



Chapter-4

Motion in a Plane :-

* **Physical Quantity :-** It is the property which can be measured and follow the law of physics.

eg = force, speed, Velocity, distance, displacement, temperature etc.

* **Vector :-**

Vectors are those Physical Quantity which have magnitude as well as direction and they must follow the law of vector addition.

eg = force, Velocity, displacement, acceleration, electric field, magnetic field etc.

* **Scalar :-**

Scalar are those Physical Quantity which have magnitude only and might have direction but they must not follow the law of vector addition.

eg :- distance, speed, work, time, power, energy, temperature etc.

Note :-

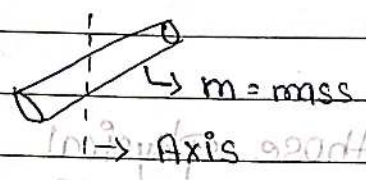
Electric current is only a Physical Quantity, which have magnitude and direction both but it is a scalar because it does not follow the law of vector addition.

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*** Tensor (Not in Syllabus) :-**
 Tensor are those Physical Quantity, whose magnitude depends on direction.

eg = Moment of inertia, refractive index, relative refractive index, relative density etc.

eg = Moment of inertia (a)



AM O I = $\frac{ml^2}{2}$ = length

(b) O O M.O.I = $\frac{ml^2}{3}$
 Axis

(c) O O M.O.I = 0

*** Representation of a Vector**

1. Symbolic Representation :-

\vec{A} = here, A is a Vector

\vec{B} = here, B is a Vector

\hat{a} or \vec{a} here, \hat{a} & \vec{a} are unit Vector.

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Simply F is also used to represent the magnitude of vector.

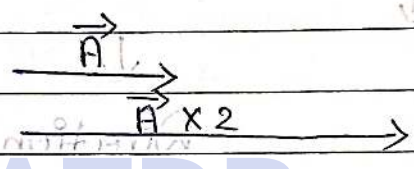
$$|F = 20 \text{ N}|$$

$\therefore F =$ This is representing the magnitude of vector \vec{F}

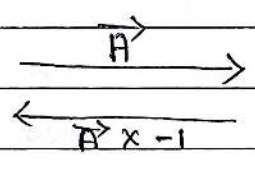
Note :-

Multiplication of a vector with **-Scaler**

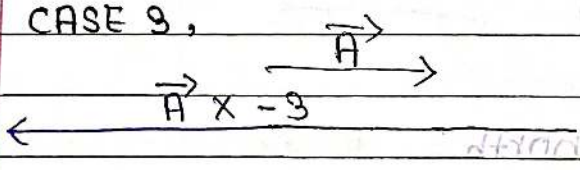
CASE 1,



CASE 2,



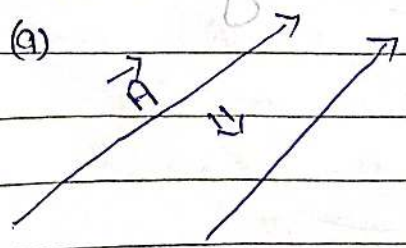
CASE 3,



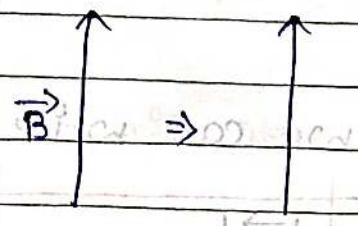
Note :-

A vector can be shifted parallel to its self.

eg = (a)



(b)

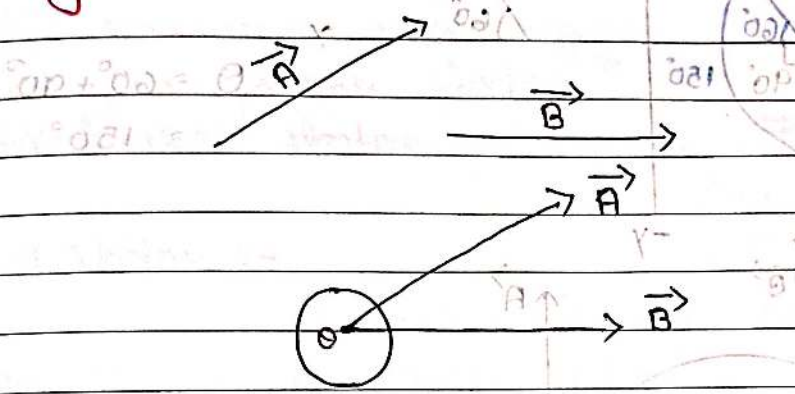


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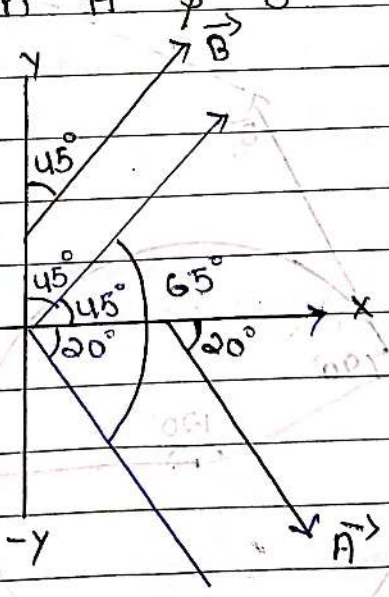


*** Angle between two vectors.**



→ when we join tails of two vectors by shifting them, then there will be the formation of two angles, among them the smaller angle will be known as angle between two vectors. here θ , is angle between \vec{A} & \vec{B} .

Q. Find Angle between \vec{A} & \vec{B}

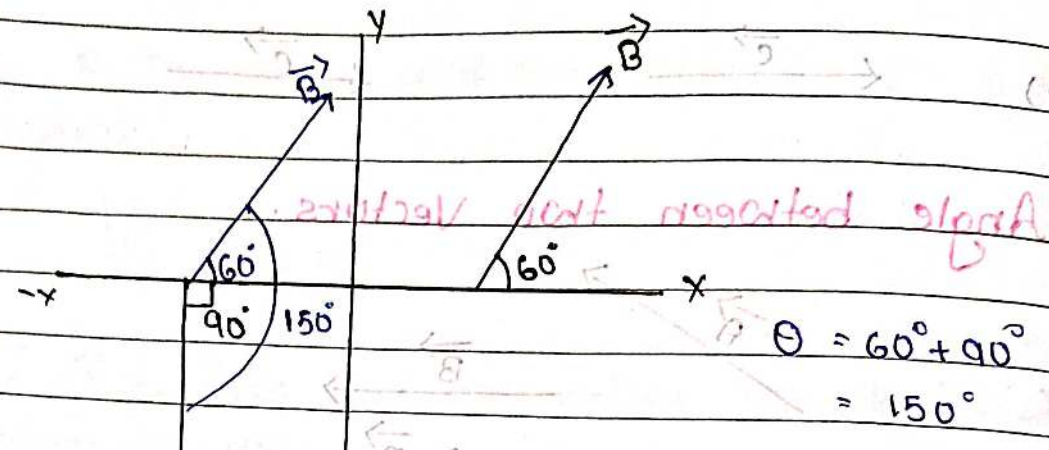


$\theta = 65^\circ$

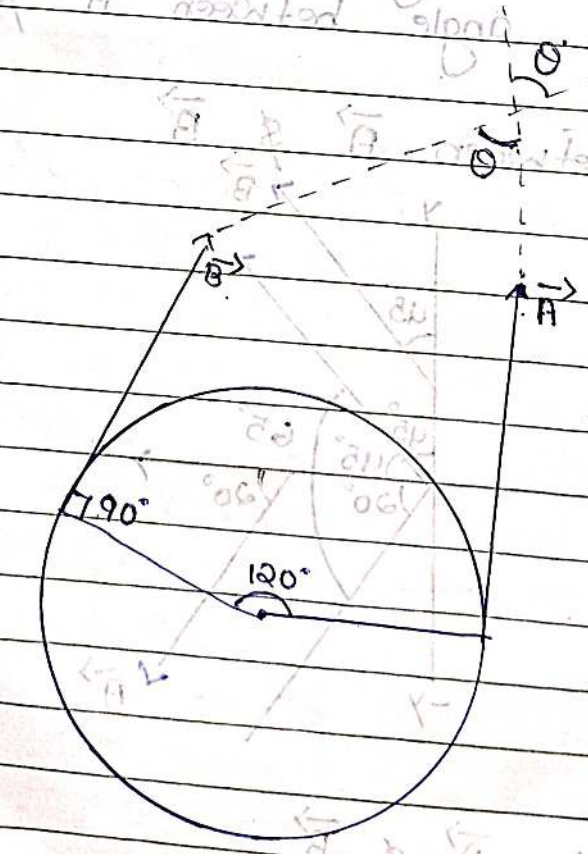
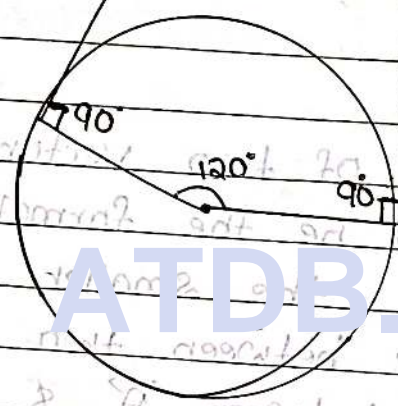
Q. Find angle b/w \vec{A} & \vec{B}

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Q.



$$360^\circ = 90 + 90 + 120 + \theta$$

$$360 = 300 + \theta$$

$$360 - 300 = \theta$$

$$60 = \theta$$

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*** Types of vector**

- (i) Null Vector or zero Vector
- (ii) collinear Vectors
- (iii) Concurrent Vectors
- (iv) Co-planer Vectors
- (v) u, v, z Unit Vectors

(i) Null Vector :-

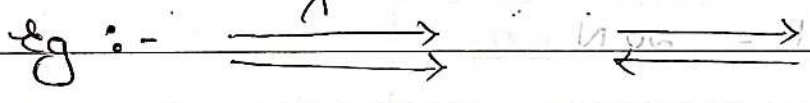
A vector having zero magnitude is called null vector or zero vector.

eg = $\vec{A} - \vec{A} = \vec{0}$ (x)

$\vec{A} \times \vec{A} = \vec{0}$ (v)

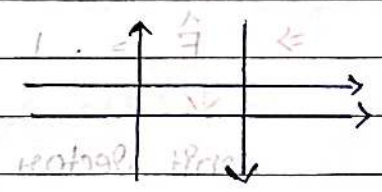
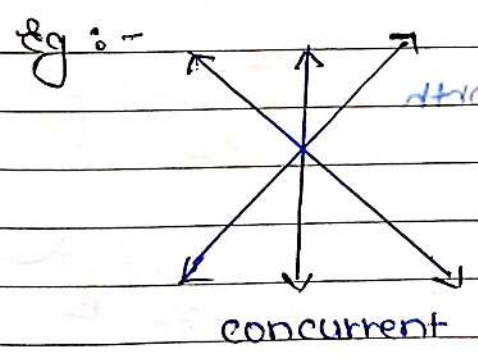
(ii) collinear Vectors :-

Vectors which are along same line, parallel or antiparallel.



