SURE SHOT QUESTIONS

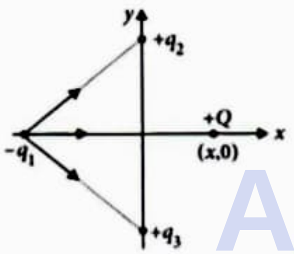
ARVIND ACADEMY

Chapter – 01

Electric Charges and Fields

➢ MCQ (1 mark)

1. Soln. (a): Since positive charge q_2 and q_3 exert a net force in the +x – direction on the charge q_1 fixed along the x-axis, the charge q_1 is negative as shown in figure. Obviously, due to addition of positive charge Q at (x, 0), the force on -q shall increase along the positive x-axis.



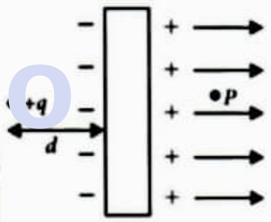
2. Soln. (a): When a point positive charge is brought near an isolated conducting sphere, then there develops some negative charge on left side of the sphere and an equal positive charge on the right side of the sphere. Electric lines of force emanating from the point positive charge end normally on the left side of the sphere. And due to positive charge on the right side of the sphere, the electric lines of force emanate normally from the right side. So the electric field is best given by figure (i).

3. Soln. (d): As per Gauss's theorem in electrostatics, the electric flux through a surface depends only on the amount of charge enclosed by the surface. It does not depend on size and shape of the surface. Therefore, electric flux through the surface is the same for all figures.

4. Soln. (b)

5. Soln. (c) : The spacing between electric lines of force increases from left to right. Therefore, £ on left is greater than E on right. Force on +q charge of dipole is smaller and to the right. Force on -q charge of dipole is bigger and to the left. Hence the dipole will experience a force towards the left,

6. Soln. (a): When a point charge +q is placed at a distance d from an isolated conducting plane, some negative charge develops on the surface of the plane towards the charge and an equal positive charge develops on opposite side of the plane. Hence, the field at a point P on the other side of the plane is directed perpendicular to the plane and away from the plane as shown in figure.



7. Soln. (a): When the point is on the diameter and away from the centre of hemisphere which is charged uniformly and positively, the component of electric field intensity parallel to the diameter cancel out. So the electric field is perpendicular to the diameter.

8. **Ans.** (c) : As X is repelled by Y, so Y is negatively charged. Now Z is attracted to Y, so either it is positively charged or neutral.

9. **Ans.** (b) : $q_1 = 3e,\ q_2 = +5e,\ q_3 = -3e$

The total charge,

$q = q_1 + q_2 + q_3 = 3e + 5e - 3e = 5e$

As the charge is conserved so,

In option (a), $5e - 4e + 5e = 6e$

In option (b), $6e + 6e - 7e = 5e$

In option (c), $-4e+3.5e+5.5e=5e$

But, from quantisation of charge, charge 3.5e and 5.5e is not possible.

In option (d), $q=5e-8e+7e=4e$

10. Ans. (d): Initial charge, q = 1C

Number of electrons gained, $n = 5 \times 10^{18}$

Charge transferred, Q = ne

$Q = -5 \times 10^{18} \times 1.6 \times 10^{-19} = -0.8$ C

So, net charge on object = q + Q = 1 – 0.8 = 0.2C

11. Ans. (b) : According to Coulomb's law, $\vec{F}_{12} = -\vec{F}_{21}$

There is a force of attraction which shows that the charges must be unlike charges.

Also, $F = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}$

For attractive force, $q_1 q_2 < 0$,

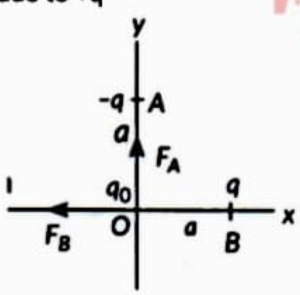
$q_1 = +ve \text{ and } q_2 = -ve$

$\therefore\ q_1 q_2 < 0$

12. Ans. (c) : Force on q_0 due to -q

$F_A = \dfrac{kqq_0}{a^2}$; along y – direction

Force on q_0 due to +q



$F_B = \dfrac{kqq_0}{a^2}$ along negative x-direction

Net force on q_0

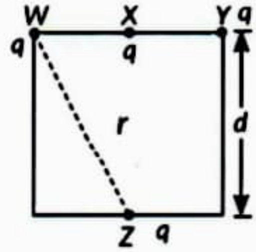
$$F = \sqrt{F_A^2 + F_B^2} = \frac{kqq_0\sqrt{2}}{a^2}$$

13. Ans. (b) :

Force on X by W is F.

$$F = \frac{Kq^2}{\left(\frac{d}{2}\right)^2} = \frac{4Kq^2}{d^2} \quad \text{............(i)}$$

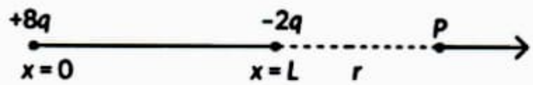
Force on Z by W,



$$r = \sqrt{d^2 + \frac{d^2}{4}} = \sqrt{\frac{5d^2}{4}}$$

$$F' = \frac{Kq^2}{\left(\sqrt{\frac{5d^2}{4}}\right)^2} = \frac{Ka^2 \times 4}{5d^2} = \frac{1}{5}F \quad \text{(using (i))}$$

14. Ans. (c) : Let P is the observation point at a distance (L + r) from 8q and at r from -2q.



Net electric field at P = 0

$\therefore\ \vec{E}_1 = EFI$ (electric field intensity) at P due to +8q

$\vec{E}_2 = EFI$ (electric field intensity) at P due to -2q

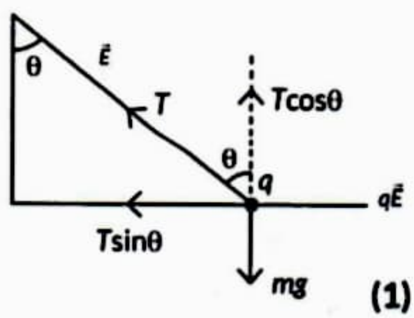
$|\vec{E_1}| = |\vec{E_2}|$

$$\therefore \frac{k(8q)}{(L+r)^2} = \frac{k(2q)}{r^2} \therefore \frac{4}{(L+r)^2} = \frac{1}{r^2} \Rightarrow 4r^2 = (L+r)$$

$\Rightarrow\ 2r = L + r \Rightarrow\ r = L$

$\therefore$ P is at x = L + L = 2L from origin.

15. Ans. (b) : From figure,



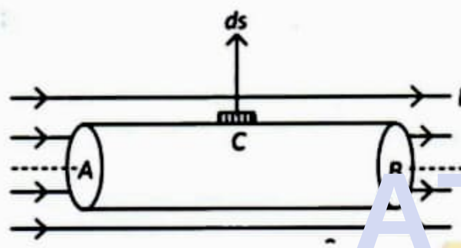
(1)

$$T\cos\theta = mg$$
$$T\sin\theta = qE$$
$$\therefore\ qE = mg\tan\theta$$
$$q = \left(\frac{mg}{E}\right)\tan\theta$$
$$\tan\theta = \frac{F_e}{mg}$$

16. Ans. (a) : Side of square sheet = a is XY plane

$$\vec{E} = cz^2\hat{k};\ Electric\ flux, \phi = \vec{E}.d\vec{s}$$
$$\phi = (cz^2\hat{k}).(a^2)\hat{k}$$
$$\phi = cz^2a^2 \qquad (put\ z = a)$$
$$\phi = a^4c$$

17. Ans. (a) :



Flux through surface A, $\phi_A = E\times\pi R^2$

Flux through surface B, $\phi_B = E\times\pi R^2$

$\therefore$ Flux through curved surface, $C = \int E.ds$

$$= \int Eds\cos 90^0 = 0$$

$\therefore$ Total flux through cylinder = $\phi_A + \phi_B + \phi_C = 0$

18. Ans. (d) : For stable equilibrium, $\theta = 0^0$

Potential energy,

$$U = -\vec{p}.\vec{E} = -pE\cos 0^0 = -pE = -2qLE$$

19. Ans. (a) : Given :

$$2l = 2cm,\ \theta = 30^0,\ E = 2\times10^5\ N/C$$
$$\tau = 8\times10^{-3}\ Nm$$
$$\tau = PE\sin\theta$$
$$8\times10^{-3} = p\times2\times10^5\times\sin 30^0$$
$$p = 8\times10^{-8}\ cm$$

Let the charge is q

$$p = q\times2l$$
$$8\times10^{-8} = q\times2\times10^{-2} \Rightarrow q = 4\mu C$$

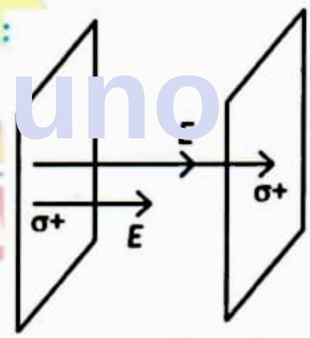
20. Ans. (c) : An electric dipole in a non-uniform electric field always experience a force and a torque.

21. Ans. (d) : The electric field over the Gaussian surface remains continuous and uniform at every point.

22. Ans. (a) : As $\phi = \frac{q}{\varepsilon_0}, \phi = 0 \Rightarrow q = 0$

23. Ans. (a) : Net charge enclosed and permittivity of the medium.

24. Ans. (d) :



Here: Surface charge density,

$$\sigma = 26.4\times10^{-12}\ C/m^2$$
$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{2\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$
$$= \frac{26.4\times10^{-12}}{8.85\times10^{-12}}\ (\because \varepsilon_0 = 8.85\times10^{-12}\ C^2/N\text{-}m^2)$$
$$\therefore\ E = 3N/C$$

25. Ans. (a) Electric field due to a uniformly charged sheet $E = \frac{\sigma}{2\varepsilon_0}$

$$E = \frac{1\times10^{-15}\times4\pi\times9\times10^9}{18\pi\times2} \qquad \left(\because \frac{1}{\varepsilon_0} = 4\pi\times9\times10^9\right)$$

E = 10^{-6} N/C outwards.

Point is Q
As, for the finite plane sheet, electric field is uniform in the middle and the edges it will be curved.

(b) $q_1 = 10\mu C,\ q_2 = -20\mu C$

$$F = \frac{kq_1q_2}{r^2} = \frac{k\times10\times20\times10^{-12}}{r^2}$$

Whey they bought in contact,

$$q'_1 = q'_2 = \frac{10-20}{2} = -5\mu C, r' = \frac{r}{2}$$

$$F = \frac{k\times5\times5\times10^{-12}\times4}{r^2}$$

$$\frac{F'}{F} = \frac{5\times5\times4\times10^{-12}}{10\times20\times10^{-12}} \Rightarrow F' = \frac{F}{2}$$

➤ Assertion-Reasoning (1 mark)

Is Assertion (A) and Reason (R) type questions. Given below are the two statements labelled as Assertion (A) and Reason (R). Select the most appropriate answer from the options given below.

26. **Ans.** (d) : Force on a negative charge in electric field $\vec{F} = -q\vec{E}$, so it moves along the opposite direction of the electric field. So, (A) and (R) both false.

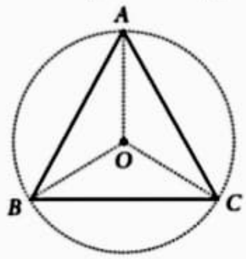
For question below two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

27. **Ans.** (c) : An electric dipole in a non-uniform electric field always experience a force and a torque.

28. **Sol.** (c): A body becomes negatively charged only when some electrons are transferred to the body i.e. the body gains some electrons. Hence its mass increases slightly. Mass of a body decreases only when body gives some electrons to some other body.

29. **Sol.** (c): If the charged particle is initially at rest in an electric field, it will move along the electric line of force. But when the initial velocity of charged particle makes some angle with the line of force then the resultant path is not along the line of force. Because electric line of force may not coincide with the line of velocity of the charge.

30. **Sol.** (a): Resultant of electric intensity at O due to B and C is equal and opposite to that due to A.



31. **Sol.**
(b): As $\sigma_1 = \sigma_2$ (Given)

$$\therefore \frac{q_1}{4\pi r_1^2} = \frac{q_2}{4\pi r_2^2}, \text{ or, } \frac{q_1}{q_2} = \frac{r_1^2}{r_2^2}$$

[Let r_1 and r_2 be two different radii]

Then the ratio of electric field intensities near the surface of spherical conductors,

$$\frac{E_1}{E_2} = \frac{q_1}{4\pi\varepsilon_0 r_1^2}\times\frac{4\pi\varepsilon_0 r_2^2}{q_2} = \frac{q_1}{q_2}\times\frac{r_2^2}{r_1^2} = \frac{q_1}{q_2}\times\frac{q_2}{q_1} = 1$$

i.e. $E_1 = E_2$

32. **Sol.** (d): Surface of a charged conductor is always an equipotential surface, whatever may be its shape. Hence σR = constant, at every point on the surface of charged conductor i.e. at the sharpest point ($R \rightarrow 0$) of the surface, charge density will be maximum. A uniformly charged conductor exerts no electrostatic force on a point charge located anywhere inside the conductor or electric field is zero.

33. **Sol.**

(b): The electric field intensity is equal to force experienced by unit positive test charge q_0 placed at that point *i.e.*

$\vec{E} = \frac{\vec{F}}{q_0}$ thus $\vec{E}$ is also a vector quality.

As, $E = \frac{F}{q} = \frac{\text{newton}}{\text{coulomb}}$.

Case Study (4 marks)

34. Ans. (i) (c) : Copper

(ii) (a) : Car

(iii) (c) : zero

(iv) (c) : Its surface must have charge equal to -q.

(i) (b) : 1.9 x 10^5 Nm^2/C, leaving the surface.

35. Ans. (c) At the centre of an electric dipole, electric field is twice the electric field due to each charge.

(b) At far away points of a dipole, $E \propto \frac{1}{r^3}$

(d) For $z >> a$, $|E_z| = \frac{2p}{4\pi\varepsilon_0 z^3}$

For $y >> a$, $|E_y| = \frac{p}{4\pi\varepsilon_0 y^3}$

For $z = y >> a$, $\frac{|E_z|}{|E_y|} = 2.$

(a) $\tau_{max} = pE\sin 90° = pE.$

(b) Restoring torque for small θ,

$\tau = -pE\sin\theta = -pE\theta$ $[\sin\theta = \theta]$

or $I\alpha = -pE\theta$

$\therefore$ $\alpha = -\frac{pE}{I}\theta$ *i.e.,* $\alpha \propto \theta$

$\therefore$ $\nu = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{pE}{I}}$

➤ Questions

26. Ans. Given q = 2 μ C, m = 1.6 = 1.6 x 10-3 kg,

$u = 4\hat{i}\ ms^{-1}, \vec{E} = 80\hat{i} + 60\hat{j}$ and t = 5 s.

$F = m\vec{a}$ (From Newton's law)

Or $q\vec{E} = m\vec{a}$

$\Rightarrow$ $2\times10^{-6}(80\hat{i} + 60\hat{j}) = (1.6\times10^{-3})\vec{a}$

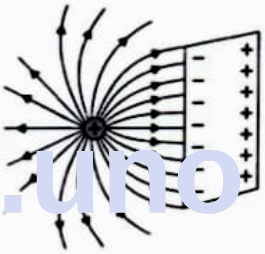
$\Rightarrow$ $\vec{a} = 100\times10^{-3}\hat{i} + 75\times10^{-3}\hat{j}$

Now from equation of motion,

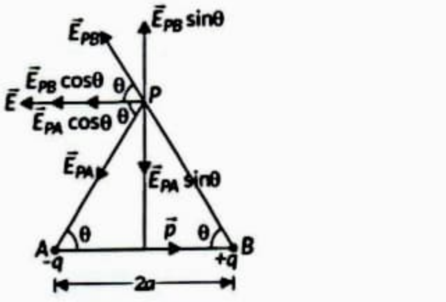
$\vec{v} = \vec{u} + \vec{a}t = 4\hat{i} + (100\times10^{-3}\hat{i} + 75\times10^{-3}\hat{j})5$

$= 4.5\hat{i} + 0.375\hat{j}$

36. AAns.



37. Ans. Electric field on the equatorial line of an electric dipole : Electric field at any point on the perpendicular bisector of an electric dipole at distance r from its centre is



$E_{net} = E_x = E_{PA}\cos\theta + E_{PB}\cos\theta$

(Vertical component cancel each other)

Or $E_{net} = 2E_{PA}\cos\theta$ $(E_{PA} = E_{PB})$

$$E_{net} = 2.\frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}.\frac{a}{(r^2+a^2)^{1/2}}$$

$$E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{q.2a}{(r^2+a^2)^{3/2}} \text{ or } E_{net} = \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$$

Directed antiparallel to dipole moment $\vec{p}$. For short dipole, when r >>>> a, then electric field at point P is

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

In vectorial form, the electric field intensity at point P on the perpendicular bisector of short electric dipole is then given by $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{-\vec{p}}{r^3}.\hat{r}$

38. **Ans.** (i) Electric flux: Total number of electric field lines crossing a surface normally is called electric flux. SI unit of electric flux is N m² C⁻¹.

(ii) The area of a surface can be represented as a vector along normal to the surface.

Here, $\vec{E} = 3\times10^3\,\hat{i}\,NC^{-1}$

Area of the square $\Delta S = 10\times10 cm^2$

$\Delta S = 100$ cm² = 10^{-2} m²

Since normal to the square is along x-axis, we have

$$\Delta S = 10^{-2}\,\hat{i}\,m^2$$

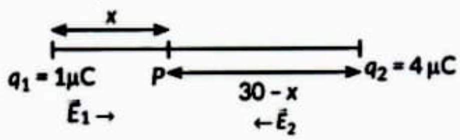
Electric flux through the square,

$$\phi = \vec{E}.\Delta\vec{S} = (3\times10^3\,\hat{i}).(10^{-2}\,\hat{j})$$

$$\phi = 30\ N m^2 C^{-1}$$

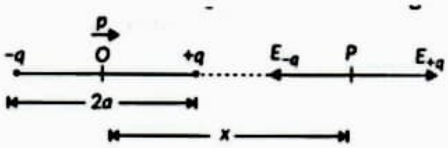
39. **Ans.** Let at point P, the net electric field is zero,

then $\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(30-x)^2}$



$$\frac{1}{x^2} = \frac{4}{(30-x)^2} \Rightarrow x = 10cm$$

40. **Ans.** (a) Electric field at an axial point of an electric dipole. Let us consider an electric dipole consisting of charges +q and -q, separated by distance 2a and placed in vacuum. Let P be a point on the axial line at distance r from the centre O of the dipole on right side of the charge +q.



Electric field at an axial point of dipole

$$\vec{E}_{-q} = \frac{-q}{4\pi\varepsilon_0(r+a)^2}\hat{p} \quad \text{(towards left)}$$

Where $\hat{p}$ is a unit vector along the dipole axis from -q to +q.

Electric field due to charge +q at point P is

$$\vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0(r-a)^2}\hat{p} \quad \text{(towards right)}$$

Hence the resultant electric field at point P is

$$\vec{E}_{axial} = \vec{E}_{-q} + \vec{E}_{+q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$= \frac{q}{4\pi\varepsilon_0}.\frac{4ar}{(r^2-a^2)^2}\hat{p}$$

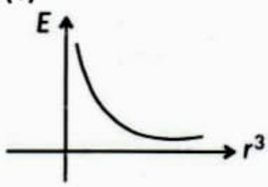
$$\text{or} \quad \vec{E}_{axial} = \frac{1}{4\pi\varepsilon_0}.\frac{2pr}{(r^2-a^2)^2}\hat{p}$$

Here p = q x 2a = dipole moment

For r >> a, a² can be neglected as compared to r².

Or $\vec{E}_{axial} = \frac{1}{4\pi\varepsilon_0}.\frac{2p}{r^3}\hat{p}$ (towards right)

(b)



41. **Ans.** (a) Surface charge density on the inner surface = $\frac{q}{4\pi r_1^2}$

On the outer surface = $\frac{Q-q}{4\pi r_2^2}$

(b) For a spherical Gaussian surface $x > r_2$

$$\oint \vec{E}\,\vec{ds} = \frac{Q-q}{\varepsilon_0}$$

$$E \times 4\pi x^2 = \frac{Q-q}{\varepsilon_0}$$

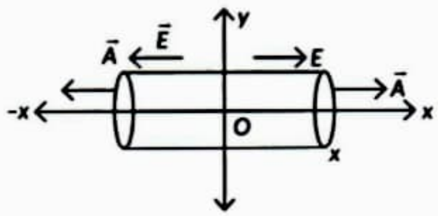
$$E = \frac{1}{4\pi\varepsilon}\frac{Q-q}{x^2}$$

42. **Ans.** (i) Given l = 20 cm, r = 5 cm = 0.05 m

Net flux,

$$\phi = \int E.dA + \int E.dA = 200\pi(0.05)^2 \cos 0 \times 2$$

$$= \pi Nm^2C^{-1}$$



(ii) The net charge enclosed, $q = \phi_1\varepsilon_0$

$$\pi Nm^2C^{-1} \times 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$$

$$= 27.789 \times 10^{-12} C$$

43. **Ans.** (i) Charge enclosed by sphere S_1 = 2Q

By Gauss law, electric flux through sphere S_1 is

$$\phi_1 = 2Q/\varepsilon_0$$

Charge enclosed by sphere,

$$S_2 = 2Q + 4Q = 6Q$$

$$\phi_2 = 6Q/\varepsilon_0$$

The ratio of the electric flux is

$$\phi_1 : \phi_2 = 2:6 = 1:3$$

(ii) When a medium of dielectric constant ε_r is introduced in sphere S_1, the flux through S_1 would be

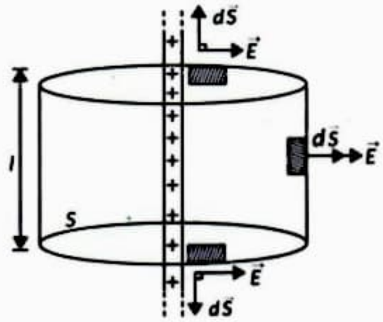
$$\phi'_1 = \frac{2Q}{\varepsilon_0\varepsilon_r}$$

44. **Ans.** According to Gauss's law, total flux over a closed surface S in vacuum is $\frac{1}{\varepsilon_0}$ times the total charge enclosed by closed surface S

$$\phi = \oint_S \vec{E}.\vec{ds} = \frac{q_{enclosed}}{\varepsilon_0}$$

Electric field intensity due to line charge or infinite long uniformly charged wire at point P at distance r from it is obtained as:

Assume a cylindrical gaussian surface S with charged wire on its axis and point P on its surface, then net electric flux through surface S is



$$\phi = \oint_S \vec{E}\cdot d\vec{S} = \int_{\text{upper plane face}} EdS\cos 90^\circ + \int_{\text{curved surface}} EdS\cos 0^\circ + \int_{\text{lower plane face}} EdS\cos 90^\circ$$

Or $\phi = 0 + EA + 0$ or $\phi = E.2\pi rl$

But by Gauss's theorem $\phi = \frac{q}{\varepsilon_0} = \frac{\lambda l}{\varepsilon_0}$

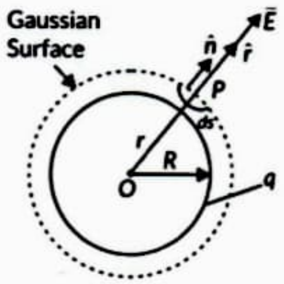
Where q is the charge on length l of wire enclosed by cylindrical surface S, and λ is uniform linear charge density of wire.

$$\therefore\ E \times 2\pi rl = \frac{\lambda l}{\varepsilon_0} \text{ or } E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

45. **Ans.** Consider a thin spherical shell of radius R carrying charge q. To find the electric field outside the shell, we consider a spherical Gaussian surface of radius r (> R), concentric with given shell. The electric field $\vec{E}$ is same at every point of Gaussian surface and directed radially outwards (as is unit vector $\hat{n}$ so that $\theta = 0^0$)

According to Gauss's theorem,

$$\oint_S \vec{E}.d\vec{s} = \oint_S \vec{E}.\hat{n}\,ds = \frac{q}{\varepsilon_0}$$



Or $E \oint_S ds = \frac{q}{\varepsilon_0}$

$$\therefore\ E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2}$$

Vectorially, $\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$

Special cases

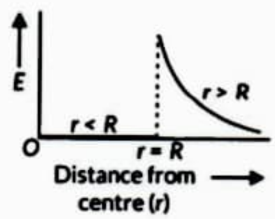
At the point on the surface of the shell, r = R

$$\therefore\ E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

If σ is the surface charge density on the shell

then $q = 4\pi R^2\sigma$

$$\therefore\ E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4\pi R^2\sigma}{R^2} = \frac{\sigma}{\varepsilon_0}$$



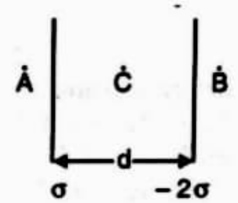
If the point P lies inside the spherical shell then the Gaussian surface encloses no cha

46. Soln.

(i) Electric field due to plane sheet toward right = $\frac{\sigma}{\varepsilon_0}$

Where, σ is charge density.

Towards left = $\frac{-\sigma}{\varepsilon_0}$



Electric field at point A i.e., to the left of first sheet and due to large plane sheet.

$$\vec{E}_A = -\frac{\sigma}{\varepsilon_0} + \left(+\frac{2\sigma}{\varepsilon_0}\right) = \frac{\sigma}{\varepsilon_0}$$

(ii) Electric field at point B i.e., to the right of second sheet,

$$\vec{E}_B = \frac{\sigma}{\varepsilon_0} - \frac{2\sigma}{\varepsilon_0} = \frac{-\sigma}{\varepsilon_0}$$

(iii) Electric field at point C i.e., between two plates,

$$\vec{E}_C = \frac{\sigma}{\varepsilon_0} + \frac{2\sigma}{\varepsilon_0} = \frac{3\sigma}{\varepsilon_0}.$$

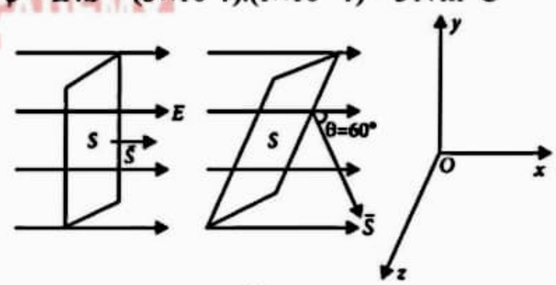
47. Soln. Given electric field $\vec{E} = 3\times10^3\,\hat{i}\,NC^{-1}$

Magnitude of area, S = 10 cm² = 1 x 10⁻³ m²

(i) When the surface is parallel to y-z plane, the normal to plane is along x-axis.

In this case $\theta = 0$; so electric flux,

$$\phi = \vec{E}.\vec{S} = (3\times10^3\,\hat{i}).(1\times10^{-3}\,\hat{i}) = 3\,\mathrm{N\,m^2\,C^{-1}}$$



(ii) In this case $\theta = 60^0$, so electric flux,

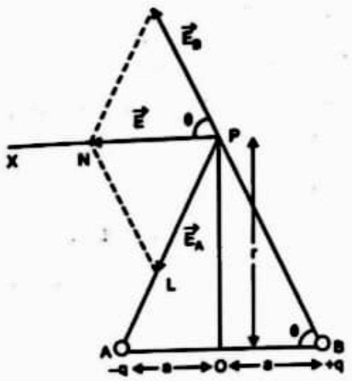
$\phi = E.S\cos\theta$

$= 3\times10^3\times1\times10^{-3}\cos60^0 = 3\times\frac{1}{2}$

$= 1.5\ N\,m^2C^{-1}$.

48. Soln.

(a) Consider an electric dipole of charges -q and +q separated by a distance 2a and placed in a free space. Let P be a point on equatorial line of dipole at a distance r from the centre of a dipole.



Let $\vec{E}_A$ and $\vec{E}_B$ be the electric field at point P due to charges -q and +q

Then resultant electric field at point P is

$$\vec{E} = \vec{E}_A + \vec{E}_B$$

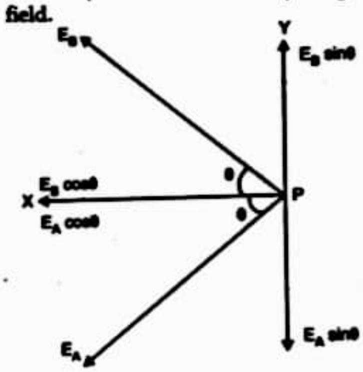
Now,

$$|\vec{E}_A| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}\text{(along PA)}$$

$$|\vec{E}_B| = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r^2+a^2)}\text{(along BP)}$$

The, resultant intensity is the vector sum of E_A and E_B.

E_A and E_B can be resolved into two components. The Y-components cancel out each other. And X-component will add up to give the resultant Field field.



$$\therefore |\vec{E}| = E_A\cos\theta + E_B\cos\theta$$

Now in right, triangle ORB

$$\cos\theta = \frac{OB}{BP} = \frac{a}{\sqrt{r^2+a^2}}$$

$$\therefore\ E = 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{q}{r^2+a^2}\frac{a}{(r^2+a^2)^{1/2}}$$

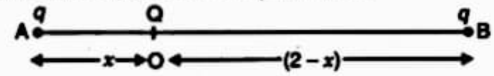
$$= \frac{2qa}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{(r^2+a^2)^{3/2}}$$

$$[\because\ 2qa = p]$$

This is the required expression.

(b) Let the two charges of +q each placed at point A and B at a distance 2 m apart in air.



Suppose, the third charge Q (unknown magnitude and charge) is placed at a point O, on the line joining the other two charges, such that OA = x and OB = 2 − x.

For the system to be in equilibrium, net force on each 3 charges must be zero.

If we assume that charge Q placed at O is positive, the force on it at O may be zero. But the force on charge q at point A or B will not be zero. It is because, the forces on a charge q due to the other two charges will act in same direction. If charge Q is negative, then the forces on q due to other two charges will act in opposite direction.

Hence, Q will be negative in nature.

For charge (-Q) to be in equilibrium

Force on charge (-q) due to charge (+q) at point A should be equal and opposite to charge (+Q) at B

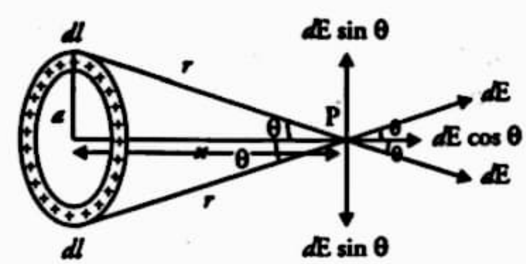
$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{(2-x)^2}$$

Or $(2-x)^2 = x^2$

$$\Rightarrow\qquad x = (2-x) \Rightarrow x = 1\,m$$

Therefore, for the system to be in equilibrium a charge -Q is placed at a mid point between the two charges of +q each.

49. Soln. Suppose we have a ring of radius a that carries a uniformly distributed positive charge q.



As the total charge q is uniformly distributed, the charge dq on the element dl is

$$dq = \frac{q}{2\pi a}.dl$$

∴ The magnitude of the electric field produced by the element dl at the axial point P is

$$dE = k.\frac{dq}{r^2} = \frac{kq}{2\pi a}.\frac{dl}{r^2}$$

The electric field dE has two components.

(i) The axial components $dE\cos\theta$ and

(ii) The perpendicular component $dE\sin\theta$.

Since the perpendicular component of any two diametrically opposite elements are equal and opposite, they cancel out in pairs. Only the axial components will add up to produce the resultant field.

E at point P is given by

$$E = \int_0^{2\pi a} dE\cos\theta$$

[∵ Only the axial components contribute towards E]

$$E = \int_0^{2\pi a} \frac{kq}{2\pi a}.\frac{dl}{r^2}.\frac{x}{r} \qquad \left[\because \cos\theta = \frac{x}{r}\right]$$

$$= \frac{kqx}{2\pi a}.\frac{1}{r^3}\int_0^{2\pi a} dl$$

$$= \frac{kqx}{2\pi a}.\frac{1}{r^3}(l)_0^{2\pi a}$$

$$= \frac{kqx}{2\pi a}.\frac{1}{(x^2+a^2)^{3/2}}.2\pi a$$

$$[\because r^2 = x^2 + a^2]$$

$$E = \frac{kqx}{(x^2+a^2)^{3/2}}$$

[Where $k = \frac{1}{4\pi\varepsilon_0}$ a = constant]

$$= \frac{1}{4\pi\varepsilon_0}.\frac{qx}{(x^2+a^2)^{3/2}}$$

If x >> a, then $x^2 + a^2 \approx x^2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(x^2)^{3/2}}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

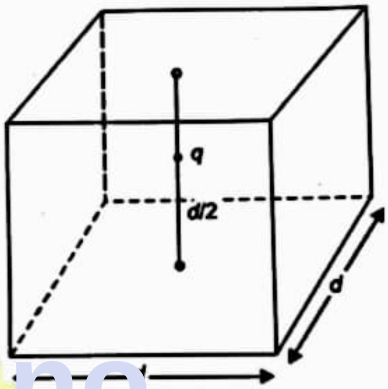
This expression is similar to electric field due to a point charge.

50. Soln. (a) Electric flux: Electric flux through an area is defined as the product of electric field strength E and area dS perpendicular to the field. It represents the field lines crossing the area. It is a scalar quantity. Imagine a cube of edge d, enclosing the charge. The square surface is one of the six faces of this cube. According to Gauss' theorem in electrostatics,

Total electric flux through the cube = $\frac{q}{\varepsilon_0}$

This is the total flux through all six surface

∴ Electric flux through the square surface = $\frac{q}{\sigma\varepsilon_0}$



(b) On moving the charge to distance d from the centre of square and making side of square 2d, does not change the flux at all because flux is independent of side of square or distance of charge in this case.

51. Soln. The force of attraction or repulsion between two point charges q_1 and q_2 separated by a distance r is directly proportional to product of magnitude of charges and inversely proportional to square of distance between charges, written as:

$$F \propto q_1 q_2 \text{ and } \quad F \propto \frac{1}{r^2}$$

$$\therefore \qquad F \propto \frac{q_1 q_2}{r^2} \qquad or \qquad F = k\frac{q_1 q_2}{r^2}$$

Where k is a constant of proportionality, called *electrostatic force constant*. The value of k depends on the nature of the medium between the two charges and the system of units chosen to measure F, q_1, q_2 and r.

For the two charges located in free space and in SI units, we have

$$k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2}$$

Where ε_0 is called *permittivity* of free space. So we can express Coulomb's law in SI units as

$$F = \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r^2}$$

In vector form, Coulomb's law may be expressed as

$\vec{F}_{21}$ = Force on charge q_2 due to q_1

$$= \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r^2}\hat{r}_{12}$$

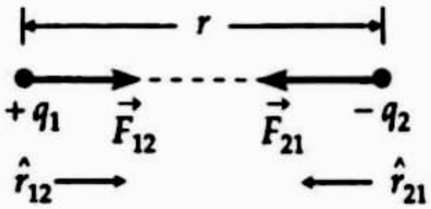
Where $\hat{r}_{12} = \frac{\vec{r}_{12}}{r}$, is a unit vector in the direction from q_1 and q_2.

Similarly, $\vec{F}_{12}$ = Force on charge q_1 due to q_2

$$= \frac{1}{4\pi\varepsilon_0}.\frac{q_1q_2}{r^2}\hat{r}_{21}$$

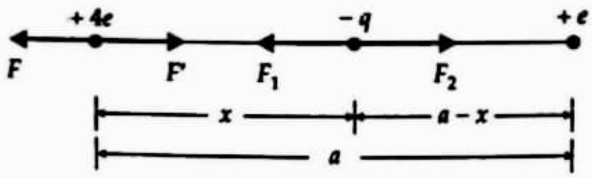
Where $\hat{r}_{21} = \frac{\vec{r}_{21}}{r}$, is a unit vector in the direction from q_2 to q_1.

The coulombian forces between unlike charges $(q_1q_2 < 0)$ are attractive, as shown in fig.



Attractive coulombian forces for $q_1q_2 < 0$.

52. Soln. Suppose the charges are placed as shown in fig.



As the charge +e exerts repulsion F on charge +4e, so for the equilibrium of charge +4e, the charge -q must exert attraction F' on +4e. This requires the charge q to be negative.

For equilibrium of charge +4e,

$$F = F'$$

$$\frac{1}{4\pi\varepsilon_0}.\frac{4e\times e}{a^2} = \frac{1}{4\pi\varepsilon_0}.\frac{4e\times q}{x^2}$$

Or $\qquad q = \frac{ex^2}{a^2}$

For equilibrium of charge -q,

Attraction F_1 between +4e and -q

= Attraction F_2 between +e and -q

$$\therefore \quad \frac{1}{4\pi\varepsilon_0}.\frac{4e\times q}{x^2} = \frac{1}{4\pi\varepsilon_0}.\frac{e\times q}{(a-x)^2}$$

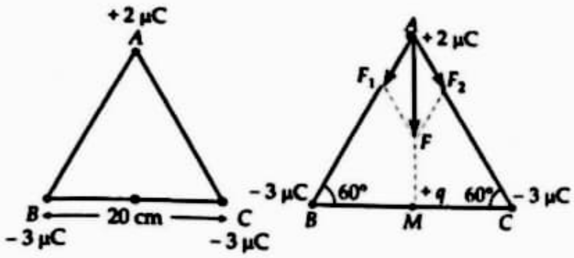
Or $\qquad x^2 = 4(a-x)^2$

$\therefore \qquad x = 2a/3$

Hence $\qquad q = \frac{ex^2}{a^2} = \frac{e}{a^2}.\frac{4a^2}{9} = \frac{4e}{9}$.

The equilibrium of the negative charge q will be unstable.

53. Soln.



(a) (b)

As shown in fig. the force exerted on charge $+2\mu C$ by charge at B,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$$

$$= \frac{9\times10^9\times2\times10^{-6}\times3\times10^{-6}}{(0.20)^2}$$

$$= 1.35\,N,\ along\ \ AB$$

Force exerted on charge $+2\mu C$ by charge at C,

$$F_2 = \frac{9\times10^9\times2\times10^{-6}\times3\times10^{-6}}{(0.20)^2}$$
$$=1.35\,N,\ along\ AC$$

Resultant force of F_1 and F_2

$$F=\sqrt{F_1^2+F_2^2+2F_1F_1\cos60^0}$$
$$=\sqrt{1.35^2+1.35^2+2\times1.35\times1.35\times0.5}$$
$$=1.35\times\sqrt{3}=2.34\,N,\ along\ AM$$

For the charge at A to be equilibrium, the charge q to be placed at point M must be a positive charge so that it exerts a force on $+2\mu C$ charge along MA.

Now, $AM=\sqrt{20^2-10^2}$

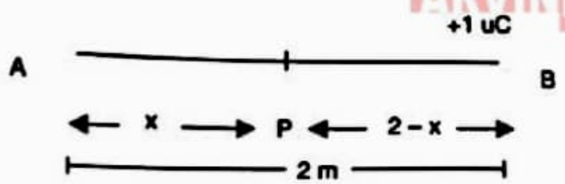
$$=\sqrt{300}=10\sqrt{3}\,cm$$
$$=0.1\times\sqrt{3}\ m$$

Net force on charge at A will be zero if

$$\frac{9\times10^9\times q\times2\times10^{-6}}{(0.1\times\sqrt{3})^2}=2.34$$

Or $q=\dfrac{2.34\times0.01\times3}{18\times10^3}=3.9\times10^{-6}C=3.9\,\mu C.$

54. Soln.



Distance between the two charges, AB = 2 m

$\therefore\quad PA=x\,m,\ PB=2-x\,m$

Net electric field at point P = E

Electric field at point P caused by $+4\mu C$ charge,

$$E_1=\frac{4\times10^{-6}}{4\pi\varepsilon_0(PA)^2}N/C \text{ along PA}$$

Magnitude of electric field at point P caused by $+1\mu C$ charge,

$$E_2=\frac{+1\times10^{-6}}{4\pi\varepsilon_0(PB)^2}N/C \text{ along PB}$$

Now, $$\frac{4\times10^{-6}}{4\pi\varepsilon_0(x)^2}=\frac{+1\times10^{-6}}{4\pi\varepsilon_0(2-x)^2}$$

Cross multiply and solve:

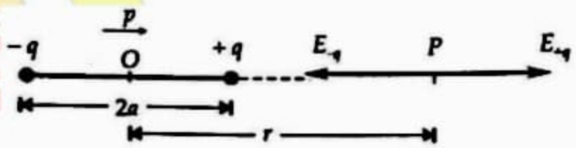
$$4\pi\varepsilon_0(2-x)^2 4\times10^{-6}=4\pi\varepsilon_0(x)^2 1\times10^{-6}$$
$$(2-x)^2 4\times10^{-6}=(x)^2 1\times10^{-6}$$
$$(2-x)^2 4=(x)^2 1$$
$$16-16x+3x^2=0$$

Now on factorizing,

$$4(4-x)-3x(4-x)=0$$
$$(4-3x)(4-x)=0$$

$x=\frac{4}{3}$, $x=4$, possible point on the line will be $x=\frac{4}{3}$.

55. Soln. Electric field at an axial point of an electric dipole. As shown in fig. consider an electric dipole consisting of charges +q and -q, separated by distance 2a and placed in vacuum. Let P be a point on the axial line at distance r from the centre O of the dipole on the side of the charge +q.



Electric field due to charge -q at point P is

$$\vec{E}_{-q}=\frac{-q}{4\pi\varepsilon_0(r+a)^2}\hat{p} \qquad \text{(towards left)}$$

Where $\hat{p}$ is a unit vector along the dipole axis from -q to +q.

Electric field due to charge +q at point P is

$$\vec{E}_{+q}=\frac{q}{4\pi\varepsilon_0(r-a)^2}\hat{p} \qquad \text{(towards right)}$$

Hence the resultant electric field at point P is

$$\vec{E}_{axial}=\vec{E}_{+q}+\vec{E}_{-q}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$$

$$= \frac{q}{4\pi\varepsilon_0}.\frac{4ar}{(r^2-a^2)^2}\hat{p}$$

Or $\overrightarrow{E_{axial}} = \frac{1}{4\pi\varepsilon_0}.\frac{2pr}{(r^2-a^2)^2}\hat{p}$

Here $p = q \times 2a =$ dipole moment.

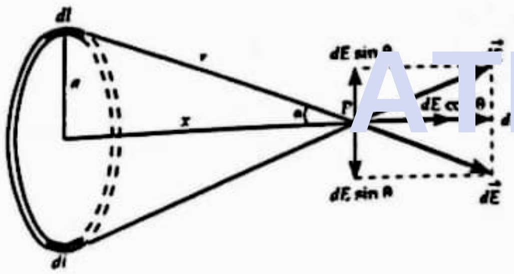
For r >> a, a^2 can be neglected compared to r^2
.

$\therefore \quad \overrightarrow{E_{axial}} = \frac{1}{4\pi\varepsilon_0}.\frac{2p}{r^3}\hat{p}$

(towards right)

Clearly, electric field at any axial point of the dipole acts along the dipole axis from negative to positive charge i.e., in the direction of dipole moment $\vec{p}$.

56. Soln.



$\therefore$ The magnitude of the field $d\vec{E}$ produced by the element dl at the field point P is

$$d\vec{E} = k.\frac{dq}{r^2} = \frac{kq}{2\pi a}.\frac{dl}{r^2}$$

As shown in fig. the field dE has two components:

1. The *axial component* dE $\cos\theta$, and
2. *The perpendicular component* dE $\sin\theta$.

Since the perpendicular components of any two diametrically opposite elements are equal and opposite, they all cancel out in pairs. Only the axial components will add up to produce the resultant field $\vec{E}$ at point P, which is given by

$E = \int_0^{2\pi a} dE\cos\theta$ [$\because$ Only the axial components contribute towards E]

$$= \int_0^{2\pi a} \frac{kq}{2\pi a}.\frac{dl}{r^2}.\frac{x}{r} = \frac{kqx}{2\pi a}.\frac{1}{r^3}\int_0^{2\pi a} dl$$

$$\left[\because \cos\theta = \frac{x}{r}\right]$$

$$= \frac{kqx}{2\pi a}.\frac{1}{r^3}[l]_0^{2\pi a} = \frac{kqx}{2\pi a}.\frac{1}{(x^2+a^2)^{3/2}}.2\pi a$$

$$\left[\because r^2 = x^2 + a^2\right]$$

Or $E = \frac{kqx}{(x^2+a^2)^{3/2}} = \frac{1}{4\pi\varepsilon_0}.\frac{qx}{(x^2+a^2)^{3/2}}$.

57. Soln. The electric flux through a given area held inside an electric field is the measure of the total number of electric lines of force passing normally through that area.

$$\Delta\phi_E = E\Delta\cos\theta = \vec{E}.\vec{\Delta S}$$

SI unit of electric flux = Vm.

58. Soln. Torque on a dipole in a uniform electric field. As shown in fig. consider an electric dipole consisting of charges +q and -q and of length 2a placed in a uniform electric field $\vec{E}$ making an angle θ with it. It has a dipole moment of magnitude,

$$p = q \times 2a$$

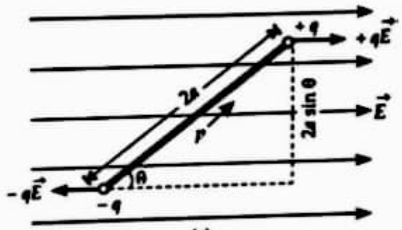
Force exerted on charge +q by field $\vec{E} = q\vec{E}$

(along $\vec{E}$)

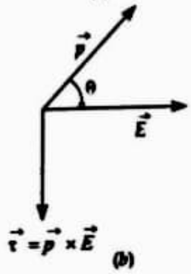
Force exerted on charge -q by field $\vec{E} = -q\vec{E}$

(opposite to $\vec{E}$)

$$\vec{E}_{Total} = +q\vec{E} - q\vec{E} = 0.$$



(a)



(b)

Hence the net translating force on a dipole in a uniform electric field is zero. But the two equal and opposite forces act at different points of the dipole.

They form a couple which exerts a torque.

Torque = Either force x Perpendicular distance between the two forces

$$\tau = qE \times 2a\sin\theta = (q \times 2a)E\sin\theta$$

Or $\tau = pE\sin\theta$
$(p = q \times 2a)$

As the direction of torque $\vec{\tau}$ is perpendicular to both $\vec{p}$ and $\vec{E}$, so we can write

$$\vec{\tau} = \vec{p} \times \vec{E}$$

The direction of vector $\vec{\tau}$ is that in which a right handed screw would advance when rotated from $\vec{p}$ to $\vec{E}$. As shown in fig. the direction of vector $\vec{\tau}$ is perpendicular to and points into the plane of paper.

When the dipole is released, the torque $\vec{\tau}$ tends to align the dipole with the field $\vec{E}$ i.e., tends to reduce angle θ to 0. When the dipole gets aligned with $\vec{E}$, the torque $\vec{\tau}$ becomes zero.

Clearly, the torque on the dipole will be maximum when the dipole is held perpendicular to $\vec{E}$. Thus

$$\tau_{max} = pE\sin 90^0 = pE.$$

Dipole moment. We know that the torque,

$$\tau = pE\sin\theta$$

If E = 1 unit, $\theta = 90^0$, then $\tau = p$

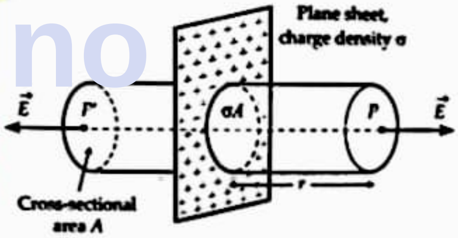
Hence *dipole moment may be defined as the toque acting on an electric dipole, placed perpendicular to a uniform electric field of unit strength.*

59. Soln. *Gauss theorem states that total flux through a closed surface is* $1/\varepsilon_0$ *times the net charge enclosed by the closed surface.*

Mathematically, it can be expressed as

$$\phi_E = \int_S \vec{E}.\vec{dS} = \frac{q}{\varepsilon_0}$$

Electric field due to a uniformly charged infinite plane sheet. As shown in fig. consider a thin, infinite plane sheet of charge with uniform surface charge density σ. We wish to calculate its electric field at a point P at distance r from it.



By symmetry, electric field E points outwards normal to the sheet. Also, it must have same magnitude and opposite direction at two points P and P' equidistant from the sheet and on opposite sides. We choose cylindrical Gaussian surface of cross-sectional area A and length 2r with its perpendicular to the sheet.

As the lines of force are parallel to the curved surface of the cylinder, the flux through the curved surface is zero. The flux through the plane – end faces of the cylinder is

$$\phi_E = EA + EA = 2EA$$

Charge enclosed by the Gaussian surface,

$$q = \sigma A$$

According to Gauss's theorem,

$$\phi_E = \frac{q}{\varepsilon_0}$$

$$\therefore \quad 2EA = \frac{\sigma A}{\varepsilon_0} \quad or \quad E = \frac{\sigma}{2\varepsilon_0}$$

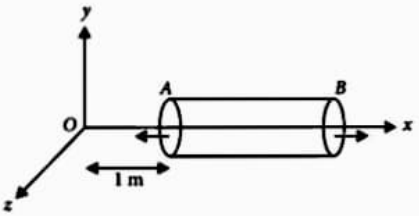
Clearly, E is independent of r, the distance from the plane sheet.

(i) If the sheet is *positively* charged $(\sigma > 0)$, the field is directed *away* from it.

(ii) If the sheet is *negatively* charged $(\sigma < 0)$, the field is directed towards it.

60. Sol.

(i)



Given, $\vec{E} = 50x\hat{i}$ and $A = 25cm^2 = 25\times10^{-4}m^2$ As the electric field is only along the x-axis, s, flux will pass only through the cross – section of cylinder.

Magnitude of electric field at cross-section A,

$$E_A = 50\times1 = 50\,NC^{-1}$$

Magnitude of electric field at cross – section B,

$$E_B = 50\times2 = 100\,NC^{-1}$$

The corresponding electric fluxes are

$$\phi_A = \vec{E}_A.\vec{A} = 50\times25\times10^{-4}\cos180^0$$
$$= -0.125\,N\,m^2C^{-1}$$

$$\phi_B = \vec{E}_B.\vec{A} = 100\times25\times10^{-4}\cos0^0$$
$$= 0.25\,N\,m^2C-1$$

So, the next flux through the cylinder,

$$\phi = \phi_A + \phi_B = -0.125 + 0.25 = 0.125\,N\,m^2C^{-1}$$

(ii) Using gauss's law

$$\oint \vec{E}.d\vec{A} = \frac{q}{\varepsilon_0} \Rightarrow 0.125 = \frac{q}{8.85\times10^{-12}}$$

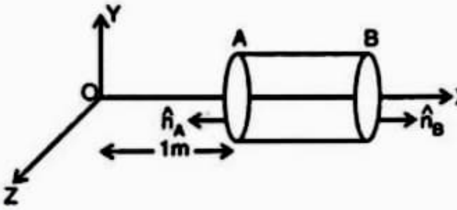
$$\Rightarrow q = 8.85\times0.125\times10^{-12} = 1.1\times10^{-12}C$$

61. Soln. (i) Given:

$$\vec{E} = 50x\hat{i}$$

And $\Delta S = 25cm^2$ or $25\times10^{-4}m^2$

As the electric field is only along the x-axis , hence, flux will pass only through the cross-section of cylinder.



Magnitude of electric field at cross-section A,

$$E_A = 50\times1 = 50\,N/C.$$

Magnitude of electric field at cross-section B,

$$E_B = 50\times2 = 100\,N/C$$

The corresponding electric fluxes are

$$\oint\phi_A = \vec{E}.\vec{\Delta S}$$
$$= 50\times25\times10^{-4}\times\cos180^0$$
$$= -0.125\,Nm^2/C^2$$

$$\oint\phi_B = \vec{E}.\vec{\Delta S}$$
$$= 100\times25\times10^{-4}\times\cos0^0$$
$$= 0.25\,Nm^2/C^2$$

So, the net flux through the cylinder,

$$\oint\phi = \oint\phi_A + \oint\phi_B$$
$$= 0.125 + 0.25$$
$$= 0.375\,Nm^2/C^2$$

(ii) Using the Gauss's law,

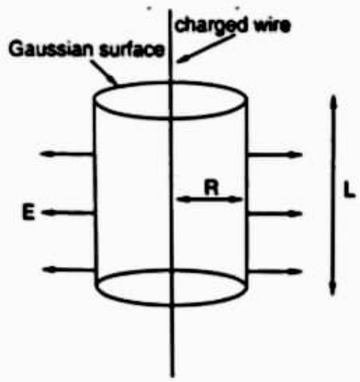
$$\phi = \oint \vec{E}.\vec{ds} = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad 0.375 = \frac{q}{8.85\times10^{-12}}$$

$$\Rightarrow \quad q = 8.85\times10^{-12}\times0.375$$

$$= 3.3\times10^{-12}C.$$

62. Soln. In a long straight wire with uniform charge per unit length λ, there should be electric field generated by charge distribution for cylindrical symmetry. Also, field to point will radially be away from the wire.



In this, cylindrical gaussian surface is co-axial with the wire of radius R and length L where symmetry implies to electric field generated by wire that will be perpendicular to curved surface of cylinder, so as per Gauss' law,

$$E(\mathrm{R})\times 2\pi\,\mathrm{RL} = \frac{\lambda L}{\varepsilon_0}$$

Where, E(R) is electric field strength which acts as perpendicular distance R from the wire.

In figure, left part shows electric flux through Gaussian surface while right part shows total charge enclosed by cylinder which is divided by ε_0.

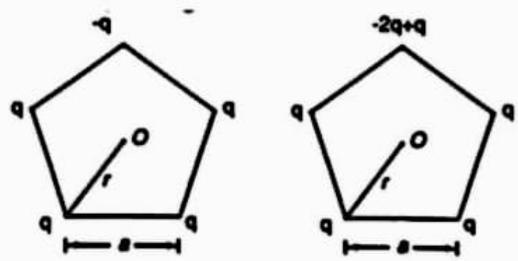
Further, $$E(\mathrm{R}) = \frac{\lambda}{2\pi\varepsilon_0 R}$$

Here, the field points are radially away from the wire then $\lambda > 0$, and radially towards the wire when $\lambda < 0$

63. Soln. (i) If a charge q is removed from point A, a negative charge is developed at A where electric field will be

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$ which is along $\overline{OA}$.

(ii) If a charge q is replaced by charge -q at point A, there generates a net electric field at point O as a result of -2q charge, so



$$E = \frac{1}{4\pi\varepsilon_0}\frac{-2q}{r^2}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2q}{r^2}\, along\ \overline{OA}$$

64. Soln. Flux of this field through a square of 10 cm

Flux of the square with normal making 30^0 angle

$\phi = EA\cos\theta$

$$\varphi = 5\times10^3\times10^{-2}\cos 0^0\ NC^{-1}m^2$$

$$\phi = 50\,NC^{-1}m^2$$

$$\phi = 5\times10^3\times10^{-2}\cos 60^0\ NC^{-1}m^2$$

$$= 25\,NC^{-1}m^2$$

65. Soln. Here, m = 10^{-3} kg, q = 5 x 10^{-6} C, E = 2 x 10^5 N/C,

U = 20 m/s, v = 0

As the particle enters opposite to the field, so it will retard.

Acceleration, $$A = -\frac{qE}{m}$$

$$= -\frac{5\times10^{-6}\times2\times10^5}{10^{-3}}$$

$$= -10^3\,m/s^2$$

Using, $$v^2 = u^2 - 2as$$

$$0 = (20)^2 - 2\times1000\times s$$

$$\Rightarrow \quad s = \frac{400}{2000} = \frac{1}{5} = 0.2\,m$$

MATERIAL

@BOARDDISCUSSION12TH

Created By Chankaya

SURE SHOT QUESTIONS

Chapter – 02

Electrostatic Potential and Capacitance

MCQ

1. Soln. (d): Current in the lower arm of the circuit,

$$I = \frac{2.5V}{2\Omega + 0.5\Omega} = 1A.$$

Potential difference across the internal resistance of cell = $(0.5\Omega)(1\text{A}) = 0.5V$ and potential difference across the $4\mu F$ capacitor $= 2.5V - 0.5V = 2V$.

Charge on the capacitor plates, Q = CV = $(4\mu\text{F})(2\text{V}) = 8\mu\text{C}$.

2. Soln. (c): In a uniform electric field, when a positively charged particle is released from rest, it moves along the electric field, i.e. is from higher potential to lower potential. Therefore, electric potential energy of charge decreases.

3. Soln. (c): In all the three figures, $V_A = 20V$ and $V_B = 40V$

Work done in carrying a charge q from A to B is

$$W = q(\text{V}_B - \text{V}_A)$$

Hence, work done is same in all figures.

4. Soln. (c): Potential at any point inside a charged conducting sphere = potential on the surface,

$$V = \frac{kq}{R} = 100V$$

Now, $E = -\frac{dV}{dr} = 0$ ($\because V$ is constant)

5. Soln. (a): For a collection of charges, whose total sum is not zero, equipotentials at large distances must be spheres only.

6. Soln. (c): The capacities of two individual condensers are $C_1 = \frac{K_1\varepsilon_0 A}{d_1}$ and $C_2 = \frac{K_2\varepsilon_0 A}{d_2}$

The arrangement is equivalent to two capacitors joined in series.

$\therefore$ Equivalent capacitance,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d_1}{K_1\varepsilon_0 A} + \frac{d_2}{K_2\varepsilon_0 A}$$

$$= \frac{1}{\varepsilon_0 A}\left[\frac{d_1}{K_1} + \frac{d_2}{K_2}\right] = \frac{1}{\varepsilon_0 A}\left[\frac{K_2 d_1 + K_1 d_2}{K_1 K_2}\right]$$

Or $C_{eq} = \varepsilon_0 A\left(\frac{K_1 K_2}{K_2 d_1 + K_1 d_2}\right)$ ....(i)

Also $C_{eq} = \frac{K\varepsilon_0 A}{d_1 + d_2}$ ....(ii)

From (i) and (ii), we get

$$\varepsilon_0 A\left(\frac{K_1 K_2}{d_2 K_1 + d_1 K_2}\right) = \varepsilon_0 A\left(\frac{K}{d_1 + d_2}\right)$$

$$\therefore \quad K = \frac{K_1 K_2(\text{d}_1 + \text{d}_2)}{d_2 K_1 + d_1 K_2}$$

7. Ans. (a) : Potential, V = $3x^2$

$$E = \frac{-dV}{dx} = -3 \times 2x = -6x$$

E(1, 0, 2) = -6 x 1 = -6 V m^{-1}

E = 6 V m^{-1} along -x axis

8. Ans. (a) : For spheres, the equipotentials at a large distance from a collection of charges, the sum is non zero.

9. Ans. (d) : Given, r_1 = 10 cm, r_2 = 15 cm

Work done = change in PE

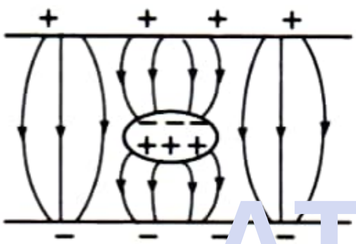
$$W = \frac{kqQ}{r_1} - \frac{kqQ}{r_2}$$

$$W = 9\times10^9 \times 5\times3\times10^{-18}\left[\frac{100}{10} - \frac{100}{15}\right]$$

$$= 9\times15\times10^{-7}\left[\frac{3-2}{30}\right]$$

$$W = \frac{9\times15\times10^{-7}}{30} = 4.5\times10^{-7}\ J$$

10. Ans. (d) : When an neutral conductor is placed between plates, a negative charge is induced on upper part and positive charge is induced on lower part of sphere so the correctly field lines are represented in (d).



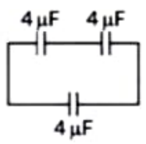
11. Ans. (c) : The electric field between the oppositely charged plates of a capacitor in twice of that due to one plate. So, when the one plate is removed the electric force reduces to half of its earlier value.

12. Ans. (a) : When they are connected in parallel, V is same

So, $\frac{q_1}{C_1} = \frac{q_2}{C_2}$

$$\frac{q_1}{q_2} = \frac{C_1}{C_2}$$

13. Ans. (c) : For the two series capacitors



$$C_{eq} = \frac{4\times4}{4+4}$$

$$C_{eq} = 2\mu F$$

Now, C_{eq} and $4\mu F$ in parallel.

$$C = 2+4 = 6\mu F$$

14. Ans. (a) : Voltage, V = 200 V,

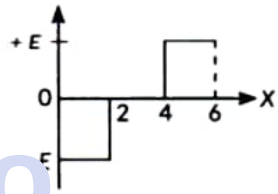
$$C_1 = 2\mu F \ to \ C_2 = X\ \mu F$$

Decrease in energy, $\Delta U = 2\times10^{-2} J$

$$\Delta U = \frac{1}{2}C_1V^2 - \frac{1}{2}C_2V^2$$

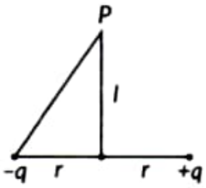
$$2\times10^{-2} = \frac{1}{2}\times200\times200(2-X)\times10^{-6}; X = 1\mu F$$

15. Ans. (a) : As, $E = -\frac{dV}{dx}$. Hence, the graph of electric field E as a function of 'x' will be shown as:



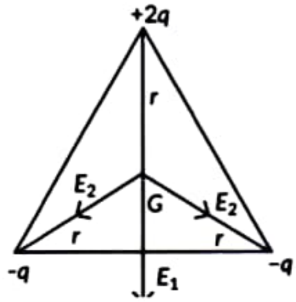
16. Ans. (d) : Both the electric potential and electric field achieve a maximum magnitude at B.

17. Ans. (d) :



Potential at 'P' is, $V_P = \frac{K(-q)}{\sqrt{r^2+l^2}} + \frac{K(q)}{\sqrt{r^2+l^2}} = 0$

18. Ans. (c) :



Net electric field intensity at centre G, $E \neq 0$

Net potential at G,

$$V = \frac{k \times 2q}{r} - \frac{kq}{r} - \frac{kq}{r}$$

$$\therefore \ V = 0$$

19. Ans. (c) : In uniform electric field, equipotential surfaces are never concentric spheres as they can never intersect but perpendicular to electric field lines.

For question number 6, two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below. [2020 – 21]

20. Ans. (b) : $W = pE(\cos\theta_1 - \cos\theta_2)$

$As, \ \theta_1 = 0^0 \ and \ \theta_2 = 90^0$

$$\therefore \ W = pE(\cos 0^0 - \cos 90^0)$$

$$= pE(1-0) = pE$$

Given below are two statements labelled as Assertion (A) and Reason (R).

21. Ans. (b) : As the three capacitors are joined in series,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = \frac{6}{6}$$

$$\therefore \ C = 1\mu F$$

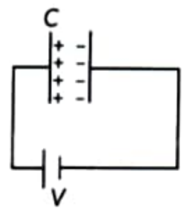
22. Ans. (b) : $\therefore \ F_p = F_e, \ \therefore \ F = qE$

E = q = same

Now, P.E. = qV(r)

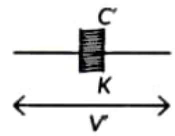
$$\therefore \ (P.E.)_p > (P.E.)_e$$

23. Ans. (c) :



$\therefore$ When battery is disconnected

Q = Charge remains constant



$$C' = KC$$

$$Q' = C'V'$$

$$Q = C'V'$$

$$Q = KCV'$$

$$\therefore \ V' = \frac{Q}{KC} = \frac{V}{K} \qquad \left(\therefore \ V = \frac{Q}{C}\right)$$

Assertion-Reasoning [1 Marks]

24. Ans. (b) : Electric field is always at right angle to equipotential surface because there is no potential gradient along any direction parallel to the surface and so an electric field parallel to surface.

Given below are two statements labelled as Assertion (A) and Reason (R).

25. Ans. (c) : Electrons move from a region of low potential to high potential.

26. **Sol.** (d):The potential of a conductor depends upon the charge given to it and there exist a potential different between two adjacent conductors.

27. **Sol.** (c): The capacitance of a capacitor filled partially with a dielectric of thickness t given by $C = \frac{\varepsilon_0 A}{d - t(1 - 1/K)}$.

For metals, $K = \infty$, $\therefore C = \frac{\varepsilon_0 A}{d-t}$.

Now if the capacitor is filled completely with a metallic slab, then $t = d$, $\therefore C = \infty$ i.e., when a charged capacitor filled fully with a metallic slab, then capacitor is short circuited i.e., it will no more work as a capacitor.

28. **Sol.** (a): The capacity of a parallel plate condenser is given by, $C = \frac{q}{V}$ ...(i)
Electric field intensity becomes $\frac{1}{K}$ times [as $K = E_0/E$], therefore potential V also becomes 1 IK times. Hence, from equation (i) capacity becomes K times. Thus electric field decreases and capacitance increases when condenser is filled with insulated medium of some dielectric constant.

29. **Sol.** (a): Capacitance of parallel plate capacitor is $C = \frac{\varepsilon_0 A}{d}$. Thus distance decreases and capacitance of capacitor increases.

30. **Sol.** (c): Earth is a good conductor of very large size. When some small charge is given to earth, its potential does not change. Hence potential of earth is assumed to be zero. It is just like sea level which does not alter materially when water is added to it or removed from it. Thus, the potential of all other bodies are measured with reference to the earth. For this, if the connection of a charged body to the ground by a metallic conductor would cause electrons to flow to that body from ground, the body is at positive potential. Conversely, is also true. In either case the conductor is neutralized and brought to zero potential. In fact the atmosphere does possess significant electric field.

31. **Sol.** (d):The whole charge of a conductor can be transferred to another isolated conductor, if it is placed inside the hollow insulated conductor and connected with it.

32. **Sol.** (c): If two points P and Q in an electric field are separated by an infinitesimal distance Ax and have a potential difference ΔV between then, $E = \frac{-\Delta V}{\Delta x}$.
Here negative sign implies that $\vec{E}$ has got a direction opposite to the potential gradient i.e. in the direction of $\vec{E}$ the potential decreases i.e. positive charge always move from a higher potential point to a lower potential point.

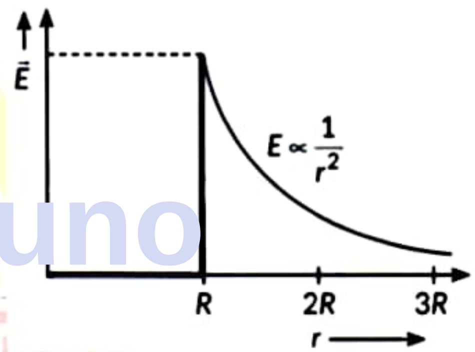
33. **Sol.** (b) Both A and R are true but R is NOT the correct explanation of A
Explanation: Both assertion and reason are true but the reason is not the correct explanation of assertion. When the pressure of a gas is lowered, the number of gas atoms become very small. Hence, the positive ions and the electrons are able to move over a long distance under the action of the electric field without any collision. Their mean free path becomes longer. The electrons rush towards the anode and ionise by collision with other gas atoms coming in their path. The positive ions move towards the cathode and eject more electrons from the atoms of the cathode. These fresh electrons also rush towards the anode, i. e., the flow of electrons and positive ions is maintained in the gas. On reducing the pressure of the gas, the discharge current increases further.

Case Study [4 Marks]

The following questions are source based/case based questions. Read the case carefully and answer the questions that follow.

34. Ans. (a) Here, a uniformly charged conducting shell of radius R.

We know that,



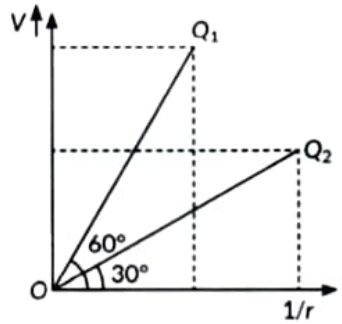
$E_1 = 0, \quad (0 < r < R)$

$E_2 = \frac{kQ}{R^2} \quad (r > R)$

$\left(\because \ E_2 = \frac{kQ}{R^2}\right)$

And at 3R, E = $\frac{kQ}{(3R)^2} = \frac{kQ}{9R^2}$

(b) Q_1 has more slope than Q_2.



So, $Q_1 > Q_2$ ....(i)

$V = \frac{kQ}{r}$ ....(ii)

$\left\{V \propto \frac{Q}{r}\right\}$

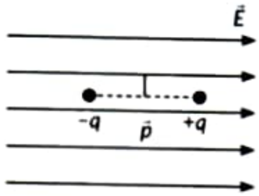
Because both are straight line passing through origin.

$y = mx$ ....(iii)

So, $Q \propto m$,

Or $\frac{Q_1}{Q_2} = \frac{\tan 60^0}{\tan 30^0} = \frac{\sqrt{3}}{1/\sqrt{3}}$ $\therefore \frac{Q_1}{Q_2} = \frac{3}{1}$

(c) Given, p = 6 x 10^{-7} C-m



E = 10^4 N/C

Since, dipole moment and electric field are parallel to each other.

So, $\theta = 0^0$.

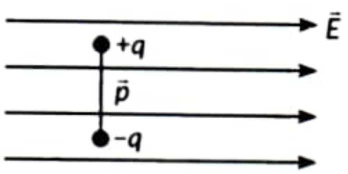
Potential energy, $U = -pE\cos\theta$

$U = -6\times10^{-7}\times10^4$

$U = -6\times10^{-3}\ Joules$

OR

An electric dipole of dipole moment $\vec{p}$ is initially kept in uniform electric field $\vec{E}$,



$U = -PE\cos\theta$

$or\ \ dU = -PE\cos\theta\ d\theta$

$\int dU = -PE\int_{\frac{\pi}{2}}^{-\pi}\cos\theta d\theta$

$\Delta U = \int dU = -pE\,|\sin\theta|_{\frac{\pi}{2}}^{-\pi} = -pE\left[\sin(-\pi) - \sin\frac{\pi}{2}\right]$

$\therefore\ \ \Delta W = \Delta U$

$-pE[0-1] = pE$

35. Soln.

1. (*c*) The Q charge on a capacitor indicates that the charges on its plates are $+Q$ and $-Q$.

2. (*b*) $V = Ed$

As E remains the same, so V increases as distance increases.

3. (*c*) $\frac{1}{C_s} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \ldots\ldots\ n$ factors $= \frac{n}{C}$

$\therefore\ \ C_s = \frac{C}{n}$

4. (*b*) $\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6}{6} = \frac{1}{1}$ $\Rightarrow C_{eq} = 1\mu F$

Charge on each capacitor is

$\therefore\ \ q = CV = 1\mu F \times 10V = 10\,\mu C.$

5. (*c*) $\mathcal{E}_{net} = 16 - 6 = 10V$

$C_{eq} = \frac{2\times3}{2+3} = \frac{6}{5}\mu F$

$q = C_{eq}\mathcal{E}_{net} = \frac{6}{5}\times10 = 12\,\mu C$

P.D. across $2\,\mu F$ capacitor,

$V_1 = \frac{q}{C_1} = \frac{12\mu C}{2\mu F} = 6\ V.$

Question

36. Ans. Let q be the charge on each droplet.

Then $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$ ....(i)

Volume of big drop = N x volume of small drop

$\frac{4}{3}\pi R^3 = N \times \frac{4}{3}\pi r^3$

Where R is the radius of the big drop.

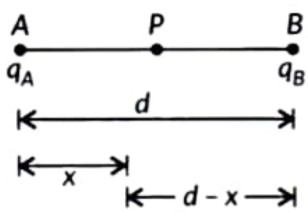
$\Rightarrow \quad R = N^{1/3} r$ ....(ii)

And Q = Nq, where Q is the charge of bigger drop

$\therefore$ Potential of larger drop,

$$V' = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Nq}{N^{1/3} r} = \frac{N}{N^{1/3}} V = N^{2/3} V$$

37. Ans. $q_A = q \text{ and } q_B = -2q$



$V_{PA} = \frac{kq_A}{x}$

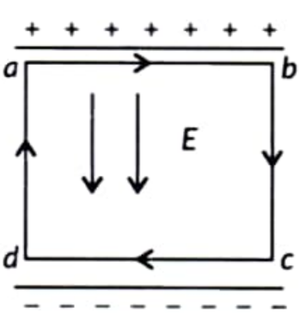
$V_{PB} = \frac{kq_B}{(d-x)}$

$V_{PA} + V_{PB} = 0$

$\frac{kq}{x} = \frac{2kq}{(d-x)}; d - x = 2x$

$3x = d;\ x = \frac{d}{3}$

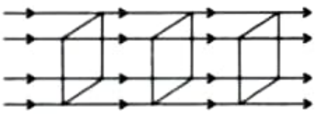
38. Ans. Electric field inside a parallel plate capacitor = E



Here, electric field is conservative. Work done by the conservative force in closed loop is zero.

So, required work done = 0.

39. Ans. Equipotential surfaces in a constant electric field in Z – direction.



For constant electric field

Electric field as gradient of potential consider a point charge +q placed at point O. Suppose that V and $V + \delta V$ are electrostatic potential at points A and B, where distance from the charge +q are r and $r - \delta r$ respectively.

$$(V + \delta V) = V + \frac{\delta W}{q_0}$$

$$\delta V = \frac{\delta W}{q_0} \quad \text{..............(i)}$$

If $\vec{E}$ is electric field at point P due to charge +q placed at point O, then the test charge q_0 experiences a force equal to $q_0\vec{E}$ and the external force required to move the test charge against the electric field $\vec{E}$ is given by

$$\vec{F} = -q_0\vec{E}$$

Therefore, work done to move the test charge through an infinitesimally small displacement

$\overrightarrow{PQ} = \overrightarrow{\delta l}$ is given by

$$\Delta W = \vec{F}.\overrightarrow{\delta l} = (-q\vec{E}).\vec{\delta l} = -q_0 E\delta l\cos 180^0 = q_0 E\delta l$$

As the distance r decreases in the direction of δl, then

$\delta W = -q_0 E\delta r$

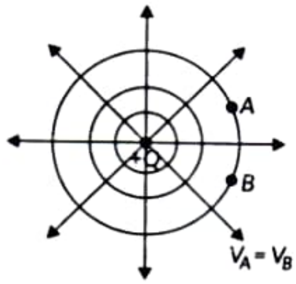
$$\frac{\delta W}{q_0} = -E\delta r \quad \text{..............(ii)}$$

From equations (i) and (ii), we get

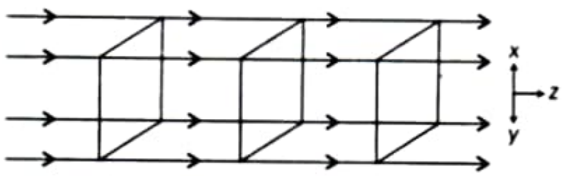
$$\delta V = -E\delta r;\ E = -\frac{\delta V}{\delta r} \qquad (1)$$

40. Ans. Equipotential surface is the surface with a constant value of potential at all points on the surface.

(i) Equipotential surface for single point charge:



(ii) Equipotential surfaces in a constant electric field in Z – direction.



For constant electric field

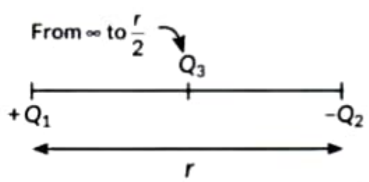
Equipotential surfaces about a single charge are not equidistant because electric potential, $V \propto \frac{1}{r}$

(iii) Electric field cannot exist tangential to an equipotential surface.

If the field lines are tangential, work will be done in moving a charge on the surface whereas on equipotential surface but we know that

$W_{AB} = q_0(V_B - V_A) = 0$

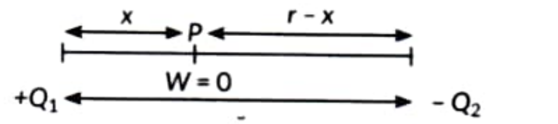
41. Ans. (a) The work done to bring the charge Q_3 from infinity to $\frac{r}{2}$,



$$W = U = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_3}{r/2} - \frac{1}{4\pi\varepsilon_0}\frac{Q_2Q_3}{r/2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{2Q_3}{r}[Q_1 - Q_2]$$

(b) Consider a point P at a distance x from Q_1 where work done is zero. Then



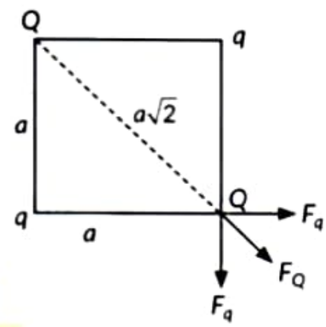
∴ Potential at P due to Q_1 = potential at P due to Q_2

$$\frac{kQ_1}{x} = \frac{kQ_2}{(r-x)} \Rightarrow (r-x)Q_1 = xQ_2$$

$$rQ_1 - xQ_1 = xQ_2 \Rightarrow rQ_1 = x(Q_1 + Q_2)$$

$$\Rightarrow x = \frac{rQ_1}{Q_1 + Q_2}$$

42. Ans. (a) Force on charge Q due to charge q.



$$F_q = \frac{1}{4\pi\varepsilon_0} \times \frac{qQ}{a^2}$$

Force on charge Q due to another charge Q.

$$F_Q = \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{(a\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}$$

Net force on charge Q is

$$F_{net} = F_Q + \sqrt{F_q^2 + F_q^2} = F_Q + F_q\sqrt{2}$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{Q^2}{2a^2} + \frac{1}{4\pi\varepsilon_0} \times \frac{qQ}{a^2}\sqrt{2}$$

$$= \frac{Q}{4\pi\varepsilon_0 a^2}\left[\frac{Q}{2} + \sqrt{2}q\right]$$ along diagonal

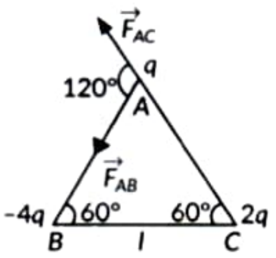
(b) Potential energy of the given system

$$U = U_{qQ} + U_{Qq} + U_{qQ} + U_{qq} + U_{QQ} = 4U_{qQ} + U_{qq} + U_{QQ}$$

$$= \frac{4qQ}{4\pi\varepsilon_0 a} + \frac{q^2}{4\pi\varepsilon_0(\sqrt{2}a)} + \frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}a)}$$

$$= \frac{1}{4\pi\varepsilon_0 a}\left[4qQ + \frac{q^2}{\sqrt{2}a} + \frac{Q^2}{\sqrt{2}a}\right]$$

43. Ans. (a) $F_{AB} = \frac{1}{4\pi\varepsilon_0}\frac{q(4q)}{l^2}$



$$= \frac{1}{4\pi\varepsilon_0}\frac{4q^2}{l^2}$$

$$F_{AC} = \frac{1}{4\pi\varepsilon_0}\frac{q(2q)}{l^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q^2}{l^2}$$

Angle between forces $\vec{F}_{AB}$ and $\vec{F}_{AC}$ is 120^0.

