PRAYAS

JEE 2026

Physics

COM and System of particles

Lecture - 15

Manish Singh Tak (Masti Sir)

Topics to be covered

A

B

C

D

COM JEE Advance Questions

ATDB.uno

Jee Advance

Question

A uniform thin rod of mass M and length L is standing vertically along the Y-axis on a smooth horizontal surface, with its lower end at the origin (0, 0). A slight disturbance at $t = 0$ causes the lower end to slip on the smooth surface along the positive X-axis, and the rod starts falling.

(A) What is the path followed by the center of mass of the rod during its fall?

(B) Find the equation of the trajectory of a point on the rod located at a distance r from the lower end. What is the shape of the path of this point? **(1993)**

Ans: (a) Straight line

(b) $\frac{x^2}{\left(\frac{L}{2}-r\right)^2} + \frac{y^2}{r^2} = 1$, ellipse

Sol: $(F_{ext})_x = 0$, $(a_{cm})_x = 0$

$\Rightarrow (v_{cm})_x = 0$

(i) $\Rightarrow$ COM will move vertically downward

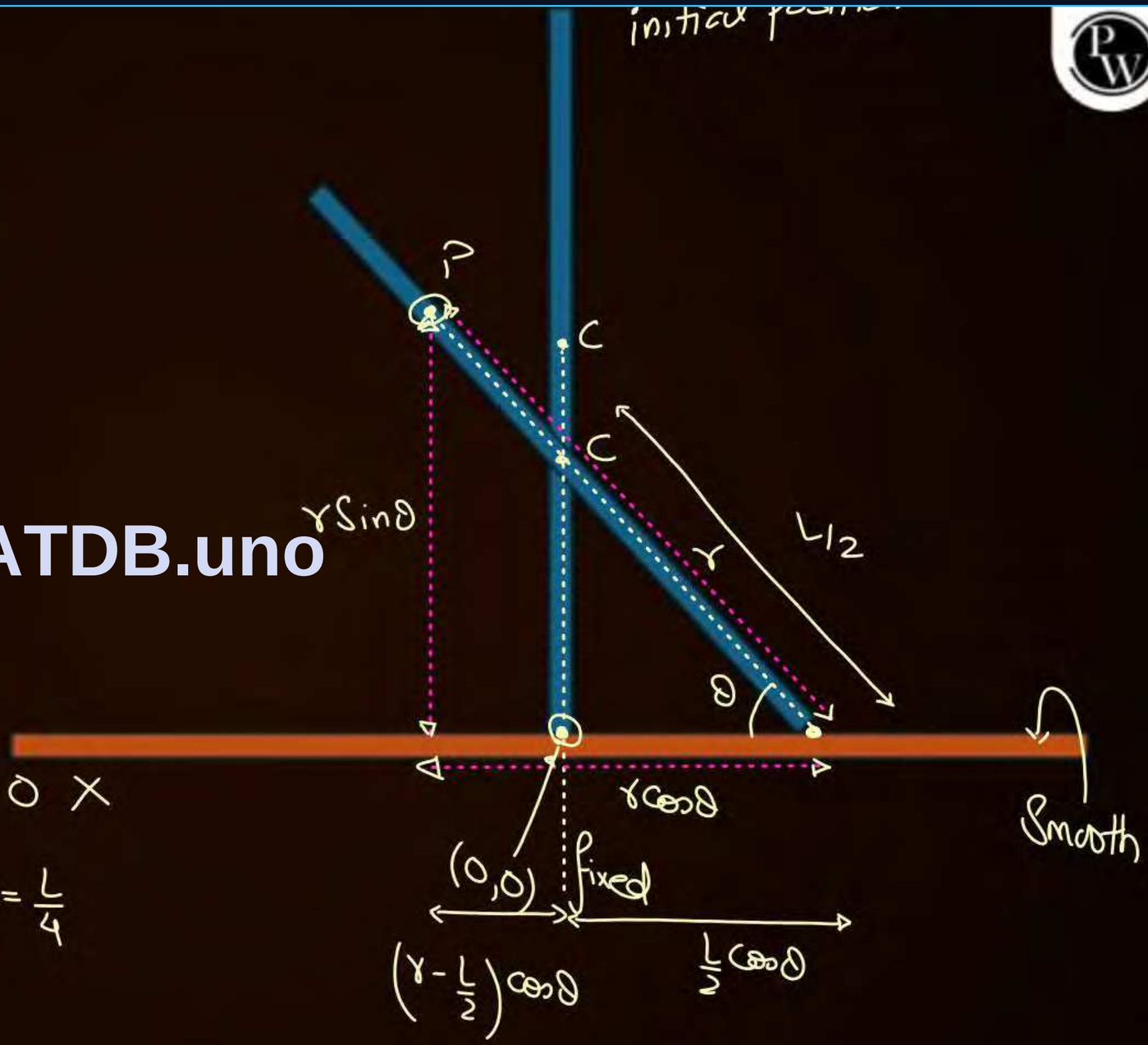
(ii) Point P $\longrightarrow$ $y = r\sin\theta$

$x = -\left(r - \frac{L}{2}\right)\cos\theta$

$$\frac{y^2}{r^2} + \frac{x^2}{\left(r - \frac{L}{2}\right)^2} = 1$$

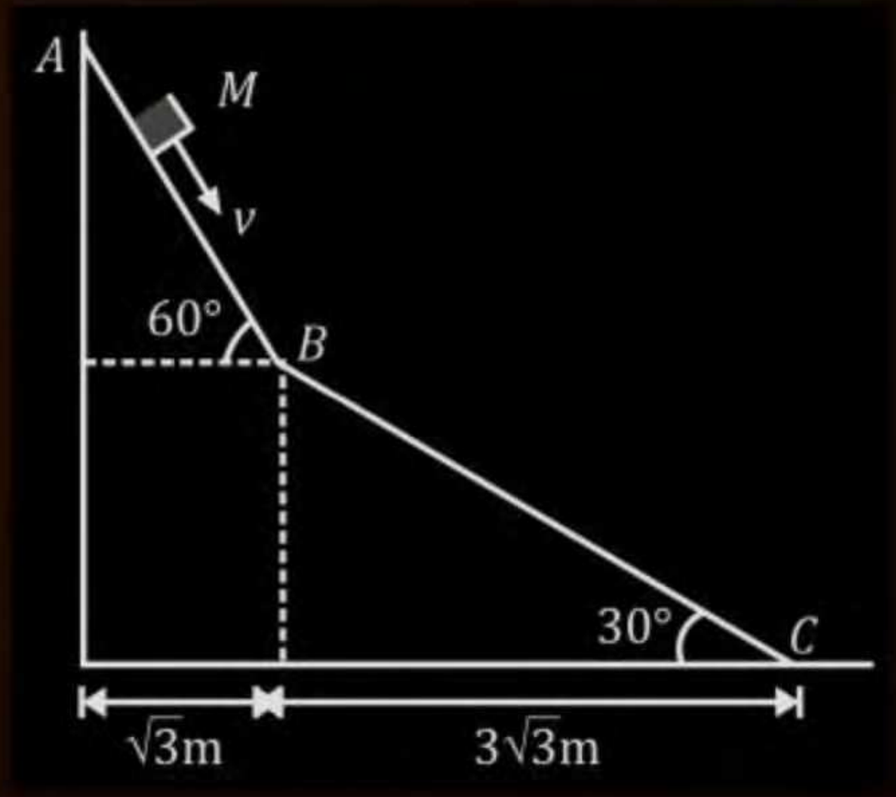
Circle $\left|r - \frac{L}{2}\right| = r \Rightarrow r - \frac{L}{2} = r \Rightarrow -\frac{L}{2} = 0$ X

$\hookrightarrow r - \frac{L}{2} = -r \Rightarrow r = \frac{L}{4}$



Passage (Next three Questions are part of Passage)

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic. $(g = 10\ m/s^2)$ **(2008)**



Question

Part of Passage

The speed of the block at point B immediately after it strikes the second incline is (2008)