(iii) Con-current Vectors :-

Vectors passing from one point.

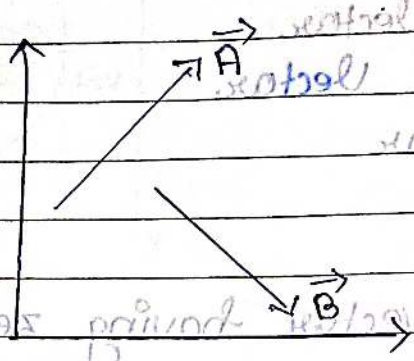


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(iv) Co-Planer Vector :-
 Vectors which are in same plane are known as co-planer vector.

eg :-



(v) Unit Vector :-
 A vector having unit magnitude is called unit vector.

- It has no SI unit.
- Symbol \hat{a} , \hat{b} , \hat{c} or \hat{a}_1 , \hat{b}_1 , \hat{c}_1 .

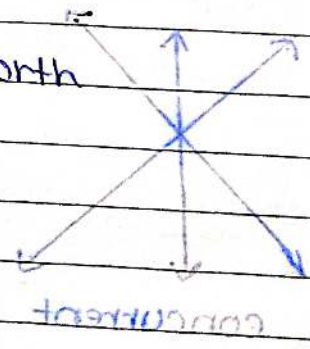
Formation of unit Vector.

eg :- $\vec{F} = 20\text{ N towards north}$
 $|\vec{F}| = 20\text{ N}$

$\Rightarrow \frac{\vec{F}}{|\vec{F}|} = \frac{20\text{ N towards north}}{20\text{ N}}$

$\Rightarrow \hat{F} = 1\text{ towards north}$

unit vector creator



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* $\frac{\vec{A}}{|\vec{A}|} = \hat{A}$

explanation of unit vector

Use of unit Vector.

→ It is used to convert a scalar into vector.

eg: $v = 5 \text{ m/s} \rightarrow \text{scalar}$

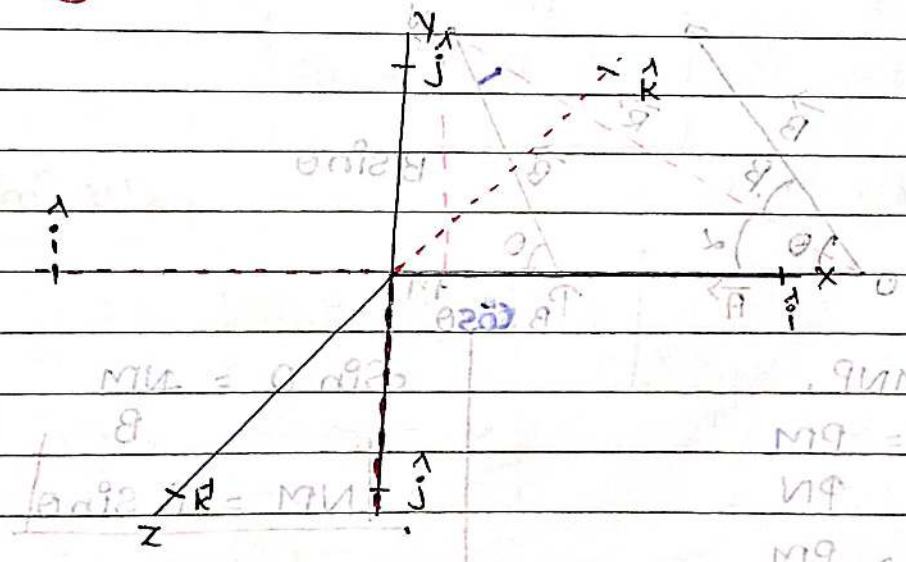
Let we have,

$\hat{v} = 1 \text{ towards north}$

$v \times \hat{v} = 5 \text{ m/s} \times 1 \text{ towards north}$

$\vec{v} = 5 \text{ m/s towards north}$

Orthogonal unit vectors



eg :- $\vec{F} = 20\hat{i} \text{ N}$

→ It means force is 20N towards x-axis.

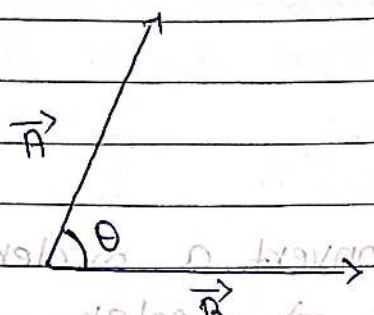
eg :- $\vec{v} = -5\hat{j} \text{ m/s}$

→ It means Velocity is 5 m/s towards -y axis.

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Addition of vectors.

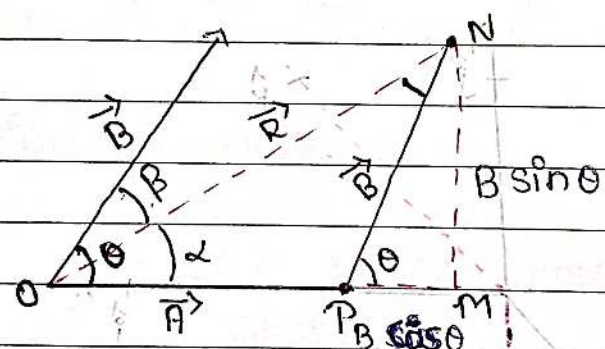


Use of unit vector

Two vectors can be added together only if they are coplanar.

$\vec{R} = \vec{A} + \vec{B}$

Resultant of \vec{A} and \vec{B}



In ΔMNP ,

$\cos \theta = \frac{PM}{PN}$

$PN = B$

$\cos \theta = \frac{PM}{B}$

$PM = B \cos \theta$

$\sin \theta = \frac{NM}{B}$

$NM = B \sin \theta$

Again,

in same ΔMNP ,

$\sin \theta = \frac{NM}{PN}$

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in Big ΔOMN ;

$$\therefore ON^2 = NM^2 + OM^2$$

$$R^2 = (B \sin \theta)^2 + (A + B \cos \theta)^2$$

$$R^2 = B^2 \sin^2 \theta + A^2 + B^2 \cos^2 \theta + 2AB \cos \theta$$

$$R^2 = B^2 \sin^2 \theta + B^2 \cos^2 \theta + A^2 + 2AB \cos \theta$$

$$R^2 = B^2 (\sin^2 \theta + \cos^2 \theta) + A^2 + 2AB \cos \theta$$

$$R^2 = B^2 + A^2 + 2AB \cos \theta$$

$$R = \sqrt{B^2 + A^2 + 2AB \cos \theta}$$

* in ΔONM

$$\sin \alpha = \frac{NM}{ON} = \frac{B \sin \theta}{R}$$

$$\sin \alpha = \frac{B \sin \theta}{R}$$

Similarly,

$$\sin \beta = \frac{A \sin \theta}{R}$$

$$\tan \alpha = \frac{NM}{OM}$$

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

Similarly,

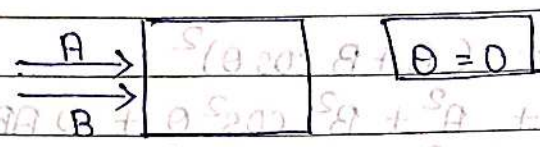
$$\tan \beta = \frac{A \sin \theta}{B + A \cos \theta}$$

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* $R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

CASE I.



$R = A + B$

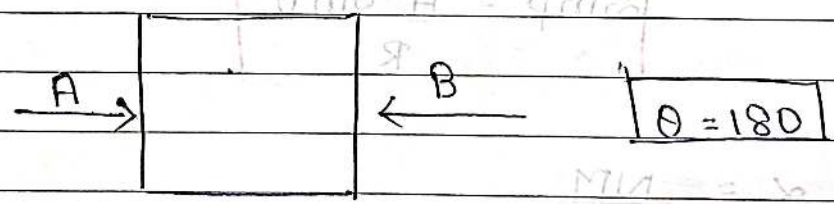
$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

$= \sqrt{A^2 + B^2 + 2AB \times 1}$

$= \sqrt{(A+B)^2}$

$= A+B$

CASE II.