Magnitude of resultant force,

$$F = \sqrt{F_{AB}^2 + F_{AC}^2 + 2F_{AB}F_{AC}\cos 120^0}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q^2}{l^2}\right)\sqrt{(4)^2 + (2)^2 + 2\times4\times2\times\left(\frac{-1}{2}\right)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\sqrt{16+4-8} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{l^2}\left(2\sqrt{3}\right)$$

(b) Required work done =Change in potential energy of the system = $U_f - U_i$

$$= 0 - (U_{AB} + U_{BC} + U_{CA})$$

$$= \frac{-1}{4\pi\varepsilon_0 l}[q(-4q) + (-4q)(2q) + (q)(2q)]$$

$$= \frac{-1}{4\pi\varepsilon_0 l}[-4q^2 - 8q^2 + 2q^2] = \frac{10q^2}{4\pi\varepsilon_0 l}$$

44. Ans. (i) Capacitance $C = \dfrac{\varepsilon_0 A}{d}$

$$= \frac{8.85\times10^{-12}\times6\times10^{-3}}{3\times10^{-4}} = 17.7\times10^{-11}F$$

(ii) Charge Q = CV = 17.7 x 10^{-11} x 100

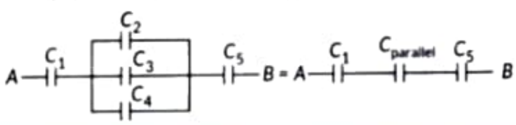
= 17.7 x 10^{-9} C

(iii) C' = KC

$$\therefore\ Q' = KQ = 10.62\times10^{-8}C$$

45. Ans. (i) In the circuit C_2, C_3 and C_4 are in parallel

$$\therefore\ C_{parallel} = C_2 + C_3 + C_4 = 2+2+2 = 6\mu F$$



∴ Equivalent capacitance between A and B is

$$\frac{1}{C_{equivalent}} = \frac{1}{C_1} + \frac{1}{C_{parallel}} + \frac{1}{C_5}$$

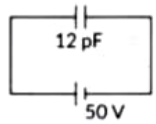
$$= \frac{1}{2} + \frac{1}{6} + \frac{1}{2} = \frac{3+1+3}{6} = \frac{7}{6}$$

$$\therefore\ C_{equivalent} = \frac{6}{7} = 0.86\,\mu F$$

(ii) $Q = C_{equivalent}V = 0.86\times7 = 6\mu C$

$$\therefore\ Energy, E = \frac{1}{2}QV = \frac{1}{2}\times6\times7 = 21J$$

46. Ans. Electrostatic energy stored in the capacitor,



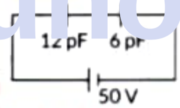
$$U = \frac{1}{2}CV^2 = \frac{1}{2}\times12\times10^{-12}\times(50)^2$$

(As C = 12 pF, V = 50V)

U = 1.5 x 10^{-8} J

When 6 pF is connected in series with 12 pF, charge stored across each capacitor,

$$Q = \frac{C_1C_2}{C_1+C_2}V$$



$$= \frac{12\times6\times10^{-24}}{(12+6)\times10^{-12}}\times50 = 200\ pC$$

Now, potential difference across 12 pF is,

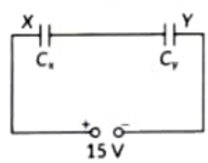
$$= \frac{Q}{C_1} = \frac{200\times10^{-12}}{12\times10^{-12}} = 16.67V$$

Potential difference across 6 pF is,

$$= \frac{Q}{C_2} = \frac{200\times10^{-2}}{6\times10^{-12}} = 33.33V$$

47. Ans. Here, $C_x = \dfrac{\varepsilon_0 A}{d}$

$$C_y = \frac{\varepsilon_0\varepsilon_r A}{d} = \varepsilon_r C_x = 4C_x$$



(i) C_x and C_y are in series, so equivalent capacitance is given by

$$C = \frac{C_x \times C_y}{C_x + C_y}$$

$$\Rightarrow \quad 4 = \frac{C_x \times 4C_x}{C_x + 4C_x} \quad (\because C = 4\mu F)$$

$$\Rightarrow \quad 4 = \frac{4C_x}{5} \quad \therefore C_x = 5\,\mu F$$

And $C_y = 4\,C_x = 20\,\mu F$

(ii) Charge on each capacitor, Q = CV

$$Q = 4 \times 10^{-6} \times 15 = 60 \times 10^{-6} C$$

Potential difference between the plates of X,

$$V_x = \frac{Q}{C_x} = \frac{60 \times 10^{-6}}{5 \times 10^{-6}} = 12V$$

Potential difference between the plates of Y,

$$V_y = V - V_x = 15 - 12 = 3V$$

(iii) Ratio of electrostatic energy stored,

$$\frac{U_x}{U_y} = \frac{\frac{Q^2}{2C_x}}{\frac{Q^2}{2C_y}} = \frac{C_y}{C_x} = \frac{4C_x}{C_x} = 4$$

48. Ans. (i) Given that energy of the 6 μF capacitor is E

Let V be the potential difference along the capacitor of capacitance 6 μF .

Since, $\frac{1}{2}CV^2 = E \quad \therefore \frac{1}{2} \times 6 \times 10^{-6} \times V^2 = E$

$$\Rightarrow V^2 = \frac{E}{3} \times 10^6 \qquad \text{.............(i)}$$

Since potential is same for parallel connection, the potential through 12 μF capacitor is also V.

Hence, energy of 12 μF capacitor is

$$E_{12} = \frac{1}{2} \times 12 \times 10^{-6} \times V^2 = \frac{1}{2} \times 12 \times 10^{-6} \times \frac{E}{3} \times 10^6 = 2E$$

(ii) Since charge remains constant in series, the charge on 6 μF and 12 μF capacitors combined will be equal to the charge on 3 μF capacitor.

Using the formula, Q = CV.

We can write

$$(6 + 12) \times 10^{-6} \times V = 3 \times 10^{-6} \times V'$$

$$V' = 6V$$

Using (i) and squaring both sides, we get

$$V'^2 = 12E \times 10^6$$

$$\therefore E_3 = \frac{1}{2} \times 3 \times 10^{-6} \times 12E \times 10^6 = 18E$$

(iii) Total energy drawn from battery is

$$E_{total} = E + E_{12} + E_3 = E + 2E + 18E = 21E$$

49. Ans. Given E = 10r + 5

Now the electric potential, $V = -\int E.dr$

$$= -\int_1^{10} (10r + 5)dr = -\left[\frac{10r^2}{2} + 5r\right]_1^{10}$$

$$= -1[5r^2 + 5r]_1^{10} = -[(5 \times 100 + 50) - (5 + 5)] = -540V$$

50. Ans. Electric field $E = -\frac{dV}{dx}$

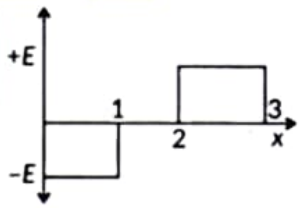
$For \;\; x = 0 \; to \; 1, V = kx$ ....(i)

$x = 1 \; to \; 2, \;\; V = k$

$x = 2 \; to \; 3, V = -kx$

Where k is some constant

So using (i) the variation of electric field is shown in figure



51. Ans. Since net force on electric dipole in uniform electric field is zero, so no work is done in moving the electric dipole in uniform electric field, however some work is done in rotating the dipole against the torque acting on it. So, small work done in rotating the dipole by an angle dθ in uniform electric field E is

$$dW = \tau d\theta = pE \sin\theta d\theta$$

Hence, net work done in rotating the dipole from angle θ_i to θ_f in uniform electric field is

$$W = \int_{\theta_i}^{\theta_f} pE \sin\theta d\theta = pE[-\cos\theta]_{\theta_i}^{\theta_f}$$

Or $W = pE[-\cos\theta_f + \cos\theta_i] = pE[\cos\theta_i - \cos\theta_f]$

If initially, the dipole is placed at an angle $\theta_i = 90^0$ to the direction of electric field, and is then rotated to the angle $\theta_f = \theta$, then net work done is

$$W = pE[\cos 90^0 - \cos\theta]$$
$$or\quad W = -pE\cos\theta$$

This gives the work done in rotating the dipole through an angle θ in uniform electric field, which gets stored in it in the form of potential energy i.e.,

$$U = -pE\cos\theta$$

This gives potential energy stored in electric dipole of moment p when placed in uniform electric field at an angle θ with its direction.

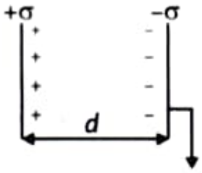
(i) When $\theta = 0^0$, then $U_{min} = -pE$

So, potential energy of an electric dipole is minimum, when it is placed with its dipole moment p parallel to the direction of electric field E and so it is called its most stable equilibrium position.

(ii) When $\theta = 180^0$, then $U_{max} = +pE$

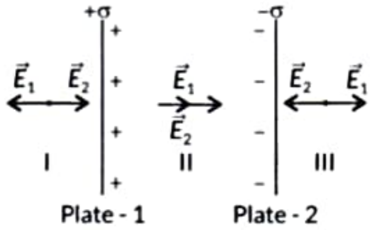
So, potential energy of an electric dipole is maximum, when it is placed with its dipole moment p anti-parallel to the direction of electric field E and so it is called its most unstable equilibrium position.

52. Ans. Capacitor is based on the principle of electrostatic induction. The capacitance of an insulated conductor increases significantly by bringing an uncharged earthed conductor near to it. This combination forms parallel plate capacitor.



(a) Magnitude of electric field intensities

$$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$$



(i) In region I (outside)

$$E_I = E_2 - E_1 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

(ii) In region II (inside)

$$E_{II} = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

(iii) In region III (outside)

$$E_{III} = E_1 - E_2 = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$$

In the region II i.e., in the space between the plates, resultant electric field $\vec{E}_{II}$ is directed normal to plates, from positive to negative charge plate.

(b) The potential difference between the plates is

$$V = E_{II}d = \frac{\sigma}{\varepsilon_0}d \quad or \quad V = \frac{Q}{A\varepsilon_0}d$$

(c) Capacitance of the capacitor so formed is

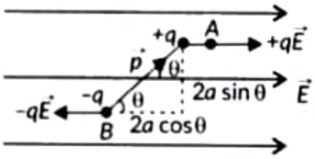
$$C = \frac{Q}{V} = \frac{Q}{Qd / A\varepsilon_0} \quad or \quad C = \frac{\varepsilon_0 A}{d}$$

53. Ans. (a) A pair of equal and opposite charges separated by a small vector distance is called an electric dipole. An ideal dipole consist of two very very large charges +q and -q separated by a very very small distance. An ideal dipole has almost no size.

Water molecule is an example of electric dipole.

(b) Torque on a dipole in uniform electric field:

When electric dipole is placed in a uniform electric field, its two charges experience equal and opposite forces, which cancel each other and hence net force on an electric dipole in a uniform electric field is zero.



However these forces are not collinear, so they give rise to some torque on the dipole given by

Torque = Magnitude of either force x Perpendicular distance between them

$$\tau = Fr_1 = qE.2a\sin\theta = q2a.E\sin\theta$$

Or $\tau = pE\sin\theta$

Where θ is the angle between the directions of $\vec{p}$ and $\vec{E}$.

In vectorial form, $\vec{\tau} = \vec{p}\times\vec{E}$

(i) When $\theta = 0^0$ or 180^0 then $\tau_{min} = 0$

(ii) When $\theta = 90^0$ then $\tau_{max} = pE$

Thus, torque on a dipole tends to align it in the direction of uniform electric field.

If the field is not uniform in that condition the net force on electric dipole is not zero.

When $\theta = 0;\ \tau = 0$ and $\vec{p}$ and $\vec{E}$ are parallel and the dipole is in a position of stable equilibrium.

(c) Torque $\tau = PE\sin\theta = Q/\sin\theta$ ...(i)

Here l is the length of the dipole. Q is the charge and E is the electric field.

Therefore Q = Torque/Esin (θ)

$$= 8\sqrt{3}/(2\times10^{-2})(10^5)\frac{\sqrt{3}}{2} = 8\times10^{-3}C$$

Potential energy, $U = -PE\cos\theta = -Ql\cos\theta$ ...(ii)

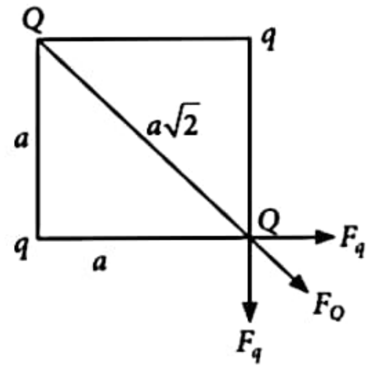
Divide equation (i) by (ii),

$$\frac{\tau}{U} = \frac{Q/\sin\theta}{-Q/\cos\theta}(where\ P = Ql)$$

$$\frac{\tau}{U} = -\tan\theta \Rightarrow U = \frac{\tau}{-\tan 60^0} = \frac{-8\sqrt{3}}{\sqrt{3}} = -8J$$

54. Soln. (a) Force on charge Q due to charge q.

$$F_q = \frac{1}{4\pi\varepsilon_0}\times\frac{qQ}{a^2}$$



Force on charge Q due to another charge Q,

$$F_Q = \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{(a\sqrt{2})^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}$$

Net force on charge Q is

$$F_{net} = F_Q + \sqrt{F_q^2 + F_q^2} = F_Q + F_q\sqrt{2}$$

$$= \frac{1}{4\pi\varepsilon_0}\times\frac{Q^2}{2a^2} + \frac{1}{4\pi\varepsilon_0}\times\frac{qQ}{a^2}\sqrt{2}$$

$$= \frac{Q}{4\pi\varepsilon_0 a^2}\left[\frac{Q}{2} + \sqrt{2}q\right] along\ diagonal$$

(b) Potential energy of the given system.

$$U = U_{qQ} + U_{Qq} + U_{qQ} + U_{Qq} + U_{qq} + U_{QQ}$$

$$= 4U_{qQ} + U_{qq} + U_{QQ}$$

$$= \frac{4qQ}{4\pi\varepsilon_0 a} + \frac{q^2}{4\pi\varepsilon_0(\sqrt{2}\,a)} + \frac{Q^2}{4\pi\varepsilon_0(\sqrt{2}\,a)}$$

$$= \frac{1}{4\pi\varepsilon_0 a}\left[4qQ + \frac{q^2}{\sqrt{2}a} + \frac{Q^2}{\sqrt{2}a}\right]$$

55. Soln. The potential at any point on the surface of the conductor having radius r and charge q is given by

$$V = \frac{1}{4\pi\varepsilon_0}.\frac{q}{r}$$

Where $\varepsilon_0 = 8.854\times10^{-12}\ C^2N^{-1}m^{-2}$

The capacitance f the spherical conductor situated in vacuum is given by

$$C = \frac{q}{V} = \frac{q}{\frac{1}{4\pi\varepsilon_0}.\frac{q}{r}}$$

$$C = 4\pi\varepsilon_0 r$$

Hence, the capacitance of an isolated spherical conductor situated in vacuum is $4\pi\varepsilon_0$ times its radius.

56. Soln. Energy stored = $\frac{1}{2}CV^2 = \frac{1}{2}\frac{Q_2}{C}$

Net capacitance with switch S closed = C + C = 2C

∴ Energy stored = $\frac{1}{2}\times 2C\times V^2 = CV^2$

After the switch S is opened, capacitance of each capacitor = KC

∴ Energy stored in capacitor $A = \frac{1}{2}KCV^2$

For capacitor B,

Energy stored

$$= \frac{1}{2}\frac{Q^2}{KC} = \frac{1}{2}\frac{C^2V^2}{KC} = \frac{1}{2}\frac{CV^2}{K}$$

∴ Total energy stored

$$= \frac{1}{2}KCV^2 + \frac{1}{2}\frac{CV^2}{K}$$

$$= \frac{1}{2}CV^2\left(K + \frac{1}{K}\right)$$

$$= \frac{1}{2}CV^2\left(\frac{K^2+1}{K}\right) \quad \text{.......(ii)}$$

On dividing (i) and (ii)

∴ Required ratio = $\frac{2CV^2K}{CV^2(K^2+1)} = \frac{2K}{(K^2+1)}$

57. Soln. Given,

$c = 100\,\mu F$

$d = 4\times10^{-3} m$

$V = 200V$

$k = 5$

$Q = CV$

= 200 x 100 x 10^{-6}

= 2 x 10^{-2} Coulomb

As dielectric of 4 mm is inserted between the plates of capacitor and the spacing between the plates is doubled then it will acts as following fig-A and fig-B.

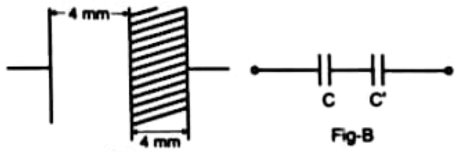


Fig-A Fig-B

Here, C′ will be capacitance with dielectric of 4 mm & 8 mm separation between the plates.

$$C' = KC = 5\times100\times10^{-6} = 0.5\times10^{-3} F$$

(i) Thus, equivalent capacitance

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C'} = \frac{1}{100\times10^{-6}} + \frac{1}{0.5\times10^{-3}}$$

$$= 10\times10^3 + 2\times10^3$$

$$= 12\times10^3$$

$$\frac{1}{C_{eq}} = 12\times10^3$$

$$C_{eq} = \frac{1}{12\times10^3}$$

$$= \frac{1}{12}\times10^{-3}$$

$$C_{eq} = 83.33\,\mu F$$

(ii) Electric field inside dielectric will be

$(E') = \frac{E}{K} = \frac{5[illegible]}{5\times10[illegible]}$

$$= \frac{50\times10^3}{5} = 10\times10^3$$

$$= 10000\, v/m$$

And Electric field inside capacitor but out of dielectric Area will be

$$E = 50\times10^3\, V/m$$

(iii) Energy density f capacitor is given by:

$$U = \frac{Q^2}{2C}$$

$$= U' + U$$

$$= \frac{Q^2}{2C} + \frac{Q^2}{2C'}$$

$$\Rightarrow \quad \frac{Q^2}{2}\left[\frac{1}{C}+\frac{1}{C'}\right] = \frac{2\times10^{-2}\times2\times10^{-2}}{2}\left[12\times10^{3}\right]$$

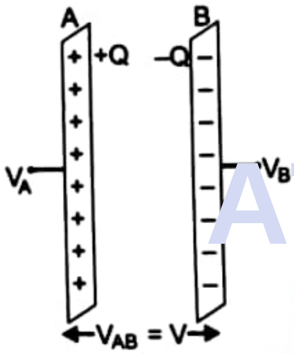
$$\left(\because \frac{1}{C}+\frac{1}{C'}=\frac{1}{C_{eq}}\right)$$

$$= 2\times10^{-1}\times12$$

$$= 2.4\text{ J}$$

58. Soln. (a) Consider a capacitor of capacitance C. Initial charge on capacitor is zero. Initial potential difference between capacitor plates is zero.

Let a charge Q be given to it in small steps. When charge is given to capacitor, the potential difference between its plates increases. Let at any instant when charge on capacitor be q, the potential difference between its plates $V=\frac{q}{C}$.



Now work done in giving an additional infinitesimal charge dq to capacitor.

$$dW = V\ dq = \frac{q}{C}dq$$

The total work done in giving charge from 0 to Q will be equal to the sum of all such infinitesimal works, which may be obtained by integration.

Therefore, total work done.

$$W = \int_0^Q V\ dq = \int_0^Q \frac{q}{C}dq$$

$$= \frac{1}{C}\left[\frac{q^2}{2}\right] = \frac{1}{C}\left(\frac{Q^2}{2}-\frac{0}{2}\right) = \frac{Q^2}{2C}$$

If V is the final potential difference between capacitor plates, then Q = CV

$$\therefore \quad W = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

This work is stored as electrostatic potential energy of capacitor i.e.,

Electrostatic potential energy,

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

Energy density: Consider a parallel plate capacitor consisting of plates, each of area A, separated by a distance d. If space between the plates is filled with a medium of dielectric constant K, then

Capacitance of capacitor, $C = \frac{K\varepsilon_0 A}{d}$

If σ is the surface charge density of plates, then electric field strength between the plates

$$E = \frac{\sigma}{K\varepsilon_0} \Rightarrow \sigma = K\ \varepsilon_0 E$$

Charge on each plate of capacitor,

$Q = \sigma A = K\ \varepsilon_0 EA$

$\therefore$ Energy stored by capacitor,

$$U = \frac{Q^2}{2C} = \frac{(K\,\varepsilon_0\,EA)^2}{2(K\,\varepsilon_0\,A/d)}$$

$$= \frac{1}{2}K\,\varepsilon_0 E^2 A\,d$$

But Ad = Volume of space between capacitor plates

$\therefore$ Energy stored, $U = \frac{1}{2}K\,\varepsilon_0 E^2 Ad$

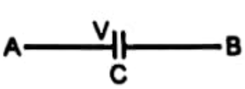
Electrostatic energy stored per unit volume,

$$u_e = \frac{U}{Ad} = \frac{1}{2}K\,\varepsilon_0 E^2$$

This is expression for electrostatic energy density in medium of dielectric constant K.

(b) Initially, if we consider a charged capacitor, then its charge would be

Q = CV

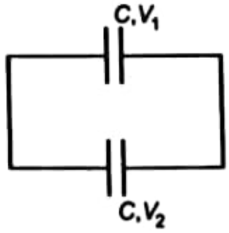


And energy stored is

$$U_1 = \frac{1}{2}CV^2 \qquad \text{.................(i)}$$

Then, this charged capacitor is connected to uncharged capacitor.

Let the common potential be V. The charge flows from first capacitor to the other capacitor unless both the capacitors attain common potential.



$$\Rightarrow \quad Q_1 = CV_1 \;\; and \;\; Q_2 = CV_2$$

Applying conservation of charge,

$$Q = Q_1 + Q_2 \Rightarrow CV = CV_1 + CV_2$$

$$\Rightarrow \quad V = V_1 + V_2 \Rightarrow V_1 = \frac{V}{2} [\because V_1 = V_2]$$

Total energy stored,

$$U_2 = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2$$

$$U_2 = \frac{1}{2}C\left(\frac{V}{2}\right)^2 + \frac{1}{2}C\left(\frac{V}{2}\right)^2$$

$$\Rightarrow \quad U_2 = \frac{1}{4}CV^2 \qquad \text{..............(ii)}$$

From Equations (i) and (ii), we get

$$U_2 < U_1$$

$\Rightarrow$ Energy stored in the combination is less than that stored initially in single capacitor.

59. Soln. When S is closed:

P.d. across C_1 = P.d. across C_2 = 6 V

$$V_1 = V_2 = 6V \qquad [\because q = CV]$$

$$\therefore \; q_1 = q_2 = 1\mu F \times 6V = 6\mu C$$

When S is open:

When dielectric slab (K = 3) are inserted,

$$C_1 = 3 \times 1\mu F = 3\mu F$$
$$C_2 = 3 \times 1\mu F = 3\mu F$$

P.d. across C_1,

$$V_1' = 6V$$

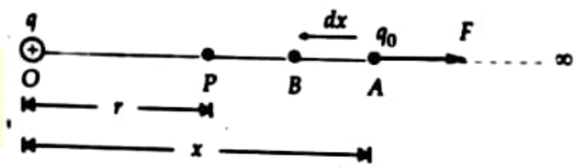
$$\therefore \qquad q_1' = 3\mu F \times 6V = 18\,\mu C$$

P.d. across C_2,

$$V_2' = \frac{q_2}{C_2} = \frac{6\mu C}{3\mu F} = 2V$$

$$\therefore \qquad V_2' = 2V$$

60. Soln. Electric potential due to a point charge. Consider a positive point charge q placed at the origin O. We wish to calculate its electric potential at a point P at distance r from it, as shown in fig. By definition, the electric potential at point P will be equal to the amount of work done in bringing a unit positive charge from infinity to the point P.



Suppose a test charge q_0 is placed at point A at distance x from O. BY coulomb's law, the electrostatic force acting on charge q_0 is

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{x^2}$$

The force $\vec{F}$ acts away from the charge q. The small work done in moving the test charge q_0 from A to B through small displacement $\vec{dx}$ against the electrostatic force is

$$W = \int dW = -\int_{\infty}^{r} F dx = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{x^2} dx$$

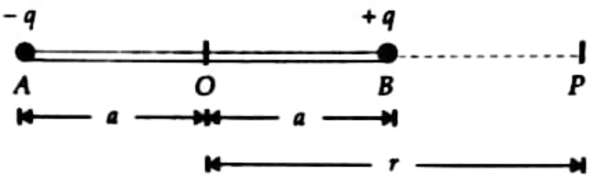
$$= -\frac{qq_0}{4\pi\varepsilon_0} \int_{\infty}^{r} x^{-2} dx = -\frac{qq_0}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_{\infty}^{r}$$

$$= \frac{qq_0}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qq_0}{r}.$$

Hence the work done in moving a unit test charge from infinity to the point P, or the electric potential at point P is

$$V = \frac{W}{q_0} \quad or \qquad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

61. Soln. Electric potential at an axial point of a dipole. As shown in fig. consider an electric dipole consisting of two point charges -q and +q and separated by distance 2a. Let P be a point on the axis of the dipole at a distance r from its centre O.



Electric potential at point P due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{BP}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r+a} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r-a}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r-a} - \frac{1}{r+a}\right]$$

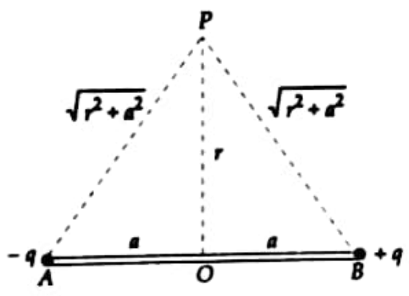
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)-(r-a)}{r^2-a^2}\right] = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\times 2a}{r^2-a^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{p}{r^2-a^2} \qquad [\because p = q\times 2a]$$

For a short dipole, $a^2 << r^2$, so $V = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}$.

Electric potential at an equatorial point of a dipole.

As shown in fig, consider an electric dipole consisting of charges -q and +q and separated by distance 2a. Let P be a point on the perpendicular bisector of the dipole at distance r from its centre O.

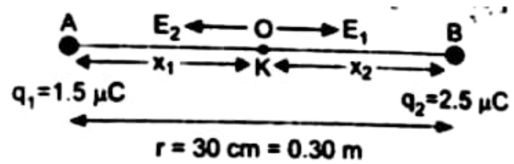


Electric potential at point P due to the dipole is

$$V = V_1 + V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-q}{AP} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{BP}$$

$$= -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{r^2+a^2}} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\sqrt{r^2+a^2}} = 0.$$

62. Soln. The potential due to similar charges is additive while electric field at a point due to individual charges are added vectorially.



(a) The electric potential at mid point O.

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{x_1} + \frac{q_2}{x_2}\right)$$

Here $x_1 = x_2 = \frac{0.30}{2} = 0.15\,m$

$$V = 9\times10^9\left[\frac{1.5\times10^{-6}}{0.15} + \frac{2.5\times10^{-6}}{0.15}\right] = 9\times10^9\left[10\times10^{-6} + \frac{50}{3}\times10^{-6}\right]$$

$$= 9\times10^9\times\frac{80}{3}\times10^{-6} = 2.4\times10^4\ volt.$$

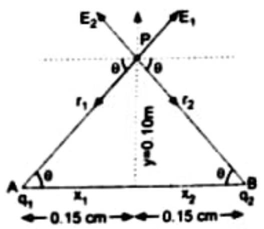
Electric field at O due to q_1 is towards $\vec{AB}$ and that due to q_2 is towards $\vec{BO}$ is net electric field at mid point O

$$E = E_2 - E_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_2}{x_2^2} - \frac{q_1}{x_1^2}\right) = 9\times10^9\left[\frac{2.5\times10^{-6}}{(0.15)^2} - \frac{1.5\times10^{-6}}{(0.15)^2}\right]$$

$= 4.0\times10^5\ N/C$ Charge from q_2 to q_1.

(b) Let P be a point at distance 10 cm = 0.10 m from O, in a plane normal to line AB.

$$AP = BP = \sqrt{(0.15)^2 + (0.10)^2} = 0.18\,m$$



Electric potential at P

$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{(AP)} + \frac{q_2}{(BP)}\right]$$

$$= 9\times10^{9}\left[\frac{1.5\times10^{-6}}{0.18}+\frac{2.5\times10^{-6}}{0.18}\right]$$

$$= \frac{9\times10^{9}\times4.0\times10^{-6}}{0.18} = 2.0\times10^{5}\ volt.$$

Electric field at P due to q_1,

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1^2}\,along\ \vec{PQ} = 9\times10^{9}\times\frac{1.5\times10^{-6}}{(0.18)^2}\,along\ \vec{PQ}$$

Electric field at P due to q_2

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2^2}\,along\ \vec{PR} = 9\times10^{9}\times\frac{2.5\times10^{-6}}{(0.18)^2}\,along\ \vec{PR}$$

Resolving E_1 and E_2 along and normal to AB.

Net electric field along

$B\vec{A},\ E_x = E_2\cos\theta - E_1\cos\theta$

$$= (E_2 - E_1)\cos\theta = (E_2 - E_1)\frac{x_1}{r}$$

$$= 9\times10^{9}\left[\frac{2.5\times10^{-6}-1.5\times10^{-6}}{(0.18)^2}\right]\times\left(\frac{0.15}{0.18}\right)$$

$$= \frac{9\times10^{9}\times1.0\times10^{-6}}{(0.18)^2}\times\left(\frac{0.15}{0.18}\right) = 2.3\times10^{5}\ N/C$$

Net electric field normal to AB,

$E_y = (E_2 + E_1)\sin\theta$

$$= 9\times10^{9}\left[\frac{2.5\times10^{-6}+1.5\times10^{-6}}{(0.18)^2}\right]\times\frac{0.10}{0.18}$$

$$= 9\times10^{9}\times\frac{4.0\times10^{-6}}{(0.18)^2}\times\frac{10}{18} = 6.2\times10^{5}\ N/C$$

Net electric field

$$E = \sqrt{E_x^2+E_y^2} = \sqrt{(2.3\times10^5)^2+(6.2\times10^5)^2} = 6.6\times10^{5}\ N/C$$

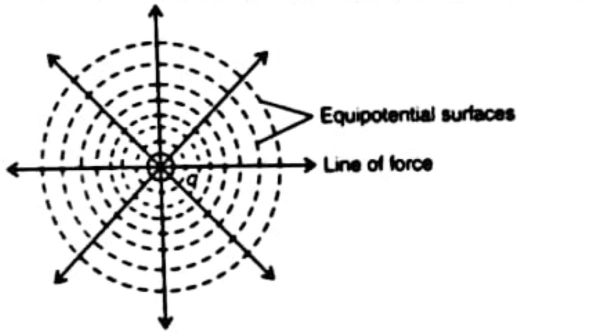
If α is the angle made by resultant field with AB, then

$$\tan\alpha = \frac{E_y}{E_x} = \frac{6.2\times10^5}{2.3\times10^5} = 2.69$$

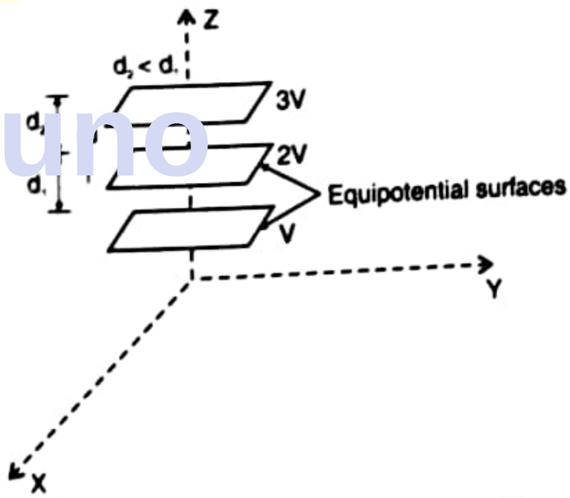
$$\Rightarrow\quad \alpha = \tan^{-1}(2.69) = 69.6^0$$

63. Soln. Equipotential surface is a surface which has equal potential at every point on it.

(i) Equipotential surfaces due to single point charge are concentric spheres having charge at the centre.



(ii) In constant electric field along z-direction, the perpendicular distance between equipotential surfaces remains same.



For single charge, equipotential surface will be series of concentric spherical shells with charge at centre,

$$dr \propto \frac{1}{E}$$

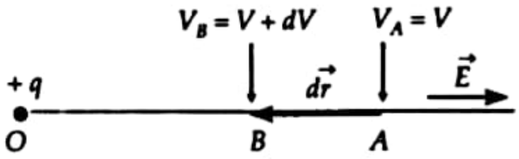
The separation dr between equipotential surface will go on increasing with decrease in electric field.

(iii) No, because if the surface is not equipotential then it would mean that there is tangential component of electric field along surface.

This component will result in motion of electrons, but since we have static fields, this is not possible.

64. Soln. Computing electric field from electric potential. As shown in fig. consider the electric field due to charge +q located at the origin O. Let A and B be two adjacent points separated by distance dr.

The two points are so close that electric field $\vec{E}$ between them remains almost constant. Let V and V + dV be the potentials at the two points.



The external force required to move the test charge q_0 (without acceleration) against the electric field $\vec{E}$ is given by

$$\vec{F} = -q_0 \vec{E}$$

The work done to move the test charge from A to B is

$$W = F.dr = -q_0 E.dr$$

Also, the work in moving the test charge from A to B is

W = Charge x potential difference

$$= q_0(V_B - V_A) = q_0\, dV$$

Equating the two works done, we get

$$-q_0 E.dr = q_0.dV$$

$$or \qquad E = -\frac{dV}{dr}$$

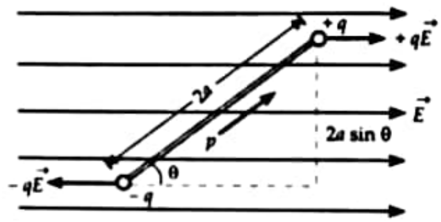
The quantity $\frac{dV}{dr}$ is he rate of change of potential with distance and is called *potential gradient*. Thus *the electric field at any point is equal to the negative of the potential gradient at that point*. The negative sign shows that the direction of the electric field is in the direction of decreasing potential. Moreover, the field is in the direction where this decrease is steepest.

65. Soln. Potential energy of a dipole placed in a uniform electric field. As shown in fig. consider an electric dipole placed in a uniform electric field $\vec{E}$ with its dipole moment $\vec{p}$ making an angle θ with the field.

Two equal and opposite forces $+q\vec{E}$ and $-q\vec{E}$ act on its two ends. The two forces form a couple. The torque exerted by the couple will be

$$\tau = qE \times 2a\sin\theta = pE\sin\theta$$

Where $q \times 2a = p$, is the dipole moment.



If the dipole is rotated through a small angle $d\theta$ against the torque acting on it, then the small work done is

$$dW = \tau\, d\theta = pE\sin\theta\, d\theta$$

The total work done in rotating the dipole from its orientation making an angle θ_1, with the direction of the field to θ_2 will be

$$W = \int dW = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$= pE[-\cos\theta]_{\theta_1}^{\theta_2} = pE(\cos\theta_1 - \cos\theta_2)$$

This work done is stored as the potential energy U of the dipole.

$$\therefore \qquad U = pE(\cos\theta_1 - \cos\theta_2)$$

If initially the dipole is oriented perpendicular to the direction of the field ($\theta_1 = 90^0$) and then brought to some orientation making an angle θ with the field ($\theta_2 = \theta$), then potential energy of the dipole will be

$$U = pE(\cos 90^0 - \cos\theta) = \mathrm{pE}(0 - \cos\theta)$$

Or $\quad U = -pE\cos\theta = -\vec{p}.\vec{E}$

66. Soln. Here 2a = 4 cm = 0.04 m, $\theta = 60^0$;

$$\tau = 4\sqrt{3}\ Nm,\ q = 8nC = 8\times10^{-9}C$$

Dipole moment,

$$p = q\times 2a = 8\times10^{-9}\times0.04 = 0.32\times10^{-9}\ Cm.$$

(i) As $\tau = pE\sin\theta$

$$\therefore\ E = \frac{\tau}{p\sin\theta} = \frac{4\sqrt{3}}{0.32\times10^{-9}\times\sin60^0}$$

$$= \frac{4\sqrt{3}\times10^9\times2}{0.32\times\sqrt{3}} = 2.5\times10^{10}\ NC^{-1}.$$

(ii) $u = -pE\cos\theta$

$$= -0.32\times10^{-9}\times2.5\times10^{10}\times\cos60^0 = -4\ J.$$

67. Soln. Here m = 5.0 g = 5 x 10^{-3} kg,

$$q = \pm4\times10^{-5}C,\ r_1 = 1.0m,\ r_2 = 50cm = 0.50m$$

Let v = speed of each particle at the separation of 50 cm.

From energy conservation principle,

K.E. of the two particles at 50 cm separation + P.E. of the two particles at 50 cm separation

= P.E. of the two particles at 1.0 m separation

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_1}$$

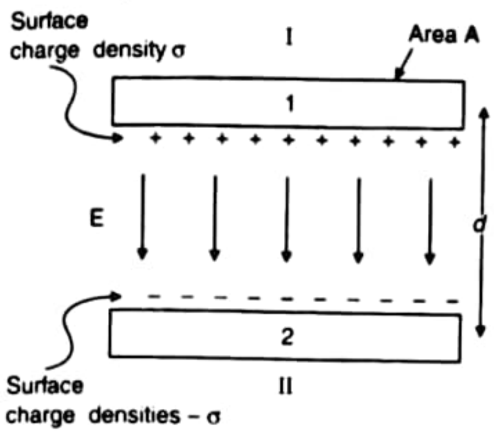
$$mv^2 = \frac{q_1q_2}{4\pi\varepsilon_0}\left[\frac{1}{r_1}-\frac{1}{r_2}\right]\ or\ v^2 = \frac{q_1q_2}{4\pi\varepsilon_0 m}\left[\frac{r_2-r_1}{r_1r_2}\right]$$

$$\therefore\ v^2 = \frac{4\times10^{-5}\times(-4\times10^{-5})\times9\times10^9}{5\times10^{-3}}\left[\frac{0.50-1.0}{1.0\times0.50}\right]$$

$$= 2880\ or\ v = 53.67\ ms^{-1}$$

68. Soln.

(i) Capacitance equals the magnitude of the charge on each plate needed to raise the potential difference between the plates by unity.



[The capacitance is defined as $C = \frac{q}{V}$]

Consider parallel plates of area A

Plate separation d, the potential difference applied across it is V. The electric field

$$E = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A}$$

Electric field = potential gradient

$$\therefore\quad E = \frac{V}{d}$$

Hence, $\frac{V}{d} = \frac{q}{\varepsilon_0 A}$

$$c = \frac{q}{v} = \frac{\varepsilon_0 A}{d}$$

(ii) Capacitance without dielectric,

$$C = \frac{A\varepsilon_0}{d}$$

Capacitance when filled with dielectric having thickness $\frac{3d}{4}$

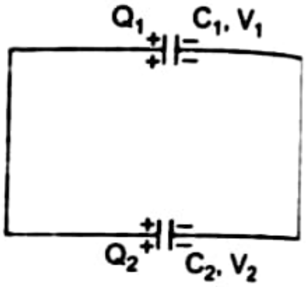
$$C = \frac{A\varepsilon_0}{\left(d-t+\frac{t}{k}\right)}$$

$$= \frac{A\varepsilon_0}{\left(d-\frac{3d}{4}+\frac{3d}{4k}\right)}\qquad As\ \ t = \frac{3d}{4}$$

Ratio $\frac{C'}{C} = \frac{A\varepsilon_0 4k}{d(k+3)}\times\frac{d}{A\varepsilon_0}$

$$= \frac{4k}{(k+3)}$$

69. Soln. Suppose two charged capacitors C_1 and C_2 charged to potentials V_1 and V_2 are connected in parallel, with their positive terminals connected together and negative terminals connected together as shown in the figure. After connection, the charge redistributes in such a way that the potential differences across C_1 and C_2 become equal.



Charges on capacitors before connection,
$Q_1 = C_1V_1,\ Q_2 = C_2V_2$

Common potential after connection, $V = \dfrac{Q_1+Q_2}{C_1+C_2}$

$$V = \frac{C_1V_1 + C_2V_2}{C_1+C_2}$$
..............(i)

If $Q_1^{'}$ and $Q_2^{'}$ are charges after sharing, then

$$Q_1^{'} = C_1V, \qquad Q_2^{'} = C_2V$$

$$\Rightarrow \quad \frac{Q_1^{'}}{Q_2^{'}} = \frac{C_1}{C_2} \qquad \text{..........(ii)}$$

This means that *after connection, the charge on capacitors are shared in ratio of their capacitances.*

Electrostatic energy stored in the system: When charges are shared between two capacitors, then some energy is dissipated as heat and hence, definitely there is a loss of energy.

The energy loss = Initial energy (U_i) – Final energy (U_f)

Initial energy: $\quad U_i = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2$

After connecting the two capacitors, their combined capacitance is (C_1+C_2) and common potential is V; therefore, final electrostatic energy

$$U_f = \frac{1}{2}(C_1+C_2)V^2$$

$$= \frac{1}{2}(C_1+C_2)\left(\frac{C_1V_1+C_2V_2}{C_1+C_2}\right) \qquad \{\text{using (i)}\}$$

$$= \frac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

∴ *Loss in energy during sharing of charges,*

$$\Delta U = U_i - U_f$$

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{\frac{1}{2}(C_1V_1+C_2V_2)^2}{C_1+C_2}$$

$$= \frac{(C_1+C_2)(C_1V_1^2+C_2V_2^2) - (C_1V_1+C_2V_2)^2}{2(C_1+C_2)}$$

$$= \frac{1}{2(C_1+C_2)}\{C_1^2V_1^2 + C_1C_2V_2^2 + C_1C_2V_1^2 + C_2^2V_2^2 - (C_1^2V_1^2 +$$

$$= \frac{1}{2(C_1+C_2)}\{C_1C_2V_2^2 + C_1C_2V_1^2 - 2C_1V_1V_2\}$$

$$= \frac{C_1C_2}{2(C_1+C_2)}\{V_1^2 + V_2^2 - 2V_1V_2\}$$

$$= \frac{C_1C_2}{2(C_1+C_2)}(V_1 - V_2)^2$$

i.e., energy loss, $\Delta U = \dfrac{C_1C_2}{2(C_1+C_2)}(V_1-V_2)^2$.

As, C_1, C_2 and $(V_1 - V_2)^2$ are all positive, therefore, ΔU is always positive. Thus, in the process of redistribution of charges, there is always a loss of energy. This energy is lost in the form of heat in connecting wire.

70. Soln.

(a) Charge on proton $q_1 = +1.6\times10^{-19}C$

Charge on electron $q_2 = -1.6\times10^{-19}C$

Separation $r = 0.53\overset{0}{A} = 0.53\times10^{-10}m$

∴ Potential energy of system $U = U\ at\ r - U$ at ∞

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r} - 0$$

$$= 9\times10^9 \times \frac{(1.6\times10^{-19})(-1.6\times10^{-19})}{0.53\times10^{-10}}$$

$$= -43.47\times10^{-19}\ \text{J}$$

As 1 eV = 1.6 x 10^{-19} J, we have

$$U = -\frac{43.47\times10^{-19}}{1.6\times10^{-19}}eV \approx -27.2eV$$

(b) Kinetic energy is always positive, so kinetic energy of electron = $\frac{27.16}{2} = +13.6eV$

Total energy of electron = -27.2 + 13.6 = -13.6 eV

Minimum work required to free the electron = -Total energy of bound electron = 13.6 eV

(c) Potential energy is at separation, $r_0 = 1.06\overset{0}{A}$ is

$$U_0 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$= 9\times10^9 \times \frac{(1.6\times10^{-19})(-1.6\times10^{-19})}{1.6\times10^{-10}}$$

= $-21.73\times10^{-19}J = -13.6eV$

∴ Potential energy of system when zero of potential energy is taken at $r_0 = 1.06\overset{0}{A}$

$$U = U(r) - U_0 = -27.2 + 13.6 = -13.6eV$$

Now total energy of hydrogen atom is zero

∴ Minimum work =

$$E - U = 0 - (-13.6)\text{eV} = 13.6\text{eV}$$

71. Soln. ∴ $C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{A\varepsilon_0}{d}\left(\frac{K_1K_2}{K_1+K_2}\right)$

Now, capacitor C_3 can be considered as made up of two capacitors C_1 and C_2, each of plate area A and separation d, connected in series.

We have $C_1' = \frac{A\varepsilon_0K_1}{d}$

And $C_2' = \frac{A\varepsilon_0K_2}{d}$

$$\Rightarrow \quad C_3 = \frac{C_1'C_2'}{C_1'+C_2'} = \frac{A\varepsilon_0}{d}\left(\frac{K_1K_2}{K_1+K_2}\right)$$

$$\therefore \quad \frac{C_3}{C_{eq}} = 1$$

Hence, the net capacitance of the combination is equal to that of C_3.

72. Soln.

(i) Capacitance of parallel plate capacitor is:

$$C = \frac{\varepsilon_0\varepsilon_r A}{d}$$

Now knowing the capacitance of X and Y plates:

$$C_x = \frac{\varepsilon_0\varepsilon_r A}{d}$$

Now knowing the capacitance of X and Y plates:

$$C_x = \frac{\varepsilon_0 A}{d}$$

And $C_y = \frac{\varepsilon_0 4A}{d}$

Hence, $C_y = 4C_x$

Now, $C_{eq} = \frac{C_xC_y}{C_x+C_y}$

$$C_{eq} = 4\mu F$$

Further, $4\mu F = \frac{C_x4(C_x)}{5C_x}$

We see that values of C_x and C_y are $5\mu F$ and $20\mu F$

(ii) Finding the potential difference between the parallel plates X and Y

$Q = CV$

Further, $C_xV_x = C_yV_y$

Now, $\frac{V_x}{V_y} = \frac{C_y}{C_x} = 4$

∴ $V_x = 4V_y$

Also it is noted that $V_y + V_x = 15$

∴ $V_y = 3V \text{ and } V_x = 12V$

(iii) Find the ratio of electrostatic energy stored in parallel plates X and Y:

Since energy stored is given as:

$$E = \frac{Q^2}{2C}$$

Now the ratio will be calculated by writing:

$$\frac{E_x}{E_y} = \frac{C_y}{C_x} = 4$$

$$\therefore \quad \frac{E_x}{E_y} = \frac{4}{1} or\ 4:1$$

73. Soln. Capacitors C_2 and C_3 form a parallel combination. Their equivalent capacitance is

$$C' = C_2 + C_3 = (4+2)\mu F = 6\mu F$$

Now C_1 and C' form a series combination, therefore, the equivalent capacitance of the entire network is

$$C = \frac{CC'}{C+C'} = \frac{3\times 6}{3+6} = 2\mu F$$

The charge on the equivalent capacitor is

$$q = CV = 2\times 10^{-6} \times 1200C = 2.4\times 10^{-3} C$$

This must be equal to the charge on C_1 and also the sum of the charges on C_2 and C_3. Thus

$$V_A - V_B = \frac{q}{C_1} = \frac{2.4\times 10^{-3}}{3\times 10^{-6}} = 800V$$

$$V_A = 1200V$$

$$\therefore \quad V_B = 1200 - 800 = 400V$$

Hence $V_C - V_B = 400 - 0 = 400V$

$$q_2 = C_2(V_C - V_B) = 4\times 10^{-6} \times 400C = 1.6\times 10^{-3} C$$
$$q_3 = C_3(V_C - V_B) = 2\times 10^{-6} \times 400C = 0.8\times 10^{-3} C$$
$$q_1 = q = 2.4\times 10^{-3} C.$$

74. Soln.

(i) $E = \frac{1}{2}CV^2 = \frac{1}{2}\times 6\times 10^{-6} V^2 = 3\times 10^{-6} V^2$

$$\therefore \quad V^2 = \frac{E}{3\times 10^{-6}}$$

Energy stored in $12\mu F$ capacitor

$$= \frac{1}{2}\times 12\times 10^{-6} \times \frac{E}{3\times 10^{-6}} = 2E.$$

(ii) Capacitance of the parallel combination $= 6 + 12 = 18\mu F$

Charge on parallel combination,

$$Q = CV = 18\times 10^{-6} V$$

Charge on $3\mu F$ capacitor = $Q = 3\times 10^{-6} V_1$

$$\therefore \quad 3\times 10^{-6} V_1 = 18\times 10^{-6} V \Rightarrow V_1 = 6V.$$

Energy stored in $3\mu F$ capacitance

$$= \frac{1}{2}CV_1^2 = \frac{1}{2}\times 3\times 10^{-6} \times (6\,V)^2$$
$$= \frac{1}{2}\times 3\times 10^{-6} \times \frac{36E}{3\times 10^{-6}} = 18E.$$

(iii) Total energy drawn = $E + 2E + 18E = 21E.$

75. Soln. Electrostatic shielding. Consider a conductor with a cavity, with no charges placed inside the cavity. Whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external fields in which it might be placed, the electric field inside the cavity is zero, i.e., the cavity inside the conductor remains shielded from outside electric influence. This is known as electrostatic shielding. Such a field free region is called a *Faraday cage*.

The phenomenon of making a region free from any electric field is called electrostatic shielding. It is based on the fact that electric field vanishes inside the cavity of a hollow conductor.

76. Soln. Capacitance of a parallel plate capacitor with a dielectric slab. The capacitance of a parallel plate capacitor of plate area A and plate separation d with vacuum between its plates is given by

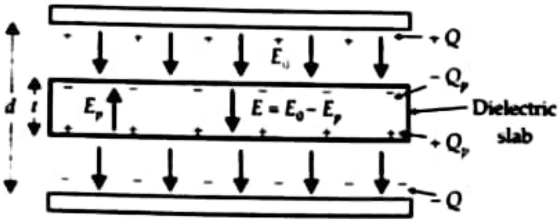
$$C_0 = \frac{\varepsilon_0 A}{d}$$

Suppose initially the charges on the capacitor plates are $\pm Q$. Then the uniform electric field set up between the capacitor plates is

$$E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

When a dielectric slab of thickness t < d is placed between the plates, the field E_0 polarises the dielectric.

This induces charge $-Q_p$ on the upper surface and $+Q_p$ on the lower surface of the dielectric. These induced charges set up a field E_p inside he dielectric in the opposite direction of $\vec{E}_0$. The induced field is given by $E_p = \frac{\sigma_p}{\varepsilon_0} = \frac{P}{\varepsilon_0}$ $\quad [\sigma_p = \frac{Q}{A} = P$, polarisation density]



The net field inside the dielectric is

$$E = E_0 - E_p = \frac{E_0}{\kappa}$$

$$\left[\because \frac{E_0}{E_0 - E_p} = \kappa\right]$$

Where κ is the dielectric constant of the slab. So between the capacitor plates, the field E exists over a distance t and field E_0 exists over the remaining distance (d – t). Hence the potential difference between the capacitor plates is

$$V = E_0(d-t) + Et = E_0(d-t) + \frac{E_0}{\kappa}t \quad \left[\because \frac{E_0}{E} = \kappa\right]$$

$$= E_0\left(d - t + \frac{t}{\kappa}\right) = \frac{Q}{\varepsilon_0 A}\left(d - t + \frac{t}{\kappa}\right)$$

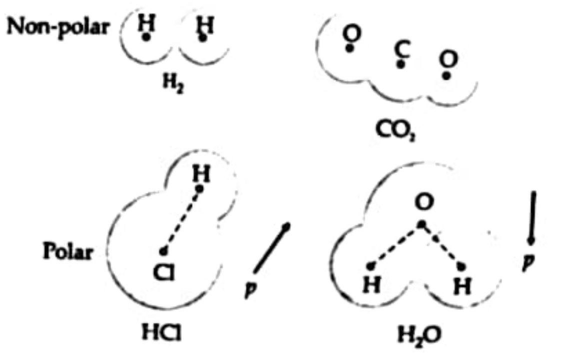
The capacitance of the capacitor on introduction of dielectric slab becomes

$$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d - t + \frac{t}{\kappa}}$$

77. Soln. *A dielectric is a substance which does not allow the flow of charges through it but permits them to exert electrostatic forces on one another through it. A dielectric is essentially an insulator which can be polarised through small localised displacements of its charges.*

Polar and non-polar dielectrics. A dielectric may consist of either polar or non-polar molecules. *A molecule in which the centre of mass of positive charges (protons) does not coincide with the centre of mass of negative charges (electrons) is called a polar molecule.*

The dielectrics made of polar molecules are called polar dielectrics. The polar molecules have unsymmetrical shapes. They have permanent dipole moments of the order of 10^{-30} cm. For example, a water molecule has a bent shape with its two O – H bonds inclined at a angle of 105^0 as shown in fig. It has a very large dipole moment of 6.1 x 10^{-30} cm. Some other polar molecules ae HCl, NH_3, CO, CH_3OH, etc.



A molecule in which the centre of mass of positive charges coincides with the centre of mass of negative charges is called a non-polar molecule. The dielectrics made of non-polar molecules are called *non-polar dielectrics.* Non-polar molecules have symmetrical shapes. They have normally zero dipole moment. Examples of non-polar molecules are H_2, N_2, O_2, CO_2, CH_4, etc.

78. Soln.

	Non – polar (O_2)	Polar (H_2O)
In absence of electric field		
Individual	No dipole moment exists.	Dipole moment exists.
Specimen	No dipole moment exists.	Dipoles are randomly oriented. Net P = 0
In presence of electric field		
Individual	Dipole moment exists (molecules become polarised.)	Torque acts on the molecules to align them parallel to E.
Specimen	Dipole moment exists.	Net dipole moment exists parallel to dipole moment exists E.

SURE SHOT QUESTIONS

Chapter – 03

Current Electricity

MCQ (1 mark)

1. Soln. (a)

2. Soln. (a): As $R = \rho\frac{l}{A}$, resistance is maximum when l is large and A is least. For the given dimensions of wire, resistance will be maximum for $l = 10$ cm and $A = 1\,cm \times \frac{1}{2}\,cm$.

3. Soln. (a): Current in conductor
$I = neAv_d \Rightarrow I \propto v_d$

Hence current in a conductor is determined by drift velocity and does not depend upon thermal velocity

4. Ans. (a) : Here, V = 200 V; R = 100 Ω; t = 1s

Let the current is I and the number of electrons are n.

$V = IR;\ I = \frac{V}{R} = \frac{200}{100} = 2A$

$q = It;\ q = 2\times 1 = 2C$

$n = \frac{q}{e} = \frac{2}{1.6\times 10^{-19}} = 1.25\times 10^{19}$

5. Ans. (b) : $\vec{J} = \sigma\vec{E} \Rightarrow \frac{\vec{J}}{\vec{E}} = \sigma$, conductivity of the material. Current density is microscopic form of Ohm's law. $J = \sigma E$.

OR

(a) : Resistivity is the ratio of electric field to current density.

6. Ans. (b) : The resistivity ρ is given by $\rho = \frac{m}{ne^2\tau}$

Resistance is given by

$\therefore\ R = \rho\frac{l}{A} = \frac{ml}{ne^2\tau A}$

7. Ans. (c) : It remains unchanged

8. Ans. (b) : As resistivity, $\rho = \frac{m}{ne^2\tau}$

$\rho \propto \frac{1}{\tau} \Rightarrow R \propto \frac{1}{\tau}$

9. Ans.(a)

10. Ans. (c) : The negative temperature coefficient of resistivity shows that when temperature increases, resistivity decreases. This is the property of semiconductor.

11. Ans. (a) : Rating of bulb = 220 V – 100 W

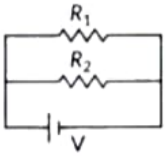
Operating voltage, V = 110 V

Resistance of bulb, $R = \frac{V^2}{P} = \frac{220^2}{100} = 484\Omega$

Power consumed by bulb,

$P = \frac{V^2}{R} = \frac{110^2}{484} = 25W$

12. Ans. b) : $R_1 = 4\Omega,\ R_2 = 6\Omega$

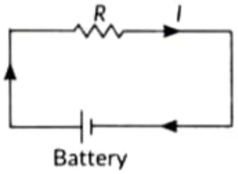


$P_1 = \frac{V^2}{R_1}, P_2 = \frac{V^2}{R_2}$

$\frac{P_1}{P_2} = \frac{R_2}{R_1} = \frac{6}{4} = \frac{3}{2}$

13. Ans. (c) : In a d.c. circuit, the current outside the battery travels from positive to negative terminal

and inside the battery, it is from negative to positive terminal.



14. Ans. (d) : The current flowing through the circuit will be maximum when the enternal resistance is zero i.e., R = 0.

15. Ans.(b)

16. Ans.(d)

17. Ans. (i) (b) : Here,

$R_1 = 10\Omega,\ R_2 = R_3 = 5\Omega, r = 0, E = 5V$

As there is no resistance between f, h, j so they are at same potential.

(ii) (c) : R_2 *and* R_3 in series, $R_s = 5 + 5 = 10\Omega$

Now, R_s *and* R_1 in parallel, $R_{eq} = \frac{10\times10}{10+10} = 5\Omega$

So, $I = \frac{E}{R_{eq}} = \frac{5}{5} = 1A$

Current in bg, $I = \frac{E}{R_1} = \frac{5}{10} = \frac{1}{2}A$

(i) (b) : Power dissipated in R_1

$P_1 = I^2 R_1 = \frac{1}{4}\times10 = 2.5W$

(ii) (c) : Current in $R_3, I_3 = \frac{1}{2}A$

Potential difference across

$R_3, V_3 = I_3 R_3 = \frac{1}{2}\times5 = 2.5V$

18. Ans. (b) : Kirchhoff's first rule or Kirchhoff's junction rule deals with the conservation of charge.

19. Ans. (d) : Same charge flows through each cross-section in the given time.

20. Ans. (c) : We know, $V_d = \frac{-eE\tau}{m} = \frac{-eV\tau}{ml}$

If temperature is kept constant, relaxation time τ will remain constant, and e, m are also constants.

$V_d \propto V \text{ and } V_d \propto 2V$

21. Ans. (b) : As copper and germanium are metal and superconductor respectively.

22. Ans. (c) : Here, $l_1 : l_2 = 3:2; r_1 : r_2 = 2:3; I_1 : I_2 = ?$

$R_1 = \rho\frac{l_1}{\pi r_1^2}$ ......(i);

$R_2 = \rho\frac{l_2}{\pi r_2^2}$ ....(ii)

Dividing eqn. (i) by (ii), we get

$$\frac{R_1}{R_2} = \frac{l_1}{l_2}\frac{\pi r_2^2}{\pi r_1^2} = \frac{l_1}{l_2}\times\frac{r_2^2}{r_1^2} = \frac{3}{2}\times\left(\frac{3}{2}\right)^2 = \frac{(3)^3}{(2)^3} = \frac{27}{8}$$

$$\therefore\ \frac{I_1}{I_2} = \frac{V/R_1}{V/R_2} = \frac{R_2}{R_1} = \frac{8}{27}$$

23. Ans. (a) : Alloys have low temperature coefficient of resistivity (α) and high specific resistance. If α = low, the value of R with temperature will not change much and specific resistance is high then required length of the wire will be less.

24. Ans. (a) : $H \propto l;\ H \propto \frac{1}{r^2}$

25. Ans.(c)

26. Ans.(c)

Assertion-Reasoning (1 mark)

27. Sol.

(c): 1 kWhr = 1 kW × 1 hour
= 1000 (joule/sec) × 3600 sec = 36 × 10^5 joule

i.e., kWhr is the unit of electric energy and used for expressing consumed electric energy.

28. Sol. (a): These alloys (constantan or manganin) are used for making standard resistance because they

possess high resistivity and low temperature coefficient of resistance.

29. **Sol.** (b):The value of temperature coefficient of resistance is positive only for metals and alloys and is negative for semiconductors and insulators.

30. **Sol.**

(a): Drift velocity of free electrons is given by

$$v_d = \frac{eE}{m}\tau$$

where, $E = \frac{\text{Potential difference}}{\text{length}} = \frac{V}{l}$

$\therefore \quad v_d = \frac{eV}{ml}\tau$ *i.e.*, $v_d \propto 1/l$ where, $\frac{eV\tau}{m}$ is constant.

It mean if l is doubled, the drift velocity will become half of the original value.

31. **Sol.** (b): On increasing temperature of wire the kinetic energy of free electrons increase and so they collide more rapidly with each other and hence their drift velocity decreases. Also when temperature increases, resistance increase and resistance is inversely proportional to conductivity of material.

32. **Sol.**

(c): Consider a conductor of length l and area of cross section A. Time taken by the free electrons to cross the conductor, $t = l/v_d$.

Hence, current, $I = \frac{q}{t} = \frac{Al \times ne}{l/v_d}$

or, $\quad I = Anev_d$

or, $\quad I \propto v_d$

Thus current is directly proportional to drift velocity.

33. **Sol.** (c): Primary cells cannot be recharged because they involve irreversible reactions. Secondary cells can be recharged because they involve reversible reactions.

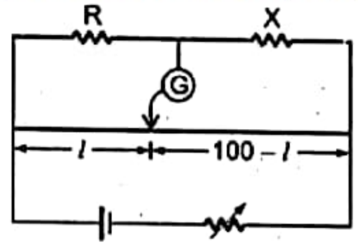
34. **Sol.** (a): In normal conductor, the direction of electrons are randomly oriented such that the total sum of their velocities is equal to zero.

35. **Sol.**

(d) A is false and R is also false

Explanation:

Both assertion and reason are false.



36. **Sol.**

(a) As we know that,

Glow = Power (P) = I²R

$$\therefore \quad \frac{dP}{P} = 2\left(\frac{dI}{I}\right) = 2 \times 0.5 = 1\%.$$

Case Study

PARAGRAPH 1: Electric Resistance

37. Answers

1. (*c*) $R = \rho\frac{l}{A} = \rho\frac{l}{\pi r^2} \quad \Rightarrow \quad R \propto \frac{1}{r^2}$.

2. (*b*) The resistance between opposite rectangular faces is

$$R = \rho\frac{h}{l\,b} = \frac{3.7\times10^{-7}\times10^{-2}}{1\times10^{-2}}\Omega = 3.7\times10^{-7}\Omega.$$

3. (*d*) The specific resistance does not depend on l and A, it depends on nature of the material only.

4. (*a*) $\frac{R'}{R} = \frac{l'}{l}\times\frac{A}{A'} = \frac{2l}{l}\times\frac{A}{A/2} = 4.$

5. (*c*) In the portion CD, the current I decreases with the increase in voltage V. This portion corresponds to negative resistance.

PARAGRAPH 2: Internal Resistance of a Cell

38. Answers

6. (*c*) Internal resistance of a cell acts only in a closed circuit and it reduces the current.

7. (*d*) $V = \varepsilon - Ir = 10 - 0.5\times3 = 10 - 1.5 = 8.5$ V.

8. (*a*) $V = IR = \left(\frac{\varepsilon}{R+r}\right)R = \frac{\varepsilon}{1+\frac{r}{R}}$

As R increases, V also increases.

When $R \to 0, \quad V = 0$

When $R \to \infty, \quad V = \varepsilon$

9. (*b*) Maximum power is drawn when $r = R$

$$\therefore \quad I = \frac{\mathcal{E}}{r+R} = \frac{\mathcal{E}}{r+r} = \frac{\mathcal{E}}{2r}$$

$$P_{max} = I^2R = \left(\frac{\mathcal{E}}{2r}\right)^2 \times r = \frac{\mathcal{E}^2}{4r}$$

10. (*d*) For maximum power, $r = R$

$$\therefore \quad I = \frac{\mathcal{E}}{R+r} = \frac{\mathcal{E}}{2r} = \frac{16}{2 \times 2} = 4 \text{ A.}$$

➢ Question

39. Ans. When an electric field is applied across a conductor then the charge carriers inside the conductor move with an average velocity which is independent of time. This velocity is known as drift velocity (v_d).

Current flowing in a conductor is given by $I = neAv_d$

Current density $J = \frac{I}{A}$

$\therefore \; J = nev_d$

40. Ans. Mobility of a charge carrier is defined as the drift velocity of the charge carrier per unit electric field.

It is generally denote by μ.

$$\mu = \frac{v_d}{E}$$

The SI unit of mobility is $m^2 V^{-1} s^{-1}$. Mobility in term of relaxation time: $\vec{v}_d = \frac{-e\vec{E}}{m}\tau$

In magnitude, $v_d = \frac{eE}{m}\tau$ or $\frac{v_d}{E} = \frac{e\tau}{m}; \mu = \frac{e\tau}{m}$

41. Ans. As we know that

$I = neAv_d$

Also current density J is given by $J = \frac{I}{A}$

$$\therefore \; |\vec{J}| = \frac{ne^2}{m}\tau|\vec{E}| \qquad \left(\because \; v_d = \frac{e\tau E}{m}\right)$$

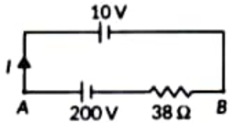
is parallel to $\vec{E}$,

$$\therefore \; \vec{J} = \frac{ne^2}{m}\tau\vec{E}$$

$$\because \; \sigma = \frac{1}{\rho} = \frac{ne^2}{m}\tau$$

$$\therefore \; \vec{J} = \frac{\vec{E}}{\rho}$$

42. Ans. As cells are connected in parallel so potential difference across terminals of each cell is same.



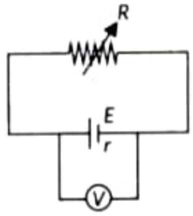
$200 - 38I = 10$

$38I = 200 - 10 = 190$

$I = \frac{190}{38} = 5A$

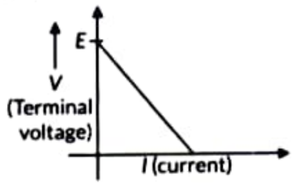
43. Ans. Terminal voltage 'V' of the cell is $V = E - Ir$

E is the emf of the cell, r is the internal resistance of the cell and I is the current through the circuit.



So, $V = -Ir + E$

Comparing with the equation of a straight line y = mx + c, we get,



$y = V; \; x = I;$

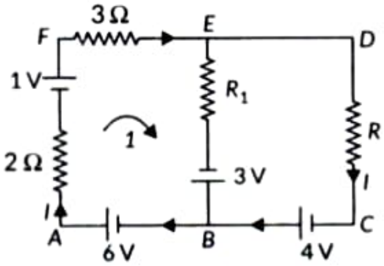
$m = -r; \; c = E$

Graph showing variation of terminal voltage 'V' of the cell versus the current 'I'.

Emf of the cell = Intercept on V axis

Internal resistance = slope of line.

44. Ans. First we need to calculate R for no current through R_1.



By Kirchhoff's law,

$3I + RI + 2I = 1 + 4 + 6$

$5I + RI = 11$ ....(i)

Also, in loop (1),

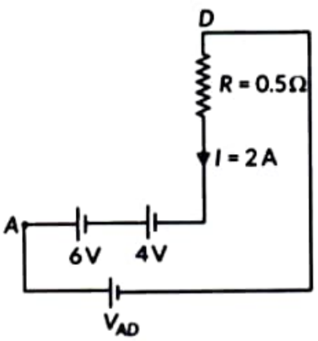
$3I + 2I = 3 + 6 + 1$

$5I = 10$ or $I = 2amp$ ....(ii)

Using in eqn.(i),

$10 + R \times 2 = 11$

$2R = 1$ or $R = 0.5\Omega$ ....(iii)



Now the determine the potential difference between A and D, we can assume a cell of required potential V_{AD} between two points.

On applying Kirchhoff's law,

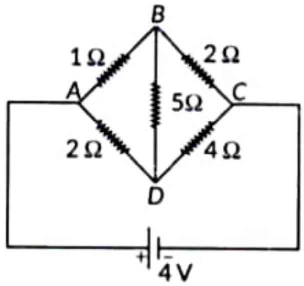
$V_{AD} - 6 - 4 = -2 \times 0.5$

$V_{Ad} - 10 = -1$

$V_{AD} = 9 volt$

45. Ans. Since the condition $\frac{P}{Q} = \frac{R}{S}$ is satisfied, it is a balanced bridge.

The equivalent Wheatstone bridge for the given combination is shown in figure.



The resistance of arm ABC, $R_{S_1} = 2 + 1 = 3\Omega$

Also, the resistance of arm ADC,

$R_{S_2} = 4 + 2 = 6\Omega$

Equivalent resistance

$$R_{eq} = \frac{R_{S_1} \times R_{S_2}}{R_{S_1} + R_{S_2}} = \frac{3 \times 6}{3 + 6} = \frac{18}{9} = 2\Omega$$

Current drawn from the battery

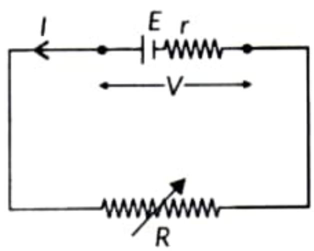
$$I = \frac{V}{R_{eq}} = \frac{4}{2} \quad \therefore \quad I = 2A.$$

46. Ans. Current is constant is non – uniform cross-section.

47. Ans. (i) Resistance of Hg below 4 K is zero so it behaves like super conductor. Between T = 4 K to 5 K resistance rises linearly and beyond T = 5K resistance becomes constant.

(ii) In region BC, the material shows negative resistance property because current decreases with increase in voltage or slope of BC is negative.

48. Ans. Given situation is shown in figure

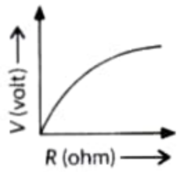


$$I = \frac{E}{r + R}$$

(i) V versus R, Terminal voltage,

$V = E - Ir$

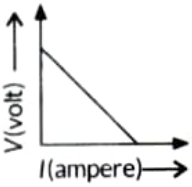
$$V = E - Ir = E - \frac{E}{r + R} r = \frac{ER}{r + R}$$



(ii) V versus I,

V = E – Ir

When $R = 4\Omega$,



Then $I_1 = 1A$

$$\therefore \quad 1 = \frac{E}{r+4}$$

$$r + 4 = E \qquad \text{.........(i)}$$

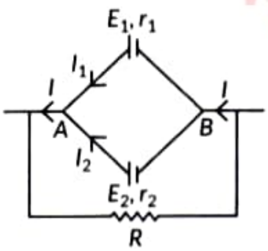
When $R = 9\Omega$, *then* $I = 0.5\,A = \frac{1}{2}A$

$$\therefore \quad \frac{1}{2} = \frac{E}{r+9} = \frac{r+4}{r+9} \qquad \text{[Using eqn. (i)]}$$

$$r + 9 = 2r + 8, \; r = 1\Omega$$

From eqn. (i), emf, E = 1 + 4 = 5V

49. Ans. (a) Here, $I = I_1 + I_2$ ....(i)



Let V = Potential difference between A and B

For cell E_1,

$$V = E_1 - I_1 r_1 \Rightarrow I_1 = \frac{E_1 - V}{r_1}$$

Similarly, for cell E_2, $I_2 = \frac{E_2 - V}{r_2}$

Putting these values in equation (i)

$$I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$

$$or \quad I = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$

$$or \quad V = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right) \qquad \text{..........(ii)}$$

Comparing the above equation with the equivalent circuit of emf 'E_{eq}' and internal resistance 'r_{eq}' then,

$$V = E_{eq} - I r_{eq} \qquad \text{..............(iii)}$$

Then, $E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$ *and* $r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$

(b) Given $E_1 = E_2 = E = 5V$

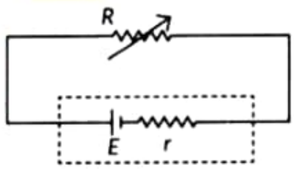
And $r_1 = r_2 = r = 2\Omega$, *and* $R = 10\Omega$

Then current, $I = \frac{E_{eq}}{R + r_{eq}} = \frac{5}{10 + 4/4} = \frac{5}{11}A$

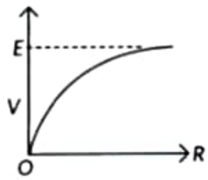
∴ Voltage across the external resistance,

$$V = IR = \frac{5}{11} \times 10 = \frac{50}{11} = 4.55V$$

50. Ans. (a)



(b) .



(c) Maximum current drawn will be at R = 0

51. Soln. In first circuit

Reading of ideal voltmeter = 6 V

Net potential difference = 9 + 6 = 15 V

Total resistance = $1 + 1 = 2\Omega$

Current in ammeter = $\frac{V}{R} = \frac{15}{2} = 7.5A$

In second Circuit

Reading of ideal volt meter = 6V

Net potential difference = 9 – 6 = 3 V

Total resistance = $1+1=2\Omega$

Current in ammeter = $\frac{V}{R}=\frac{3}{2}=1.5A$

52. Soln. Current is constant in non-uniform cross-section.

53. Soln. Drift velocity $v_d=\frac{e\vec{E}}{m}\tau$, where E is electric field strength. And the relation between current and drift velocity is $I=neAv_d$.

$$\therefore \quad \frac{I}{A}=\frac{ne^2\tau}{m}E \Rightarrow j=\sigma AE$$

$$\sigma=\frac{ne^2\tau}{m}=\frac{1}{\rho} \; or, \; \rho=\frac{m}{ne^2\tau}$$

With rise of temperature, the rate of collision of electrons with ions of lattice increases, so relaxation time decreases. As a result resistivity of the material increases with the rise of temperature, hence the resistance.

54. Soln.

(a) Drift velocity is defined as the average velocity with which the free electrons are drifted towards the positive terminal under the effect of applied electric field. Thermal velocities are randomly distributed and average thermal velocity is zero.

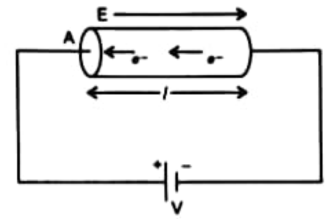
$$\frac{\vec{u}_1+\vec{u}_2+.....+\vec{u}_N}{N}=0$$

$$i.e., \quad v_d=-\frac{eE\tau}{m}$$

(b) We know that the current flowing through the conductor is:

$I=nAev_d$

$$\therefore \quad I=neA\left(-\frac{eE\tau}{m}\right)$$



$$\text{Using} \quad E=-\frac{V}{l}$$

$$I=neA\left(\frac{eV}{ml}\right)\tau$$

$$=\left(\frac{ne^2A\tau}{ml}\right)V=\frac{1}{R}V$$

$I\propto V \rightarrow$ by Ohm's law

Where, $R=\frac{ml}{nAe^2\tau}$ is a constant for a particular conductor at a particular temperature and is called the resistance of the conductor.

$$R=\left(\frac{m}{ne^2\tau}\right)\frac{l}{A}=\frac{\rho l}{A}$$

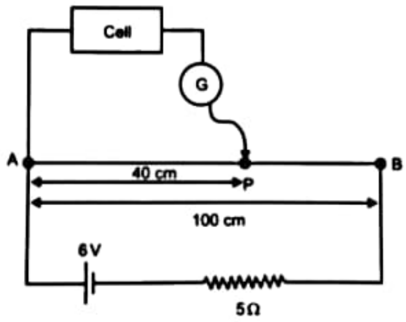
$$\rho=\left(\frac{m}{ne^2\tau}\right)$$

Where ρ is the specific resistance or resistivity of the material of the wire. It depends on number of free electron per unit volume and temperature.

(c) They are used to make standard resistors because:

1. They have high value of resistivity.
2. Temperature coefficient of resistance is less.
3. They are least affected by temperature.

55. Soln. From the figure below:



Total resistance of the circuit,

$$R=(R_{AB}+5)=(10+5)\Omega=15\Omega$$

Current in the circuit, $I = \frac{V}{R} = \frac{6}{15} A$

$$V_{AB} = 6 - \frac{6}{15} \times 5$$
$$= 6 - 2 = 4V$$

$\therefore$ Voltage across AB,

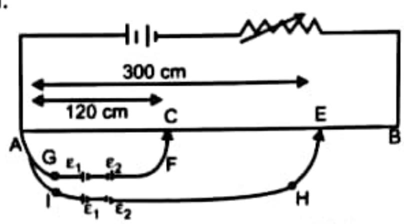
Emf of the cell, $e = \left(\frac{l}{L}\right) V_{AB}$

Here: $l = 40m$ (balance point)

AB = L = 1m = 100 cm (total length of the wire)

$\therefore$ $e = \left(\frac{40}{100}\right) 4 = 1.6V$

56. Soln.



(i) Apply Kirchhoff's law in loop ACFGA:

$k(120) = \varepsilon_1 - \varepsilon_2$

k = potential drop per unit length

Or $\varepsilon_1 = \varepsilon_2 + k(120)$ ....(i)

For loop AEHIA:

$k(300) = \varepsilon_2 + \varepsilon_1$

By substituting value of ε_1 from equation (i),

$$\varepsilon_2 + [\varepsilon_2 + k(120)] = k(300)$$
$$2\varepsilon_2 = k(300 - 120)$$

Or, $\varepsilon_2 = 90k$ ....(ii)

Thus, $\varepsilon_1 = 90k + 120k$

$\varepsilon_1 = 210k$ ....(iii)

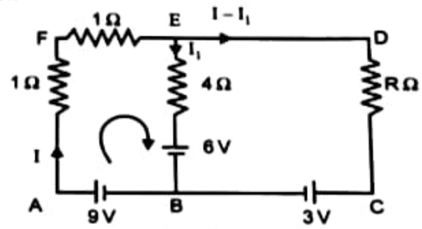
Hence, $\frac{\varepsilon_1}{\varepsilon_2} = \frac{210}{90} = \frac{7}{3}$

(ii) As we know, $\varepsilon = kl$
Thus, from equations (ii) and (iii),
Null point for cell ε_2 is 90 cm
And for cell ε_1, it is 210 cm.

Sensitivity of the potentiometer can be increased by:

(a) Increasing the length of the potentiometer wire.
(b) Decreasing the resistance in the primary circuit.

57. Soln.



Apply Kirchhoff's law in loop AFEBA:

$$I + I + 4I_1 = 9 - 6$$
$$2I + 4I_1 = 3 \qquad ....(i)$$

As there is no current flowing through the 4Ω resistance,

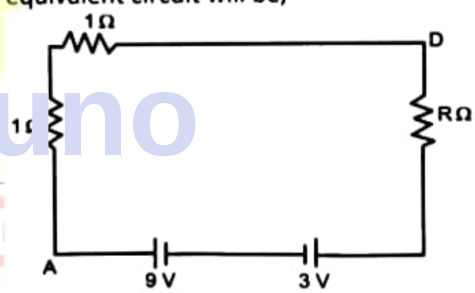
$\therefore$ $I_1 = 0$

Or $2I = 3$

Or $I = 1.5A$

Thus, the current through resistance R is 1.5 A.

As there is no current through branch EB, thus equivalent circuit will be,



By applying Kirchhoff's loop law, we get

$$1.5 + 1.5 + R(1.5) = 9 - 3$$
$$R = 2\Omega$$

Potential difference between A and D = $2 \times 1.5 = 3V$

58. Soln. Drift velocity may be defined as the average velocity gained by the free electrons of a conductor in the opposite direction of the externally applied electric field.

The average time that elapses between two successive collisions of an electron is called relaxation time.

59. Soln. Deduction of Ohm's law. When a potential difference V is applied across a conductor of length l, the drift velocity in terms of V is given by

$$v_d = \frac{eE\tau}{m} = \frac{eV\tau}{ml}$$

If the area of cross-section of the conductor is A and the number of electrons per unit volume or the electron density of the conductor is n, then the current through the conductor will be

$$I = enAv_d = enA.\frac{eV\tau}{ml}$$

Or $$\frac{V}{I} = \frac{ml}{ne^2\tau A}.$$

At a fixed temperature, the quantities m, l, n, e, τ and A, all have constant values for a given conductor.