a. $\sqrt{60}$ m/s

b. $\sqrt{45}$ m/s

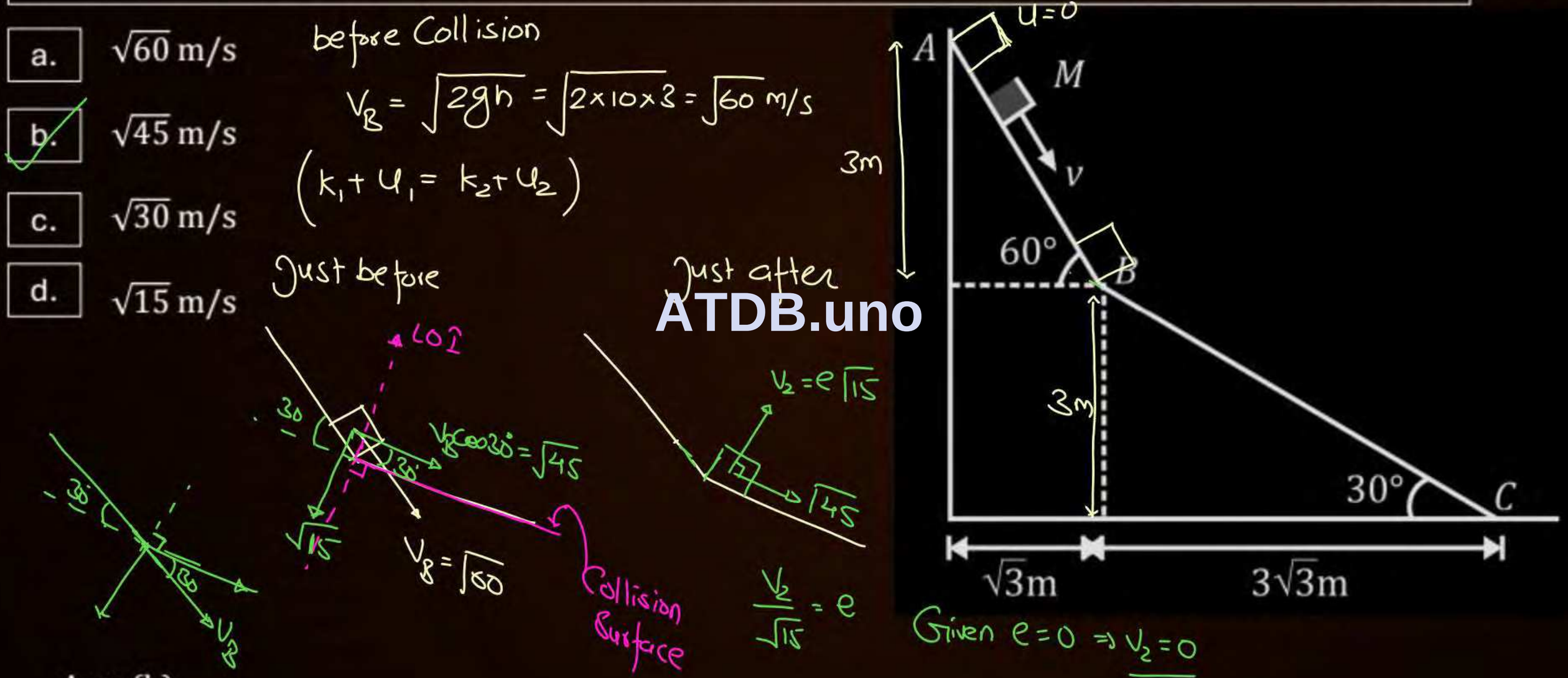
c. $\sqrt{30}$ m/s

d. $\sqrt{15}$ m/s

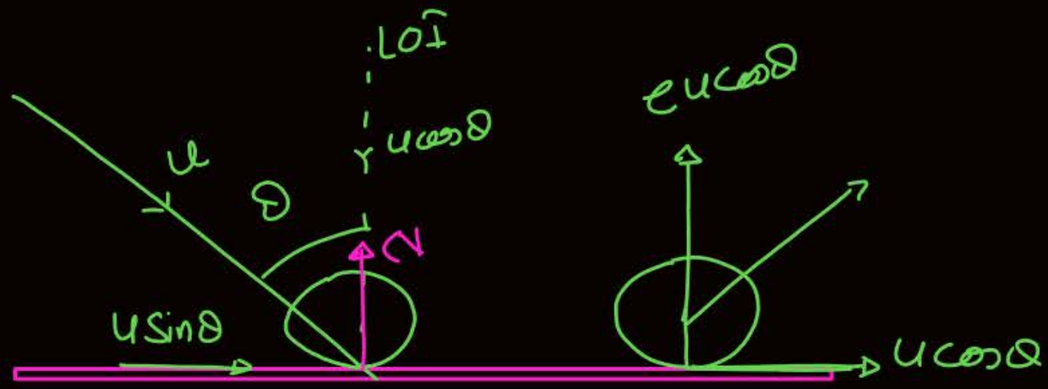
before Collision

$V_B = \sqrt{2gh} = \sqrt{2\times10\times3} = \sqrt{60}$ m/s

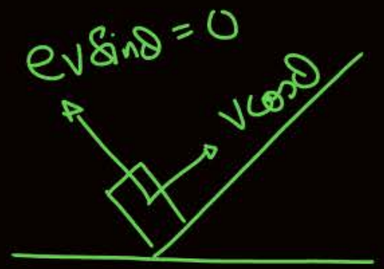
$(K_1 + U_1 = K_2 + U_2)$

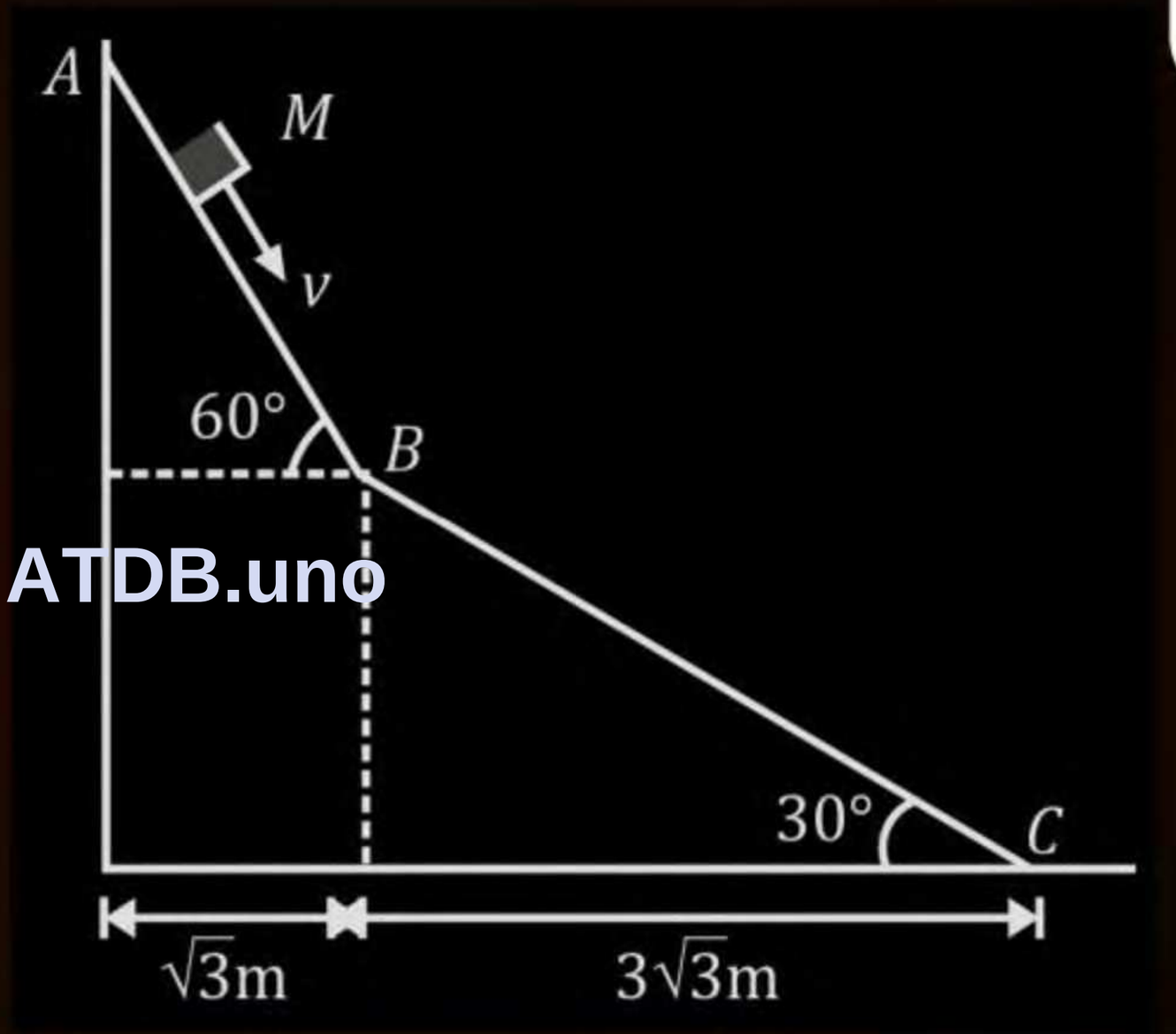


Ans: (b)



Speed of block just after Collision





Question

Part of Passage

The speed of the block at point C, immediately before it leaves the second incline is **(2008)**