$R = A - B$

$R = \sqrt{A^2 + B^2 + 2AB \cos 180^\circ}$

$\because \cos 180^\circ = -1$

$= \sqrt{A^2 + B^2 + 2AB \times (-1)}$

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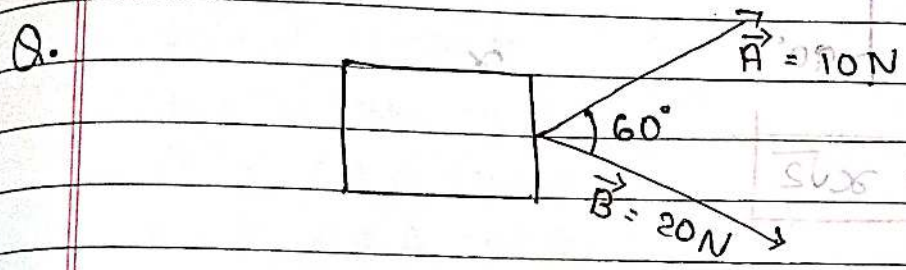
$$= \sqrt{A^2 + B^2 - 2AB}$$

$$= \sqrt{(A-B)^2}$$

$$= R = A - B$$

Maximum Resultant = $A + B$

Minimum Resultant = $A - B$



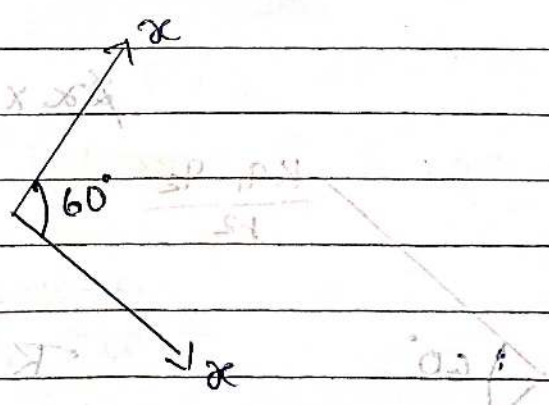
$$\rightarrow R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$= \sqrt{10^2 + 20^2 + 2 \times 10 \times 20 \times \frac{1}{2}}$$

$$= \sqrt{100 + 400 + 200}$$

$$= \sqrt{700} = 26.45$$

Trick ① :-



$$R = \sqrt{x^2 + x^2 + 2x \times x \times \cos 60^\circ}$$

$$= \sqrt{2x^2 + 2x^2 \times \frac{1}{2}}$$

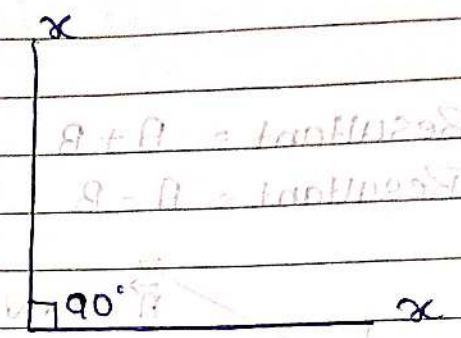
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$$= \sqrt{2x^2 + x^2}$$

$$= \sqrt{3x^2} = x\sqrt{3}$$

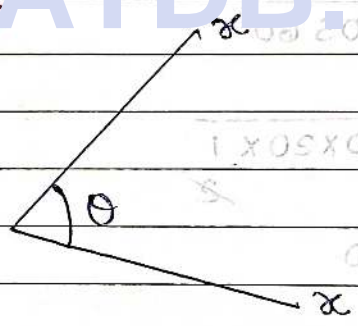
Trick 2.



$$R = x\sqrt{2}$$

Main trick

ATDB.uno



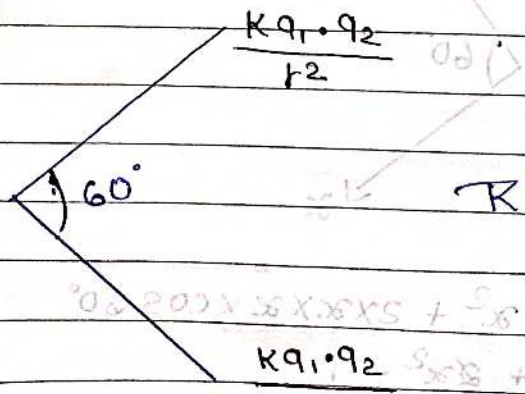
$$R = 2x \cos\left(\frac{\theta}{2}\right)$$

$$\because \theta = 60^\circ$$

$$2x \cos 60^\circ$$

$$\frac{2x \times \sqrt{3}}{2} = x\sqrt{3}$$

*



$$\frac{kq_1 \cdot q_2}{r^2}$$

$$R = \frac{kq_1 \cdot q_2 \sqrt{3}}{r^2}$$

$$\frac{kq_1 \cdot q_2}{r^2}$$

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Q. If Resultant of two unit vector is also a unit vector then find the angle between that two unit vectors.

⇒ $A = 1, B = 1, R = 1$

$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$1 = \sqrt{1 + 1 + 2 \cos \theta}$$

$$1 = \sqrt{2 + 2 \cos \theta}$$

$$1 = \sqrt{2 * 2 \cos \theta}$$

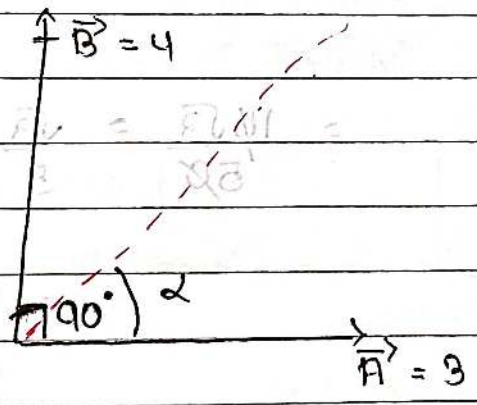
$$1^2 = 2 * 2 \cos \theta$$

$$1 = 2 * 2 \cos \theta$$

$$1 - 2 = 2 \cos \theta$$

$$\frac{-1}{2} = \cos \theta$$

$$\theta = 120^\circ \text{ Ans}$$



find angle between \vec{A} & resultant

$$\tan \alpha = \frac{B \sin \theta}{A + B \cos \theta}$$

$$\tan \alpha = \frac{4 \sin 90^\circ}{3 + 4 \cos 90^\circ}$$

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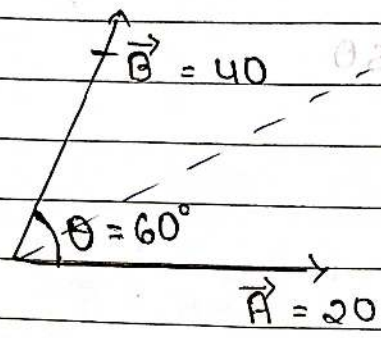
$$\tan \alpha = \frac{4 \times 1}{3 + 4 \times 0}$$

$$= \frac{4}{3}$$

$$\tan \alpha = \frac{4}{3}$$

$$\alpha = 53^\circ$$

Q.



Find angle between Resultant & \vec{B} .

$$\rightarrow \tan \beta = \frac{A \sin \theta}{B + A \cos \theta}$$

$$= \frac{20 \sin 60}{40 + 20 \cos 60}$$

$$= \frac{20 \times \frac{\sqrt{3}}{2}}{40 + 20 \times \frac{1}{2}}$$

$$= \frac{10\sqrt{3}}{50} = \frac{\sqrt{3}}{5}$$

$$\tan \beta = \frac{\sqrt{3}}{5}$$

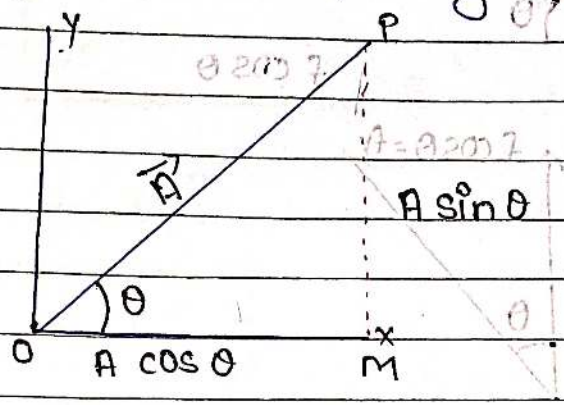
$$\beta = \tan^{-1} \left(\frac{\sqrt{3}}{5} \right)$$

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* component of Vector

→ It is the process of splitting a vector.



in ΔOMP ,

$$\sin \theta = \frac{PM}{OP}$$

$$\cos \theta = \frac{OM}{OP}$$

$$\sin \theta = \frac{A \sin \theta}{A}$$

$$\cos \theta = \frac{A \cos \theta}{A}$$

$$A \sin \theta = PM$$

$$A \cos \theta = OM$$

eg: $\frac{10}{20} \times \sqrt{3} = 10\sqrt{3}$
 $f \sin 60^\circ$

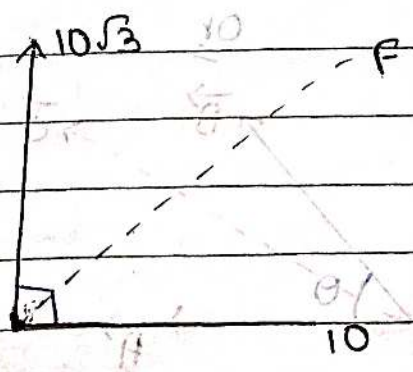
$f = 20N$

$$f \cos 60^\circ = 20 \times 1 = 10N$$

$$10\sqrt{3} = 10 \times 1.732$$

$$= 17.32$$

check,

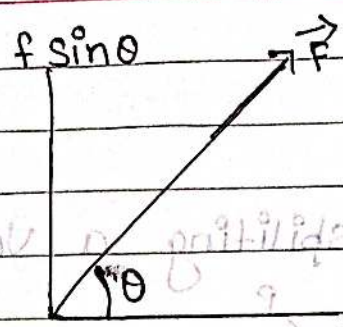


$$C = \sqrt{(10)^2 + (10\sqrt{3})^2 + 2 \times 10 \times 10\sqrt{3} \cos 60^\circ}$$

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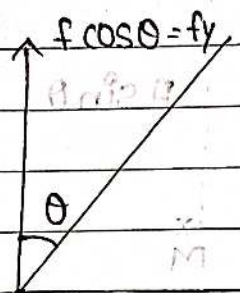
*



Component of Vector

It is the process of splitting a vector into its components

*

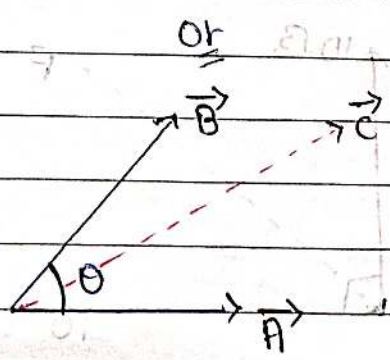


$f \sin \theta = f_x$

Note :- $\vec{f} = f \sin \theta \hat{i} + f \cos \theta \hat{j}$

Note :- अगर किसी Question में ऐसा लिखा हो कि \vec{C} is the resultant of \vec{A} & \vec{B} तो इसका मतलब हम यह समझेंगे कि $\vec{C} = \vec{A} + \vec{B}$