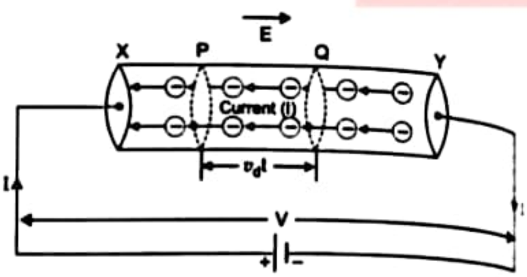
Therefore,

$$\frac{V}{I} = \text{a constant, R}$$

This proves Ohm's law for a conductor and here $R = \frac{ml}{ne^2\tau A}$

Is the resistance of the conductor.

60. Soln. Consider a uniform metallic wire XY of length l and cross-sectional area A. A potential difference V is applied across the ends X and Y of the wire. This causes an electric field at each point of the wire of strength



$$E = \frac{V}{l}. \quad \text{.........(i)}$$

Due to this electric field, the electrons gain a drift velocity v_d opposite to direction of electric field. If q be the charge passing through the cross-section of wire in t seconds, then

Current in wire $I = \frac{q}{t}$

..................(ii)

The distance traversed by each electron in time t = average velocity x time = $v_d t$

If we consider two planes P and Q at a distance v_d t in a conductor, then the total charge flowing in time t will be equal to the total charge on the electrons present within the cylinder PQ.

The volume of this cylinder = cross sectional area x height

$$= A\, v_d\, t$$

If n is the number of free electrons in the wire per unit volume, then the number of free electrons in the cylinder $= n(\mathrm{A\, v_d t})$

If charge on each electron is $-e(\mathrm{e} = 1.6\times10^{-19}\ \mathrm{C})$, then the total charge flowing through a cross-section of the wire

$$q = (\mathrm{nA\, v_d t})(-\mathrm{e}) = -\mathrm{neAv_d}\, t$$

..........(iii)

∴ Current flowing in the wire,

$$= \frac{q}{t} = \frac{-neAv_d t}{t}$$

i.e., current I = $-neAv_d$

..........(iv)

This is the relation between electric current and drift velocity. Negative sign shows that the direction of current is opposite to the drift velocity.

Numerically $I = neAv_d$

........(v)

Current density, ∴ $J = \frac{I}{A} = nev_d$

$\Rightarrow \quad J \propto v_d.$

That is, current density of a metallic conductor is directly proportional to the drift velocity.

61. Soln. In a conductor, current density at particular point is the current flowing per unit area in conductor provided the area is in direction normal to current.

$$J = \frac{I}{A}$$

Current density is a vector quantity. Its direction is the direction of motion of positive charge. The unit of current density is ampere/metre^2, or [Am^{-2}].

Relation between J and E:

$$I = nAev_d = nAe\left(\frac{eE}{m}\tau\right)$$

$$I = \frac{nAe^2\tau E}{m}$$

Or $$\frac{I}{A} = \frac{ne^2\tau E}{m}$$

Or $$J = \frac{1}{\rho}E$$

$$\left[\because J = \frac{I}{A} \text{ and } \rho = \frac{m}{ne^2\tau}\right]$$

$$J = \sigma E \qquad \left[\because \sigma = \frac{1}{\rho}\right]$$

62. Soln. The conductivity of any material is due to its mobile charge carriers. These may be electrons in metals, positive and negative ions in electrolytes; and electrons and holes in semiconductors.

The mobility of a charge carrier is the drift velocity acquired by it in a unit electric field. It is given by

$$\mu = \frac{v_d}{E}$$

63. Soln. $E_1 = 1.5V, \quad r_1 = 0.2\Omega$

$E_2 = 2.0V, \quad r_2 = 0.3\Omega$

Emf of equivalent cell

$$E = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_1}$$

$$= \left(\frac{1.5\times 0.3 + 2\times 0.2}{0.2+0.3}\right) = \frac{0.45+0.40}{0.5}V = 1.7V$$

Internal resistance of equivalent cell

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} = \frac{r_1 r_2}{r_1 + r_2} = \left(\frac{0.2\times 0.3}{0.2+0.3}\right)\Omega = \frac{0.06}{0.5}\Omega = 0.12\Omega$$

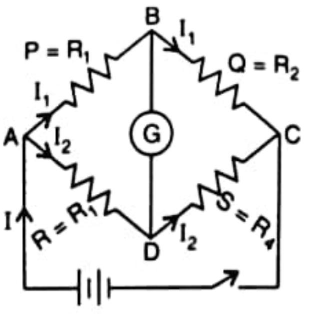
64. Soln. Kirchhoff's first law is known as junction rule which states that for a given junction or node in a circuit, sum of the currents entering will be equal to sum of current leaving.

Kirchhoff's second law is also known as loop rule which shows that around any closed loop in a circuit, sum of the potential differences across all elements will be zero

Justification The junction rule is in accordance with the conservation of charge that serves as basis of current rule while loop rule is based on law of conservation of energy.

65. Soln. Applying Kirchhoff's loop rule to closed loop ADBA

$$-I_1 R_1 + 0 + I_2 R_2 = 0 (I_g = 0) \qquad ....(i)$$



For loop CBDC,

$$-I_2 R_4 + 0 + I_1 R_3 = 0$$

From equation (i)

$$\frac{I_1}{I_2} = \frac{R_1}{R_2}$$

From equation (ii)

$$\frac{I_1}{I_2} = \frac{R_4}{R_3}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{R_4}{R_3}$$

66. Soln. Applying Kirchhoff's laws,

For closed loop ADCBA

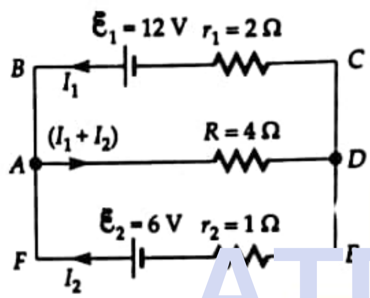
$$12 = 4(I_1 + I_2) + 2I_1 = 6I_1 + 4I_2$$

..........(i)

For closed loop ADEFA,

$$6 = 4(I_1 + I_2) + I_1 = 4I_1 + 5I_2$$

...........(ii)



Solving (i) and (ii), we get

$$I_1 = \frac{18}{7}A \;\; and \;\; I_2 = -\frac{6}{7}A$$

P.D. across R = V

$$= (I_1 + I_2)R$$

$$= \left(\frac{18-6}{7}\right) \times 4\,volt = \frac{48}{7}V$$

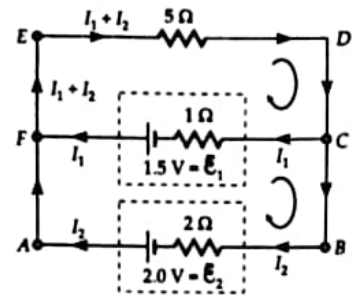
P.D. across each cell = P.D. across $R = \frac{48}{7}$V

Energy dissipated in $R = 4\Omega$ resistor

$$= (I_1 + I_2)^2 R = \left(\frac{12}{7}\right)^2 \times 4J$$

$$= \frac{576}{49}J = 11.75\,J$$

67. Soln. (i)



(ii) (a) Let I_1 and I_2 be the currents as shown in fig. Using Kirchhoff's second law for the loop AFCBA, we get

$$2I_2 - 1I_1 = \varepsilon_2 - \varepsilon_1 = 2 - 1.5$$

Or $2I_2 - I_1 = 0.5$ ....(i)

For loop CFEDC, we have

$$1I_1 + 5(I_1 + I_2) = \varepsilon_1 = 1.5$$

Or $5I_2 + 6I_1 = 1.5$ ....(ii)

Solving equations (i) and (ii), we get

$$I_1 = \frac{1}{34}A, \qquad I_2 = \frac{9}{34}A$$

∴ Current through branch BA,

$$I_1 = \frac{1}{34}A$$

Current through branch CF,

$$I_2 = \frac{9}{34}A$$

Current through branch DE,

$$I_1 + I_2 = \frac{10}{34}A.$$

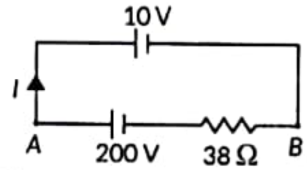
(b) P.D. across the 5Ω resistance

$$= (I_1 + I_2) \times 5 = \frac{10}{34} \times 5V = 1.47\,V.$$

68. Soln. (i) Resistance of Hg below 4 K is zero so it behaves like a superconductor. Between T = 4 K to 5 K resistance rises linearly and beyond T = 5 K resistance becomes constant.

(ii) In region BC, the material shows negative resistance property because current decreases with increase in voltage or slope of BC is negative.

69. Soln. As cells are connected in parallel so potential difference across terminals of each cell is same.



$200 - 32I = 10$

$38I = 200 - 10 = 190$

$I = \frac{190}{38} = 5A$

70. Soln. (a) $\because H = \frac{V^2}{R}t \Rightarrow \frac{H}{t} = \frac{V^2}{R}$

$\therefore \quad \frac{H}{t} \propto V^2$

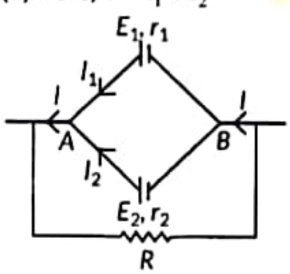
Given heat produce per second $\frac{H}{t}$, increases by a factor of 9.

Hence, applied potential difference V increased by factor of 3.

(b) $I = \frac{E}{R+r} = \frac{12}{4+2} = \frac{12}{6} = 2A$

$V = E - Ir = 12 - 2\times 2 = 8V$

71. Soln. (a) Here, $I = I_1 + I_2$ ....(i)



Let V = Potential difference between A and B

For cell E_1,

$$V = E_1 - I_1 r_1 \Rightarrow I_1 = \frac{E_1 - V}{r_1}$$

Similarly, for cell E_2, $I_2 = \frac{E_2 - V}{r_2}$

Putting these values in equation (i)

$$I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$

Or $\quad I = \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$

Or $\quad V = \left(\frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}\right) - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$

........(ii)

Comparing the above equation with the equivalent circuit of emf 'E_{eq}' and internal resistance 'r_{eq}' then,

$V = E_{eq} - Ir_{eq}$ ....(iii)

Then

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \; and \; r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

(b) Given $E_1 = E_2 = E = 5V$

And $r_1 = r_2 = r = 2\Omega$, and $R = 10\Omega$

Then current, $I = \frac{E_{eq}}{R + r_{eq}} = \frac{5}{10 + 4/4} = \frac{5}{11}A$

$\therefore$ Voltage across the external resistance,

$$V = IR = \frac{5}{11}\times 10 = \frac{50}{11} = 4.55V$$

SURE SHOT QUESTIONS

ARVIND ACADEMY

Chapter – 04

Moving Charges and Magnetism

MCQ (1 mark)

1. Soln. (a): Magnetic field produced by charges moving with velocity $\vec{v}$, at a distance r is

$\vec{B} = \left(\frac{\mu_0}{4\pi}\right) . q \frac{\vec{v} \times \hat{r}}{r^2}$. Therefore $\vec{B} \perp \vec{v}$.

2. Soln. (a): For a circular loop of radius R carrying current I placed in x-y plane, the magnetic moment $M = I \times \pi R^2$. It acts perpendicular to the loop i.e., along z-direction. When half of the current loop is bent in y-z plane, then magnetic moment due to half current loop in x-y plane, $M_1 = I(\pi R^2 / 2)$ acting along z-direction.

Magnetic moment due to half current loop in y-z plane, $M_2 = I(\pi R^2 / 2)$ along x-direction.

Effective magnetic moment due to entire bent current loop,

$$M' = \sqrt{M_1^2 + M_2^2} = \sqrt{(I\pi R^2/2)^2 + (I\pi R^2/2)^2} = \frac{I\pi R^2}{2}\sqrt{2} < M$$

i.e., magnetic moment diminishes.

3. Soln. (d): Let the electron (e) is projected with a uniform velocity (v) in a uniform magnetic field B. The magnitude of force on it is

$$|\vec{F}| = -e|\vec{v} \times \vec{B}| = -evB\sin\theta$$

As $\theta = 0^0, |\vec{F}| = -evB\sin 0^0 = 0$

Hence the electron will continue to move with a uniform velocity along the axis of the solenoid.

4. Soln. (d): Rotation of loop by 30^0 about an axis perpendicular to its plane does not change the angle between magnetic moment and magnetic field. Hence, no work is done.

5. Ans. (c) : Length of conductor = l, mass = m

Magnetic field = B.
Let the current is I. Tension force = mg

$$IBl = mg \Rightarrow I = \frac{mg}{lB}$$

6. Ans. (a) : Force on a current carrying conductor placed in magnetic field is,

$$\vec{F} = I(\vec{L} \times \vec{B}); \ F = IBL\sin\theta$$

The force is maximum when $\theta = 90^0$.
So, the current carrying conductor placed perpendicular to the magnetic field will experience a maximum force.

7. Ans. ([illegible]) : Particle mass = m

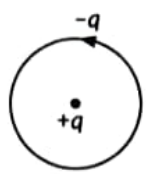
Particle charge = q
Force between the positive and negative charged particles,

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q(-q)}{r^2}$$

Or $F = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$ ('-' means force is attractive)

Centripetal force, $F_C = \frac{mv^2}{r}$

Now, centripetal force = Magnitude of electric force



$$\Rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

$$\Rightarrow r = \frac{q^2}{4\pi\varepsilon_0}\cdot\frac{1}{mv^2}$$

$$or \quad r = \frac{1}{4\pi\varepsilon_0 m}\left(\frac{q}{v}\right)^2$$

Hence, option (b) is correct.

8. Ans.(b) : As, $qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$ ......(i)

Now, $qV = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2qV}{m}}$ ....(ii)

Substituting (ii) in (i),

$$r = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \sqrt{\frac{2mV}{qB^2}} \quad \text{.............(iii)}$$

Now,

$$V' = 2V, \ \therefore \ r' = \sqrt{\frac{2m(2V)}{qB^2}} = \sqrt{2}\sqrt{\frac{2mV}{qB^2}} = \sqrt{2}r$$

9. Ans.(c) : $r = \frac{mv}{Bq} = \frac{m}{Bq}\sqrt{\frac{2K}{m}} = \frac{\sqrt{2mK}}{Bq}$

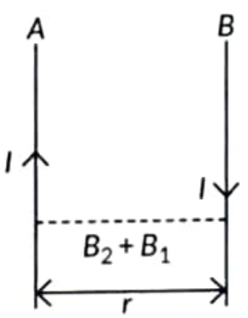
$m \propto r^2$

As B, q and K is constant

So, $\frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^2$

10. Ans.(a)

11. Ans.(*): The direction of both the magnetic fields is into the plane of paper.



$$B_1 = \frac{\mu_0}{2\pi}\cdot\frac{2I}{r} \quad \text{............(i)}$$

$$B_2 = \frac{\mu_0}{2\pi}\cdot\frac{2I}{r} \quad \text{............(ii)}$$

$$B_{net} = B_1 + B_2 = \frac{\mu_0}{2\pi}\times\frac{4I}{r} = \frac{2\mu_0 I}{\pi r}$$

Here no option is correct.

12. Ans.(a) : Magnetic field due to loop at its centre,

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{R} \quad \text{............(i)}$$

Magnetic field due to current carrying loop on its axis at a distance x is, $B = \frac{\mu_0}{2}\cdot\frac{I x^2}{(R^2+x^2)^{3/2}}$

Put x = R, we get

$$B_2 = \frac{\mu_0}{2}\cdot\frac{IR^2}{(2R^2)^{3/2}} = \frac{\mu_0}{2}\cdot\frac{IR^2}{2^{3/2}R^3} = \frac{\mu_0 I}{4\sqrt{2}R} \quad \text{....(ii)}$$

Divide eqn (i) by (ii), we get

$$\text{So, } \frac{B_1}{B_2} = \frac{\mu_0}{4\pi}\frac{2\pi I\times 4\sqrt{2}R}{RI\mu_0} = 2\sqrt{2}$$

13. Ans.(d) : Magnetic field due to straight wire

$$B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I}{R}$$

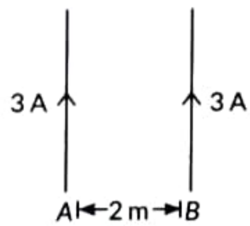
Magnetic field due to circular wire

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I}{R} \otimes$$

$$B_{net} = B_2 - B_1 = \frac{\mu_0}{4\pi}\cdot\frac{2I}{R}[\pi - 1]$$

$$B_{net} = \frac{\mu_0 I}{2R}\left(1 - \frac{1}{\pi}\right)$$

14. Ans.(c*) : Two long parallel wires kept 2 m apart carry a current of 3 A each.



Force

$$F = \frac{\mu_0}{2\pi}\frac{I_1 I_2 L}{d}$$

$$I_1 = I_2 = 3A, \ d = 2m$$

$$F = \frac{4\pi\times 10^{-7}}{2\pi}\times\frac{3\times 3}{2}\times L$$

$\because$ Force per unit length,

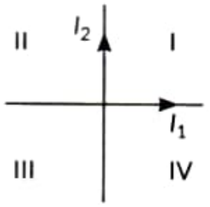
$$\frac{F}{L}=\frac{4\pi\times10^{-7}}{2\pi}\times\frac{9}{2}=9\times10^{-7}N/m$$

Force will be attractive.

* The most appropriate answer is (c).

15. Ans.(b)

16. Ans.(d) : $I_1=4A,\ I_2=10A,$



$r=2.5\,cm$

The force per unit length between them is

$$F=\frac{\mu_0}{4\pi}\cdot\frac{2I_1I_2}{r}=\frac{10^{-7}\times2\times4\times10}{2.5\times10^{-2}}$$

$=3.2\times10^{-4}\,N\,m^{-1}$

17. Ans.(d) : Magnetic dipole moment dos not depend upon the material of the turns of the coil.

18. Ans.(d) : As the ammeter is connected in place of voltmeter, so, its resistance should be high. So, we have to connect a high resistance in series.

19. Ans.(c) : Area of square = a^2, Also, here

$$l=4a\Rightarrow a=\frac{l}{4}$$

$$\therefore\quad Area=\frac{l^2}{16}$$

$$\Rightarrow\quad A_1=\frac{l^2}{16}\qquad\qquad \text{........(i)}$$

$\therefore$ Area of a circle = πr^2

Also here, $2\pi r=l\ \Rightarrow\ r=\frac{l}{2\pi}$

Now, area = $\pi\left(\frac{l}{2\pi}\right)^2$

$$\Rightarrow\quad A_2=\frac{l^2}{4\pi}\qquad\qquad \text{...........(ii)}$$

Now magnetic moment = IA

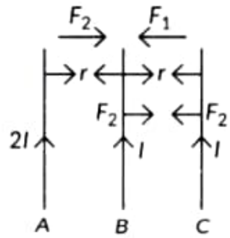
$\therefore\ M_1=IA_1$ and $M_2=IA_2$

Since I (current) is same in both.

Using eqn. (i) and (ii)

$$\therefore\quad \frac{M_1}{M_2}=\frac{A_1}{A_2}=\frac{l^2}{16}\times\frac{4\pi}{l^2}=\frac{\pi}{4}$$

20. Ans.(a) : Let F_1 is force per unit length between A and C

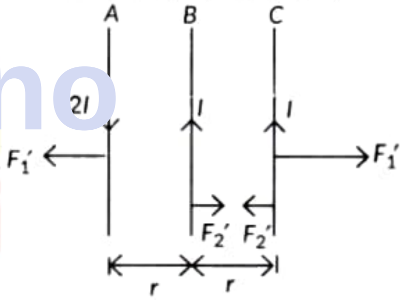


$$\therefore\ F_1=\frac{\mu_0}{2\pi}\frac{2I\times I}{2r}$$

And F_2 is force per unit, length between B and C

$$\therefore\quad F_2=\frac{\mu_0}{2\pi}\frac{I\times I}{r}$$

Now, net force on C is per unit length



$$F_1+F_2=\frac{\mu_0}{2\pi}\frac{I^2}{r}(1+1)$$

$$=\frac{2\mu_0}{2\pi}\frac{I^2}{r}=F(\text{given})$$

Now, from figure

F'_1 = repulsive force between A and C

$$=\frac{\mu_0}{2\pi}\frac{2I^2}{2r}$$

$F'_2=F_2=A$ reactive force between B and C

$\therefore$ Net force on C, $F'_1-F'_2=0$

$$\therefore\ F'_1=F'_2=\frac{\mu}{2\pi}\frac{2I^2}{2r}$$

$\therefore$ Net force on C is zero.

21. Ans.(d) : The coil of a moving coil galvanometer is wound over metallic frame to provide electromagnetic damping so it becomes dead beat galvanometer.

22. Ans.(d) : Given, $I'_g = I_g + \frac{20}{100} I_g = \frac{120}{100} I_g = 1.2 I_g$

$$R' = R + \frac{25}{100} R = \frac{125}{100} R = 1.25R$$

$$V'_g = ?$$

$$V'_g = \frac{I'_g}{R'} = \frac{1.2 I_g}{1.25R} = \frac{120}{125} V_g = \frac{24}{25} V_g$$

$$\% change = \frac{V'_g - V_g}{V_g} \times 100 = \frac{\left(\frac{24}{25} V_g - V_g\right)}{V_g} \times 100$$

$$= \frac{(24-25)}{25} \times 100 = \frac{-1}{25} \times 100 = -4\%$$

$\therefore$ Voltage sensitivity decreases by 4%.

Assertion-Reasoning (1 mark)

23. Ans.(d) : A circular loop has more area than a square loop with same perimeter.

The torque is directly proportional to the area.

Perimeter of wire = L

One side of square loop, $l = \frac{L}{4}$

Area of square, $l^2 = \frac{L^2}{16}$

Perimeter of circular loop = L

Radius of circular loop, $r = \frac{L}{2\pi}$

Area of circular loop,

$$A' = \pi r^2 = \frac{\pi L^2}{4\pi^2} \text{ or } A' = \frac{L^2}{4\pi}$$

Hence, area of circular loop (A') > Area of square loop (A)

$$\frac{L^2}{4\pi} > \frac{L^2}{16}$$

So, option (d) is correct.

24. Ans.(c) : Assertion : True, $\vec{\tau} = \vec{M} \times \vec{B}$, here the angle between $\vec{M}$ and $\vec{B}$ is zero.

Reason : false, it is given by cross product.

Assertion : True, Reason false.

25. Ans.(a) : The magnetic moment of current carrying loop, $\vec{M} = I\vec{A}$

Where, I is current and A is area.

$A = \pi r^2$, when radius is doubled the magnetic moment is 4 times.

26. Ans.(c) : When the resistance is lower, the range of ammeter is higher. So, (A) is true.

To increase the range of ammeter, additional shunt is added in parallel. So, (R) is false.

Given below are two statements labelled as Assertion (A) and Reason (R).

27. Ans.(b) : We know, $\frac{mv^2}{r} = Bqv\sin\theta$

Centripetal force = magnetic Lorentz force

$\sin\theta = \sin 90^0$

$(\because angle\ between\ \vec{v} \text{ and } \vec{B} = 90^0)$

$$\frac{mv^2}{r} = Bqv \quad or \quad \frac{mv}{r} = Bq$$

$$r = \frac{mv}{Bq} = \frac{p}{Bq} = \frac{linear\ momentum}{Bq}$$

$$Since,\ r = \frac{p}{Bp}$$

Given, p, B are same.

Also, q for proton and electron is same except its sign.

$\therefore$ Radius is same. So assertion is correct but reason is not the correct explanation of assertion.

Given below are two statements labelled as Assertion (A) and Reason (R).

28. Ans.(d) : To increase the range of an ammeter, suitable low resistance (or shunt) should be connected in parallel to it. The ammeter with increased range has low resistance.

Given below are two statements labelled as Assertion (A) and Reason (R).

29. Ans.(a) : When we increase current sensitivity by increasing number of turns, then resistance of coil also increases. So increasing current sensitivity does not necessarily imply that voltage sensitivity will increase because $V_g = \frac{I_g}{R}$

$\therefore$ If I_g and R increases, by different amounts, then V_g may increase or decrease.

30. **Sol.** (c): An ammeter is a low resistance galvanometer. It is used to measure the current in amperes. To measure the current of a circuit, the ammeter is connected in series in the circuit so that the current to be measured must pass through it. Since, the resistance of ammeter is low, so its inclusion in series in the circuit does not change the resistance and hence the main current in the circuit.

31. **Sol.** (c): In a conductor, the average velocity of electrons is zero. Hence no current flows through the conductor. Hence, no force acts on this conductor.

32. **Sol.**

(a): The magnetic field at the centre of circular coil is given by .

$$B = \frac{\mu_0}{4\pi}\frac{2\pi nI}{a}$$

So if current through coil is doubled then magnetic field is $B' = 2B$.

The magnetic field also get doubled. The magnetic field is directly proportional to the current in conductor

33. **Sol.** (d): When a charged particle moves perpendicular to magnetic field, it experiences a force which changes the direction of motion of the particle without changing the magnitude of velocity of the particle. Hence kinetic energy remains constant but momentum of electron changes.

Case study Question

34. . Answers

1. (b) $\oint \vec{B}.\vec{dl} = \mu_0 I$

2. (c) $B \propto \frac{1}{r}$

$$\therefore \quad \frac{B'}{B} = \frac{20}{5} \quad \Rightarrow \quad B' = \frac{B}{4}$$

3. (c) $B = 10^{-7} \times \frac{2I}{r} = \frac{10^{-7}\times 2\times 1}{1} = 2\times 10^{-7}\text{T}$

4. (a) $B_{inside} \propto r$ and $B_{outside} \propto \frac{1}{r}$.

5. (c) $B_{inside} = \frac{\mu_0 Ir}{2\pi a^2}$ i.e., $B_{inside} \propto r$

35. Answers

6. (a) $F = IlB$, F acts to the right.

7. (d) $v^2 = 2ad$

$$\Rightarrow \quad d = \frac{v^2}{2a} = \frac{v^2 m}{2F} = \frac{v^2 m}{2IlB}$$

8. (b) $F = IlB\sin 90° = 8\times 5\times 1.5\times 1 = 60$ N.

9. (d) For equilibrium of wire in mid-air,

Weight of wire = Force exerted by magnetic field

$$mg = IlB\sin 90°$$

$$B = \frac{mg}{Il} = \frac{200\times 10^{-3}\times 9.8}{2\times 1.5} = 0.65 \text{ T.}$$

10. (d) $\vec{F} = i(\vec{l}\times\vec{B}) = i[l\hat{i}\times B_0(\hat{i}+\hat{j}+\hat{k})]$

$$= ilB_0[\hat{i}\times\hat{i} + \hat{i}\times\hat{j} + \hat{i}\times\hat{k}] = ilB_0[\hat{k}-\hat{j}]$$

$$|\vec{F}| = ilB_0\sqrt{(1)^2+(-1)^2} = \sqrt{2}\,ilB_0.$$

Questions

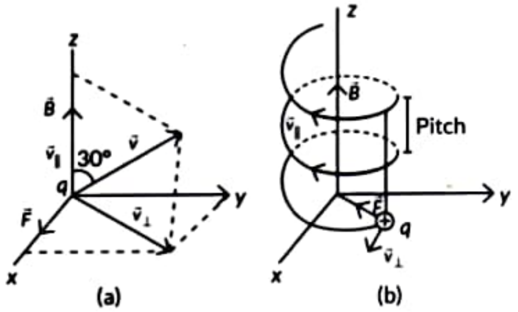
36. Ans.Mass of wire, m = 200 g = 0.2 kg, length of wire, ℓ = 1.5 m, current in the wire, I = 2A

In the equilibrium position, the net force on the wire will be zero.

Thus, $mg = BI\ell$

$$\Rightarrow B = \frac{mg}{I\ell} \Rightarrow B = \frac{0.2\times 9.8}{2\times 1.5} \Rightarrow B = 0.65\,T$$

37. Ans.A charged particle moving in a uniform magnetic field has two motions.



A linear motion in the direction of $\vec{B}$ (along z-axis) as shown in figure (a) and a circular motion in a plane perpendicular to $\vec{B}$ (in xy-plane). Hence, the resultant path of the charged particle will be a helix, with its axis along the direction of $\vec{B}$, as shown in figure (b).

38. Ans.The magnetic field at an axial point due to circular loop is given by

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I a^2}{(a^2+r^2)^{3/2}}$$

Where, I = current through the loop

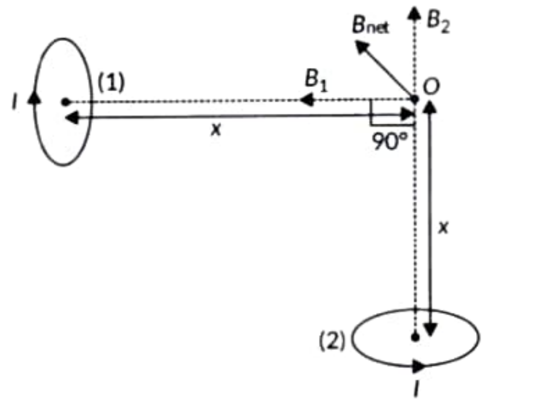
a = radius of the loop

r = distance of O from the centre of the loop.

Since I, a and r = x are the same for both the loops, the magnitude of B will be the same and is given by

$$B_1 = B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi I a^2}{(a^2+x^2)^{3/2}}$$

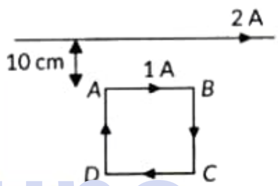
The direction of magnetic field due to loop (1) will be away from O and that due to loop (2) will be towards O as shown. The direction of the net magnetic field will be as shown in the figure.



The magnitude of the net magnetic field is given by $B_{net} = \sqrt{B_1^2 + B_2^2}$

$$B_{net} = \frac{\mu_0}{4\pi}\,\frac{2\sqrt{2}\pi I a^2}{4\pi(a^2+x^2)^{3/2}}$$

39. Ans. Force between two parallel current carrying wires,



$$F = \frac{\mu_0 I_1 I_2 l}{2\pi r}$$

Force on arm AB,

$$F_{AB} = \frac{\mu_0\times 2\times 1\times 20\times 10^{-2}}{2\pi\times 10\times 10^{-2}}$$

$= \frac{2\mu_0}{\pi} N$ (Attractive, towards the wire)

Force on arm CD,

$$F_{CD} = \frac{\mu_0\times 2\times 1\times 20\times 10^{-2}}{2\pi\times 30\times 10^{-2}}$$

$= \frac{2\mu_0}{3\pi} N$ (Repulsive, away from the wire)

Force on arms BC and DA are equal and opposite. So, they cancel out each other.

Net force on the loop is $F = F_{AB} - F_{CD}$

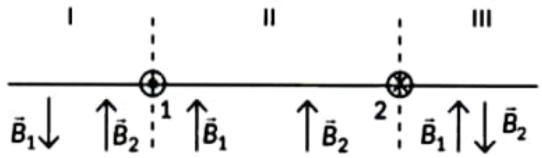
$$= \frac{\mu_0}{\pi}\left[2 - \frac{2}{3}\right] = \frac{4\mu_0}{3\pi} = \frac{4\times 4\pi\times 10^{-7}}{3\pi}$$

$= 5.33\times 10^{-7} N$ (Attractive, towards the wire)

40. Ans.Let the resistance is R. So, V = IR

$$1\times 0.8 = 5\times\frac{0.8R}{R+0.8};\; R = 0.2\Omega$$

41. Ans.For a straight current carrying wire magnetic field is $B = \frac{\mu_0 I}{2\pi r}$



From the figure, we see magnetic field can be zero in region I or III. But for region I, $r_2 > r_1$ and for region III, $r_1 > r_2$. As $I_2 > I_1$, so we can conclude that magnetic field will vanish in region I only.

42. Ans.When a proton, a deuteron and an alpha particle are accelerated through potential difference V, then their energies are

$$E_p = eV,\ E_d = eV,\ E_\alpha = 2eV$$

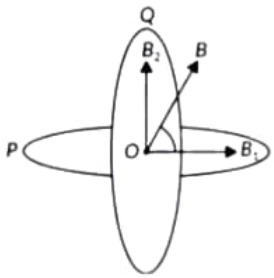
(i) $KE_p : KE_d : KE_\alpha = 1:1:2$

(ii) $r = \frac{mv}{qB} = \frac{\sqrt{2mKE}}{qB}$

$$r_p : r_d : r_\alpha \frac{\sqrt{m_p}}{e} : \frac{\sqrt{2m_p}}{e} : \frac{\sqrt{4m_p}}{2e} = 1 : \sqrt{2} : 1$$

$$As\ r_p = 5cm \ \therefore\ r_d = 5\sqrt{2}\,cm, r_\alpha = 5cm$$

43. Ans. Magnetic field induction due to vertical loop at the centre O is,



$$B_1 = \frac{\mu_0 I_1}{2R} = \frac{4\mu_0}{10^{-1}}$$

$(\because\ R = 5cm)$

Magnetic field induction due to horizontal loop at the centre O is,

$$B_2 = \frac{\mu_0 I_2}{2R} = \frac{4\mu_0}{10^{-1}}$$

∵ B_1 and B_2 are perpendicular to each other, therefore the resultant magnetic field induction at the centre O is,

$$B_{net} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{4\mu_0}{10^{-1}}\right)^2 + \left(\frac{3\mu_0}{10^{-1}}\right)^2} = \frac{\mu_0}{10^{-1}}\sqrt{9+16} = \frac{5\mu_0}{10^{-1}}$$

$$= 50 \times 4\pi \times 10^{-7} = 62.8 \times 10^{-6} T = 62.8\mu T$$

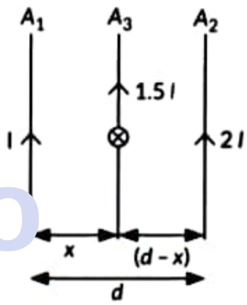
Direction of resultant magnetic field,

$$\tan\theta = \frac{B_2}{B_1} = \frac{3\mu_0 \times 10^{-1}}{4\mu_0 \times 10^{-1}}$$

$$\tan\theta = \frac{3}{4}\ or\ \theta \approx 37^0$$

Resultant magnetic field B making an angle 37^0 with B_1.

44. Ans. Force on A_3 due to A_1



$$f_1 = \frac{4\pi \times 10^{-7} \times I \times 1.5I}{2\pi x}$$

Force on A_3 due to A_2

$$f_2 = \frac{4\pi \times 10^{-7} \times 2I \times 1.5I}{2\pi(d-x)}$$

When there is no net force on A_3 $f_1 = f_2$

$$\frac{4\pi \times 10^{-7} \times I \times 1.5I}{2\pi x} = \frac{4\pi \times 10^{-7} \times 2I \times 1.5I}{2\pi(d-x)}$$

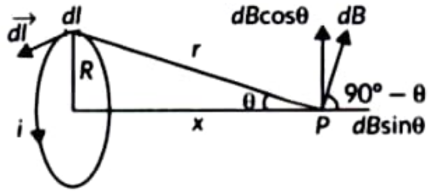
$$d - x = 2x \Rightarrow x = \frac{d}{3}$$

Hence, from A_1 at $\frac{d}{3}$ there is no net force on A_3.

Also from the above result we can say that net force is independent of current flowing on A_3.

45. Ans.(a) The magnetic moment associated with a current (I) carrying circular coil of radius r having N turns, is given by, $M = NIA = NI\pi r^2$.

(b) Magnetic field at a distance x from the centre of the ring due to element



$$dl,\ dB = \frac{\mu_0}{4\pi}\frac{idl\sin 90^0}{r^2}$$

Since, angle between $d\vec{l}$ and $\vec{r}$ is 90^0. The component $dB\cos\theta$ will get cancelled due to symmetry

$$B = \int dB\sin\theta = \int\left(\frac{\mu_0}{4\pi}\frac{idl}{r^2}\right)(\sin\theta)$$

Here, r and θ are constants and $\sin\theta = \frac{R}{r}$

$$B = \int\frac{\mu_0}{4\pi}\frac{idl}{r^2}\left(\frac{R}{r}\right) = \int\frac{\mu_0}{4\pi}\frac{idl}{r^3}$$

$$= \frac{\mu_0}{4\pi}\frac{iR}{r^3}\int dl = \frac{\mu_0 iR}{4\pi r^3}(2\pi\text{R}) = \frac{\mu_0 iR^2}{2r^3}$$

Putting $r = (\text{R}^2+\text{x}^2)^{1/2}$, we get

$$B = \frac{\mu_0 iR^2}{2(\text{R}^2+\text{x}^2)^{3/2}}$$

For N turns, $B = \frac{\mu_0 NiR^2}{2(\text{R}^2+\text{x}^2)^{3/2}}$

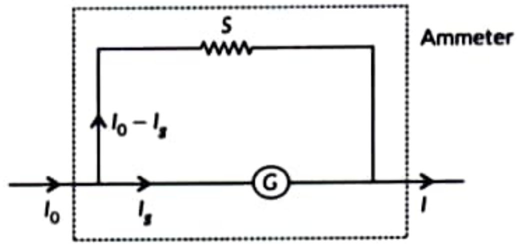
46. Ans.(a) Current sensitivity is defined as the deflection of coil per unit current flowing in it, i.e.,

$$I_S = \frac{\theta}{I} = \frac{NAB}{k}$$

(b) A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer, whose value is given by

$$S = \left(\frac{I_g}{I_O - I_g}\right)G$$

Where I_g is the current for full scale deflection of galvanometer, I_0 is the current to be measured by the galvanometer and G is the resistance of galvanometer.

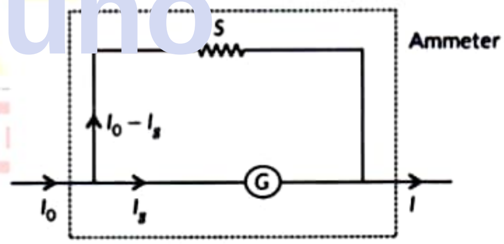


In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is S = G/(n – 1).

47. Ans.(a) A galvanometer can be converted into an ammeter of given range by connecting a suitable low resistance S called shunt in parallel to the given galvanometer, whose value is given by

$$S = \left(\frac{I_g}{I_o - I_g}\right)G$$

Where I_g is the current for full scale deflection of galvanometer, I_0 is the current to be measured by the galvanometer and G is the resistance of galvanometer.



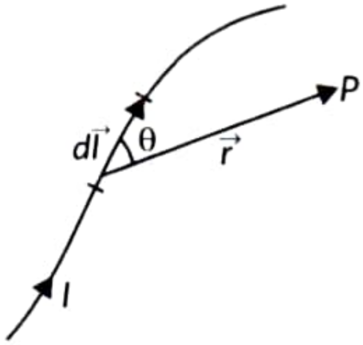
In order to increase the range of an ammeter n times, the value of shunt resistance to be connected in parallel is S = G/(n – 1).

(b) For $I = 6A$

$$S = \left(\frac{I_g}{I - I_g}\right)G = \frac{4\times10^{-3}}{6-0.004}\times15\Omega$$

$$\approx \frac{2}{3}\times15\times10^{-3}\Omega \,\square\, 0.01\Omega$$

48. Ans. A current carrying wire produces a magnetic field around it. Biot – Savart law states that magnitude of intensity of small magnetic field $d\vec{B}$ due to current I carrying element $d\vec{l}$ at any point P at distance r from it is given by



$$|d\vec{B}| = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\sin\theta}{r^2}$$

Where θ is the angle between $\vec{r}$ and $d\vec{l}$ and $\mu_0 = 4\pi \times 10^{-7}\, T\, m\, A^{-1}$ is called permittivity of free space.

In vector form,

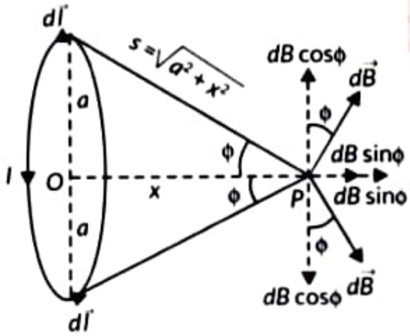
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3}$$

So, the direction of $d\vec{B}$ is perpendicular to the plane containing $\vec{r}$ and $d\vec{l}$.

S.I. unit of magnetic field strength is tesla denoted by 'T' and cgs unit is gauss denoted by 'G', where

1 T = 10^4 G

Magnetic field on the axis of a circular coil



Small magnetic field due to a current element of circular coil of radius r at point P at distance x from its centre is

$$dB = \frac{\mu_0}{4\pi}\frac{Id\vec{l}\sin 90^0}{s^2} = \frac{\mu_0}{4\pi}\frac{Id\vec{l}}{(a^2+x^2)}$$

Component $dB\cos\phi$ due to current element at point P is cancelled by equal and opposite component $dB\cos\phi$ of another diametrically opposite current element, whereas the since components $dB\sin\phi$ add up to give net magnetic field along the axis. So, net magnetic field at point P due to entire loop is

$$B = \oint dB\sin\phi = \int_0^{2\pi r}\frac{\mu_0}{4\pi}\frac{Id\bar{l}}{(a^2+x^2)}\cdot\frac{r}{(a^2+x^2)^{1/2}}$$

$$B = \frac{\mu_0 Ir}{4\pi(a^2+x^2)^{3/2}}\int_0^{2\pi r} dl \quad \text{or} \quad B = \frac{\mu_0 Ir}{4\pi(a^2+x^2)^{3/2}}2\pi r$$

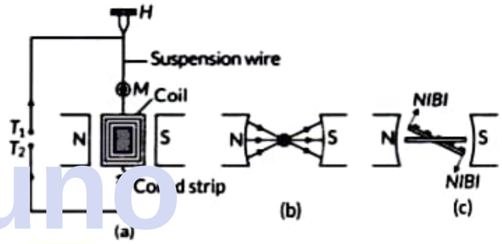
Or $B = \dfrac{\mu_0 I r^2}{2(a^2+x^2)^{3/2}}$ directed along the axis,

(a) Towards the coil if current in it is in clockwise direction

(b) Away from the coil if current in it is in anticlockwise direction.

49. Ans. A galvanometer is used to detect current in a circuit. Principle and working : When current (I) is passed in the coil, torque τ acts on the coil, given by

$$\tau = NIAB\sin\theta$$



Where θ is the angle between the normal to plane of coil and the magnetic field of strength B, N is the number of turns in a coil.

When the magnetic field is radial, as in the case of cylindrical pole pieces and soft iron core, then in every position of coil, the plane of the coil, is parallel to the magnetic field lines, so that $\theta = 90^0$ and $\sin 90^0 = 1$

Deflecting torque, $\tau = NIAB$

If C is the torsional rigidity of the wire and θ is the twist of suspension strip, then restoring torque $= C\theta$

For equilibrium, deflecting torque = restoring torque

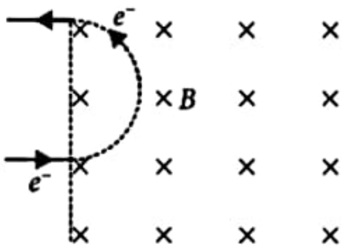
i.e., $NIAB = C\theta$

$$\therefore \quad \theta = \frac{NAB}{C}I \text{ i.e., } \theta \propto I$$

Deflection of coil is directly proportional to current flowing in the coil and hence we can construct a linear scale.

The uniform radial magnetic field keeps the plane of the coil always parallel to the direction of the magnetic field i.e., the angle between the plane of the coil and the magnetic field is zero for all the orientations of the coil.

50. Soln.



Trajectory of electron

Let the time taken by the electron to come out of the region of magnetic field be t.

Velocity of the electron, $v = 4\times10^4 m/s$

Magnetic field, $B = 10^{-5} T$

Mass of the electron, $m = 9\times10^{-31} kg$

We know

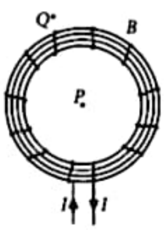
$t = \frac{\pi r}{v}$ where $r = \frac{mv}{q\mathrm{B}}$

Now, $t = \frac{\pi m}{Bq} = \frac{3.14\times9\times10^{-31}}{10^{-5}\times1.6\times10^{-19}}$

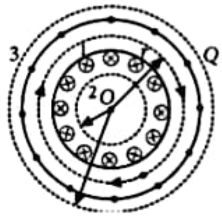
$\Rightarrow \quad t = 17.66\times10^{-7} s = 1.77\,\mu s$

Thus, the time taken by the electron to come out of the region of magnetic field is 1.77 μs .

51. Soln. (a) A solenoid bent into the form of closed loop is called toroid. The magnetic field B has a constant magnitude everywhere inside the toroid.



(b) Let magnetic field inside the toroid is B along the considered loop (1) as shown in figure.



Applying Ampere's circuital law,

$$\int_{loop1} \vec{B}.\vec{dl} = \mu_0(\mathrm{NI})$$

Since, toroid of N turns, threads the loop 1, N times, each carrying current I inside the loop.

Therefore, total current threading the loop 1 is NI.

$$\Rightarrow \quad \oint_{loop1} \vec{B}.\vec{dl} = \mu_0(\mathrm{NI})$$

$$B\oint_{loop} dl = \mu_0(\mathrm{NI})$$

$$\mathrm{B}\times 2\pi \mathrm{r} = \mu_0 NI \text{ or } B = \frac{\mu_0 NI}{2\pi r}$$

(c) (i) Magnetic field inside the open space interior the toroid. Let the loop (2) as shown in figure experience magnetic field B.

No current threads the loop 2 which lies in the open space inside the toroid.

$\therefore$ Ampere's circuital law

$$\oint_{loop3} \vec{B}.\vec{dl} = \mu_0(\mathrm{NI - NI}) = 0 \Rightarrow \mathrm{B} = 0$$

(ii) Magnetic field in the space exterior of toroid. Let us consider a coplanar loop (3) in the open space of exterior of toroid. Here, each turn of toroid threads the loop two times in opposite directions.

Therefore, net current threading the loop

= NI – NI = 0

$\therefore$ By Ampere's circuital law

$$\oint_{loop3} \vec{B}.\vec{dl} = \mu_0(\mathrm{NI - NI}) = 0 \Rightarrow \mathrm{B} = 0$$

Thus, there is no magnetic field in the open space interior and exterior the toroid.

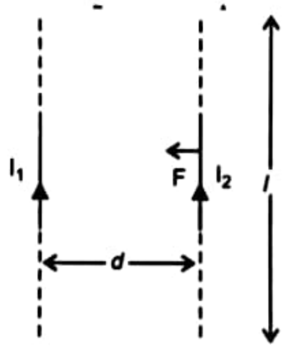
52. Soln. (a) $\vec{F} = q(\vec{v}\times\vec{B}) = \mathrm{qvBsin}\,\theta$

(i) If the angle between v and B is 90^0 then it will move in circular path.

(ii) If the angle is other than, 0^0, 90^0 and 180^0 the path will be helical.

(b) Since the work done on the charged particle moving in the magnetic field is zero because force experienced is perpendicular cannot bring any charge in speed of charged particular. Hence the change in K.E. is zero.

53. Soln. Magnetic field produced on the wire (carrying current I_2) due to I_1 will be



Force acting at l length is

$$F = I_2 l B$$

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ towards } I_1$$

If $l = 1m$, $d = 1m$, $I_1 = I_2 = I$ and

$F = 2\times10^{-7} N$

$\Rightarrow \quad I = 1A$

So one ampere is defined as the current, which when maintained in two parallel infinite length conductors, held at a separation of one metre will produce a force of 2×10^{-7} N per metre on each conductor.

54. Soln. (a) Force acting on a charge 'q' moving with velocity $\vec{v}$ in the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$ is given by,

$$\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B})$$

Consider a region in which magnetic field, electric field and velocity of charge particle are perpendicular to each other.

To move charge particle undeflected, the net force acting on the particle must be zero i.e., the electric force must be equal and opposite to the magnetic force.

$$q\vec{E} = -q(\vec{v}\times\vec{B})$$

$$\vec{E} = -(\vec{v}\times\vec{B})$$

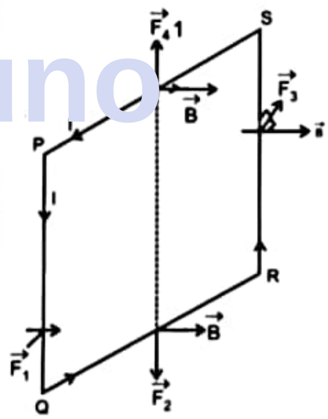
$$\vec{E} = \vec{B}\times\vec{v}$$

$$\vec{E} = Bv\sin\theta = Bv \qquad (\because \theta = 90^0)$$

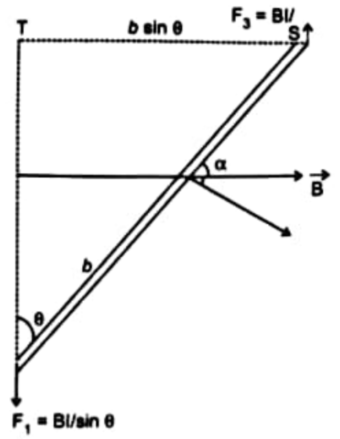
$$v = \frac{E}{B}$$

The direction of electric and magnetic forces are in opposite direction. Their magnitudes are in such a way that they cancel out each other to give net force zero and so the charge particle does not deflect.

(b) When an electric current flows in closed loop of wire, placed in a uniform magnetic field, the magnetic forces produce a torque which tends to rotate the loop so that area of the loop is perpendicular to the direction of the magnetic field.



Consider a rectangular coil PQRS placed in an external magnetic field as shown in fig.



Let T be the current flowing through the coil. Each part of the coil experiences a Lorentz force. Forces of each part is $\vec{F_1}, \vec{F_2}, \vec{F_3}$ and $\vec{F_4}$ as shown. The $\vec{F_4}$ and $\vec{F_2}$ are equal in magnitude but act in opposite directions along the same straight line. Hence, they cancel out each other.

The force $\vec{F_1} = I(\vec{PQ} \times \vec{B})$

$$F_1 = I / B \qquad (\because \theta = 90^0)$$

($\vec{F_1}$ acts in direction perpendicular to the plane of paper)

Similarly, $\vec{F_3} = I(\vec{RS} \times \vec{B})$

$$F_3 = I / B$$

These two forces constitute a couple and so rotates the coil in the anticlockwise direction.

The torque

$\tau =$ force x arm of couple

$$\tau = Fb\cos\theta$$

$$\tau = I / Bb\cos\theta$$

$$\tau = IAB\cos\theta \quad (\because l \times b = A)$$

If the coil has N turns then

$$\tau = NIAB\cos\theta$$

The area vector A makes an angle α with $\vec{B}$ so

$$\theta + \alpha = 90^0$$

$$\cos\theta = \cos(90 - \alpha) = \sin\alpha$$

$\therefore \quad \tau = NIAB\sin\alpha$

$$\tau = mB\sin\alpha$$

or $\quad \vec{\tau} = \vec{m} \times \vec{B}$

Where m = NIA is called the magnetic dipole moment of the loop.

55. Soln. The standard formula for field at an axial point is given as

$$\frac{\mu_0 I a^2}{2(a^2 + x^2)^{3/2}}$$

Field at O due to loop P

$$B_p = \frac{\mu_0 I(r^2)}{2\left[r^2 + \left(\frac{2r}{2}\right)^2\right]^{3/2}} = \frac{\mu_0 I}{2\pi}$$

And, field at O due to loop Q

$$B_Q = \frac{\mu_0 I(r^2)}{2\left[r^2 + \left(\frac{2r}{2}\right)^2\right]^{3/2}} = \frac{\mu_0 I}{2\pi}$$

Now, as the current flowing in loop P is clockwise, by using right hand thumb's rule, the direction of the magnetic field will be towards left and as the current in loop Q is clockwise then the direction of magnetic field is towards left. So the net magnetic field at point O will be the sum of the magnetic fields due to loops P and Q?

So, net field

$$B = B_P + B_Q = \frac{2\mu_0 I}{2r} = \frac{\mu_0 I}{r}$$

56. Soln. Equivalent magnetic moment of the coil

$$\vec{m} = IA\hat{n}$$

$\therefore \quad \vec{m} = I / b\hat{n}$

($\hat{n}$ = unit vector $\perp$ to the plane of the coil)

$\therefore \quad$ Torque = $\vec{m} \times \vec{B}$

$$= I_l b\hat{n} \times \vec{B}$$

$$= 0$$

(as $\hat{n}$ and $\vec{B}$ are parallel or antiparallel to each other)

57. Soln. We have:

$$N_1.2\pi R = N_2.2\pi(R/2)$$

$\therefore \quad N_2 = 2N_1$

Magnetic moment of a coil, M = NAI

For the coil of radius 'R',

$$M_1 = N_1 I A_1 = N_1 I \pi R^2$$

For the coil of radius R/2,

$$M_2 = N_2 I A_2 = 2N_1 I \pi R^2 / 4 = N_1 \, I \pi R^2 / 2$$

$$\Rightarrow \quad M_2 : M_1 = 1:2$$

58. Soln. According to Biot-Savart law, the magnitude of the field $\vec{dB}$ is

1. Directly proportional to the current I through the conductor,
   $dB \propto I$
2. Directly proportional to the length dl of the current element,
   $dB \propto dl$
3. Directly proportional to sin θ,
   $dB \propto \sin\theta$
4. Inversely proportional to the square of the distance r of the point P from the current element,
   $dB \propto \dfrac{1}{r^2}$

Combining all these four factors, we get

$$dB \propto \frac{I\,dl \sin\theta}{r^2}$$

Or $$dB = K.\frac{I\,dl\sin\theta}{r^2}$$

The proportionality constant K depends on the medium between the observation point P and the current element and the system of units chosen. For free space and in SI units,

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\,T\,mA^{-1}(\text{or Wbm}^{-1}\,\text{A}^{-1})$$

Here μ_0 is a constant called *permeability* of free space. So the Biot – Savart law in SI units may be expressed as

$$dB = \frac{\mu_0}{4\pi}.\frac{I\,dl\sin\theta}{r^2}$$

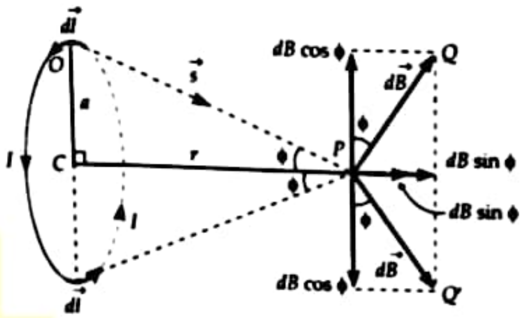
We can write the above equation as

$$dB = \frac{\mu_0 I}{4\pi}\frac{dl\,r\sin\theta}{r^3}$$

As the direction of $\overrightarrow{dB}$ is perpendicular to the plane of $\overrightarrow{dl}$ and $\overrightarrow{r}$, so from the above equation, we get the vector form of the Biot – Savart law as

$$\overrightarrow{dB} = \frac{\mu_0 I}{4\pi}\frac{\overrightarrow{dl}\times\overrightarrow{r}}{r^3}$$

59. Soln. Magnetic field along the axis of a circular current loop. Consider a circular loop of wire of radius a and carrying current I, as shown in fig. Let he plane of the loop be perpendicular to the plane of paper. We wish to find field $\vec{B}$ at an axial point P at a distance r from the centre C.



Consider current element $\vec{dl}$ at the top of the loop.

It has an outward coming current.

If $\vec{s}$ be the position vector of point P relative to the element $\vec{dl}$, then from Biot – Savart law, the field at point P due to the current element is

$$dB = \frac{\mu_0}{4\pi}.\frac{I\,dl\sin\theta}{s^2}$$

Since $\vec{dl} \perp \vec{s}$, i.e., $\theta = 90^0$, therefore

$$dB = \frac{\mu_0}{4\pi}.\frac{I\,dl}{s^2}$$

The field $d\vec{B}$ lies in the plane of paper and is perpendicular to $\vec{s}$, as shown by $\vec{PQ}$. Let ϕ be the angle between OP and CP. Then dB can be resolved into *two* rectangular components.

1. $dB\sin\phi$ along the axis,
2. $dB\cos\phi$ perpendicular to the axis.

For any two diametrically opposite elements of the loop, the components perpendicular to the axis of the loop will be equal and opposite and will cancel out. Their axial components will be in the same direction, i.e., along CP and get added up.

∴ Total magnetic field at the point P in the direction CP is

$$B = \int dB \sin\phi$$

But $\sin\phi = \frac{a}{s}$ and $dB = \frac{\mu_0}{4\pi}.\frac{I\,dl}{s^2}$

∴ $B = \int \frac{\mu_0}{4\pi}.\frac{I\,dl}{s^2}.\frac{a}{s}$

Since μ_0 and I are constant, and s and a are same for all points on the circular loop, we have

$$B = \frac{\mu_0 Ia}{4\pi s^3}\int dl = \frac{\mu_0 Ia}{4\pi s^3}.2\pi a = \frac{\mu_0 Ia^2}{2s^3}$$

[∵ $\int dl =$ circumference = $2\pi a$]

Or

$$B = \frac{\mu_0 Ia^2}{2(r^2+a^2)^{3/2}} \qquad [\because s = (r^2+a^2)^{1/2}]$$

As the direction of the field is along +ve X-direction, so we can write

$$\vec{B} = \frac{\mu_0 Ia^2}{2(r^2+a^2)^{3/2}}\hat{i}$$

If the coil consists of N turns, then

$$B = \frac{\mu_0 N Ia^2}{2(r^2+a^2)^{3/2}}$$

60. Soln. Since the point O lies on lines SP and QR, so the magnetic field at O due to these straight portions is zero.

The magnetic field at O due to the circular segment PQ is

$$B_1 = \frac{\mu_0}{4\pi}\frac{I}{a^2}l$$

Here, l = length of arc PQ = $\alpha\ a$

∴ $B_1 = \frac{\mu_0}{4\pi}\frac{I\alpha}{a}$, directed normally upward

Similarly, the magnetic field at O due to the circular segment SR is

$B_2 = \frac{\mu_0}{4\pi}.\frac{I\alpha}{b}$, directed normally downward.

The resultant field at O is

$$B = B_1 - B_2 = \frac{\mu_0 I\alpha}{4\pi}\left[\frac{1}{a} - \frac{1}{b}\right]$$

Or $B = \frac{\mu_0 I\alpha(b-a)}{4\pi ab}$

61. Soln. $\vec{B}_P = \frac{\mu_0 I}{2R}$, vertically upwards,

$\vec{B}_Q = \frac{\mu_0\sqrt{3}I}{2R}$, along horizontal

Resultant field at the centre is

$$B = \sqrt{B_P^2 + B_Q^2} = \left[\left(\frac{\mu_0 I}{2R}\right)^2 + \left(\frac{\mu_0\sqrt{3}I}{2R}\right)^2\right]^{1/2}$$

$$= \frac{\mu_0 I}{2R}(1+3)^{1/2} = \left(\frac{\mu_0 I}{R}\right)$$

$$\tan\theta = \frac{B_P}{B_Q} = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30^0$$

62. Soln. *Ampere's circuital law states that the line integral of the magnetic field* $\vec{B}$ around way closed circuit is equal to μ_0 (permeability constant) times the total current I threading or passing through this closed circuit. Mathematically,

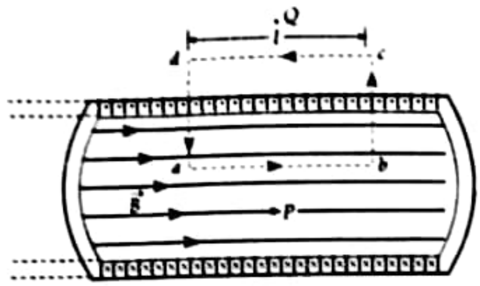
$$\oint \vec{B}.\vec{dl} = \mu_0 I$$

In a simplified form, Ampere's circuital law states that if field $\vec{B}$ is directed along the tangent to every point on the perimeter L of a closed curve and its magnitude is constant along the curve, then

$$BL = \mu_0 I$$

Where I is the net current enclosed by the closed circuit. The closed curve is called *Amperean loop* which is a *geometrical entity* and *not a real* wire loop.

It fig. shows the sectional view of a long solenoid. At various turns of the solenoid, current comes out of the plane of paper at points marked Θ and enters the plane of paper at points marked $\otimes$. To determine the magnetic field $\vec{B}$ at any inside point, consider a rectangular closed path abcd as the Amperean loop.



According to Ampere's circuital law,

$$\int \vec{B}.\vec{dl}$$

$= \mu_0 \times$ Total current through the loop abcd

Now

$$\oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl} + \int_b^c \vec{B}.\vec{dl} + \int_c^d \vec{B}.\vec{dl} + \int_d^a \vec{B}.\vec{dl}$$

But $\int_b^c \vec{B}.\vec{dl} = \int_b^c B\,dl \cos 90^0 = 0$

$$\int_d^a \vec{B}.\vec{dl} = \int_d^a B\,dl \cos 90^0 = 0$$

$$\int_c^d \vec{B}.\vec{dl} = 0$$

As B = 0 for points outside the solenoid.

$$\therefore \oint \vec{B}.\vec{dl} = \int_a^b \vec{B}.\vec{dl}$$

$$= \int_a^b B\,dl \cos 0^0 = B\int_a^b dl = Bl$$

Where,

l = length of the side ab of the rectangular loop abcd.

Let number of turns per unit length of the solenoid = n

Then number of turns in length l of the solenoid

$$= nl$$

Thus the current I of the solenoid threads the loop bcd, nl times.

$\therefore$ Total current threading the loop abcd = nlI

Hence $Bl = \mu_0 nlI$ or $B = \mu_0 nI$

63. Soln. Lorentz force. *The total force experienced by a charged particle moving in a region where both electric and magnetic fields are present, is called Lorentz force.*

A charge q in an electric field $\vec{E}$ experiences the electric force,

$$\vec{F_e} = q\vec{E}$$

This force acts in the direction of field $\vec{E}$ and is independent of the velocity of the charge.

The magnetic force experienced by the charge q moving with velocity $\vec{v}$ in the magnetic field $\vec{B}$ is given by

$$\vec{F_m} = q(\vec{v} \times \vec{B})$$

This force acts perpendicular to the plane of $\vec{v}$ and $\vec{B}$ and depends on the velocity $\vec{v}$ of the charge.

The total force, or the *Lorentz force*, experienced by the charge q due to both electric and magnetic field is given by

$$\vec{F} = \vec{F_e} + \vec{F_m}$$

Or $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

64. Soln. (i) Magnitude of magnetic field at A

Direction of magnetic field at A

(ii) Magnitude of magnetic force on conductor 2

Direction of magnitude force on conductor 2

(i) $B_2 = \frac{\mu_0}{4\pi}\frac{2(3I)}{r} = \frac{\mu_0}{4\pi}\left(\frac{6I}{r}\right)$ into the plane of the paper/ (⊗)

$B_3 = \frac{\mu_0}{4\pi}\left(\frac{2(4I)}{3r}\right) = \frac{\mu_0}{4\pi}\left(\frac{8I}{3r}\right)$ out of the plane of the paper/ (*)

$B_A = B_2 - B_3$ into the paper

$= \frac{\mu_0}{4\pi}\left(\frac{10I}{3r}\right)$ into the paper/ (⊗)

(ii) $F_{21} = \frac{\mu_0}{4\pi}\frac{2I(3I)}{r}$ away from wire 1 (/towards 3)

$F_{23} = \frac{\mu_0}{4\pi}\frac{2I(3I)}{r}$ away from 3 (towards 1)

$F_{net} = F_{23} - F_{21}$ towards wire 1

$= \frac{\mu_0}{4\pi}\frac{6(I)^2}{r}$ towards wire 1

65. Soln. (i) Behaviour of revolving electron as a tiny magnetic dipole.

(ii) Proof of the relation $\vec{\mu} = -\frac{e}{2m_e}\vec{L}$

(iii) Significance of negative sign

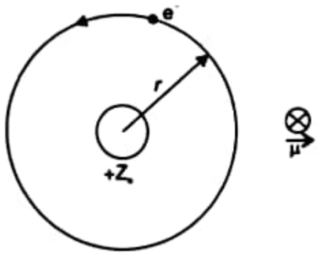
Electron, in circular motion around the nucleus constitutes a current loop which behaves like a magnetic dipole.

Current associated with the revolving electron.

$$I = \frac{e}{T}$$

And $$T = \frac{2\pi r}{v}$$

$$\therefore \quad I = \frac{e}{2\pi r}v$$



Magnetic moment of the loop, $\mu = IA$

$$\mu = IA = \frac{ev}{2\pi r}\pi r^2$$

$$= \frac{evr}{2} = \frac{e.m_e vr}{2m_e}$$

Orbital angular momentum of the electron,

$$L = m_e vr$$

$$\vec{\mu} = \frac{-e}{2m_e}\vec{L}$$

-ve sign signifies that the angular momentum of the revolving electron is opposite in direction to the magnetic moment associated with it.

66. Soln.

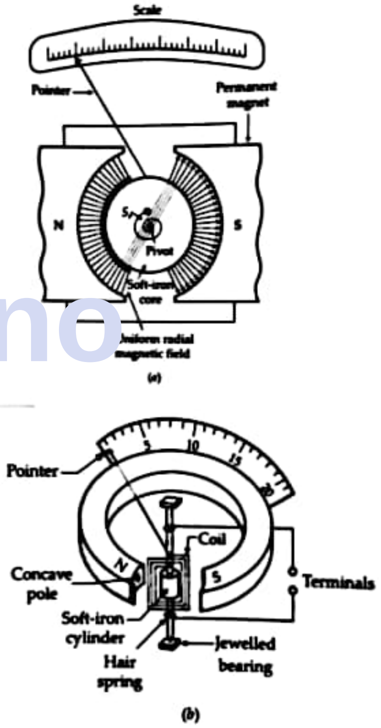


Fig. 4.94 (a) Top view (b) Front view of a pivoted-type galvanometer.

I = current flowing through the coil PQRS

a,b = sides of the rectangular coil PQRS

A = ab = area of the coil

N = number of turns in the coil.

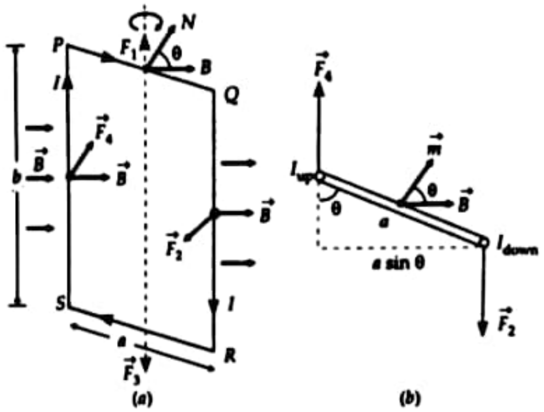


Fig. 4.95 (a) Rectangular loop PQRS in a uniform magnetic field. (b) Top view of the loop.

Since the field is radial, the plane of the coil always remains parallel to the field $\vec{B}$. The magnetic forces on sides PQ and SR are equal, opposite and collinear, so their resultant is zero. According to Fleming's left rule, the side PS experiences a normal inward force equal to NIbB while the side QR experiences an equal normal outward force. The two forces on sides PS and QR are equal and opposite. They form a couple and exert a torque given by

τ = Force x Perpendicular distance

$$= NIbB \times a \sin 90^0 = NIB(\text{ab}) = \text{NIBA}$$

Here $\theta = 90^0$, because the normal to the plane of coil remains perpendicular to the field $\vec{B}$ in all positions.

The torque τ deflects the coil through an angle α. A restoring torque is set up in the coil due to elasticity of the springs such that

$$\tau_{restoring} \propto \alpha \quad or \quad \tau_{restoring} = k\alpha$$

Where k is the *torsion constant* of the springs i.e., torque required to produce unit angular twist. In equilibrium position.

Restoring torque = Deflecting torque

$$k\alpha = NIBA$$

Or $$\alpha = \frac{NBA}{k}.I$$

Or $$\alpha \propto I$$

Thus the deflection produced in the galvanometer coil is proportional to the current flowing through it. Consequently, the instrument can be provided with a scale with equal divisions along a circular scale to indicate equal steps in current. Such a scale is called *linear scale*.

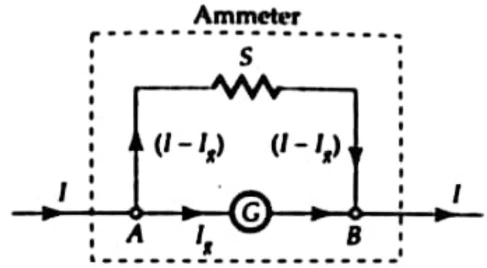
Also, $$I = \frac{k}{NBA}.\alpha = G\alpha$$

The factor $G = k / \text{NBA}$ is constant for a galvanometer and is called *galvanometer constant* or *current reduction factor* of the galvanometer.

Fig. of merit of a galvanometer. *It is defend as the current which produces a deflection of one scale division in the galvanometer and is given by*

$$G = \frac{I}{\alpha} = \frac{k}{NBA}$$

Ammeter



Let G = resistance of the galvanometer

I_g = the current with which galvanometer gives full scale deflection

$0 - I$ = the required current range of the ammeter

S = shunt resistance

$I - I_g$ = current through the shunt.

As galvanometer and shunt are connected in parallel, so

P.D. across the galvanometer = P.D. across the shunt

$$I_g G = (\text{I} - \text{I}_g)\text{S}$$

Or $$S = \frac{I_g}{I - I_g} \times G$$

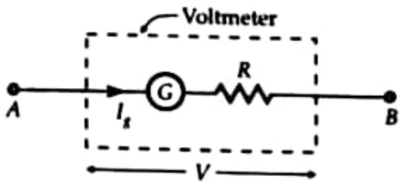
So by connecting a shunt of resistance S across the given galvanometer, we get an ammeter of desired range. Moreover,

$$I_g = \frac{S}{G + S} \times I$$

The deflection in the galvanometer is proportional to I_g and hence to I. So the scale can be graduated to read the value of current I directly.

Hence *an ammeter is a shunted or low resistance galvanometer.* Its effective resistance is

$$R_A = \frac{GS}{G+S} < S$$



A galvanometer can be converted into a voltmeter by connecting a high resistance in series with it. The value of this resistance is so adjusted that only current I_g which produces full scale deflection in the galvanometer, passes through the galvanometer.

Let

G = resistance of the galvanometer

I_g = the current with which galvanometer gives full scale deflection

$0-V$ = required range of the voltmeter, and

R = the high series resistance which restricts the current to safe limit I_g.

∴ Total resistance in the circuit = R + G

By Ohm's law,

$$I_g = \frac{Potential\ difference}{Total\ resistance} = \frac{V}{R+G}$$

So by connecting a high resistance R in series with the galvanometer, we get a voltmeter of desired range. Moreover, the deflection in the galvanometer is proportional to current I_g and hence to V. The scale can be graduated to read the value of potential difference directly.

Hence a *voltmeter is a high resistance galvanometer.* Its effective resistance is

$$R_V = R + G >> G.$$

67. Soln. Current sensitivity. It is defined as the deflection produced in the galvanometer when a unit current flows through it.

Current sensitivity, $I_S = \frac{\alpha}{I} = \frac{NBA}{k}$

Voltage sensitivity. *It is defined as the deflection produced in the galvanometer when a unit potential difference is applied across its ends.*

Voltage sensitivity, $V_S = \frac{\alpha}{V} = \frac{\alpha}{IR} = \frac{NBA}{k\ R}$

Clearly, Voltage sensitivity = $\frac{Current\ sensitivity}{R}$

68. Soln.

$$A = 5cm \times 2cm = 10\times10^{-4} m^2 = 10^{-3} m^2$$

$$B = 2.5\times10^{-2} Wb\, m^{-2},\ k = 1.5\times10^{-8} Nm\ rad^{-1}$$

$$\theta = 0.2\, rad,\ I = 2\mu A = 2\times10^{-6} A$$

As $I = \frac{k}{NBA}.\alpha$

∴ $N = \frac{k}{I\,B\,A}.\alpha$

$$= \frac{1.5\times10^{-8}\times0.2}{2\times10^{-6}\times2.5\times10^{-2}\times10^{-3}} = 60.$$

69. Soln. The given ammeter can be regarded as the galvanometer.

∴ $I_g = 1.0\,A,\quad R_g = 0.80\Omega$

(i) Total current in the circuit, $I = 5.0A$
The required shunt resistance,

$$R_s = \frac{I_g}{I-I_g}\times R_g = \frac{1.0}{5.0-1.0}\times0.80 = 0.20\Omega.$$

(ii) The combined resistance R_A of the ammeter and the shunt is given by

$$\frac{1}{R_A} = \frac{1}{R_g} + \frac{1}{R_s} = \frac{1}{0.8} + \frac{1}{0.2} = \frac{1+4}{0.8} = \frac{25}{4}$$

Or $R_A = 4/25 = 0.16\Omega$.

70. Soln. The current for full scale deflection of voltmeter is given by

$$I_g = \frac{V}{R_g + R}$$

In first case, $I_g = \frac{B}{R_g + 980}$

In second case, $I_g = \frac{V/2}{R_g + 470}$

$\therefore \quad \frac{V}{R_g + 980} = \frac{V}{2(R_g + 470)}$

Or $\quad 2R_g + 940 = R_g + 980$

Or $\quad R_g = 40\Omega$.

SURE SHOT QUESTIONS

Chapter – 05

Magnetism and Matter

MCQ (1 mark)

1. Soln. (c)
2. Soln. (b): According to Gauss's law in magnetism $\oint \vec{B}.d\vec{S} = 0$, which implies that number of magnetic field lines entering the Gaussian surface is equal to the number of magnitude field lines leaving it. Therefore, case (ii) is not possible.

3. Soln. (b): Here, $M_1 = 8\,Am^{-1}$, $B_1 = 0.6\,T$, $T_1 = 4K$, $B_2 = 0.2\,T$, $T_2 = 16K$

Then, for paramagnetic materials,

Magnetization $(\text{M}) = \text{C}\dfrac{B}{T}$ (Curie's law)

Now, in the first case, $M_1 = \dfrac{CB_1}{T_1}$ ....(i)

And in the second case, $M_2 = \dfrac{CB_2}{T_2}$ ....(ii)

Dividing equation (i) by (ii), we get

$$\frac{M_1}{M_2} = \frac{B_1}{B_2}.\frac{T_2}{T_1} \Rightarrow \frac{M_1}{M_2} = \frac{0.6}{0.2}\times\frac{16}{4}; \frac{8}{M_2} = 3\times 4$$

$$M_2 = \frac{8}{12} = \frac{2}{3}\,Am^{-1}$$

4. Ans. (c) : Given that Magnetic moment $\vec{M}$ is parallel to Magnetic field $\vec{B}$ i.e., $\vec{M} \parallel \vec{B}$.

Torque, $\vec{\tau} = |\vec{M}\times\vec{B}| = MB\sin\theta = MB\sin 0^0 = 0$

Net force = 0, as two forces are equal and opposite.

5. Ans. (c) : The direction of magnetic field at any point is given by the direction of tangent at that point on the magnetic field line.

6. Ans. (a) : Copper has less than one relative permeability. Copper relative permeability,

$$\mu_r = \frac{\mu}{\mu_0} = 0.999994$$

7. Ans. (d) : As copper is diamagnet, so the magnetic field lines do not pass through it.

8. Ans. (c) : Water has relative magnetic permeability between 0 and 1 as it is a diamagnetic material.

9. Ans.(b)

10. Ans. (a) : The magnetic permeability of a substance is given by $\mu = \dfrac{B}{H}$. Here, B is magnetic field inside the material, H is applied external magnetic field. When external applied magnetic field H is increased, the internal magnetic field for a ferromagnetic material remains the same. Thus, the magnetic permeability decreases

11. Ans. (i) Specimen A – diamagnetic,
Specimen B – paramagnetic
(ii) The magnetic susceptibility of A is small negative and that of B is small positive.

Assertion-Reasoning (1 mark)

12. Ans. (a) : Magnetic poles cannot be separated as monopoles of a magnetic does not exists.

Gauss's law in magnetism:

$$\phi = \sum_{\substack{all\ area \\ element}} \vec{B}.\Delta\vec{S} = 0$$, this indicates monopole does not exist.

13. **Sol.** (a): The field magnet used in a moving coil galvanometer is very strong. The earth's magnetic field is quite weak as compared to the magnetic field produced by the field magnet. Practically the coil rotates under the effect of the strong magnetic field due to the field magnet and the weak magnetic field due to the earth does not affect the working of the moving coil galvanometer.

14. **Sol.**

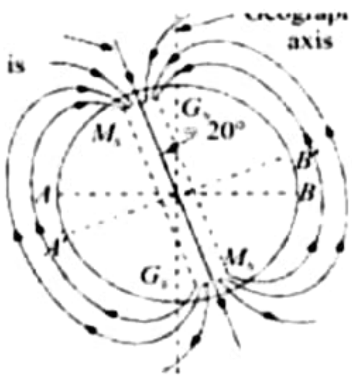
(d): The angle of dip is the angle between the axis of the dip needle in the magnetic meridian and the horizontal direction.

$$\tan\theta = \frac{\text{Vertical component of the earth's magnetic field}}{\text{Horizontal component of the earth's magnetic field}}$$

$$= \frac{B_V}{B_H} \cdot \text{when } B_V = B_H, \tan\theta = 1 \Rightarrow \theta = 45°$$

Lines joining points of zero dip are called aclinic lines. Lines joining points of equal dip are called isoclinic lines.

15. **Sol.** (d): From the compass we are able to know the direction of the magnetic poles. The north of compass points towards the magnetic south pole.



If we know the magnetic declination at that particular place (which is angle between geographic meridian and magnetic meridian) we can easily find out the true geographic north-south direction. Imaginary lines drawn along the earths surface in the direction of the horizontal component of the magnetic field of the earth at all points passing through the north and south magnetic poles. This is similar to the longitudes of the earth, which pass through the geographic north and south poles.

16. **Sol.** (b): There will be only one neutral point on the horizontal board. This is because field of earth magnetic field is from south to north; and the field of pole on the board is radially outwards. At any point towards south of magnetic pole, field of earth and field of pole will cancel out to give a neutral point.

17. **Sol.** (d): The temperature inside the earth is so high that it is impossible for iron core to behave as a magnet and act as a source of magnetic field. The magnetic field of earth is considered to be due to circulating electric current in the iron (in molten state) and other conducting materials inside the earth.

18. **Sol.** (a): Earths magnetic field can be represented as the field of a huge bar magnet. If the magnet is freely suspended its north-pole points towards geographic north pole (really a south magnet pole of earth)

19. **Sol.** (d): The earth has only vertical component of its magnetic field at the magnetic poles. Since compass needle is only free to rotate in horizontal plane. At north pole the vertical component of earths field will exert torque on the magnetic needle so as to align it along its direction. As the compass needle can not rotate in vertical plane, it will rest horizontally, when placed on the magnetic north pole of the earth.

20. **Sol.** (a): A neutral point in the magnetic field of a bar magnet is that point, where the field due to magnet is completely neutralised by the horizontal component of earths magnetic field. The net horizontal field is zero at such a point. If a compass needle is placed at such a point, it can stay in any position.