a. $\sqrt{120}$ m/s

b. $\sqrt{105}$ m/s

c. $\sqrt{90}$ m/s

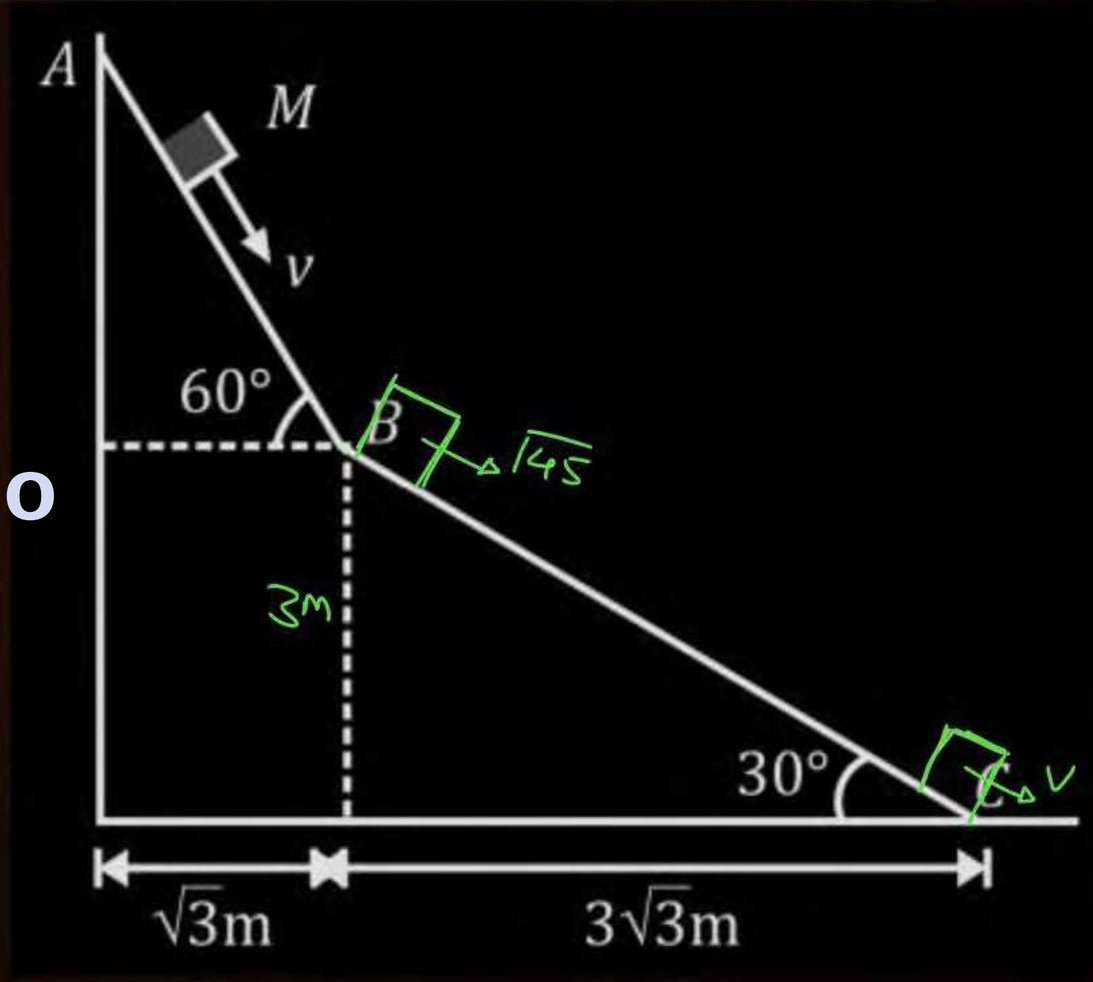
d. $\sqrt{75}$ m/s

$K_1 + U_1 = K_2 + U_2$

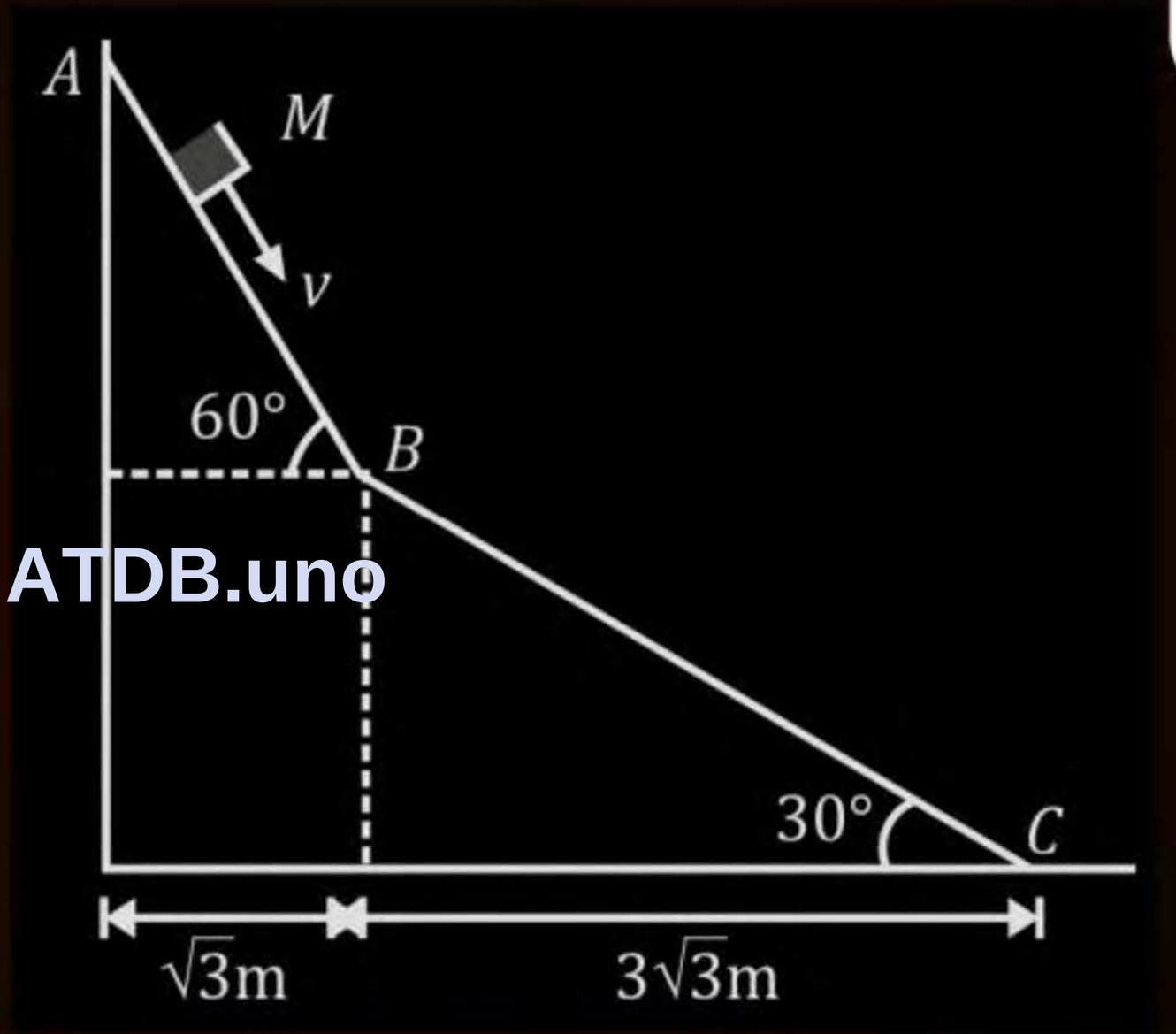
$\frac{1}{2}m(45) + mg(3) = \frac{1}{2}mv^2 + 0$

$v^2 = 45 + 60$

$V = \sqrt{105}$ m/s



Ans: (b)



Question

Part of Passage

If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is – **(2008)**

a. $\sqrt{30}$ m/s

b. $\sqrt{45}$ m/s

c. 0 m/s

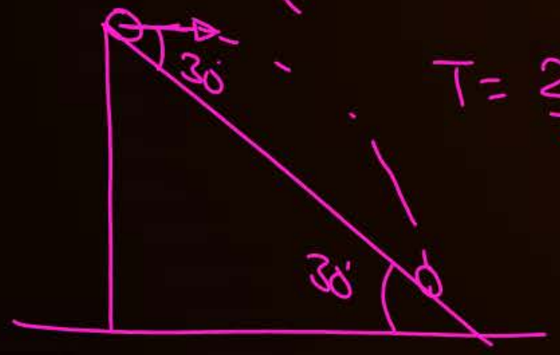
d. $\sqrt{15}$ m/s

net $V_y = 0$

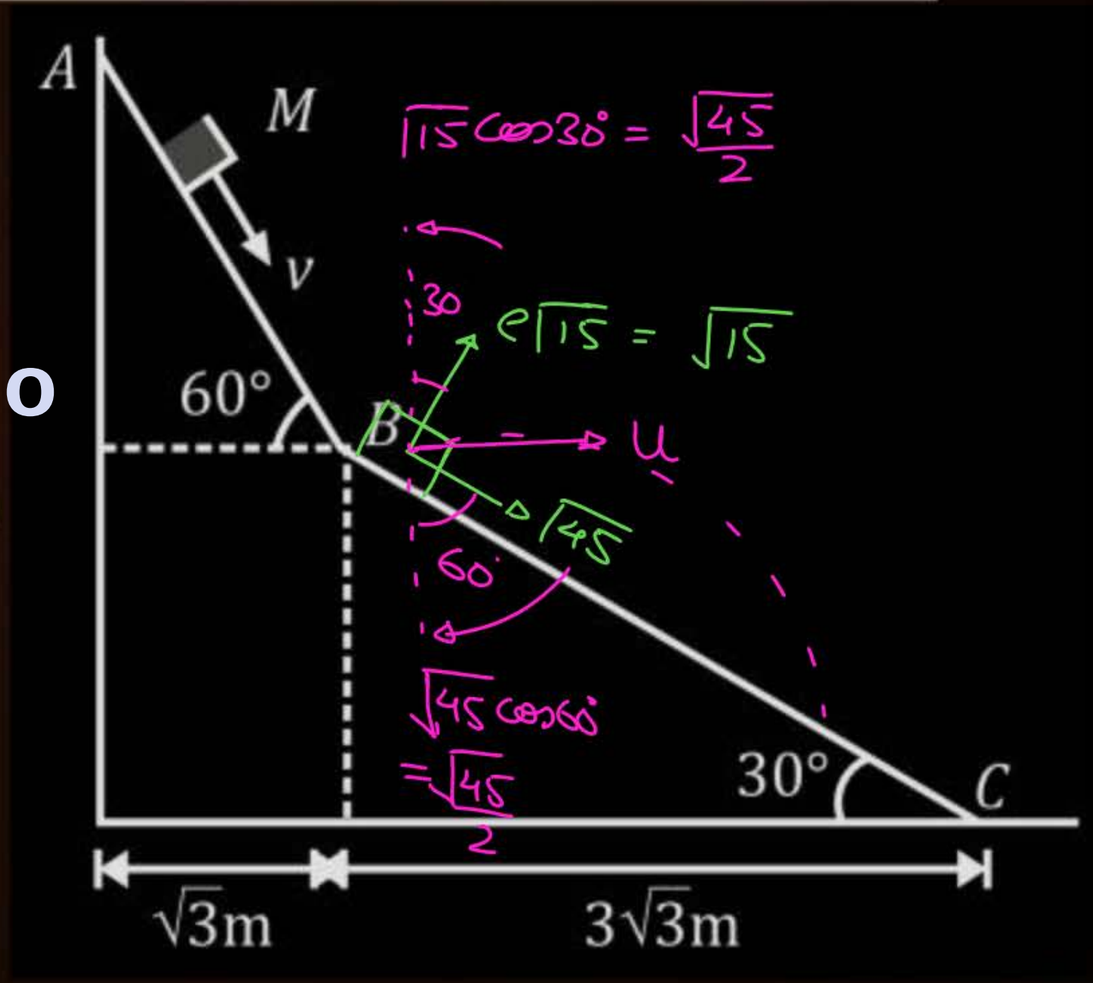
$u = \sqrt{45}\sin 60 + \sqrt{15}\sin 30$

$= \frac{3\sqrt{15}}{2} + \frac{\sqrt{15}}{2} = 2\sqrt{15}$

(Speed before Collision $= \sqrt{60}$ m/s)



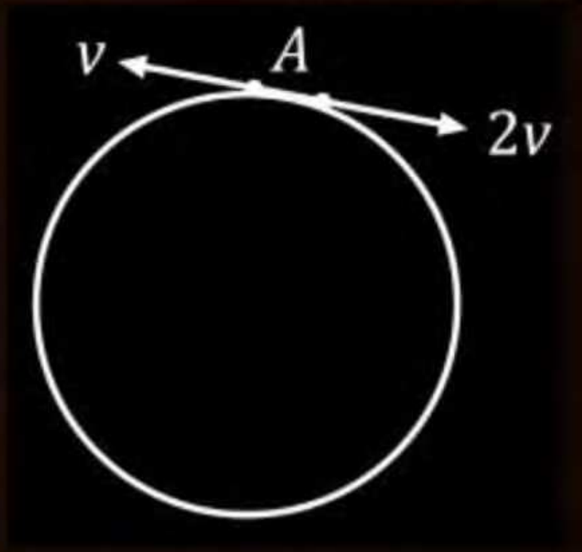
$T = \frac{2u\sin 30}{g\cos 30}$



Ans: (c)