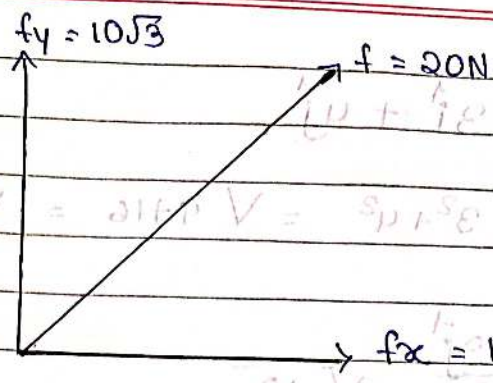
Or $C = \sqrt{A^2 + B^2 + 2AB \cos \theta}$



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Note :-



$$\vec{f} = f_x \hat{i} + f_y \hat{j}$$

$$\vec{f} = 10\hat{i} + 10\sqrt{3}\hat{j}$$

$$f = \sqrt{10^2 + (10\sqrt{3})^2 + 2 \times 10 \times 10\sqrt{3} \cos 90^\circ}$$

$$f = \sqrt{10^2 + (10\sqrt{3})^2}$$

$$f = \sqrt{f_x^2 + f_y^2}$$

It means

* Magnitude of vector, given in the form of i, j .

$$\vec{f} = 10\hat{i} + 10\sqrt{3}\hat{j}$$

$$f = \sqrt{10^2 + (10\sqrt{3})^2}$$

$$f = 20$$

eg :- $\vec{A} = 2\hat{i} + 3\hat{j}$

$$\begin{aligned} A &= \sqrt{2^2 + 3^2} \\ &= \sqrt{4 + 9} \\ &= \sqrt{13} \end{aligned}$$

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eg:- $\vec{B} = 3\hat{i} + 4\hat{j}$

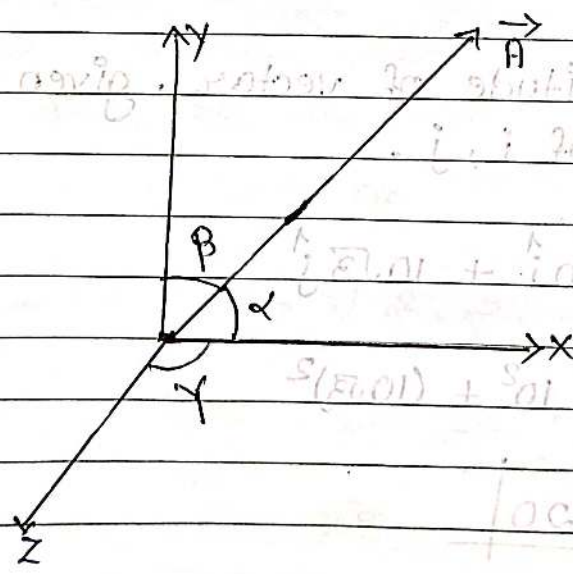
$B = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$

* $\vec{A} = 2\hat{i} + 3\hat{j}$
 $A = \sqrt{2^2 + 3^2} = \sqrt{13}$

* $\vec{A} = 2\hat{i} + 3\hat{j} + \hat{k}$
 $= \sqrt{2^2 + 3^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$

* $\vec{B} = 2\hat{i} - 3\hat{j} + 4\hat{k}$
 $= \sqrt{2^2 + (-3)^2 + 4^2} = \sqrt{4 + 9 + 16} = \sqrt{29}$

* Component of vector in 3-D



$A_x = A \cos \alpha$
 $A_y = A \cos \beta$
 $A_z = A \cos \gamma$

$\hat{i} \cos \alpha + \hat{j} \cos \beta + \hat{k} \cos \gamma = \frac{\vec{A}}{A}$
 $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

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$$\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$A = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

* Addition of vectors given in the form of $\hat{i}, \hat{j}, \hat{k}$.

$$\vec{A} = 2\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{B} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{R} = \vec{A} + \vec{B} = 3\hat{i} - \hat{j} - 2\hat{k}$$

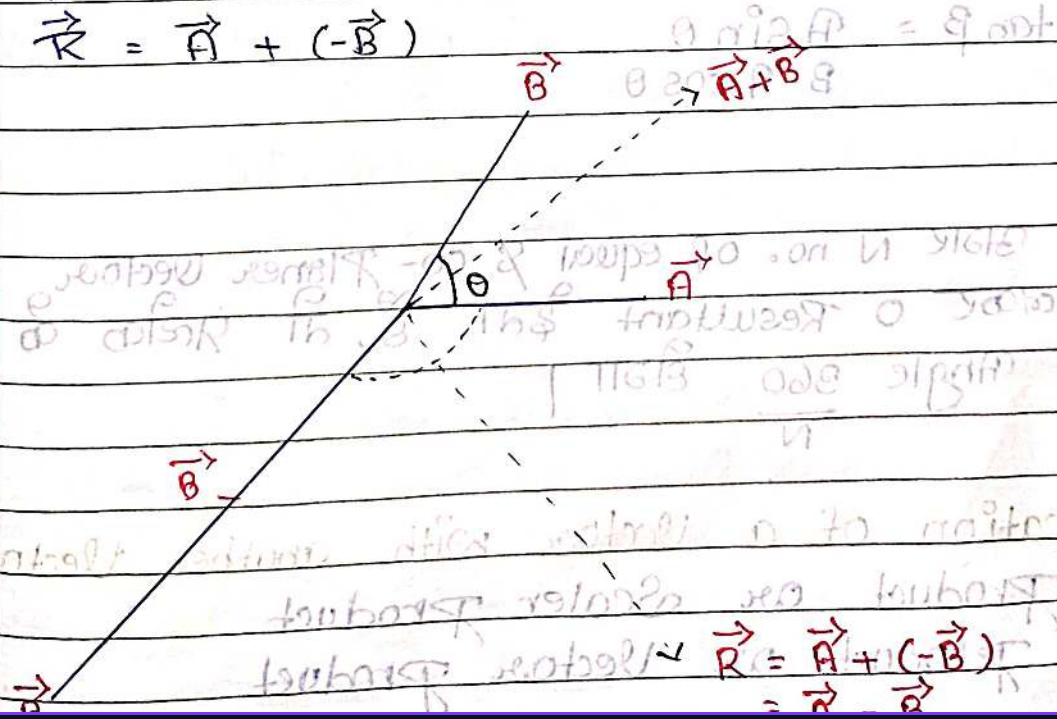
* $\vec{A} = 2\hat{i} + 3\hat{j}$
 $\vec{B} = -\hat{i} + \hat{k}$

$$\vec{A} + \vec{B} = \hat{i} + 3\hat{j} + \hat{k}$$

* Subtraction of vector

$$\vec{R} = \vec{A} - \vec{B}$$

$$\vec{R} = \vec{A} + (-\vec{B})$$



$$\vec{R} = \vec{A} + (-\vec{B})$$

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$$R = \sqrt{A^2 + B^2 + 2AB \cos \theta}$$

$$R = \sqrt{A^2 + B^2 + 2AB \cos (180 - \theta)}$$

$$R = \sqrt{A^2 + B^2 + 2AB \cos (90 + 90 - \theta)}$$

$$R = \sqrt{A^2 + B^2 + 2AB \times -\sin (90 - \theta)}$$

$$R = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

Note :- $\vec{R} = \vec{A} - \vec{B}$
 it means,

$$R = \sqrt{A^2 + B^2 - 2AB \cos \theta}$$

$$\therefore \sin \alpha = \frac{B \sin \theta}{R}$$

$$\therefore \cos \beta = \frac{A \sin \theta}{R}$$

$$\therefore \tan \alpha = \frac{B \sin \theta}{A - B \cos \theta}$$

$$\therefore \tan \beta = \frac{A \sin \theta}{B - A \cos \theta}$$

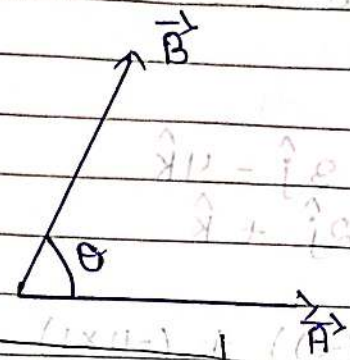
Note :- अगर N no. of equal & co-Planer Vectors
 मिनकर 0 Resultant है, तो प्रत्येक के बीच
 का Angle $\frac{360}{N}$ होगा।

- * Multiplication of a Vector with another Vector.
- Dot product or scalar product
- cross product or vector product

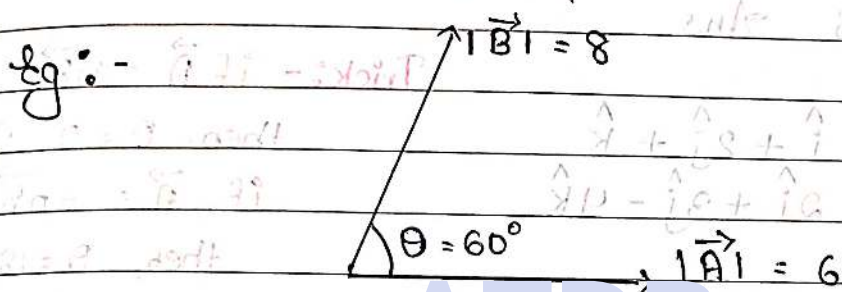
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• Dot Product :-



$$\vec{A} \cdot \vec{B} = AB \cos \theta$$



$$\begin{aligned} \vec{A} \cdot \vec{B} &= AB \cos \theta \\ &= 8 \times 6 \times 1 \\ &= 24 \end{aligned}$$

Note :- If, $\theta = 0$
 $\vec{A} \cdot \vec{B} = AB \cos 0^\circ = AB$ (maximum)

If, $\theta = 90^\circ$
 $\vec{A} \cdot \vec{B} = AB \cos 90^\circ = 0$

If, $\theta = 180^\circ$
 $\vec{A} \cdot \vec{B} = AB \cos 180^\circ$

$\hookrightarrow -1$
 $= -AB$

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* Dot Product of two Vectors Given in the form of $\hat{i}, \hat{j}, \hat{k}$:-

eg :- $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$
 $\vec{B} = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{A} \cdot \vec{B} = (2 \times 1) + (3 \times -2) + (-4 \times 1)$
 $= 2 - 6 - 4$
 $= -8$ Ans