21. **Sol.**

(a): Magnetic moment $= \frac{\text{joule}}{\text{tesla}} = \frac{W}{B} = \frac{W}{F/qv}$

$= \frac{Wqv}{F} = \frac{[ML^2T^{-2}][AT][LT^{-1}]}{[MLT^{-2}]}$

$= [AL^2] = \text{amp m}^2$

Case Study Question

22. Answers

1. (*b*) At the magnetic equator,

$\delta = 0°$ and $B_H = B\cos 0° = B$.

2. (*d*) The angle of dip gives the direction of the earth's magnetic field.

3. (*a*) Angle of dip is 90° at poles.

4. (*a*) $B_V = \sqrt{B^2 - B_H^2} = \sqrt{(0.5)^2 - (0.3)^3}$

$= \sqrt{0.8 \times 0.2} = 0.4\,G$

$\tan\delta = \frac{B_V}{B_H} = \frac{0.4}{0.3} = \frac{4}{3}$

$\Rightarrow \quad \delta = \tan^{-1}\left(\frac{4}{3}\right)$.

5. (*b*) At any geomagnetic pole, $B_H = 0$. A compass needle, which is free to rotate in a horizontal plane only, will stay in any position.

23. Answers

6. (*a*) Diamagnetism is a universal property among all substances.

7. (*a*) A diamagnetic material is repelled by a strong magnetic field.

8. (*b*) Magnetic susceptibility of a diamagnetic substance is independent of temperature.

9. (*c*) For a paramagnetic material, $\chi \propto \frac{1}{T}$.

10. (*d*) In a superconductor, the magnetic lines of force get completely expelled.

For this material, $\chi = -1$.

Question

24. Ans. .

S.No.	Paramagnetic substances	Diamagnetic substances
1.	A paramagnetic substance is feebly attracted by a magnet,	A diamagnetic substance is feebly repelled by a magnet.
2.	For a paramagnetic	For a diamagnetic substance, the

	substance, the intensity of magnetisation has a small positive value.	intensity of magnetization hasa small negative value.

25. Ans. Properties of magnets:

(i) Attractive property: When a magnet is dipped into iron filings, it is found that the concentration of iron fillings is maximum at the ends. It means attracting power of the magnet is maximum at two points near the ends and minimum at the centre. The places in a magnet where at attracting power is maximum are known as poles while the place of minimum attracting power is known as the neutral region.

(ii) Directive property: When a magnet is suspended, its length becomes parallel to N – S direction. The pole at the end pointing north is known as north pole while the other pointing south is known as south pole.

(iii) Magnetic pole always exist in pairs i.e., an isolated magnetic pole does not exist.

(iv) Like poles repel each other and unlike poles attract each other.

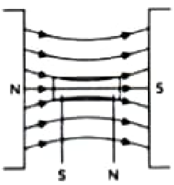
26. A bar magnet of magnetic moment 6 J T^{-1} is aligned at 60^0 with a uniform external magnetic field of 0.44 T. Calculate (a) the work done in turning the magnet to align its magnetic moment (i) normal to the magnetic field, (ii) opposite to the magnetic field, and (b) the torque on the magnet in the final orientation in case (ii). [2018]

27. .

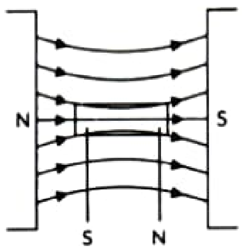
S.No.	Paramagnetic	Diamagnetic	Ferromagnetic
1.	Substances are feebly attracted by the magnet. Na, K, Mg, Mn, Al, Cr, Sn and liquid oxygen are paramagnetic.	Substances are feebly repelled by the magnet. Bi, Cu, Ag, Hg, water, hydrogen, He, Ne etc are diamagnetic.	Substances are strongly attracted by the magnet. Fe, Co, Ni and their alloys are ferromagnetic.
2.	χ_m is small, positive and varies inversely with temperature, *i.e.*, $\chi_m \propto (1/T)$.	Susceptibility χ_m is small, negative and temperature independent.	χ_m is very large, positive and temperature dependent.
3.	μ_r is slightly greater than unity, *i.e.*, $\mu > \mu_0$.	Relative permeability μ_r is slightly lesser than unity, *i.e.*, $\mu < \mu_0$	μ_r is much greater than unity, *i.e.*, $\mu >> \mu_0$.

28. Ans. As $\chi = 0.9853$, so material is paramagnetic.

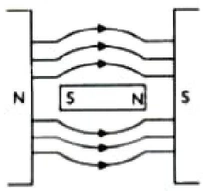
The behaviour of magnetic field lines in the presence of a paramagnetic substance is shown.



29. Ans. (i) The behaviour of magnetic field lines in the presence of a paramagnetic substance is shown:



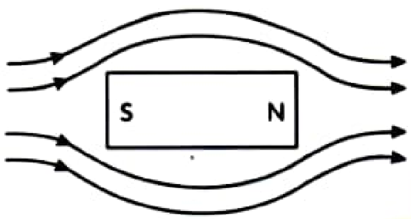
(ii) The behaviour of magnetic field lines in the presence of a diamagnetic substance is shown:



This distinguishing feature is because of the difference in their relative permeabilities. The relative permeability of the diamagnetic substance is negative; so, the magnetic lines of force do not prefer passing through the substance. The relative permeability of a paramagnetic substance is greater than 1; so, the magnetic lines of force prefer passing through the substance.

30. Ans. A diamagnetic specimen would tend to move towards the region of weaker magnetic field while a paramagnetic specimen would tend to move towards the region of stronger magnetic field.

31. Ans. Behaviour of magnetic field lines when a diamagnetic substance is placed in an external field.



32. Ans. The relative permeability is an intrinsic property of a magnetic material. A related quantity is the magnetic susceptibility, denoted by χ_m.

$\mu_r = 1 + \chi_m$ $\quad\quad$ $[\because \mu_r = 0.5]$

Here, $\mu_r < 1$ (χ_m negative), so the material is termed as diamagnetic.

33. Soln. When current I is passed through the wire having n turns per unit length and of mean radius r, then magnetic field lines set up inside the ring in the form of concentric circles. Let one such loop be of radius r, then line integral of magnetic field over that closed loop is

$$\oint \vec{H}.\vec{dl} = \oint H\,dl\cos 0^0 = H\oint dl = H.2\pi r$$

.......(i)

But by Ampere's circuital law,

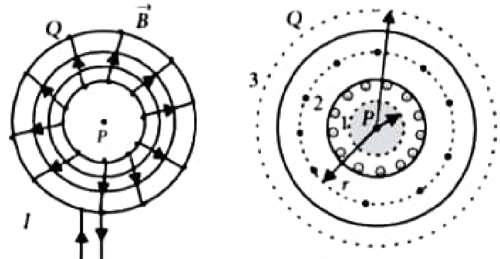
$\oint \vec{H}.\vec{dl} =$ total current threading the toroid

= total number of turns in toroid $\times I$

= $n(2\pi r)I$ ....(ii)

By equations (i) and (ii), $H.2\pi r = n\,2\pi r I$ or $B = \mu_0\mu_r nI$.

This gives magnetic field at any point inside the ring, directed at any point along the tangent to concentric circular magnetic line of force at that point inside the toroid.



Cross sectional view

34. Soln.

(i) Attractive Property: When a magnetic is dipped into iron filings, it is found that the concentration of iron fillings is maximum at the ends. It means attracting power of the magnet is maximum at two points near the ends and minimum at the centre. The places in a magnet where its attracting power is maximum are known as poles while the place of minimum attracting power is known as the neutral region.

(ii) Directive property: When a magnet is suspended, its length becomes parallel to N-S direction. The pole at the end pointing north is known as north pole while the other pointing south is known as south pole.

(iii) Magnetic poles always exist in pairs i.e., an isolated magnetic pole does not exist.

(iv) Like poles repel each other and unlike poles attract each other.

35. Soln. Given, χ_{mg} at 300 K = 1.2 x 10^5

χ'_{mg} at t temp. = 1.44 x 10^5

$t = ?$

From Curies law,

$$\chi \propto \frac{1}{T}$$

$$\frac{\chi'_{mg}}{\chi_{mg}} = \frac{300}{t}$$

$$\Rightarrow \quad \frac{1.44\times10^5}{1.2\times10^5} = \frac{300}{t}$$

$$t = \frac{300 \times 1.2}{1.44}$$

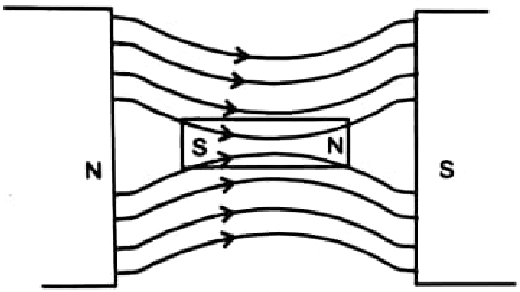
$= 250$ K

OR

Diamagnetic as $1 < \chi < 0$.

36. Soln. Given, susceptibility, $\chi = 0.9853$.

The material is paramagnetic in nature. If a piece of this material is kept in uniform magnetic field, then field pattern gets modified as follows:



The lines of force tend to pass through the material rather tan the surrounding air.

37. Soln. Given, magnetic momentum,

$m = 6\ JT^{-1}$

External magnetic field, $B = 0.44\,T$

$$\theta_1 = 60^0 \Rightarrow \cos\theta_1 = \cos 60^0 = \frac{1}{2}.$$

(a) Work done in turning the magnet normal to the field,

$W = -mB(\cos\theta_2 - \cos\theta_1)$

(i) Here, $\theta_2 = 90^0$

$\therefore \quad W = +mB\cos\theta_1$

$$= 6 \times 0.44 \times \frac{1}{2} = 1.32\,J$$

(ii) Here $\theta_2 = 180^0$

$\therefore \quad W = -mB(\cos\theta_2 - \cos\theta_1)$

$$W = -6 \times 0.44\left(-1 - \frac{1}{2}\right)$$

= 3.96 J

(b) Torque on magnet when moment is aligned opposite to the field,

Torque $= |\vec{m} \times \vec{B}|$

$$\tau = mB\sin\theta$$

$$= 6 \times 0.44 \times \sin 180^0$$

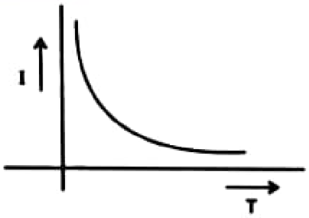
$$= 0 \quad (\because \sin 180^0 = 0)$$

38. Soln. Gauss law for magnetism: It a closed surface is imagined in a magnetic field, the number of lines of force emerging from the surface must be equal to the number entering it. That is, the net magnetic flux out of any closed surface is zero. Gauss law signifies that magnetic monopoles does not exist.

1. In a bar magnet, each lines of force, starts from a north poles and reaches the south pole externally and then goes from south pole to a north pole internally. Thus, magnetic line of force forms a closed loop.
2. No two lines of force will never intersect each other.
3. In a uniform field, the lines are parallel and equidistant from each other.
4. The lines of force are crowded near the poles.

39. Soln. (i) P – Paramagnetic material

(ii) Q – Ferromagnetic material



In paramagnetic materials, I decreases with temperature as, $I \propto \frac{H}{T}$, where, H is the magnetising field.

40. Soln. *The magnetic moment developed per unit volume of a material when placed in a magnetising field is called intensity of magnetisation or simply magnetisation.* Thus

$$\vec{M} = \frac{\vec{m}}{V}$$

If I_M is the surface magnetisation current set up in a solenoid of cross-sectional area A and having n turns per unit length, then magnetic moment developed per unit length of the solenoid is $nI_M A$. Therefore, magnetic moment developed per unit volume or the magnetisation $\vec{M}$ is given by

$$M = \frac{m}{V} = \frac{n I_M A}{A} = n I_M$$

Hence $B_M = \mu_0 n I_M = \mu_0 M$

Again, consider a bar of magnetic material having cross-sectional area a and length $2l$. Its volume is

$$V = a \times 2l$$

$$\therefore \quad M = \frac{m}{V} = \frac{q_m \times 2l}{a \times 2l} = \frac{q_m}{a}$$

Hence *intensity of magnetisation may also be defined as the pole strength developed per unit cross-sectional area of a material.*

Magnetising field intensity. The ability of magnetising field to magnetise a material medium is expressed by a vector $\vec{H}$, called *magnetising field intensity* or magnetic intensity. *Its magnitude may be defined as the number of ampere – turns (nI) flowing round the unit length of the solenoid required to produce the given magnetising field.* Thus

$$H = nI$$

$$\therefore \quad B_0 = \mu_0 nI = \mu_0 H \quad or \quad H = \frac{B_0}{\mu_0}$$

Magnetic Permeability. Permeability is the measure of the extent to which a material can be penetrated or permeated by a magnetic field. *The magnetic permeability of a material may be defined as the ratio of its magnetic induction B to the magnetic intensity H.*

$$\mu = \frac{B}{H}$$

Clearly, SI unit of μ

$$= \frac{tesla}{ampere\,metre^{-1}}$$

$$= tesla\ metre\ ampere^{-1} \quad or \quad TmA^{-1}$$

$\therefore$ Dimensions of $\mu = [\mathrm{MLT^{-2}A^{-2}}]$.

Magnetic susceptibility. Magnetic susceptibility measures the ability of a substance to take up magnetisation when placed in a magnetic field. *It is defined as the ratio of the intensity of magnetisation M to the magnetising field intensity H.* It is denoted by χ_m. Thus,

$$\chi_m = \frac{M}{H}$$

As magnetic susceptibility is the ratio of two quantities having the same units (Am^{-1}), so it has no units.

41. Soln. Curie's law. From experiments, it is found that the intensity of magnetisation (M) of a paramagnetic material is

(i) Directly proportional to the magnetising field intensity H, because the latter tends to align the atomic dipole moments.

(ii) Inversely proportional to the absolute temperature T, because the latter tends to oppose the alignment of the atomic dipole moments.

Therefore at low H/T values, we have

$$M \propto \frac{H}{T}$$

Or $M = C.\dfrac{H}{T}$

Or $\dfrac{M}{H} = \dfrac{C}{T}$ or $\chi_m = \dfrac{C}{T}$

Here C is curie constant and χ_m is the susceptibility of the material. The above relation is called *Curie's law*. This law states that far away saturation, the susceptibility of a paramagnetic material is inversely proportional to the absolute temperature.

Temperature at which a ferromagnetic substance becomes paramagnetic is called curie temperature of Curie point T_C.

Above the curle point i.e., the paramagnetic phase, susceptibility varies with temperature as

$$\chi_m = \frac{C}{T - T_C} \qquad (T > T_C)$$

Where C is a constant. This is modified Curie's law for a ferromagnetic material above the Curie temperature. It is also known as Curie – Weiss law. This law states that the susceptibility of a ferromagnetic substance above its Curie temperature is inversely proportional to the excess of temperature above the Curie temperature.

42. Soln. (i) Low coercivity and high permeability

(ii) Gauss's law in magnetism: The net magnetic flux through any closed surface is zero.

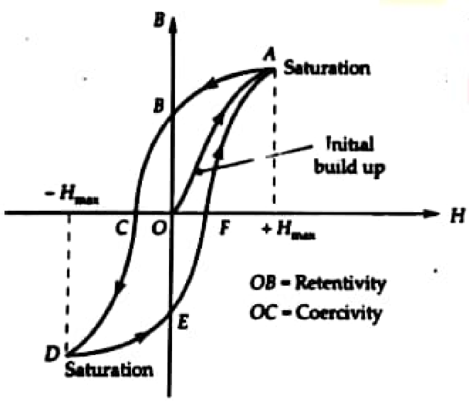
$$\oint B.ds = 0$$

Gauss's Law in electrostatics: The net electric flux through any closed surface is $\frac{1}{\varepsilon_0}$ times the net charge.

$$\oint E.ds = \frac{q}{\varepsilon_0}$$

The difference between the Gauss's law of magnetism and that for electrostatic is a reflection of the fact that magnetic monopole do not exist i.e., magnetic poles always exist in pairs.

43. Soln.



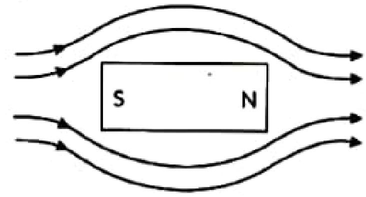
Hysteresis loop for a ferromagnetic sample.

Magnetic induction (=BA) left behind in the same after the magnetising field has been removed is called residual magnetism or retentivity or remanence.

The value of reverse magnetising field intensity H required for the residual magnetism of a sample to become zero is called coercivity of the sample.

44. Soln. A diamagnetic specimen would tend to move towards the region of weaker magnetic field while a paramagnetic specimen would tend to move towards the region of stronger magnetic field.

45. Soln. Behaviour of magnetic field lines when a diamagnetic substance is placed in an external field.



46. Soln. The relative permeability is an intrinsic property of a magnetic material. A related quantity is the magnetic susceptibility, denoted by χ_m.

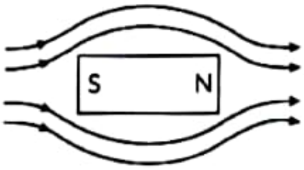
$$\mu_r = 1 + \chi_m \qquad [\because \mu_r = 0.5]$$

Here, $\mu_r < 1$ (χ_m negative), so the material is termed as diamagnetic.

47. Soln.

S.No.	Paramagnetic substance	Diamagnetic substance
1.	A paramagnetic substance is feebly attracted by a magnet.	A diamagnetic substance is feebly repelled by a magnet.
2.	For a paramagnetic substance, the intensity of magnetisation has a small positive value.	For a diamagnetic substance, the intensity of magnetisation has a small negative value.

48. Soln.
(i) Behaviour of magnetic field lines when a diamagnetic substance is placed in an external field.
(ii) Behaviour of magnetic field lines when a paramagnetic substance is placed in an external field.

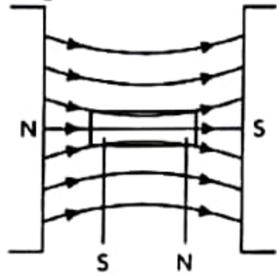


Atoms/molecules of a diamagnetic substance contain even number of electrons and these electrons form the pair of opposite spin; while the atoms/molecules of a paramagnetic substance have excess of electrons spinning in the same direction.

49. Soln.

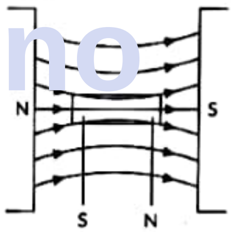
S. No.	Paramagnetic	Diamagnetic	Ferromagnetic
1.	Substances are feebly attracted by the magnet. Na, K, Mg, Mn, Al, Cr, Sn and liquid oxygen are paramagnetic.	Substances are feebly repelled by the magnet. Bi, Cu, Ag, Hg, Pb, water, hydrogen He, Ne, etc., are diamagnetic.	Substances are strongly attracted by the magnet. Fe, Co, Ni and their alloys are ferromagnetic.
2.	χ_m is small, positive and varies inversely with temperature, i.e., $\chi_m \propto (1/T)$.	Susceptibility χ_m is small, negative and temperature independent.	χ_m is very large, positive and temperature dependent.
3.	μ_r is slightly greater than unity, i.e., $\mu > \mu_0$	Relative permeability μ_r is slightly lesser than unity, i.e., $\mu < \mu_0$	μ_r is much greater than unity, i.e., $\mu >> \mu_0$.

50. Soln. As $\chi = 0.9853$, so material is paramagnetic.



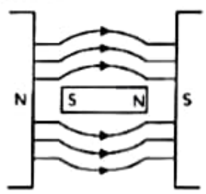
The behaviour of magnetic field lines in the presence of a paramagnetic substance is shown:

51. Soln. (i) The behaviour of magnetic field lines in the presence of a paramagnetic substance is shown:



(ii) The behaviour of magnetic field lines in the presence of a diamagnetic substance is shown:

This distinguishing feature is because of the difference in their relative permeabilities. The relative permeability of the diamagnetic substance is negative; so, the magnetic lines of force do not prefer passing through the substance. The relative permeability of a paramagnetic substance is greater than 1; so, the magnetic lines of force prefer passing through the substance.



52. Soln. Given magnetic moment of samples is

$$M_1 = 20\% of \left(2.0\times10^{24}\times1.5\times10^{-23}\right)$$

Or

$$M_1 = \frac{20}{100} \times 2.0 \times 10^{24} \times 1.5 \times 10^{-23} \; or \; M_1 = 6.0\,Am^2$$

As $\chi_m = \frac{C}{T}$ or $\frac{I}{H} = \frac{C}{T}$ or $\frac{M/V}{B_0/\mu_0} = \frac{C}{T}$

As volume V, Curie's constant C and μ_0 are constants, so

$$\frac{M}{B_0} = \frac{1}{T} \; or \; M = \frac{B_0}{T} \; \therefore \; \frac{M_2}{M_1} = \frac{B_2/T_2}{B_1/T_1} = \frac{B_2}{B_1} \times \frac{T_1}{T_2}$$

$$or \quad M_2 = \frac{1.00}{0.84} \times \frac{4.2}{2.1} \times 6$$

$$M_2 = 14.3\,Am^2$$

53. Soln. (i) For $\theta = 0^0$ between $\vec{M}$ and $\vec{B}$, dipole is in stable equilibrium.

(ii) For $\theta = 180^0$ between $\vec{M}$ and $\vec{B}$, dipole is in unstable equilibrium.

Potential energy, $U = -\vec{M}.\vec{B}$

At $\theta = 0^0$, $U_i = -MB\cos 0^0 = -MB$

$= -0.30 \times 0.50 = -0.15\,J$

At

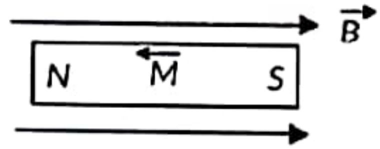
$\theta = 180^0$, $U_f = -MB\cos 180^0 = MB = +0.15\,J$

Torque on magnet is $\tau = MB\sin\theta$

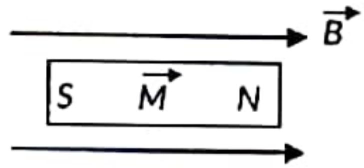
At $\theta = 0^0$, $\tau = MB\sin 0^0 = 0$

At $\theta = 180^0$, $\tau = MB\sin 180^0 = 0$

54. Soln. $\theta_1 = 180^0$ between $\vec{M}$ and $\vec{B}$ of bar magnet P



Whereas $\theta_2 = 0^0$ between $\vec{M}$ and $\vec{B}$ of bar magnet Q



So, force and toque on both the magnets due to magnetic field is zero, and the hence both are in equilibrium.

But $U = -MB\cos\theta$

i.e., Potential energy possessed by P is +MB, whereas by Q is -MB. So magnet Q with less potential energy is in stable equilibrium.

55. Soln. Magnetic field lines. No, if they intersect at a point, it will show two magnetic fields with difference directions at a point, which is never possible.

56. Soln.

$$T = 2\pi \times \sqrt{\frac{I}{MB}},\; 0.67 = 2\pi\sqrt{\frac{7.5\times10^{-6}}{6.7\times10^{-2}(B)}}$$

$$B = 0.01\text{T}$$

SURE SHOT QUESTIONS

ARVIND ACADEMY

Chapter – 06

Electromagnetic Induction

MCQ (1 mark)

1. Soln.(c) : Here, $\vec{B} = B_0(2\hat{i}+3\hat{j}+4\hat{k})\text{T}$

Area of the square = $L^2\hat{k}\,m^2$

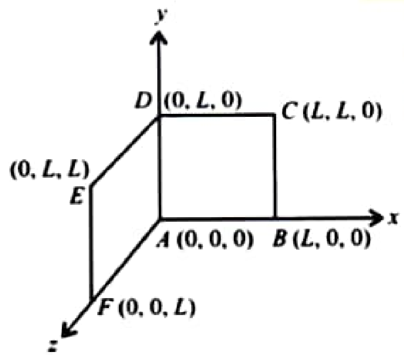
∴ Flux passing through the square.

$$\phi = \vec{B}.\vec{A} = B_0(2\hat{i}+3\hat{j}+4\hat{k}).L^2\hat{k} = 4B_0L^2Wb$$

2. Soln. (b): Here, $\vec{B} = B_0(\hat{i}+\hat{k})\text{T}$

Area vector of ABCD = $L^2\hat{k}$

Area vector of DEFA = $L^2\hat{i}$



Total area vector, $\vec{A} = L^2(\hat{i}+\hat{k})$

Total magnetic flux, $\phi = \vec{B}.\vec{A}$

$$= B_0(\hat{i}+\hat{k}).L^2(\hat{i}+\hat{k}) = B_0L^2(1+1) = 2B_0L^2Wb$$

3. Soln. (b): As there is no change in magnetic flux associated with the circuit, no current is induced in the circuit. The ammeter A shows no deflection.

4. Soln. (d): Coil A must be carrying a constant current in counter clockwise direction. Because of that, when A moves towards B, current induced in B is counterclockwise direction as per Lenz's law. The current in B would stop when A stops moving.

5. Soln. (a)

6. Soln. (b): The self inductance L of a solenoid of length l and area of cross section A with fixed number of turns N is

$$L = \frac{\mu_0 N^2 A}{l}$$

So, L increases when l decreases and A increases.

7. Ans. (c) : Along abc if I increases. When current I in the straight conductor XY is increased, then induced current in loop will be in clockwise direction.

8. Ans. (d) :

$$\frac{dB}{dt} = 1T/s,\ side,\ l = 10\,cm\ or\ 0.1m,\ N = 100$$

$$e = Na\frac{dB}{dt} = 100\times0.1\times0.1\times1 = 1V$$

9. Ans. (a) : Here, area, A = 100 cm^2 = 100 x 10^{-4} m^2

Magnetic field, B = 10^{-1} T

Angle, $\theta = 30^0$, time $t = 10^{-4}s$

The induced emf is

$$\Rightarrow e = \frac{-d\phi}{dt} = \frac{d}{dt}\left(\vec{B}.\vec{A}\right)$$

$$e = -A\frac{dB}{dt}\cos\theta = -A\cos\theta \times \frac{dB}{dt}$$

$$e = -100\times10^{-4}\times\cos30^0\times\frac{\left(0-10^{-1}\right)}{10^{-4}}$$

$$e = 100\times\frac{\sqrt{3}}{2}\times\frac{1}{10} = 5\sqrt{3}\ V$$

10. Ans. (a) : Change in current, $dI = 7A - 3A = 4A$

Time, dt = 0.04 s

Mutual inductance, M = 0.5 H

The induced emf is given by

$$e = \frac{MdI}{dt} = 0.5\times\frac{4}{0.04} = 50V$$

11. Ans. (d) : Self inductance, L = 108 mH

Number of turns N = 600

Now, N' = 500

Let the new self inductance is L'.

The self inductance of a solenoid is, $L = \frac{\mu_0 N^2 A}{l}$

So,

$$\frac{L'}{L} = \left(\frac{N'}{N}\right)^2 = \left(\frac{500}{600}\right)^2 ;\ L' = 108\times\frac{5^2}{6^2} = 75mH$$

12. Ans. (c) : Current in solenoid = I

When iron rod is inserted in the solenoid, the magnetic field increases, flux linked increases and self inductance also increases. So, only the rate of heating does not change.

13. Ans. (b) Given : $\phi = 5t^2 + 3t + 16$

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}[5t^2+3t+16] = -(10t+3)$$

At t = 4, e = -43V

14. Ans. (b):Out of the four given loops, when circular and elliptical loops come out of the field, equal areas will not trace out in equal interval of time. So, for circular and elliptical loops, induced emf will not remain constant.

15. Ans. (d) : Current induced is $I = \frac{|e|}{R}$

Now, $|e| = \frac{d\phi}{dt}$

But there is no change of flux with time, as $\vec{B}$, $\vec{A}$ and θ all remain constant with time.

$\therefore$ No current is induced.

16. Ans. b) : Mutual inductance of a pair of two coils depends on the relative position and orientation of two coils.

17. Ans. (d) : Magnetic field inside a solenoid,

$$B = \mu_0\frac{N}{l}i$$

Flux linked with N turns

Initial flux, $\phi_1 = NBA = N\mu_0\frac{N}{l}iA$

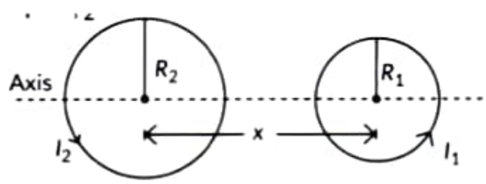
$$= \mu_0\frac{N^2}{l}iA = \frac{4\pi\times10^{-7}\times800\times800\times2.5\times25\times10^{-4}}{0.30}$$

$$= 16.74\times10^{-3}\,Wb \quad \therefore\ Final\ flux,\ \phi_2 = 0$$

Average back emf,

$$|e| = \frac{d\phi}{dt} = \frac{16.74\times10^{-3}-0}{10^{-3}} = 16.74V$$

18. Ans. (d) : Let flux linked with smaller loop is ϕ_1 and with bigger loop is ϕ_2.



Given, $R_2 = 0.2m,\ R_1 = 0.3cm = 0.003m$

$x = 15cm = 0.15m$

Now, $\phi_1 = B_2 A_1 = \frac{\mu_0}{4\pi}\left[\frac{2\pi R_2^2 I_2}{(R_2^2+x^2)^{3/2}}\right]\pi R_1^2$

$$M = \frac{\phi_1}{I_2} = \frac{\mu_0}{4\pi}\frac{2\pi R_2^2 \pi R_1^2}{(R_2^2+x^2)^{3/2}}$$

Now, $\phi_2 = MI_1 = \frac{\mu_0}{4\pi}\frac{2\pi R_2^2\pi R_1^2}{(R_2^2+x^2)^{3/2}}I_1$

On putting the values, we get

$\therefore\ \phi_2 = 9.1\times10^{-11}$ weber

19. Ans. (b) : $L = \mu_0 \dfrac{N^2}{l} A$ ....(i)

$$L' = \mu_0 \frac{(2N)^2}{2l} A \quad \text{............(ii)}$$

On solving (i) and (ii), we get

$$L' = 2\mu_0 \frac{N^2}{l} A = 2L$$

Assertion-Reasoning (1 mark)

20. **Sol.**

(a): The coefficient of self inductance of the coil is given by $L = \dfrac{\mu_0 N^2 A}{l}$

where N is number of turns, l is length of the coil and A is area of coil, so $L \propto N^2$.

21. **Sol.** (c): The coil in the resistance boxes are made from double wire. Due to this current in two wires flows in opposite directions i.e., magnetic flux linked with the each coil cancel each other. Thus, no e.m.f. is induced in the resistance.

22. **Sol.**

(d): Both assertion and reason are false.
When an alternating current is sent through the coil then effective resistance of the coil will be $\sqrt{R^2+(\omega L)^2}$ while it was only R for direct current *i.e.*, effective resistance of coil will increases for A.C.

23. **Sol.**

(b): According to Faraday's law, the induced e.m.f. (ε) is given by $\varepsilon = \dfrac{-d(N\phi)}{dt} = -\dfrac{d(NBA)}{dt} = -NA\dfrac{dB}{dt}$.
Thus the induced e.m.f. depends on the rate of change of magnetic flux, number of turns of coil and area of the coil. If any of these factor increases (or decreases) then induced e.m.f. also increases (or decreases).

24. **Sol.** (a): The inductance coils made of copper will have very small ohmic resistance. Due to change in magnetic flux a large induced current will be produced in such an inductance, which will offer appreciable opposition to the flow of current.

25. **Sol.** (a): Self-inductance of a coil is its property by virtue of which the coil opposes any change in the current flowing through it. It is because, the induced emf produced opposes the change in current. For this reason, self induction is called inertia of electricity.

26. **Sol.** (a): An induced current develop in a conductor cannot moved in a direction parallel to magnetic field. This is because when the conductor moved in a direction parallel to magnetic field, amount of flux linked with the conductor does not change. Thus the induced current develops only when conductor cuts the lines of magnetic force. The direction of flow of induced current can also be found by applying Flemings right hand rule, when the direction of motion of conductor inside the magnetic field and the direction of magnetic field action on it are known.

27. **Sol.** (a): When we pull a copper plate out of the magnetic field or push it into the magnetic field, magnetic flux linked with the plate changes. As a result of this eddy currents are produced in the plate which oppose its motion (according to Lenz's law).

28. **Sol.** (b): E.m.f. induces, when there is change in magnetic flux. The magnitude of induced e.m.f. depends upon the rate at which the magnetic flux changes. When magnetic flux is steady or constant no e.m.f. is induced. Faraday did experiment in which, there is relative motion between the coil and magnet, the flux linked with the coil changes and e.m.f. induces.

29. **Sol.**

(c) A is true but R is false

Explanation: A is true but R is false

➤ Case Study Question

30. Answers

1. (*c*) As the outward flux is increasing, the induced current must flow clockwise so as to produce downward flux. Hence the induced current in the wire flows due south. By Fleming's left hand rule, magnetic force on the wire acts towards west.

2. (*b*) $\mathcal{E} = Blv = 0.01 \times 0.50 \times 4\,V = 0.02$ V.

3. (*a*) $\mathcal{E} = Blv$

$= 4 \times 10^{-5} \times 35 \times 90$

$= 126 \times 10^{-3}$ V $= 0.126$ V.

4. (*b*) $\mathcal{E} = Blv$

$= 6 \times 10^{-5} \times 1 \times \frac{36 \times 1000}{3600}$ V $= 6 \times 10^{-4}$ V.

5. (*d*) Here $L = 0.50$ m, $B = 0.40G = 0.40 \times 10^{-4}$ T

$$f = 120 \frac{\text{rev}}{\text{min}} = \frac{120}{60}\frac{\text{rev}}{\text{sec}} = 2 \text{ rps}$$

Induced emf,

$$\mathcal{E} = B\pi L^2 f = 0.40 \times 10^{-4} \times 3.14 \times (0.50)^2 \times 2$$

$= 6.28 \times 10^{-5}$ V.

31. Answers

6. (*a*) The magnitude of flux linked with the oscillating pendulum changes. Eddy currents are set up in it which damp its oscillations in the magnetic field.

7. (*b*) As the magnet falls, magnetic flux linked with the cylinder changes. Eddy currents set up in the cylinder oppose the motion of the magnet.

8. (*d*) The oscillating coil produces changing magnetic field. This changes the flux through the aluminium plate. Strong eddy currents set up in the plate cause electromagnetic damping of the coil.

9. (*b*) Eddy currents are induced by varying magnetic field in a metal.

10. (*d*) Laminated core reduces losses due to eddy currents.

➤ Questions

32. Soln. Let ON be x at some instant.

The emf induced in the loop = e.

$$e = \frac{d\phi}{dt} = \frac{-d(\text{Blx})}{dt}$$

$$e = Bl\left(\frac{-dx}{dt}\right)$$

$$= Blv = 0.5 \times 0.2 \times 10 \quad \left[\because v = \frac{dx}{dt}\right]$$

= 1 V

Current in the arm,

$$I = \frac{e}{R} = \frac{1}{5} = 0.2A$$

33. Soln. Given:

Energy $W = \frac{1}{2}LI^2$

A solenoid having magnetic field B, area A, & length l and having n numbers of turns per unit length.

Self – inductance of the solenoid is given by:

$$L = \mu_0 n^2 lA$$

$$B = \mu_0 nI$$

$$\therefore \quad W = \frac{1}{2}\mu_0 n^2 lAI^2$$

$$\therefore \quad B^2 = \mu_0^2 n^2 I^2$$

$$V = AL(\text{volume})$$

$$\Rightarrow \quad W = \frac{1}{2\mu_0}\mu_0^2 n^2 lAI^2$$

$$\Rightarrow \quad W = \frac{1}{2\mu_0}B^2 V$$

Energy density $= \frac{W}{V} = \frac{B^2}{2\mu_0}$

34. Ans. (i) We know that, $I = \frac{dq}{dt} \Rightarrow dq = Idt$

$\therefore \; q = \int Idt =$ Area under the I-t curve

$$= \frac{1}{2} \times 0.4 \times 1 = 0.2\text{C}$$

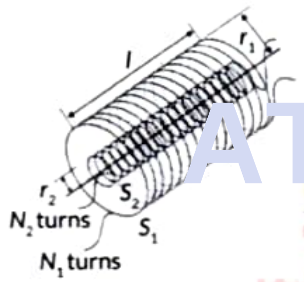
(ii) As we know $|\varepsilon| = \frac{d\phi}{dt} \Rightarrow \frac{dq}{dt} \times R = \frac{d\phi}{dt}$

$\Rightarrow \phi = qR = 0.2 \times 10 = 2Wb$

(iii) Now, $\phi = B.A = B(10 \times 10^{-4})$

$$\Rightarrow B = \frac{2}{10 \times 10^{-4}} = 2000T$$

35. Ans. Mutual Inductance: The phenomenon according to which an opposing emf is produced as a result of change in current or magnetic flux linked with another coil. Consider two long coaxial solenoids each of length l. Let n_1 be the number of turns per unit length of inner solenoid S_1 of radius r_1, n_2 be the number of turns per unit length of outer solenoid S_2 of radius r_2. Let N_1 and N_2 be the total number of turns of solenoids S_1 and S_2 respectively.



When a current i is passed through S_2, the magnetic flux linked with solenoid S_1 is

$N_1\phi_1 = M_{12}i_2$ ....(i)

Where M_{12} is called the mutual inductance of solenoid S_1 with respect to solenoid S_2.

It is also referred as the coefficient of mutual induction.

The magnetic field due to current i in S_2 is

$B_2 = \mu_0 n_2 i$ ....(ii)

$\therefore$ The magnetic flux linked with S_1 is

$N_1\phi_1 = B_2(\pi r_1^2) n_1 l = \mu_0 n_1 n_2 \pi r_1^2 l i_2$ ....(iii)

Where $n_1 l$ is the total number of turns in solenoid S_1.

From (i) and (iii), we get

$M_{12} = \mu_0 n_1 n_2 \pi r_1^2 l$ ....(iv)

Which is required expression.

Similarly, $M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$ ....(v)

From (iv) and (v), we get

$M_{12} = M_{21} = M$

Hence, coefficient of mutual induction between two coaxial solenoid is

$$M = \mu_0 n_1 n_2 \pi r_1^2 l \ or, \ M = \frac{\mu_0 N_1 N_2 \pi r_1^2}{l}$$

36. Ans. (i) The phenomenon of inducing current in a circuit by changing the current or flux in a neighbouring circuit is called mutual induction. S.I. unit of mutual inductance is henry denoted by H.

(ii) Here, M = 1.5 H,

$\Delta I_1 = 20A, \Delta t = 0.5s, \Delta\phi = ?$

We know, emf induced in the second coil,

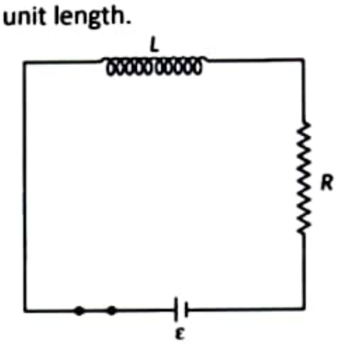
$$\varepsilon = -\frac{(\Delta\phi)_2}{\Delta t} = -\frac{M\Delta I_1}{\Delta t}$$

$$\therefore \quad (\Delta\phi)_2 = M\Delta I_1 = 1.5 \times 20 = 30Wb$$

37. Ans. The phenomena of induced emf in a solenoid due to change in current or magnetic flux linked with the solenoid is called self inductance of the solenoid.

The self inductance of a long solenoid, the core of which consists of a magnetic material of permeability μ is given by $L = \mu l_0 n^2 Al$

Where, A is the area of cross-section of the solenoid, l is the length and n is the number of turns per unit length.



Consider the circuit shown here, consisting of a inductor L and a resistor R, connected to a source of emf e. As the connections are made, the current grows in the circuit and the magnetic field increases in the inductor. Part of the work done by the battery during the process is stored in the inductor as magnetic field energy and the rest appears as thermal energy in the resistor. After sufficient time, the current, and hence the magnetic field, becomes constant and further work done by the battery appears completely as thermal energy. If, I be the current in the circuit at time t, we have

Self induced emf $\varepsilon = L\dfrac{dI}{dt}$

$$dW = \varepsilon I dt$$

$$dW = L\frac{dI}{dt} I dt$$

$$dW = LIdI$$

Work done by source of emf to supply current I for a small time dt.

Now total work done by cell to establish current I_0 in inductor

$$W = \int dW = L\int_0^{I_0} I\,dI = \frac{1}{2}LI_0^2$$

Total work done is stored as magnetic energy in the solenoid.

38. Ans. (i) Induced voltage $V = L\dfrac{di}{dt}$

$$\frac{V_1}{V_2} = \frac{L_1}{L_2}\left(as\ \frac{di}{dt}\ is\ same\right)$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{16}{12} = \frac{4}{3}$$

(ii) Power P = iV

$$\frac{i_1}{i_2} = \frac{V_2}{V_1} = \frac{3}{4}\ (\text{as P is same})$$

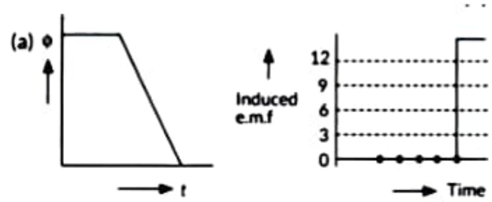
$$\Rightarrow \frac{i_1}{i_2} = \frac{3}{4}$$

(iii) Energy stored $E = \dfrac{1}{2}Li^2$

$$\frac{E_1}{E_2} = \frac{L_1 i_1^2}{L_2 i_2^2} = \frac{16}{12}\times\frac{9}{16} = \frac{3}{4}$$

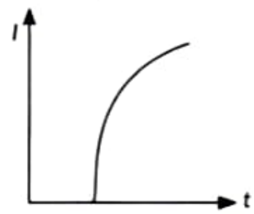
$$\Rightarrow \frac{E_1}{E_2} = \frac{3}{4}$$

39. Ans. (a)

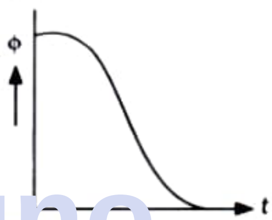


Induced current and power, sketch is same as shown above.

(b) In case of circular coil, rate of change of area of the loop during its passage out of field is not constant, hence induced current varies accordingly.



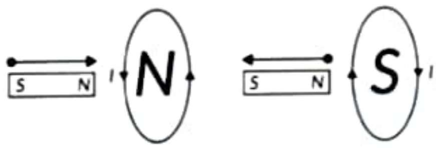
Variation of flux with time (in case of circular loop):



40. Ans. Lenz's law states that the direction of the induced emf and the direction of induced current are such that they oppose the cause which produces them. Here, the North pole is approaching the loop, so the induced current in the face of loop viewed from the left side will flow in such a way that it will behave like North pole, so South pole developed on loop when viewed from right hand side of the loop, The flow of induced current is clockwise, hence A acquires positively polarity and B negative.

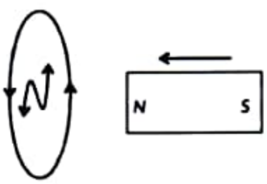
Whenever magnetic flux linked with a circuit changes, it induces an EMF in it. The induced current set up in the circuit flows in such a direction that it opposes the change in magnetic flux linked with the circuit.

In order to continue the change in magnetic flux linked with the circuit, some work is to be done or some energy is to be spent against the opposition offered by induced EMF. This energy spent by the external source ultimately appears in the circuit in the form of electrical energy.



This is why a magnet is moved near the closed loop with its N – pole towards the loop, then current is produced in loop on the side of a magnet in anticlockwise direction so as to develop the north pole which applies repulsive force on magnet opposing motion of magnet towards the loop. Similarly when a magnet is moved away from the closed loop with its N – pole towards the loop, the current is produced in the loop on the side of magnet in clockwise direction, so as to develop the south pole which attracts the bar magnet opposing its motion away from the loop.

41. . Ans. Lenz's law states that the direction of the induced emf and the direction of induced current are such that they oppose the cause which produces them.

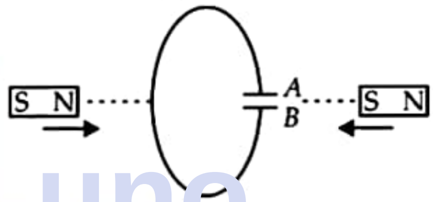


When the N pole of a magnet is moved towards a coil, the induced current in the coil flows in anticlockwise direction on the side of magnet, so as to acquire north polarity and oppose the motion of the magnet towards the coil, by applying repulsive force on it.
In order to continue the change in magnetic flux linked with the circuit, some work is to be done or some energy has to be spent against the opposition offered by induced EMF. This energy spent by the external source ultimately appears in the circuit in the form of electrical energy.
Suppose that the Lenz's law is not valid. Then the induced current flows through the coil in a direction opposite to one dictated by Lenz's law. The resulting force on the magnet makes it move faster and faster, i.e., the magnet gains speed and hence kinetic energy without expanding an equivalent amount of energy. This sets up a perpetual motion machine, violating the law of conservation of energy. Thus Lenz's law is valid and is a consequence of the law of conservation of energy.

42. Soln. Lenz's law states that the direction of the induced emf and the direction of induced current are such that they oppose the cause which produces them.

The rod held along east west direction will fall in a perpendicular magnetic field B_H present in N-S direction. Hence an emf will be induced in the rod following the relation for the motional emf $\varepsilon = B_H vl$.

Predict the polarity of the capacitor in the situation described below:

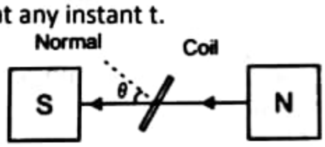


43. Soln.

(a) As the armature coil is rotated in the magnetic field, angle θ between the field and normal t the coil changes continuously. Therefore, magnetic flux linked with the coil changes. An e.m.f. is induced in the coil. According to Fleming's right hand rule, current induced in AB is from A to B and it is from C to D in CD in the external circuit current flows from B_2 to B_1.

To calculate the magnitude of e.m.f. induced:
Suppose,

$A \longrightarrow$ Area of each turn of the coil
$N \longrightarrow$ Number of turns in the coil
$B \longrightarrow$ Strength of magnetic field
$\theta \longrightarrow$ Angle which normal to the coil makes with $\vec{B}$ at any instant t.



$\therefore$ Magnetic flux linked with the coil in this position.

$$\phi = N(\vec{B}.\vec{A}) = NBA\cos\theta = NBA\cos\omega t \quad ....(i)$$

Where, ω is angular velocity of the coil

As the coil rotates, angle θ changes. Therefore, magnetic flux ϕ linked with the coil changes and hence, an e.m.f. is induced in the coil. At this instant t, if e is the e.m.f. induced in the coil, then

$$e = -\frac{d\theta}{dt} = -\frac{d}{dt}(\text{NAB}\cos\omega\text{t})$$

$$= -NAB\frac{d}{dt}(\cos\omega\text{t})$$

$$= -NAB(-\sin\omega t)\omega$$

$$\therefore\ e = NAB\omega\sin\omega t$$

$$e = \varepsilon_0 \sin\omega t\ (\text{Here } \varepsilon_0 = \text{NBA}\,\omega)$$

(b) We have number spokes (N) = 100

Length of each spoke (L) = 0.5 m

Magnetic field (B) = $0.4\times10^{-4}T = 4\times10^{-5}T$

Frequency (f) = 120 rpm = 2 rps

Induced e.m.f. between axle and rim is given by

$$e = N\times B\times l^2\times\pi\times f$$

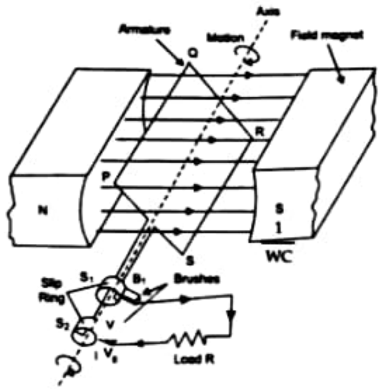
$$= 100\times4\times10^{-5}\times(0.5)^2\times3.14\times2$$

$$= 6.28\times10^{-3}V$$

44. Soln.

(a) Principle of ac generator: The ac generator is based on the principle of electromagnetic induction. When closed coil is rotated in a uniform field with its axis perpendicular to field, then magnetic flux changes and emf is induced.

Working: When the armature coil rotates, the magnetic flux linked with it changes and produces induced current. If initially, coil PQRS is in vertical position and rotated clockwise, then PQ moves down and SR moves up. By Fleming's right hand rule, induced current flows from Q to P and S to R which is the first half rotation of coil. Brush B_1 is positive terminal and B_2 is negative. In second half rotation, PQ moves up and SR moves down. So induced current reverses and the alternating current is produced in this manner by the generator.



Expression for emf : If number of turns in coil = N, cross-section = A, angular speed of rotation = ω, magnetic field $\vec{B}$, then to find emf induced,

Flux through the coil when its normal makes angle θ with the field,

$$\phi = BA\cos\theta$$

When coil rotates with angular velocity ω and turns through θ in time 't' then $\theta = \omega t$

$$\Rightarrow\quad \phi = BA\cos\omega t$$

When coil rotates, ϕ changes to set in induced emf.

$$\varepsilon = \frac{-d\phi}{dt} = -\frac{d}{dt}(\text{BA}\cos\omega\text{t})$$

$$= BA\omega\sin\omega t$$

For N turns, total induced emf,

$$\varepsilon = NBA\omega\sin\omega t$$

(b) Given: Velocity (v)

$$= 900\,km/h = 900\times\frac{5}{18}m/s$$

$$= 250\ ms^{-1}$$

Wing span (l) = 20 m

Horizontal component of earth's field (B_H) = $5\times10^{-4}T$

Angle of dip $(\delta) = 30^0$

Potential difference

$$B_v = B_H\tan\delta$$

$$= 5\times10^{-4}\times\tan30^0$$

$$= 5\times10^{-4}\times\frac{1}{1.732}$$

Emf induced = $B_v lv$

$$= \frac{5\times10^{-4}}{1.732}\times20\times250$$

= 1.44 V

45. Soln. Let ON be x at some instant.
The emf induced in the loop = e.

$$e=\frac{d\phi}{dt}=\frac{-d(\text{Blx})}{dt}$$

$$e=Bl\left(\frac{-dx}{dt}\right)$$

$$=Blv=0.5\times0.2\times10$$

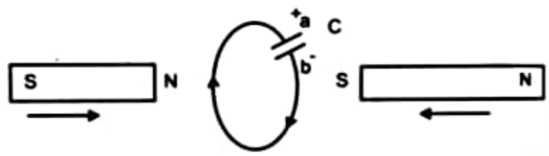
$$\left[\because v=\frac{dx}{dt}\right]$$

= 1 V

Current in the arm,

$$I=\frac{e}{R}=\frac{1}{5}=0.2A$$

46. Soln. In this situation, a will become positive with respect to b, as current induced is in clockwise direction.



47. Soln. From an observer, the direction of current in the solenoid is anti-clockwise. On displacing it towards the loop, current in the loop will get induced in a direction in order to oppose the approach of solenoid, hence direction of induced current as seen by the observer will be clockwise.

48. Soln. Magnetic flux. *The magnetic flux through any surface placed in a magnetic field is the total number of magnetic lines of force crossing this surface normally*. It is measured as the product of the component of the magnetic field normal to the surface and the surface area.

Magnetic flux is a scalar quantity, denoted by $\phi \, or \, \phi_B$.

SI unit of magnetic flux. The SI unit of magnetic flux is weber (Wb). *One weber is the flux produced when a uniform magnetic field of one tesla acts normally over an area of 1 m²*.

1 weber = 1 tesla x 1 metre²

Or 1 Wb = 1 Tm²

49. Soln. When current is increasing magnetic flux linked with the coils also increases. The magnetic field due to the current element in 2 is into the plane and 1 is out of the plane. Since flux increases, the current induced is such that it opposes the cause due to which it is produced thus the induced emf in the loop 1 is clockwise and in coil 2 is anticlockwise.

50. Soln.
(i) Current in the loop PQRS.

$$I=\frac{\varepsilon}{r}$$

Since $\varepsilon=\frac{d\phi}{dt}=Blv$ So, $I=\frac{Blv}{r}$

(ii) The force required to keep the arm PQ in constant motion

$$F=BI\,l=B\left(\frac{Blv}{r}\right)l=\frac{B^2l^2v}{r}$$

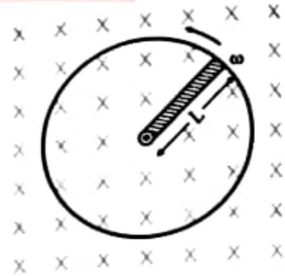
(iii) Power required to move the arm PQ

$$P=|\text{F}|\,|\text{v}|=\left(\frac{B^2l^2v}{r}\right)|\text{v}|=\left(\frac{B^2l^2v^2}{r}\right)$$

51. Soln. Suppose the rod completes one revolution in time T. Then change in flux

$= B\times$ Area swept $= B\times\pi L^2$

Induced emf $=\frac{Change\,in\,flux}{Time}$



Or $\varepsilon=\frac{B\times\pi L^2}{T}=B\pi L^2 f$ $\left[\because T=\frac{1}{f}\right]$

As $f=\frac{\omega}{2\pi}$, therefore

$$\varepsilon=B\pi L^2.\frac{\omega}{2\pi}=\frac{1}{2}BL^2\omega.$$

Induced current,

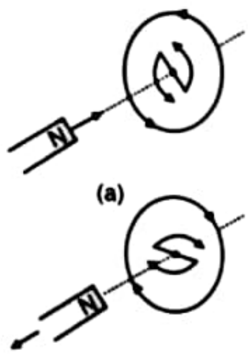
$$I=\frac{\varepsilon}{R}=\frac{1}{2}\frac{BL^2\omega}{R}$$

Heat dissipation in time t,

$$Q=\frac{\varepsilon^2 t}{R}=\frac{1}{4}\frac{B^2L^4\omega^2 t}{R}.$$

52. Soln. Lenz's law: The direction of an induced emf always opposes the change in magnetic flux which produces it.

Explanation:



When the north pole of a bar magnet is pushed towards the close coil, the magnetic flux through coil increases and the current is induced in the coil in such a direction that it opposes the increase in flux. This is possible when the induced current in the coil is in the anticlockwise direction. Just the opposite happens when the north pole is moved away from the coil.

In either case, it is the work done against the force of magnetic repulsion/attraction that gets 'converted' into the induced emf.

53. Soln. The emf induced across the ends of a conductor due to its motion in a magnetic field is called motional emf. As suppose a length x of the loop lies inside the magnetic field at any instant of time t. Then the magnetic flux linked with the rectangular loop PQRS is

$$\phi = BA = Blx$$

According to Faraday's law of electromagnetic induction, the induced emf is

$$\varepsilon=-\frac{d\phi}{dt}=-\frac{d}{dt}(\mathrm{Blx})=-\mathrm{Bl}\frac{dx}{dt}$$

Or $\varepsilon = Blv$

Where $dx/dt=-v$, because the velocity v is in the decreasing direction of x. The induced emf Blv is called motional emf because this emf is induced due to the motion of a conductor in a magnetic field.

54. Soln.

(i) Brightness will decrease when an iron rod is inserted in the solenoid.

(ii) When an iron rod is inserted in the solenoid with a velocity, the iron ad rod will cut the magnetic field lines of an inductor, so as per Kirchoof's and Faraday's Laws, cutting of magnetic field will tend to induce a current inside the Inductor which opposes the direction of it's cause. So when the current is being induced by the moving rod, it opposes the current flow of existing current in the circuit, causing the bulb's brightness to go down as there is less current passing through it.

55. Soln. Self-Inductance is the property by which an opposing induced emf is produced in a coil due to a change in current, or magnetic flux, linked with the coil.

OR

Self – inductance of a coil is numerically equal to the flux linked with the coil when the current through the coil is 1 A.

OR

Self - inductance of coil is equal to the induced emf developed in the coil when the rate of change of current in the coil is one ampere per second.

Energy stored in an inductor:

Consider a source of emf connected to an inductor L. As the current starts growing, the opposing induced emf is given by

$$\varepsilon=-L\frac{dI}{dt}$$

If the source of emf flows a current i through the inductor for a small time dt, then the amount of work done by the source, is given by

$$dW = |\varepsilon| \mathrm{Idt}$$

$$=Li\frac{di}{dt}dt$$

$$=Lidi$$

Hence the total amount of work done (by the source of emf) when the current increases from its initial value $(\mathrm{i}=0)$ to its final value (I) is given by

$$W = \int_0^I Li\,di = L\int_0^I i\,di = L\left[\frac{i^2}{2}\right]_0^I = \frac{1}{2}LI^2$$

This work done gets stored in the inductor in the form of energy.

$$\therefore \quad U = \frac{1}{2}LI^2$$

56. Soln. Given:

Energy $W = \frac{1}{2}LI^2$

A solenoid having magnetic field B, area A, & length l and having n numbers of turns per unit length.

Self – inductance of the solenoid is given by:

$$L = \mu_0 n^2 lA$$

$$B = \mu_0 nI$$

$$\therefore \quad W = \frac{1}{2}\mu_0 n^2 lAI^2$$

$$\therefore \quad B^2 = \mu_0^2 n^2 I^2$$

$$V = AL(\text{volume})$$

$$\Rightarrow \quad W = \frac{1}{2\mu_0}\mu_0^2 n^2 lAI^2$$

$$\Rightarrow \quad W = \frac{1}{2\mu_0}B^2 V$$

Energy density = $\frac{W}{V} = \frac{B^2}{2\mu_0}$

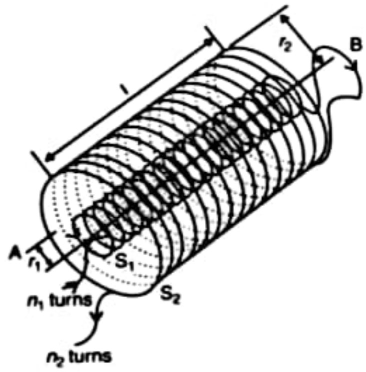
57. Soln.

(i) Mutual inductance, is numerically equal to the induced emf in the secondary coil when the current in the primary coil changes by unity.

Alternatively,

Mutually inductance in numerically equal to the magnetic flux linked with one coil/secondary coil when unit current flows through the other coil/primary coil.

(ii)



Let a current I_2, flow in the secondary coil

$$\therefore \quad B_2 = \frac{\mu_0 N_2 i_2}{l}$$

$\therefore$ Flux linked with the primary coil

$$= \frac{\mu_0 N_2 N_1 A_1 i_2}{l}$$

$$= M_{12} i_2$$

Hence, $M_{12} = \frac{\mu_0 N_2 N_1 A_2}{l}$

$$= \mu_0 n_2 n_1 A_1 l$$

$$\left(n_1 = \frac{N_1}{l}; n_2 = \frac{N_2}{l}\right)$$

58. Soln.

(i) The phenomenon of production of induced emf in a coil due to change in current in neighbouring coil.

(ii) Given:

Mutual inductance of a pair of coils, $\mu = 1.5H$

Initial current, $I_1 = 0A$

Final current $I_2 = 20A$

Change in current will be:

$$\Delta I = I_2 - I_1$$

$$20 - 0 = 20A$$

And we know,

$$\Delta\phi = 1.5 \times 20$$

$$= 30Wb$$

Hence, change in the flux linkage will be 30 Wb.

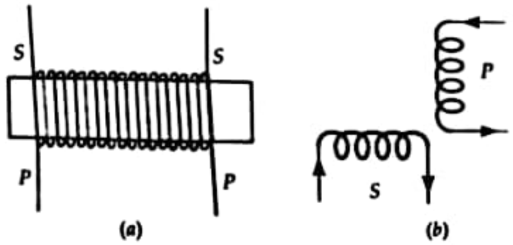
59. Soln.

1. Number of turns. Larger the number of turns in the two solenoids, larger will be their mutual inductance.

$M \propto N_1 N_2$

2. Common cross-sectional area. Larger the common cross-sectional area of two solenoid, larger will be their mutual inductance.

3. Relative separation. Larger the distance between two solenoids, smaller will be the magnetic flux linked with the secondary coil due to current in the primary coil. Hence smaller will be the value of M.
4. Relative orientation of the two coils. M is maximum when the entire flux of the primary is linked with the secondary, i.e., when the primary coil completely envelopes the secondary coil. M is minimum when the two coils are perpendicular to each other, as shown in fig.



(a) M is maximum when primary envelopes secondary, (b) M is minimum when primary is perpendicular to secondary.

5. Permeability of the core material. It the two coils are wound over an iron core of relative permeability μ_r, their mutual inductance increases μ_r times.

Coefficient of coupling. *The coefficient of coupling of two coils gives a measure of the manner in which the two coils are couple together. If L_1 and L_2 are the self – inductances of two coils and M is their mutual inductance, then their coefficient of coupling is given by*

$$K = \frac{M}{\sqrt{L_1 L_2}}$$

The value of K lies between 0 and 1.

When the coupling is perfect i.e., the entire flux of primary is linked with the secondary, M is maximum and K = 1.

When there is no coupling, M = 0 and K = 0.

60. Soln. Eddy currents. Currents can be induced, not only in conducting coils, but also in conducting sheets or blocks. Whenever the magnetic flux linked with a metal sheet or block changes, an emf is induced in it. The induced currents flow in closed paths in planes perpendicular to the lines of force throughout the body of the metal. These currents look like eddies or whirl – pools in water and so they are known as eddy currents. As these currents were first discovered by Focault in 1895, so eddy currents are also known as Focault currents.

Applications

1. Induction furnace. If a metal specimen is placed in a rapidly changing magnetic field (produced by high frequency a.c.), very large eddy currents are set up. The heat produced is sufficient to even melt the metal. This process is used in the extraction of some metals from their ores.
2. Electromagnetic damping. When a current is passed through a galvanometer, its coil suffers few oscillations before coming to rest in the final position. As the coil moves in the magnetic field, induced current is set in the coil which opposes its motion. The oscillations of the coil are damped. The electromagnetic damping can be further increased by winding the coil on a light copper or aluminium frame. As the frame moves in the magnetic field, eddy currents are set up in the frame which resist the motion of the col. This is how a galvanometer is rendered dead beat, i.e., the coil does not oscillate – It deflects and stays in the final position immediately.

61. Soln. l = 30 cm = 0.30 m,

A = 25 cm^2 = 25 x 10^{-4} m^2, N = 500, dt = 10^{-3}s,

$$dI = 0 - 2.5 = -2.5A$$

$$\text{Back emf} = -L\frac{dI}{dt} = -\frac{\mu_0 N^2 A}{l}\cdot\frac{dI}{dt}$$

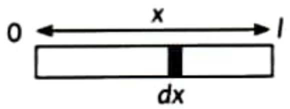
$$= -\frac{4\pi \times 10^{-7} \times (500)^2 \times 2.5 \times 10^{-4} \times (-2.5)}{0.30 \times 10^{-3}}$$

= 6.542 V.

62. Soln.

Self – Inductance	Mutual
1. Self – Inductance is the phenomenon of production of induced emf in a coil when a changing current passes through it.	Mutual induction is the phenomenon of production of induced emf in one coil due to a change of current in the neighboring coil.
2. Self – inductance depends upon the size, shape and the number of turns of the col. Larger the number of turns and area of cross-section, larger is the self – inductance.	The mutual inductance of two coils depends on the number of turns in the two coils, their geometrical shape and their relative separation.

63. Ans. Induced EMF, $\varepsilon = (\vec{v}\times\vec{B})l = vBl$



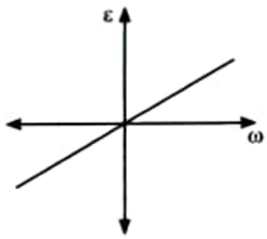
For the element dx, induced EMF,

$d\varepsilon = Bvdx = B\omega x dx$

$\therefore \quad \varepsilon = \int d\varepsilon = \int_0^l B\omega x\, dx = \frac{B\omega l^2}{2}$

$\Rightarrow \quad \varepsilon \propto \omega$ and $\varepsilon \propto l^2$

(i)



(ii)

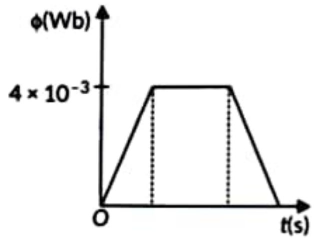
64. Ans. Given l = 20 cm = 0.2 m,

B = 0.1 T, v = 10 cm s^{-1} = 0.1 m s^{-1}

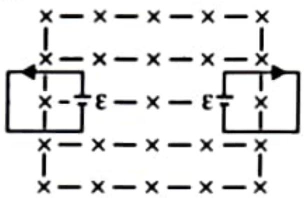
(i) Magnetic flux through loop

$\phi = B.A = Blx$

$\phi_{max} = 0.1\times0.2\times0.2 = 0.004\,Wb = 4\times10^{-3}\,Wb$



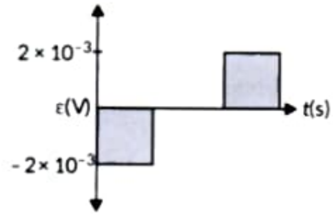
(ii) Induced emf,



$\varepsilon = \frac{-d\phi}{dt} = -Blv$

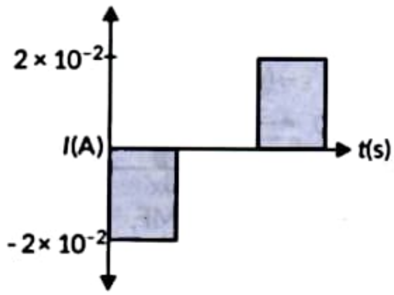
$|\varepsilon|_{max} = 0.1\times0.2\times0.1$

$= 0.002\,V = 2\times10^{-3}\,V$



(iii) Induced current,

$I = \frac{|\varepsilon|}{R} = \frac{2\times10^{-3}}{0.1} = 2\times10^{-2}\,A$

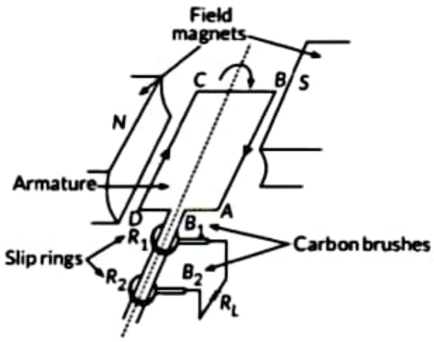


65. Ans. (i) Given r_1 = 1 cm = 1 x 10^{-2} m and r_2 = 20 cm = 20 x 10^{-2} m Mutual inductance of two concentric and coplanar coils is given by, $M = \frac{\mu_0 N_1 N_2 \pi r_1^2}{2r_2}$

$= \frac{4\pi\times10^{-7}\times1\times1\times\pi(1\times10^{-2})^2}{2(20\times10^{-2})} = \pi^2\times10^{-10}\,H$

(ii) The induced emf,

66. Ans. a) Principle : AC generator is based on the principle of electromagnetic induction. It converts mechanical energy into electrical energy.



It consists of

(i) Armature coil of large number of turns of copper wire wound over soft iron core. Soft iron core is used to increase magnetic flux.

(ii) Field magnets used to apply magnetic field, in which armature coil is rotated with its axis perpendicular to field lines.

(iii) Slip rings used to provide movable contact of armature coil with external circuit containing load.

(iv) Brushes which are the metallic pieces used to pass on electric current from armature coil to the external circuit containing load.

When armature is rotated in the magnetic field, due to change in orientation of the coil magnetic flux through it changes. Due top change in flux an e.m.f. is induced.

$$\varepsilon = -N\frac{d\phi}{dt}$$

$$\varepsilon = NBA\omega \sin\omega t \qquad [\because \phi = BA\cos\omega t]$$

$$i = \frac{\varepsilon}{R} = \frac{NBA\omega}{R}\sin\omega t$$

Direction of induced current is given by Fleming's right hand rule.

(b) Radius of coil, r = 10 cm = 0.1 m

Area, $A = \pi r^2 = 3.14\times(0.1)^2 = 0.0314\,m^2$

Number of turns, N = 20

Angular speed, $\omega = 50\,rad\,s^{-1}$

Magnetic field, B = 3.0 x 10^{-2} T

(i) Maximum induced emf $\varepsilon_{max} = NAB\omega$

$$= 20\times0.0314\times3.0\times10^{-2}\times50 = 0.942\,V$$

Average induced emf,

$$\varepsilon = \frac{\varepsilon_{max}+\varepsilon_{min}}{2} = \frac{0.942+0}{2} = 0.471\,V$$

(ii) Given, R = 10 Ω

Maximum current in the coil,

$$I_0 = \frac{\varepsilon_0}{R} = \frac{0.942}{10} = 0.0942\,A$$

Average current,

$$I = \frac{\varepsilon}{R} = \frac{0.471}{10} = 0.0471\,A$$

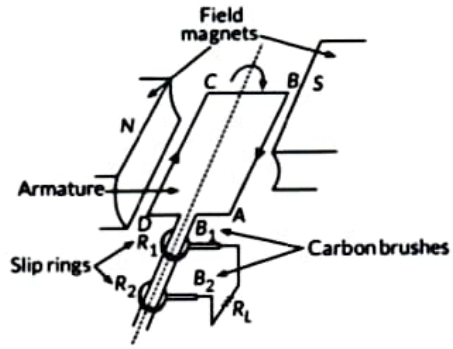
Now average power loss, $P = I^2R$

$$= (0.0471)^2\times10 = 0.022\,W$$

$$|\varepsilon| = -M\left|\frac{dI}{dt}\right| = \pi^2\times10^{-10}\times5\times10^3$$

$$= 4.93\times10^{-6}\,V = 4.93\,\mu V$$

67. Ans. (a) : Principle : AC generator is based on the principle of electromagnetic induction. It converts mechanical energy into electrical energy.



It consists of

(i) Armature coil of large number of turns of copper wire wound over soft iron core. Soft iron core is used to increase magnetic flux,

(ii) Field magnets used to apply magnetic field, in which armature coil is rotated with its axi perpendicular to field lines,

(iii) Slip rings used to provide movable contact of armature coil with external circuit containing load,

(iv) Brushes which are the metallic pieces used to pass on electric current from armature coil to the external circuit containing load. When armature is rotated in the magnetic field, due to change in orientation of the coil magnetic flux through it changes. Due to change in flux an e.m.f. is induced.

$$\varepsilon = -N\frac{d\phi}{dt}$$

$$\varepsilon = NBA\omega\sin\omega t \qquad [\because \phi = BA\cos\omega t]$$

$$i = \frac{\varepsilon}{R} = \frac{NBA\omega}{R}\sin\omega t$$

Direction of induced current is given by Fleming's right hand rule.

(b) Here, A = 200 cm² = 2 x 10^{-2} m²

$$N = 20,\ \omega = 50\,rad\,s^{-1},\ B = 3\times10^{-2}\,T,\ I_{max} = ?$$

Maximum emf induced in the coil,

$$\varepsilon_0 = NBA\omega = 20\times3\times10^{-2}\times2\times10^{-2}\times50 = 0.6V$$

If R is the resistance of the coil, the maximum value of the current is,

$$I_{max} = \frac{\varepsilon_0}{R} = \frac{0.6V}{R}$$

SURE SHOT QUESTIONS

ARVIND ACADEMY

Chapter – 07

Alternating Current

➢ MCQ (1 mark)

1. Soln. (b): Here, $I_{rms} = 5\,A,\ \nu = 50\,Hz,\ t = \frac{1}{300}\,s$

$$I_0 = \sqrt{2}\,I_{rms} = 5\sqrt{2}\,A,$$

$$I = I_0 \sin \omega t = I_0 \sin 2\pi \nu t = 5\sqrt{2} \sin\left(2\pi \times 50 \times \frac{1}{300}\right)$$

$$= 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2}\,\frac{\sqrt{3}}{2} = 5\sqrt{\frac{3}{2}}\,A$$

2. Soln. (c): For maximum power to be delivered from the generator to the load, the total reactance must vanish. i.e., $X_L + X_g = 0$ or $X_L = -X_g$.

3. Soln. (c): The voltmeter connected to ac mains is calibrated to read root mean square value or virtual value of ac voltage.

4. Soln. (b): Resonant frequency in a series LCR circuit is

$$\nu_r = \frac{1}{2\pi\sqrt{LC}}$$

If capacitance C increases the resonant frequency will reduce, which can be achieved by adding another capacitor in parallel to the first.

5. Soln. (c): For better tuning of an LCR circuit used for communication the circuit should possess high quality factor of resonance.

i.e., $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ should be high.

For it R should be low, L should be high and C should be low, therefore combination in option (c) is correct.

6. Soln. (c): Here, $X_L = 1\Omega,\ R = 2\Omega,\ V_{rms} = 6V$

Impedance of the circuit

$$Z = \sqrt{X_L^2 + R^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{5}\,\Omega$$

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{6}{\sqrt{5}}\,A$$

Power dissipated

$$P = V_{rms} I_{rms} \cos\phi = V_{rms} I_{rms} \frac{R}{Z}$$

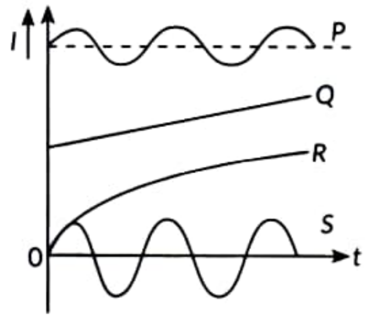
$$= 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} = \frac{72}{5} = 14.4\,W$$

7. Soln. (a): Here, $V_s = 24V,\ P_s = 12W$

$$I_s = \frac{P_s}{V_s} = \frac{12}{24} = 0.5\,A;\ I_m = \sqrt{2}\,I_s = \sqrt{2} \times 0.5 = \frac{1}{\sqrt{2}}\,A$$

8. Ans. (d) There are four devices connected with different sources.

In alternating current, the current changes its magnitudes sinusoidally with time and passing through origin.
Here, it is clear that Q and R are not alternating current flow.



Here, S – device shows the variation of current with time and P – device's current does not passing through origin.
Hence, option (d) is correct.

9. Ans. (a) : Here, $I_{rms} = 15A,\ f = 50\,Hz,\ t = \frac{1}{600}s$

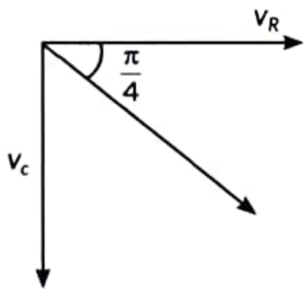
The equation of current is

$I = I_0 \sin(\omega t) = I_{rms}\sqrt{2}\sin(2\pi f t)$

So, $I = 15\sqrt{2}\sin\left(2\times\pi\times 50\times\frac{1}{600}\right)$

$I = 15\sqrt{2}\sin\left(\frac{\pi}{6}\right) = \frac{15\sqrt{2}}{2} \Rightarrow I = \frac{15}{\sqrt{2}}A$

10. Ans. (d) Given, voltage, $v = v_0 \sin\omega t$ is applied to a series combination of a resistor R and an element X.



Also, instantaneous current in the circuit,

$I = I_0 \sin\left(\omega t + \frac{\pi}{4}\right)$

Since, current is leading the voltage with phase difference $\frac{\pi}{4}$. So, capacitor must be present in the given circuit.

Thus, the circuit must be RC.

$\because\ \tan\frac{\pi}{4} = \frac{v_c}{v_r}$

Or $v_c = v_R$ or $X_c = R$

$Z = \sqrt{R^2 + X_c^2}$

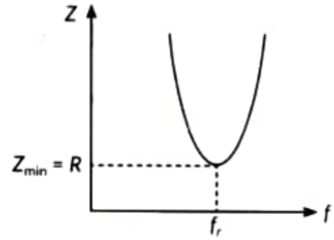
$Z = \sqrt{R^2 + R^2}$

$Z = \sqrt{2}R$

Hence, X is a capacitor and $X_c = R$. So, option (d) is correct.

11. Ans. (a) : Impedance (Z) versus frequency (f) is shown below:

$$Z = \sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}$$



12. Ans. (a) : $X_L = \omega L,\ X_C = \frac{1}{\omega C}$

$\frac{X_L}{X_C} = \frac{\omega L\times\omega C}{1} = \omega^2 LC$

13. Ans. (c) : Given, the phase difference, is $\phi = \frac{\pi}{2}$

(between source voltage and alternating current)

Phase difference of $\pi/2$ is possible when L or C is present.

So, L and R cannot be present.

14. Ans.(d)

15. Ans. (c) : Voltage across resistor, $V_R = 20V$

Voltage across inductor, $V_L = 15V$

Voltage across capacitor, $V_C = 30V$

The resultant voltage is given by

$V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{20^2 + (15-30)^2} = 25V$

16. Ans. (a) : In an a.c. circuit containing L, R and C, the impedance is

$Z = \sqrt{R^2 + (X_L - X_C)^2};\ X_L = 2\pi f L,\ X_C = \frac{1}{2\pi f C}$

We know, $I = \frac{V}{Z}$.

When frequency increases, the current first increases and then decreases.

17. Ans. (b) : Here, resistance, $R = 15\Omega$, inductance, L = 80 mH, capacitance = C, frequency, f = 50 Hz

When, the current and voltage are in same phase,

So, $X_L = X_C$

$$2\pi fL = \frac{1}{2\pi fC}$$

$$C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\times(3.14)^2\times 50^2\times 80\times 10^{-3}}$$

$$C = 1.27\times 10^{-4} F = 127\,\mu F$$

18. Ans. (b) : Here, resistance, $R = 300\Omega$,

Capacitance, $C = \frac{25}{\pi}\mu F$

$$X_C = \frac{1}{2\pi vC} = \frac{1}{2\pi\times 50\times\frac{25}{\pi}}\times 10^6 = 400F$$

$$Z = \sqrt{R^2 + X_C^2}$$

$$Z = \sqrt{300^2 + 400^2} = 500\Omega$$

$$Current,\ I = \frac{V}{Z} = \frac{200}{500} = 0.4A$$

19. Ans. (c) : Selectivity depends on the quality factor,

$$Q = \frac{\omega_0 L}{R}.$$

20. Ans. (b) : Three electric bulbs of power 200 W, 100 W and 50 W connected in series.

$\because$ We know that, $P \propto \frac{V^2}{R}$ or $R \propto \frac{1}{P}$

As we know that, $P = i^2 R$. In a series circuit, the current remain same for all three electric bulbs, but resistance is maximum for 50 W bulbs.

Resistance of bulbs = 50 W > 100 W > 200 W.

In a series circuit potential difference across each bulb will be different.

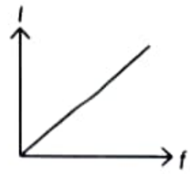
It is not correct explanation of the assertion.

Hence, option (b) is correct.

21. Ans. (a) : Power factor, $\cos\phi = \frac{R}{Z} = 1$.

22. Ans. (d) : The reduce the eddy currents, the core of a transformer is laminated.

23. Ans. (c) : In pure capacitor, $I = \frac{V}{X_C}$



$$\therefore\ I = \frac{V}{\frac{1}{2\pi fC}} = V\times 2\pi fC$$

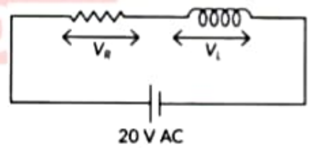
$$\therefore\ I \propto f$$

24. Ans. (b) : When the current passes through the inductor, magnetic field is produced and it opposes the current flow. When the iron coil is removed, self – inductance of the coil decreases i.e., the current flow will be increase and hence brightness of the bulb will increase.

