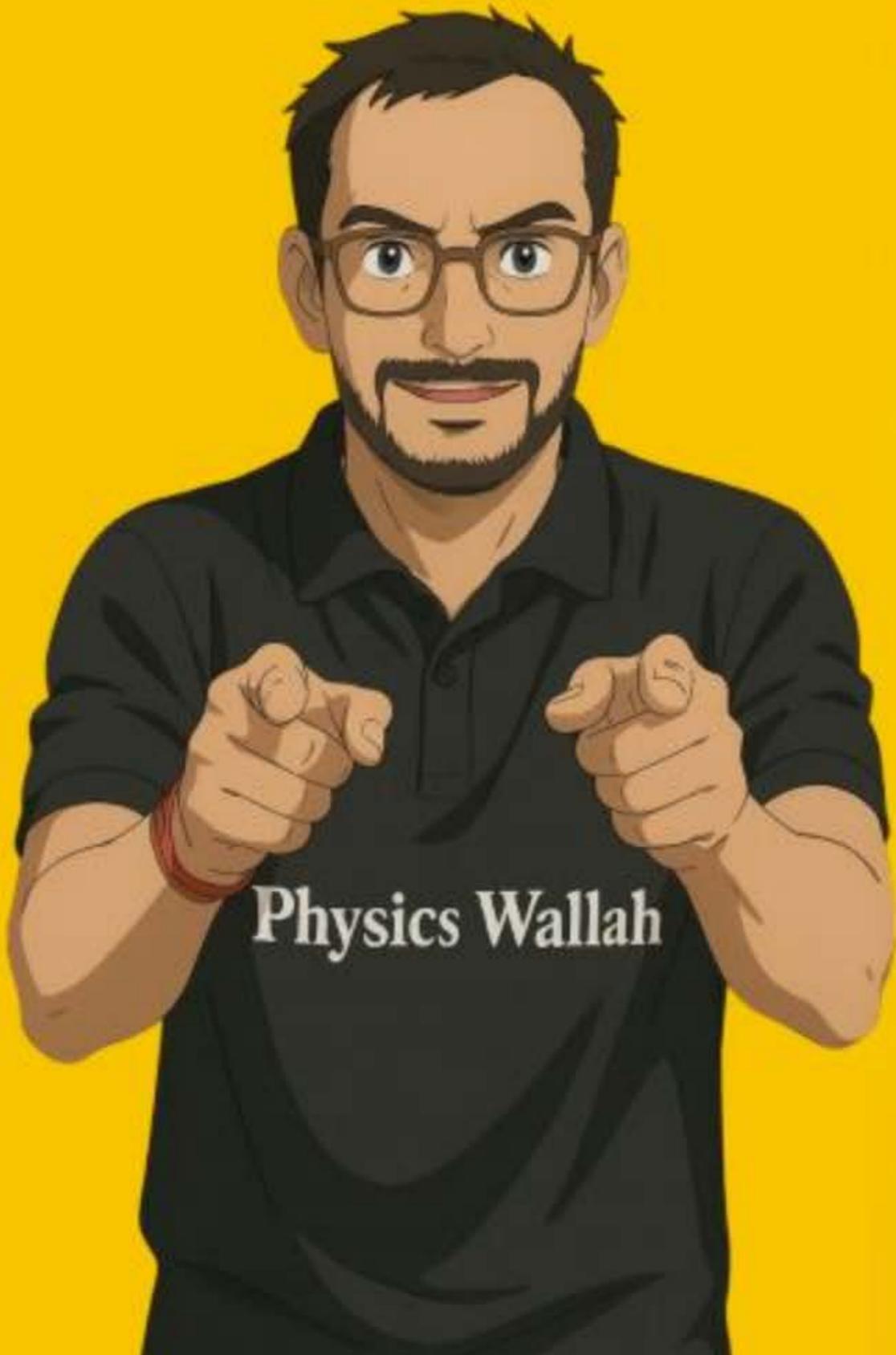
The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 ms^{-2} . The gases come out at a relative speed of 500 ms^{-1} with respect to the rocket:

[Use $g = 10 \text{ m/s}^2$]

[26 Aug, 2021 (Shift-I)]

- 1 500 kgs^{-1}
- 2 10 kgs^{-1}
- 3 $6.0 \times 10^2 \text{ kg s}^{-1}$
- 4 60 kg s^{-1}

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THANK YOU
BAWWAL
BACCCHA
PARTY

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