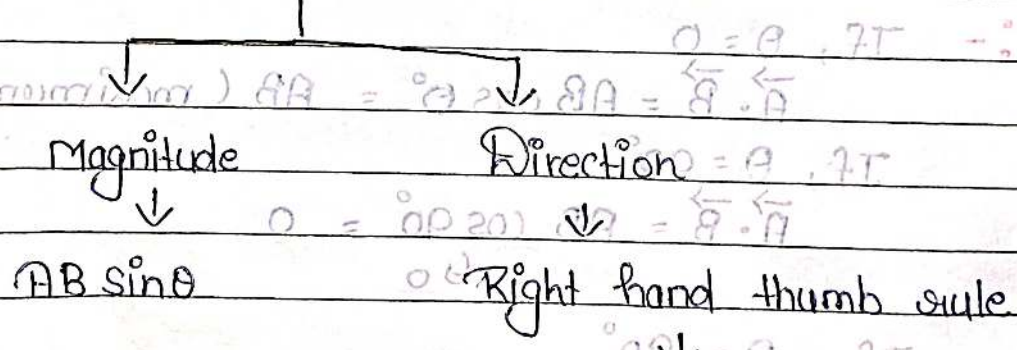
eg :- $\vec{A} = \hat{i} + 2\hat{j} + \hat{k}$
 $\vec{B} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$\vec{A} \cdot \vec{B} = 2 + 6 - 4$
 $= 4$ Ans

Trick:- if $\vec{A} = n\vec{B}$ then, $\theta = 0^\circ$ Always
 if, $\vec{A} = -n\vec{B}$ then, $\theta = 180^\circ$

• Cross Product (Vector Product)

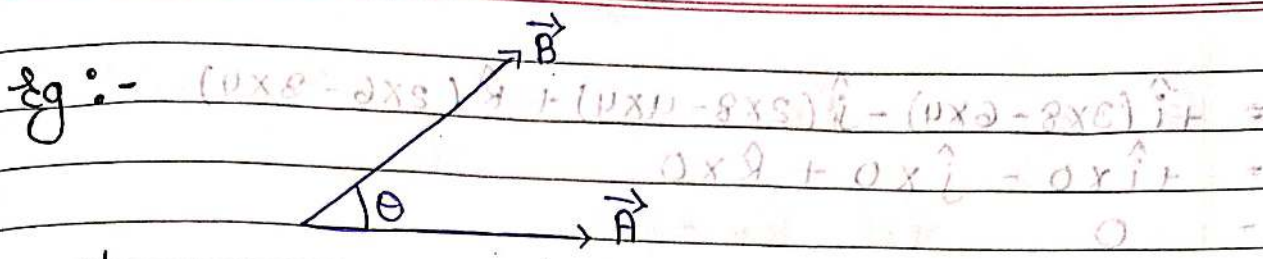
$\vec{A} \times \vec{B} = \text{Vector}$



this rule if we put our fingers of right hands on the vector which is written first & curl our fingers towards another vector then, thumb will give the direction

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(saathi)



$|\vec{A} \times \vec{B}| = AB \sin \theta$

Direction of $\vec{A} \times \vec{B} = \odot$ (Outward)

Direction of $\vec{B} \times \vec{A} = \otimes$ (Inward)

Note :- जब दो Vectors का cross product किया जाता है, तो जो उसका result होता है वह हमेशा दोनों Vectors पर Perpendicular होता है।

or

साथ ही साथ उस पूरे Plane पर Perpendicular होता है जिसमें दोनों Vectors मौजूद हैं।

cross product of two vectors given in the form of $\hat{i}, \hat{j}, \hat{k}$.

Determinant :-

$\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$
 $\vec{B} = 4\hat{i} + 6\hat{j} + 8\hat{k}$

$\vec{A} \times \vec{B} =$	\hat{i}	\hat{j}	\hat{k}
	2	3	4
	4	6	8

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$$= +\hat{i}(3 \times 8 - 6 \times 4) - \hat{j}(2 \times 8 - 4 \times 4) + \hat{k}(2 \times 6 - 3 \times 4)$$

$$= +\hat{i} \times 0 - \hat{j} \times 0 + \hat{k} \times 0$$

$$= 0$$

* Dot & cross product

$$|\vec{A} \times \vec{B}| = AB \sin \theta$$

Dot :-

$$\hat{i} \times \hat{j} = 0 \quad (\because \theta = 90^\circ)$$

$$-\hat{i} \cdot \hat{k} = 0$$

$$\hat{k} \cdot \hat{j} = 0$$

$$\hat{i} \cdot \hat{i} = |\hat{i}| \times |\hat{i}| \cos \theta = 1 \times 1 \times 1 = 1$$

$$\hat{j} \cdot \hat{j} = 1$$

$$\hat{k} \cdot \hat{k} = 1$$

cross :-

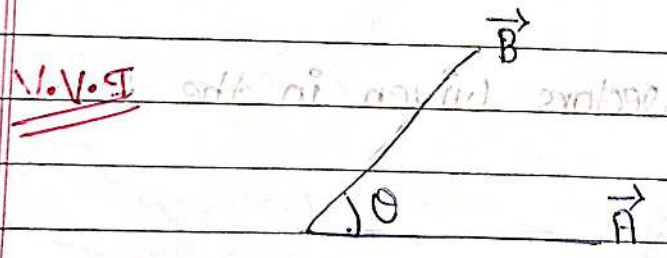
$$\hat{i} \times \hat{i} = 0$$

$$\hat{j} \times \hat{j} = 0$$

$$\hat{k} \times \hat{k} = 0$$

$$\hat{i} \times \hat{j} = 1 \times 1 \times \sin 90^\circ = 1$$

$$-\hat{j} \times \hat{i} = 1 \times 1 \times \sin 90^\circ = 1$$



$$|\vec{A} \times \vec{B}| = |\vec{B} \times \vec{A}|$$

(मान) (मान)

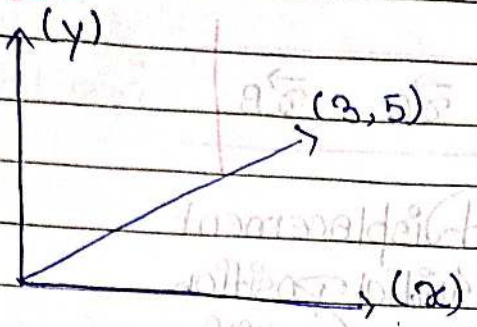
$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

but $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ (✓) $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$ (✓)

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*** Position Vector**



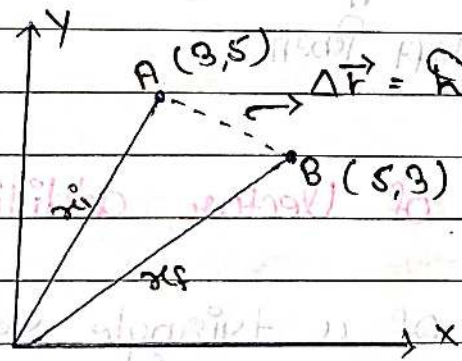
$$\vec{r} = 3\hat{i} + 5\hat{j}$$

→ Complete in figure formation of location.

*** Displacement :-**

change in position.

*** Displacement = final position - Initial position. ***



$\Delta \vec{r} =$ Displacement Vector.

Q. Find displacement? -

→ $\vec{r}_i = 3\hat{i} + 5\hat{j}$, $\vec{r}_f = 5\hat{i} + 3\hat{j}$

*** $\Delta \vec{r} = \vec{r}_f - \vec{r}_i$**
 $\Delta \vec{r} = (5\hat{i} + 3\hat{j}) - (3\hat{i} + 5\hat{j})$
 $= 5\hat{i} + 3\hat{j} - 3\hat{i} - 5\hat{j}$

2nd Method
 $\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$
 ↓
 Position of B

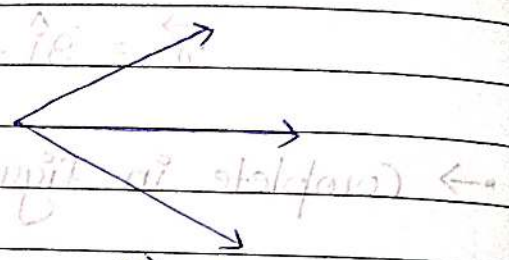
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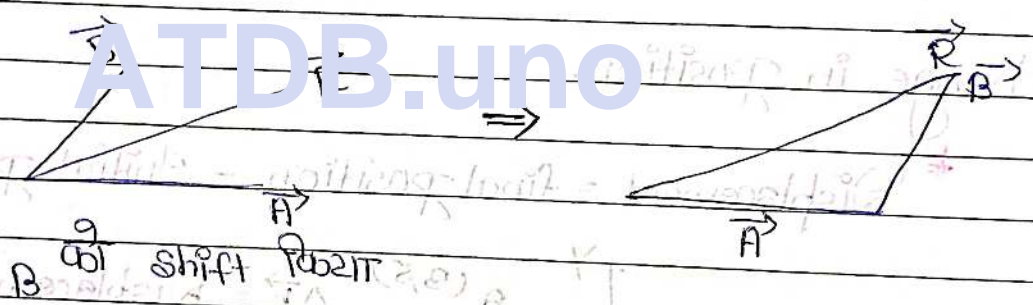
* $\vec{r}_{BA} = \vec{r}_B - \vec{r}_A$

$\vec{r}_{BA} + \vec{r}_A = \vec{r}_B$

∴ \vec{r}_{BA} = Displacement
 \vec{r}_A = Initial Position
 \vec{r}_B = Final Position

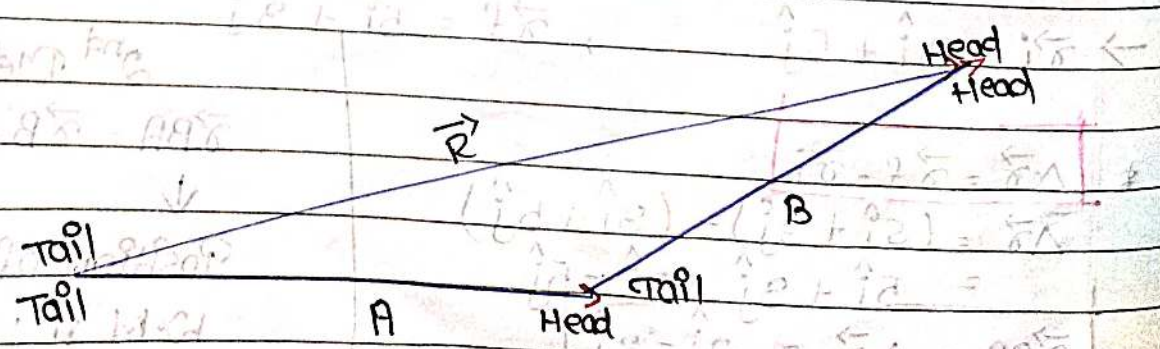


V.V.I
 Baat Alag se :-



* Triangle law of Vector addition :-

→ If two sides of a triangle represent the Vectors in particular order, then the 3rd remaining sides will represent the resultant that two Vectors.