25. Ans. (a) : $V_{rms} = \frac{50\sqrt{2}}{\sqrt{2}} = 50$

$$V_2 = \sqrt{V_{rms}^2 - V_1^2} = \sqrt{(50)^2 - (40)^2} = 30V$$

26. Ans. (a) :



V_R = effective voltage across R

$\therefore\ V_R = I_{eff}\times R$

V_L = effective voltage across L

$V_L = I_{eff}\times L$

So, $V = \sqrt{V_R^2 + V_L^2} = \sqrt{I_{eff}^2 R^2 + I_{eff}^2 L^2}$

$$20 = \sqrt{(12)^2 + V_L^2}\ or\ (20)^2 = (12)^2 + V_L^2$$

$$\Rightarrow 400 = 144 + V_L^2 \Rightarrow V_L = \sqrt{400 - 144}$$

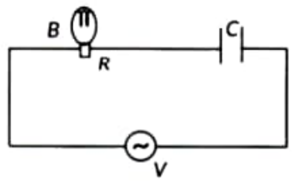
$$= \sqrt{256} = 16\text{V}$$

27. Ans. (d) : As, $E = E_0 \sin\omega t$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{3}\right)$$

As I can lead the voltage in RC and LCR circuit, so it can be RC or LCR circuit.

28. Ans. (b) : $X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}$



When angular frequency ω increased, then impedance of circuit decreases and current increases. So, the bulb will glow brighter.

29. Ans. (c) : Here : $V_0 = 283V,\ f = 50\,Hz, R = 3\Omega$

$L = 25.48\ mH$

$C = 796\,\mu F,\ P|_{at\ resonance} = ?$

Power dissipated, $P = I^2R$

$$I = \frac{I_0}{\sqrt{2}} = \frac{1}{\sqrt{2}}\left(\frac{283}{3}\right) = 66.7\ A$$

$$P = I^2R = (66.7)^2 \times 3 = 13.35\ kW$$

30. Ans. (d) : As primary coil made of thick copper wire has very low resistivity. Therefore power loss is negligible. Rest all options are reasons for power losses in a transformer.

➢ Assertion-Reasoning (1 mark)

31. Ans. (d) : As,

$$E = E_0 \sin \omega t,\ I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right);\ \phi = \frac{\pi}{2}$$

32.

Sol.

(b): The phase angle for the LCR series circuit is given by $\tan\theta = \frac{X_L - X_C}{R} = \frac{\omega L - 1/\omega C}{R}$

where X_L, X_C are inductive reactance and capacitive reactance respectively. When $X_L > X_C$ then $\tan\theta$ is positive *i.e.* θ is positive (between 0 and $\pi/2$). Hence emf leads the current.

33.

Sol.

(b): At resonance $X_L = X_C$ or $\omega L = \frac{1}{\omega C}$. Because of this impedance of LCR series circuit become equal to resistance of circuit $\left(Z = \sqrt{R^2 + (X_L - X_C)^2}\right)$. Therefore from $I = \frac{E}{Z} = \frac{E}{R}$, at resonance, current in LCR series circuit is maximum. Correspondingly phase angle is also equal to zero. Therefore emf and current are in phase in LCR series circuit.

34. **Sol.**

(c): The transmission is done at high voltage due to which current through the wire is reduced. By reduction in current corresponding dissipation of energy is also reduced (as $H \propto I^2R$). If transmission is done at low voltage then we have to use thick wire in order to reduce the dissipation of energy. This increase the cost of transmission lines wires. In order to reduce both energy dissipation and cost of transmission wire, transmission is done at high voltage by using step-up transformers.

35. **Sol.**

(a): Capacitive reactance $X_C = \frac{1}{\omega C}$. When capacitance C increases, the capacitive reactance decreases. Due to decrease in its values, the current in the circuit will increase $\left(I = \frac{E}{\sqrt{R^2 + X_C^2}}\right)$ and hence brightness of source (or electric lamp) will also increases.

36. **Sol.** (a): Transformer works on the principle of mutual induction i.e., if two coils are inductively coupled and when current or magnetic flux is changed through one of the two coils, then induced e.m.f. is produced in the other coil. So whenever there is change in current or magnetic flux, only then e.m.f. is induced. But in case of D.C. current or voltage, e.m.f. is not induced because it remains constant throughout and never changes its direction and magnitude. Therefore, transformer cannot work when D.C. is applied.

37. **Sol.**

(c): When pure inductor is connected to source of an alternating emf, then instantaneous value of alternating emf is given by $E = E_0 \sin\omega t$, and corresponding alternating current is given by $I = I_0 \sin(\omega t - \pi/2)$. From these two equations, it follows that alternating current lags behind e.m.f. by a phase angle of $\pi/2$. The inductive reactance, $X_L = \omega L = 2\pi f L$, so when frequency increases correspondingly inductive reactance also increases.

38. Sol.

(a): The capacitive reactance of capacitor is given by

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}.$$

So this is infinite for D.C. ($f = 0$) and has a finite value for A.C. Therefore a capacitor blocks D.C. and offers an easy path for A.C.

➤ Case Study Question

39. Ans. (i) (d) : Step – down transformer decreases the AC voltage.

(ii) (a) : $\because \dfrac{N_S}{N_p} = \dfrac{E_S}{E_p}$

i.e., number of turns of secondary coil are more than number of turns in primary coil then voltage is increased or stepped-up in secondary coil. So, it is called step-up transformer.

(iii) (a) : Current is reduced of voltage is stepped – up so corresponding I^2R losses are cut down.

(iv) (c) : Given,

$E_i = 2300V,\ E_0 = 230V,\ N_p = 4000,\ N_S = ?$

$$\frac{E_i}{E_o} = \frac{N_p}{N_s} \Rightarrow \frac{2300}{230} = \frac{4000}{x}$$

$\Rightarrow x = 400 = N_S$ = number of turns in the secondary coil

40. **Answers**

6. (*b*) Frequency,

$$f = \frac{c}{\lambda} = \frac{3\times10^8}{360} = \frac{1}{12}\times10^7 \text{ Hz}$$

Required inductance,

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2\left(\frac{1}{12}\times10^7\right)^2\times1.2\times10^{-6}}$$

$$= 3.07\times10^{-8}\text{ H}.$$

7. (*b*) $Q = \dfrac{\omega L}{R}$.

8. (*c*) $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{3}\times\sqrt{\dfrac{1}{9}} = \dfrac{1}{9}$.

9. (*c*) Clearly, Q can be increased by decreasing R.

10. (*b*) Time constant, $\tau = \dfrac{L}{R}$

$\therefore$ R/L has the dimension of frequency.

Questions

41. Ans. $V = V_0 \sin \omega t,\ C = \dfrac{q}{V}$

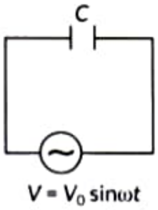
$$q = CV_0 \sin \omega t,\ i = \frac{dq}{dt} = \frac{d}{dt}(CV_0 \sin \omega t)$$

$$= \omega C V_0 \cos \omega t$$

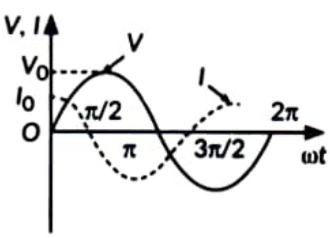
$$= \frac{V_0}{\frac{1}{\omega C}} \cos \omega t$$

$$i = \frac{V_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$or\ \ i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$



In pure capacitive circuit current leads voltage by $\frac{\pi}{2}$.



42. Ans. For the RC circuit,

Impedance, $Z = \sqrt{R^2 + (1/\omega C)^2}$

Current, $I = \dfrac{\varepsilon_0}{Z}$ ....(i)

(i) When a dielectric slab is introduced between the plates of the capacitor, then its capacitance increases. Hence, from equation (i), impedance of the circuit is decreased and the current through it is increased. So, brightness of the bulb will increase.

(ii) When the resistance R is increased and capacitance is same, then from equation (i), impedance of the circuit is increased and the current flowing through it is decreased. So, brightness of the bulb will decrease.

43. Ans. (i) $L = 50 \times 10^{-3}$ H, $C = 80 \times 10^{-6}$ F, $R = 40\Omega$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50\times10^{-3}\times80\times10^{-6}}}$$

$$\omega = \frac{10^3}{2} = 500\, rad\, s^{-1} \Rightarrow \nu = \frac{500}{2\pi} = 80\, Hz$$

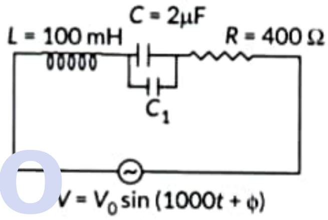
(ii)

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{40}\sqrt{\frac{50\times10^{-3}}{80\times10^{-6}}} = \frac{1}{40}\times\sqrt{625} = 0.625$$

44. Ans. Given L

$$V = V_0 \sin(1000t + \phi), R = 400\Omega,\ L = 100\,\text{mH},$$

$$C = 2\mu F$$



The standard equation is given as

$$V = V_0 \sin(\omega t + \phi)\ \therefore\ \omega = 1000$$

$$X_L = \omega L = 1000 \times 100 \times 10^{-3} = 10^2 = 100\Omega$$

$$\therefore\ X_C = \frac{1}{\omega C} = \frac{1}{1000\times2\times10^{-6}} = 500\Omega$$

Phase difference between the current and the voltage in the series LCR circuit is given as,

$$\phi = \tan^{-1}\frac{X_C - X_L}{R}$$

$$\therefore\ \phi = \tan^{-1}\left(\frac{500-100}{400}\right) = \tan^{-1}(1)$$

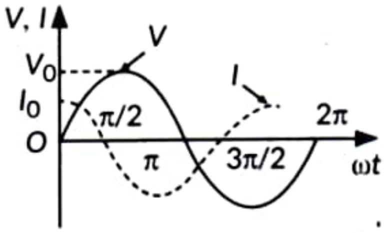
$$\phi = 45^0$$

Since, $X_C > X_L$, therefore current leads in phase.

45. Ans. (a) Here device X is a capacitor.

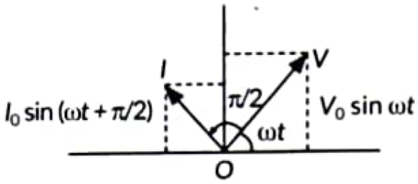
Capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi\nu C}$

(b) In pure capacitive circuit current leads voltage by $\frac{\pi}{2}$.



(c) The capacitive reactance varies inversely with the frequency. As ν increases, X decreases. Graph shows the variation of X_C with ν.

(d)



46. Ans. (a) Device X is a capacitor.

(b) $B \rightarrow$ Voltage (Because it is sine wave)

$C \rightarrow$ Current (Because current leads voltage by $\pi/2$)

$A \rightarrow$ Power (Average power over one cycle is zero)

(c) The capacitive reactance varies inversely with the frequency. As ν increases, X decreases. Graph shows the variation of X_C with ν.

(d) $V = V_0 \sin \omega t$

$C = \frac{q}{V}$

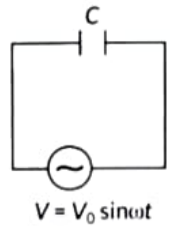
$q = CV_0 \sin \omega t$

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV_0 \sin \omega t)$$

$$= \omega CV_0 \cos \omega t$$

$$= \frac{V_0}{\frac{1}{\omega C}} \cos \omega t$$

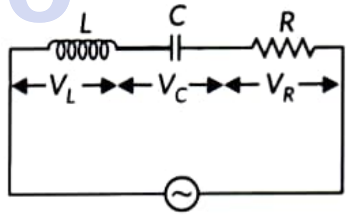
$$i = \frac{V_0}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right) or\ i = i_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$



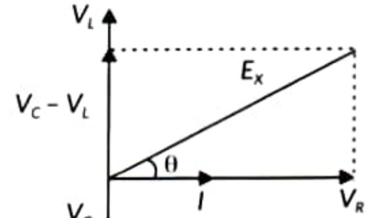
In pure capacitive circuit current leads voltage by $\frac{\pi}{2}$.

47. Ans. (a) AC circuit containing inductor, capacitor and resistor in series [Series LCR circuit]

If I is the current in the circuit containing inductor of inductance L, capacitor of capacitance C and resistor of resistance R in series, then the voltage drop across inductor is $V_L = I \times X_L$



Which leads current I by phase angle of $\pi/2$, and voltage drop across the capacitor is $V_C = I \times X_C$



Phasor diagram

Which lags behind current I by phase angle of $\pi/2$, and voltage drop across the resistor is $V_R = IR$

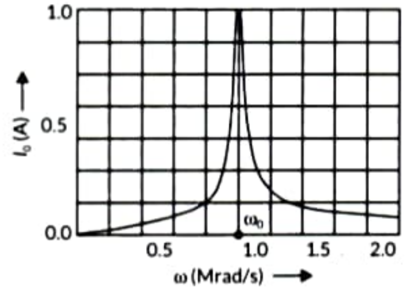
Which is in phase with current I. So the net voltage E, across the circuit is

$E=\sqrt{V_R^2+(V_L-V_C)^2}$

or $E=I\sqrt{R^2+(X_L-X_C)^2}$ or $E=IZ$

Where Z is the effective resistance offered by ac circuit containing inductor, capacitor and resistor in series, known as impedance in series LCR circuit. Hence in series LCR circuit, phase difference ϕ between the current I and the voltage E is

$$\tan\phi=\frac{X_L-X_C}{R}=\frac{\omega L-\frac{1}{\omega C}}{R}$$



With increase in ω, current first increases (upto ω_0) and then decreases.

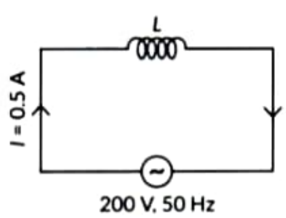
(b) At resonance, $X_L=X_C$

$\therefore\ \tan\phi=\frac{X_L-X_C}{R}=0$

$\therefore\ \phi=0^0$

$\therefore$ There is no phase difference between voltage across inductor and capacitor at resonance in the LCR circuit.

(a) Whenever an inductor is connected to an a.c. source then it produces inductive reactance as impedance, that reduces the amount of current flowing through it. When inductor is connected flow in a circuit is 1 A and when in same inductor is connected to a.c. source, current will be reduced so, we can say that power consumption is more in case of d.c. circuit.



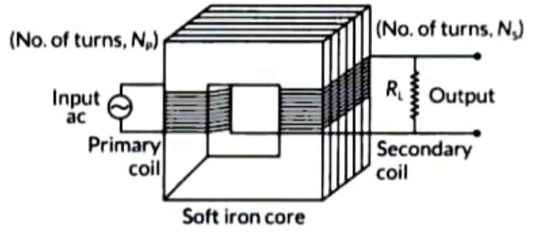
Here, $I=0.5A$, $V=200V$, $\nu=50\,Hz$

$\therefore$ Inductive reactance, $X_L=\omega L=2\pi\nu L$

Also, $I=\frac{V}{X_L}$ or $0.5=\frac{200}{2\times3.14\times50\times L}$

$\Rightarrow L=\frac{200}{0.5\times2\times3.14\times50}=1.27H$

48. Ans. Step – up transformer (or transformer) is based on the principle of mutual induction.



An alternating potential (V_p) when applied to the primary coil induced an emf in it.

$\varepsilon_p=-N_p\frac{d\phi}{dt}$

If resistance of primary coil is low $V_p=\varepsilon_p$.

i.e., $V_p=-N_p\frac{d\phi}{dt}$

as same flux is linked with the secondary coil with the help of soft iron core due to mutual induction, emf is induced in it.

$\varepsilon_S=-N_S\frac{d\phi}{dt}$

If output circuit is open $V_s=\varepsilon_s$

$V_S=-N_S\frac{d\phi}{dt}$

Thus $\frac{V_S}{V_P}=\frac{N_S}{N_P}$

For an ideal transformer, $P_{out}=P_{in}$

$\Rightarrow I_SV_S=I_PV_P$

$\therefore\ \frac{V_S}{V_P}=\frac{I_P}{I_s}=\frac{N_S}{N_P}$

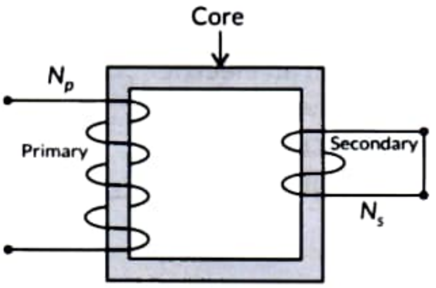
For step – up transformer, $\frac{N_S}{N_P}>1$

In case of dc voltage, flux does not change. Thus no emf is induced in the circuit.

(i) The core of the transformer is laminated to reduce eddy current losses.

Thick copper wire is used in winding of transformers because of its low resistivity i.e., low resistance.

49. Ans. (a) Step down transformer (or transformer):



Principle: When the current flowing through the primary coil changes, an emf is induced in the secondary coil due to the change in magnetic flux linked with it i.e., it works on the principle of mutual induction.

There are number of energy losses in a transformer,

(i) Copper losses due to Joule's heating produced across the resistances of primary and secondary coils. It can be reduced by using copper wires.

(ii) Hysteresis losses due to repeated magnetization and demagnetization of the core of transformer. It is minimized by using soft iron core, as area of hysteresis loop for soft iron is small and hence energy loss also becomes small,

(iii) Iron losses due to eddy currents produced in soft iron core. It is minimized by using laminated iron core.

(iv) Flux losses due to flux leakage or incomplete flux linkage and can be minimised by proper coupling of primary and secondary coils.

(b) Power required, P = 1200 kW = 1200 x 10^3 W

Total resistance of two wire lines,

$R = 2 \times 20 \times 0.5 = 20\Omega$

$E_v = 4000\, volt$

As, $P = E_V I_V \quad \therefore 1200 \times 10^3 = 4000 \times I_v$

$$\Rightarrow I_V = \frac{1200 \times 10^3}{4000} = 300A$$

Where I_V is the rms value of current.

Line power loss in the form of heat is,

$= (I_P)^2 \times$ Resistance of wire line

$= (300)^2 \times 20 = 1800\, kW$

50. Soln. (i) On increasing capacitance, current will increases. It also increases the brightness of bulb.

(ii) There will no flow of current and hence bulb will not glow.

51. Soln. Since the reactance of an inductor is zero for d.c. circuit. Therefore, the current decreases for the same inductor when it is connected with an a.c. source.

When inductor is connected in a.c. circuit:

$V = 200V$

$f = 50\, Hz$

$i = 0.5\, A$

$$i = \frac{V}{R} = \frac{V}{X_L} = \frac{V}{\omega L}$$

$$0.5 = \frac{200}{2\pi \times 50\, L} \qquad [\because \omega = 2\pi f]$$

$$L = \frac{200}{100 \times 0.5 \times 3.14}$$

$$= \frac{4}{3.14} = 1.27 \text{ H}$$

52. Soln. Length of wire line = 20 x 2 = 40 km

Resistance of wire line, r = 40 x 0.5 = 20Ω

Power to be supplied = 1200 kW = 1200 x 10^3 Ω

Voltage at which power supplied = 4000 V

Since, $P = VI$

$$\Rightarrow I = \frac{P}{V}$$

$$\Rightarrow I = \frac{1200 \times 10^3}{4000}$$

= 300 A

Therefore, line power loss

$= I^2 \times R$

$= (300)^2 \times 200$

$= 1800000\, w$

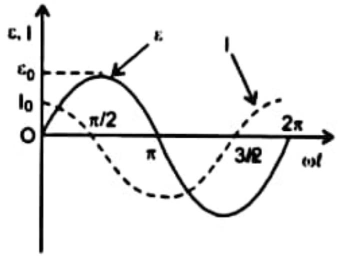
$\therefore$ Line power loss in the form of heart is 18000 Kw

53. Soln. $V = V_0 \sin \omega t;\ I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.

(a) Since current leads the voltage by $\frac{\pi}{2}$ radians, X is a capacitor.

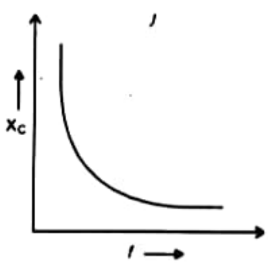
Capacitive reactance, $X_C = \frac{1}{2\pi fC}$ where, ω = angular velocity, C = capacitance.

(b) .

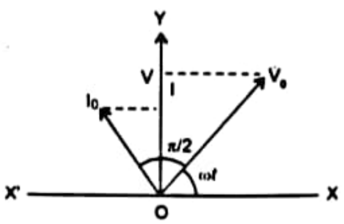


(c) Capacitive reactance varies inversely with frequency as $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

$X_c \times \frac{1}{f}$



(d)



54. Soln.

(a) Given:

$L = 1000 \times 10^{-3} H$

$C = 2 \times 10^{-6} F$

$R = 400\Omega$

$\omega = 1000$

Now, $X_L = \omega L$

$X_L = (1000 \times 100 \times 10^{-3})\Omega$

$X_L = 100\Omega$

And $X_C = \frac{1}{\omega C}$

$= \frac{1}{1000 \times 2 \times 10^{-6}}$

$= \frac{10^6}{2 \times 10^3}$

$X_C = \frac{1000}{2} = 500\Omega$

Here $X_C > X_L$ so, the current will lead.

$\tan\phi = \frac{X_L - X_C}{R}$

$= \frac{100 - 500}{400}$

$\tan\phi = -\frac{400}{400}$

$\tan\phi = -1$

$\Rightarrow$ Phase ongle $\phi = -45^0$

(b) For unity power factor,

$X_L = X_C$

$\omega L = \frac{1}{\omega C_{eq}}$

$C_{eq} = \frac{1}{l\omega^2}$

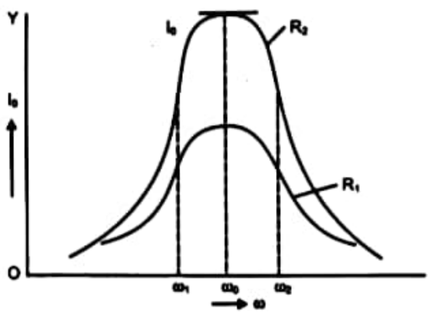
$C_{eq} = \frac{1}{(1000)^2 \times 100 \times 10^{-3}} = 10\mu F$

Now, $C_{eq} = C + C_1$

$10 = 2 + C_1$

$C_1 = 8\mu F$

55. Soln. Figure shows the variation of i_m with ω in a LCR series circuit for two values of Resistance R_1 and R_2 ($R_1 > R_2$),



The condition for resonance in the LCR circuit is,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

We can observe that the current amplitude is maximum at the resonant frequency ω_0. Since $i_m = V_m / R$ at resonance, the current amplitude for case R_2 is sharper to that for case R_1.

Quality factor or simply the Q-factor of a resonant LCR circuit is defined as the ratio of voltage drop across the capacitor (or inductor) to that of applied voltage.

It is given by $Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

The Q factor determines the sharpness of the resonance curve and if the resonance is less sharp, the maximum current decreases and also the circuit is close to the resonance for a larger range $\Delta\omega$ of frequencies and the regulation of the circuit will not be good. So, less sharp the resonance, less is the selectivity of the circuit while higher is the Q, sharper is the resonance curve and lesser will be the loss in energy of the circuit and circuit will be more selective.

56. Soln. $\cos\phi = \frac{1}{2} \Rightarrow \phi = \cos^{-1}\left(\frac{1}{2}\right) = \cos 60^0 = \frac{\pi}{3}$

$\therefore$ Phase difference between current and voltage = $\frac{\pi}{3}$.

57. Soln. Root mean square or virtual or effective value of a.c. It is defined as that value of a direct current which produces the same heating effect in a given resistor as is produced by the given alternating current when passed for the same time. It is denoted by I_{rms}, I_v or by I_{eff}.

Relation between the effective and peak value of a.c. Suppose an alternating current $I = I_0 \sin\omega t$ be passed through a circuit of resistance R. Then the amount of heat produced in small time dt will be

$$dH = I^2 R dt$$

If T is the time period of a.c., then heat produced in one complete cycle will be

$$H = \int_0^T I^2 R dt$$

Let I_{eff} be the effective value of a.c. Then heat produced in time T must be

$$H = I_{eff}^2 RT$$

$$\therefore \quad I_{eff}^2 RT = \int_0^T I^2 R\, dt \quad or \quad I_{eff}^2 = \frac{1}{T}\int_0^T I^2 dt$$

But $\frac{1}{T}\int_0^T I^2 dt$ is the mean of the squares of the instantaneous values of a.c. over one complete cycle, hence the effective or virtual value of a.c. equals its root mean square value, i.e.,

$$I_{eff} = I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2 dt}$$

Now $\int_0^T I^2 dt = \int_0^T I_0^2 \sin^2\omega t \; dt$

$$= I_0^2 \int_0^T \frac{1-\cos 2\omega t}{2} dt$$

$$= \frac{I_0^2}{2}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{I_0^2}{2}\left[(T-0) - \frac{1}{2\omega}\left|\sin\frac{4\pi}{T}t\right|_0^T\right]$$

$$= \frac{I_0^2}{2}\left[T - \frac{1}{2\omega}(\sin 4\pi - \sin 0)\right]$$

$$= \frac{I_0^2}{2}[T-0] = \frac{I_0^2 T}{2}$$

$$\therefore \qquad I_{eff} \text{ or } I_{rms} = \sqrt{\frac{1}{T} \cdot \frac{I_0^2 T}{2}}$$

Or $$I_{eff} \text{ or } I_{rms} = \frac{1}{\sqrt{2}} I_0 = 0.707\, I_0$$

Thus the effective or rms value of an a.c. is $\frac{1}{\sqrt{2}}$ times its peak value.

58. Soln. Any two of the following (or any other correct) reactions:

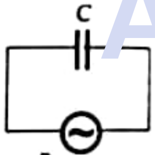
(i) ac can be transmitted with much lower energy losses as compared to dc.

(ii) ac voltage can be adjusted (stepped-up or stepped down) as per requirement.

(iii) ac current in a circuit can be controlled using (almost) wattless devices like choke coil.

(iv) ac is easier to generate.

59. Soln. A.C. circuit containing only a capacitor. As shown in fig. consider a pure capacitor C connected across a source of alternating emf ∈ given by



$$\varepsilon = \varepsilon_0 \sin \omega t$$

.................(1)

Due to the continuous charging and discharging of the capacitor plates, a continuous but alternating current exists in the circuit.

At any instant,

P.D. across the capacitor plates= Applied emf

i.e., $V = \varepsilon = \varepsilon_0 \sin \omega t$

$\therefore$ Current at any instant is

$$I = \frac{dQ}{dt} = \frac{d}{dt}(C\,\varepsilon_0 \sin \omega t) = \omega C\, \varepsilon_0 \cos \omega t$$

Or $$I = I_0 \cos \omega t = I_0 \sin(\omega t + \pi/2)$$

........(2)

Where $I_0 = \omega C \varepsilon_0 = \frac{\varepsilon_0}{1/\omega C}$ = the current amplitude.

Phase relationship between ε and I. On comparing equations (1) and (2), we find that in a capacitive a.c. circuit, the current leads the voltage or the voltage lags behind the current in phase by $\pi/2$ radian. The phase relationship between ε and I is shown graphically in fig. We see that the current reaches its maximum value earlier than the voltage by one-fourth of a period.

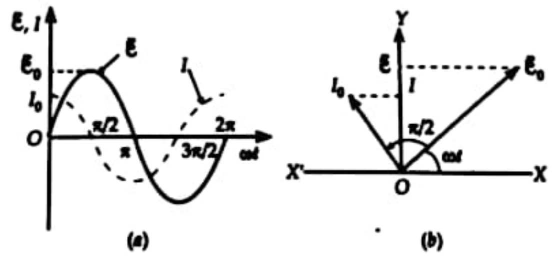
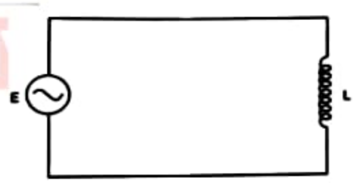


Fig. shows the phasor diagram for a capacitive a.c. circuit. The phasor $\vec{\varepsilon}$ makes an angle ωt with X-axis in anticlockwise direction. As the current leads the emf in phase by $\pi/2$ rad, so the current phasor $\vec{I}$ makes an angle $\pi/2$ rad with phasor $\vec{\varepsilon}$ in anticlockwise direction.

60. Soln. (i)



Induced emf $e = -L\frac{dI}{dt}$

Hence, Net voltage in the circuit = $V - L\frac{dI}{dt}$

According to Kirchoff's Rule

$$V - L\frac{dI}{dt} = 0$$

$$V_m \sin \omega t = L\frac{dI}{dt}$$

$$dI = \frac{V_m}{L}\sin \omega t\ dt$$

$$I = -\frac{V_m}{\omega L}\cos \omega t$$

$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\therefore \quad i = i_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Hence current lags by $\frac{\pi}{2}$

(ii) Inductance of the inductor = 100 mH

Average power dissipation,

$$P = V_{rms} I_{rms} \cos\phi$$

$$= 10 \times 1 \times \cos\frac{\pi}{4}$$

$$= \frac{10}{\sqrt{2}} W$$

$$= 5\sqrt{2}\ Watts\ (7.05\ \text{W})$$

61. Soln. The instantaneous induced emf in an inductor when current changes through it

$$e = -L\frac{dI}{dt}$$

Hence, instantaneous applied voltage

$$e = V = L\frac{dI}{dt}$$

Work done $dW = V.dq = VI\ dt$

$$\therefore \quad dW = LI\ dI$$

$$\Rightarrow \quad \int dW = \int_0^I LI\ dI$$

$$W = \frac{1}{2}LI^2$$

Energy density, $(u) = \dfrac{total\ energy\ stored}{volume}$

$$u = \frac{\left(\frac{1}{2}\right)LI^2}{Al} = \frac{\frac{1}{2}(\text{LI})\text{I}}{Al}$$

Flux = NBA = LI

And $\quad B = \frac{\mu_0 NI}{l} \Rightarrow I = \frac{Bl}{\mu_0 N}$

$$\therefore \quad u = \frac{\frac{1}{2}(\text{NBA}).\frac{Bl}{\mu_0 N}}{Al} = \frac{B^2}{2\mu_0}$$

62. Soln. (i) The device X is a capacitor

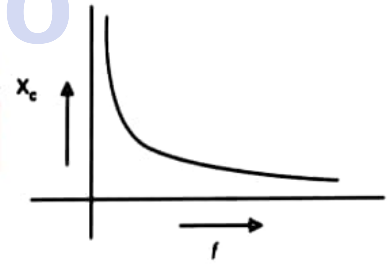
(ii) Curve $B \longrightarrow voltage$

Curve $C \longrightarrow current$

Curve $A \longrightarrow power$

Reason: The current leads the voltage in phase, by $\frac{\pi}{2}$, for a capacitor.

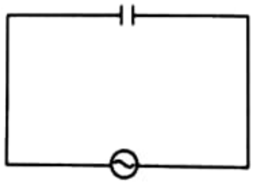
(iii) $X_c = \frac{1}{\omega C}\left(X_c \propto \frac{1}{\omega}\right)$



(iv) $V = V_0 \sin \omega t$

$$Q = VC = CV_0 \sin \omega t$$

$$I = \frac{dq}{dt} = \omega C V_0 \cos \omega t$$



$$V = V_0 \sin \omega t$$

$$= I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Current leads the voltage, in phase, by $\frac{\pi}{2}$

63. Soln. (i) A, Power's frequency is double of current or Voltage frequency.

(ii) From the graph, the phase difference between V and I is $\frac{\pi}{2}$ so device 'X' may be an inductor (L) or capacitor (C) or series combination of L and C, but since the average power is zero so the device has to capacitor.

64. Soln. (i) When a dielectric slab is introduced in between the plates of capacitor, its capacitance will increase which tends to decrease the potential drop across the capacitor, i.e., $V = \frac{Q}{C}$. As a result of this potential drop across bulb will increase as they are connected in series, so the brightness of the bulb will increase.

(ii) As the resistance R increases, potential drop across the resistor will also increase which lead to decrease in potential drop across the bulb, since it is connected in series. Hence, the brightness of the bulb will decrease.

65. Soln. For the applied voltage

$V = 70.7\ \sin(1000t)$, we have

$V = 70.7\ volts$

$\omega = 1000\, S^{-1}$

For the inductor

$$i_0 = \frac{V_0}{\omega L} = \frac{70.7}{1000\times 200\times 10^{-3}} A$$
$$= 35.35\times 10^{-2} A$$
$$= 0.3535\,\text{A}$$

∴ Expression for current is

$$i = (0.3535)\sin\left(1000t - \frac{\pi}{2}\right)$$

For the capacitor

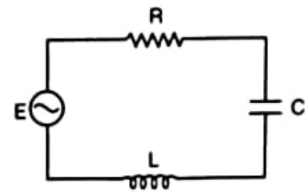
$$i_0 = \frac{V_0}{\left(\frac{1}{\omega C}\right)} = V_0.\omega C$$

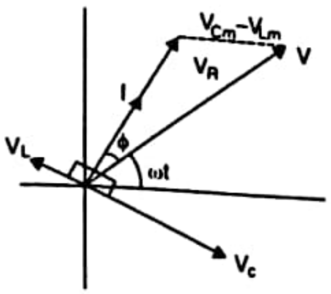
$$= 70.7\times 1000\times 5\times 10^{-6} A$$
$$= 353.5\times 10^{-3} A = 0.3535\, A$$

∴ Expression for current is

$$I = 0.3535\ \sin\left(1000t + \frac{\pi}{2}\right)$$

66. Soln. (i) In a series LCR circuit shown,



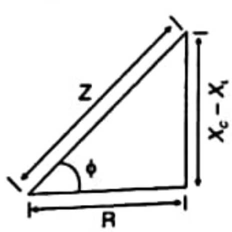


From the phasor relation, voltages $V_L + V_R + V_C = V$, as V_C and V_L are along the same line and in opposite directions, so they will combine in single phasor $(V_C + V_L)$ having magnitude $|V_{Cm} - V_{Lm}|$. Since voltage V is shown as hypotenuse of right triangle with sides as V_R and $(V_C + V_L)$, so the Pythagorean Theorem results as:

$$V_m^2 = V_m^2 + (V_{Cm} - V_{Lm})^2$$
$$V_m^2 = (I_m R)^2 + (I_m X_C - I_m X_L)^2$$
$$V_m^2 = I_m^2(R^2 + (X_C - X_L)^2)$$

Now current in the circuit:

$$I_m = \frac{V_m}{\sqrt{(R^2 + (X_C - X_L)^2}}$$
$$I_m = \frac{V_m}{Z}\ as\ Z = \sqrt{\left[R^2 + (X_C - X_L)\right]^2}$$



As phasor I is always parallel to phasor V_R, the phase angle ϕ is the angle between V_R and V and can be determine from figure.

$$\tan\phi = \frac{V_{Cm} - V_{Lm}}{V_{Rm}}$$

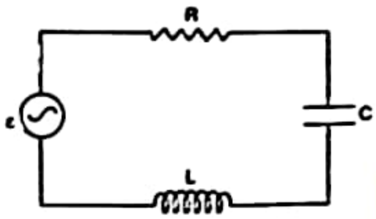
$$\tan\phi = \frac{X_C - X_L}{R}$$

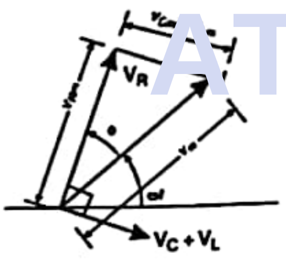
(ii) Resonance Frequency

Frequencies at which the response amplitude is relative maximum are known as system's resonant frequencies. It is shown as:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

67. Soln. (i)





From figure,

$$\vec{V} = \vec{V}_L + \vec{V}_R + \vec{V}_C$$

Where $|\vec{V}_R| = i_m R$

$$|\vec{V}_L + \vec{V}_C| = V_{Cm} - V_{Lm}$$

$$= i_m(X_C - X_L)$$

$$\Rightarrow \quad V_m^2 = V_{Rm}^2 + (V_{Cm} - V_{Lm})^2$$

$$I_m^2 Z^2 = I_m^2 R^2 + i_m^2 (X_C - X_L)^2$$

$$\Rightarrow \quad Z = \sqrt{R^2 + (X_C - X_L)^2}$$

From figure

$$\tan\phi = \frac{V_{Cm} - V_{Lm}}{V_{Rm}} = \frac{i_m(X_C - X_L)}{i_m R}$$

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

Condition for current and voltage are in phase:

$$V_L = V_C \ or \ X_L = X_C$$

Circuit is called Resonant circuit.

(ii) Power factor

$$P_1 = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + R^2}}$$

$$= \frac{1}{\sqrt{2}} \text{ (as } X_L = R)$$

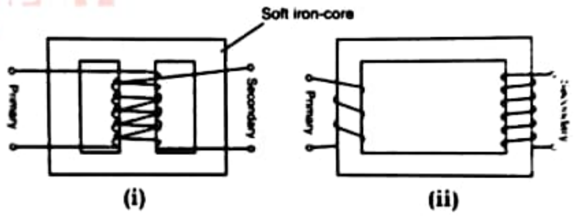
Power factor when capacitor C of Reactance $X_C = X_L$ is put in series in the circuit

$$P_2 = \frac{R}{Z} = \frac{R}{R} = 1$$

As Z = R at resonance

$$\therefore \frac{P_1}{P_2} = \frac{\frac{1}{\sqrt{2}}}{1} = \frac{1}{\sqrt{2}}$$

68. Soln.



Working principle

Whenever current in one coil changes an emf gets induced in the neighbouring coil

(Principle of mutual induction)

Voltage across secondary

$$V_s = e_s = -N_s \frac{d\phi}{dt}$$

Voltage across primary

$$V_p = e_p = -N_p \frac{d\phi}{dt}$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \qquad (\text{here } N_s > N_p)$$

In an Ideal transformer

Power Input = Power output

$$I_p V_p = I_s V_s$$

$$\therefore \qquad \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$$

(ii) Input power,

$$P_i = I_i \times V_i = 15 \times 100 = 1500 W$$

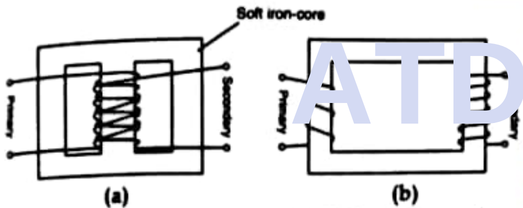
Power output

$$P_0 = P_i \times \frac{90}{100} = 1350 \text{ W}$$

$$\Rightarrow \qquad I_0 V_0 = 1350 \ W$$

Output voltage, $V_0 = \frac{1350}{3} V = 450$ V

69. Soln.



(Any one of the above diagram)

(i) Conversion of ac of low voltage into ac of high voltage & vice versa.

Working Principle:

Mutual induction: When alternating voltage is applied to primary windings, emf is induced in the secondary windings.

Energy losses:

- Leakage of magnetic flux
- Eddy currents
- Hysteresis loss
- Copper loss (Any two)

(ii) $N_p = 100$

Transformation ratios = 100

(a) Number of turns in secondary coil
$N_S = 100 \times 100 = 10000$

(b) Input Power = Input voltage x current in primary

1100 = 220 x I_p

$$\Rightarrow \qquad I_p = 5A$$

(c) $\frac{V_S}{V_P} = \frac{N_S}{N_P}$

$$\frac{V_S}{220} = 100$$

$$\Rightarrow \qquad V_S = 2.2 \times 10^4 \ volts$$

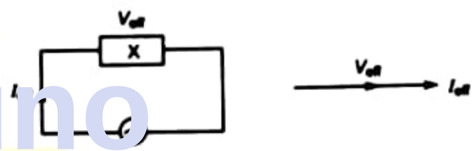
(d) $\frac{I_P}{I_S} = \frac{N_S}{N_P}$

$$\frac{5}{I_S} = 100$$

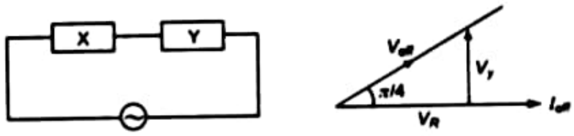
$$\Rightarrow \qquad I_s = \frac{5}{100} = 0.05A$$

(e) Power in secondary = Power in Primary = 1100 W

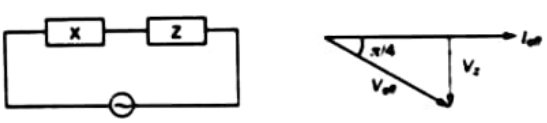
70. Soln.



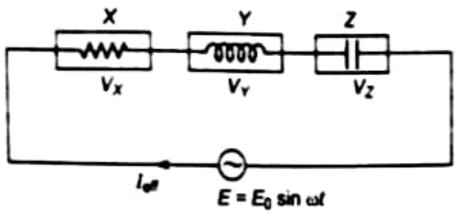
Since the phase angle between the current I_{eff} and voltage drop V_{eff} is zero. Hence, the element X is a resistor.



The voltage drop across the combination X and Y is ahead of the current flow. Hence, the element Y is ac inductor.



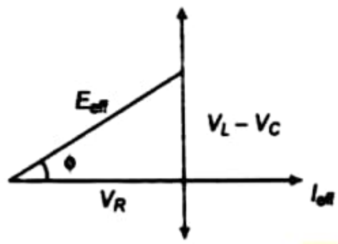
The voltage drop across the combination X and Z lags behind the current flow. Hence, the element Z is a capacitor.



Let I_{eff} be the current flows through each element X, Y and Z. The voltage V_X, V_Y and V_Z drop across the elements. However,

$$V_X + V_Y + V_Z > E_{eff}.$$

It is not possible. So, on plotting phasor diagram, we have



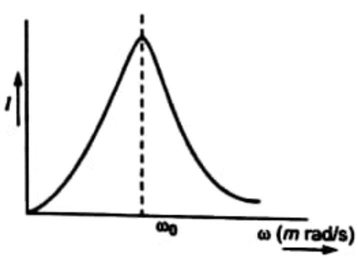
$$|E_{eff}|^2 = v_R^2 + (V_L - V_C)^2$$

$$E_{eff} = I_{eff}\sqrt{R^2 + (X_L - X_C)^2}$$ ....(1)

Since $E_{eff} = I_{eff} Z$ ....(2)

The impedance of the circuit can be given

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



The flow of the current in series LCR circuit varies as a function of frequency of a.c. source is shown in figure.

Current amplitude is maximum at resonant frequency ω_0 : and can be given as $I_{max} = \frac{V_{eff}}{R}$.

So, we can not have resonance in a R.L. or R.C. circuits.

71. Soln. Given V = 230 V, v = 50 Hz, L = 80 mH = 80 x 10^{-3} H, C = 60 $\mu F = 60 \times 10^{-6} F$

(a) Inductive reactance $X_L = \omega L = 2\pi v L$

$$= 2 \times 3.14 \times 50 \times 80 \times 10^{-3} = 25.1\Omega$$

Capacitive reactance

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C} = \frac{1}{2 \times 3.14 \times 50 \times 60 \times 10^{-6}} = 53.1\Omega$$

Impedance, Z = Net reactance =

$$\left|\frac{1}{\omega C} - \omega L\right| = 53.1 - 25.1 = 28.0\Omega$$

Current amplitude

$$I_0 = \frac{V_0}{Z} = \frac{V\sqrt{2}}{Z} = \frac{230 \times 1.41}{28.0} = 11.6\ A$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{11.6}{1.41} = 8.23\,\text{A}$$

(b) RMS value of potential drops across L and C are

$$V_L = X_L\, I_{rms} = 25.1 \times 8.23 = 207\,\text{V}$$

$$V_C = X_C\, I_{rms} = 53.1 \times 8.23 = 437\,\text{V}$$

Net voltage = $V_C - V_L = 230\,\text{V}$

(c) The voltage across L leads the current by angle $\frac{\pi}{2}$, therefore, average power

$$P_{av} = V_{rms}\, I_{rms} \cos\frac{\pi}{2} = 0(\text{zero}).$$

(d) The voltage across C lags behind the current by angle $\frac{\pi}{2}$,

$$\therefore \quad P_{av} = V_{rms}\, I_{rms} \cos\frac{\pi}{2} = 0$$

(e) As circuit contains pure L and pure C, average power consumed by LC circuit is zero.

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SURE SHOT QUESTIONS

ARVIND ACADEMY

Chapter – 08

Electromagnetic Waves

MCQ (1 mark)

1. Soln. (c): Here, E = 11 eV = 11 x 1.6 x 10^{-19} J = hv

$$\therefore \ v = \frac{11\times1.6\times10^{-19}}{h} = \frac{11\times1.6\times10^{-19}}{6.62\times10^{-34}} = 2.6\times10^{15}\ Hz$$

This frequency belongs to ultraviolet region.

2. Soln. (b): As the wall is perfectly reflecting, there is no change in amplitude E_0.

Also the wall is optically inactive, so, there is no phase change.

After reflection, the wave travels along -ve z direction,

$$\therefore \ \vec{E}_r = E_0 \hat{i}\cos(-kz-\omega t) = E_0 \hat{i}\cos(kz+\omega t)$$

$$(\because \ \cos(-\theta) = \cos\theta)$$

3. Soln. (b): For complete absorption, $p = \frac{U}{c}$

Where U = Energy carried by light = IAt

$$\therefore \qquad p = \frac{IAt}{c}$$

Here,
$I = 20W/cm^2, A = 30cm^2, t = 30\min = 1800s$

So, $p = \frac{20\times30\times1800}{3\times10^8} = 36\times10^{-4}$ kg m s^{-1}

4. Soln. (a): Electric field intensity on a surface due to the incident radiation is

$$E = \frac{U}{At} = \frac{P}{A} \qquad \left(\because \frac{U}{t} = P\right)$$

As, $\quad E \propto P \quad$ (for the given area of the surface)

$$\therefore \ \frac{E'}{E} = \frac{P'}{P} = \frac{50}{100} = \frac{1}{2} \ \therefore \quad E' = \frac{E}{2}$$

5. Soln. (d): The direction of propagation of electromagnetic wave is perpendicular to both electric field vector $\vec{E}$ and magnetic field vector $\vec{B}$ i.e., in the direction of $\vec{E}\times\vec{B}$.

6. Soln. (c): Intensity of electromagnetic wave, $I = U_{av}c$

In terms of electric field, $U_{av} = \frac{1}{2}\varepsilon_0 E_0^2$

In terms of magnetic field, $U_{av} = \frac{1}{2}\frac{B_0^2}{\mu_0}$

Now U_{av} (due to electric field) = $\frac{1}{2}\varepsilon_0 E_0^2$

$$= \frac{1}{2}\varepsilon_0 (cB_0)^2 = \frac{1}{2}\varepsilon_0 \times \frac{1}{\mu_0\varepsilon_0}B_0^2 \quad \left(\because \frac{E_0}{B_0} = c\right)$$

$$= \frac{1}{2}\frac{B_0^2}{\mu_0} = U_{av} \qquad \text{(due to magnetic field)}$$

Therefore, the ratio of contributions by the electric field and magnetic field components to the intensity of electromagnetic wave is 1 : 1.

7. Soln. (c): From a dipole antenna, the electromagnetic waves are radiated outwards. The amplitude of electric field vector E_0 which transports significant energy from the source falls off inversely as the distance r from the antenna i.e., $E_0 \propto \frac{1}{r}$.

8. Ans. (a) : Displacement current exist only when electric field is changing.

9. Ans. (c) : The oscillating electric and magnetic field vectors in electromagnetic wave are perpendicular to each other and in the same phase.

10. Ans. (d) : Infrared waves are also known as heat waves because they raise the temperature of the object on which they fall.

11. Ans. (a) : The wavelength of gamma rays is least among all.

12. Ans. (c) : Welder wears special glasses to protect his eyes from harmful effect of ultraviolet rays.

13. Ans. (a) : X-rays are used as diagnostic tool in medicine.

14. Ans. (d) : Electromagnetic waves are the combination of mutually perpendicular electric and magnetic fields.

So, option (d) is not correct.

➢ Assertion-Reasoning (1 mark)

15. **Sol.** (a): Accelerated charges radiate electromagnetic waves.

16. **Sol.** 16. (b): Electromagnetic waves transport linear momentum as well as energy. When electromagnetic waves strike a surface, a pressure is exerted on the surface. If the intensity of wave is 7, the radiation pressure P (force per unit area) exerted on the perfectly absorbing surface is $P = I/c$.

17. **Sol.**

(d): The speed of em waves in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

where $\mu_0 = 4\pi \times 10^{-7}$ N s^2/C^2 is permeability constant of vacuum and $\varepsilon_0 = 8.85419 \times 10^{-12}$ $C^2/N\ m^2$ is the permittivity of free space. After substituting these value, the value of c (= 2.99792×10^8 m/s) which is same as the speed of light in vacuum. From this it is concluded that light is an electromagnetic wave.

18. **Sol.** (b): In case of a linearly polarized plane electromagnetic wave, the average values of electric field and magnetic field are equal and average values of electric energy and magnetic energy are also equal.

19. **Sol.** (a): Light being electromagnetic wave do not require any material medium for its propagation. Hence light can travel in vacuum. On the other hand sound is a mechanical wave and requires a material medium for its propagation. Hence sound cannot travel in vacuum.

20. **Sol.** (b): Microwaves are the electromagnetic waves of wavelength of the order of a few millimetres, which is less than those of T.V. signals. On account of smaller wavelength, the microwaves can be transmitted as beam signals in a particular direction and are much better than radiowaves because microwaves do not spread or bend around the corners of any obstacle coming in their way. Therefore, microwaves are better carriers of signals than radiowaves.

21. **Sol.** (d): Velocity of light has different values in different media. It depends on the refractive index of the medium.
Related by formula

$$v_{medium} = \frac{velocity\ in\ vacuum}{refractive\ index\ of\ medium}$$

22. **Sol.** (d): All electromagnetic waves including X-rays travels with same velocity in vacuum. The energy of X-rays is greater than energy of the light because energy is inversely proportional to wavelength ($E = hc/\lambda$) and wavelength of X-rays are smaller than light waves.

23. **Sol.**(b): At every instant the ratio of the magnitudes of the electric field to the magnetic field of an em wave is given by $E/B = c$.
From this equation, the magnitude of electric vector is much greater than the magnitude of magnetic vector. Also electromagnetic waves carry energy which is equally shared by electric and magnetic field.

24. **Sol.** (b): The electromagnetic waves consist of sinusoidally time varying electric and magnetic fields acting at right angles to each other as well as at right angles to the direction of propagation of waves i.e., electromagnetic waves are transverse in nature.

➤ Case Study

25. Answers

1. (*a*) Electromagnetic waves are produced by accelerated charged particles.

2. (*b*) Light travels in vacuum due to its electro-magnetic nature.

3. (*a*) $\lambda = \frac{c}{\nu} = \frac{3\times10^8}{8.2\times10^6} = 35.6$ m.

4. (*a*) The direction of propagation is along $\vec{E}\times\vec{B}$ i.e., along $\hat{i}\times\hat{j} = \hat{k}$.

5. (*b*) Velocity of light in any medium is $v = \frac{1}{\sqrt{\mu\varepsilon}}$.

6. (*d*) All e.m. waves travel with the speed of light in free space.

26. Answers

6. (*d*) All e.m. waves travel with the speed of light in free space.

7. (*c*) In the order of increasing frequency,

radiowaves < visible light < X-rays < γ-rays

8. (*b*) The frequency of the microwaves matches with the vibrational frequency of water molecules, so their energy is transferred efficiently to the kinetic energy of water molecules.

9. (*a*) Energy from the microwaves is transferred efficiently to the kinetic energy of water molecules at their resonant frequency.

10. (*b*) X-rays are used to investigate the structure of solids.

➢ Question

27. Ans. When an ideal capacitor is charged by DC battery, no current flows as capacitor offers infinite resistance to DC. Whereas since a capacitor offers finite resistance to AC, when an AC source is connected then conduction current $I_C = \frac{dQ}{dt}$ flows in the connecting wire. Due to charging current, charge deposited on the plates of the capacitor changes with time. Charging charge produces varying electric field between the plates of capacitor, giving rise to displacement current $I_d = \varepsilon_0 \frac{d\phi_E}{dt}$.

[As displacement current is proportional to the rate of flux variation].
The electric field between the plates is

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

Electric flux, $\phi_E = EA = \frac{Q}{A\varepsilon_0} A = \frac{Q}{\varepsilon_0}$

So, $I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}\left(\frac{Q}{\varepsilon_0}\right) = \frac{dQ}{dt} = I_c$

Displacement current brings continuity in the flow of current between the plates of the capacitor.

28. Ans. According to Ampere – Maxwell law,

The total current is the sum of displacement current and the conduction current, i.e.;

$$i = i_c + i_d = i_c + \varepsilon_0 \frac{d\phi_E}{dt}$$

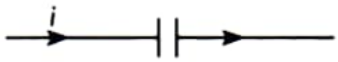
When a capacitor is charged through a battery then inside the capacitor plates there is no conduction current, i.e., $i_C = 0$ and there is only displacement current, so that $i_d = i$. The displacement current is,

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

29. Ans. Maxwell's generalization of Ampere's circuital law,

$$\oint \vec{B}.\vec{dl} = \mu_0(\mathrm{i}+\mathrm{i}_d) = \mu_0\left(i + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

In the process of charging the capacitor there is change in electric flux between the capacitor plates.



$$\frac{d\phi_E}{dt} = \frac{d}{dt}(\mathrm{EA})$$

$E \rightarrow$ Electric field between the plats = $\frac{q}{A\varepsilon_0}$

$A \rightarrow$ Area of the plates

So, $\frac{d\phi_E}{dt} = \frac{d}{dt}\left(\frac{q}{A\varepsilon_0} \times A\right) = \frac{1}{\varepsilon_0}\frac{dq}{dt} = \frac{i_d}{\varepsilon_0}$

$$\therefore\ id = i = \varepsilon_0 \frac{d\phi_E}{dt}$$

30. Ans. The speed of electromagnetic wave in a medium, $v = \frac{1}{\sqrt{\mu\varepsilon}} =$

$$= \frac{1}{\sqrt{\mu_0\mu_r\varepsilon_0\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}, where, c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} =$$

$3\times10^8\, ms^{-1}$

Is the speed of light.

31. Ans. In an electromagnetic wave, both $\vec{E}$ and $\vec{B}$ fields vary sinusoidally in space and time. The average energy density u of an e.m. wave can be obtained by replacing $\vec{E}$ and $\vec{B}$ by their rms value

$$u = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2\mu_0}B_{rms}^2 \text{ or } u = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4\mu_0}B_0^2$$

$$\left[\because\ E_{rms} = \frac{E_0}{\sqrt{2}},\ B_{rms} = \frac{B_0}{\sqrt{2}}\right]$$

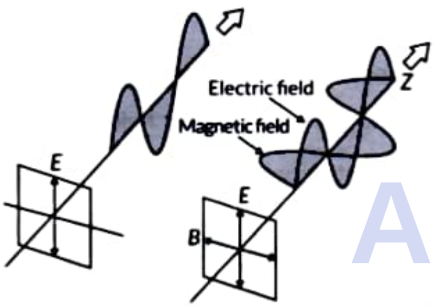
Moreover, $E_0 = cB_0$ and $c^2 = \frac{1}{\mu_0\varepsilon_0}$, therefore

$$u_E = \frac{1}{4}\varepsilon_0 E_0^2 = \frac{1}{4}\varepsilon_0(\mathrm{cB}_0)^2;$$

$$u_E = \frac{1}{4}\varepsilon_0 . \frac{B_0^2}{\mu_0 \varepsilon_0} = \frac{1}{4\mu_0} B_0^2 = u_B$$

32. Ans. An oscillating or accelerated charge is supposed to be source of an electromagnetic wave. An oscillating charge produces an oscillating electric field in space which further produces an oscillating magnetic field which in turn is a source of electric field. These oscillating electric and magnetic field hence, keep on regenerating each other and an electromagnetic wave is produced.

A plane electromagnetic wave is said to be linearly polarized. The transverse electric field wave accompanied by a magnetic field wave is illustrated.



33. Ans. (b) (i) Microwaves – $10^{-3} - 10^{-1}$ m

(ii) Infra red – $7.5 \times 10^{-7} - 10^{3}$ m

34. Ans.

	Uses	Part of electromagnetic spectrum	Frequency range
(i)	In radar system	Microwaves	3×10^8 Hz To 3×10^{11} Hz
(ii)	In eye surgery	Ultraviolet	8×10^{14} Hz to 8×10^{16} Hz

35. Ans. (a) Microwaves are suitable for radar system used in aircraft navigation.

(b) X – rays are produced by bombarding a metal target by high speed electrons.

36. Ans. (a) Microwaves are suitable for the radar system used kin aircraft navigation. Range of frequency of microwaves is 10^8 Hz to 10^{11} Hz.

(b) If the Earth did not have atmosphere, then there would be absence of green house effect of the atmosphere. Due to this reason, the temperature of the earth would be lower than what it is now.

(c) An e.m. wave carries momentum with itself and given by

$$p = \frac{Energy\ of\ wave(U)}{Speed\ of\ the\ wave(c)}$$

When it is incident upon a surface it exerts pressure on it.

37. Ans. (i) Consider a plane perpendicular to the direction of propagation of the wave. An electric charge, on the plane will be set in motion by the electric and magnetic fields of e.m. wave, incident on this plane. This illustrates that e.m. waves carry energy and momentum,

(ii) Microwaves are produced by special vacuum tube like the klystron, magnetron and Gunn diode. The frequency of microwaves is selected to match the resonant frequency of water molecules, so that energy is transformed efficiently to the kinetic energy of the molecules,

(iii) Uses of infra red waves :

(a) They are used in night vision devices during warfare. This is because they can pass through haze, fog and mist,

(b) Infra red waves are used in remote switches of household electrical appliances.

38. Soln. (a) Microwaves are suitable for the radar system used in aircraft navigation. Range of frequency of microwaves is 10^8 Hz to 10^{11} Hz.

(b) If the Earth did not have atmosphere, then there would be absence of green house effect of the atmosphere. Due to this reason, the temperature of the earth would be lower than what it is now.

(c) An e.m. wave carries momentum with itself and given by

$$p = \frac{Energy\ of\ wave\,(U)}{Speed\ of\ the\ wave\,(c)}$$

When it is incident upon a surface it exerts pressure on it.

39. Soln. (i) An e.m. wave propagating along z-axis is,

(ii) Speed of e.m. wave can be given as the ratio of magnitude of electric field (E_0) to the magnitude of magnetic field (B_0), i.e., $c = \frac{E_0}{B_0}$

Electromagnetic waves or photons transport energy and momentum. When an electromagnetic wave interacts with a small particle, it can exchange energy and momentum with the particle. The force exerted on the particle is equal to the momentum transferred per unit time. Optical tweezers use this force to provide a non – invasive technique for manipulating microscopic – sized particles with light.

40. Soln. (a) Descending order of wavelength for given electromagnetic wave are:

Microwaves ($10^{-3} - 10^{-1}$)

Infra – red rays $(7.5\times10^{-7} - 10^{-3})$

Ultra – red rays $(10^{-9} - 4\times10^{-7})$

Gamma rays $(<10^{-12})$

(b) Microwaves:

Frequency range $\rightarrow$ $3\times10^{8}\ Hz - 3\times10^{11}\ Hz$.

These are suitable for the radar system, used in aircraft navigation.

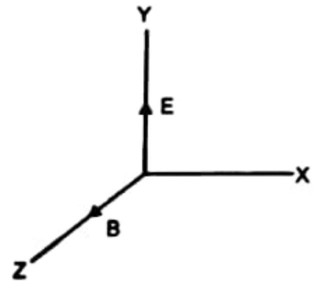
Gamma rays:

Frequency range $\longrightarrow\ >3\times10^{21}\ Hz$,

These wave are used for the treatment of cancer cells.

41. Soln. Infrared waves incident on a substance increase the internal energy and hence the temperature of the substance. That is why they are called heat waves.

42. Soln. Since electromagnetic waves and transverse in nature. We have electric and magnetic fields associated with an electromagnetic wave perpendicular to each other and perpendicular to the direction of propagation of electromagnetic waves.



Let the direction of electric field and magnetic field is along Y and Z-axis then the direction of propagation of EM waves will be along positive X-axis.

43. Soln. (a) Given,

Velocity, $v = v\hat{i}$ and electric field, E along Y-axis and magnetic field, B along Z-axis.

The propagation of EM wave is shown below:

(b) Speed of EM wave can be given as the ratio of magnitude of electric field (E_0) to the magnitude of magnetic field (B_0),

$$c = \frac{E_0}{B_0}$$

44. Soln. (a) Gamma rays are used for the treatment of certain forms of cancer. Their frequency range is 3 x 10^{19} H_3 to 5 x 10^{20} Hz.

(b) The momentum transported by electromagnetic waves is given by

$$p = \frac{U}{c} = \frac{hv}{c}$$

Where U is the energy transported by electromagnetic waves in a given time and c is speed of electromagnetic waves in free space.

Now, h = $6.62\times10^{-34} J\ s,\ c = 3\times10^{8}\ m s^{-1}$

Therefore, even for γ-rays ($v \approx 10^{20}$ Hz),

$$p = \frac{6.62\times10^{-34}\times10^{20}}{3\times10^{8}} = 2.2\times10^{-22}\ kg\ m s^{-1}$$

Thus, the amount of the momentum transferred by the e.m. waves incident on a surface is very small.

45. Soln. (a) Microwaves are suitable for radar systems used in aircrafts navigation. The range of frequency for these waves is 10^9 Hz to 10^{12} H.

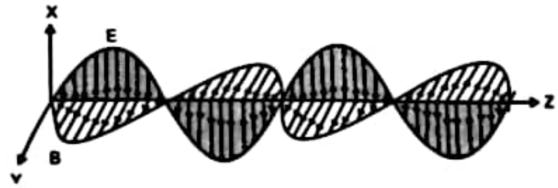
(b) In the absence of atmosphere, there would be no greenhouse effect on the surface of the Earth. As a result, the temperature of the Earth would decrease rapidly, making it difficult for human survival.

(c) The momentum transported by electromagnetic waves is given by

$$p = \frac{U}{c}$$

Where U is the energy transported by electromagnetic waves in a given time and c is speed of electromagnetic waves in a given time and c is speed of electromagnetic waves in free space. As a result, when these waves strike a surface, pressure and hence force is exerted by them on the surface.

46. Soln. A charge oscillating with some frequency, produces an oscillating electric field in space, which in turn produces an oscillating magnetic field perpendicular to the electric field. This process goes on repeating, producing e.m. waves in space perpendicular to both the fields.



The direction of electric and magnetic fields are perpendicular to each other and are also perpendicular to the direction of propagation of the wave.

47. Soln. Infrared rays are heat radiations, Microwaves are used for long distance transmission.

48. Soln. (i) 10^{-1} m = 10 cm belongs to short radiowaves.

(ii) 10^{-12} m = 0.01 $\overset{0}{A}$ belongs to gamma rays.

49. Soln. Infrared waves are produced by hot bodies and molecules, so are referred to as heat waves.

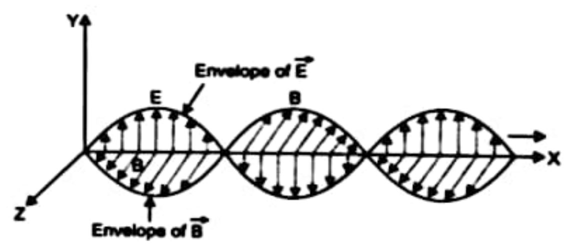
(i) Em wave having short wavelength than infrared waves are visible, UV, X-rays and γ-rays.

(ii) Em wave having longer wavelength than infrared waves are microwaves, radio waves.

50. Soln. Transverse Nature of Electromagnetic Waves:

In an electromagnetic wave, the electric and magnetic field vectors oscillate, perpendicular to the direction of propagation of wave. This is called transverse nature of electromagnetic wave.

In an electromagnetic wave, the three vectors $\vec{E}, \vec{B}$ and $\vec{K}$ form a right-handed system. Accordingly if a wave is propagating along X-axis, the electric field vector oscillates along Y-axis and magnetic field vector oscillates along Z-axis. Diagram is shown in figure.



51. Soln. (a) Microwave

(b) When a charge oscillates with some frequency. It produces an oscillating electric field and magnetic field in space. So, an electromagnetic wave is produced.

The frequency of the em wave is equal to the frequency of oscillation of the charge.

Hence energy associated with the em wave comes at the expense of the energy of the source.

If the em wave of energy U strikes on a surface and gets completely absorbed, total momentum delivery to the surface is $p = \frac{U}{E}$.

Hence em wave also carry momentum.

(c) The em wave consists of oscillating electric and magnetic fields, So net energy density of em wave is

$$U = U_E + U_B$$

$$U = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

52. Soln. (a) 10^{-3} nm ⟶ gamma radiation.

Application: Radio therapy ... to initiate nuclear reactions.

(b) $10^{-3}\ m$ ⟶ microwaves

Application: In RADAR for aircraft navigation.

(c) $1\,nm \longrightarrow X-ray$.

Application: In medical science for detection of fractures, stones in kidney, gallbladder etc.

53. Soln. (i) Microwaves are used in RADAR because they go straight and are not absorbed by the atmosphere.

(ii) X-rays are used to photograph the internal parts of human body because they can penetrate light elements (flesh).

(iii) Infrared radiations are used for taking photographs of sky during light and foggy conditions because they penetrate fog and are not absorbed by the atmosphere.

(i) It absorbs ultraviolet radiations from sun and prevents them from reaching on the earth's surface causing damage to life.

Identification: ultraviolet radiations

One correct application: santization, forensics

(ii) Water molecules present in most materials readily absorbs infra red waves. Hence, their thermal motion increases, therefore, they heat their surroundings.

They are produced by hot bodies and molecules. Incoming visible light is absorbed by earth's surface and radiated as infrared radiations. These radiation are trapped by green house gases.

54. Soln. (i) Microwaves are produced by special vacuum tubes like the Klystron / Magnetron / Gunn diode.

The frequency of microwaves is selected to match the resonant frequency of water molecules, so that energy is transferred efficiently to the kinetic energy of the molecules.

(ii) (a) Associated with the greenhouse effect.

(b) In remote switches of household electrical appliances.

55. Soln. Given: $E_{rms} = 6\,V\,m^{-1}$

$$\frac{E_{rms}}{B_{rms}} = c \quad or \quad B_{rms} = \frac{E_{rms}}{c}$$

$$B_{rms} = \frac{6}{3\times10^8} = 2\times10^{-8}\,T$$

Since, $B_{rms} = \frac{B_0}{\sqrt{2}}$

Where B_0 is the peak value of magnetic field.

$$\therefore \quad B_0 = B_{rms}\sqrt{2} = 2\times10^{-8}\times\sqrt{2}T;\ B_0 \approx 2.83\times10^{-8}$$

56. Soln. Here, $\vec{E} = 6.3\,\hat{j}\ V/m$

The magnitude of $\vec{B}$ is

$$B = \frac{E}{c} = \frac{(6.3\,\text{V/m})}{(3\times10^8\,\text{m/s})} = 2.1\times10^{-8}\,T$$

$\vec{E}$ is along y-direction and the wave propagates along x-axis. Therefore, $\vec{B}$ should be in a direction perpendicular to both x and y-axis. Using vector algebra $\vec{E}\times\vec{B}$ should be along x-direction.

Since $(+\hat{j})\times(+\hat{k}) = \hat{i}$, $\vec{B}$ is along z-direction.

Thus $\vec{B} = 2.1\times10^{-8}\,\hat{k}\,T$

57. Soln. Here, $\vec{B} = 1.2\times10^{-8}\hat{k}\,T$

The magnitude of $\vec{E}$ is

$$E = Bc = (1.2\times10^{-8}\,\text{T})(3\times10^{8}\,\text{ms}^{-1}) = 3.6\,\text{V m}^{-1}$$

$\vec{B}$ is along Z-direction and the wave propagates along X-direction. Therefore $\vec{E}$ should be in a direction perpendicular to both X and Z axes. Using vector algebra $\vec{E}\times\vec{B}$ should be along X-direction.

Since $(+\hat{j})\times(+\hat{k}) = \hat{i}$, $\vec{E}$ is along the Y-direction.

Thus, $\vec{E} = 3.6\,\hat{j}\ V\ m^{-1}$

SURE SHOT QUESTIONS

Chapter – 09

Ray Optics and Optical Instruments

➢ MCQ (1 mark)

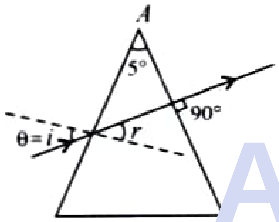
1. Soln. (a): According to the question, ray emerges from other surface of prism normally,

∴ Angle of incidence at second face, $r' = 0^0$

Now, $r + r' = A$

$\Rightarrow r = A - r' = 5^0 - 0^0 = 5^0$

Using snell's law,



$$\mu = \frac{\sin i}{\sin r}$$

Or $\sin i = \mu \sin r = 1.5 \times \sin 5^0 = 0.131$

$\Rightarrow \theta = i = \sin^{-1}(0.131) = 7.5^0$

2. Soln. (d): Since $\lambda_r > \lambda_g > \lambda_b > \lambda_v$,

Then $\mu_r < \mu_g < \mu_b < \mu_v$ $\quad (\because \mu = a + b/\lambda^2)$

$\therefore v_r > v_g > v_b > v_v$ $\quad (\because v = c/\mu)$

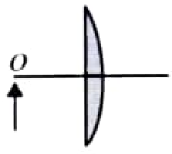
After travelling through the slab, red colour will emerge first because it has largest velocity in slab.

3. Soln. (c): When an object approaches a convergent lens from the left of the lens with a uniform speed of 5 m s^{-1}, the image moves away from the lens with a non-uniform acceleration.

4. Soln. (c): Here, for yellow light, r = 90^0 when $i = C$. As i is kept same, C must be smaller for total internal reflection. From $\mu = \frac{1}{\sin C}$, C will be smallest for blue colour. Therefore, blue light would undergo total internal reflection.

5. Soln. (c): Here $\mu = 1.5$

If object lies on plane side as shown in figure (a), then $R_1 = \infty$, $R_2 = -20\,cm$



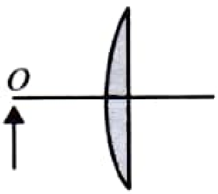
(a)

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{\infty} + \frac{1}{20}\right) = \frac{1}{40}$$

$f = +40$ cm. The lens behaves as convex lens.

If object lies on its curved side as shown in figure (b), then $R_1 = 20\,cm$, $R_2 = \infty$



(b)

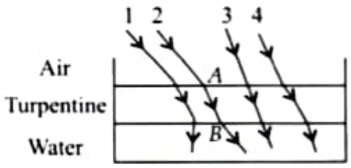
$$\frac{1}{f'} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= (1.5 - 1)\left(\frac{1}{20} - \frac{1}{\infty}\right) = \frac{1}{40}$$

$f' = 40\,cm$. The lens also behaves as convex lens.

6. Soln. (b): In spherical mirrors, the incident ray passing through the focus of mirror becomes parallel to principal axis after reflection, which is shown by ray 2.

7. Soln. (b): In the figure, the path shown for the ray 2 is correct. The ray suffers two refractions: At A, ray goes from air to turpentine, bending towards normal. At B, ray goes from turpentine to water (i.e., from denser to rarer medium), bending away from normal.



8. Soln. (d): In the side mirror, the speed of the approaching car would appear to increase as the distance between the cars decreases.

9. Soln. (a): Using Snell's law

$$\mu = \frac{\sin i}{\sin r} \Rightarrow \sin r = \frac{\sin i}{\mu}$$

10. Ans. (a) : $\frac{d}{4}$, as image at 2F formed of same size.

Here, $4F = d \Rightarrow F = d/4$

11. A

12. Ans. b) For bi-concave lens,

$$\frac{1}{F} = (\mu - 1)\left[\frac{1}{-R} - \frac{1}{R}\right] = \frac{-2(\mu - 1)}{R}$$

For plano – concave lens,

$$\frac{1}{F'} = (\mu - 1)\left[\frac{1}{-R} - \frac{1}{\infty}\right] = \frac{-(\mu - 1)}{R}$$

$$\Rightarrow F' = 2F \Rightarrow P' = \frac{P}{2}$$

13. Ans. (a) : $\delta = (\mu - 1)A$, since red is having smallest refractive index compared to other colours, it has the smallest angle of minimum deviation.

➤ Assertion-Reasoning (1 mark)

14. Ans. (c) : A concave mirror is a converging mirror. Convex mirror is a diverging mirror and can never form a real image.

15. Ans.(b)

16. Ans. (A) $\delta_{m(\text{Red})} < \delta_{m(\text{Blue})}$

$\mu_{Blue} < \mu_{Red}$

So, assertion is correct and reason is correct explanation of assertion.

17. **Sol.**

(b): The light travelling from air to glass cannot suffer total internal reflection because for total internal reflection, the essential condition is that light should travel from a denser medium to a rarer medium with incidence angle more than the critical angle.

18. **Sol.**

(b): The light gathering power (or brightness) of a telescope is directly proportional to the area of the objective lens.

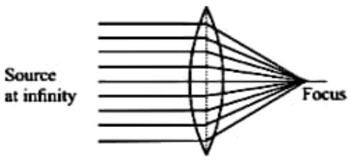
i.e., light gathering power $\propto \pi r^2 = \frac{\pi D^2}{4}$, where D is the diameter of the objective. Thus telescope will have large light gathering power if the diameter of the objective lens is large. So by increasing the objective diameter even far off stars may produce images of optimum brightness.

19. **Sol.**

(c): Since the lens is made of two different materials of different refractive indices, there will be two different focal lengths of the lens. Hence two images will be formed. The image formed by convex lens is always real except in case when object is placed between optical centre and focus.

20. **Sol.**

(a): Image formed by convex lens



21. Sol.

(a): Due to the variation of the refractive index of the material of the lens, the focal length also varies accordingly. Now as white light is composed of different colours of light, each colour will produce its own image based on the focal length for that colour. This particular phenomenon for a single lens is known as chromatic aberration.

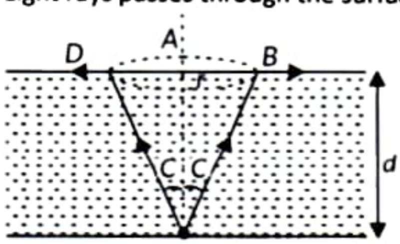
22. Sol.

(d): The distance between the object and its real image is minimum when magnification is 1. We know that the magnification of convex lens is given by $(m) = -v/u$ for $m = 1$, $v = -u$.
Now from lens formula, $v = u = 2f$
Hence, minimum distance $v + u = 4f$.

➢ Case Study (5 marks)

23. Ans. (i) (a) : Since refractive index of medium is more than that of water, medium is denser than water. Here, light travels from medium (denser) to water (rarer), so it will bends away from the normal. This is correctly shown in (a). Here total internal reflection will not take place because angle of incidence (18^0) is les than critical angle (20^0).

(ii) (b) : Light rays passes through the surface if,



$$\mu = \frac{1}{\sin C}$$

$$Also,\ \sin C = \frac{r}{\sqrt{r^2 + d^2}}$$

$$\frac{1}{\mu} = \frac{r}{\sqrt{r^2 + d^2}} \Rightarrow r^2 + d^2 = \mu^2 r^2$$

$$\Rightarrow d^2 = (\mu^2 - 1)r^2 \ or \ r^2 = \frac{d^2}{\mu^2 - 1}$$

$$Area,\ A = \pi r^2 = \frac{\pi d^2}{\mu^2 - 1}$$

(iii) (c) : Refractive index is minimum for water. Thus, water with respect to air has maximum critical angle.

(iv) (b) : $\sin C = \frac{1}{\mu}$ or $\sin C = \frac{1}{2} \Rightarrow C = 30^0$

(v) (b) : Critical angle, C = 30^0

$$\mu = \frac{1}{\sin C} = \frac{1}{\sin 30^0} \Rightarrow \mu = 2$$

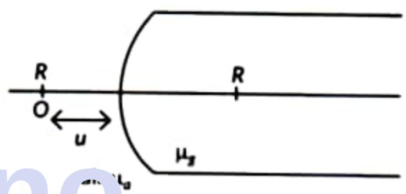
Also, $\mu = \frac{c}{v} \Rightarrow v = \frac{c}{\mu} = \frac{3\times10^8}{2} = 1.5\times10^8\ ms^{-1}$

Here, question 18 (i) to (v) is a case study base question of 5 marks.

24. Ans. (i) : (d) For the spherical refracting surface,

$$\frac{n_1}{v} - \frac{n_2}{u} = \frac{n_1 - n_2}{R}$$

(ii) : (d)



If v is the image distance

$$\frac{\mu_g}{v} - \frac{\mu_a}{-R} = \frac{\mu_g - \mu_a}{R} \ or \ \frac{\mu_g}{v} + \frac{1}{R} = \frac{\mu_g}{R} - \frac{1}{R}$$

$$\Rightarrow \frac{\mu_g}{v} = \frac{1}{R}(\mu_g - 2) \Rightarrow \frac{\mu_g}{v} = \frac{1}{R}(1.5 - 2) = -\frac{1}{2R}$$

$$\Rightarrow v = -2\times\frac{3}{2}R = -3R$$

Therefore, virtual image will be formed in the air.

(iii) (a) : If an object is kept at 2F from an equi-convex lens, then image will be real, inverted and of same size as object.

(iv) (b) : $f_1 = 10cm,\ f_2 = -20cm$

If two lens are kept in contact.

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} - \frac{1}{20} \Rightarrow f = +20cm$$

$$P = \frac{100}{f(\text{cm})} = \frac{100}{20} = +5D$$

(v) (c) : If the lens is cut along the horizontal line as shown in figure, then focal length (f) of each part will remains same, Since, $\frac{1}{f} = (\mu - 1)\left(\frac{2}{R}\right)$

25. Ans. (a) : As we know that, refractive index decrease with an increase in wavelength of light. Therefore, focal length of lens will be increased with increasing wavelength of light.

Focal length $\propto \frac{1}{power} \propto$ wavelength

So, power $\propto \frac{1}{wavelength}$

As we increase the wavelength of light, the power of lens will decrease.

(b) The radius of curvature of two surfaces of a convex lens = R

The refractive index of material μ, the focal length becomes R.

So, len's maker formula,

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$R_1 = +R$ and $R_2 = -R$

So, $\frac{1}{R} = (\mu - 1)\left[\frac{2}{R}\right]$; $\mu - 1 = \frac{1}{2} \Rightarrow \mu = \frac{3}{2}$

(a) Focal length of a concave lens = -20 cm

Refractive index of concave lens,

$\mu_1 = 1.5 or \frac{3}{2}$

Immersed in water, $\mu_2 = \frac{4}{3}$

Focal length in air,

$$\frac{1}{f} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{..........(i)}$$

Focal length in water,

$$\frac{1}{f_w} = \left(\frac{\mu_1}{\mu_2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{.............(ii)}$$

Put the respective values

$$-\frac{1}{20} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{.............(iii)}$$

$$\frac{1}{f_w} = \left(\frac{3/2}{4/3} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{9}{8} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

...........(iv)

Equation (iii) ÷ (iv),

$$\frac{-\frac{1}{20}}{\frac{1}{f_w}} = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}{\left(\frac{1}{8}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}; \quad -\frac{f_w}{20} = \frac{8}{2}$$

$f_w = -80 \ cm$

So, focal length of concave lens is -80 cm in water.