Question

Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? **(2009)**



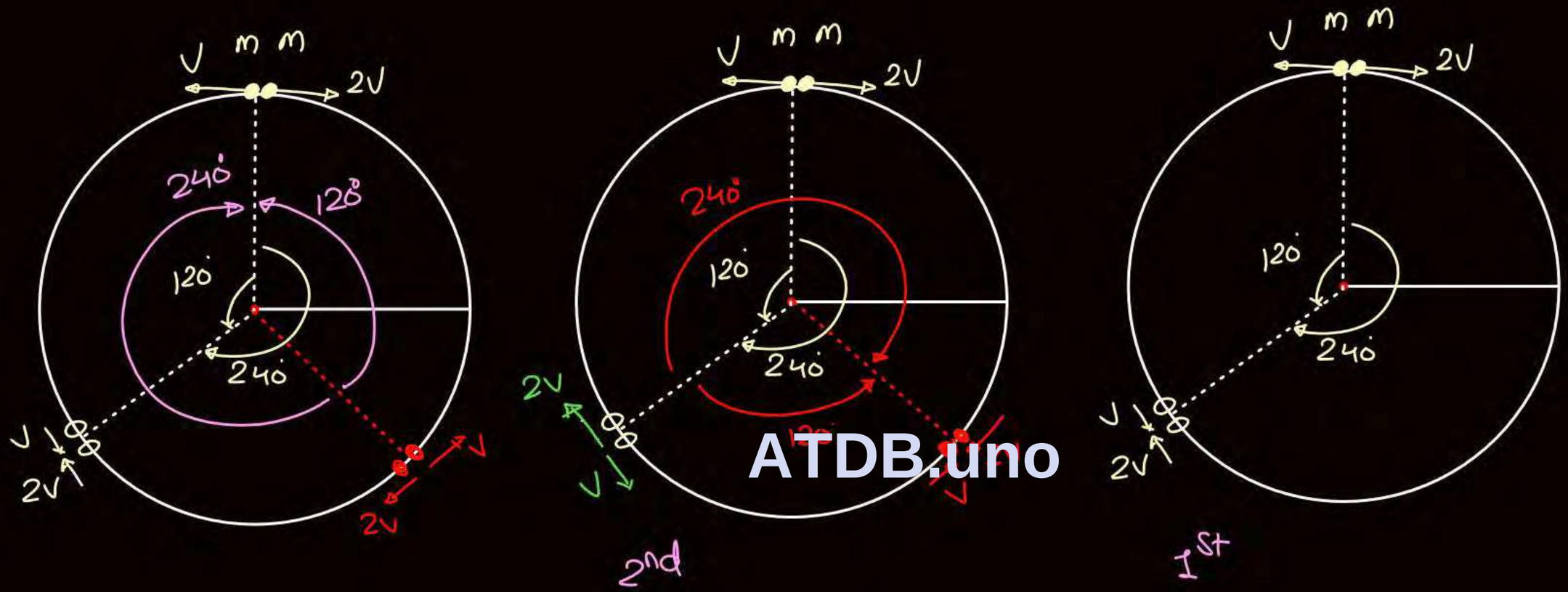
a. 4

b. 3

c. 2

d. 1

Ans: (c)

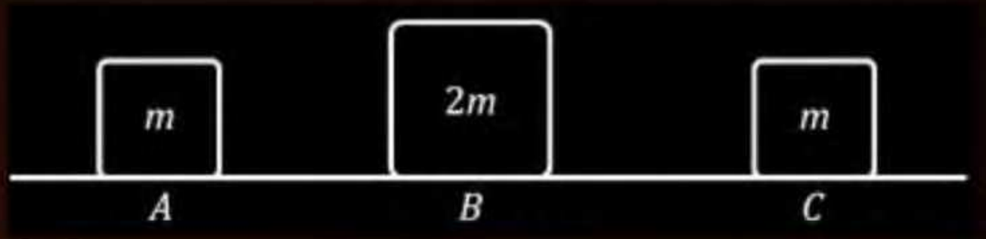


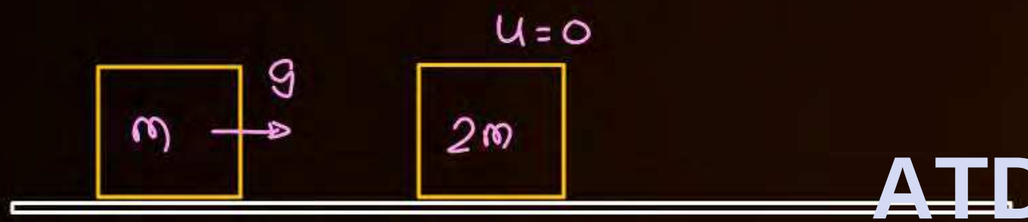
2nd

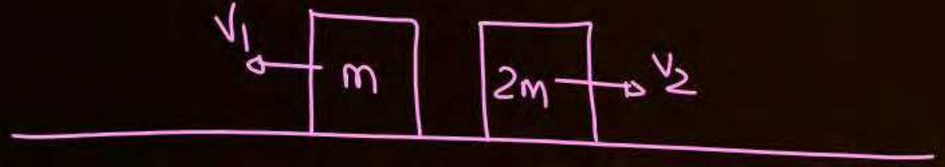
1st

Question

Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 ms^{-1} and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in ms^{-1}) of the object C. **(2009)**





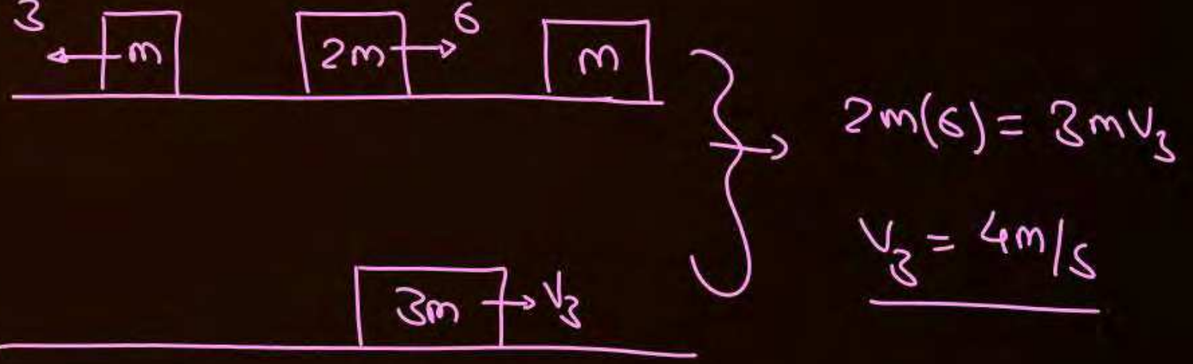


$2mv_2 - mv_1 = 9m \Rightarrow 2v_2 - v_1 = 9 \quad -(i)$

$v_2 + v_1 = 9$

$v_2 = 6$, $v_1 = 3 m/s$

Ans: (4)



$2m(6) = 3mv_3$

$v_3 = 4 m/s$

Question

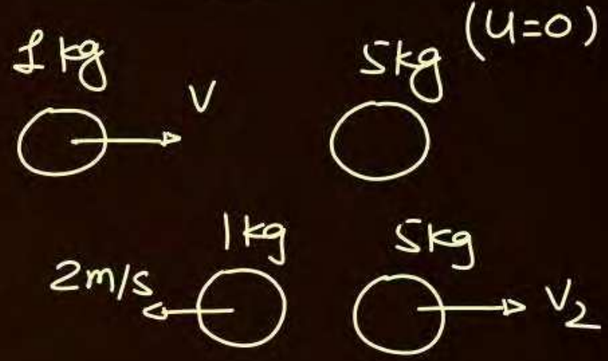
A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is/are correct for the system of these two masses **(2010)**

a. Total momentum of the system is 3 kg-ms^{-1}

b. Momentum of 5 kg mass after collision is 4 kg -ms^{-1}

c. Kinetic energy of the center of mass is 0.75 J

d. Total kinetic energy of the system is 4 J



5kg - ms^{-1}

(c) $\frac{1}{2}MV_{cm}^2 = \frac{1}{2}\times 6\left(\frac{1}{2}\right)^2 = \frac{3}{4}$ J

(d) $K.E. = \frac{1}{2}\times 1\times 3^2 + 0 = \frac{9}{2}$ J

$\frac{1}{2}\times 5\times 1^2 + \frac{1}{2}\times 1\times 4 = \frac{9}{2}$ J

Ans: (a, c)

$\longrightarrow 1(v) + 0 = 5V_2 - 1(2) \Rightarrow 5V_2 - V = 2$

$\longrightarrow V_2 + 2 = V$

$5V_2 - (V_2 + 2) = 2 \Rightarrow 4V_2 = 4 \Rightarrow V_2 = 1 m/s$

$V = 3 m/s$

Question

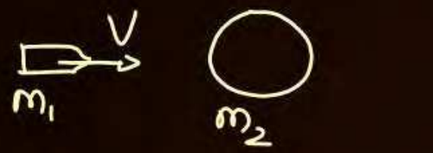
A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, travelling with a velocity v m/s in a horizontal direction, hits the center of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity v of the bullet is **(2011)**

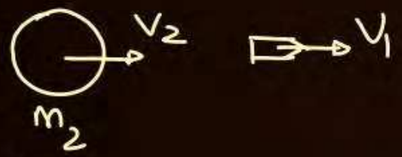
a. 250 m/s

b. $250\sqrt{2}$ m/s

c. 400 m/s

d. 500 m/s





$\vec{P}_i = \vec{P}_f$

$(10^{-2})(v) + 0 = (10^{-2})(100) + 2\times10^{-1}(20)$

$V = 100 + \frac{4}{10^{-2}} = 100 + 400 = 500\,m/s$

$T = \sqrt{\frac{2\times5}{10}} = 1\,Sec.$

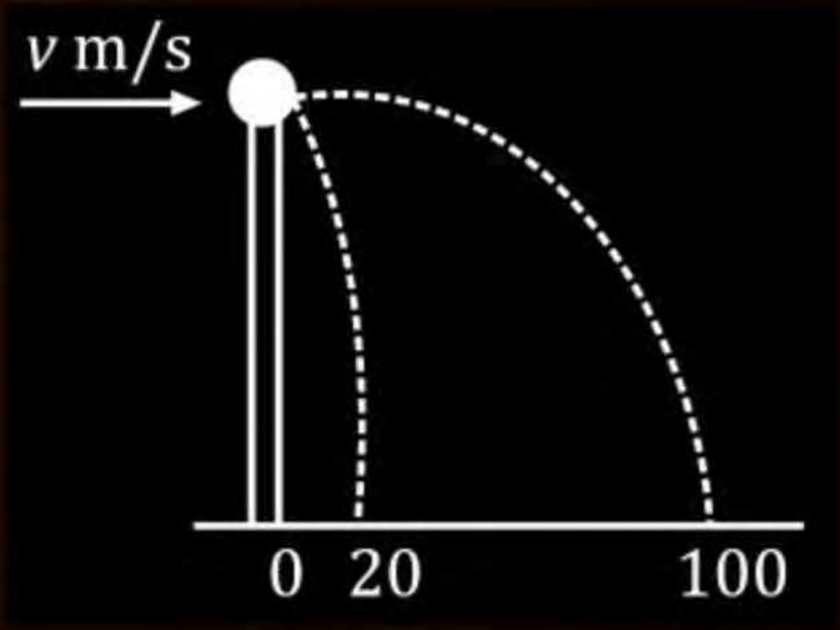
$R = UT$

$R_1 = 100 = V_1(1)$

$V_1 = 100\,m/s$

$R_2 = 20 = V_2(1)$

$V_2 = 20\,m/s$



Ans: (d)