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Velocity

$$\vec{r} = r_x \hat{i} + r_y \hat{j}$$

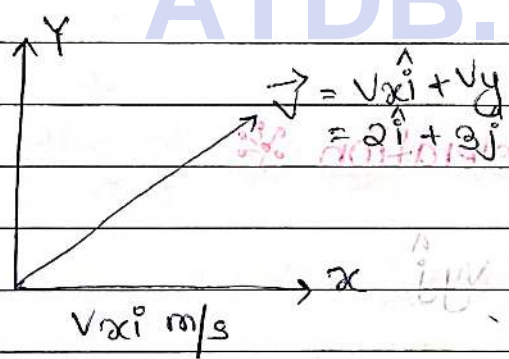
Position Vector

Now, Differentiating w.r.t time,

$$\frac{d\vec{r}}{dt} = \frac{dr_x}{dt} \hat{i} + \frac{dr_y}{dt} \hat{j}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

eg:-



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→ It means Velocity is 2 m/s towards x-axis & 3 m/s towards y-axis.

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

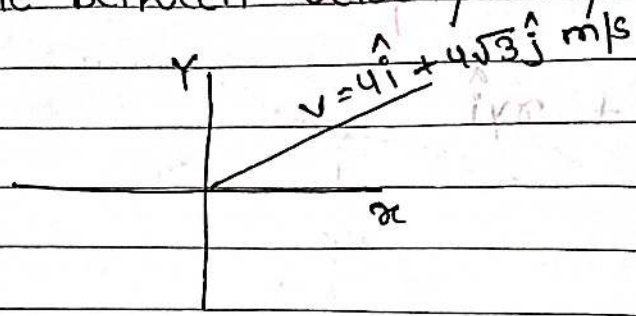
$$\Rightarrow v = \sqrt{2^2 + 3^2}$$

Note:- $\tan \theta = \text{slope}$
 $\tan \theta = \frac{v_y}{v_x}$

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Q. Find angle between Velocity \vec{v} & x-axis?



Solⁿ:- $\tan \theta = \frac{v_y}{v_x}$

$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right)$

$\theta = \tan^{-1} \left(\frac{4\sqrt{3}}{4} \right)$

$\theta = \tan^{-1} \sqrt{3}$

$\theta = 60^\circ$

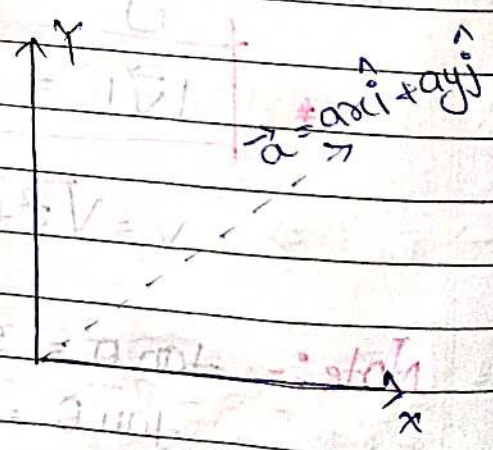
✱ Acceleration ✱

$\therefore \vec{v} = v_x \hat{i} + v_y \hat{j}$

Now, differentiating w.r.t time

$\frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$

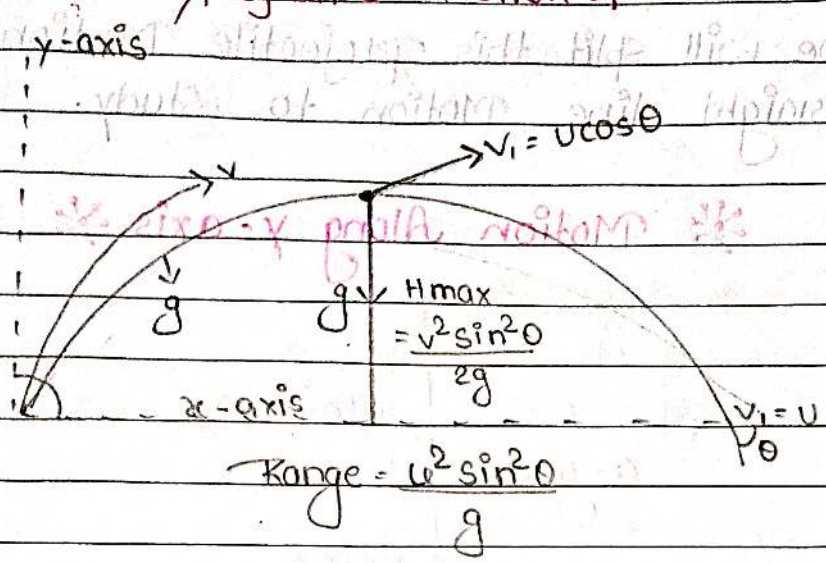
$\vec{a} = a_x \hat{i} + a_y \hat{j}$



saathi

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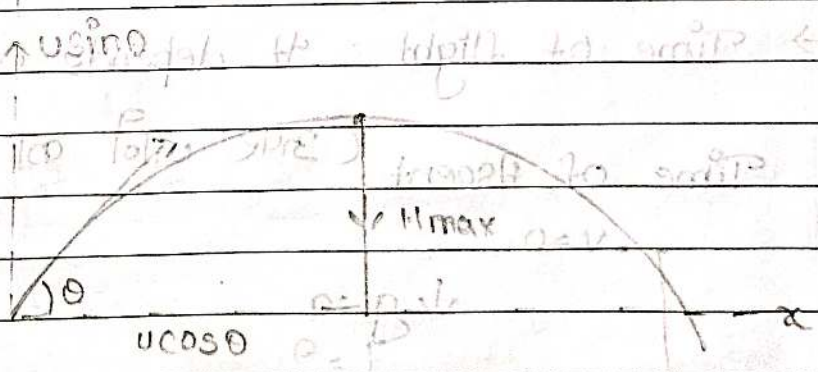
* Projectile Motion *



→ It is a two dimensional acdⁿ motion in which particles moves on parabolic path.

$t_f = \frac{2u \sin \theta}{g}$
↓
time of flight

∴ Deviation :-

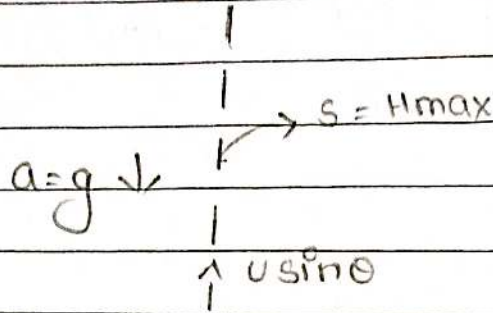


- $u \cos \theta$ → Responsible for motion along x-axis.
- $u \sin \theta$ → Responsible for motion along y-axis.
- H_{max} → It depends on only $u \sin \theta$

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→ We will split this Projectile Motion in two straight line motion to study.

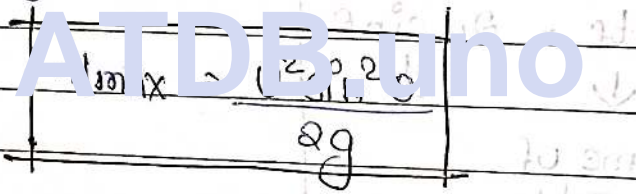
✱ Motion Along y-axis ✱



$$v^2 = u^2 - 2as$$

$$0 = (u \sin \theta)^2 - 2 \times g \times H_{\max}$$

$$2g H_{\max} = u^2 \sin^2 \theta$$

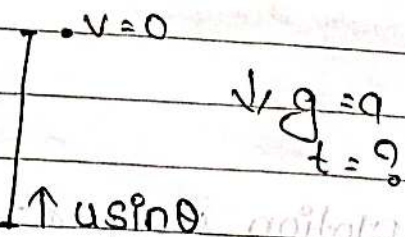


→ At H_{\max} , Velocity = $u \cos \theta$ along x-axis or range.

→ At H_{\max} , Kinetic Energy = $\frac{1}{2} m (u \cos \theta)^2$

→ Time of flight = It depends on $u \sin \theta$.