OR

(c) An object is placed in front of lens which forms its erect image of magnification, m = 3.

The power of lens, P = 5D

Magnification, $m = \frac{v}{u} = 3 \ or \ v = +3u$ .....(i)

The focal length of lens,

$$f = \frac{1}{P} = \frac{100}{5} cm = +20 cm$$

Lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \frac{1}{3u} - \frac{1}{u} = \frac{1}{20} \Rightarrow \frac{1-3}{3u} = \frac{1}{20}$$

$$\Rightarrow \frac{-2}{3u} = \frac{1}{20} \ or \ u = \frac{-40}{3} cm.$$

$$u = \frac{40}{3} cm \left(object \ is \ \frac{40}{3} cm \ away \ in \ front \ of \ lens.\right)$$

$$v = 3u = -3 \times \frac{40}{3} cm = -40 cm$$

$v = -40 cm$ (image formed at a distance of 40 cm on same side as object)

26.

I. (B) real, virtual

II. (A) the aperture of the objective and the eye-piece

III. (D) The microscop can be used as a ~~telesc~~ telescope by interchanging the two lenses

IV. (D) 200

V. (C) 200

[Topper's Answer, 2022]

➤ Questions

27. Ans. Radius of curvature R = 60 cm

Focal length f = 30 cm

By Mirror formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{F}$

4 = -20 cm, f = -30 cm (concave mirror)

$$\frac{-1}{20}+\frac{1}{v}=\frac{-1}{30} \Rightarrow \frac{1}{v}=\frac{1}{20}-\frac{1}{30}=\frac{10}{600}$$

$v = 60\,cm$

Thus image formed is virtual, erect and magnified in nature.

28. Ans. f = -15 cm, m = 3, u = -x cm

$$m=-\frac{v}{u}=3 \text{ or } v=-3u=3x$$

Using mirror formula,

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f};\ \frac{1}{3x}+\frac{1}{-x}=\frac{1}{-15} \text{ or, } -\frac{2}{3x}=\frac{1}{-15}$$

$\Rightarrow \quad x=10\,cm$

∴ Distance of object from the mirror, x = 10 cm.

29. Soln. At plane AC, the incident angle for ray 1 and ray 2 = 45^0

Let critical angle for total internal reflection for ray 1 = C_1

$$1.35=\frac{1}{\sin C_1}$$

$$\Rightarrow \quad \sin C_1=\frac{1}{1.35}$$

$$=0.74$$

Hence,

$C_1 > 45^0 \qquad (\sin 45^0 = 0.707)$

Let critical angle for total internal reflection for ray 2 = C_2

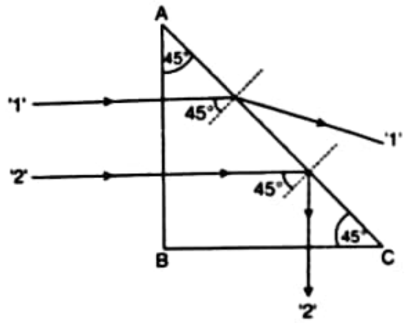
$$1.45=\frac{1}{\sin C_2}$$

$$\Rightarrow \quad \sin C_2=\frac{1}{1.45}=0.689$$

Hence $C_2 < 45^0 \qquad (\sin 45^0 = 0.707)$

As in case of ray 1, incident angle is less than critical angle, it would emerge out from AC. In the figure path of the ray 1 is shown.

In case of ray 2, incident angle is greater than critical angle, it would get total internal reflection at AC and emerge from BC. In the figure path of the ray 2 is shown.



30. Ans. (a) Here, R = -20 cm, f = R/2 = -10 cm

$m=-2$ (image is real)

u = object distance, v = image distance

$$m=-\frac{v}{u} \Rightarrow v=2u$$

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{2u}+\frac{1}{u}=\frac{1}{-10} \Rightarrow \frac{3}{2u}=\frac{1}{-10}$$

$\therefore \quad u=-15\,cm$

Hence, v = 2u = -30 cm

(b) For convex mirror: $f>0,\ u<0$

Using mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{u}=\frac{1}{f}-\frac{1}{(-u)} \Rightarrow \frac{1}{v}=\frac{1}{f}+\frac{1}{u}$$

$$\Rightarrow v=\frac{f\times u}{f+u}$$

$\therefore \quad v>0$

This implies that image of object placed in front of a convex mirror is always formed behind the mirror which is virtual in nature.

31. Ans. For plano -convex lens, $R_1=\infty$ and

$R_2=-20\,cm$

Given that, $\mu=1.5$

Using lens maker's formula,

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=(1.5-1)\left[\frac{1}{\infty}-\frac{1}{(-20)}\right]=\frac{1}{40}$$

Or f = 40 cm

Given that, u = 30 cm

Using lens formula,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\ or\ \frac{1}{v}-\left(\frac{1}{-30}\right)=\frac{1}{40}\ or\ \frac{1}{v}=-\frac{1}{120}$$

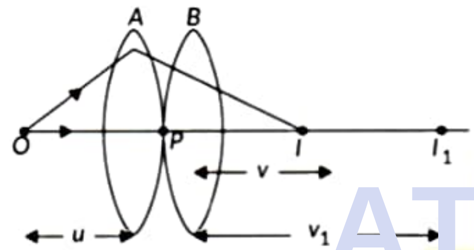
Or $v=-120cm$

$\therefore$ The image is formed at 120 cm on the same side as the object. So, the image is virtual and erect.

Magnification, $m=-\frac{v}{u}=\frac{-(-120)}{30}=4$

Thus, image is enlarge by four times the size of the object

32. Ans. (a)



An object is placed at point O. The lens A produces an image at I_1 which serves as a virtual object for lens B which produces final image at I. Given, the lenses are thin. The optical centres (P) of the lenses A and B coincide with each other.

For lens A, we have

$$\frac{1}{v_1}-\frac{1}{u}=\frac{1}{f_1} \qquad ....(i)$$

For lens B, we have $\frac{1}{v}-\frac{1}{v_1}=\frac{1}{f_2}$ ....(ii)

Adding equations (i) and (ii),

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f_1}+\frac{1}{f_2} \qquad ....(iii)$$

If two lenses are considered as equivalent to a single lens of focal length f, then

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \qquad ....(iv)$$

From equation (iii) and equation (iv), we can write

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

(b) For lens $L_1:\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

Where, $f=+10cm$

$$\frac{1}{v_1}=\frac{1}{10}-\frac{1}{30};\ \frac{1}{v_1}=\frac{3-1}{30}=\frac{2}{30}\Rightarrow v_1=15cm$$

For lens L_2 :

$v_1=15cm,\ u=10cm,\ f=-10cm$

Position of final image,

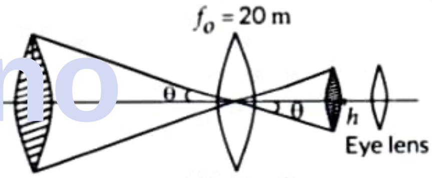
$$\frac{1}{v_2}=\frac{1}{f}+\frac{1}{u}=\frac{1}{10}-\frac{1}{10}\Rightarrow v_2=\infty$$

$\therefore$ For third lens L_3 object is at infinity, hence final image is formed at focus of L_3 at a distance of 30 cm.

33. Ans. (a) Angular magnification,

$$m=\frac{f_0}{f_e}=\frac{2000cm}{1.0cm}=2000$$

(b) The image of the moon by the objective lens formed on its focus only the as the moon in nearly at infinite distance as compared to focal length.



i.e., Radius of moon $R_m=\frac{3.5}{2}\times10^6\ m$

$R_m=1.75\times10^6\ m$

Distance of object = Radius of lunar orbit

$R=3.8\times10^8\ m$

Distance of image for objective lens is the focal length of objective lens, $f_0=20m$

Radius of image of moon by objective lens can be calculated, as,

$$\tan\theta=\frac{R_m}{R_0}=\frac{h}{f_0}$$

$$h=\frac{R_m\times f_0}{R_0}=\frac{1.75\times10^6\times20}{3.8\times10^8}=9.21\times10^{-2}m$$

Diameter of the image of the moon,

$$=2h=18.42\times10^{-2}m=18.42cm$$

34. Soln. From Snell's law, we have: $\frac{\sin(i)}{\sin(r)} = \mu$

At A, $i = 60^0$; $\mu = \sqrt{3}$

Now, $\sin(r) = \frac{\sin(i)}{\mu}$

$$\Rightarrow \quad \sin(r) = \frac{\sin(60^0)}{\sqrt{3}} = \frac{1}{2}$$

$$\Rightarrow \quad r = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\therefore \quad r = 30^0$$

35. Soln. Clearly, equivalent focal length of equiconvex lens and water lens, f =x

Focal length of equiconvex lens $f_1 = y$

Focal length f_2 for water lens is given by

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{1}{x} - \frac{1}{y} = \frac{y-x}{xy}$$

Or $\quad f_2 = \frac{xy}{y-x}$

The water lens formed between the plane mirror and the equiconvex lens is a planoconcave lens.

For this lens,

$$R_1 = -R \text{ and } R_2 = \infty$$

Using lens marker's formula,

$$\frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\text{or } \frac{y-x}{xy} = (\mu - 1)\left[\frac{1}{-R} - \frac{1}{\infty}\right]$$

$$\text{or } \mu - 1 = \frac{(x-y)R}{xy} \text{ or } \mu = 1 + \frac{(x-y)R}{xy}$$

36. Soln. $f_1 = f_{convex} = 30cm$

$$f_2 = f_{concave} = -20cm$$

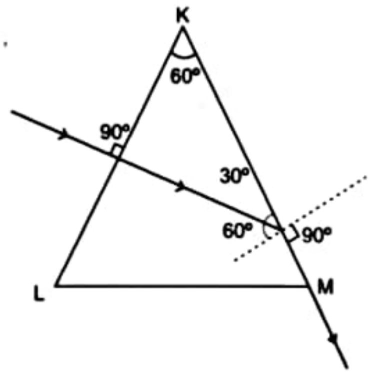
$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= \frac{1}{30} - \frac{1}{20}$$

$$= \frac{2-3}{60}$$

$$\frac{1}{f} = -\frac{1}{60} [\therefore \text{concave (diverging) lens}]$$

37. Soln. From diagram it is clear that incidence angle at face KM is 60⁰.



Hence, critical angle is also 60⁰.

Therefore, incident light ray will not emerge from KM face due to total internal reflection at this face. Hence, it will move along face KM Angle of emergence = 90⁰.

Hence angle of deviation = 30⁰ (from fig.)

38. Soln. The rainbow is an example of the dispersion of sunlight by the water drops in the atmosphere. This is a phenomenon due to combined effect of dispersion, refraction and reflection of Sunlight by spherical water droplets of rain. The conditions for observing a rainbow are that the Sun should be shining in one part of the sky (say near western horizontal while it is raining in the opposite part of the sky (say eastern horizon).

Difference between Primary and Secondary Rainbow:

S.No.	Primary Rainbow	Secondary Rainbow
1.	Three Step process (Re-fraction-Reflection	Four step process (Refraction-Reflection and Refraction)

	and Refraction)	
2.	Appearance intensity better than Second ary.	Appearance intersity lesser than Primary.
3.	Single reflection occurs.	Double reflection occurs.
4.	2 Degree range occurs.	3 Degree range.
5.	Fig. 1	Fig. 2

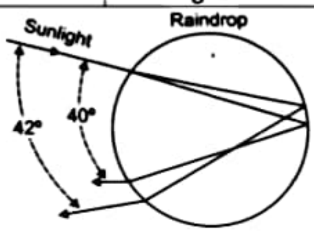
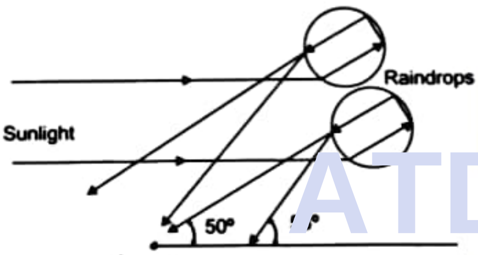
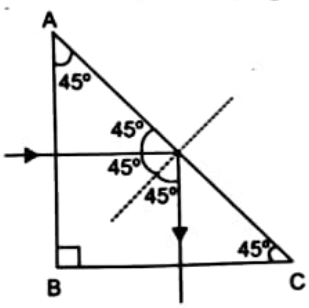


Fig. (1)

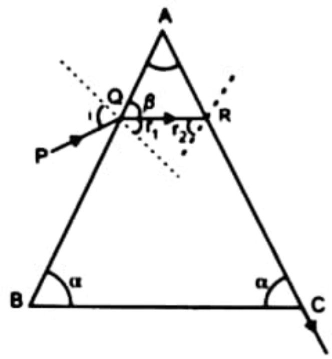


39. Soln. The phenomenon of total internal reflection occurs when,
1. Angle of incidence is equal or greater than critical angle.
   $i \geq C$
2. When light travels from more denser medium to less denser medium.
   In case of right angle isosceles triangle if light rays fall normally on AB then light incident of face AC with angle of incidence > critical angle.



Hence, total internal reflection will occur with normal to the surface of BC.

40. Soln.



$$\frac{\sin i}{\sin r} = \mu$$

(i) Condition for minimum deviation:
1. $A = 180 - 2\alpha$
2. $\dfrac{\sin i}{\sin(90-\beta)} = \mu$

When, $r_1 = r_2 = r >$ critical angle

$$r_1 + r_2 = 180 - 2\alpha$$
$$2r = 180 - 2\alpha \quad [\because r_1 = r_2]$$
$$r_1 = r = 90 - \alpha$$
$$\beta = 90 - r_1$$
$$= 90 - 90 + \alpha$$
$$\beta = \alpha$$

Condition when QR have total internal reflection:

$\angle QRC \geq$ critical angle for the prism

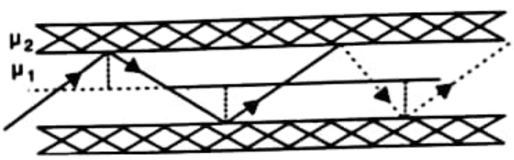
$\angle 180^0 - \beta \geq$ critical angle

Or $\angle 180^0 - \alpha \geq$ critical angle

$\because \quad \angle 180^0 - \alpha = \angle BAC$

$\therefore \quad \angle BAC \geq$ critical angle

41. Soln.



Optical fibre works on principle of total internal reflection. When angle of incidence is greater than Critical angle then incident rays are totally reflected back in same media.

When, $\theta_i > \theta_C$, Total internal reflection occurs and if $\theta_i < \theta_C$, refraction occurs.

Application: Optical fibre are used for communication due to very high bandwidth of media.

42. Soln. Here for convex lens

$$u = -60\,cm$$

$$f = +20\,cm$$

(convex lens)

Using lens formula

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{60} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{60} = \frac{2}{60}$$

$$So, \quad v = 30\,cm$$

Positive sign shows that image will be at the right hand side of the lens.

Now this image would act as virtual source for convex mirror. Its distance from convex mirror is

$$u = 30 - 15 = 15\,cm$$

(+ve sign means virtual source)

$$f = +10\,cm$$

(convex mirror)

Applying mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$$

Hence v = 30 cm

So image would be at 30 cm from the mirror. As v is positive, image would be virtual, magnified and erect.

43. Soln. From the lens maker formula, it is clear that n_{21} decreases then focal length increase.

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \left(\because\ n_{21} = \frac{n_2}{n_1}\right)$$

Here refractive index of the glass with respect to surrounding material decreases. Hence, focal length increases which will also increase the image distance.

44. Soln. (i) Refractive index of a medium is the ratio of speed of light (c) in free space to the speed of light (v) in that medium.

$$\mu = \frac{c}{v}$$

(ii) $\mu = \frac{c}{v} = \frac{1}{\sin i_c}$

$$= \frac{3 \times 10^8}{v} = \frac{1}{\frac{30}{50}}$$

$$v = \frac{30}{50} \times 3 \times 10^8$$

$$= 1.8 \times 10^8\ m/s$$

45. Soln. (i) Refractive index of glass for different wavelengths (colours) of light ray is different. Hence different colours bend with different angles. This give dispersion of white light when it passes through a glass prims.

(ii) As the refractive index of the medium with respect to air (medium 1) depends on the wavelength or colour of light, focal length of the lens would change with colour.

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

46. Soln. At plane AC, the incident angle for ray 1 and ray 2 = 45^0

Let critical angle for total internal reflection for ray 1 = C_1

$$1.35 = \frac{1}{\sin C_1}$$

$$\Rightarrow \quad \sin C_1 = \frac{1}{1.35}$$

$$= 0.74$$

Hence,

$$C_1 > 45^0 \qquad (\sin 45^0 = 0.707)$$

Let critical angle for total internal reflection for ray 2 = C_2

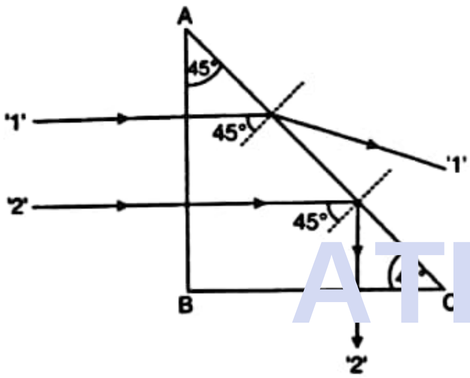
$$1.45 = \frac{1}{\sin C_2}$$

$$\Rightarrow \quad \sin C_2 = \frac{1}{1.45} = 0.689$$

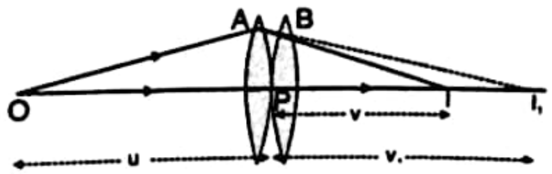
Hence $C_2 < 45^0$ $(\sin 45^0 = 0.707)$

As in case of ray 1, incident angle is less than critical angle, it would emerge out from AC. In the figure path of the ray 1 is shown.

In case of ray 2, incident angle is greater than critical angle, it would get total internal reflection at AC and emerge from BC. In the figure path of the ray 2 is shown.



47. Soln.



(i) Two thin lenses, of focal length f_1 and f_2 are kept in contact. Let O be the position of object and let u be the object distance. The distance of the image (which is at I_1), for the first lens is v_1.

This image serves as object for the second lens.

Let the final image be at I. We then have

$$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u}$$

$$\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1}$$

Adding, we get

$$\frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$P = P_1 + P_2$$

(ii) At minimum deviation

$$r = \frac{A}{2} = 30^0$$

We are given that, $i = \frac{3}{4}A = 45^0$

$$\therefore \quad \mu = \frac{\sin 45^0}{\sin 30^0} = \sqrt{2}$$

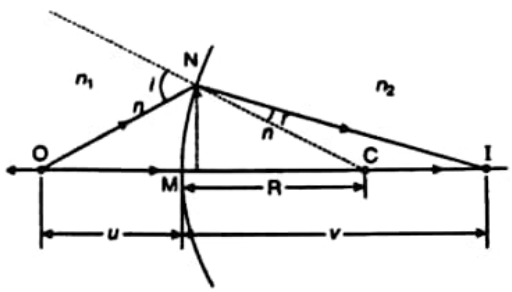
$\therefore$ Speed of light in the prism = $\frac{c}{\sqrt{2}}$

$(\cong 2.1 \times 108\, ms^{-1})$

[Award ½ mark if the student writes the formula:

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$ but does not do any calculations.]

48. Soln. (i)



For small angles

$$\tan\angle NOM = \frac{MN}{OM}$$

$$\tan\angle NCM = \frac{MN}{NC}$$

And $$\tan\angle NIM = \frac{MN}{NC}$$

For ΔNOC, i is exterior angle, therefore

$$i = \angle NOM + \angle NCM$$
$$= \frac{MN}{OM} + \frac{MN}{MC}$$

Similarly, $$r = \frac{MN}{MC} - \frac{MN}{MI}$$

For small angles, Snells law can be written as

$$n_1 i = n_2 r$$

$$\therefore \quad \frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC}$$

$$\therefore \quad OM = -u, \quad MI = +v$$
$$MC = +R$$

(using sign convention)

$$\therefore \quad \frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

(ii) Lens Maker's formula is

$$\frac{1}{f} = \left(\frac{n_2}{n_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ Focal length in an is

$$\frac{1}{20} = (1.6-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \quad \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{20\times 0.6} = \frac{1}{12}$$

Let f' be the focal length of the lens in water

$$\therefore \quad \frac{1}{f'} = \left(\frac{1.6-1.3}{1.3}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$
$$= \frac{0.3}{12\times 1.3}$$

$$or \quad f' = \frac{120\times 1.3}{3} = 52\,cm$$

49. Soln. (i)

From fig $\angle A + \angle QNR = 180^0$ ....(i)

From triangle ΔQNR, $r_1 + r_2 + QNR = 180^0$ ....(ii)

Hence from eqn. (i) & (ii)

$$\therefore \quad \angle A = r_1 + r_2$$

The angle of deviation

$$\delta = (\mathrm{i} - r_1) + (\mathrm{e} - \mathrm{r}_2)$$
$$= \mathrm{i} + \mathrm{e} - \mathrm{A}$$

At minimum deviation $i = e$ and $r_1 = r_2$

$$\therefore \quad r = \frac{A}{2}$$

$$and \quad i = \frac{A+\delta_m}{2}$$

Hence, refractive index,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}$$

(ii) From snell's law, $\mu_1 \sin i = \mu_2 \sin r$

Given $\mu_1 = \sqrt{2}, \ \mu_2 = 1$

And $r = 90^0$ (just grazing)

$$\therefore \quad \sqrt{2}\sin i = 1\times\sin 90^0$$

$$\Rightarrow \quad \sin i = \frac{1}{\sqrt{2}}$$

$$or \quad i = 45^0$$

50. Soln. Magnifying power is defined as the angle subtended at the image to the angle subtended (at the unaided eye) by the object.

Alternatively: Also accept this definition in the form of formula

$$m = m_0 \times m_e = \frac{L}{f_0} \times \frac{D}{f_e}$$

The increase the magnifying power both the objective and eyepiece must have short focal lengths $\left(as\ m = \frac{L}{f_0} \times \frac{D}{f_e} \right)$

51. Soln. Given, $f_0 = 1.25cm,\ f_e = 5cm$ (In microscope, focal length of objective lens should be very small)

Magnification, m= 30,

It we set these lens for minimum distance for distinct vision, then for

$$m = \frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$$

$$30 = \frac{L}{1.25}\left(1 + \frac{25}{5}\right)$$

$$L = 1.25 \times 30 \times \frac{5}{30}$$

$$L = 5 \times 1.25$$

$$L = 6.25cm$$

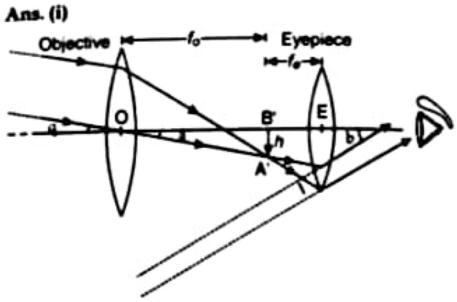
Hence, distance between two lenses is

$$= f_0 + 6.25 + f_e$$
$$= 1.25 + 6.25 + 5$$
$$= 12.5cm$$

This is a required separation between the objective and the eyepiece.

52. Soln.

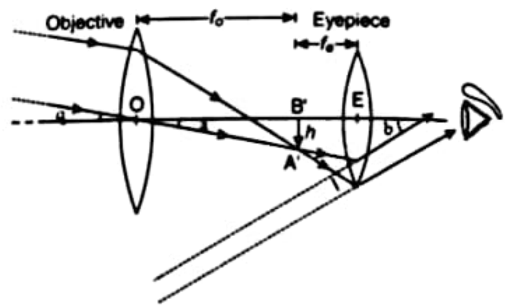
Ans. (i)



For a large magnifying power, f_0 should be large and f_e should be small.

For a higher resolution, the diameter of the objective should be large.

53. Soln. (i)



[Note: deduct ½ mark if not labelled]

Magnifying power,

$$m = \frac{\tan\beta}{\tan\alpha} \cong \frac{\beta}{\alpha}$$

∵ The angles are small

Final image is formed at infinity when the image A'B' is formed by the objective lens at the focus of the eyepiece,

$$m = \frac{h}{f_e} \times \frac{f_0}{h}$$

$$m = \frac{f_0}{f_e}$$

(ii) Given

$$f_0 + f_e = 105,\ f_0 = 20 f_e$$
$$20 f_e + f_e = 105$$
$$f_e = \frac{105}{21} = 5cm$$
$$f_0 = 20 \times 5 = 100cm$$

∴ Magnification, $m = \frac{f_0}{f_e} = \frac{100}{5} = 20$

54. Soln. (i) $m = \frac{f_0}{f_e}$

By increasing f_0 or decreasing f_e

(ii) (a) No chromatic aberration.

(b) No spherical aberration.

(c) Mechanical advantage – low weight, easier to support.

(d) Mirrors are easy to prepare.

(e) More economical.

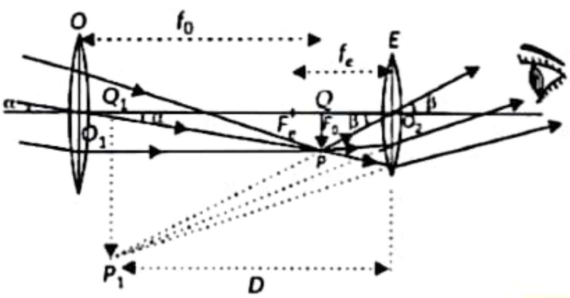
55. Soln. Large focal length : to increase magnifying power

$$\left(\because\ m = \frac{f_0}{f_e}\right)$$

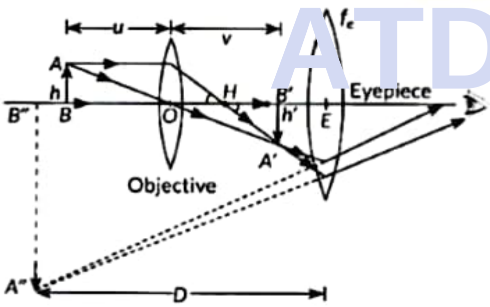
Large aperture: to increase resolving power

$$\left(\because\ RP = \frac{2a}{1.22\lambda}\right)$$

56. Soln. Construction for astronomical telescope.



Construction for compound microscope:



Angular magnification, $m = \frac{v_0}{u_0}\left(1 + \frac{D}{f_e}\right)$

$$\Rightarrow \quad 30 = \frac{v_0}{u_0}\left(1 + \frac{25}{5}\right)$$

$$\Rightarrow \quad v_0 = 5u_0 \qquad \text{.........(i)}$$

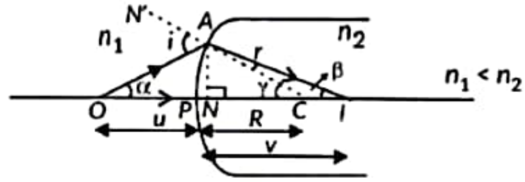
From lens formula,

$$\frac{1}{f_0} = \frac{1}{v_0} - \frac{-1}{(-u_0)} \Rightarrow \frac{1}{1.25} = \frac{u_0 + v_0}{v_0 u_0} \qquad \text{......(ii)}$$

Substituting (i) in (ii), u_0 = 1.5 cm

57. Soln. Refraction at convex spherical surface, see figure.

When object is in rarer medium and image formed is real.



In $\Delta OAC,\ i = \alpha + \gamma$

And in $\Delta AIC,\ \gamma = r + \beta$ or $r = \gamma - \beta$

$\therefore$ By Snell's law ${}^1n_2 = \frac{\sin i}{\sin r} = \frac{i}{r} = \frac{\alpha + \gamma}{\gamma - \beta}$

Or $\frac{n_2}{n_1} = \frac{\alpha + \gamma}{\gamma - \beta}$ or $n_2\gamma - n_2\beta = n_1\alpha + n_1\gamma$

Or $(n_2 - n_1)\gamma = n_1\alpha + n_2\beta$ ....(i)

As α, β and γ are small and P and N lie close to each other,

So, $\alpha \approx \tan\alpha = \frac{AN}{NO} \approx \frac{AN}{PO}$

$\beta \approx \tan\beta = \frac{AN}{NI} \approx \frac{AN}{PI}$

$\gamma \approx \tan\gamma = \frac{AN}{NC} \approx \frac{AN}{PC}$

On using them in equation (i), we get

$$(n_2 - n_1)\frac{AN}{PC} = n_1\frac{AN}{PO} + n_2\frac{AN}{PI}$$

Or $\frac{n_2 - n_1}{PC} = \frac{n_1}{PO} + \frac{n_2}{PI}$ ....(ii)

Where, PC = +R, radius of curvature

PQ = -u, object distance

$PI = +v$, image distance

So $\frac{n_2 - n_1}{R} = \frac{n_1}{-u} + \frac{n_2}{v}$ or $\frac{n_2 - n_1}{R} = \frac{n_2}{v} - \frac{n_1}{u}$

This gives formula for refraction at spherical surface when object is in rarer medium.

58. Soln. As the image of the object is formed by the lens on the screen, therefore the image is real.

Let the object is placed at a distance x from the lens.

As the distance between the object and the screen is 90 cm. Therefore, the distance of the image from the lens is (90 – x)

According to new Cartesian sign conventions,

$u = -x,\ v = +(90 - x)$

Magnification $m = \dfrac{v}{u}$

$\therefore \quad -2 = \dfrac{(90 - x)}{-x} \Rightarrow x = 30\,cm$

$\therefore \quad u = -30\,cm,\ m = \dfrac{v}{u} = 60\,cm$

Let f be focal length of the lens.

According to thin lens formula,

$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f};\ \dfrac{1}{60} - \dfrac{1}{-(30)} = \dfrac{1}{f}$

$\dfrac{1}{60} + \dfrac{1}{30} = \dfrac{1}{f}$

$f = +20\,cm$

A convex lens of focal length 20 cm is required.

SURE SHOT QUESTIONS

ARVIND ACADEMY

Chapter – 10

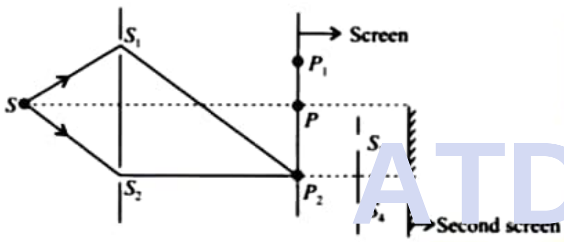
Wave Optics

MCQ (1 mark)

1. Soln. (a)

2. Soln. (c): The light from two slits of Young's double slit experiment is of different colours/wavelengths/frequencies. Hence, there shall be no interference fringes.

3. Soln. (d):



In given figure there is a hole at minima point P_2, the hole will act as a source of fresh light for the slits S_3 and S_4. Hence, there will be a regular two slit pattern on the second screen.

4. Ans. (a) : According to Huygen's principle each point on the wavefront acts as a source of secondary wavelets and the amplitude of these secondary wavelets is equal in both the forward and backward directions.

   The new wavefront is the tangential surface of all these secondary wavelets.

5. Ans. (c) : If a plane wavefront is incident on a concave mirror, then the emergent wavefront will be spherical with centre of the wavefront at its focus.

6. Ans. (c) : (i) Angular separation = $\frac{\lambda}{d}$

   Remain same

   (ii) Fringe width, $\beta = \frac{D\lambda}{d}$

   So, as D increases fringe width increases.

   (i) Remains same, but

   (ii) Increases.

7. Ans. (a) : $\lambda = 700\ nm,\ \lambda' = ?$

   $y_3 = y_5$

   $$\frac{3\times 700D}{d} = \frac{5\times\lambda' D}{d}$$

   $$\lambda' = \frac{2100}{5} = 420\,nm$$

8. Ans. (c) : Fringe width is given as

   $\beta = \frac{\lambda D}{d}$

   Now, entire apparatus is placed in liquid of refractive index μ.

   Thus, wavelength of incident wave changes to

   $\lambda' = \frac{\lambda}{\mu}$

   $\therefore$ Now fringe width, $\beta' = \frac{\lambda' D}{d}$

   $$\beta' = \frac{\lambda}{\mu}\frac{D}{d} = \frac{\beta}{\mu}$$

   So, answer is option (c).

9. Ans. (d) : Given, path difference, $\Delta x = \frac{\lambda}{8}$

   Phase difference, $\Delta\phi = \frac{2\pi}{\lambda}.\Delta x \Rightarrow \Delta\phi = \frac{\pi}{4}$

At any point, intensity $I = I_{max} \cos^2\left(\frac{\Delta\phi}{2}\right)$

$$I = I_{max} \cos^2\left(\frac{\pi/4}{2}\right)$$

$$\frac{I}{I_{max}} = \left[\cos\left(\frac{\pi}{8}\right)\right]^2 = (0.9239)^2 = 0.853$$

➢ Assertion-Reasoning (1 mark)

10. Ans. (c) : Fringe width,

$$\beta = \frac{\lambda D}{d}. \text{ As, } d' = \frac{d}{3} \text{ and } D' = \frac{D}{3}$$

$\therefore$ New fringe width, $\beta' = \frac{\lambda D'}{d'} = \frac{\lambda \times \frac{D}{3}}{\frac{d}{3}} = \frac{\lambda D}{d}$

11. **Sol.** (b):When slit is wide ($i.e.\, a » \lambda$), bending of light becomes so small that it cannot be detected up to a certain distance of screen from the slit. Hence, practically, no diffraction occurs.

12. **Sol.** (d): When a light wave travel from a rarer to a denser medium it loses speed, but energy carried by the wave does not depend on its speed. Instead, it depends on the amplitude of wave. The frequency also remain constant.

13. **Sol.** (c): If both the slits are illuminated by two bulbs of same power, no interference pattern will be observed on the screen. This is because waves reaching at any point on the screen do not have a constant phase difference, as phase difference from two incoherent sources changes randomly. Therefore, maxima and minima would also change their positions randomly and in quick succession. This will result in general illumination of the screen.

14. **Sol.** (c): When source in Young's double slit experiment is of white light, the central fringe is white as all colours meet there in phase.

15. **Sol.** (a): For reflected system of the film, the maxima or constructive interference is $2\mu t \cos r =$ *while* the maxima for transmitted system of film is given by equation $2\mu t \cos r = n\lambda$ where t is thickness of the film and r is angle of refraction. From these two equations we can see that condition for maxima in reflected system and transmitted system are just opposite.

16. **Sol.** (c): When one of slits is covered with cellophane paper, the intensity of light emerging from the slit is decreased (because this medium is translucent). Now the two interfering beam have different intensities or amplitudes. Hence, intensity at minima will not be zero and fringes will become indistinct.

17. **Sol.** (a): As a broad source is equivalent to a large number of narrow sources lying side by side. Each set of these sources will produce an interference pattern of its own which will overlap on another to such an extent that all traces of a fringe system is lost and results in general illumination. Because of this reason, for interference a narrow slit should be used.

➢ Case Study

18. Answers

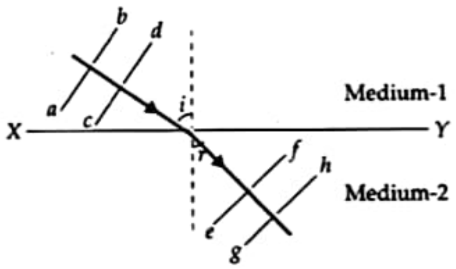
1. (*b*) Maxwell proved theoretically that light has a transverse wave nature.

2. (*b*) Medium-2 is denser because the refracted ray bends towards the normal.

3. (*d*) Points c and d lie on the same wavefront, so $\phi_c = \phi_d$.

Similarly, $\phi_e = \phi_f \quad \therefore \quad \phi_d - \phi_f = \phi_c - \phi_e$

4. (*a*) Light travels as a parallel beam in each medium.



5. (*d*) During refraction, the speed and wavelength change but frequency remains unchanged.

19. Answers

7. (*c*) When $p = \frac{\lambda}{4}, \phi = \frac{\pi}{2}$

$$I = 4I_0 \cos^2 \frac{\pi}{4} = 4I_0 \cdot \frac{1}{2} = 2I_0$$

8. (*d*) $\lambda_G < \lambda_O < \lambda_R$

Fringe width $\omega = \frac{D\lambda}{d} \Rightarrow \omega \propto \lambda$

$\therefore \omega_G < \omega_O < \omega_R \Rightarrow \omega_2 < \omega_1 < \omega_3 \Rightarrow \omega_3 > \omega_1 >$

9. (*c*) For first bright fringe,

$$x_1 = \frac{D\lambda}{d} \Rightarrow 0.4 = \frac{500 \times \lambda}{0.8}$$

$$\Rightarrow \lambda = \frac{0.4 \times 0.8}{500} \text{mm} = \frac{32 \times 10^6}{500 \times 100} \text{nm} = \mathbf{640\ nm.}$$

10. (*c*) Angular separation, $\theta = \frac{\beta}{D} = \frac{\lambda}{d}$

θ is independent of D because as D changes, β also changes in the same ratio.

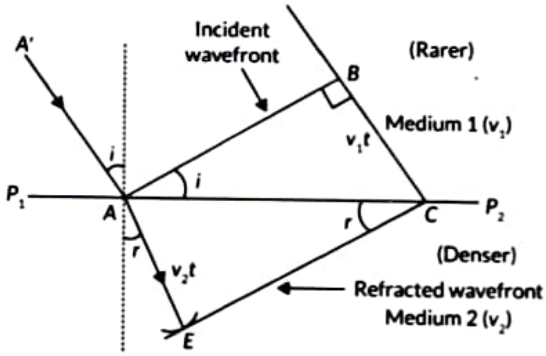
$$\theta \propto \frac{1}{d}$$

As separation d between the two slits is increased, angular separation θ decreases.

Questions

20. Ans. A source of light sends the disturbance in all the directions and continuous locus of all the particles vibrating in same phase at any instant is called as wavefront.

Given figure shows the refraction of a plane wavefront at a rarer medium i.e., $v_1 > v_2$



The incident and refracted wavefronts are shown in figure.

Let the angles of incidence and refraction be i *and* r respectively.

From right ΔABC, we have,

$$\sin\angle BAC = \sin i = \frac{BC}{AC}$$

From right ΔAEC, we have,

$$\sin\angle ECA = \sin r = \frac{AE}{AC}$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{BC}{AE} = \frac{v_1 t}{v_2 t} \; or \; \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = {}^1\mu_2 \quad (\text{a constant})$$

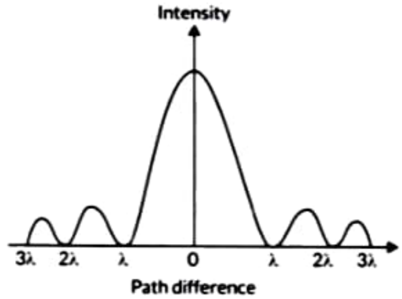
This verifies Snell's law of refraction. The constant ${}^1\mu_2$ is called the refractive index of the second medium with respect to first medium.

21. Ans. Width of central maximum is given by

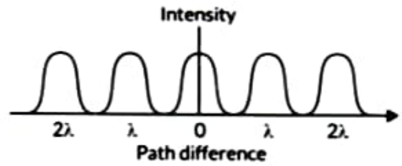
$$\beta_0 = \frac{2D\lambda}{a}$$

If width of slit is reduced then (i) size of central maxima will increase and (ii) intensity of central maximum will decrease.

22. Ans. (i) Single slit diffraction:



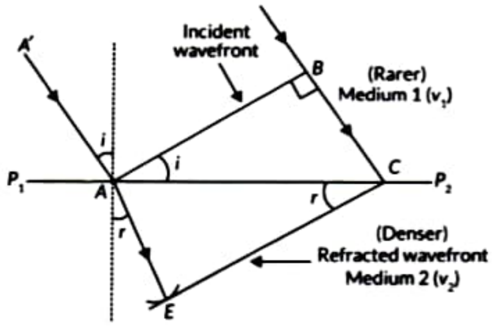
Double slit interference:



(ii) Difference between interference and diffraction

	Interference		Diffraction
1.	Interference is caused by superposition two waves starting from two coherent sources	1.	Diffraction is caused by superposition of a number of waves starting from the slit.
2	All bright and dark fringes are of equal width.	2.	Width of central bright fringe is double of all other maxima.
3.	All bright fringes are of same intensity.	3.	Intensity of bright fringes decreases sharply as we move away from central bright fringe.
4.	Dark Fringes are perfectly dark.	4.	Dark fringes are not perfectly dark.

23. Ans. Snell's law of refraction : Let P_1P_2 represents the surface separating medium 1 and medium 2 as shown in figure.



Let v_1 and v_2 represents the speed of light in medium 1 and medium 2 respectively. We assume a plane wavefront AB propagating in the direction A'A incident on the interface at an angle i. Let t be the time taken by the wavefront to travel the distance BC.

$\therefore \; BC = v_1 t$ [∵ distance = speed x time]

In order to determine the shape of the refracted wavefront, we draw a sphere of radius v_2t from the point A in the second medium (the speed of the wave in second medium is v_2).

Let CE represents a tangent plane drawn from the point C. Then

AE = v_2t

∴ CE would represent the refracted wavefront.

In ΔABC and ΔAEC, we have

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC} \; and \; \sin r = \frac{AE}{AC} = \frac{v_2 t}{AC}$$

Where i and r are the angles of incident and refraction respectively.

$$\therefore \; \frac{\sin i}{\sin r} = \frac{v_1 t}{AC} \cdot \frac{AC}{v_2 t} \Rightarrow \frac{\sin i}{\sin r} = \frac{v_1}{v_2}$$

If c represents the speed of light in vacuum, then

$$\mu_1 = \frac{c}{v_1} \; and \; \mu_2 = \frac{c}{v_2} \Rightarrow v_1 = \frac{c}{\mu_1} \; and \; v_2 = \frac{c}{\mu_2}$$

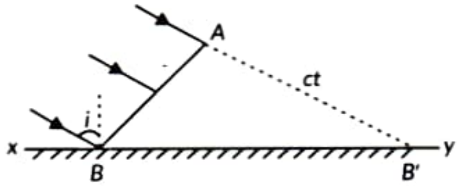
Where μ_1 and μ_2 are the refractive indices of medium 1 and medium 2.

$$\therefore \; \frac{\sin i}{\sin r} = \frac{c/\mu_1}{c/\mu_2} \Rightarrow \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

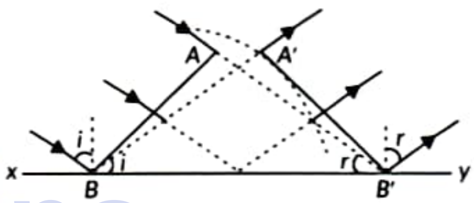
$$\Rightarrow \mu_1 \sin i = \mu_2 \sin r$$

This is the Snell's law of refraction.

24. Ans. A source of light sends the disturbance in all the directions and continuous locus of all the particles vibrating in same phase at any instant is called as wavefront. Laws of reflection by Huygens' principle : Let us consider a plane wavefront AB incident on the plane reflecting surface xy. Incident rays are normal to the wavefront AB.



Let in time t the secondary wavelets reaches B' covering a distance ct. Similarly from each point on primary wavefront AB. Secondary wavelets start growing with the speed c. To find reflected wavefront after time t, let us draw a sphere of radius ct taking B as center and now a tangent is drawn from B' on the sphere the tangent B'A' represents reflected wavefront after time t.



For every point on wavefront AB a corresponding point lie on the reflected wavefront A'B'.

So, comparing two triangle $\Delta BAB'$ and $\Delta B'A'B$

We find that

$AB' = A'B = ct$

$BB' = common$

$\angle A = \angle A' = 90^0$

Thus two triangles are congruent, hence

$\angle i = \angle r$

This proves first law of reflection.

Also incident rays, reflected rays and normal to them all lie in the same plane. This gives second law of reflection.

25. Ans. (i) Reflection and refraction arise through interaction of incident light with atomic constituents of matter which vibrate with the same frequency as that of the incident light. Hence frequency remains unchanged.

(ii) Energy carried by a wave depends on the frequency of the wave, not on the speed of wave propagation.

26. Ans. (a) We know, $\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$

According to question, $I_2 = 50\%\ of\ I_1$

$I_2 = 0.5 I_1;\ \ a_2^2 = 0.5 a_1^2 \quad (\because\ I \propto a^2)$

$a_2 = \frac{a_1}{\sqrt{2}}$

Hence,

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_1/\sqrt{2})^2}{(a_1-a_1/\sqrt{2})^2} = \frac{(1+1/\sqrt{2})^2}{(1-1/\sqrt{2})^2} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 \approx 34$$

(b) The central fringes are white. On the either side of the central white fringe the coloured bands (few coloured maxima and minima) will appear. This is because fringes of different colours overlap.

27. Ans. (a) Angular width, $\theta = \frac{\lambda}{d}\ or\ d = \frac{\lambda}{\theta}$

Here, $\lambda = 600\ nm = 6\times10^{-7} m$

$\theta = 0.1^0 = \frac{0.1\times\pi}{180} rad = \frac{\pi}{1800} rad,\ d = ?$

$\therefore\ d = \frac{6\times10^{-7}\times1800}{\pi} = 3.4\times10^{-4} m$

(b) Frequency of a light depends on its source only. So, the frequencies of reflected and refracted light will be same as that of incident light.

Reflected light is in the same medium (air) so its wavelength remains same as $500\ \overset{0}{A}$.

Wavelength of refracted light, $\lambda_r = \frac{\lambda}{\mu_w}$

μ_w = refractive index of water.

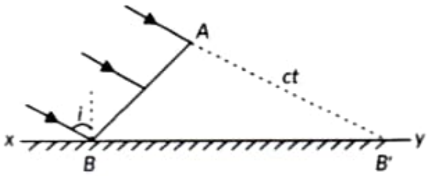
So, wavelength of refracted wave will be decreased.

28. Ans. Two independent monochromatic sources cannot produce sustained interference pattern because the phase difference between the light waves from two independent sources keeps on changing continuously.

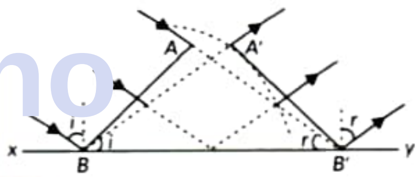
29. Ans. A source of light sends the disturbance in all the directions and continuous locus of all the particles vibrating in same phase at any instant is called as wavefront. Phase speed is the speed with which a wavefront moved outwards from the source.

Laws of reflection by Huygens' principle:

Let us consider a plane wavefront AB incident on the plane reflecting surface xy. Incident rays are normal to the wavefront AB.



Let in time t the secondary wavelets reaches B' covering a distance ct. Similarly from each point on primary wavefront AB. Secondary wavelets start growing with the speed c. To find reflected wavefront after time t, let us draw a sphere of radius ct taking B as center and now a tangent is drawn from B' on the sphere the tangent B'A' represents reflected wavefront after time t.



For every point on wavefront AB a corresponding point lie on the reflected wavefront A'B'.

So, comparing two triangle $\Delta BAB'$ and $\Delta B'A'B$

We find that

$AB' = A'B = ct$

$BB' = common$

$\angle A = \angle A' = 90^0$

Thus two triangles are congruent, hence

$\angle i = \angle r$

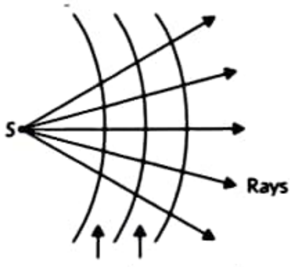
This proves first law of reflection.

Also incident rays, reflected rays and normal to them all lie in the same plane. This gives second law of reflection.

30. Ans. (a) A wavefront is defined as the locus of all the particles vibrating in same phase at any instant.

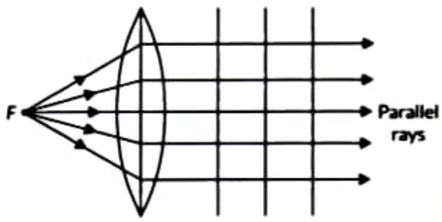
A line perpendicular to the wavefront in the direction of propagation of light wave is called a ray.

(b) (i) The wavefront will be spherical of increasing radius as shown in figure.



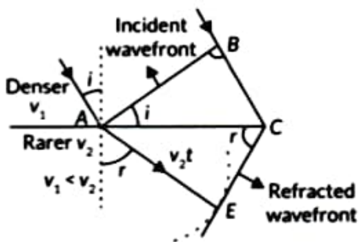
Spherical wavefront

(ii) When source is at the focus, the rays coming out of the convex lens are parallel, so wavefront is plane as shown in figure.



Plane wavefront

(iii)



31. Ans. (a) Condition for constructive interference,

$$\cos\Delta\phi = +1$$

$$2\pi\frac{\Delta x}{\lambda} = 0, 2\pi, 4\pi......$$

$$or \quad \Delta x = n\lambda; \quad n = 0, 1, 2, 3, ...$$

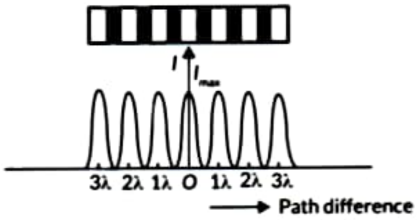
(b) Condition for destructive interference,

$$\cos\Delta\phi = -1$$

$$2\pi\frac{\Delta x}{\lambda} = \pi, 3\pi, 5\pi....$$

$$or \quad \Delta x = (2n-1)\lambda/2$$

Where n = 1, 2, 3.....



32. Soln. (a) We know, $\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$

According to question, $I_2 = 50\%$ of I_1

$$I_2 = 0.5I_1; \qquad a_2^2 = 0.5a_1^2 \qquad (\because I \propto a^2)$$

$$a_2 = \frac{a_1}{\sqrt{2}}$$

Hence,

$$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_1/\sqrt{2})^2}{(a_1-a_1/\sqrt{2})^2} = \frac{(1+1/\sqrt{2})^2}{(1-1/\sqrt{2})^2}$$

$$= \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 \approx 34$$

(b) The central fringes are white. On the either side of the central white fringe the coloured bands (few coloured maxima and minima) will appear. This is because fringes of different colours overlap.

33. Soln. (a) Coherent sources are necessary to produce a sustained interference pattern otherwise the phase difference changes very rapidly with time and hence no interference will be observed.

(b) Intensity at a point, $I = 4I_0\cos^2\left(\frac{\phi}{2}\right)$

Phase difference = $\frac{2\pi}{\lambda}$ × Path difference

At path difference λ,

Phase difference, $\phi = \frac{2\pi}{\lambda}\times\lambda = 2\pi$

$\therefore$ Intensity, $K = 4I_0\cos^2\left(\frac{2\pi}{2}\right)$

[$\because$ Given $I = K$, at path difference λ]

$K = 4I_0$ ....(i)

If path difference is $\frac{\lambda}{3}$, then phase difference will be

$$\phi' = \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

$\therefore$ Intensity,

$$I' = 4I_0 \cos^2\left(\frac{2\pi}{6}\right) = \frac{K}{4} \text{ (Using (i))}$$

34. Soln. Position of first minimum in diffraction pattern

$$y = \frac{D\lambda}{a}$$

So, slit width

$$a = \frac{D\lambda}{y} = \frac{1\times 500\times 10^{-9}}{2.5\times 10^{-3}} = 2\times 10^{-4}\, m$$

35. Soln. Here, $\lambda = 620\, nm = 620\times 10^{-9}\, m$

$$a = 3\times 10^{-3} m, \quad D = 1.5 m$$

Distance of first order minima from the centre,

$$y_1 = \frac{D\lambda}{a} = \frac{1.5\times 620\times 10^{-9}}{3\times 10^{-3}}$$
$$= 3.1\times 10^{-4} m$$

Distance of third order maxima on the same side,

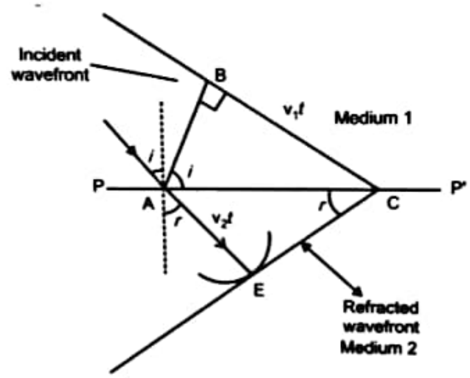
$$y_2 = \frac{7D\lambda}{2a} = \frac{7\times 1.5\times 620\times 10^{-9}}{2\times 3\times 10^{-3}}$$
$$= 10.85\times 10^{-4}\, m$$

Separation between them.

$$y = y_2 - y_1$$
$$= 10.85\times 10^{-4} - 3.1\times 10^{-4}$$
$$= 7.75\times 10^{-4}\, m$$

36. Soln. Wavefront: The continuous locus of all the particles of a medium, which are vibrating in the same phase is called wavefront.

Laws of refraction: Let PP' represent the surface separating medium 1 and medium 2 as shown in fig.



From ΔABC,

$$\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}$$

From ΔAEC,

$$\sin r = \frac{AE}{AC} = \frac{v_2 t}{AC}$$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{v_1 t}{AC} \times \frac{AC}{v_2 t} = \frac{v_1}{v_2} = \mu$$

$$\therefore \quad \frac{\sin i}{\sin r} = \mu$$

Which is Snell's law of refraction of light (first law).

Second law: Incident wavefront, refracted wavefront, normal all lie in the same plane.

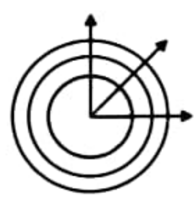
37. Soln. (i) A wavefront is defined as a surface of constant phase.

[Alternatively, A wavefront is the locus of all points in the medium that have the same phase.]

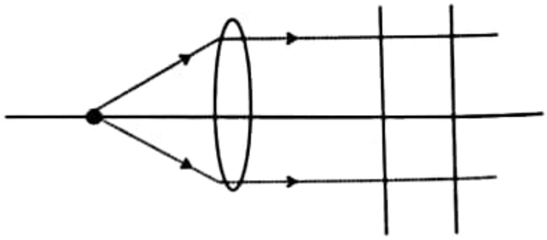
Difference from a ray:

(a) The ray, at each point of a wavefront, is normal to the wavefront at that point.
(b) The ray indicates the direction of propagation of wave while the wavefront is the surface of constant phase.
(c) The shape of the wavefront, in the three cases, are as shown.

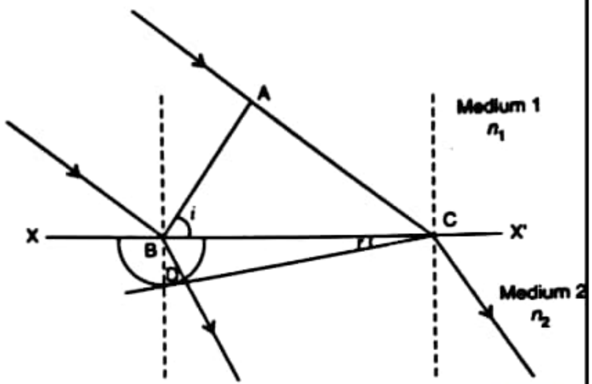
(ii) (a)



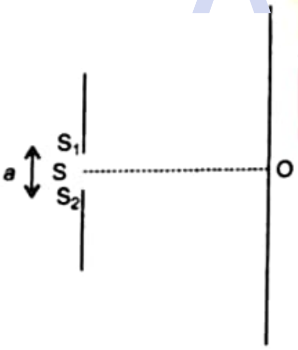
(b)



(c)

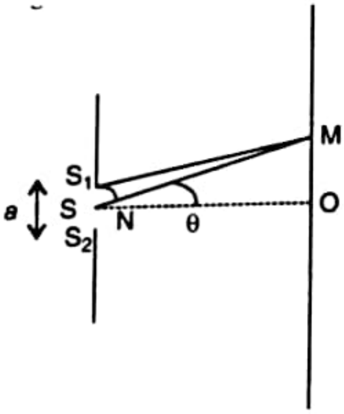


38. Soln. Explanation: As per Huygens Principle, net effect at any point = Sum total of contribution of all wavelets with proper phase difference.



At the central point (O) the contribution from each half in SS_1 is in phase with that from the corresponding part bin SS_2. Hence, O is a maxima.

From the figure,



At the point M where $S_2M - S_1M = \lambda/2$

Phase difference between each wavelet from SS_1

And corresponding wavelet from SS_2 = $\lambda/2$

Hence, M would be a minima.

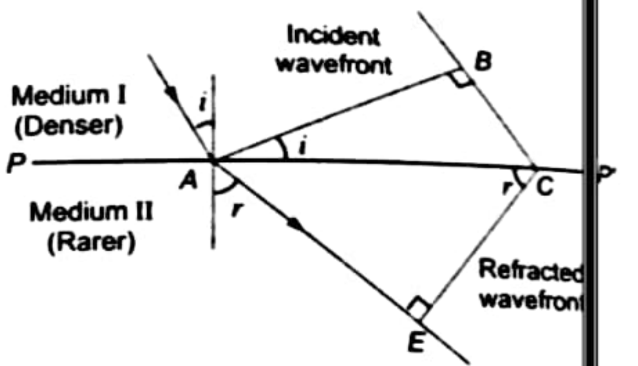
All such points (path difference = $n\lambda/2$) are also minima.

Similarly, all points, for which path difference = $(2n+1)\lambda/2$, are maxima but with decreasing intensity.

The angular width of central maxima = λ/a

∴ The size of central maxima will be reduced to half and intensity of central maxima will be four times if slit is made double the original width.

39. Soln. We assume a plane wavefront AB propagating in denser medium incident on the interface PP' at angle $\hat{i}$ as shown in fig. Let τ be the time taken by the wave front to travel a distance BC. If v_1 is the speed of the light in medium I.



So, $BC = v_1\ \tau$

In order to find the shape of the refracted wavefront, we draw a sphere of radius AE = $v_2 \tau$, where v_2 is the speed of light in medium II (rarer medium). The tangent plane CE represents the refracted wave front

In ΔABC, $\sin i = \frac{BC}{AC} = \frac{v_1 t}{AC}$

And in ΔACE, $\sin r = \frac{AE}{AC} = \frac{v_2 t}{AC}$

$$\therefore \quad \frac{\sin i}{\sin r} \frac{BC}{AE} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2}$$

.............(1)

Let c be the speed of light in vacuum

So, $\mu_1 = \frac{C}{v_1}$ and $\mu_2 = \frac{C}{v_2}$

$$\frac{\mu_2}{\mu_1} = \frac{v_1}{v_2}$$

...........(2)

From equations (1) and (2), we have

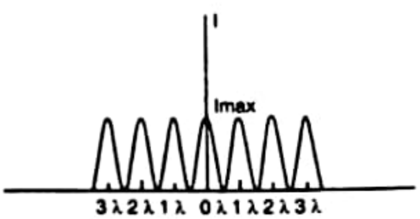
$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

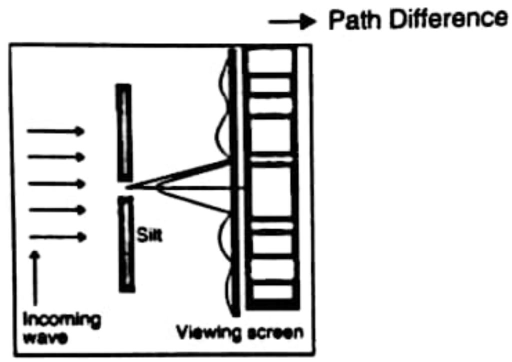
$$\mu_1 \sin i = \mu_2 \sin r$$

It is known as Snell's law.

40. Soln. Two monochromatic sources, which produce light waves, having a constant phase difference, are known as coherent sources.

41. Soln.





Interference	Diffraction
Fringe width is constant.	Fringe width is varied.
Fringes are obtained with the coherent light coming from two slits.	Fringes are obtained with the monochromatic light coming from single slit.
It is superposition of fewer waves.	It is superposition of many waves.
It depends upon the distance between two openings.	It depends upon the aperture of single slit opening.
Many fringes are visible.	Fewer fringes are visible.
All fringes are of same brightness.	Central fringe has maximum brightness, then it reduces gradually.

42. Soln. Phase difference = $\frac{2\pi}{\lambda}$ × Path difference

Path difference = $\frac{\lambda}{6} \Rightarrow$ Phase difference = $\frac{\pi}{3}$

Path difference $\frac{\lambda}{2} \Rightarrow$ Phase difference = π

$$I = 4I_0 \cos^2\left(\frac{\phi}{2}\right)$$

(i) $I_1 = 4I_0 \times \frac{3}{4} = 3I_0$

(ii) $I_2 = 4I_0 \times 0 = 0$

43. Soln. Diffraction due to a Single Slit:

(i) It is produced due to different parts of same wavefronts.

(ii) Central fringe is twice as wide as other fringes.

(iii) Intensity of fringes decreases as we go to successive maxima away from the centre.

(iv) At an angle 1/a, first minima is obtained.

Interference Fringe Young's Double Slit:

(i) It is produced due to two different wavefronts.

(ii) Fringe width is of same size.

(iii) Fringes have same intensity.

(iv) At an angle λ / a, maxima is obtained.

44. Soln. Separation between two dark bands on each side of central bright fringe = width of bright fringe,

We know width of the central fringe

$$=\frac{2D\lambda}{a}$$

Where, D = distance of slit from screen,

λ = wavelength of the light,.

a = width of the slit.

According to question

Width of central fringe = $\frac{1.5\times2\times6000\times10^{-10}}{1\times10^{-4}}$

So the distance between the two dark lines on either side of the central maximum

= 18 mm

Angular spread of the first diffraction minimum $=\frac{\lambda}{a}$

$$=\frac{6000\times10^{-10}}{1\times10^{-4}}$$

$$=6\times10^{-3}\ rad$$

45. Soln. (i) Reflection and refraction arise through interaction of incident light with atomic constituents of matter which vibrate with the same frequency as that of the incident light. Hence frequency remains unchanged.

(ii) No. [Energy carried by a wave depends on the amplitude of the wave, not on the speed of wave propagation].

(iii) For a given frequency, intensity of light in the photon picture is determined by the number of photon incident normally on a crossing an unit area per unit time.

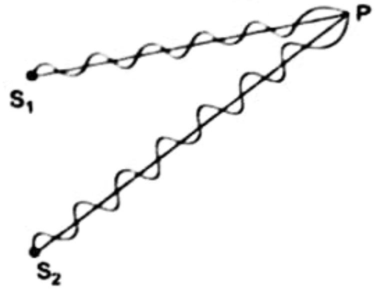
46. Soln. Coherent sources are those which have exactly the same frequency and are in this same phase or have a zero or constant difference.

Conditions:

(i) The sources should be monochromatic and originating from common single source.

(ii) The amplitudes of the waves should be equal.

(a) Condition for formation of bright and dark fringes.



Suppose a narrow slit S is illuminated by monochromatic light of wavelength λ

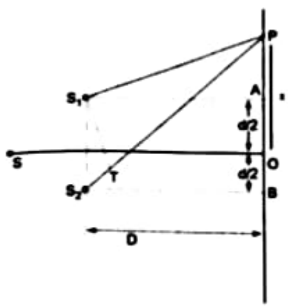
The light rays from two coherent sources S_1 and S_2 are reaching a point P, have a path different $(S_2P - S_1P)$.

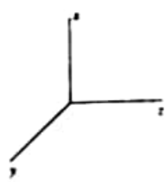
(i) If maxima (bright fringe) occurs at point P, then

$S_2P - S_1P = n\lambda$ (n = 0, 1, 2, 3......)

(ii) If minima (dark fringe) occurs at point P, then

$$S_2P - S_1P = (2n+1)\frac{\lambda}{2} \qquad (n = 0, 1, 2, 3...)$$





Light waves starting from S and fall on both slits S_1 and S_2. Then S_1 and S_2 behave like two coherent sources. Spherical waves emanating from S_1 and S_2 superpose on each other, and produces interference pattern on the screen. Consider a point P at a distance x from 0, the centre of screen. The position of maxima (or minima) depends on the path difference. ($S_2T = S_2P - S_1P$).

From right angled ΔS_2BP and ΔS_1AP,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(x + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(x - \frac{d}{2}\right)^2\right] = 2xd$$

$$(S_2P + S_1P)(S_2P - S_1P) = 2xd$$

$$\Rightarrow \quad S_2P - S_1P = \frac{2xd}{(S_2P + S_1P)}$$

In practice, the point P lies very close to O, therefore

$$S_2P = S_1P = D$$

$$S_2P - S_1P = \frac{2xd}{2D} = \frac{xd}{D} \quad ....(i)$$

For constructive interference (Bright fringes)

Path difference, $\frac{dx}{D} = n\lambda$ where n = 0, 1, 2, 3............

$$x = \frac{nD\lambda}{d}$$

For n = 0, $x_0 = 0$ Central bright fringe

For n = 1, $x_1 = \frac{D\lambda}{d}$ 1st bright fringe

For n = 2, $x_2 = \frac{2D\lambda}{d}$ 2nd bright fringe

For n = n, $x_n = \frac{nD\lambda}{d}$ nth bright fringe

The distance between two consecutive bright fringes is

$$\beta = x_n - x_{n-1}$$

$$= \frac{nD\lambda}{d} - \frac{(n-1)D\lambda}{d} = \frac{D\lambda}{d}$$

For destructive interference (dark fringes)

Path difference $\frac{dx}{D} = (2n-1)\frac{\lambda}{2}$

$$x = (2n-1)\frac{D\lambda}{2d} \text{ where } n = 1, 2, 3, ...$$

For n = 1, $x'_1 = \frac{D\lambda}{2d}$ for 1st dark fringe

For n = 2, $x'_2 = \frac{3D\lambda}{2d}$ for 2nd dark fringe

For n = n, $x'_n = (2n-1)\frac{D\lambda}{2d}$ for nth dark fringe.

The distance between two consecutive dark fringe is

$$\beta' = (2n-1)\frac{D\lambda}{2d} - [2(n-1)-1]\frac{D\lambda}{2d} = \frac{D\lambda}{d}$$

The distance between two consecutive bright or dark fringes is called fringe width (w).

$\therefore$ Fringe width = $\frac{D\lambda}{d}$

The expression for fringe width is free from n. Hence the width of all fringes of red light are broader than the fringes of blue light.

(b) Intensity of light (using classical theory) is given as $I \propto$ (Width of the slit) $\propto$ (Amplitude)^2

$$\frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \frac{25}{9}$$

$$\frac{a_1 + a_2}{a_1 - a_2} = \frac{5}{3} \quad \Rightarrow \quad \frac{a_1}{a_2} = \frac{4}{1}$$

Intensity ratio

$$\frac{I_1}{I_2} = \frac{w_1}{w_2} = \frac{a_1^2}{a_2^2}$$

$$\frac{I_1}{I_2} = \left(\frac{4}{1}\right)^2 = \frac{16}{1}$$

(c) The condition for the interference fringes to be seen is

$$\frac{s}{b} < \frac{\lambda}{d}$$

When s is the size of the source and b is the distance of this source from plane of the slit.

47. Soln. (a) Conditions of Constructive and Destructive Interference:

When two waves of same frequency and constant initial phase difference travel in the same direction along a straight line simultaneously, then superpose in such a way that the intensity of the resultant wave is maximum at certain points and minimum at certain other points. The phenomenon of redistribution of intensity due to superposition of two waves of same frequency and constant initial phase difference is called the interference. The waves of same frequency and constant initial phase difference are called coherent waves. At points of medium where the waves arrive in the same phase, the resultant intensity is maximum and the interference at these points is said to be constructive. On the other hand, at points of medium where the waves arrive in opposite phase, the resultant intensity is minimum and the interference at these points is said to be destructive. The positions of maximum intensity are called maxima while those of minimum intensity are called minima. The interference takes place in sound and light both.

Mathematical Analysis: Suppose two coherent waves travel in the same direction along a straight line the frequency of each wave is $\frac{\omega}{2\pi}$ and amplitudes of electric field are a_1 and a_2 respectively. If at any time t, the electric fields of waves at a point are y_1 and y_2 respectively and phase difference is ϕ, then equation of waves may be expressed as

$$y_1 = a_1 \sin \omega t$$
..........(i)

$$y_2 = a_2 \sin(\omega t + \phi)$$
..........(ii)

According to Young's principle of superposition, the resultant displacement at that point will be

$$y = y_1 + y_2$$
...........(iii)

Substituting values of y_1 and y_2 from (i) and (ii) in (iii), we get

$$y = a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

Using trigonometric relation

$$\sin(\omega t + \phi) = \sin \omega t \cos \phi + \cos \omega t \sin \phi$$

We get

$$y = a_1 \sin \omega t + a_2(\sin \omega t \cos \phi + \cos \omega t \sin \phi)$$

$$= (a_1 + a_2 \cos \phi) \sin \omega t + (a_2 \sin \phi) \cos \omega t$$
...........(iv)

Let $a_1 + a_2 \cos \phi = A \cos \theta$
...........(v)

And $a_2 \sin \phi = A \sin \theta$
........(vi)

Where A and θ are new constants.

Then equation (iv) gives

$$y = A \cos \theta \sin \omega t + A \sin \theta \cos \omega t = A \sin(\omega t + \theta)$$
....(vii)

This is the equation f the resultant disturbance. Clearly the amplitude of resultant disturbance is A and phase difference from first wave is θ. The values of A and θ are determined by (v) and (vi). Squaring (v) and (vi) and then adding, we get

$$(a_1 + a_2 \cos \phi)^2 + (a_2 \sin \phi)^2 = A^2 \cos^2 \theta + A^2 \sin^2 \theta$$

Or

$$a_1^2 + a_2^2 \cos^2 \phi + 2a_1 a_2 \cos \phi + a_2^2 \sin^2 \phi = A^2(\cos^2 \theta + \sin^2 \theta)$$

As $\cos^2 \theta + \sin^2 \theta = 1$, we get

$$A^2 = a_1^2 + a_2^2(\cos^2 \phi + \sin^2 \phi) + 2 a_1 a_2 \cos \phi$$

Or $A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$

$\therefore$ Amplitude,

$$A = \sqrt{a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi}$$
......(viii)

As the intensity of a wave is proportional to its amplitude in arbitrary units $I = A^2$

$\therefore$ Intensity of resultant wave

$$I = A^2 = a_1^2 + a_2^2 + 2a_1 a_2 \cos \phi$$
...........(ix)

Clearly the intensity of resultant wave at any point depends on the amplitudes of individual waves and the phase difference between the waves at the point.

Constructive Interference: For maximum intensity at any point $\cos\phi = +1$

Or phase difference $\phi = 0,\ 2\pi, 4\pi, 6\pi.....$

$$= 2n\pi\ (n = 0,1,2,.....)$$

..........(x)

The maximum intensity,

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$$

.........(xi)

Path difference $\Delta = \frac{\lambda}{2\pi} \times$ Phase difference =

$\frac{\lambda}{2\pi} \times 2n\pi = n\lambda$ ....(xii)

Clearly the maximum intensity is obtained in the region of superposition at those points where waves meet in he same phase or the phase difference between the waves is even multiple of π or path difference between them is the integral multiple of λ and maximum intensity is $(a_1 + a_2)^2$ which is greater than the sum intensities of individual waves by an amount $2a_1a_2$.

Destructive Interference: For minimum intensity at any point $\cos\phi = -1$

Or phase difference, $\phi = \pi, 3\pi, 5\pi, 7\pi.....$

$$= (2n-1)\pi,\ n = 1,2,3...$$

.......(xiii)

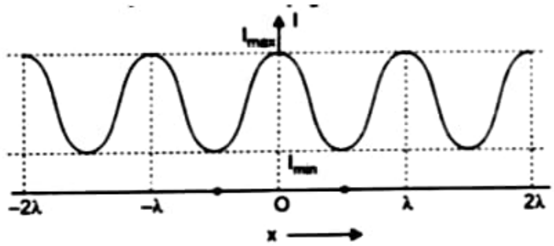
In this case the minimum intensity,

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2 = (a_1 - a_2)^2$$

Path difference, $\Delta = \frac{\lambda}{2\pi} \times$ Phase difference

$$= \frac{\lambda}{2\pi} \times (2n-1)\pi = (2n-1)\frac{\lambda}{2}$$

Clearly, the minimum intensity is obtained in the region of superposition at those points where waves meet in opposite phase or the phase difference between the waves is odd multiple of π or path difference between the waves is odd multiple of $\frac{\lambda}{2}$ and minimum intensity = $(a_1 - a_2)^2$ which is less than the sum of intensities of the individual waves by an amount $2a_1a_2$.



Fro equations (xii) and (xvi) it is clear that the intensity $2a_1a_2$ is transferred from positions of minima to maxima. This implies that that the *interference is based on conservation of energy* i.e., *there is no wastage of energy.*

Variation of intensity of light with position x is shown in fig.

(a) Comparison of two Slit Young's Interference pattern and Single slit diffraction pattern
Both patterns are the result of wave nature of light; both patterns contain maxima and minima. Interference pattern is the result of superposing two coherent wave while the diffraction pattern is the superposition of large number of waves originating from each point on a single slit.
Differences: (i) In Young's two slit experiment; all maxima are of same intensity while in diffraction at a single slit, the intensity of central maximum is maximum and it falls rapidly for first, second order secondary maxima on either side of it.
(ii) In Young's interference the fringes are of equal width while in diffraction at a single slit, the central maximum is twice as wide as other maxima. The intensity falls as we go to successive maxima away from t7he centre on either side.
(iii) In a single slit diffraction pattern of width a, the first minimum occurs at λ / a; while in two slit interference pattern of slit separation a, we get maximum at the same angle $\frac{\lambda}{a}$.

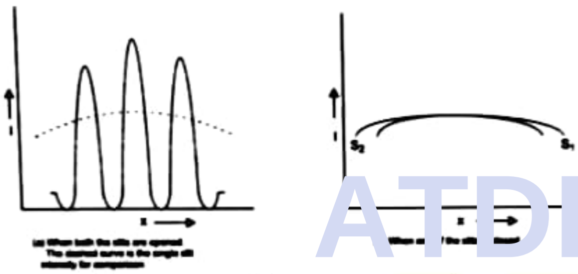
48. Soln. Interference of light: When two waves of same frequency and constant initial phase difference travel in the same direction along a straight line simultaneously, They superpose in such a way that the intensity of the resultant wave is maximum at certain points and minimum at certain other points. This phenomenon of redistribution of energy due to superposition of two waves of same frequency and constant initial phase difference is called interference.

Conditions for Sustained Interference of Light Waves

To obtain sustained (well-defined and observable) interference pattern, the intensity must be maximum and zero at points corresponding to constructive and destructive interference. For the purpose following conditions must be fulfilled:

(i) *The two interfering sources must be coherent and of same frequency, i.e., the sources should emit light of the same wavelength or frequency and their initial phase should remain constant.* If this condition is not satisfied the phase difference between the interfering wave will vary continuously. As a result the resultant intensity at any point will vary with time being alternately maximum and minimum, just like the phenomenon of beats in sound.

(ii) *The interfering waves must have equal amplitudes.* Otherwise the minimum intensity will not be zero and there will be general illumination.



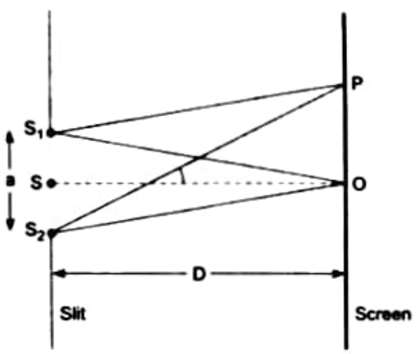
The variation of intensity I versus the position x on the screen in Young's experiment.

Fringe width, $\beta = \dfrac{D\lambda}{d}$.

(i) $\beta \propto D$, therefore with the decrease of separation between the plane of slits and screen, the fringe width decreases.

(ii) On increasing the separation between two slits (d), the fringe separation decreases as β is inversely proportional to d $\left(i.e.,\ \beta \propto \dfrac{1}{d}\right)$

49. Soln.



According to Huygen's principle. "The net effect at any point due to a number of wavelets is equal to sum total of contribution of all wavelets with proper phase difference. The point O is maxima because contribution from each half of the slit S_1S_2 is in phase, i.e., the path difference is zero.

At point p

(i) If $S_2P - S_1P = n\lambda \Rightarrow$ the point P would be minima.

(ii) If $S_2P - S_1P = (2n+1)\dfrac{\lambda}{2} \Rightarrow$ the point would be maxima but with decreasing intensity.

The width of central maxima = $\dfrac{2\lambda D}{a}$

When the width of the slit is made double the original width, then the size of central maxima will be reduced to half and intensity will be four times.

50. Soln. Resultant intensity at any point having a phase difference ϕ is given by $I = 4I_0\cos^2\dfrac{\phi}{2}$

When path difference is λ, phase difference is 2π

$\therefore I = 4I_0\cos^2\pi = 4I_0 = K$ (given) ....(i)

When path difference, $\Delta = \dfrac{\lambda}{3}$, the phase difference

$$\phi' = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}\times\frac{\lambda}{3} = \frac{2\pi}{3}$$

$$I' = 4I_0\cos^2\frac{2\pi}{3} \qquad (\text{since } K = 4I_0)$$

$$= K\cos^2\frac{2\pi}{3} = K\times\left(-\frac{1}{2}\right)^2 = \frac{1}{4}K$$

51. Soln. Here, $n = 1,\ \lambda = 6\times10^{-5}\,cm$

Distance of screen from slit = 100 cm

Distance of first minimum from central maxima = 0.1 cm

$$\sin\theta = \frac{\text{Distance of } 1^{st} \text{ minima from the central maxima}}{\text{Distance of the screen from the slit}}$$

$$\theta_1 = \frac{0.1}{100} = \frac{1}{1000}$$

We know that

$$a\sin\theta = n\lambda \Rightarrow a = \frac{\lambda}{\theta_1} = 0.06\,cm$$

52. Soln. For minima in diffraction pattern,

$d\sin\theta = n\lambda$

For first minima,

$$d\sin\theta_1 = (1)\lambda_1 \Rightarrow \sin\theta_1 = \frac{\lambda_1}{d}$$

For first maxima,

$$d\sin\theta_2 = \frac{3}{2}\lambda_2 \Rightarrow \sin\theta_2 = \frac{3\lambda_2}{2d}$$

The two will coincide if, $\theta_1 = \theta_2$ or

$\sin\theta_1 = \sin\theta_2$

$$\therefore \frac{\lambda_1}{d} = \frac{3\lambda_2}{2d} \Rightarrow \lambda_2 = \frac{2}{3}\lambda_1 = \frac{2}{3}\times 660\,nm = 440\,nm$$

53. Soln. (i) Here $a = 1\times10^{-4}\,m,\ D = 1.5\,m$

$$\lambda = 6000\,\overset{0}{A} = 6000\times10^{-10}\,m$$

The distance between the two dark bands on each side of central band is equal to width of the central bright band,

i.e., $$\frac{2D\lambda}{a} = \frac{2\times1.5\times6000\times10^{-10}}{1\times10^{-4}} = 18\,mm$$

54. Soln. Given that distance between the two slits, d = 0.15 mm

Wavelength of monochromatic light,

$\lambda = 450\,nm$

Distance between the screen and slits, D = 1 m

(a) (i) Distance of nth bright fringe from central maximum = $\frac{n\lambda D}{d}$

$$= 2\times\frac{450\times10^{-9}\times1}{0.15\times10^{-3}} \qquad [\because n = 2]$$

= 6×10^{-3} m = 6 mm

(ii) Distance of n^{th} dark fringe from central maximum

$$= (2n-1)\frac{\lambda D}{2d}$$

$$= (2\times2-1)\times\frac{450\times10^{-9}\times1}{2\times0.15\times10^{-3}}\ [\because n = 2]$$

$$= \frac{3}{2}\times3\times10^{-3} = 4.5\,nm$$

(b) Since, width of bright or dark fringes is given by $\beta = \frac{\lambda D}{d}$,

Thus when screen is moved away, D increases and hence fringe width increases.