Question

A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is **(2013 Adv.)**

a. $\frac{\pi}{4}$

b. $\frac{\pi}{4} + \alpha$

c. $\frac{\pi}{4} - \alpha$

d. $\frac{\pi}{2}$

$$H = \frac{u_0^2 \sin^2\alpha}{2g}$$

Just before

$u_0\cos\alpha$ (m) $\quad$ $V = u_0\cos\alpha$ (m)

Just after

$\frac{u_0\cos\alpha}{2}$ (2m), V', $\pi/4$, $\frac{u_0\cos\alpha}{2}$

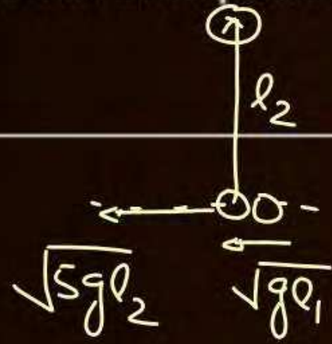
$$V^2 = u^2 + 2as = u_0^2 - 2g\left(\frac{u_0^2\sin^2\alpha}{2g}\right) = u_0^2\cos^2\alpha$$

$$V = u_0\cos\alpha$$

Ans: (a)

Question

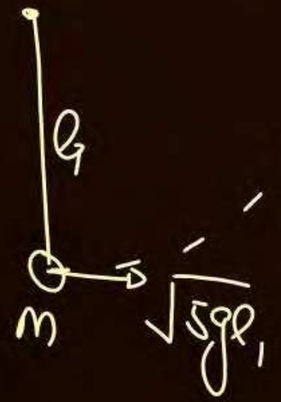
A bob of mass m, suspended by a string of length ℓ_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length ℓ_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum sped required to complete a full circle in the vertical plane, the ratio $\frac{\ell_1}{\ell_2}$ is. [JEE Advanced-2013]



$$\sqrt{5g\ell_2} = \sqrt{g\ell_1}$$

$$5g\ell_2 = g\ell_1 \Rightarrow 5\ell_2 = \ell_1$$

$$\frac{\ell_1}{\ell_2} = 5$$

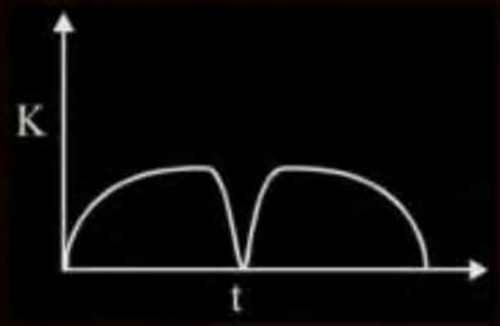


Ans: (5)

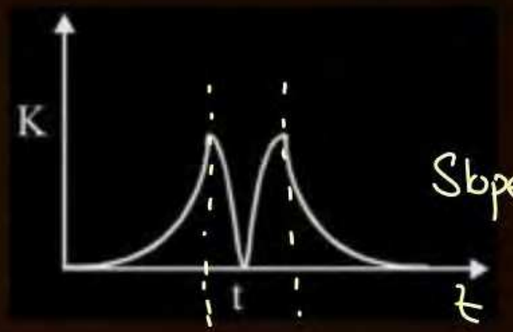
Question

A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figures are only illustrative and not to the scale.
[JEE Advanced-2014]

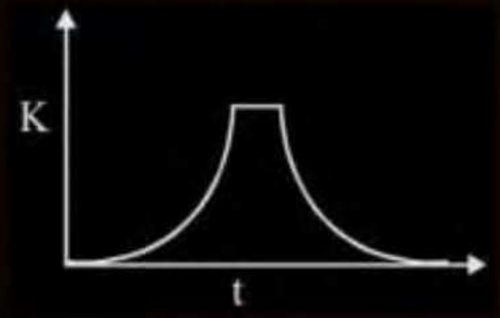
a.



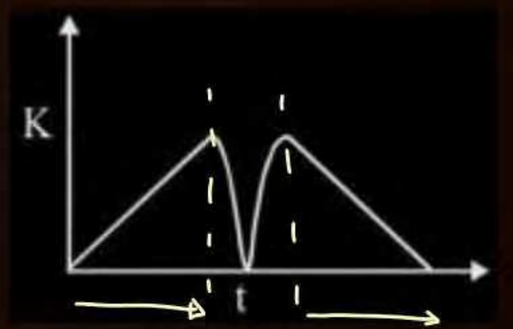
b.

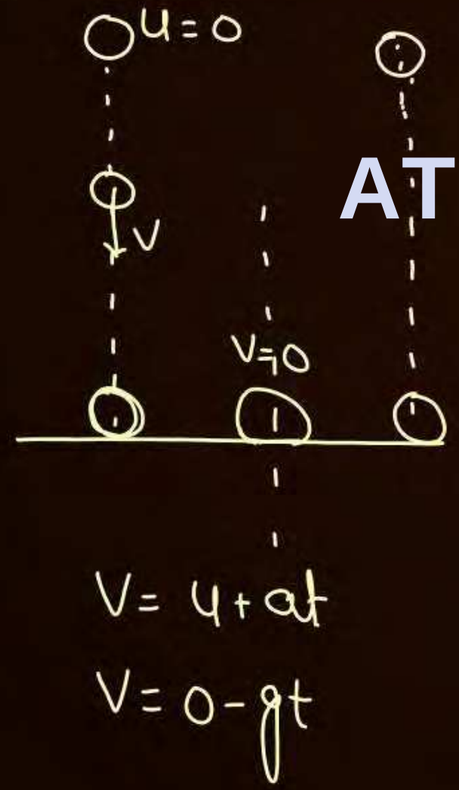


c.



d.





$K \cdot E = \frac{1}{2} m g^2 t^2$

$\text{Slope} = \frac{d(K \cdot E)}{dt} = mg^2 t$

↓

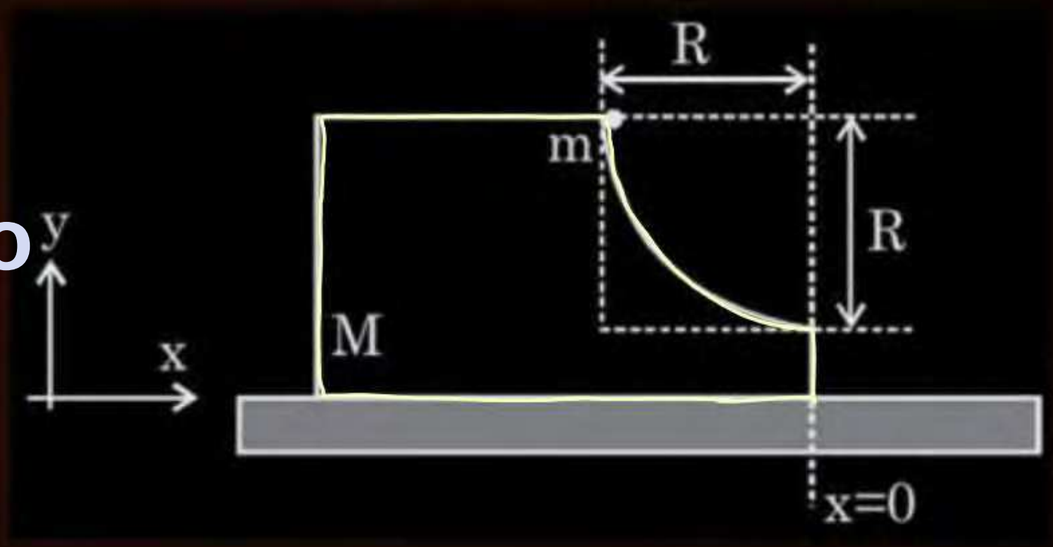
+ive, increases with time

$t=0$ Slope $= 0$

Ans: (B)

Question

A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at $x = 0$, in a coordinate system fixed to the table. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v . At that instant, which of the following options is/are correct ? [JEE Advanced-2017]



a. The x component of displacement of the center of mass of System is Zero

b. The position of the point mass is : $x = -\sqrt{2}\frac{mR}{M+m}$

c. The velocity of the point mass m is : $v = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$

d. The velocity of the block M is: $V = -\frac{m}{M}\sqrt{2gR}$

Ans: (a, c)

$\rightarrow F_{ext.})_x = 0 \Rightarrow (\vec{P}_f)_x = (\vec{P}_i)_x$

$mV_1 - MV_2 = 0 \Rightarrow mV_1 = MV_2 \quad -(i)$

$\rightarrow F_{ext.})_x = 0$ and $U_{cm})_x = 0$

$\Rightarrow \underbrace{\vec{S}_{cm} = 0 = m_1\vec{S}_1 + m_2\vec{S}_2}_{x\text{-dir}^n}$

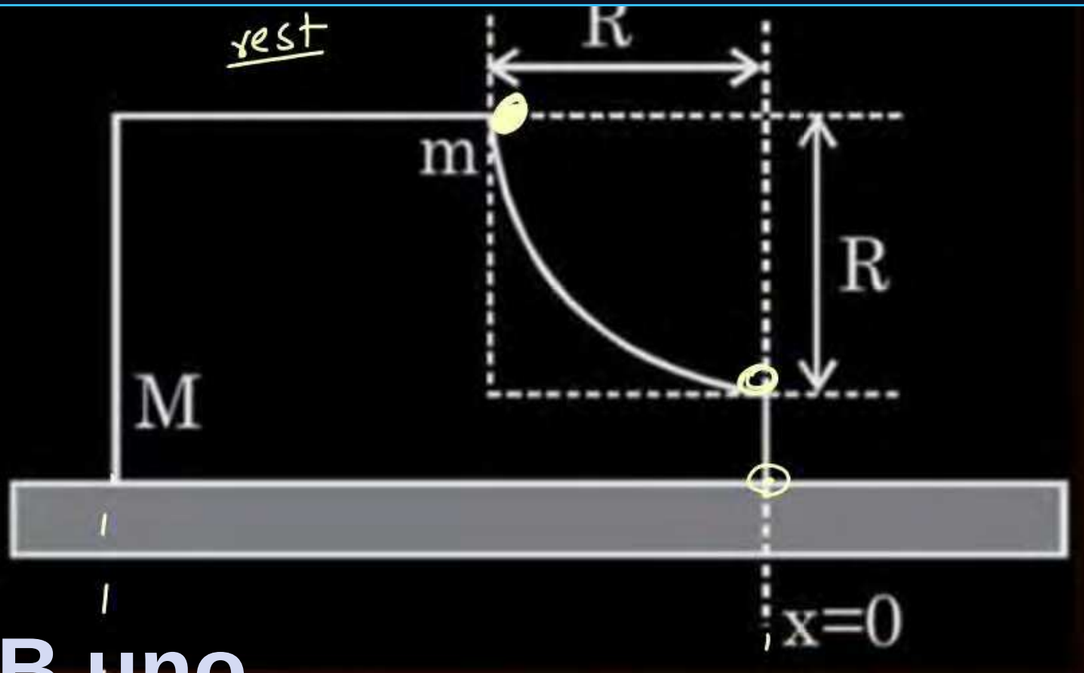
$m(R-x) + Mx = 0 \Rightarrow x = \frac{mR}{m+M}$