• Time of Ascent (आर जान का time t_a):-



$$v = u - at$$

$$0 = u \sin \theta - g t_a$$

$$g t_a = u \sin \theta$$

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Time of ascent = Time of descent
(t_a) (t_d)

∴ total time of flight = t_a + t_d
= U sin θ + U sin θ

t_f = $\frac{2U \sin \theta}{g}$

* Distance along x-axis = Range = Speed x Time

= U cos θ x t_f
= U cos θ x $\frac{2U \sin \theta}{g}$

R = $\frac{2 \sin \theta \times \cos \theta \times U^2}{g}$

R = $\frac{U^2 \sin 2\theta}{g}$

Q. If a ball is projected at an angle 30° with surface of earth with velocity 80 m/s then calculate. (m = 1 kg)

- i) H_{max}
- ii) Time of flight
- iii) Range
- iv) Velocity at H_{max}
- v) Kinetic energy at H_{max}
- vi) Velocity of landing

i) H_{max} = $U^2 \sin^2 \theta$ = $80^2 \times \sin^2 30$ = $6400 \times \frac{1}{4}$

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= 80

ii) $t_f = \frac{2u \sin \theta}{g} = \frac{2 \times 80 \times 1}{2} = 8 \text{ sec}$

iii) $R = \frac{u^2 \sin 2\theta}{g} = \frac{6400 \times \sqrt{3}}{2} = 3200\sqrt{3}$

iv) Velocity of $H_{\text{max}} = u \cos \theta = \frac{80 \times \sqrt{3}}{2} = 40\sqrt{3}$

v) K.E of $H_{\text{max}} = \frac{1}{2} \times m (u \cos \theta)^2$
 $= \frac{1}{2} \times 1 \times (40\sqrt{3})^2$
 $= \frac{1}{2} \times 4800$

= 2400 J

vi) Velocity of landing = $t_a = t_d = 80 \text{ m/s}$

Q. Find the angle for which range & max^m height are equal.

⇒ Range = H_{max}
 $\frac{u^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$

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$$\Rightarrow \frac{2 \sin \theta \cdot \cos \theta}{2} = \frac{\sin \theta \times \sin \theta}{2} \quad \text{--- (i)}$$

$$\Rightarrow \frac{4 \cos \theta}{4} = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow 4 = \tan \theta$$

$$\Rightarrow \theta = \tan^{-1} 4$$

$$\Rightarrow \theta = 76^\circ$$

Q. Find the angle for which range of projectile will be max^m.

$$\Rightarrow R = \frac{u^2 \sin 2\theta}{g}$$

∴ Value of sin will be maximum at sin 90°

∴ In sin 2θ ; θ = 45°

→ Range will be maximum at θ = 45°

Q. Find the angle for which H_{max} will be max^m.

$$\Rightarrow \theta = 90^\circ ; H_{\max} \text{ will be max}^m$$

Q. If a ball is projected at an angle 60° with velocity 80 m/s then find H_{max} & range.

$$\begin{aligned} \Rightarrow H_{\max} &= \frac{u^2 \sin^2 \theta}{2g} \\ &= \frac{80^2 \times \sin^2 60^\circ}{2 \times 10} \\ &= \frac{6400 \times 3}{4} \\ &= 80 \times 3 = 2400 \end{aligned}$$

$$\begin{aligned} \because u &= 80 \\ \theta &= 60^\circ \\ R &= \frac{u^2 \sin 2\theta}{g} \\ &= \frac{6400 \times \sin 2 \times 60^\circ}{10} = \frac{6400 \times \sqrt{3}}{2} \end{aligned}$$

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Note :-

(i) अगर किसी object को θ angle या $90^\circ - \theta$ angle पर फेंका जाए तो Range बराबर होगा।
 H_{max} अलग-अलग होगा।

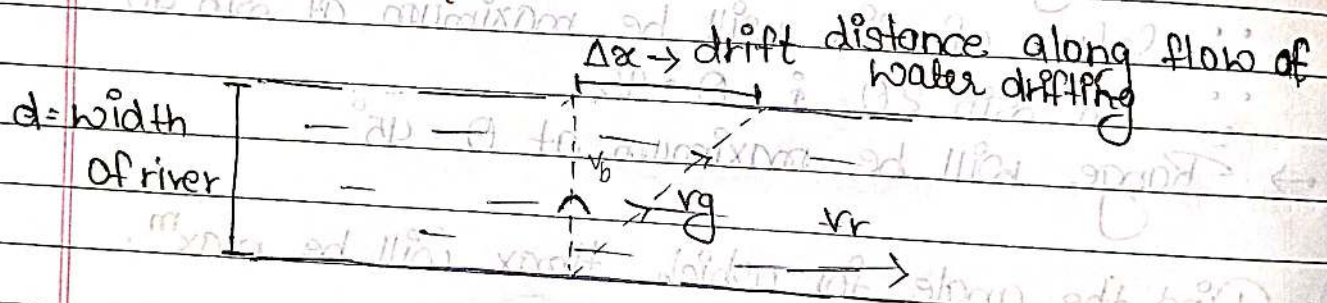
Or,

इस situation में range बराबर $R = 4u \cdot H_1 \cdot H_2$

(ii) जब range maxm होता है ($\theta = 45^\circ$) तो

$H_{max} = R_{max}$ Or, $R_{max} = u \times H_{max} \times \cot \theta$
 जहाँ $\frac{4}{u}$ होता है।

River Problem



v_r = Velocity of river.
 v_b = Velocity of boat.

This is the situation of minimum possible time of crossing the river.

$t_c = \frac{\text{distance}}{\text{speed}} = \frac{d}{v_b}$

time of crossing

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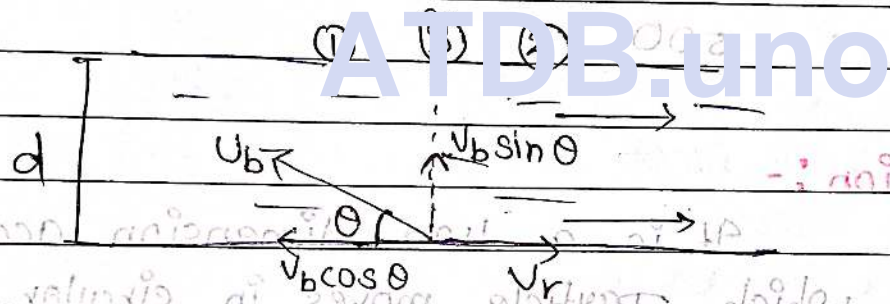
Now, drift distance = $\Delta x = \text{Speed} \times \text{time}$
 $= V_r \times t$

$$\Delta x = V_r \times \frac{d}{V_b}$$

$\therefore V_g =$ Velocity of boat w.r.t ground.

CASE II

→ To reach just opposite side of river.



$V_b \sin \theta =$ responsible for crossing the river.

$t_c = \frac{\text{dist}}{\text{speed}}$

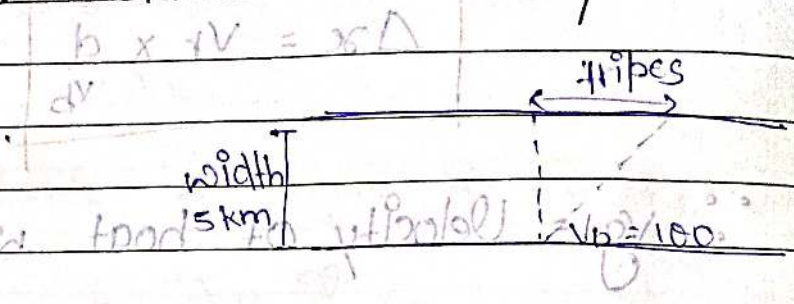
$$t_c = \frac{d}{V_b \sin \theta}$$

- ① $V_b \cos \theta > V_r$
- ② $V_r > V_b \cos \theta$
- ③ $V_b \cos \theta = V_r$

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Q. If a boat can move with a maximum speed of 100 m/s then calculate minimum possible time to cross the river of width 5 km and also find the drift distance if velocity of river is 10 m/s.



$$t_c = \frac{d}{v_b}$$

$$= \frac{5000}{100}$$

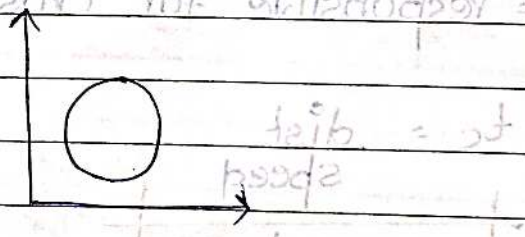
$$= 50 \text{ sec}$$

Distance = speed \times time
 $= 10 \times 50$

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Circular motion :-

It is a two dimension accelerated motion in which particle moves in circular motion.



- * Types of circular motion
- (i) Uniform circular motion
- (ii) non-uniform circular motion

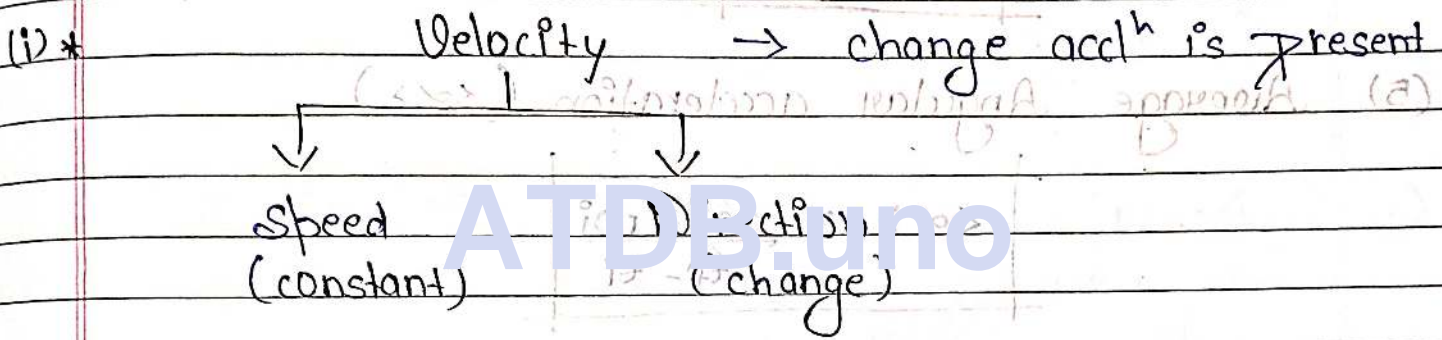
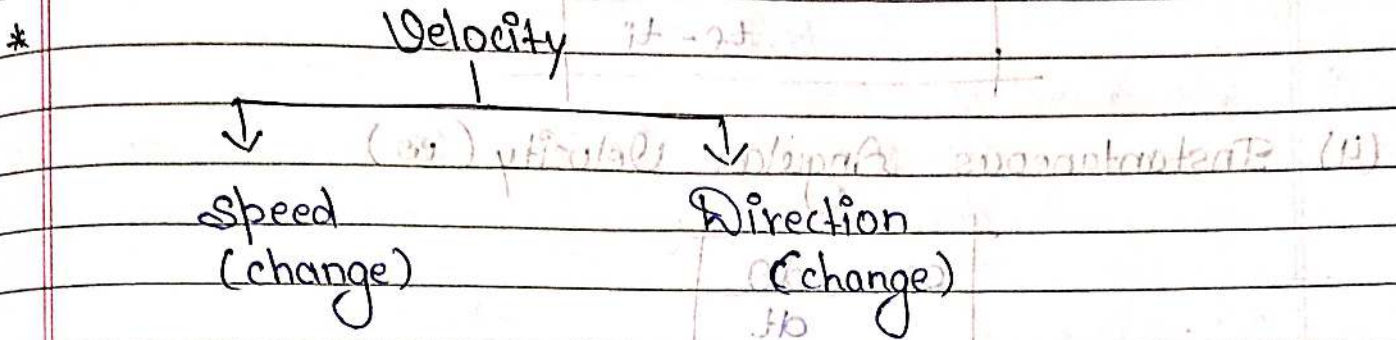
(i) Uniform circular motion :-

In this case speed of object constant but direction always change

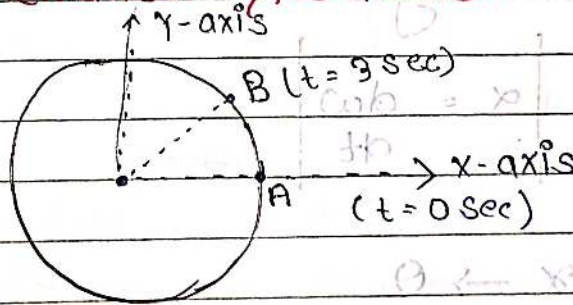
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(ii) non-uniform circular motion:- In this case speed & direction both change with time.