55. Soln.

(a) Yes, Huygen's principle is valid for longitudinal as well as transverse waves and for all wave phenomena.

(b) The diffraction effect is more pronounce if the size of the aperture or the obstacle is of the order of wavelength of wave. As wavelength of light ($\square 10^{-6}$ m) is much more smaller than size of object around us so diffraction of light is not easily seen but sound wave has large wavelength (15 mm < λ < 15 m), they get easily diffracted by objects around us.

(c) In young's double slit experiment, if one slit is fully closed, the new pattern has larger central maximum in angular size.

SURE SHOT QUESTIONS

ARVIND ACADEMY

Chapter – 11

Dual Nature of Radiation and Matter

MCQ (1 mark)

1. Soln. (d): Velocity acquired by a particle while falling from a height H is

$v = \sqrt{2gH}$ ....(i)

As $\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2gH}}$ (Using (i))

Or $\lambda \propto \frac{1}{\sqrt{H}}$

2. Soln. (b): Here, E = 1 MeV = 10^6 eV, h = 6.63 x 10^{-34} J s, c = 3 x 10^8 m s^{-1}

$\therefore\ hc = \frac{6.63\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}} \approx 1240\, eV\, nm$

As $E = \frac{hc}{\lambda}$ or $\lambda = \frac{hc}{E} = \frac{1240\, eV\, nm}{10^6\, eV} = 1.24\times10^{-3}$ nm

3. Soln. (d): When a beam of electrons of energy E_0 is incident on a metal surface kept in an evacuated chamber, electrons can be emitted with maximum energy E_0 (due to elastic collision) and with any energy less than E_0, when part of incident energy of electron is used in liberating the electrons from the surface of metal.

4. Soln. (b): Kinetic energy of particle, $K = \frac{1}{2}mv^2$ or $mv = \sqrt{2mK}$

de Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$

for the given value of K, $\lambda \propto \frac{1}{\sqrt{m}}$

$\therefore\ \lambda_p : \lambda_n : \lambda_e : \lambda_\alpha = \frac{1}{\sqrt{m_p}} : \frac{1}{\sqrt{m_n}} : \frac{1}{\sqrt{m_e}} : \frac{1}{\sqrt{m_\alpha}}$

Since $m_p = m_n$, hence $\lambda_p = \lambda_n$

As $m_\alpha > m_p$, therefore $\lambda_\alpha < \lambda_p$

As $m_e < m_n$, therefore $\lambda_e > \lambda_n$

Hence $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$

5. Soln. (a): Here $\vec{v} = v_0\hat{i}$, $\vec{B} = B_0\hat{j}$

Force on moving electron due to magnetic field is

$\vec{F} = -e(\vec{v}\times\vec{B}) = -e(v_0\hat{i}\times B_0\hat{j}) = -ev_0B_0\hat{k}$

As this force is perpendicular to $\vec{v}$ and $\vec{B}$, so the magnitude of $\vec{v}$ will not change. i.e., momentum (= mv) will remain constant in magnitude.

Therefore, de Broglie wavelength, $\lambda\left(=\frac{h}{mv}\right)$ remains constant.

6. Soln. (c): Initial de Broglie wavelength of electron, $\lambda_0 = \frac{h}{mv_0}$

Force on electron in electron field,

$\vec{F} = -e\vec{E} = -eE_0\hat{j}$

Acceleration of electron, $\vec{a} = \frac{\vec{F}}{m} = -\frac{eE_0}{m}\hat{j}$

It is acting along negative y – axis.

The initial velocity of electron along x-axis $\vec{v}_{x_0} = v_0\hat{i}$.

Initial velocity of electron along y-axis $\vec{v}_{y_0} = 0$.

Velocity of electron after time t along x-axis, $\vec{v}_x = v_0 \hat{i}$

($\because$ there is no acceleration of electron along x-axis)

Velocity of electron after time t along y-axis,

$$\vec{v}_y = 0 + \left(-\frac{eE_0}{m}\hat{j}\right)t = -\frac{eE_0}{m}t\hat{j}$$

Magnitude of velocity of electron after time t is

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2 + \left(\frac{-eE_0}{m}t\right)^2} = v_0\sqrt{1 + \frac{e^2E_0^2t^2}{m^2v_0^2}}$$

de Broglie wavelength associated with electron at time t is

$$\lambda = \frac{h}{mv} = \frac{h}{mv_0\sqrt{1 + \frac{e^2E_0^2t^2}{(m^2v_0^2)}}} = \frac{\lambda_0}{\sqrt{1 + \frac{e^2E_0^2t^2}{(m^2v_0^2)}}}$$

7. Ans. (b): Energy of photon of wavelength λ.

$$E = hv$$

$$\because \quad c = v\lambda \quad or \quad v = \frac{c}{\lambda} \quad or \quad E = \frac{hc}{\lambda}$$

8. Ans. (b): According to Einstein's photoelectric equation,

$$eV = hv - \phi$$
$$\phi = hv - eV$$

Here, energy of photon = 3.2 eV
Stopping potential = 1.5 eV

$$\therefore \quad \phi = 3.2eV - 1.5\,eV = 1.7\,eV$$

9. Ans. (b) : Kinetic energy of emitted electron

$$K = hv - W_0 \quad \text{............(i)}$$
$$\text{hv} = \text{K} + \text{W}_0 \; or \; hv - W_0 \quad \text{......(ii)}$$

Work function is constant for every metal surface and if we double the frequency of incident light, then new KE will be

$$K' = h(2\text{v}) - \text{W}_0 = 2hv - W_0$$

Using equation (ii), we get

$$K' = 2(\text{K} + \text{W}_0) - \text{W}_0 = 2K + W_0$$

$\therefore$ Kinetic energy will be more than 2K, hence option (b) is correct.

10. Ans. (c) : $E = hv = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{E}$$

11. Ans. (c) : $(\text{KE})_{\text{max}} = hv - \phi$

$$\frac{(\text{KE})_1}{(\text{KE})_2} = \frac{hv_1 - \phi}{hv_2 - \phi} = \frac{1-0.5}{2-0.5} = \frac{0.5}{1.5} = \frac{1}{3}$$

12. Ans. (b) : $E = hv = hv_{OA} + (\text{KE})_{\text{max}_A} =$

$$h_{v_{O_B}} + (KE)_{\text{max}_B}$$

$$\therefore hv = \frac{hv}{2} + K_A = \frac{hv}{3} + K_B$$

$$\Rightarrow \frac{K_A}{K_B} = \frac{hv}{2} \times \frac{3}{2hv} = \frac{3}{4}$$

13. Ans. (b) : De Broglie wavelength is given as

$$\lambda = \frac{h}{p}$$

But, $p = \sqrt{2mE}$

$$\therefore \lambda = \frac{h}{\sqrt{2mE}} \therefore \lambda \propto \frac{1}{m} \Rightarrow \frac{\lambda_{proton}}{\lambda_{alpha}} = \sqrt{\frac{m_{alpha}}{m_{proton}}}$$

$$m_{alpha} = 4m_{proton}$$

$$\therefore \quad \frac{\lambda_{proton}}{\lambda_{alpha}} = \sqrt{\frac{m_{alpha}}{m_{proton}}} = \sqrt{\frac{4}{1}} = 2$$

So, correct option is (b).

14. Ans. (d) : $\lambda = \frac{h}{p} \Rightarrow \lambda \propto \frac{1}{p}$

The graph is rectangular hyperbola.

15. Ans. (d) de Broglie wavelength is given by:

$$\lambda = \frac{h}{mv}$$

(i) When velocity decreases, wavelength increases

(ii) $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2meV_0}}; \lambda \propto \frac{1}{\sqrt{V_0}}$

When accelerating potential increases, the de-broglie wavelength decreases.

16. Ans. (c) : By momentum of conservation

$$MV = m_1v_1 + m_2v_2 + ...$$

Particle is at rest initially $\therefore m_1v_1 = -m_2v_2$

$|P_1| = |P_2|$

According to de-Broglie wavelength $\lambda = \frac{h}{P}$

$$\frac{\lambda_1}{\lambda_2} = \frac{P_2}{P_1} = 1$$

17. Ans. (b) : Photoelectric emission from a given surface of metal can take place when its work function is less than the energy of incident photon.

18. Ans. (c) : Work function for a metal surface,

$$\phi = \frac{hc}{\lambda_0}$$

Given that, $\phi = 4.14 eV = 4.14 \times 1.6 \times 10^{-19} J$

$$\therefore \lambda_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4.14 \times 1.6 \times 10^{-19}} = 3000 Å$$

➢ Assertion-Reasoning (1 mark)

19. Ans. (a) : Electrons being emitted as photoelectrons have different velocities. As all the electrons do not occupy the same level of energy but they occupy continuous band and levels. So, electrons ejected out from different levels come out with different energies.

20. Sol. (a)

21. Sol. (a): The photoemissive cell contain two electrodes are enclosed in a glass bulb which may be evacuated or contain an inert gas at low pressure. An inert gas in the cell gives greater current but causes a time lag in the response of the cell to very rapid changes of radiation which may make it unsuitable for some purpose.

22. Sol. (b):Less work function means less energy is required for ejecting out the electrons.

23. Sol.

(a): Equivalent mass of photon (m) is given from equation

$$E = mc^2 = h\upsilon \quad \therefore \quad m = \frac{h\upsilon}{c^2}$$

where E is energy, m is mass, c is speed of light, h is Planck's constant, υ is frequency.

$\therefore$ Momentum of photon $= \frac{h\upsilon}{c^2} \times c = \frac{h\upsilon}{c}$

24. Sol.

(b): Mass of moving photon

$$m = \frac{h\upsilon}{c^2} = \frac{h}{c\lambda} \text{ and } E = mc^2.$$

25. Sol. (b)

➢ Case Study

26. Answers

1. (*d*) The saturation value of the photoelectric current depends on the intensity of incident light.

2. (*c*) The kinetic energy of the fastest photoelectrons is

$$K_{max} = h(\nu - \nu_0)$$

other electrons have kinetic energy < K_{max}.

3. (*c*) Saturation increases with the increase in intensity of incident radiation for fixed ν.

4. (*b*) Stopping potential increases with the increase in frequency of incident radiation.

5. (*b*) $E_k = h\nu - W_0$; $y = mx + c$

The graph of E_k versus ν is a straight line with slope, m = Planck's constant (h).

27. Answers

6. (*b*) Momentum, $p = h/\lambda$

As both electron and proton have same λ, so they have the same momentum.

7. (*a*) $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$\Rightarrow \quad p = \sqrt{2mK} \qquad \therefore \qquad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

8. (*a*) $\lambda = \frac{12.3}{\sqrt{V}}\,\text{Å} = \frac{12.3}{\sqrt{10 \times 10^3}} = 0.12\ \text{Å}$

9. (*b*) $\lambda = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{2m_\alpha q_\alpha V}{2m_p q_p V}} = \sqrt{\frac{2 \times 4m_p \times 2e \times V}{2m_p \times e \times V}} = 2\sqrt{2} : 1.$$

10. (*d*) $\lambda = \frac{h}{\sqrt{2mK}}$

For same λ, mK = constant

As $m_e < m_p \simeq m_n < m_d \quad \therefore K_e > K_p \simeq K_n > K_d$.

➤ Questions

28. Ans. (i) Given, $\nu = 6.0 \times 10^{14}\ Hz$

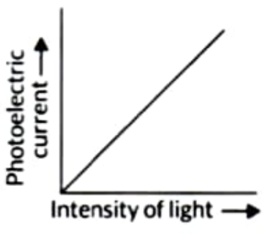
P = 2.0 x 10^{-3} W

Let n is the number of photons emitted by the source per second.

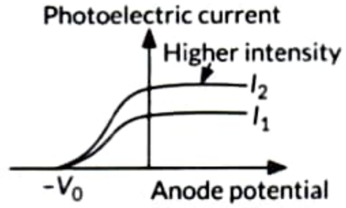
$$n = \frac{P}{E} = \frac{P}{h\nu} = \frac{2 \times 10^{-3}}{6.63 \times 10^{-34} \times 6.0 \times 10^{14}}$$

$= 0.0502 \times 10^{17} = 5 \times 10^{15}$ photons per second.

(ii)



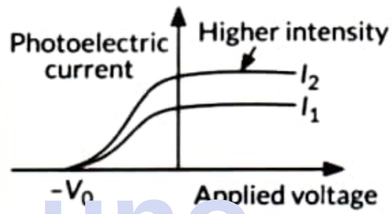
29. Ans. For a given frequency, Photoelectric current varies with anode potential is as shown in graph.



30. Ans. (a) (i) Threshold Frequency: The minimum frequency of incident light which is just capable of ejecting electrons from a metal is called the threshold frequency. It is denoted by ν_0.

(ii) Stopping Potential: The minimum retarding potential applied to anode of a photoelectric tube which is just capable of stopping photoelectric current is called the stopping potential. It is denoted by V_0 (or V_S).

(b)



31. Ans. According to Louis de-Broglie, wave is associated with every moving particle. These ae called matter waves.

∵ de-Broglie wavelength of a charged particle accelerated through a potential difference V is given by, $\lambda = \frac{h}{\sqrt{2mqV}}$

∴ For proton and α particle charges are q and 2q respectively,

$$\therefore \qquad \lambda_p = \frac{h}{\sqrt{2m_p qV}} \ ....(i)$$

$$\lambda_\alpha = \frac{h}{\sqrt{4m_\alpha qV}} \ ....(ii)$$

From eqn. (i) and (ii)

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \left(\frac{4m_\alpha}{2m_p}\right)^{1/2} = 2\sqrt{2} \qquad (\text{As } m_\alpha = 4m_p)$$

32. Ans. (a) Wavelength, $\lambda = 331.5\,nm$

Energy, $E = \frac{hc}{\lambda} = \frac{1240nm - eV}{331.5\,nm} = 3.74\,eV$

Momentum,

$$p = \frac{E}{c} = \frac{3.74\times1.6\times10^{-19}}{3\times10^{8}} = 2\times10^{-27}\ kg\ m\ s^{-1}$$

(b) For hydrogen atom, momentum,

$$p = 2\times10^{-27}\ kg\ ms^{-1}$$

Speed, $v = \frac{p}{m} = \frac{2\times10^{-27}}{1.6\times10^{-27}} = 1.24\ m\ s^{-1}$

33. Ans. De-Broglie wavelength of a charged particle accelerated through a potential difference V is given by,

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$\therefore$ For proton and α particle charges are q and 2q respectively,

$$\therefore\quad \lambda_p = \frac{h}{\sqrt{2m_p qV}} \quad ....(i)$$

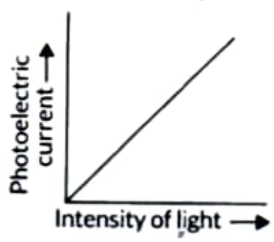
$$\lambda_\alpha = \frac{h}{\sqrt{4m_\alpha qV}} \quad ....(ii)$$

From eqn (i) and (ii)

$$\therefore\quad \frac{\lambda_\alpha}{\lambda_p} = \left(\frac{2m_p}{4m_\alpha}\right)^{1/2} = \frac{1}{2\sqrt{2}}$$

34. Ans. Variation of photoelectric current with intensity of light for a given frequency of incident radiation

Given that $\lambda = 3300\times10^{-10}\,m$,



$$\phi_{Na} = 2.75eV,\ \phi_{Mo} = 4.175eV$$

Then energy of the laser beam is

$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{3300\times10^{-10}\times1.6\times10^{-19}} = 3.75eV$$

Since $E < \phi_{Mo}$ therefore there will be no emission of photoelectrons for molybdenum (Mo).

Bringing the source nearer will cause to emit more photoelectrons as intensity on the plate will increase.

35. Ans. (a) Yes, all emitted photoelectrons have same kinetic energy as the kinetic energy of emitted photoelectrons depends upon frequency of the incident radiation for a given photosensitive surface,

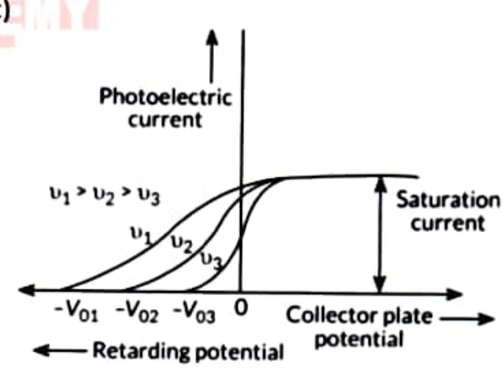
(b) No, the kinetic energy of emitted electrons does not depend on the intensity of incident radiation. If the intensity is increased, number of photons will also increase but energy of each photon remains same as the frequency is also same. The maximum kinetic energy depends on frequency not on intensity,

(c) The number of emitted photoelectrons depends only on intensity of incident light. For a given frequency of incident radiation, its intensity depends on the number of photons.

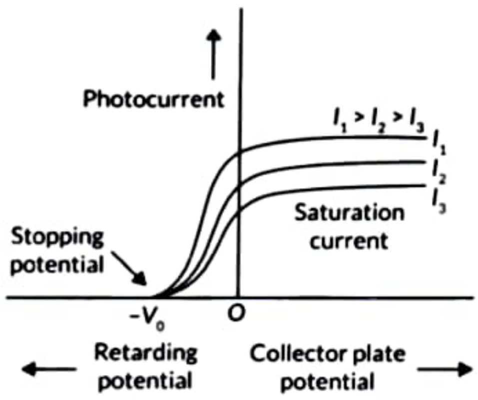
36. Ans. (a) The variable X on the horizontal axis is collector plate potential.

(b) The point A on the horizontal axis represents stopping potential.

(c)



(d)



37. Ans. (a) (i) Zinc, cadmium

(ii) Lithium, sodium

(b) Work function of metal, $\phi = 4.50 eV$

Kinetic energy = 6.06 x 10^{-19} J

$= \frac{6.06\times10^{-19}}{1.6\times10^{-19}} eV = 3.78 eV$

Now, $K.E. = h\nu - \phi$

$\Rightarrow h\nu = K.E. + \phi = 3.78 eV + 4.50 eV = 8.28 eV$

$\nu = \frac{8.28 eV}{h} = \frac{8.28 eV}{4.135\times10^{-15} eVs}$

$= 2.002\times10^{15}\ Hz$

38. Ans. (i) Two features of Einstein's photoelectric equation:

(a) Below threshold frequency ν_0 corresponding to W_0, no emission of photoelectrons takes place.

(b) As the number of photons in light depend on its intensity, and one photon liberates one photoelectron. So number of emitted photoelectrons depends only on the intensity of incident light for a given frequency.

(ii) Below threshold frequency no emission takes place. As there is no photoemission from surface P i.e., the frequency of incident radiation is less than the threshold frequency for surface P.

From surface Q photoemission is possible i.e., the frequency of incident radiation is equal or greater than threshold frequency. As the kinetic energy of photo electrons is zero i.e., the energy of incident radiation is just sufficient to pull out the electron from the surface Q.

Work function for surface Q, $W_Q = h\nu$.

As K.E> = 0; $\nu = \nu_0 = 10^{15}\ Hz$

$W_Q = 6.6\times10^{-34}\times10^{15} = 6.6\times10^{-19} J = 4.125 eV$

39. Ans. (i) For P, threshold frequency

$\nu_P = 3\times10^{14}\ Hz$

For Q, threshold frequency $\nu_Q = 6\times10^{14}\ Hz$

So, metal Q has higher threshold frequency.

(ii) Work function for Q<

$W_Q = h\nu_Q = 6.6\times10^{-34}\times6\times10^{14}$

$= 39.6\times10^{-20}\ J = 2.47\ eV$

(i) The maximum kinetic energy, of electron emitted by light of frequency 8×10^{14} Hz is,

$\therefore K_{max} = h(\nu - \nu_0)$

$= 6.6\times10^{-34}(8\times10^{14} - 6\times10^{14})$

$= 13.2\times10^{-20} J = 0.825\ eV$

40. Ans. For a given frequency, intensity of light in the photon picture is determined by

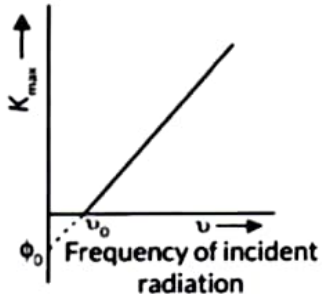
$I = \frac{Energy\ of\ photons}{area\times time} = \frac{n\times h\nu}{A\times t}$

Where n is the number of photons incident normally on crossing area A in time t.

41. Ans. (a) The main features of photons are as follows:

(i) In the interaction of photons with free electrons, the entire energy of photon is absorbed.

(ii) Energy of photon is directly proportional to frequency. Intensity of incident radiation depends on the number of photons falling per unit area per unit time for a given frequency.



(iii) In photon electron collision, the total energy and momentum remain constant.

Einstein's photoelectric equation is

$$K_{max} = h\nu - \phi_0$$

(b) Einstein's photoelectric equation : According to Einstein, when light is incident on metal surface, incident photons are absorbed completely by valence electrons of atoms of metal on its surface. Energy hv of each photon is partially utilized by an electron to become free or to overcome its "work function" W_0 and rest of the absorbed energy provides the maximum kinetic energy to the photoelectron during the emission. i.e.,

$$h\nu = \frac{1}{2}mv_{max}^2 + W_0$$

The minimum value of the frequency of incident radiation below which the photoelectric emission stops i.e., kinetic energy of photoelectron is zero is called threshold frequency (ν_0).

Threshold frequency, $\nu_0 = \frac{W}{h}$

$$\frac{1}{2}mv_{max}^2 = K.E._{max} = h\nu - W_0$$

$$or, \quad K.E._{max} = eV_0$$

When work done by collecting electrode potential on a photoelectron is equal to its maximum kinetic energy then the electrode potential is known as stopping potential.

Stopping potential, $V_0 = \frac{K.E._{max}}{e}$

42. Ans. De-Broglie wavelength of a particle of mass m and charge q accelerating through a potential V is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}} \quad ....(i)$$

Here,

$$m_p = m,\ q_p = e,\ m_\alpha = 4m_p = 4m, q_\alpha = 2q_p = 2e$$

From eqn. (i)

$$\therefore \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}}; 1 = \sqrt{\frac{4m \times 2e \times V_\alpha}{m \times e \times V_p}}$$

$(\because \lambda_p = \lambda_\alpha)$

$$\therefore \frac{V_p}{V_\alpha} = \frac{8}{1};\ V_p : V_\alpha = 8:1$$

43. Ans. (a) (i) For same accelerating potential, a proton and an electron have same kinetic energy. The de-Broglie wavelength associated with same potential V is given by,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m(qV)}}$$

$$So, \quad \lambda = \frac{1}{\sqrt{m}}$$

As electron's mass is lesser than proton. Thus $\lambda_e > \lambda_p$.

(ii) Momentum, $p = \sqrt{2mK}$ or $p \propto \sqrt{m}$

As electron's mass is lesser than proton. Thus momentum of electron is lesser than proton.

(b) De-Broglie wavelength of a particle

$$\lambda = \frac{h}{p} \text{ or } \lambda p = h = \text{constant}$$

It shows a rectangular hyperbola.

44. Ans. Main implications:

(i) Kinetic energy of emitted electrons depends upon frequency, but not on intensity of radiation.

(ii) There exist a frequency of radiation below which no photoemission takes place, how high intensify of radiation may be known as threshold frequency. Explanation of wave nature of radiation fails to explain photoelectric effect. According to wave theory, when light falls on a metal surface, energy is continuously distributed over the surface. All electrons may be ejected only when it acquires energy more than the work function. So, if we use low intensity source, it should take hours for photoelectric emission, but photoelectric effect is almost a spontaneous process.

45. Ans. Given, $\lambda = 2000\,\overset{0}{A} = 2000\times10^{-10}\,m$

$W_0 = 4.2\ eV$

$h = 6.63\times10^{-34}\ J\ s$

(a) $\frac{hc}{\lambda} = W_0 + K.E.\ \ or\ \ K.E. = \frac{hc}{\lambda} - W_0$

$= \frac{(6.63\times10^{-34})\times(3\times10^{8})}{(2000\times10^{-10})}\times\frac{1}{1.6\times10^{-19}}$ eV – 4.2 eV

$= (6.2-4.2)\text{eV} = 2.0\text{eV}$

(b) The energy of the emitted electrons does not depend upon intensity of incident light, hence the energy remains unchanged.

For this surface, electrons will not be emitted as the energy of incident light (6.2 eV) is less than the work function (6.5 eV) of the surface.

46. Ans. (a) De-broglie reasoned out that nature was symmetrical and two basic physical entities – mass and radiation must be symmetrical. If radiation shows dual aspect than matter should do so.

De-broglie equation, $\lambda = \frac{h}{P}$

For photon, $p = \frac{hv}{c}$

Therefore, $\frac{h}{p} = \frac{c}{v} = \lambda$, wavelength of electromagnetic radiation.

(b) As $\lambda = \frac{h}{\sqrt{2mK}}$

So, alpha particle will be having shortest De-Broglie wavelength compared to deutrons.

$k = qV$

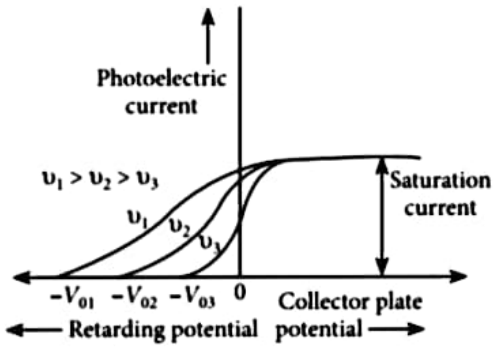
$\frac{\lambda_d}{\lambda_\alpha} = \frac{\sqrt{m_\alpha q_\alpha V}}{\sqrt{m_d q_d V}} = \sqrt{\frac{2m_d\times 2q_d}{m_d q_d}}\qquad \begin{pmatrix} m_\alpha = 2m_d \\ q_\alpha = 2q_d \end{pmatrix}$

$\frac{\lambda_d}{\lambda_\alpha} = \frac{2}{1} \Rightarrow \lambda_d : \lambda_\alpha = 2:1$

47. Soln. (a) The variable X on the horizontal axis is collector plate potential.

(b) The point A on the horizontal axis represents stopping potential.

(c)



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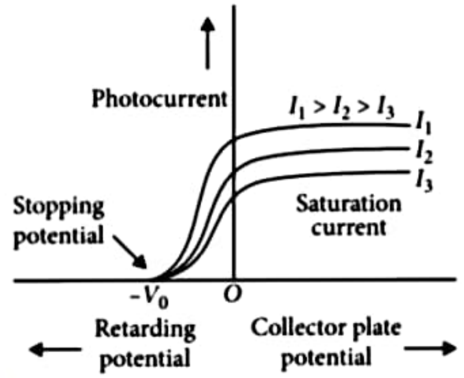


48. Soln. $\lambda_{pr} = \frac{h}{m_p v}$ de-Broglie wavelength of proton

$\lambda_{ph} = \frac{hc}{E},\qquad \left(\because E_{ph} = \frac{hc}{\lambda}\right)$

(de – Broglie wavelength of photon)

$KE_{proton} = \frac{1}{2}m_p v^2$

$\Rightarrow\qquad \lambda_{pr} = \lambda_{ph}$

Hence, $\frac{hc}{E} = \frac{h}{m_p v}$

$$v = \frac{h}{m_p} \cdot \frac{E}{hc} = \frac{E}{m_p c}$$

$$K.E. = \frac{1}{2} \cdot \frac{E}{m_p c} \cdot \frac{E}{m_p c} m_p$$

$$= \frac{1}{2} \cdot \frac{E.E}{m_p c.c}$$

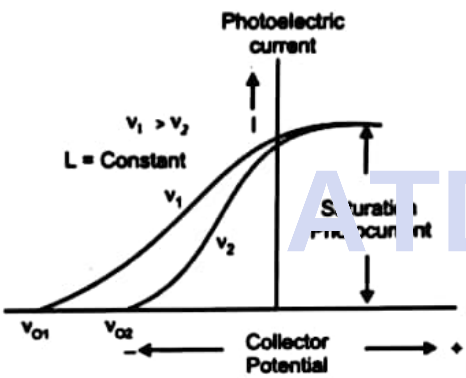
$$= \frac{1}{2} \cdot \frac{E}{m_p} \cdot \frac{hc}{\lambda c.c}$$

$$K.E. = \frac{1}{2} \frac{E}{m_p} \cdot \frac{h}{c\lambda}$$

$$K.E.\left(\frac{2m_p c\lambda}{h}\right) = E$$

$$K.E.\left(\frac{2mc\lambda}{h}\right) = E$$

49. Soln.



Stopping potential is more for the curve corresponding to the frequency ν_2 ($\because$ $\nu_1 > \nu_2$)

This is due to the fact that with increase in the frequency, the kinetic energy of emitted the frequency, the kinetic energy of emitted photoelectrons also increases. Therefore, we need more negative potential to stop these electrons.

50. Soln. de Broglie postulated that the material particles may exhibit wave aspect. Accordingly a moving material particle behaves as wave and the wavelength associated with material particle is

$$\lambda = \frac{h}{mv}$$

Where h = Planck's constant

m = mass of the object

v = velocity of the object

51. Soln. In terms of kinetic energy, wavelength is given by

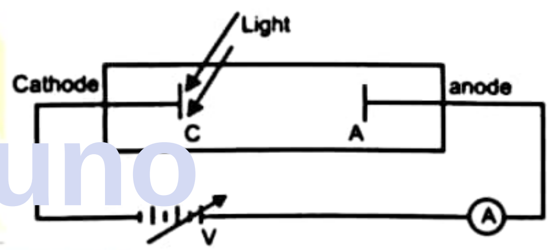
$$\lambda = \frac{h}{\sqrt{2m E_k}}$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{m}}$$

So wavelength is inversely proportional to $\sqrt{m}$, i.e., more the mass, less the wavelength and vice-versa.

So, for same kinetic energy, as a proton has a larger mass than an electron, thus a proton has smaller de-Broglie wavelength than a electron.

52. Soln. When light of suitable frequency is incident on a metal surface, electrons are ejected from the metal. This phenomenon is called the photoelectric effect.



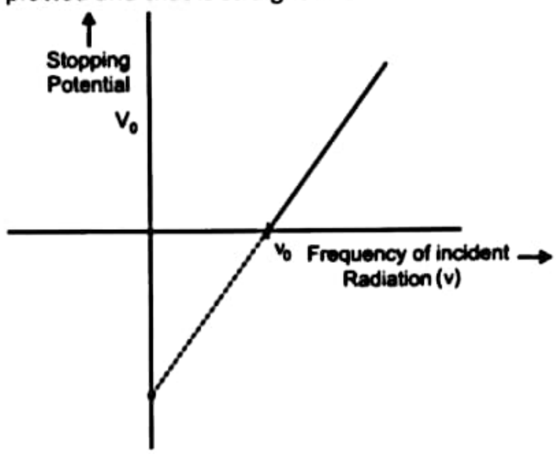
(i) The cathode is illuminated with light of some fixed frequency ν and fixed intensity I_1. A small photoelectric current is observed due to few electrons that reach anode just because they have sufficiently large velocity of emission. If we make the potential of the anode negative with respect to cathode then the electrons emitted by cathode are repelled. Some electrons even go back to the cathode so that the current decreases. At a certain value of this negative potential, the current is completely stopped. The least value of this anode potential which just stops he photocurrent is called cut off potential or stopping potential.

(ii) For a given material, there is a certain minimum frequency that if the incident radiation has a frequency below this threshold, no photoelectric emission will take place, howsoever intense the radiation may be falling. This minimum frequency is called threshold frequency.

According to Einstein's photoelectric equation, maximum K.E. is given as

$$K.E_{max} = \frac{hc}{\lambda} - \phi = h\nu - \phi$$

Where λ is wavelength of light and ν is corresponding frequency and ϕ is work function. We expose a material to lights of various frequencies and thus photoelectric current is observed and cut off potential needed to reduce this current to zero is noted. A graph is plotted and that is straight line.



According to Einstein's photoelectric equation

$$K.E_{max} = \frac{hc}{\lambda} - \phi = h\nu - \phi$$

$$K.E_{max} = eV_0$$

$$\therefore \quad eV_0 = h\nu - \phi$$

$$V_0 = \left(\frac{h}{e}\right)\nu - \frac{\phi}{e} \text{ ....(i)}$$

We can read the value of threshold frequency from graph. From equation (i), we can find the value of stopping potential (V_0).

53. Soln. (a) Wave nature of radiation cannot explain the following:

1. The immediate ejection of photoelectrons.
2. The presence of threshold frequency for a metal surface.
3. The fact that kinetic energy of the emitted electrons is independent of the intensity of light and depends upon its frequency.

Thus, the photoelectric effect cannot be explained on the basis of wave nature of light.

(b) Photon picture of electromagnetic radiation on which Einstein's photoelectric equation is based on particle nature of light.

Its basic features are:

1. In interaction with matter, radiation behaves as if it is made up of particles called photons.
2. Each photon has energy (E=hν). Momentum $\left(p = \frac{h\nu}{c}\right)$, where c is the speed of light.
3. All photons of light of a particle frequency ν, or wavelength λ, have the same energy $\left(E = h\nu = \frac{hc}{\lambda}\right)$ and momentum $\left(p = \frac{h\nu}{c}\right)$
4. By increasing the intensity of light of given wavelength, there is only an increase in the number of photons emitted per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.
5. Photons are electrically neutral and are not deflected by electric and magnetic fields.
6. In a photon – particle collision (such as photon electron collision), the total energy and total momentum are conserved. However, number of photons may not be observed.

54. Soln. (1) de-Broglie wavelength of a particle is dependent on its mass and charge for same accelerating potential, such that

$$\lambda \propto \frac{1}{\sqrt{(\text{Mass})(\text{Charge})}}$$

Mass and charge of a deuteron are $2m_p$ and e respectively and mass and charge of an alpha particle are $4m_p$ and 2e respectively.

Where,

m_p = the mass of a proton.

And e = the charge of an electron.

$$\frac{\lambda_D}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_D q_D}} = \sqrt{\frac{(4m_p)(2e)}{(2m_p)(e)}} = \frac{2}{1}.$$

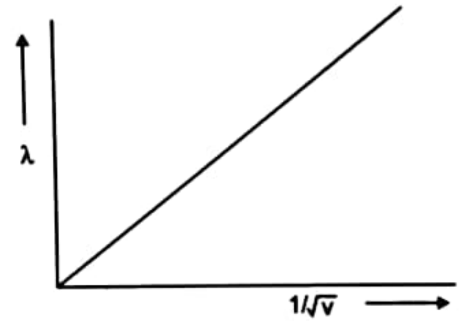
Thus, de-Broglie wavelength related with deuteron is twice of the de-Broglie wavelength of alpha particle.

(2) $K.E. \propto q$

(for same accelerating potential)

Charge of a deuteron is less as compared to an alpha particle. So, deuteron will have less kinetic energy.

55. Soln.



Since, $\lambda = \dfrac{h}{\sqrt{2meV}}$

I.e., $\lambda \propto \dfrac{1}{\sqrt{e}}$

Therefore, more the wavelength lesser is the charge.

56. Soln. (i) Einstein's photoelectric equation, K.E. of photoelectron = Incident energy of photons – Work function

Or $K.E. = h\nu - \phi_0$

Or $K.E. = h\nu - h\nu_0$

Where, ν_0 is called threshold frequency.

Threshold Frequency: For a given metal, there exists a certain minimum frequency of the incident radiation below which no emission of photoelectrons takes place. This frequency is called threshold frequency. It is denoted by ν_0.

(ii) Stopping Potential: It is that minimum negative potential given to anode in a photocells for which the photoelectric current becomes zero. It is denoted by V_0. It is independent of the intensity of the incident light.

57. Soln. The work function of a metal is the minimum energy by it to eject an electron. It is given for metal X = 3 x 10^{-19} J

Energy of a photon of wavelength 26.52 nm = hν

$$= \frac{hc}{\lambda}$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{26.52\times10^{-9}}$$

$$= 0.746\times10^{-17}\,J$$

If number of photons = n; for total energy $3\times10^{-19}J$.

Then,

$$n\times0.746\times10^{-17} = 3\times10^{-19}$$

Hence, $$n = \frac{3\times10^{-19}}{0.746\times10^{-17}}$$

$$= 4.021\times10^{-2}$$

58. Soln. Energy of one photon

$$E = \frac{hc}{\lambda}$$

$$E = \frac{6.6\times10^{-34}\times3\times10^{8}}{6\times10^{-7}}$$

$$\cong 3.3\times10^{-19}J$$

∴ Number of photons emitted by the source in 1s,

$$n = \frac{P}{E},\ or\ n = \frac{66}{3.3\times10^{-19}} = 2\times10^{20}$$

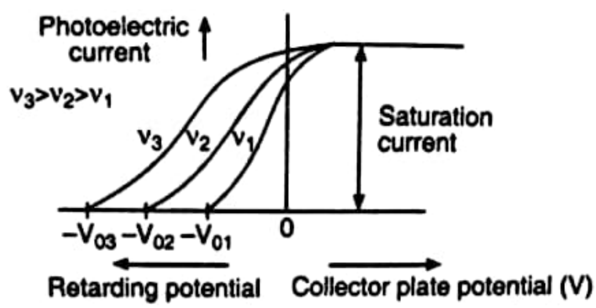
∴ Total number of photons emitted by source in 2 minutes

$$(N) = n\times2\times60$$

$$= 2\times10^{20}\times120$$

$$= 2.4\times10^{22}\ photons$$

59. Soln. (i)



(ii) According to Einstein's photoelectric equation

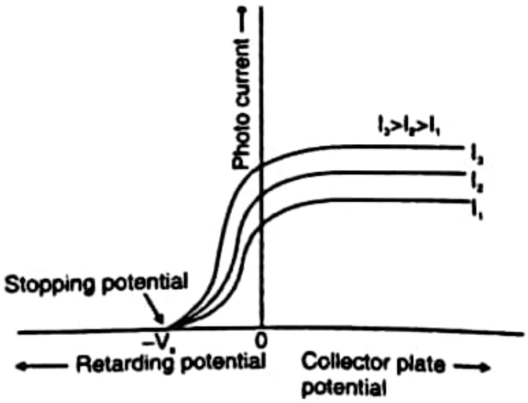
$$K_{max} = h\nu - \phi_0$$

If V_0 is stopping potential then

$$eV_0 = h\nu - \phi$$

Thus, for different values of frequency (ν) there will be different values of cut off potential V_0.

(iii)



60. Soln. (i) Q has higher threshold frequency

(ii) Work function, $\phi_0 = h\nu_0$

$$h\nu_0 = (6.6\times10^{-34})\times\frac{6\times10^{14}}{1.6\times10^{-19}}eV$$

$$= 2.5\ eV$$

(ii) $K_{max} = h(\nu-\nu_0)$

$$= \frac{6.6\times10^{-34}\times2\times10^{14}}{1.6\times10^{-19}}eV$$

$$K_{max} = 0.82eV$$

61. Soln. Cut off Frequency: It is the minimum frequency of incident radiation at which photoelectric emission start to take place from photoelectric material. At this frequency kinetic energy of emitted electron is zero.

Einstein's photoelectric equation

$$K.E. = h\nu - h\nu_0$$

Where, ν_0 is threshold frequency

According to the question

$$\nu = 2f$$

And $\nu_0 = f$ then velocity $= v_1$

$$\frac{1}{2}mv_1^2 = 2hf - hf$$

$$\frac{1}{2}mv_1^2 = hf$$

$$v_1^2 = \frac{2hf}{m}$$

In the second case

$\nu = 5f$ then velocity = v_2

Hence, $\frac{1}{2}mv_2^2 = 5hf - hf$

(threshold frequency is constant for a metal)

$$v_2^2 = \frac{8hf}{m}$$

Hence, $v_1^2 : v_2^2 = 2:8$ or $1:4$

Hence $v_1 : v_2 = 1:2$ (positive values of square root is taken as velocity can not be negative)

62. Soln. de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Mass of a proton = u

Charge of proton = e

Hence, Kinetic energy of a proton when accelerated through a potential = eV

Putting these values in de-Broglie wavelength formula to calculated wave length of proton

$$\lambda_p = \frac{h}{\sqrt{2ueV}}$$

Now,

- Mass of an α -particle = 4u
- Charge of an α −particle = 2 e

Hence, Kinetic energy of an α −particle when accelerate through a potential = 2 eV

Putting these values in de-Broglie wavelength formula to calculated wave length of α −particle

$$\lambda_\alpha = \frac{h}{\sqrt{8u\times2eV}} = \frac{h}{\sqrt{16ueV}}$$

Hence ratio of $\frac{\lambda_p}{\lambda_\alpha} = \frac{1}{\sqrt{8}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{1}{2\sqrt{2}}$$

63. Soln. (a) All photons of light of a particular frequency 'ν' have same energy and momentum whatever the intensity of radiation may be.

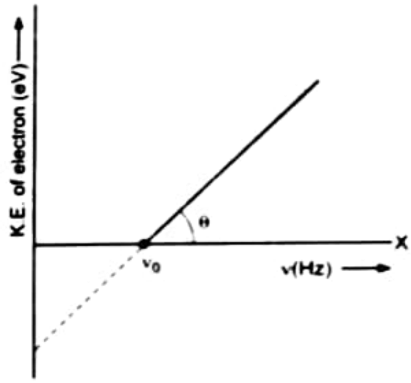
(b) Photons are electrically neutral and are not affected by presence of electric and magnetic fields,

(i) From this graph, the Planck constant can be calculated by the slope of the current

$$h = \frac{\Delta(\text{KE})}{\Delta v}$$

(ii) Work function is the minimum energy required to eject the photo-electron from the metal surface.

$\phi = h\nu_0$, where ν_0 = Threshold frequency



64. Soln. (i) de Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

For same V, $\lambda \alpha \frac{1}{\sqrt{mq}}$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4m_p}{m_p} \cdot \frac{2e}{e}}$$

$$= \sqrt{8} = 2\sqrt{2}$$

Clearly, $\lambda_p > \lambda_\alpha$.

Hence, proton has a greater de-Broglie wavelength.

(ii) Kinetic energy, $K = qV$

For same V, $K \alpha q$

$$\frac{K_p}{K_\alpha} = \frac{q_p}{q_\alpha} = \frac{e}{2e} = \frac{1}{2}$$

Clearly, $K_p < K_\alpha$.

Hence, proton has less kinetic energy.

65. Soln. Expression for de Broglie Wavelength associated with Accelerated Electrons:

The de Broglie wavelength associated with electrons of momentum p is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad \text{.........(i)}$$

Where m is mass and v is velocity of electron. If E_k is the kinetic energy of electron, then

$$E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$$

$$\left(\text{since } p = mv \Rightarrow v = \frac{p}{m}\right)$$

$$\Rightarrow \quad p = \sqrt{2m\,E_K}$$

$$\therefore \quad \text{Equation (i) gives } \lambda = \frac{h}{\sqrt{2mE_K}}$$

.....................(ii)

If V volt is accelerating potential of electron, then kinetic energy,

$$E_K = eV$$

$\therefore$ Equation (ii) gives $$\lambda = \frac{h}{\sqrt{2meV}}$$

..............(iii)

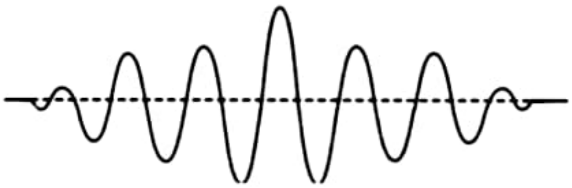
Substituting m = $9.1\times10^{-31}kg$, $e = 1.6\times10^{-19}C$, $h = 6.62\times10^{-34}$ Js, we get

$$\lambda = \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}V}} = \frac{12.27}{\sqrt{V}}\times10^{-10}m$$

$$\lambda = \frac{12.27}{\sqrt{V}}\overset{0}{A} \qquad \text{..........(iv)}$$

This is the required expression for de Broglie wavelength associated with electron accelerated to potential of V volt.

The diagram of wave packet describing the motion of a moving electron is shown.



66. Soln. According to Bohr's quantum condition "Only those atomic orbits are allowed as stationary orbits in which angular momentum of an electron is the integral multiple of $\frac{h}{2\pi}$.

If m is the mass, v velocity and r radius of orbit, then angular momentum of electron L = mvr. According to Bohr's quantum condition

$$mvr = n\frac{h}{2\pi} \quad ....(i)$$

According to de Broglie quantum condition only those atomic orbits are allowed as stationary orbits in which circumference of electron-orbit is the integral multiple of de Broglie wavelength associated with electron, i.e.,

$$2\pi r = n\lambda \quad ....(ii)$$

According to de Broglie hypothesis

$$\lambda = \frac{h}{mv} \quad ....(iii)$$

Substituting this value in (ii), we get

$$2\pi r = n\left(\frac{h}{mv}\right) \Rightarrow mvr = n\frac{h}{2\pi}$$

This is Bohr's quantum condition.

67. Soln. Given $\lambda = 1.00\,nm = 1.00\times10^{-9}\,m$

(a) Momenta of electron and photon are equal; given by

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}}{1.00\times10^{-9}} = 6.63\times10^{-25}\,kg\ ms^{-1}$$

(b) Energy of photon, $E = h\nu = h.\frac{c}{\lambda} = \frac{h}{\lambda}c$

$$= pc = 6.63\times10^{-25}\times3\times10^{8}J = 19.89\times10^{-17}J$$

$$= \frac{19.89\times10^{-17}}{1.6\times10^{-19}}eV = 1.24\times10^{3}eV = 1.24\,keV$$

(c) Kinetic energy of electron $E_k = \frac{1}{2}m_e v^2 = \frac{p^2}{2m_e}$

$$= \frac{(6.63\times10^{-25})^2}{2\times9.1\times10^{-31}}J$$

$$= 2.42\times10^{-19}J = \frac{2.42\times10^{-19}}{1.6\times10^{-19}}eV = 1.51eV$$

68. Soln. According to de Broglie a particle behaves as a wave; but it is now established that a particle cannot be equivalent to a single wave; but it is equivalent to a group of waves or a wave packet. The velocity of a single wave is called the phase velocity ($u = \nu\lambda$), which is not equal to particle velocity. The velocity of wave packet (called group velocity) is equal to particle velocity.

Group velocity $v_g = \frac{dw}{dk} = v$ (velocity of particle) = $\frac{p}{m}$

As $p = \frac{h}{\lambda}$; hence in the discussion of matter waves, a wave packet is significant and hence only wavelength (λ) is significant. As a single wave is insignificant, phase velocity (velocity of a single wave) is insignificant and hence frequency ν is also insignificant.

69. Soln. (i) Frequency of violet light $(\nu_v) >$ frequency of blue light (ν_b) i.e., $\left(\frac{\nu_v}{\nu_b}\right) > 1$

As both light have same intensity, so

$$n_v\nu_v = n_b\nu_b \Rightarrow \frac{n_v}{n_b} = \frac{\nu_b}{\nu_v} < 1$$

$$\therefore \quad n_b > n_v$$

(i) Since $n_b > n_v$, hence number of electrons emitted per second corresponding to blue light will be more than that for violet light.

(ii) Since $\nu_v > \nu_b$, hence maximum kinetic energy of the electrons ($K_{max} = h\nu - \phi_0$) for violet light will more than that for blue light.

70. Soln. Since

$V_S = 3V$ and $K_{max} = eV_S$, so $K_{max} = 3eV$

71. Soln. $I = 1400\,W/m^2$; $\lambda = 6000\,\overset{0}{A}$

(a) Energy of the photon, $E = h\nu = \frac{hc}{\lambda}$

$(c = 3\times10^8$ m/ sec)

Let n be the number of photons received/sec per unit area.

$$n = \frac{IA}{E_{Photon}} = \frac{(1400\times1)\times(6000\times10^{-10})}{6.63\times10^{-34}\times3\times10^{8}}$$

$$= 4.22\times10^{21}.$$

(b) Total energy emitted per second = power (watt)

$$n/\sec = \frac{Power\ of\ sun\,(\mathrm{W})}{E/photon}$$

$$= \frac{I\times(4\pi R^{2})\times(6000\times10^{-10})}{6.63\times10^{-34}\times3\times10^{8}}$$

(R is the average radius of earth's orbit)

$$= 1.178\times10^{45}$$

SURE SHOT QUESTIONS

ARVIND ACADEMY

Chapter – 12

Atoms

MCQ (1 mark)

1. Soln. (c): Here, a_0 = 53 pm, n = 1 for ground state

For Li^{++} ion, Z = 3

Radius of n^{th} orbit

$$r = \frac{n^2h^2}{4\pi^2 m KZe^2} = \frac{a_0 n^2}{Z}$$

$$\therefore \quad r = \frac{53\times(1)^2}{3} \qquad \left[\because a_0 = \frac{h^2}{4\pi^2 mKe^2} = 53\,pm\right]$$

$$= \quad 17.66 \approx 18\ pm$$

2. Soln. (c): In the frame of reference, where electron is at rest the given expression cannot be true for binding energy as the frame in which electron is at rest would not be inertial. In hydrogen atom, electron revolving around a fixed proton nucleus has some centripetal acceleration.

3. Soln. (a): In atoms with many electrons, electrons are not being subjected to one single central force.

4. Soln. (a): In the given oxygen molecule, nuclear force between the nuclei of two atoms is not important because nuclear forces being short ranged are confined only within one particular nucleus. The distance between the nuclei of two atoms is large. So nuclear forces between two nuclei is not effective.

5. Soln. (a)

6. Ans.(d)

7. Ans. (a): radius, $r = n^2 r_0$

8. Ans. (b) : PE of electron in second excited state in Hydrogen atom

So, second excited n_1 = 3

Total energy in second excited state,

$$TE = \frac{-13.6}{n^2} = \frac{-13.6}{9} = -1.5\,eV$$

We know, TE = PE + KE

And PE = -2 x KE

$$\Rightarrow \quad -1.51 = -2KE + KE$$

$$\Rightarrow \quad -KE = -1.51\,eV$$

$$\because \quad PE = -2\,KE = -2\times 1.51$$

$$PE = -3.02\,eV$$

So, option (b) is correct.

9. Ans. (a) :

$$\frac{1}{\lambda_1} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) R\left(1 - \frac{1}{9}\right) = 8\frac{R}{9} = 0.88R$$

$$\frac{1}{\lambda_2} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = R\left(\frac{1}{9} - \frac{1}{16}\right) = \frac{7R}{144} = 0.0486$$

$$\frac{1}{\lambda_3} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = R\left(1 - \frac{1}{4}\right) = 3\frac{R}{4} = 0.75R$$

$$\frac{1}{\lambda_4} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36} = 0.139R$$

Energy $\propto \frac{1}{\lambda}$, so energy is highest for I.

10. Ans. (c) : The (I) transition is of absorption. Thus maximum energy photon transition will take place in transition (III) as

$$E = -3.4 - (-13.6) = 10.2\,\text{eV}$$

11. Ans. (d): Given, r = 5.3 x 10^{-11} m

Let r_1 be the radius at n = 3,

$$r_1 = n^2 r = (3)^2 \times 5.3\times10^{-11} m = 4.77\times10^{-10} m$$

12. Ans. (b) : As per question, $12.1 = 13.6 - \frac{13.6}{n^2}$

$$\Rightarrow \quad \frac{13.6}{n^2} = 1.5$$

$$\Rightarrow \quad n^2 = 9$$

$$\Rightarrow \quad n = 3$$

➢ Assertion-Reasoning (1 mark)

13. **Sol.**

(*a*) Both assertion and reason are true and the reason is the correct explanation of the assertion.

For Balmer series,

$$\frac{1}{\lambda_{max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5R}{36}$$

$$\therefore \quad \lambda_{max} = \frac{36}{5R} = \frac{36}{5\times1.097\times10^7} = 6563 \text{ Å}$$

$$\frac{1}{\lambda_{min}} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) = \frac{R}{4}$$

$$\lambda_{min} = \frac{4}{R} = \frac{4}{1.097\times10^7} = 3646 \text{ Å}$$

Both of these wavelengths lie in the visible region of the spectrum.

14. **Sol.**

(*a*) Electron is bounded to the nucleus by electrostatic forces of attraction. So total energy is negative.

15. **Sol.**

(*c*) Assertion is true but the reason is false.

16. **Sol.** (a): We know that an electron is very light particle as compared to an a particle. Hence electron cannot scatter the a-particle at large angles, according to law of conservation of momentum. On the other hand, mass of nucleus is comparable with the mass of a-particle, hence only the nucleus of atom is responsible for scattering of a-particles.

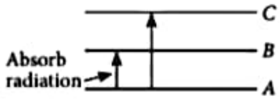
17. **Sol.** (c): Emission transitions can take place between any higher energy level and any energy level below it while absorption transitions start from the lowest energy level only and may end at any higher energy level. Hence number of absorptions transitions between two given energy levels is always less than the number of emission transitions between same two levels.

18. **Sol.** (b): According to classical electromagnetic theory, an accelerated charge continuously emits radiation. As electrons revolving in circular paths are constantly experiencing centripetal acceleration, hence they will be losing their energy continuously and the orbital radius will go on decreasing and form spiral and finally the electron will fall into the nucleus.

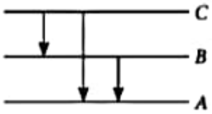
19. **Sol.** (b): Every atom has certain definite energy level. In the normal state, the electron in the hydrogen atom stays in lowest energy level. When the atom gets appropriate energy from outside, then this electron rises to some higher energy level i.e., atom is excited. Within nearly 10^{-8} sec, the electron leaves the higher energy level. Now, it can return either directly to the lowest energy level (or the ground state) or come to the ground state after passing through other lower energy levels. Since there are a large number of atoms in a light - source (hydrogen lamp), hence all possible transitions take place in the source and many lines are seen in the spectrum. The slit gives the shape of the spectrum and the large number of lines are obtained because a large number of atoms are getting excited and de-excited to different energy levels.

20. **Sol.**

(a): Absorption transition



Two possibilities in absorption transition.
Emission transition



Three possibilities in emission transition. Therefore number of absorption transition < number of emission transition.
For any two states A and B such that $E_A < E_B$ we have absorption spectrum for $A \to B$ transition and emission $B \to A$. But most of the time atoms are in ground state, absorption is only from the ground state.

21. **Sol.** (b): When the flame of Bunsen burner is smoky, the carbon particles in it are in the incandescent state. Hence the flame gives a continuous spectrum. But when the Burner gives a blue flame, then it has carbon, cynogen etc., in the molecular state. Hence it gives band spectrum.

22. **Sol.** (b): According to classical physics, all moving charged particle radiate electromagnetic radiation. So moving electrons will also radiate energy. If we see the atomic structure we find that electrons revolve around the nucleus in some particular orbits. Bohr termed these orbits as the stationary orbits as the electrons do not radiate energy as long as they are moving in these orbits. This is one of Bohr's postulates. This postulate is based on the fact that if the moving electrons radiate thereby losing energy, they have got a chance to finally fall back onto the nucleus and the atom will be collapsed.

➢ Case Study

23. **Answers**

1. (*c*) Most of the space within the atoms is empty.

2. (*d*) Trajectory of particle 4 not physically possible.

3. (*d*) Most of the space within atoms is empty, so most of the α-particles pass straight through the foil.

4. (*a*) At the distance of closed approach,
Initial K.E. of the particle
= P.E. of the particle and the massive nucleus

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze\times e}{r_0} \quad \text{or} \quad r_0 = \frac{Ze^2}{2\pi\varepsilon_0 mv^2}$$

5. (*c*) Radius of atomic nucleus $\simeq 10^{-15}$m.

24. **Answers**

6. (*d*) Bohr model of atom assumes all the three conditions (*a*), (*b*) and (*c*).

7. (*d*) $\frac{h}{2\pi}$ is the angular momentum an electron in the first orbit of hydrogen atom.

8. (*c*) $r_n \propto n^2$.

9. (*d*) $E_2 = -\frac{13.6}{2^2} = -3.4\text{eV}$; $E_3 = -\frac{13.6}{3^2} = -1.5\text{eV}$

$$E = E_3 - E_2 = -1.5 + 3.4 = \mathbf{1.9\ eV}.$$

10. (*b*) Lyman series lies in the ultraviolet region of the em spectrum.

➤ Questions

25. Ans. In the α -particle scattering experiment, if a thin sheet of solid hydrogen is used in place of a gold foil, then the scattering angle would not be large enough because the mass of hydrogen (1.67×10^{-27}) is less than the mass of incident α -particle (6.64×10^{-27}). Thus, the mass of scattering particle is more than the target nucleus. As a result, α -particles would not bounce back if solid hydrogen is used in the α -particle scattering.

26. Ans. The distance from the nucleus, where all kinetic energy of α -particles is completely converted into potential energy is known as the distance of closest approach.

$$r=\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{K} \ or\ r\propto\frac{1}{K}$$

If kinetic energy will be doubled, then the distance of closest approach will become half.

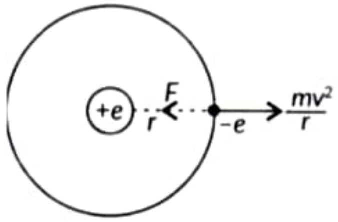
27. Ans. The two important limitations of Rutherford nuclear model of the atom are:

(i) This model cannot explain about the stability of matter.

It cannot explain the characteristic line spectra of atoms of different elements.

28. Ans. Radius of nth orbit of hydrogen atom: In H – atom, an electron having charge -e revolves around the nucleus of charge +e in a circular orbit of radius r, such that necessary centripetal force is provided by the electrostatic force of attraction between the electron and nucleus.

i.e., $$\frac{mv^2}{r}=\frac{1}{4\pi\varepsilon_0}\frac{e.e}{r^2} \ or\ mv^2=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \quad .....(i)$$



From Bohr's quantization condition

$$mvr=\frac{nh}{2\pi} \ or\ v=\frac{nh}{2\pi mr} \quad ....(ii)$$

Using equation (ii) in (i), we get

$$m.\left(\frac{nh}{2\pi mr}\right)^2=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \ or\ \frac{m.n^2h^2}{4\pi^2m^2r^2}=\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Or $$r=\frac{n^2h^2\varepsilon_0}{\pi me^2} \quad ....(iii)$$

Where n = 1, 2, 3, ... is principal quantum number.

Equation (iii), gives the radius of nth orbit of H – atom. So the radii of the orbits increase proportionally with n^2 i.e., $[r\propto n^2]$. Radius of first orbit of H – atom is called Bohr radius a_0 and is given by

$$a_0=\frac{h^2\varepsilon_0}{\pi me^2} \ for\ n=1 \ or\ a_0=0.529\,\overset{0}{A}$$

So, radius of nth orbit of H – atom then becomes

$$r=n^2\times0.529\,\overset{0}{A}$$

29. Ans. Limitation of Rutherford's model:

Rutherford's atomic model is inconsistent with classical

physics, that is why, Rutherford's model is not able to explain the spectrum of even most simplest H-spectrum. Bohr's postulates to resolve observed features of atomic spectrum:

Bohr's quantization condition: Of all the possible circular orbits allowed by the classical theory, the electrons ae permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, h being Planck's constant.

Therefore, for any permitted orbit,

$$L=mvr=\frac{nh}{2\pi}, n=1,2,3.....,$$

Where n is called the principal quantum number, and this equation is called Bohr's quantisation condition.

30. Ans. Bohr's quantization condition : The electron can resolve around the nucleus only in those circular orbits in which angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$ i.e.,

$$mvr = \frac{nh}{2\pi}, n = 1, 2, 3, ...$$

The shortest wavelength of Brackett series is given as

$$\frac{1}{\lambda} = 1.097\times10^7\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right] = \frac{1.097\times10^7}{16}$$

$$\Rightarrow \lambda = 1.4585\times10^{-6} m$$

This wavelength lies in the infrared region of electromagnetic spectrum.

31. Ans. The energy levels of H_2 atom is given as

$$E_n = \frac{-13.6}{n^2}$$

$$\Rightarrow -1.51 = \frac{-13.6}{n^2}$$

$$\Rightarrow n^2 = \frac{13.6}{1.51} \approx 9 \Rightarrow n = 3$$

$$E_n = \frac{-13.6}{n^2} \Rightarrow -3.4 = \frac{-13.6}{n^2}$$

$$n^2 = \frac{13.6}{3.4} \Rightarrow n^2 = 4 \Rightarrow n = 2$$

Thus an electron makes a transition from n = 3 energy level to n = 2 energy level.

$$\therefore \frac{hc}{\lambda_{32}} = \frac{me^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_2^2} - \frac{1}{n_3^2}\right)$$

$$\frac{hc}{\lambda_{32}} = 21.76\times10^{-19}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\lambda_{32} = \frac{hc}{21.76\times10^{-19}\left(\frac{1}{4} - \frac{1}{9}\right)}$$

$$= \frac{6.625\times10^{-34}\times3\times10^8\times36}{21.76\times10^{-19}\times5}$$

$$= \frac{715.5\times10^{-26}}{108.85\times10^{-19}} = 6.57\times10^{-7} m$$

It belongs to Balmer series.

32. Ans. Given, short wavelength limit of Lyman series,

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right) \Rightarrow \frac{1}{913.4\overset{0}{A}} = R\left(\frac{1}{1^2} - \frac{1}{\infty}\right)$$

$$\lambda_L = \frac{1}{R} = 913.4\overset{0}{A}$$

For the short wavelength limit of Balmer series,

$$n_1 = 2,\ n_2 = \infty$$

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right) \Rightarrow \lambda_B = \frac{4}{R} =$$

$$4\times913.4\overset{0}{A} = 3653.6\overset{0}{A}$$

33. Ans. (a) Let the minimum distance of approach be r_0. At this distance, the whole of the kinetic energy of the alpha – particle will be converted into the electrical potential energy.

The positive charge on the gold nucleus = Ze = 79e and the positive charge on the α -particle = 2e

At $r = r_0$, $KE = PE$

$$2.56\times10^{-12} = \frac{1}{4\pi\varepsilon_0}\frac{(79e)(2e)}{r_0}$$

$$\therefore r_0 = \frac{(9\times10^9)(79)(2)(1.6\times10^{-19})^2}{2.56\times10^{-12}} = 14.2\times10^{-15} m$$

(b) If proton is used instead of alpha particle, r_0 will become half.

$$\therefore r'_0 = \frac{r_0}{2} = 7.1\times10^{-15} m$$

34. Ans. Energy of hydrogen atom in nth state

$$E_n = -\frac{13.6eV}{n^2}$$

According to question, $h\nu = E_4 - E_1$

$$h\nu = -13.6\left(\frac{1}{16} - 1\right)eV = 13.6\times\frac{15}{16}eV$$

$$\nu = 13.6\times\frac{15}{16}\times\frac{1.6\times10^{-19}}{6.63\times10^{-34}} = 3\times10^{15} Hz$$

35. Ans. Here, $\Delta E = 12.5 eV$

Energy of an electron in n[th] orbit of hydrogen atom is,

$$E_n = -\frac{13.6}{n^2} eV$$

In ground state, n = 1

E_1 = -13.6 eV

Energy of an electron in the excited state after absorbing a photon of 12.5 eV energy will be

$$E_n = -13.6 + 12.5 = -1.1 eV$$

$$\therefore \quad n^2 = \frac{-13.6}{E_n} = \frac{-13.6}{-1.1} = 12.36 \Rightarrow n = 3.5$$

Here, state of electron cannot be fraction.

So, n = 3 (2[nd] excited state).

The wavelength λ of the first member of Lyman series is given by

$$\frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = \frac{3}{4} R$$

$$\Rightarrow \lambda = \frac{4}{3R} = \frac{4}{3 \times 1.097 \times 10^7}$$

$$\Rightarrow \lambda = 1.215 \times 10^{-7} m$$

$$\Rightarrow \lambda = 121 \times 10^{-9} m \Rightarrow \lambda = 121 nm$$

The wavelength λ' of the first member of the Balmer series is given by

$$\frac{1}{\lambda'} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36} R$$

$$\Rightarrow \lambda' = \frac{36}{5R} = \frac{36}{5 \times (1.097 \times 10^7)}$$

$$= 6.56 \times 10^{-7} m = 656 \times 10^{-9} m = 656 \ nm$$

36. Ans. de-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$,

where K is the kinetic energy.

Now, energy of electron,

$$K = \frac{13.6 z^2}{n^2} = \frac{13.6}{3^2} = 1.51 eV = 2.41 \times 10^{-19} J$$

$$\therefore \lambda = \frac{h}{\sqrt{2mK}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9 \times 10^{-31} \times 2.41 \times 10^{-19}}}$$

$$= 1 \times 10^{-9} m = 1 \ nm$$

37. Ans. From Bohr's theory, the frequency ν of the radiation emitted when an electron de-excites from level n_2 to level n_1 is given as

$$\nu = \frac{E_2 - E_1}{h}$$

$$\nu = \frac{me^4}{8\varepsilon_0^2 h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

Given $n_1 = n - 1, \ n_2 = n$,

$$\nu = \frac{me^4}{8\varepsilon_0^2 h^3} \frac{2n - 1}{(n-1)^2 n^2}$$

For large n, 2n – 1 = 2n, n – 1 = n

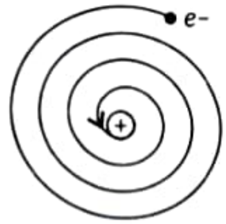
Thus, $\nu = \frac{me^4}{4\varepsilon_0^2 h^3 n^3}$

$$\nu = \frac{v}{2\pi r} = \frac{me^4}{4\varepsilon_0^2 h^3 h^3}$$

Which is same as orbital frequency of electron in nth orbit.

38. Ans. (a) Limitation of Rutherford's model;

Rutherford's atomic model is inconsistent with classical physics. According to electromagnetic theory, an electron is a charged particle moving in the circular orbit around the nucleus and is accelerated, so it should emit radiation continuously and thereby loose energy. Due to this, radius of the electron would decrease continuously and also the atom should then produce continuous spectrum, and ultimately electron will fall into the nucleus and atom will collapse in 10^{-8} s. But the atom is fairly stable and it emits line spectrum.



(ii) Rutherford's model is not able to explain the spectrum of even most simplest H – spectrum.

Bohr's postulates to resolve observed features of atomic spectrum:

(i) Quantum condition: Of all the possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$.

h being Planck's constant. Therefore, for any permitted orbit,

$$L = mvr = \frac{nh}{2\pi},\ n = 1, 2, 3, \ldots\ldots,$$

Where n is called the principal quantum number, and this equation is called Bohr's quantisation condition.

(ii) Stationary orbits: While resolving in the permissible orbits, an electron does not radiate energy. These non-radiating orbits are called stationary orbits.

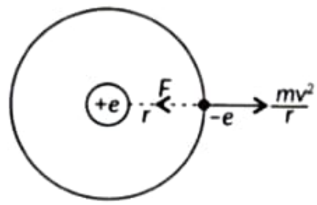
(iii) Frequency condition: An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to lower orbit or from a lower to a higher orbit respectively.

$$h\nu = E_i - E_f$$

Where v is frequency of radiation emitted, E_i and E_f are the energies associated with stationary orbits of principal quantum number n_i and n_f respectively (whre $n_i > n_f$).

(b) Radius of nth orbit of hydrogen atom: In H-atom, an electron having charge -e revolves around the nucleus of charge +e in a circular orbit of radius r, such that necessary centripetal force is provided by the electrostatic force of attraction between the electron and nucleus.

i.e., $\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e.e}{r^2}$ or $mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ ………(i)



from Bohr's quantization condition

$mvr = \frac{nh}{2\pi}$ or $v = \frac{nh}{2\pi mr}$ …………..(ii)

Using equation (ii) in (i), we get

$$m.\left(\frac{nh}{2\pi mr}\right)^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r} \text{ or } \frac{m.n^2h^2}{4\pi^2m^2r^2} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$$

Or $r = \frac{n^2h^2\varepsilon_0}{\pi me^2}$ …………..(iii)

Where n = 1, 2, 3, ….. is principal quantum number.

Equation (iii), gives the radius of nth orbit of H – atom. So the radii of the orbits increase proportionally with n^2 i.e., $[r \propto n^2]$. Radius of first orbit of H – atom is called Bohr radius a_0 and is given by

$$a_0 = \frac{h^2\varepsilon_0}{\pi me^2}\ (\text{for } n = 1) \text{ or } a_0 = 0.529\ \overset{0}{A}$$

So, radius of n^{th} orbit of H – atom then becomes

$$r = n^2 \times 0.529\ \overset{0}{A}$$

39. Ans. (i) According to Bohr's postulates, in a hydrogen atom, as single electron revolves around a nucleus of charge +e. For an electron moving with a uniform sped in a circular orbit of a given radius, the centripetal force is provided by coulomb force of attraction between the electron and the nucleus. The gravitational attraction may be neglected as the mass of electron and proton is very small.

So, $\frac{mv^2}{r} = \frac{ke^2}{r^2}$ $\left(Where,\ k = \frac{1}{4\pi\varepsilon_0}\right)$

Or $mv^2 = \frac{ke^2}{r}$ …………..(i)

Where, m = mass of electron

r = radius of electronic orbit

v = velocity of electron

Again, by Bohr's second postulates

$$mvr = \frac{nh}{2\pi}$$

Where, n = 1, 2, 3........ or $v = \frac{nh}{2\pi mr}$

Putting the value of v in eq.(i)

$$m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{ke^2}{r} \Rightarrow r = \frac{n^2h^2}{4\pi^2kme^2} \quad .....(ii)$$

Kinetic energy of electron,

$$E_k = \frac{ke^2}{2}\frac{4\pi^2kme^2}{n^2h^2} = \frac{2\pi^2k^2me^4}{n^2h^2}$$

Potential energy of electron,

$$E_p = -\frac{k(e)\times(e)}{r} = -\frac{ke^2}{r}$$

Using eq. (ii), we get

$$E_p = -ke^2 \times \frac{4\pi^2kme^2}{n^2h^2} = -\frac{4\pi^2kme^4}{n^2h^2}$$

Hence, total energy of the electron in the n^{th} orbit

$$E = E_p + E_k$$

$$= -\frac{4\pi^2k^2me^4}{n^2h^2} + \frac{2\pi^2k^2me^4}{n^2h^2} = -\frac{2\pi^2k^2me^4}{n^2h^2}$$

$$-\frac{13.6}{n^2}eV$$

When the electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength. It is called a spectral line.

40. Soln. Assumptions of Rutherford's atomic model:

(i) Every atom consists of a tiny central core called the atomic nucleus, in which the entire positive charge and atmost entire mass of the atom are concentrated.

(ii) The size of nucleus is of the order of 10^{-15} m, which is very small as compared to the size of the atom which is of the order of 10^{-10} m.

(iii) The atomic nucleus is surrounded by certain number of electrons. As atom on the whole is electrically neutral, the total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.

(iv) The electrons revolve around the nucleus in various circular orbits.

According to electromagnetic theory, electron revolving around the nucleus are continuously accelerated. Since an accelerated charge emits energy, the radius of the circular path of a revolving electron should go on decreasing and ultimately it should fall into the nucleus. So, it could not explain the structure of the atom. As matter is stable, we cannot expect the atoms to collapse.

41. Soln. Bohr's quantization condition: The electron can revolve round the nucleus only in those circular orbits in which angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$ i.e.,

$$mvr = \frac{nh}{2\pi}, n = 1, 2, 3, ...$$

The shortest wavelength of Brackett series is given as

$$\frac{1}{\lambda} = 1.097\times10^7\left[\frac{1}{4^2} - \frac{1}{\infty^2}\right] = \frac{1.097\times10^7}{16}$$

$$\Rightarrow \quad \lambda = 1.4585\times10^{-6}m$$

This wavelength lies in the infrared region of electromagnetic spectrum.

42. Soln. Given: $\Delta E = 12.5\ eV$

Let the electron jump from n = 1 to n = n level.

$$\Delta E = E_n - E_1$$

$$\therefore \quad 12.5 = -\frac{13.6}{n^2} - \left(\frac{-13.6}{1^2}\right)$$

$$12.5 = 13.6\left(1 - \frac{1}{n^2}\right)$$

$$1 - \frac{12.5}{13.6} = \frac{1}{n^2}$$

$$\frac{1.1}{13.6} = \frac{1}{n^2}$$

$$\frac{13.6}{1.1} = n^2$$

$$12.36 = n^2$$

$$n = 3.5$$

$$n = 3^{rd}$$

43. Soln. (a) From Bohr's model – An atom has a number of stable orbits in which an electron can reside without the emission of radiant energy. Each orbit corresponds to a certain energy level.

∵ Electron revolves is circular orbit

$$\therefore \quad \frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

The motion of an electron in circular orbits is restricted in such a manner that its angular momentum is an integral multiple of $h/2\pi$

Thus, $$L = mvr = \frac{nh}{2\pi}$$

$$E_n = \frac{-13.6}{r^2} z^2 eV$$

$$Z = 1 \; for \; H_2 \; atom$$

$$E_n = \frac{-13.6}{r^2} eV$$

From de-Broglie hypothesis

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

And from Bohr model

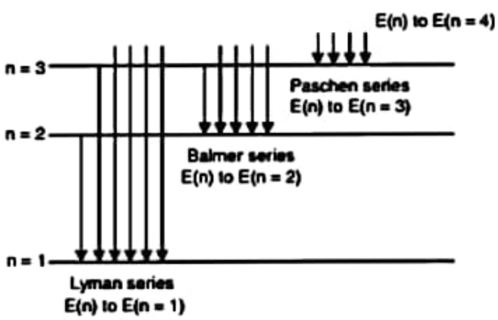
$$n\lambda = 2\pi r$$

$$\lambda = \frac{2\pi r}{r}$$

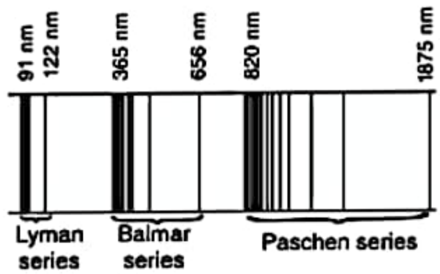
$$\frac{h}{mv} = \frac{2\pi r}{n}$$

$$\frac{nh}{2\pi} = mvr = L$$

44. Soln. (a) Energy level diagram showing lyman and balmer series:



Spectrum wavelengths of both series for hydrogen atom



(b) Rydberg formula,

$$\frac{1}{\lambda} = 1.1\times10^7\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda_1} = 1.1\times10^7\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\frac{1}{\lambda_1} = 1.1\times10^7 \times 0.1389$$

$$\lambda_1 = \frac{1}{1.1}\times10^7\times0.1389$$

$$\lambda_1 = \frac{100\times10^{-9}}{0.153}$$

$$\lambda_{max}(\lambda_1) = 653.6\,\text{nm}$$

$$\frac{1}{\lambda_2} = \frac{1.1\times10^7}{4}$$

$$\lambda_2 = \frac{4}{1.1\times10^7}$$

$$= \frac{400}{1.1}\times10^{-9}$$

$$\lambda_{min}(\lambda_2) = 363.6\,\text{nm}$$

45. Soln. We know that, according to de Broglie relation,

$$2\pi r = n\lambda$$

For first orbit, n = 1

$$2\pi r = n\lambda$$

$$\lambda = 2\pi\times0.53\,\overset{0}{A} \qquad [\because r = 0.53\,\overset{0}{A}]$$

$$\lambda = 3.331\,\overset{0}{A}$$

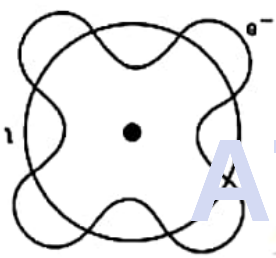
46. Soln. (a) Quantization condition: Of all possible circular orbits allowed by the classical theory, the electrons are permitted to circulate only in those orbits in which the angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$; h being Planck's constant.

Therefore, for any permitted orbit,

$$L = mvr = \frac{nh}{2\pi};\ n = 1, 2, 3, ...$$

Where L, m and v are the angular momentum, mass and speed of the electron respectively, r is the radius of the permitted orbit and n is positive integer called principle quantum number.

The above equation is Bohr's famous quantum condition. When an electron of mass m is confined to move in a line of length l with velocity v, the de-Broglie wavelength λ associated with electron is:



$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

Or p = Linear momentum

$$\Rightarrow \quad p = \frac{h}{\lambda} = \frac{h}{2l/n} = \frac{nh}{2l}$$

When electron revolves in a circular orbit of radius 'r' then $2l = 2\pi r$.

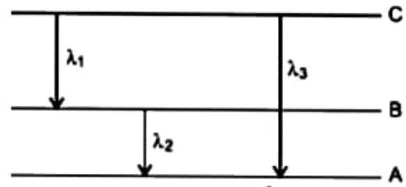
$$\therefore \quad p = \frac{nh}{2\pi r} \ or \ p \times r = \frac{nh}{2\neq}$$

Or angular momentum $|\vec{L}|$= p× r is an integral

Multiple of $h/2\pi$, which is Bohr's quantisation of angular momentum.

(b) $E_{CB} = \frac{hc}{\lambda_1}$

$E_{BA} = \frac{hc}{\lambda_2}$



$E_{CA} = \frac{hc}{\lambda_3}$

Now, $E_{CA} = E_{CB} + E_{BA}$

Where E_{CB} = Energy gap between level B and C,

E_{BA} = Energy gap between level A and B,

E_{CA} = Energy gap between level A and C.

$$\frac{hc}{\lambda_3} = \frac{hc}{\lambda_1} + \frac{hc}{\lambda_2}$$

$$\frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$$

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_2 + \lambda_1}$$

47. Soln. In a hydrogen atom,

Radius of electron orbit,

$$r = \frac{n^2h^2}{4\pi^2 k\, me^2} \quad ....(i)$$

And angular momentum

$$mvr = \frac{nh}{2\pi}$$

Or $$v = \frac{nh}{2\pi\, mr}$$

On putting value of r we get value of v as = $\frac{2\pi ke^2}{nh}$

So, kinetic energy,

$$E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2\pi ke^2}{nh}\right)^2$$

$$= \frac{4\pi^2k^2e^4m}{2n^2h^2} = \frac{2\pi^2k^2e^4m}{n^2h^2}$$

Potential energy

$$E_p = \frac{-k(e)\times(e)}{r} = -\frac{ke^2}{r}$$

Using equation (i), we get

$$E_p = -ke^2 \times \frac{4\pi^2 kme^2}{n^2h^2}$$
$$= -\frac{4\pi^2k^2me^4}{n^2h^2}$$

Hence, total energy of the electron in the n^th^ orbit

$$E = E_p + E_k$$
$$= -\frac{4\pi^2k^2me^4}{n^2h^2} + \frac{2\pi^2k^2me^4}{n^2h^2}$$
$$= -\frac{2\pi^2k^2me^4}{n^2h^2}$$

We know $k = \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ N\,m^2C^{-2}$

h (Planck's constant) = $6.6\times10^{-34}\,Js$

m for H – atom = $1.67\times10^{-27}\,kg$

$$e = 1.6\times10^{-19}\,C$$

Substituting these values, we get

$$E = \frac{-13.6}{n^2}eV$$

When the electron in a hydrogen atom jumps from higher energy level to the lower energy level, the difference of energies of the two energy levels is emitted as a radiation of particular wavelength. It is called a spectral line.