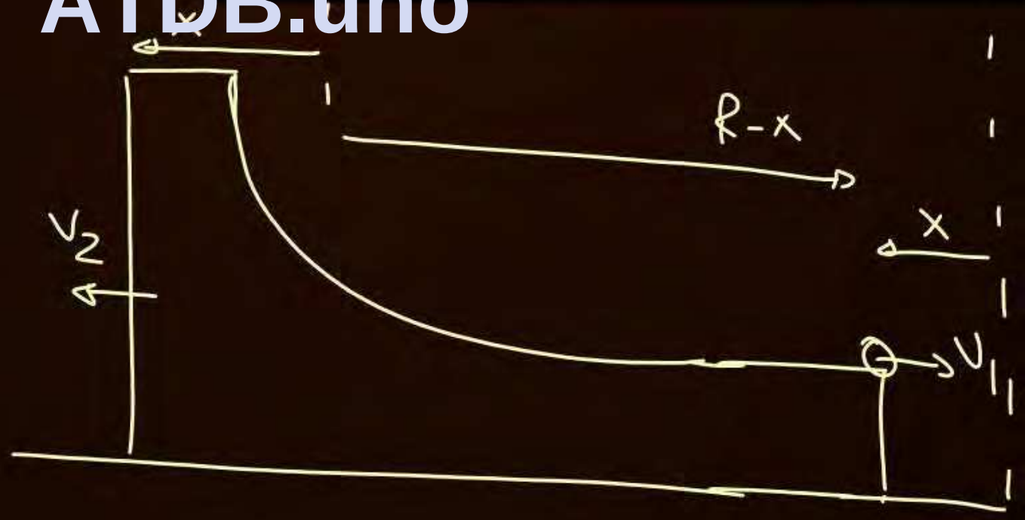
$x_{p\,M} = -x = -\frac{mR}{m+M}$

$\rightarrow K_1 + U_1 = K_2 + U_2$

$0 + mgR = \frac{1}{2}mV_1^2 + \frac{1}{2}MV_2^2 + 0$

$mgR = \frac{1}{2}mV_1^2 + \frac{1}{2}M\left(\frac{mV_1}{M}\right)^2 \Rightarrow \frac{1}{2}mV_1^2\left[\frac{M+m}{M}\right]$





$$V_1 = \sqrt{\frac{2MgR}{M+m}} = \sqrt{\frac{2gR}{1+\frac{m}{M}}}$$

$$V_2 = \frac{mV_1}{M} = \sqrt{\frac{2mgR}{M+m}}$$

Question

Consider regular polygons with number of sides $n = 3,4,5 \ldots$ as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as :
[JEE Advanced-2017]

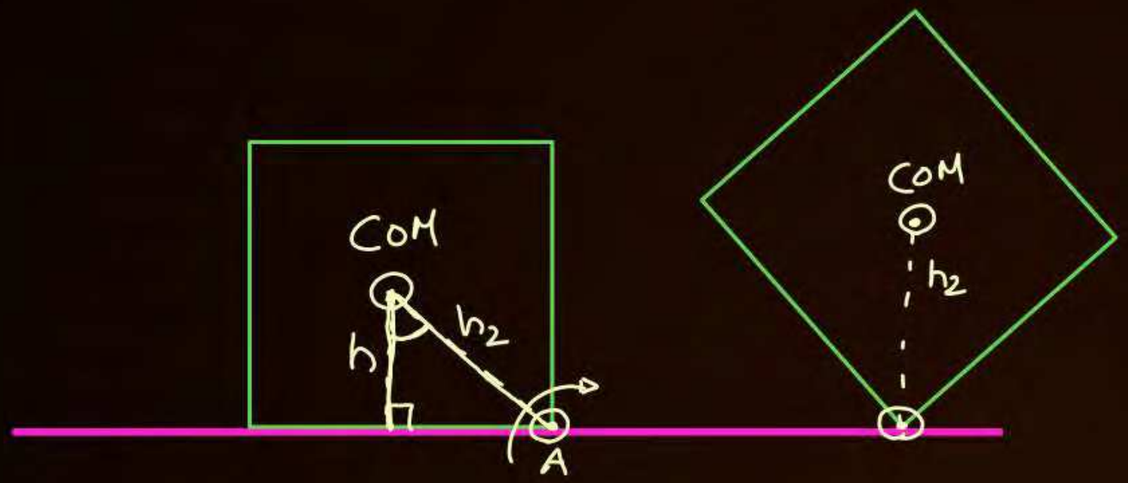
a. $\Delta = h\sin^2\left(\frac{\pi}{h}\right)$

b. $\Delta = h\sin\left(\frac{2\pi}{h}\right)$

c. $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

d. $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

Ans: (c)



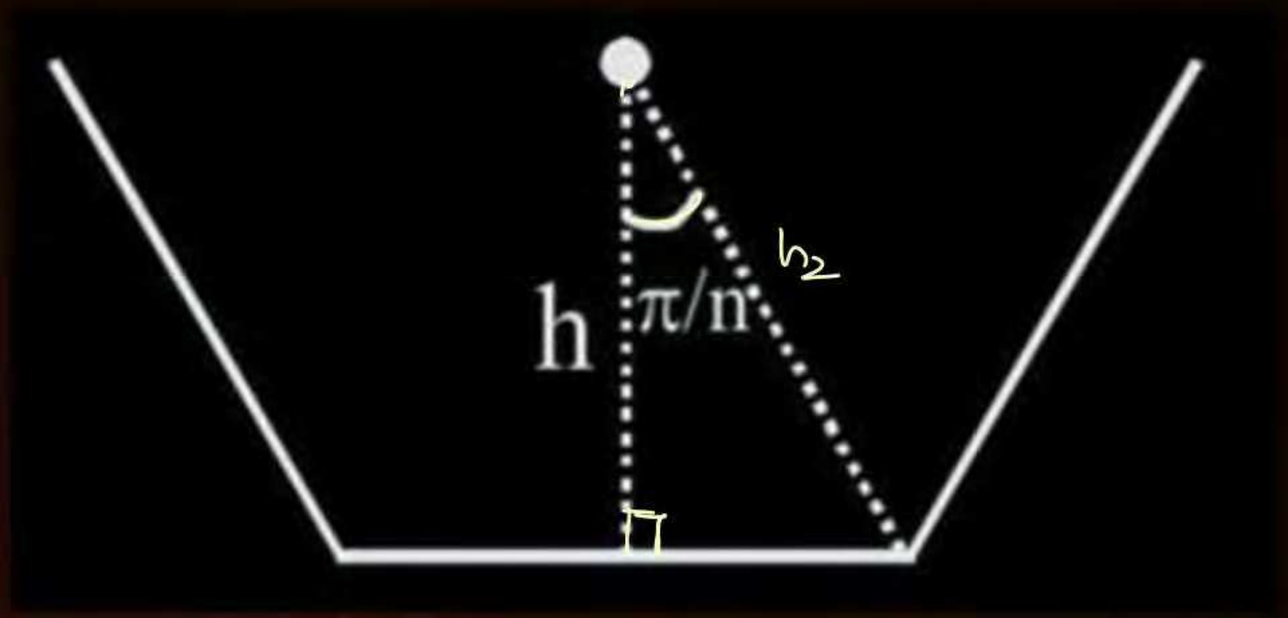
ATDB.uno

$$\frac{h}{h_2} = \cos\frac{\pi}{n}$$

$$h_2 = h\sec\frac{\pi}{n}$$

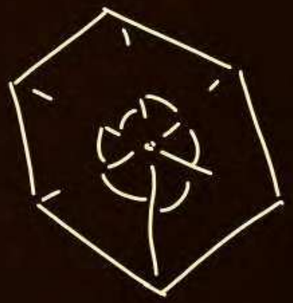
$$\Delta h = h_2 - h$$

$$= h\left(\sec\frac{\pi}{n} - 1\right)$$



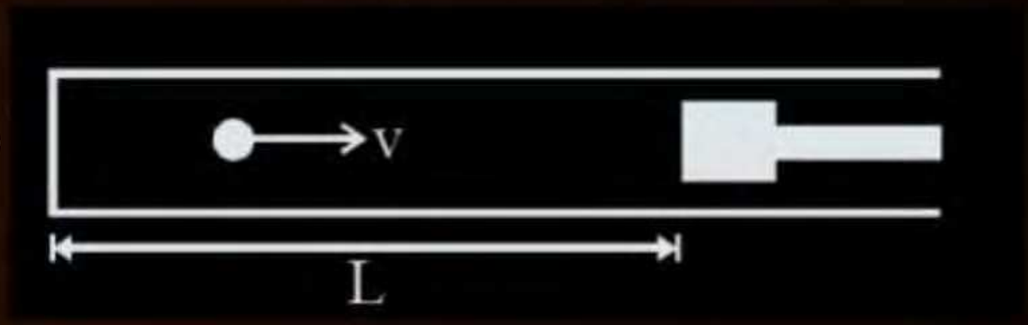
Angle made by one Side
(n Sided Polygon)

$$= \frac{2\pi}{n}$$



Question

A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction, and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V \ll \frac{dL}{L} v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ? [JEE Advanced-2019]



a. The rate at which the particle strikes the piston is v/L

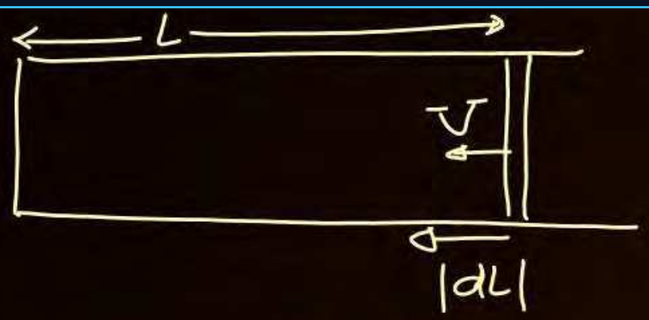
b. After each collision with the piston, the particle speed increases by 2 V

c. The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2} L_0$

d. If the piston moves inward by dL, the particle speed increases by $2v \frac{dL}{L}$

Ans: (B,C)