Circular kinematical Parameters



(1) Angular position (θ):-
 at $t = 0$, $\theta = 0^\circ$
 at $t = 3 \text{ sec}$, $\theta = 60^\circ$

(2) Angular Displacement:-
 $\Delta\theta = \theta_f - \theta_i = T$
 $= 60 - 0$
 $= 60^\circ$

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(3) Average Angular Velocity ($\langle \omega \rangle$)

$$\langle \omega \rangle = \frac{\theta_f - \theta_i}{t_f - t_i}$$

(4) Instantaneous Angular Velocity (ω)

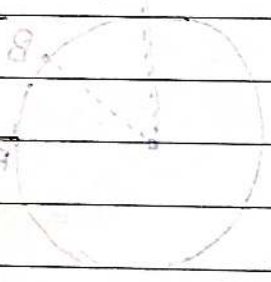
$$\omega = \frac{d\theta}{dt}$$

(5) Average Angular acceleration ($\langle \alpha \rangle$)

$$\langle \alpha \rangle = \frac{\omega_f - \omega_i}{t_f - t_i}$$

(6) Instantaneous Angular acceleration (α)

$$\alpha = \frac{d\omega}{dt}$$



Note :- $\alpha \rightarrow \theta$
 $v \rightarrow \omega$
 $a \rightarrow \alpha$



Time Period :-

It is the time taken in one circulation.

$$T = \frac{360^\circ}{\omega} \Rightarrow T = \frac{2\pi}{\omega}$$

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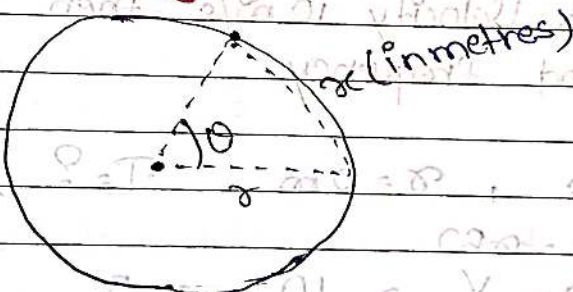
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Frequency :- ν ::
 No. of circulation in 1 sec.
 $\nu = \frac{1}{\text{Time Period}}$

$$\nu = \frac{\omega}{2\pi}$$

W.O.I

Relation between Angular & linear quantity.



- $r = \text{radius}$
- $x = \text{linear distance}$
- $\theta = \text{angular distance}$
- $\therefore \text{Angle} = \frac{\text{Arc}}{\text{radius}}$
- $\therefore \text{Arc} = \text{Angle} \times \text{radius}$
- $x = \theta \times r \quad (r = \text{constant})$

differentiating both side w.r.t time
 $\frac{dx}{dt} = \frac{d\theta}{dt} \times r$

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$$v = r \times \omega$$

$\therefore v =$ linear velocity
 $r =$ radius
 $\omega =$ angular velocity
 in (rad/sec)

* We have $v = r\omega$

differentiating both side w.r.t time

$$\frac{dv}{dt} = r \times \frac{d\omega}{dt}$$

$$a = r \times \alpha$$

$\therefore a =$ linear acceleration
 $\alpha =$ angular acceleration

Q. If an object is moving in a circular path of radius 2m with velocity 10 m/s then calculate its time period and frequency.

$v = 10 \text{ m/s}$, $r = 2 \text{ m}$, $T = ?$, $F = ?$

$\therefore v = r\omega$
 $\omega = \frac{v}{r} = \frac{10}{2} = 5$

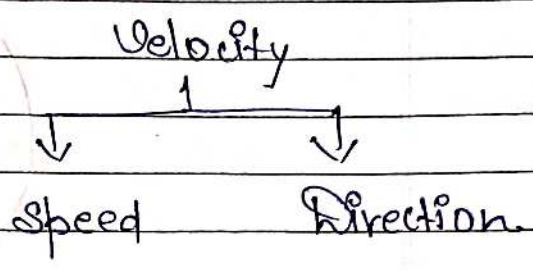
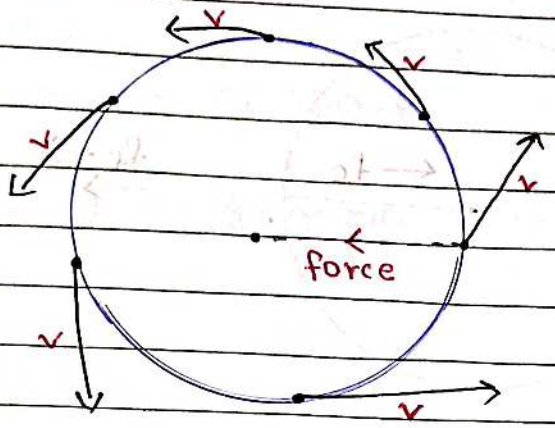
$T = \frac{2\pi}{\omega} = \frac{2\pi}{5}$

$F = \frac{\omega}{2\pi} = \frac{5}{2\pi}$

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Centripetal force & centripetal acclⁿ



force acting towards centre is called centripetal force.

$$f_c = \frac{mv^2}{r}$$

or,

$$f_c = m\omega^2 r$$

Acceleration towards centre is called Centripetal acclⁿ.

$$a_c = \frac{v^2}{r}$$

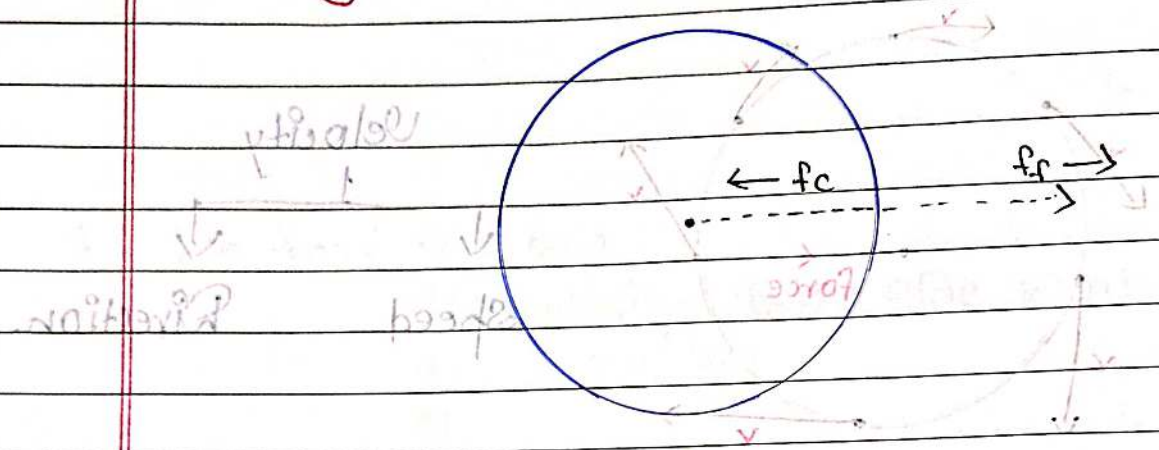
or,

$$a_c = \omega^2 r$$

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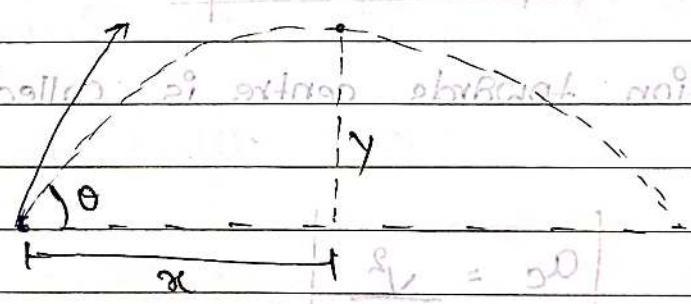
* Centrifugal force; (centrifugal force)



Magnitude of mv^2 = Magnitude of Centrifugal force.

→ Both force opposite direction.

* Equation of Trajectory



$$y = x \tan \theta - \frac{gx^2}{2u^2 \cos^2 \theta}$$

Q. If a boat can move with maximum speed of 360 km/h and speed of river is 5 m/s then calculate minimum time of crossing and also calculated drift distance, in this case. (width = 1 km)

Saathi

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$V_b = 360 \text{ km/h} = \frac{360 \times 5}{18} = 100 \text{ m/s}$

$V_r = 5 \text{ m/s}$

$d = 100 \text{ km} = 100000 \text{ m}$

(a) $t_c = \frac{d}{V_b} = \frac{100000}{100} = 1000 \text{ sec}$

(b) $\Delta x = V_r \times t_c$
 $= V_r \times t_c$
 $= 5 \times 1000$
 $= 5000 \text{ m}$

Q. If a boat reaches at just opposite side of the river at angle $\theta = 30^\circ$ with surface and velocity of river is 5 m/s then calculated velocity of boat and also calculated time of crossing in this case. (width = 100 km)

⇒ Given,

$\theta = 30^\circ$

$V_r = 5 \text{ m/s}$

$d = 100 \text{ km} = 100000 \text{ m}$

(a) $V_b \cos \theta = V_r$

$V_b \cos 30^\circ = 5$

$V_b \times \frac{\sqrt{3}}{2} = 5$