In H – atom, when an electron jumps from the orbit n_i to orbit n_f, the wavelength of the emitted radiation is given by,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Where,

$R \rightarrow$ Rydberg's constant = $1.09678\times10^7\ m^{-1}$

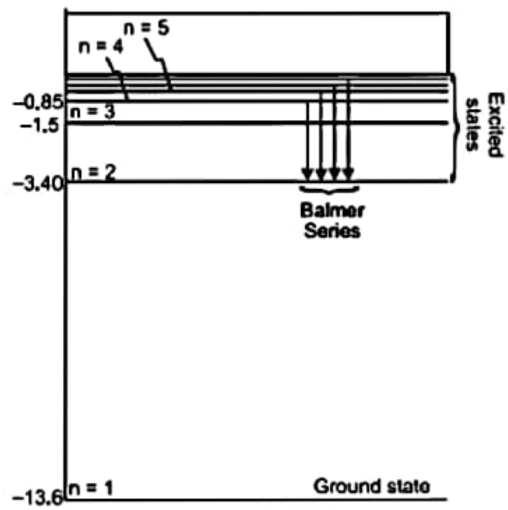
For Balmer series, $n_f = 2$ and $n_i = 3, 4, 5, ...$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

Where, $n_i = 3, 4, 5, ...$

These spectral lines lie in the visible region.

Total energy, E(eV)



48. Soln. Energy difference = $E_f - E_i$

$$= 3.4eV - 1.51eV$$
$$= 1.89eV$$
$$= 1.89\times1.6\times10^{-19}\,J$$

$$E = h\nu$$
$$= h\frac{c}{\lambda}$$

$$1.89\times1.6\times10^{-19} = \frac{6.6\times10^{-34}\times3\times10^8}{\lambda}$$

$$\lambda = \frac{6.6\times10^{-34}\times3\times10^8}{1.89\times1.6\times10^{-19}}$$
$$= \frac{6.6\times10^{-7}\times3}{1.89\times1.6}$$
$$= 6.54\times10^{-7}\,m$$
$$= 654\,nm$$

As this spectrum is in visible range. This radiation lies in Balmer series.

49. Soln. (i) According to Rutherford model, electron orbiting around the nucleus, continuously radiated energy due to the acceleration; hence the atom will not remain stable.

(ii) As electron spirals inwards; its angular velocity and frequency change continuously; therefore it will emit a continuous spectrum.

50. Soln. It is the distance of charged particle from the centreb of the nucleus, at which the whole of the initial kinetic energy of the (far off) charged particle gets converted into the electric potential energy of the system.

Distance of closest approach (r_c) is given by

$$r_c = \frac{1}{4\pi\varepsilon_0}.\frac{2Ze^2}{K}$$

'K' is doubled, $\therefore$ r_c becomes $\frac{r}{2}$

51. Soln. The minimum energy, required to free the electron from the ground state of the hydrogen atom, is know as Ionization energy.

$$E_0 = \frac{me^4}{8\varepsilon_0^2 h^2} \text{ i.e., } E_0 \propto m$$

Therefore, Ionization Energy will become 200 times.

52. Soln. Energy of a photon corresponding to wavelength λ,

$$E = h\frac{c}{\lambda}$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{275\times10^{-9}} J$$

$$= \frac{6.6\times10^{-34}\times3\times10^{8}}{275\times10^{-9}\times1.6\times10^{-19}} eV$$

$$= \frac{6.6\times3\times10^{2}}{275\times1.6}$$

$$= 4.5 eV$$

(ii) the calculated energy of the photon matches with the transition B.

53. Soln. For second excited state, n = 3

Hence two possible transition of the Lyman series: $3\to1$ *and* $2\to1$.

Wavelength for transition $3\to1, n_f = 1, n_i = 3$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = 1.1\times10^7\left(\frac{1}{1} - \frac{1}{9}\right)$$

$$= 1.1\times10^7\left(\frac{8}{9}\right)$$

$$\Rightarrow \quad \lambda = \frac{9}{8\times1.1\times10^{-7}}$$

$$= 1.023\times10^{-7}$$

$$= 102.3 nm$$

For transition $2\to1, n_f = 1, n_i = 2$

$$\frac{1}{\lambda} = 1.1\times10^7\left(1 - \frac{1}{4}\right)$$

$$\Rightarrow \quad \lambda = 212 nm$$

54. Soln. For hydrogen atom

The coulomb force provides the required centripetal force.

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r^2} = \frac{mv^2}{r}$$

$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

Electron has kinetic energy; $K = \frac{1}{2}mv^2$.

Putting the value of mv^2 in the above equation

$$K = \frac{e^2}{8\pi\varepsilon_0 r}$$

P.E. of an electron; $u = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}$ (negative sign shows that its due to attraction force)

Total energy, $E = K + U$

$$E = \frac{e^2}{8\pi\varepsilon_0 r} + \left(-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}\right)$$

$$= -\frac{e^2}{8\pi\varepsilon_0 r}$$

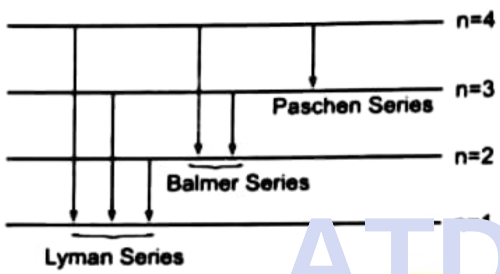
The significance of total negative energy is electron is bound to nucleus and revolve around it. This energy is known as binding energy of electron.

55. Soln. (i) Bohr's third postulate: It states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so a photon is emitted having energy equal to the energy difference between the initial and final states. The frequency of the emitted photon is given by

$$hv = E_i - E_f$$

Where E_i and E_f are the energies of t8he initial and final states and $E_i > E_f$.

(ii) Electron jumps from fourth to first orbit in an atom



$\therefore$ Maximum number of spectral lines can be ${}^4C_2 = \frac{4!}{2!2!} = \frac{4\times3}{2} = 6$

In diagram, possible way in which electron can jump (above).

The line responds to Lyman series (e^- jumps to 1st orbit), Balmer series (e^- jumps to 2nd orbit), Paschen series (e^- jumps to 3rd orbit).

56. Soln. $\Delta E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^8}{102.7\times10^{-9}} J$

$$= \frac{6.6\times10^{-34}\times3\times10^8}{102.7\times10^{-9}\times1.6\times10^{-19}} eV$$

$$= \frac{66\times3000}{1027\times16} = 12.04\,eV$$

Now, $\Delta E = |-13.6-(-1.50)|$

$$= 12.1\,eV$$

Hence, transition shown by arrow D corresponds to emission of $\lambda = 102.7$ nm.

57. Soln. Transition C and E belong to Lyman series.

Reason: In Lyman series, the electron jumps to lowest energy level from any higher energy levels Transition B and D belong to Balmer series.

Reason: The electron jumps from any higher energy level to the level just above the ground energy level. The wavelength associated with the transition is given by

$$\lambda = \frac{hc}{\Delta E}$$

Ratio of the shortest wavelength

$$\lambda_L : \lambda_B = \frac{hc}{\Delta E_L} : \frac{hc}{\Delta E_B}$$

$$= \frac{1}{0-(-10)} : \frac{1}{0-(-3)} = 3:10$$

58. Soln. (i) Radius of orbit

$$r_n = n^2 r_0$$

Where, r_0 is Bohr's radius = $5.3\times10^{-11} m$

radius of n = 3 orbit

$$r_3 = (3)^2 \times 5.3\times10^{-11} m$$
$$= 47.7\times10^{-11} m$$
$$= 4.77\times10^{-10} m$$

(ii) Given total energy $E = -\frac{e^2}{8\pi\varepsilon_0 r} = -3.4\,eV$

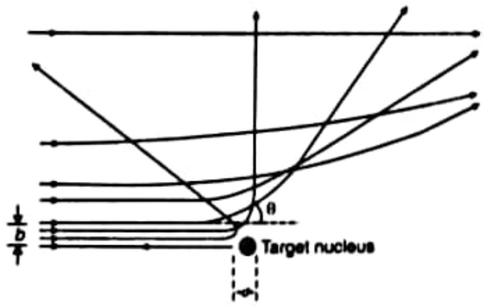
(a) Kinetic energy, $K = \frac{e^2}{8\pi\varepsilon_0 r} = -$ Total energy

Hence Kinetic energy, $K = -(-3.4)\,eV = 3.4\,eV$

(b) Potential energy, $P = -\frac{e^2}{4\pi\varepsilon_0 r} = 2\times$ total energy

= -6.8 eV

59. Soln. (i) The trajectory, traced by the α -particles in the Coulomb field of target nucleus, has the form shown below.



The size of the nucleus was estimated by observing the distance (d) of closest approach, of the α -particles. This distance is given by:

$$d = \frac{1}{4\pi\varepsilon_0}\frac{2eZe}{K}$$

Where, K = kinetic energy of the α -particles when they are far away from the target nuclei.

(ii) The wave nature of moving electrons was established through the Davission – Germer experiment.

In this experiment, it was observed that a beam of electrons, when scattered by a nickel target, showed 'maxima' in certain direction (like the 'maxima' observed in interference/diffraction experiments with light)

(iii) $\lambda = \frac{h}{p}$

$$\lambda = \frac{h}{mv}$$

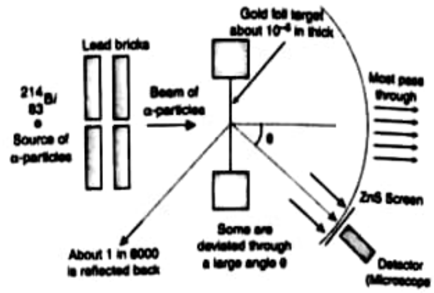
$$\lambda = \frac{h}{\sqrt{2mqV}}$$

Hence, $\frac{\lambda_d}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_d q_d}}$

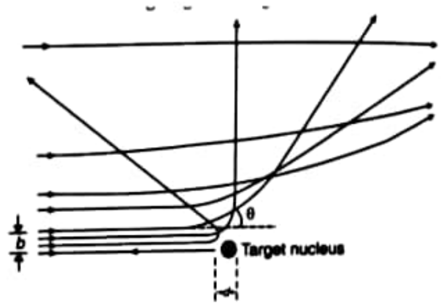
(accelerated potential is same for both particles)

$$\frac{\lambda_d}{\lambda_\alpha} = \sqrt{\frac{4\times2}{2\times1}} = 2$$

60. Soln. (i)



For most of the α -particles, impact parameter is large, hence they suffer very small repulsion due to nucleus and go right through the foil.



It gives an estimate of the size of nucleus.

(ii) K.E. of the α -particle = potential energy Possessed by beam at distance of closest approach.

$$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(2e)(Ze)}{r_0}$$

$$7.7\times1.6\times10^{-3} = \frac{9\times10^9\times2\times2.56\times10^{-38}\times80}{r_0}$$

$$r_0 = \frac{9\times10^9\times2\times2.56\times10^{-38}\times80}{7.7\times1.6\times10^{-13}}m$$

61. Soln. According to Bohr's second postulate quantization of angular momentum

$$mv_n r_n = n\frac{h}{2\pi}$$

Or $r_n = \frac{nh}{2\pi m v_n}$ ....(i)

Where h is the Planck's constant

Circumference of the electron in the n^{th} orbital state in hydrogen atom.

$$2\pi r_n = 2\pi \frac{nh}{2\pi m v_n}$$ (Using (i))

$$2\pi r_n = n\frac{h}{m v_n}$$ ....(ii)

But de Broglie wavelength of the electron

$$\lambda = \frac{h}{m v_n}$$ ....(iii)

From (ii) and (iii), we get

$$2\pi r_n = n\lambda$$

62. Soln. As per question, $\frac{v_h}{v_g} = \frac{1}{3}$ ....(i)

Where subscripts h and g denotes higher energy state and ground state.

Orbital velocity of electron in the nth orbit is

$$v_n = \frac{e^2}{2\varepsilon_0 nh} \text{ or } v_n \propto \frac{1}{n}$$

For ground state, $n = 1$, $\frac{v_h}{v_g} = \frac{1}{n}$ ....(ii)

Equating eqns. (i) and (ii), we get n = 3

Radius of n[th] orbit is $r_n = \frac{n^2 h^2 \varepsilon_0}{\pi e^2 m^2}$

Or $r_n \propto n^2$

$$\therefore \quad \frac{r_3}{r_1} = \frac{(3)^2}{(1)^2} = 9$$

$$r_3 = 9r_1 = 9R \qquad (\because r_1 = R(\text{Given}))$$

63. Soln. (a) Bohr's quantization condition: The electron can revolve around the nucleus only in those circular orbits in which angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$ i.e.,

$$mvr = \frac{nh}{2\pi}, n = 1, 2, 3, ...$$

(b) (i) The kinetic energy (E_k) of the electron in an orbit is equal to negative of its total energy (E)

$$E_k = -E = -(1.5) = 1.5\text{eV}$$

(ii) The potential energy (E_p) of the electron in an orbit is equal to twice of its total energy (E)

$$E_p = 2E = -1.5 \times 2 = -3.0eV$$

(iii) Here, ground state energy of the H-atom = -13.6 eV

When the electron goes from the excited state to the ground state, energy emitted is given by

$$E = -1.5 - (-13.6) = 12.1\text{eV} = 12.1 \times 1.6 \times 10^{-19}\text{J}$$

Now, $E = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{E} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{12.1 \times 1.6 \times 10^{-19}}$$

$$\lambda = 1.025 \times 10^{-7}$$

$$\lambda = 1025 \text{ Å}$$

64. Soln. According to Rydberg formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Frequency of the emitted radiation is

$$v = \frac{c}{\lambda} = Rc\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Here, $n_i = 4 \text{ and } n_f = 2$

$$\therefore \quad v = 10^7 \times 3 \times 10^8 \left(\frac{1}{2^2} - \frac{1}{4^2}\right) = \frac{9}{16} \times 10^{15} Hz$$

$$= 5.625 \times 10^{14} Hz$$

SURE SHOT QUESTIONS

ARVIND ACADEMY

Chapter – 13

Nuclei

MCQ (1 mark)

1. Soln.

$$\therefore Q_2 = [m_N({}_Z X^A) - m_N({}_{Z-1}Y^A) - m_e]c^2$$
$$= [m_N({}_Z X^A) + Zm_e - m_N({}_{Z-1}Y^A) - (Z-1)m_e - 2m_e]c^2$$
$$= [m({}_Z X^A) - m({}_{Z-1}Y^A) - 2m_e]c^2 = (M_x - M_y - 2m_e)c^2$$

2. Soln. (b)

3. Ans. (d): Nuclear density is constant and independent of mass number so the ratio is 1.

4. Ans. (a) : When two lighter nuclei fuse to form a heavier nucleus, its binding energy per nucleon increases.

5. Ans. (a) : The difference in mas ^7X nucleus and total mass of its constituent nucleus is 21.00 u

   Total mass of constituent nucleus = 21.00 u
   No. of nucleons present in ^7X, n = 7

   So, Binding energy per nucleon = $\frac{21.00}{7}u = 3.0u$

   So, option (a) is correct.

6. Ans. (c) : Nuclear forces are not always attractive as range below 1 fermi, it becomes repulsive in nature.

7. Ans. (b) : The nuclear force is much stronger than the Coulomb force acting between charges. So, option (b) is not true.

➢ Assertion-Reasoning (1 mark)

8. Sol.

   (b) Nuclei having mass number around 60 have maximum binding energy per nucleon (≈ 8.7 MeV), so they are most stable.

9. Sol.

   (b) Both assertion and reason are true but the reason is not a correct explanation of the assertion. An α-particle carries 2 units of positive charge. So the atomic number of the daughter nucleus decreases by 2 on α-decay.

10. Sol.

    (c) Assertion is true but the reason is false. Electrons are emitted as β-particles when a neutron decays into a proton during a radioactive disintegration.

11. Sol.

    (d) Both assertion and reason are false.

12. Sol.

    (a) Both the assertion and reason are true and the reason is the correct explanation of the assertion.

13. Sol.

    (c) Assertion is true but the reason is false.

14. Sol.

    (b) Both assertion and reason are true but reason is not the correct explanation of the assertion.

15. Sol.

    (c) Assertion is true but the reason is false. Much more energy is released in a single fission than in a single fusion.

16. Sol.

    (a) Both assertion and reason are true and the reason is the correct explanation of the assertion.

➤ Case Study (5 marks)

17. Ans. (i) (b) Radius of nucleus is given by $R = R_0 A^{1/3}$

(ii) (c) As, nuclear density is given by $\rho = \dfrac{3m}{4\pi R_0^3}$.

This means ρ is independent of A. Sor ratio would be 1 : 1.

(iii) (b)

(iv) (d) The saturation property of the nuclear forces is due to the fact that they are short range forces.

(v) (a) In Geiger –Mardsen scattering experiment thin gold foil is used to scatter alpha particle because alpha particle will not suffer more than one scattering and gold nucleus is 50 times heavier than alpha particle.

18. **Answers**

1. (*c*) The mass of a nucleus is sometimes equal to its atomic number.

For example, for $^1_1$H nucleus, $A = Z = 1$.

2. (*a*) In $^{23}_{11}$Na nucleus,

Number of protons = $Z = 11$

Number of neutrons = $A - Z = 23 - 11 = 12$.

Number of electrons = 0.

3. (*a*) The nuclei $^{13}_{6}$C and $^{14}_{7}$N are isotones. Both have 7 neutrons each.

4. (*a*) $^{74}_{34}$Se and $^{71}_{31}$Ga are isotones. Each nucleus has 40 neutrons.

5. (*c*) Suppose natural boron has x% of $^{10}_{5}$B isotope and $(100 - x)$% of $^{11}_{5}$B isotope. Then

$$\frac{10 \times x + 11(100 - x)}{100} = 10.81$$

or $$x = 19\%$$

$$\therefore \quad ^{10}_{5}B : ^{11}_{5}B = 19 : 81$$

➤ Questions

19. Ans. A certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass, and energy ΔE_b will be released in this process. The energy ΔE_b is called the binding energy of the nucleus.

If we separate a nucleus into its nucleons we would have to transfer a total energy equal to ΔE_b, to the nucleons.

Example: $_{92}U^{235} + {_0}n^1 \rightarrow {^{141}_{56}}Ba + {^{92}_{36}}Kr + 3{^1_0}n + Q$

The energy (Q) released was estimated to be 200 MeV per fission (or about 0.9 MeV per nucleon) and its equivalent to the difference in masses of the nuclei before and after the fission.

20. Ans. (a)

	Nuclear Fission		Nuclear Fusion
1.	The process of splitting of a heavy nucleus into two nuclei of nearly comparable masses with liberation of energy is called nuclear fission. Example: $^{235}_{92}U + {^1_0}n \rightarrow {^{141}_{56}}Ba + {^{92}_{36}}Kr + 3{^1_0}n + Q$	1.	When two or more than two light nuclei fuse together to form heavy nucleus with the liberation of energy, the process is called nuclear fusion. Example: $^2_1H + {^2_1}H \rightarrow {^3_2}He + {^1_0}n + 3.2MeV$
2.	A suitable bullet or projectile like neutron is needed to initiate nuclear fission	2.	The lighter nuclei have to be brought very close to each other against electrostatic repulsion
3.	Fission of single nucleus of U^{235}_{92} produces approx. 200 MeV energy.	3.	Four protons combine to form helium nucleus which produces approx. 24 MeV energy.

(b) Given: m = 100 g, P = 500 W

Here two deuterium nuclei produce 3.27 MeV energy

= 5.232×10^{-13} J

∴ Energy per nuclei = $\frac{5.232\times10^{-13}}{2}$

$= 2.616\times10^{-13}\ J$

No, of deuterium atoms in 100 g

$= \frac{6.023\times10^{23}\times100}{2} = 3.011\times10^{25}\ atoms$

∴ Total energy = $3.011 \times 10^{25} \times 2.616 \times 10^{-13}$

= 7.88×10^{12} J

$$Power = \frac{Energy}{Time} \Rightarrow t = \frac{7.88\times10^{12}}{500} = 1.58\times10^{10}\ s$$

$$= \frac{1.58\times10^{10}}{365\times24\times60\times60} = 500\ years$$

21. Ans. Given m = 2 kg, P = 800 W.

Here, two deuterium nuclei produce 3.27 MeV energy = 5.232×10^{-13} J

∴ Energy per nuclei =

$\frac{5.232\times10^{-13}}{2} = 2.616\times10^{-13}\ J$

Number of deuterium atom in 2 kg

$= \frac{6.023\times10^{23}\times2000}{2} = 6.023\times10^{26}$ atom

∴ Total energy = $6.023\times10^{26}\times2.616\times10^{-13}$

$= 15.75\times10^{13}\ J$

$$Power = \frac{Energy}{Time} \Rightarrow t = \frac{15.75\times10^{13}}{800} = 1.96\times10^{11}\ s$$

$$= \frac{1.96\times10^{11}}{365\times24\times60\times60} = 6.2\times10^{3}\ years$$

22. Ans. Nucleus was first discovered in 1911 by Lord Rutherford and his associates by experiments of scattering of α -particle by atoms. He found that the scattering result could be explained.

Atoms consists of a small, central, massive and positive core surrounded by orbiting electron. The experiment results indicated that the size of the nucleus is of the order of 10^{-14} metres and it thus 10,000 times smaller than the size of atom. Relation between the radius and mass number of the nucleus $R = R_0 A^{1/3}$

If m is the average mass of a nucleon and R is the nuclear radius, then mas of nucleus = mA, where A is the mass number of the element.

Volume of the nucleus, $V = \frac{4}{3}\pi R^3$

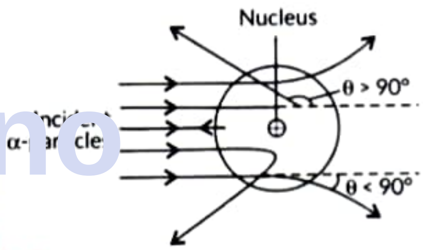
$$\therefore\ V = \frac{4}{3}\pi\left(R_0 A^{1/3}\right)^3 \Rightarrow V = \frac{4}{3}\pi R_0^3 A$$

Density of nuclear matter, $\rho = \frac{mA}{V}$

$$\Rightarrow \rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A} \Rightarrow \rho = \frac{3m}{4\pi R_0^3}$$

This shows that the nuclear density is independent of A.

23. Ans. Trajectory of α -particles in coulomb field of target nucleus shows that only a small fraction of the number of incident α -particles (1 in 8000) rebound back.



This shows that the number of α -particles undergoing head – on collision is small. This implies that the entire positive charge of the atom is concentrated in a small volume. So, this experiment is an important way to determine an upper limit on the size of nucleus.

Density of nucleus = $\frac{Mass\ of\ nucleus}{Volume}$

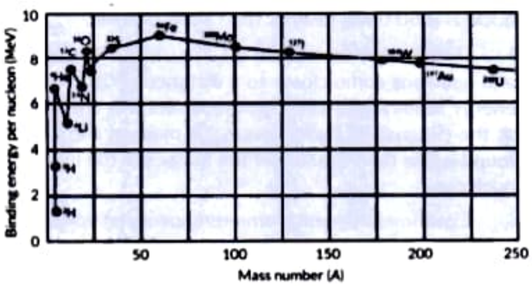
$$\rho = \frac{A\times m}{\frac{4}{3}\pi R^2};\ where\ R = R_0 A^{1/3}$$

$$Density\ \rho = \frac{A\times m}{\frac{4}{3}\pi R_0^3 A} = \frac{m}{\frac{4}{3}\pi R_0^3};\ \rho = \frac{3m}{4\pi R_0^3}$$

$$\rho = 2.97\times10^{17}\ kg\ m^{-3}$$

So, nuclear density is constant irrespective of mass number of size.

24. Ans. Binding energy curve:



Two salient features of the curve

(i) The binding energy per nucleon, E_{bn}, is practically constant i.e., practically independent of the atomic number for nuclei of middle mass number (30 < A < 170).

The curve has a maximum value of about 8.75 MeV for A = 56 and has a value of 7.6 MeV for A = 238.

(ii) E_{bn} is lower for both light nuclei (A < 30) and heavy nuclei (A > 170).

Nuclear fission: Binding energy per nucleon is smaller for heavier nuclei than the middle ones, i.e., heavier nuclei are less stable. When a heavier nucleus splits into the lighter nuclei, the B.E./nucleon changes from about 7.6 MeV to 8.4 MeV. Greater binding energy of the product nuclei results in the liberation of energy. This is what happens in nuclear fission which is the basis of the atom bomb.

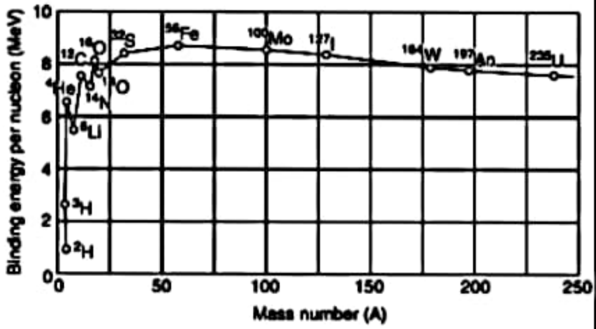
Nuclear fusion: The binding energy per nucleon is small for light nuclei, i.e., they are less stable. So when two light nuclei combine to form a heavier nucleus, the higher binding energy per nucleon of the latter results in the release of energy. This is what happens in a nuclear fusion which is the basis of the hydrogen bomb.

25. Soln. (a) Drawing the plot

Explaining the process of Nuclear fission and Nuclear fussion

(b) Finding the required time

(a) The plot of (B.E./nucleon) verses mass number is as shown.



[Note: Also accept the diagram that just shows the general shape of the graph].

From the plot we note that

(i) During nuclear fission

A heavy nucleus in the larger mass region (A > 200) breaks into two middle level nuclei, resulting in an increase in B.E./nucleon. This results in a release of energy.

(ii) During nuclear fusion

Light nuclei in the lower mass region (A < 20) fuse to form a nucleus having higher B.E./nucleon. Hence energy gets released.

[Alternatively: As per the plot: During nuclear fission as well as nuclear fusion, the final value of B.E./nucleon is more than its initial value. Hence energy gets released in both these processes.]

(b) We have

$$3.125\% = \frac{3.125}{100} = \frac{1}{32} = \frac{1}{2^5}$$

Half life = 10 years

$\therefore$ Required time = 5×10 years = 50 Years

26. Soln. We have $^{240}X = {}^{110}Y + {}^{130}Z$

Binding energy for X = 7.6 MeV

Binding energy of two fragments Y and Z = 8.5 MeV.

Energy released, Q = 240 (8.5 – 7.6) MeV = 216 MeV

27. Soln. Total Binding energy of Initial System

i.e.,

$${}^{1}_{2}H + {}^{1}_{2}H = (2.23 + 2.23)\,\text{MeV}$$

$$= 4.46\ MeV$$

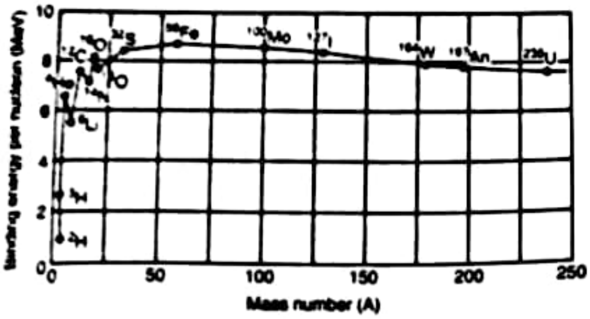
Binding energy of final system i.e., ${}^{2}_{3}He$

= 7.73 MeV

Hence energy released = 7.73 MeV – 4.46 MeV

= 3.27 MeV

28. Soln. (i)



From the above graph, it's clear that binding energy per nucleon is low for very light and for very heavy nuclei. If a nucleus of lower binding energy is converted into higher binding energy then energy is released. There are two methods of converting lower binding energy into higher binding energy.

Fission: A heavy nucleus (low BEN) is broken into two lower nucleus (higher BEN) and energy is released. This process is known as Fission.

Fusion: Two light nucleus (low BEN) are joined and form one nucleus of higher BEN, energy is released. This process is known as Fusion.

(ii) $$\frac{N}{N_0} = \frac{3.125}{100}$$

$$= (1/2)^5$$

Comparing this with the following standard equation

We may write $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$ where, $n = \frac{t}{T_{\frac{1}{2}}}$

$$\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^n$$; hence n = 5

Half life of the radioisotope is given 10 years,

Putting the value of n and $T_{1/2}$ in $\left[n = \frac{t}{T_{1/2}}\right]$

$$5 = \frac{t}{10}$$

$$t = 50 \ years$$

29. Soln. Nuclear Fission is the breaking down of heavier nucleus into smaller fragments while nuclear fusion is combining of lighter nuclei to form heavier nucleus. We see that bonding energy per nucleon of daughter nuclei in both fission and fusion processes is more than that of parent nuclei. Further, the difference in binding energy is released in form of energy while in both the process certain masses gets converted into energy.

In both processes, some mass gets converted into energy.

Energy Released

$$Q = \left[m\left({}^{2}_{1}H\right) + m\left({}^{3}_{1}H\right) - m\left({}^{4}_{2}He\right) - m(\text{n})\right] \times 931.5\,MeV$$

$$= [2.014102 + 3.016049 - 4.002603 - 1.008665] \times 931.5$$

$$= 0.018883 \times 931.5\,MeV$$

$$= 17.59\ MeV$$

30. Soln. (a) Nuclear radius $R = R_0(\text{A})^{1/3}$

Where A is the mass number of a nucleus.

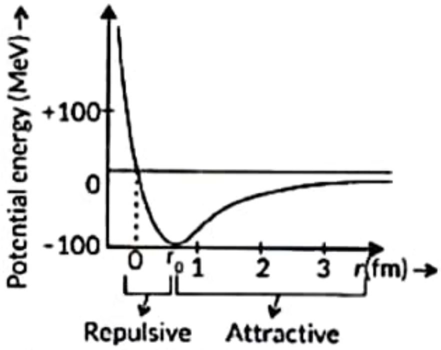
Given: R = 3.6 fm

$$\therefore\ 3.6\ fm = (1.2\ \text{fm})(\text{A}^{1/3}) \qquad [\because \text{R}_0 = 1.2\ \text{fm}]$$

Or $A = (3)^3 = 27$

31. Soln. If fact the number of protons and number of neutrons are same before and after a nuclear reaction but the binding energies of nuclei present before and after nuclear reaction are different. This difference is called the mass defect. This mass defect appears as energy of reaction. In this sense a nuclear reaction is an example of mass-energy interconversion.

32. Soln. Plot of potential energy of a pair of nucleons as a function of their separation is given in the figure.



Conclusions: (i) The nuclear force is much stronger than the coulomb force acting between charges or the gravitational forces between masses.

(ii) The nuclear force between two nucleons falls rapidly to zero as their distance is more than a few fermies.

(iii) For a separation greater than r_0, the force is attractive and for separation less than r_0, the force is strongly repulsive.

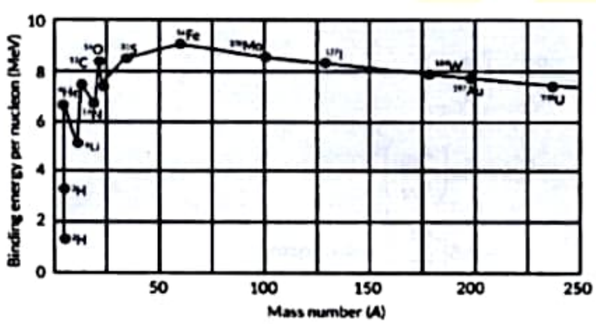
33. Soln. We have ${}^{240}X = {}^{110}Y + {}^{130}Z$

Binding energy for X = 7.6 MeV

Binding energy of two fragments Y and Z = 8.5 MeV

Energy released, Q = 240 (8.5 – 7.6) MeV = 216 MeV

34. Soln. (a) Binding energy curve:



Two salient features of the curve

(i) The binding energy per nucleon, E_{bn}, is practically constant, i.e., practically independent of the atomic number for nuclei of middle mass number (30 < A < 170).
The curve has a maximum value of about 8.75 MeV for A = 56 and has a value of 7.6 MeV for A = 238.

(ii) E_{bn} is lower for both light nuclei (A < 30) and heavy nuclei (A < 30) and heavy nuclei (A > 170).
(b) The binding energy curve can be used to explain the phenomena of nuclear fission and nuclear fusion.
Nuclear fission: Binding energy per nucleon is smaller for heavier nuclei than the middle ones, i.e., heavier nuclei are less stable. When a heavier nucleus splits into the lighter nuclei, the B.E./nucleon changes from about 7.6 MeV to 8.4 MeV. Greater binding energy of the product nuclei results in the liberation of energy. This is what happens in nuclear fission which is the basis of the atom bomb.
Nuclear fusion: The binding energy per nucleon is small for light nuclei, i.e., they are less stable. So when two light nuclei combine to form a heavier nucleus, the higher binding energy per nucleon of the latter results in the release of energy. This is what happens in a nuclear fusion which is the basis of the hydrogen bomb.

35. Soln. Energy released = $\Delta m \times 931$ MeV

$$\Delta m = 4m({}^1_1\text{H}) - m({}^4_2\text{He})$$

Energy released

$$Q = [4m({}^1_1\text{H}) - m({}^4_2\text{He})] \times 931\,\text{MeV}$$
$$= [4 \times 1.007825 - 4.002603] \times 931\,MeV = 26.72\,MeV$$

36. Soln. $\Delta m = Zm_p + (A - Z)m_n - m_N$

$$= 85 \times 1.007825 + 124 \times 1.008665 - 208.980388$$
$$= 1.75924$$

B.E. per nucleon =

$$\frac{B.E.}{A} = \frac{\Delta m \times 931.5}{209}\,MeV = 7.84\,MeV$$

37. Soln. The potential energy is minimum at a distance r_0 of about 0.8 fm.

(i) For values less than r_0, potential energy V becomes positive, so the force between the nucleons becomes repulsive.

(ii) For values more than r_0, potential energy V becomes negative, so the force between the nucleons becomes attractive.

38. Soln. Number of atoms present in 1 mole i.;e.,

239 g of ${}^{239}_{94}Pu$ = 6.023×10^{23}

∴ Number of atoms present in 1000 g of ${}^{239}_{94}Pu$

$$= \frac{6.023 \times 10^{23} \times 1000}{239} = 2.52 \times 10^{24}$$

Energy released per fission = 180 MeV

Total energy released = $2.52 \times 1024 \times 180\,MeV$

$$= 4.54 \times 1026\,MeV.$$

39. Soln. As $4\times10^{-3}\,kg$ of He consists of 6.023×10^{23} He atoms,

So $5\times10^{32}\,kg$ of He consists of

$$\frac{6.023\times10^{23}\times5\times10^{32}}{4\times10^{-3}}=7.52875\times10^{58}\,atoms$$

Now, 3 atoms of He produce energy

$$=7.27\times10^{6}\times1.6\times10^{-19}\,J.$$

So, all He atoms in star produce

Total energy =

$$\frac{7.27\times1.6\times10^{-13}}{3}\times7.52875\times10^{58}\,J$$

$$=29.2\times10^{45}\,J$$

As power generated is $P=5\times10^{30}\,W$

$\therefore$ Time taken to convert all the atoms into carbon

$$=\frac{29.2\times10^{45}}{5\times10^{30}}=5.84\times10^{15}\,seconds$$

$$=\frac{5.84\times10^{15}}{365\times24\times60\times60}=1.85\times10^{8}\,years$$

40. Soln. Let us first find the binding energy of

No. of protons in Fe = Z = 26

Mass of protons = 26 x 1.007825 u = 26.203450 u

No. of neutrons in Fe n = A – Z = 56 – 26 = 30

Mass of neutrons = 30 x 1.008665 u = 30.259950 u

Total theoretical mass of nucleus

= 26.206450 u + 30.259950 u = 56.463400 u

Actual mass of Fe nucleus = 55.934939 u

Mass defect Δm = Total mass – Actual mass = 0.528461 u

B.E. of ${}^{56}_{26}Fe$. Nucleus

$$E=\Delta mc^2=0.528461(931.5)\,\text{MeV}$$

$$=492.26\,MeV$$

$$\frac{B.E.}{nucleon}\,of\,{}^{56}_{26}Fe=\frac{492.26}{56}\,MeV=8.79\,MeV$$

(b) Now binding energy of ${}^{56}_{26}Fe$

No. of protons in Bi = Z = 83

No. of neutrons in

$$Bi\Rightarrow n=A-Z=209-83=126$$

Mass of protons =

$$83\times1.007825u=83.649475u$$

Mass of neutrons =

$$126\times1.008665u=127.091790u$$

Total theoretical mass of nucleus = $210.741265u$

Actual mass of Bi nucleus = 208.980388 u

Mass defect $\Delta m=1.760877\,u$

B.E. of ${}^{209}_{83}Bi$ nucleus =

$$\Delta mc^2=1.760877\times931.5\,MeV$$

$$=1640.3\,MeV$$

$$\frac{B.E.}{nucleon}\,of\,{}^{209}_{83}Bi=\frac{1640.3}{209}\,MeV=7.85\,MeV$$

So, ${}^{56}_{26}Fe$ is much more stable than ${}^{209}_{83}Bi$ due to more binding energy per nucleon.

41. Soln.

(a) Fusion reactions taking place within core of sun, 4 hydrogen nuclei combine to form a helium nucleus with the release of 26 MeV of energy.

$$4\,{}^{1}_{1}H\longrightarrow{}^{4}_{2}He+2e^{+}+26\,MeV$$

Number of atoms in 1 kg of ${}^{1}_{1}H$

$$n=\frac{1000g\times6\times10^{23}}{Atomic\ mass}=\frac{1000\,g}{1g}\times6\times10^{23}$$

$$n=6\times10^{26}\,atoms$$

Energy released in the fusion of 1 kg of ${}^{1}_{1}H$

$$E=\frac{6\times10^{26}\times26}{4}\,MeV$$

$$E=39\times10^{26}\,MeV.$$

(b) Energy released per fission of U-235 is 200 MeV.

Number of atoms in 1 kg U – 235

$$n=\frac{1000g\ \times6\times10^{23}}{235\,g}=25.53\times10^{23}$$

Total energy released for fission of 1 kg of uranium

$$E=25.53\times1023\times200\,MeV$$

$$=5.1\times1026\,MeV$$

So the energy released in fusion of 1 kg of Hydrogen is nearly 8 times the energy released in fission of 1 kg of uranium – 235.

SURE SHOT QUESTIONS

ARVIND ACADEMY

Chapter – 14

Semiconductor Electronics: Materials, Devices and Simple Circuits

MCQ (1 mark)

1. Soln. (d)

2. Soln. (b): Height of potential barrier decreases when p – n junction is forward biased and it increases when junction is reverse biased.

3. Soln. (b): Diode D_1 is reverse biased as p side is connected to negative potential and n side to ground.

Diode D_2 is forward biased as p side is grounded and n side is at negative potential.

4. Soln. (d): Potential difference across capacitor,

V = peak voltage = $V_{rms}\sqrt{2} = 220\sqrt{2}\,V$.

5. Soln. (b): A vacancy is created when an electron leaves a covalent bond. This vacancy is known as hole.

6. Soln. (c): During positive half cycle of input ac voltage, the p-n junction is forward biased. The resistance of p-n junction is low. The current in the circuit is maximum. In this situation, maximum potential difference will appear across resistance connected in series of the given circuit. Due to it, there is no output voltage across p-n junction.

During the negative half – cycle of input ac voltage, the p-n junction is reverse biased. The resistance of p-n junction becomes high which will be more than resistance in series. Due to it, there will be voltage across p-n junction with negative cycle in output.

7. Soln. (b): Let V be the potential difference between A and B, then V – 0.3 = (5 + 5) x 10^3 x (0.2 x 10^{-3}) = 2

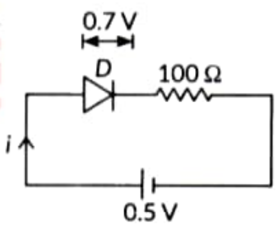
Or V = 2 + 0.3 = 2.3 V

8. Ans. (d): For silicon it is 1.1 eV.

9. Ans. (b) : In n-type semiconductor, the donor energy level lies just below the conduction band.

10. Ans. (b):As the temperature increases the carrier concentration increases significantly. This is because extra electrons are excited from the valence band to the conduction band, due to which the number of free electron - hole pairs increases.

11. Ans (a):



Here, the applied voltage (0.5 V) is less than barrier potential (0.5 V) is less than barrier potential (0.7 V).

Thus, it is an example of forward biasing and there is no flow of current.

So, answer is (a) forward biasing, 0 Amp.

12. Ans. (c) : Zero as diffusion and drift current are equal and opposite.

Assertion-Reasoning (1 mark)

13. Ans. (a) : Assertion : Correct.

Reason : correct, the number of e^- or hole increase in conduction band. They jump from valence band to conduction band.

14. Ans. (a): When a atom is ionised, it creates a free electron and also it creates a positively ionised donor atom. The charge on the free electron and the ionised donor are equal and opposite. So, as long as the electron doesn't go anywhere, the net charge remains zero.

15. Ans. (c):The electrical conductivity of semiconductor increases on adding an appropriate amount of suitable impurity or doping. It can be done with an impurity which is electron rich or electron deficient.

16. **Sol.** (c): In a semiconductor, there are no free electrons at 0 K. The number of free electrons increases with increase in temperature because with increase in temperature the electron get sufficient energy to cross forbidden band and reach conduction band. But total number of free electrons in a semiconductor is less than that in a conductor.

17. **Sol.** (b) : In semiconductor there may be energy bands due to donor impurities (E[illegible]) near the conduction band or acceptor impurities (E_D) near the valence band. When electron falls from higher to lower energy level containing holes, the energy is released in the form of radiation. The energy of radiation emitted by LED is equal or less than the band gap of the semiconductor used. The radiation released lies in range of visible light whose colour depends on the semiconductor used.

18. **Sol.** (b): A reverse bias on a p-n junction opposes the movement of the majority charge carriers thus stopping the diffusion current. It makes the free electrons and holes to drift cross the junction. Therefore a small current in pA flows even when the p-n junction is reverse biased. The drift current is due to the thermal excitations of the electrons and holes.

19. **Sol.** (c): A small increase in forward voltage across p-n junction shows large increase in forward current. Hence the resistance (= voltage / current) of p-n junction is low when forward biased. Also the width of depletion layer of p-n junction decreases in forward bias. A large increase in reverse voltage across p-n shows small increase in reverse current. Hence the resistance of p-n junction is high when reverse biased. Also the width of the depletion layer of p-n junction increases in reverse biased.

20. **Sol.** (d): In p-n junction, the diffusion of majority carriers takes place when junction is forward biased and drifting of minority carriers takes place across the junction, when reverse biased. The reverse bias opposes the majority carriers but makes the minority carriers to cross the p-n junction. Thus the small current in μA flows during reverse bias.

21. **Sol.** (a): In insulator, the forbidden energy gap is quite large. When electric field is applied to such a solid, the electron find it difficult to acquire such a large amount of energy. Thus no electron flow occurs.

22. **Sol.** (a): For electron to jump from valence band to conduction band needs energy equal or more than the forbidden band between these two band. As the energy of band gap increases, it becomes difficult for electron to get that equivalent energy.

➢ Case Study

23. Answers

1. (*a*) In semiconductors at a room temperature, the valence band is partially empty and the conduction band is partially filled.

2. (*d*) In insulators, the conduction band is empty and valence band is completely filled with electrons.

3. (*d*) For an insulator,

$$E_g = 5\text{eV}.$$

4. (*d*) In a semiconductor, $E_g = kT$, *i.e.*, the energy gap between valence and conduction bands corresponds to the thermal energy at room temperature.

5. (*c*) For a germanium crystal,

$$E_g = 0.7 \text{ eV} = 0.7 \times 1.6 \times 10^{-19} \text{ J}$$
$$= 1.12 \times 10^{-19} \text{ J}.$$

24. Answers

6. (*a*) Section *A* represents *n*-type germanium because it has electrons as the majority charge carriers.

7. (*d*) *p*-*n* junction is forward biased because its *n*-section is connected to the –ve terminal and *p*-section is connected to the +ve terminal of the battery.

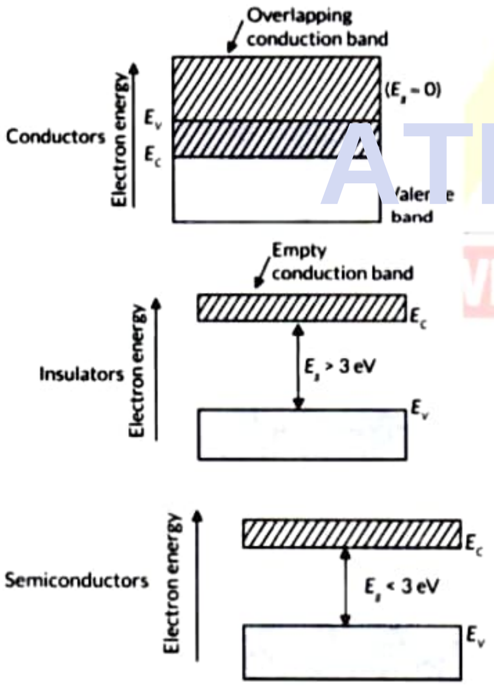
8. (*d*) Only in case (*d*), the *p*-side is at higher potential (+10V) and *n*-side is at lower potential (+5V).

9. (*c*) Holes diffuse towards the junction because hole concentration is more in *p*-region than in *n*-region.

10. (*b*) $I = \frac{(3.2-3)\text{V}}{100\,\Omega} = 2 \times 10^{-3}\,\text{A}.$

➤ Questions

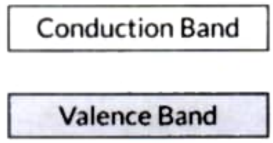
25. Ans. The energy gap between valence band and conduction band is known as energy band gap in a solid.



26. Ans. Metals ; For metals, the valence band is completely filled and the conduction band can have two possibilities either it is partially filled with an extremely small energy gap between the valence and conduction bands or it is empty, with the two bands overlapping each other as shown in the figure.

On applying even a small electric field, metals can conduct electricity.

Insulators : For insulators, the energy gap between the conduction and valence bands is very large. Also, the conduction band is completely empty as shown in the figure.



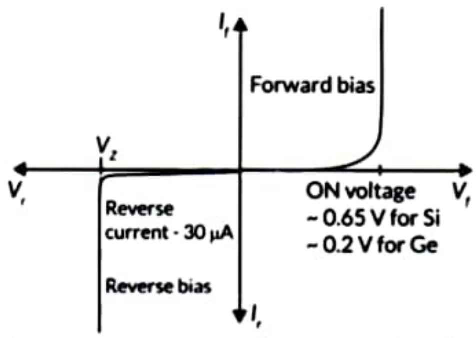
When an electric field is applied across such a solid, the electrons find it difficult to acquire such a large amount of energy to reach the conduction band. Thus, the conduction band continues to be empty. That is why no current flows through insulators.

27. Ans. Depletion layer: The small region in the vicinity of the junction which is depleted of free charge carriers and has only immobile ions is called the depletion layer. Barrier potential : Due to accumulation of negative charges in the p-region and positive charges in the n-region sets up a potential difference across the junction sets up. This acts as a barrier and is called potential barrier Vg which opposes the further diffusion of electrons and holes across the junction,

(a) In forward biasing the width of depletion layer reduced and the external applied field is able to overcome the Strong electric field of depletion layer. In reverse biasing the width of depletion layer increases and the electric field of depletion layer become more stronger,

(b) As forward voltage opposes the potential barrier and effective barrier potential decreases. It makes the width of the depletion layer smaller.

28. Ans.



The reverse current is due to minority charge carriers and even a small voltage is sufficient to sweep the minority carriers from one side of the junction to the other side of the junction. Here the current is not limited by the magnitude of the applied voltage but is limited due to the concentration of the minority carrier on either side of the junction.

29. Ans. (i) From the given curve, we have

Voltage, V = 0.8 volt for current, I = 20 mA

Voltage, V = 0.7 volt for current, I = 15 mA

$\Rightarrow \Delta I = (20-15)\,\text{mA} = 5\times10^{-3}\,A$

$\Rightarrow \Delta V = (0.8-0.7) = 0.1\,\text{V}$

$\therefore$ Resistance, $R = \dfrac{\Delta V}{\Delta I}$

$\Rightarrow R = \dfrac{0.1}{5\times10^{-3}}$

$\Rightarrow R = 20\Omega$

(ii) For V = -10 V, we have

$I = -1\mu A = -1\times10^{-6}\,A$

$\Rightarrow R = \dfrac{10}{1\times10^{-6}} = 1.0\times10^{7}\,\Omega$

30. Ans. Differences between half wave rectifier and full wave rectifier are:

	Half wave rectifier		Full wave rectifier
(i)	A half wave rectifier is an electronic circuit which converts only one-half of the AC cycle into pulsating DC. It utilizes only half of AC cycle for the conversion process.	(i)	Full wave rectifier is an electronic circuit which converts entire cycle of AC into pulsating DC.
(ii)	Output frequency of half wave rectifier is equal to the input frequency.	(ii)	Output frequency of full wave rectifier is double the input frequency.

31. Soln. Conductors:

(i) In case of conductors, the valence band is completely filled and the conduction band can have two cases – either it is partially filled with an extremely small energy gap between the valence and conduction bands or it is empty, with the two bands overlapping each other as shown below:

Case 1	Case 2
Conduction Band	Conduction Band
Valence Band	Valence Band

(ii) Even when a small current is applied, conductors can conduct electricity.

Insulators:

(i) In case of insulators, the energy gap between the conduction and valence bands is very large and the conduction band is practically empty.

Conduction Band

↑

Forbidden Energy Gap

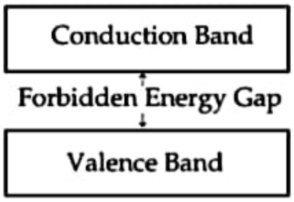
↓

Valence Band

(ii) When an electric field is applied to such kind of material, the electrons find hard to receive such a large amount of energy to reach the conduction band. Thus, the conduction band remains empty. That is why no current flows through insulators.

Semiconductors:

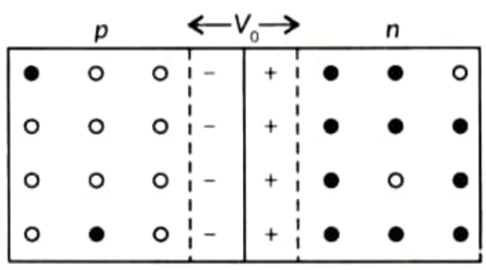
(i) In case of semiconductor, the energy band structure of semiconductors is similar to insulators, But in this case, the size of forbidden energy gap is quite smaller than that of the insulators.



When an electric field is applied to a semiconductor, the electrons in the valence band fid it relatively easier to jump to the conduction band. So, the conductivity of semiconductors lie between the conductivity of conductors and insulators.

32. Ans. (a) In forward biasing, the forward voltage opposes the potential barrier. As a result, potential barrier height is reduced and the width of depletion region decreases. Small increase in forward voltage shows large increase in forward current. Thus resistance in forward bias is reduced. In reverse bias, barrier potential height increases. For large increase in reverse voltage shows small increase in reverse current. Thus, resistance of p-n junction is high to flow of current when reverse biased, (b) Intrinsic semiconductors have very small conductivity at room temperature. When certain impurities in small amount (□ 1 part per million) are added to an intrinsic semiconductor, electrical conductivity of doped semiconductor increases to a great extent and made it useful in many practical applications.

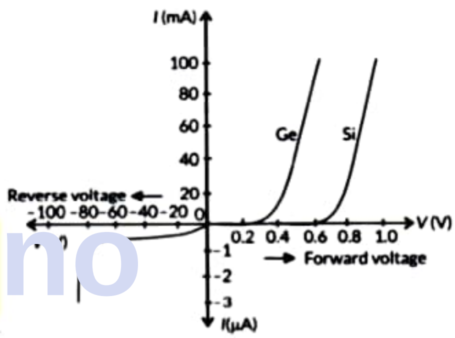
33. Ans. Two processes that take place in the formation of p-n junction are diffusion and drift.



when p-n junction is formed, then junction is formed, then at the junction free electrons from n-type diffuse over to p-type, thereby filling in the holes in p-type. Due to this a layer of positive charge is built on n-side and a layer of negative charge is built on p-side of the p-n junction. This layer sufficiently grows up within a very short time of the junction being formed, preventing any further movement of charge carriers (i.e., electrons and holes) across the junction.

Thus a potential difference V_0 of the order of 0.1 to 0.3 V is set up across the p – n junction called potential barrier or junction barrier. The thin region around the junction containing immobile positive and negative charges is known as depletion layer.

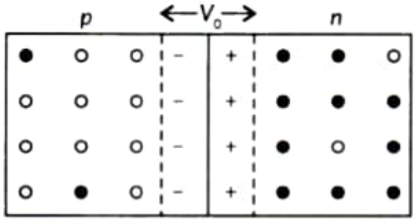
34. Ans. I – V characteristics of a p-n junction: The I-V characteristics of a p-n junction do not obey Ohm's law. The I – V characteristics of a p-n junction are as shown in the figure.



(a) The current of order in reverse bias is due to the drifting of minority charge carriers from one region to another through the junction. A small amount of applied voltage is sufficient to sweep the minority charge carriers through the junction. So, reverse current is almost independent of critical voltage.

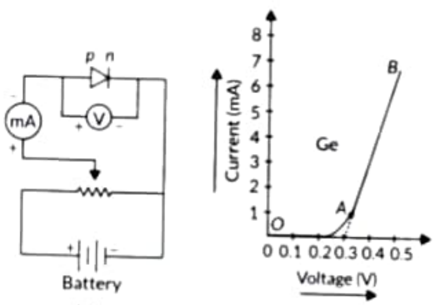
(b) During breakdown voltage, enormous covalent bond breaks. As a result large number of charge carriers increases. Therefore current increases at breakdown voltage.

35. Ans. Two processes that take place in the formation of a p-n junction are diffusion and drift.



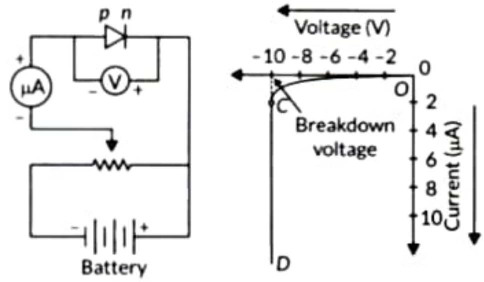
When p-n junction is formed, then at the junction free electrons from n-type diffuse over to p-type, thereby filling in the holes in p-type. Due to this a layer of positive charge is built on n-side and a layer of negative charge is built on p-side of the p-n junction. This layer sufficiently grows up within a very short time of the junction being formed, preventing any further movement of charge carriers (i.e., electrons and holes) across the junction. Thus a potential difference Vq of the order of 0.1 to 0.3 V is set up across the p-n junction called potential barrier or junction barrier. The thin region around the junction containing immobile positive and negative charges is known as depletion layer.

36. Ans. Forward biased characteristics ; The circuit diagram for studying forward biased characteristics is shown in the figure. Starting from a low value, forward bias voltage is increased step by step (measured by voltmeter) and forward current is noted (by ammeter). A graph is plotted between voltage and current. The curve so obtained is the forward characteristic of the diode.



(a) (b)

At the start when applied voltage is low, the current through the diode is almost zero. It is because of the potential barrier, which opposes the applied voltage. Till the applied voltage exceeds the potential barrier, the current increases very slowly with increase in applied voltage (OA portion of the graph). With further increase in applied voltage, the current increases very rapidly (AB portion of the graph), in this situation, the diode behaves like a conductor. The forward voltage beyond which the current through the junction starts increasing rapidly with voltage is called threshold or cut-in voltage. If line AB is extended back, it cuts the voltage axis at potential barrier voltage. Reverse biased characteristics : The circuit diagram for studying reverse biased characteristics is shown in the figure.

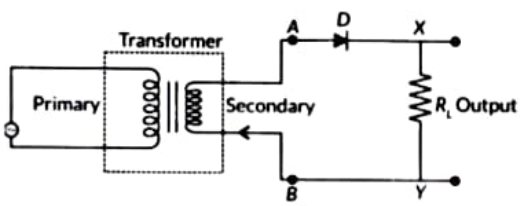


(a) (b)

In reverse biased, the applied voltage supports the flow of minority charge carriers across the junction. So, a very small current flows across the until due to minority charge carriers. Motion of minority charge carriers is also supported by internal potential barrier, so all the minority carriers cross over the junction.

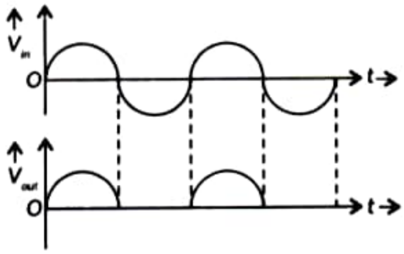
Therefore, the small reverse current remains almost constant over a sufficiently long range of reverse bias, increasing very little with increasing voltage (OC portion of the graph). This reverse current is voltage independent upto certain voltage known as breakdown voltage and this voltage independent current is called reverse saturation current.

37. Ans. Half wave rectifier:



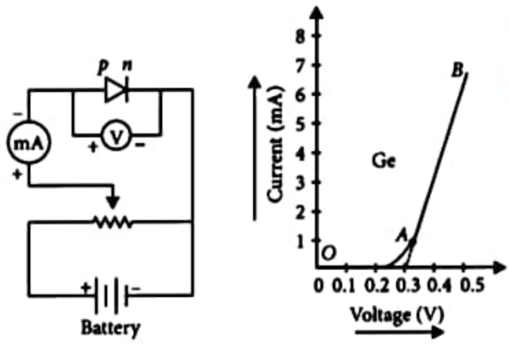
It consists of a diode D connected in series with load resistor R_L across the secondary windings of a step-down transformer. Primary of transformer is connected to a.c. supply. During positive half cycle of input a.c., end A of the secondary winding becomes positive and end 6 negative. Thus, diode

D becomes forward biased and conducts the current through it. So, current in the circuit flows from A to B through load resistor R_L.



During negative half cycle of input a.c., end A of the secondary winding becomes negative and end B positive. Thus, diode D becomes reverse biased and does not conduct any current. So, no current flows in the circuit. Since electric current through load R_L flows only during positive half cycle, in one direction only i.e., from A to B, so d.c. is obtained across R_L.

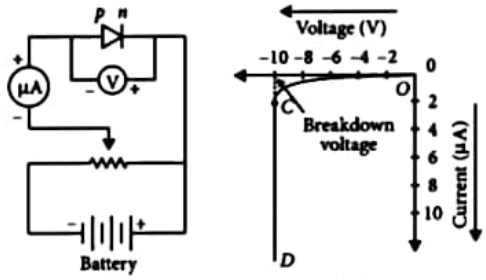
38. Soln. Forward biased characteristics: The circuit diagram for studying forward biased characteristics is shown in the figure. Starting from a low value, forward bias voltage is increased step by step (measured by voltmeter) and forward current is noted (by ammeter). A graph is plotted between voltage and current. The curve so obtained is the forward characteristic of the diode.



(a) (b)

At the start when applied voltage is low, the current through the diode is almost zero. It is because of the potential barrier, which opposes the applied voltage. Till the applied voltage exceeds the potential barrier, the current increases very slowly with increase in applied voltage (OA portion of the graph). With further increase in applied voltage, the current increases very rapidly (AB portion of the graph), in this situation, the diode behaves like a conductor. The forward voltage beyond which the current through the junction starts increasing rapidly with voltage is called threshold or cut-in voltage. If line AB is extended back, it cuts the voltage axis at potential barrier voltage.

Reverse biased characteristics: The circuit diagram for studying reverse biased characteristics is shown in the figure.



(a) (b)

In reverse biased, the applied voltage supports the flow of minority charge carriers across the junction. So, a very small current flows across the junction due to minority charge carriers.

Motion of minority charge carriers is also supported by internal potential barrier, so all the minority carriers cross over the junction.

Therefore, the small reverse current remains almost constant over a sufficiently long range of reverse bias, increasing very little with increasing voltage (OC portion of the graph). This reverse current is voltage independent upto certain voltage known as breakdown voltage and this voltage independent current is called reverse saturation current.

39. Soln. (a) Given, $\lambda = 600\,nm = 6\times10^{-7}\,m$

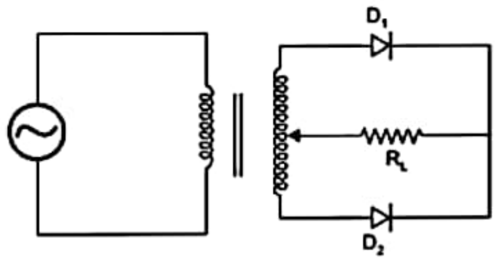
$$E = \frac{hc}{\lambda} = \frac{6.6\times10^{-34}\times3\times10^{8}}{6\times10^{-7}\times1.6\times10^{-19}}\,eV$$
$$= 2.06\,eV$$

As, energy gaps of diodes D_1 and D_3 are greater than the given energy of the incident radiation.

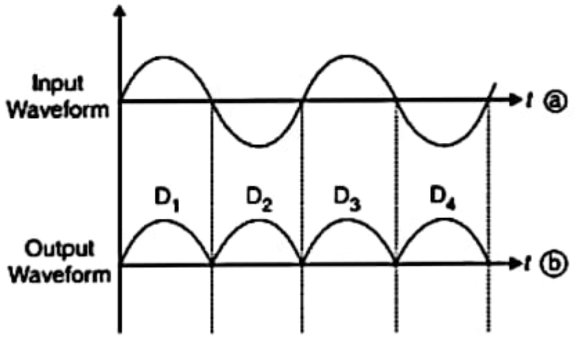
Hence diodes D_1 and D_3 will not be able to detect light of wavelength 600 nm.

(b) In reverse bias condition of photodiode, the change in saturation reverse current is directly proportional to the change in the incident light flux or light intensity, which can be measured accurately. It is not so when photodiode is forward biased.

40. Soln.



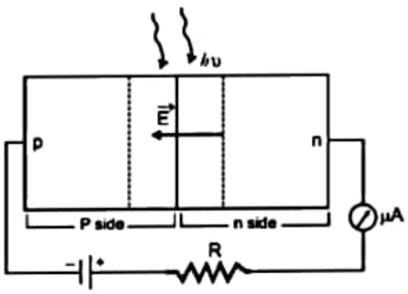
Full wave rectifier



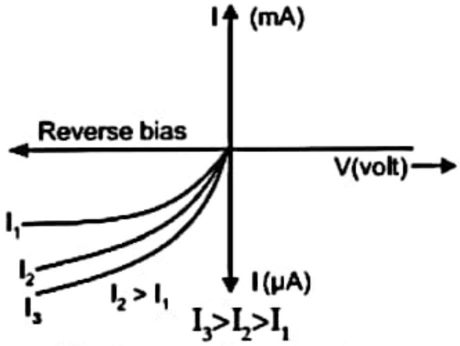
During first half of input sinusoidal ac signal diode D_1 conducts as it is forward bias and during second half of input ac signal diode D_2 conducts as it is forward bias now. D_2 and D_1 are inverse bias conditions during first and second half respectively and doesn't conduct. Due to this output appears as waveform (b)

41. Soln. (a) The value of R has to be increase because on heating, the conductivity of a semiconductor increases. i.e., resistance of S decreases on heating.

(b)

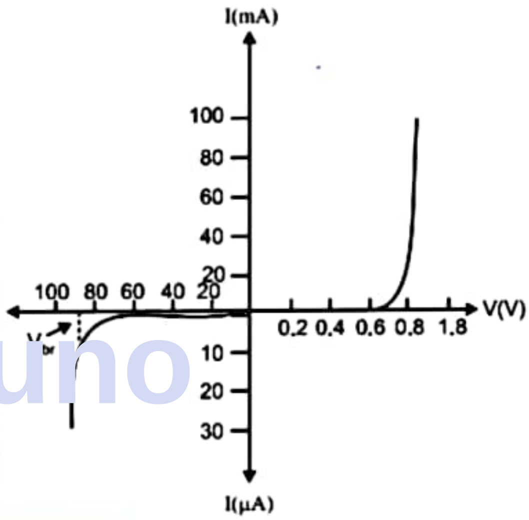


Working: In photodiode an electric field exists across the junction from n-side to p-side. When visible light with energy hv greater than energy gap (E_g) illuminates the junction, then electron-hole pairs are generated in the depletion layer. Due to electric field electron moves towards n side and hole towards p-side give rise to an emf. When an external load is connected current flows.



I-V Characteristics of the diode

42. Soln. V – I characteristics of p-n junction diode:



(i) Under the reverse bias condition, the holes of p-side are attracted towards the negative terminal of the battery and the electrons of the n-side are attracted towards the positive terminal of the battery. This increases the depletion layer and the potential barrier. However the minority charge carriers are drifted across the junction producing a small current. At any temperature the number of minority carriers is constant and very small so there is the small current at any applied potential. This is the reason for the current under reverse bias to be almost independent of applied potential. At the critical voltage, avalanche break down takes place which results in a sudden flow of large current.

(ii) At the critical voltage, the holes in the n-side and conduction electrons in the p-side are accelerated due to the reverse-bias voltage. These minority carriers acquire sufficient kinetic energy from the electric field an collide with valence electrons. Thus, the bond is finally broken and the valence electrons move into the conduction band resulting in enormous flow of electrons and thus result in formation of hole – electron pairs. Thus, there is a sudden increase in the current at the critical voltage. Zener diode is a semiconductor device which operates under the reverse bias condition in the breakdown region.

43. Soln. (i) Forward biased: Potential drop across the junction decreases and diffusion of holes and electrons across the junction increases. It makes the width of the depletion layer smaller.

(ii) Reverse biased: Potential drop across the junction increases and diffusion of holes and electrons across the junction decreases. It makes the width of the depletion layer larger.

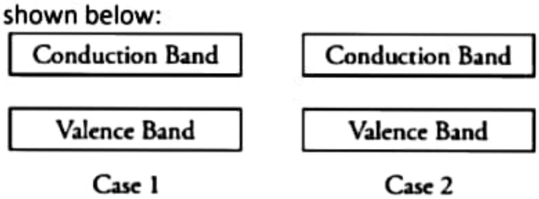
44. Soln. Forward Biasing: The positive terminal of the external battery is connected to p-side and negative terminal of battery to n-side of p-n junction. The forward bias voltage oppose the potential barrier. Due to this, the potential barrier is reduced and hence the depletion becomes thin. The resistance of p-n region junction decreases.

Reverse biasing: The negative terminal of the external battery is connected to p-side and positive terminal of battery to n-side of p-n junction. The reverse bias voltage supports the potential barrier. Due to this, the potential barrier is increased. The resistance of p-n junction becomes high.

45. Soln. This is he characteristic of solar cell.

46. Soln. Conductors:

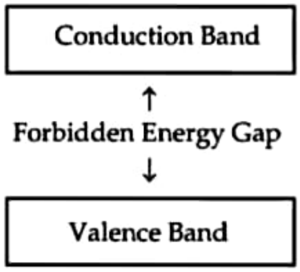
(iii) In case of conductors, the valence band is completely filled and the conduction band can have two cases – either it is partially filled with an extremely small energy gap between the valence and conduction bands or it is empty, with the two bands overlapping each other as shown below:



(iv) Even when a small current is applied, conductors can conduct electricity.

Insulators:

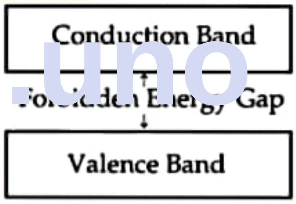
(iii) In case of insulators, the energy gap between the conduction and valence bands is very large and the conduction band is practically empty.



(iv) When an electric field is applied to such kind of material, the electrons find hard to receive such a large amount of energy to reach the conduction band. Thus, the conduction band remains empty. That is why no current flows through insulators.

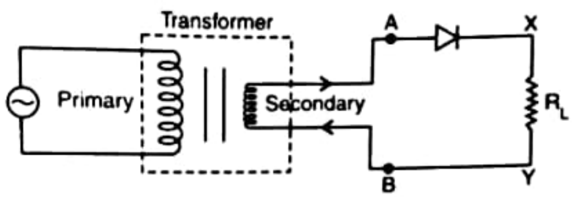
Semiconductors:

(ii) In case of semiconductor, the energy band structure of semiconductors is similar to insulators, But in this case, the size of forbidden energy gap is quite smaller than that of the insulators.



(iii) When an electric field is applied to a semiconductor, the electrons in the valence band fid it relatively easier to jump to the conduction band. So, the conductivity of semiconductors lie between the conductivity of conductors and insulators.

47. Soln.

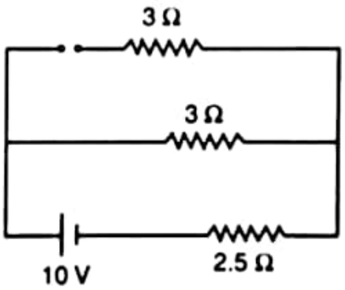


Working: During one half of the input a.c., the diode is forward biased and a current flows through R_L.

During the other half of the input a.c., the diode is reverse biased and no current flows through the load R_L.

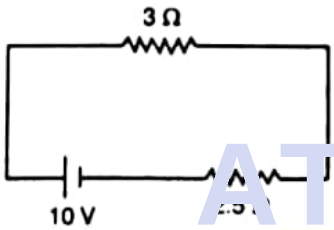
Hence, the given a.c. input is rectified.

48. Soln. In the cycle, if D_1 and D_2 are short then equivalent circuit will result as:



D_2 is reverse biased

$\therefore$ D_1 conducts



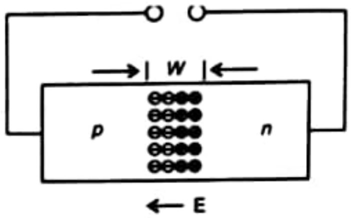
$$\therefore \quad I=\frac{10}{3+2.5}=\frac{10}{5.5}=1.8\text{ A}$$

49. Soln. Diffusion: It is the process of movement of majority charge carriers from their majority zone (i.e., electrons from $n \rightarrow p$ and holes from $p \rightarrow n$) due to the electric field developed at the junction.

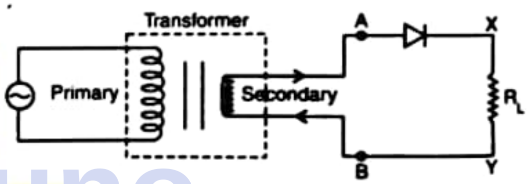
Drift: Process of movement of minority charge carriers (i.e., holes from $n \rightarrow p$ and electrons from $p \rightarrow n$) due to the electric field developed at the junction.

Barrier potential: The loss of electrons from the n-region and gain of electrons by p-region causes a difference of potential across the junction, whose polarity is such as to oppose and then stop the further flow of charge carriers. This (stopping) potential is called Barrier potential.

50. Soln. (i) Due to diffusion and drift, the electrons and holes move across the junction, creating a final stage in which a region is crated across the junction wall, which gets devoid of the mobile charge carriers. This region is called depletion region; the potential difference across the region is called Barrier potential.



(ii)



Working: An alternating voltage is applied across a diode in series with a load, a pulsating voltage will appear across the load only during that half cycle of the ac input during which the diode is forward biased.

Therefore, in the positive half – cycle of ac input there is current through the load resistor R_L and we get an output voltage whereas there is no output during the negative half cycle. Thus, the output voltage is restricted to only one direction and is said to be rectified.

51. Soln.

	Intrinsic semiconductor	Extrinsic semiconductor
	It is a semiconductor in pure form.	It is a semiconductor doped with trivalent or penetrates impurity atoms.

i)	Intrinsic charge carriers are electrons and holes with equal concentration.	The two concentrations are unequal in it. There is excess of electrons in n-type and excess of holes in p-type semiconductors.
ii)	Current due to charge carriers is feeble (of the order of μA).	Current due to charge carriers is significant (of the order of mA).

(b) If p-type and n-type semiconductor are heavily doped. Then due to diffusion of electrons from n-region to p-region, and of holes from p-region to n-region, a depletion region formed of size of order less than $1\mu m$. The electric field directing from n-region to p-region produces a reverse bias voltage of about 5V and electric field becomes very large.

$$E = \frac{\Delta V}{\Delta x} = \frac{5V}{1\mu m} \approx 5\times10^6\ V/m$$

52. Soln. (i) The doped semiconductor is n-type

(ii) $nenh = ni2$

$$nh = ni2/ne = \frac{(6\times10^8)^2}{9\times10^{12}} = 4\times10^4\ per\ m^3$$

53. Soln.

(a) In forward biasing, the forward voltage opposes the potential barrier. As a result, potential barrier height is reduced and the width of depletion region decreases. Small increase in forward voltage shows large increase in forward current. Thus resistance in forward bias is reduced. In reverse bias, barrier potential height increases. For large increase in reverse voltage shows small increase in reverse current. Thus, resistance of p-n junction is high to flow of current when reverse biased.

(b) Intrinsic semiconductors have very small conductivity at room temperature. When certain impurities in small amount (□ 1 part per million) are added to an intrinsic semiconductor, electrical conductivity of doped semiconductor increases to a great extent and made it useful in many practical applications.

54. Soln.

	Intrinsic semiconductor	p-type semiconductor
(i)	It is a semiconductor in pure form.	It is a semiconductor doped with trivalent (like Al, In) impurity
(ii)	Intrinsic charge carriers are electrons and holes with equal concentration.	Majority charge carriers are holes an minority charge carriers are electrons.
(iii)	Conductivity depends on temperature.	Conductivity depends on temperature as well as dopant concentration.

In p-type semiconductor, trivalent impurity is doped with tetravalent pure semiconductor. Both type of atom (impurity and host semiconductor) are electrically neutral and hence, so produced p-type semiconductor is electrically neutral.

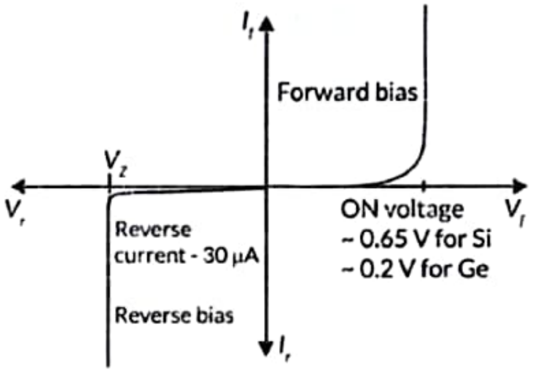
55. Soln. Depletion layer: The small region in the vicinity of the junction which is depleted of free charge carriers and has only immobile ions is called the depletion layer.

Barrier potential : Due to accumulation of negative charges in the p-region and positive charges in the n-region sets up a potential difference across the junction sets up. This acts as a barrier and is called potential barrier V_B which opposes the further diffusion of electrons and holes across the junction.

(a) In forward biasing the width of depletion layer reduced and the external applied field is able to overcome the strong electric field of depletion layer. In reverse biasing the width of depletion layer increases and the electric field of depletion layer become more stronger.

(b) As forward voltage opposes the potential barrier and effective barrier potential decreases. It makes the width of the depletion layer smaller.

56. Soln.



The reverse current is due to minority charge carriers and even a small voltage is sufficient to sweep the minority carriers from one side of the junction to the other side of the junction. Here the current is not limited by the magnitude of the applied voltage but is limited due to the concentration of the minority carrier on either side of the junction.

57. Soln. (i) From the given curve, we have
Voltage, V = 0.8 volt for current, $I = 20\,mA$

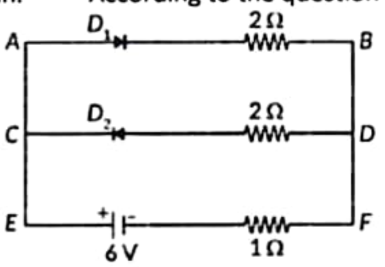
Voltage, V = 0.7 volt for current, $I = 15\,mA$

$$\Rightarrow \quad \Delta I = (20-15)\,\text{mA} = 5\times10^{-3}\,A$$
$$\Rightarrow \quad \Delta V = (0.8-0.7) = 0.1\,\text{V}$$
$$\therefore \quad Resistance, R = \frac{\Delta V}{\Delta I}$$
$$\Rightarrow \quad R = \frac{0.1}{5\times10^{-3}} \Rightarrow R = 20\Omega$$

(ii) For V = -10 V, we have

$$I = -1\,\mu A = -1\times10^{-6}\,A$$
$$\Rightarrow \quad R = \frac{10}{1\times10^{-6}} = 1.0\times10^{7}\Omega$$

58. Soln. According to the question



$$R = R_{AB} + R_{EF}$$
$$= 2 + 1 = 3\Omega$$
$$I = \frac{V}{R} = \frac{6}{3} = 2A$$

59. Soln. Voltage at p side is less than voltage at n side of the diode so it is in reverse bias.

60. Soln. $n_i^2 = n_e n_h$

$$n_e = \frac{n_i^2}{n_h} = \frac{\left(1.5\times10^{16}\right)^2}{4.5\times10^{22}} = \frac{2.25\times10^{32}}{4.5\times10^{22}}; n_e = 5\times10^{9}\,m^{-3}$$

61. Soln. The diode is reverse biased, but the voltage across it is given as 5 V. R_2 is in parallel with the diode, so current in $R_2 = \frac{5V}{1000\Omega} \Rightarrow$

Current in R_2 = 5 mA.

62. Soln. Forward resistance

$$R_f = \frac{\Delta V_f}{\Delta I} = \frac{(2.4-2.0)\,\text{V}}{(80-60)\,\text{mA}}$$

Or $R_f = \frac{0.4V}{20\times10^{-3}A} = \frac{400}{20} = 20\Omega$

Reverse resistance

$$R_r = \frac{\Delta V_r}{\Delta I_r} = \frac{(2-0)\,\text{V}}{(0.25-0)\,\mu\text{A}}$$

Or $R_r = \frac{2V}{0.25\times10^{-6}A} = 8\times10^{6}\Omega$

63. Soln.
(i) $V_A - V_B = 7 - 5 = +2V$ Forward Biased
(ii) $V_A - V_B = 0 - 2 = -2V$ Reverse Biased
(iii) $V_A - V_B = -10 - 0 = -10V$ Reverse Biased
(iv) $V_A - V_B = -5 + 12 = +7V$ Forward Biased.

64. Soln.
(i) $p-type$
(ii) $n_e = \frac{n_i^2}{n_h} = \frac{(2\times10^{6})^2}{4\times10^{10}} = 100$

Decrease

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