(C)



$$\text{time } dt = \frac{|dL|}{V}$$

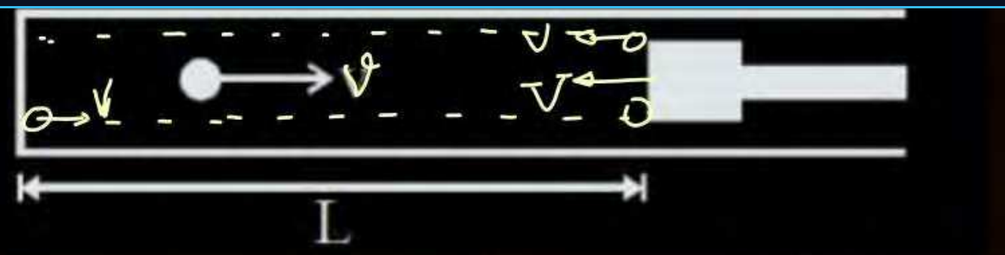
(Speed increase per Collision = $2V$)

$$\text{frequency of Collision} = \frac{\vartheta}{2L}$$

$$\text{Collisions in } dt \text{ time} = \left(\frac{\vartheta}{2L}\right)\left(\frac{|dL|}{V}\right)$$

$$\text{Change in Speed in } dt \text{ time} = \left(\frac{\vartheta}{2L}\right)\left(\frac{|dL|}{V}\right)(2V)$$

$$d\vartheta = \frac{\vartheta\,|dL|}{L}$$



(A) time b/w 2 Sucessive Collisions = $\frac{2L}{\vartheta}$

$$\text{frequency of Collision} = \frac{1}{2L/\vartheta} = \frac{\vartheta}{2L}$$

(B)

$$e = 1 = \frac{\vartheta_2 - V}{\vartheta + V} \Rightarrow \vartheta_2 = \vartheta + 2V$$

$\Delta\vartheta$ in every Collision = $2V$

$(\Delta\vartheta = \vartheta_2 - \vartheta_1)$

$$d\vartheta = \frac{\vartheta |dL|}{L} = -\frac{\vartheta dL}{L}$$

$$\int_{V_0}^{V} \frac{d\vartheta}{\vartheta} = -\int_{L_0}^{L_0/2} \frac{dL}{L}$$

$$\ln\left(\frac{V}{V_0}\right) = -\ln\left(\frac{L_0/2}{L_0}\right) = -\ln\frac{1}{2} = \ln 2$$

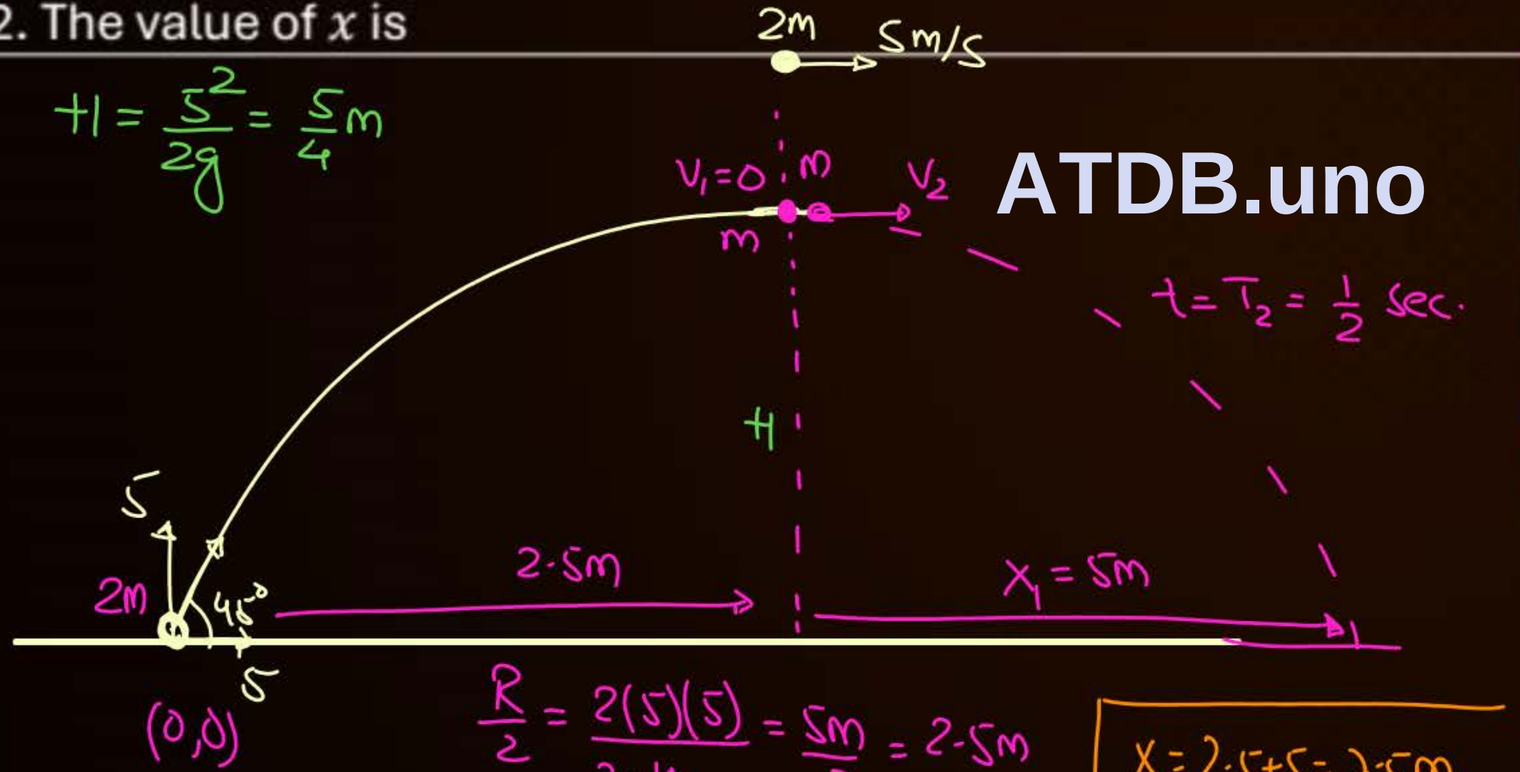
$$V = 2V_0$$

$$k\cdot\mathcal{E} = 4k\cdot\mathcal{E}_0$$

Question

A projectile is thrown from a point O on the ground at an angle 45° from the vertical and with a speed $5\sqrt{2}\ m/s$. The projectile at the highest point of its trajectory splits into two equal parts. One part falls vertically down to the ground, 0.5 s after the splitting. The other part, t seconds after the splitting, falls to the ground at a distance x meters from the point O. The acceleration due to gravity $g = 10\ m/s^2$. [JEE Advanced-2021]

1. The value of t is
2. The value of x is



Particle-1

$S = ut + \frac{1}{2}at^2$

$-\frac{5}{4} = V_1\left(\frac{1}{2}\right) - \frac{1}{2}(10)\frac{1}{4}$

$\frac{V_1}{2} = -\frac{5}{4} + \frac{5}{4} = 0$

$V_1 = 0$

$(\vec{P}_f)_x = (\vec{P}_i)_x$

$mV_2 = 2m(5)$

$V_2 = 10\ m/s$

$X_1 = V_2 T_2 = 10 \times \frac{1}{2} = 5m$

Ans: (0.50)
Ans: (7.50)

Question

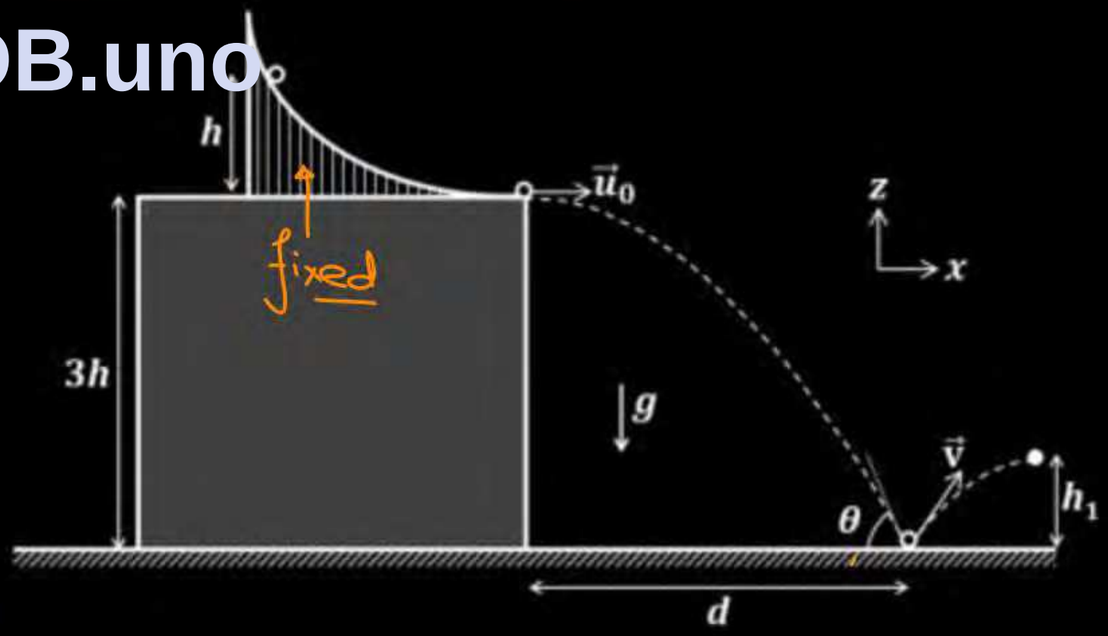
A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height $3h$ from the ground, as shown in the figure. A spherical ball of mass m is released on the slide from rest at a height h from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0\hat{x}$ and falls on the ground at a distance d from the building making an angle θ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height h_1. The acceleration due to gravity is g and the coefficient of restitution of the ground is $1/\sqrt{3}$. Which of the following statement(s) is(are) correct? [JEE Advanced-2023]

(A) $\vec{u}_0 = \sqrt{2gh}\hat{x}$

(B) $\vec{v} = \sqrt{2gh}(\hat{x} - \hat{z})$ $\{\sqrt{2gh}(\hat{x}+\hat{z})$

(C) $\theta = 60°$

(D) $d/h_1 = 2\sqrt{3}$



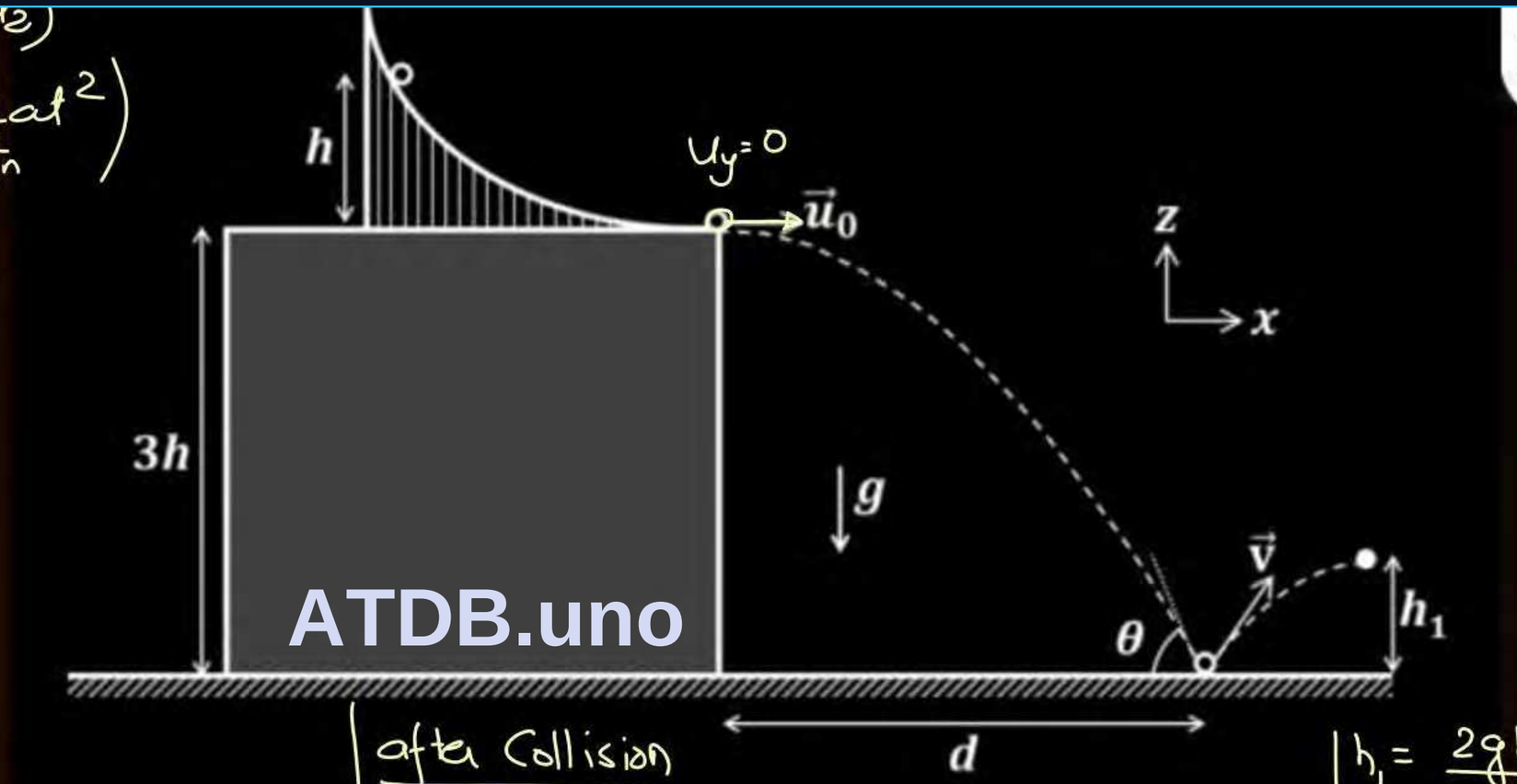
Ans: (A,C,D)

$u_0 = \sqrt{2gh}$ ($K_1 + U_1 = K_2 + U_2$)

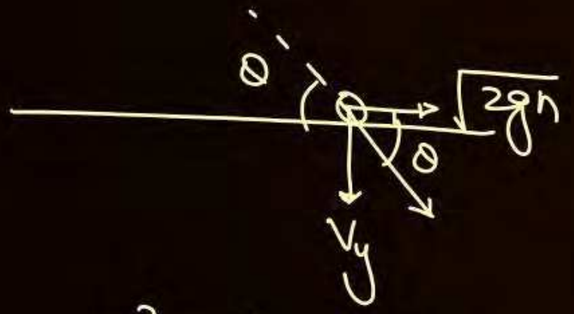
$T = \sqrt{\frac{2\times 3h}{g}}$ $\left(S = ut + \frac{1}{2}at^2 \text{ in } y\text{-dirn}\right)$

$d = u_0 T = \sqrt{2gh}\cdot\sqrt{\frac{2\times 3h}{g}}$

$d = 2\sqrt{3}h$



before



$V_y^2 = 0^2 - 2g(-3h)$

$V_y = \sqrt{6gh}$

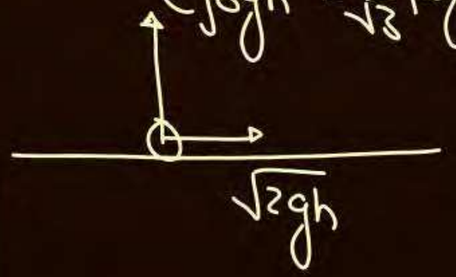
$\tan\theta = \frac{\sqrt{6gh}}{\sqrt{2gh}}$

$\tan\theta = \sqrt{3}$

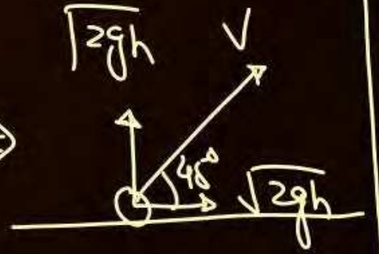
$\theta = 60^\circ$

after Collision

$e\sqrt{6gh} = \frac{1}{\sqrt{3}}\sqrt{6gh} = \sqrt{2gh}$



$\Rightarrow$



$h_1 = \frac{2gh}{2g}$

$h_1 = h$

$\frac{d}{h_1} = 2\sqrt{3}$

Question

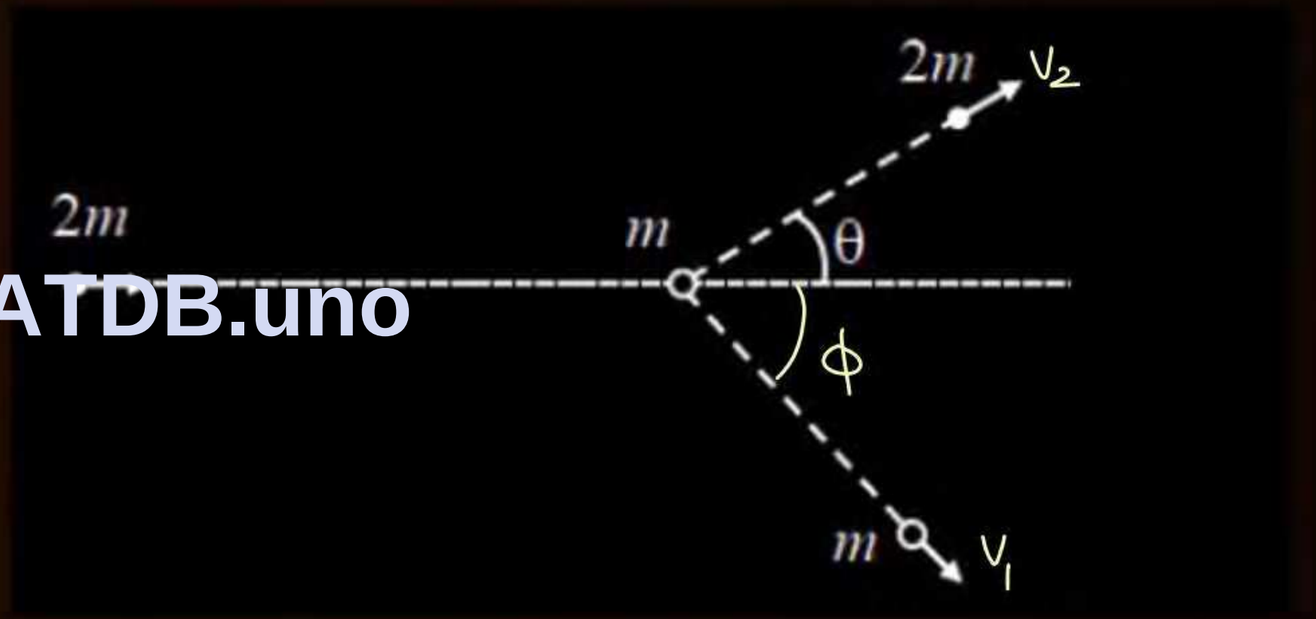
In a scattering experiment, a particle of mass $2m$ collides with another particle of mass m, which is initially at rest. Assuming the collision to be perfectly elastic, the maximum angular deviation θ of the heavier particle, as shown in the figure, in radians is: [JEE Advanced-2025]

(A) π

(B) $\tan^{-1}\left(\frac{1}{2}\right)$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{6}$



Ans: (D)

Math

$$V_1\cos\phi = 2v - 2v_2\cos\theta$$

$$V_1\sin\phi = 2v_2\sin\theta$$

$$V_1^2 = 4v^2 + 4v_2^2\cos^2\theta - 8vv_2\cos\theta + 4v_2^2\sin^2\theta$$

$$V_1^2 = 4v^2 + 4v_2^2 - 8vv_2\cos\theta = 2v^2 - 2v_2^2$$

$$6v_2^2 - 8vv_2\cos\theta + 2v^2 = 0$$

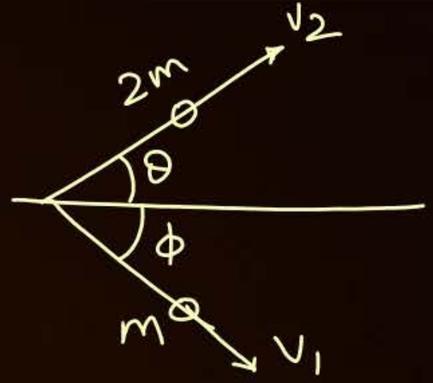
Real roots $D \geq 0$

$$64v^2\cos^2\theta - 4(6)(2v^2) \geq 0$$

$$\cos^2\theta \geq \frac{48}{64} = \frac{3}{4}$$

$$\cos\theta \geq \frac{\sqrt{3}}{2} \Rightarrow \underline{\theta \leq 30^\circ}$$

Physics



$$\vec{P}_f = \vec{P}_i \Rightarrow x\text{-dirn} \qquad 2mv_2\cos\theta + mv_1\cos\phi = 2mv$$

$$2v_2\cos\theta + v_1\cos\phi = 2v \quad -(i)$$

$$\Downarrow$$

y-dirn

$$2mv_2\sin\theta - mv_1\sin\phi = 0$$

$$2v_2\sin\theta = v_1\sin\phi \quad -(iv)$$

$e = 1$

$$\frac{1}{2}2mv_2^2 + \frac{1}{2}mv_1^2 = \frac{1}{2}2mv^2$$

$$2v_2^2 + v_1^2 = 2v^2 \quad -(iii)$$