श्री BalaJi

Advanced Problems in Coordinate Geometry for JEE Main & Advanced

Vikas Gupta

12th edition

श्री BalaJi

ADVANCED
Problems in
COORDINATE GEOMETRY

for

JEE (MAIN & ADVANCED)

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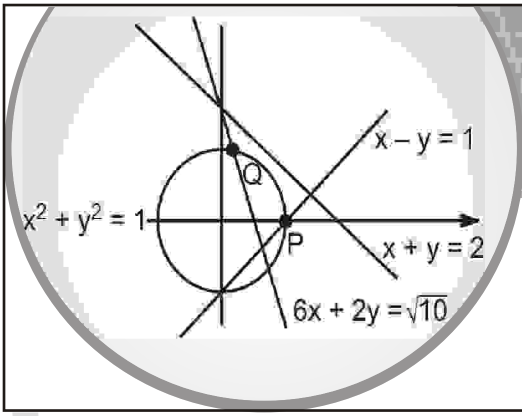


1

STRAIGHT LINE

KEY CONCEPTS

1. Distance Formula

The distance between the points (Ax_1, y_1) and $B(x_2, y_2)$ is $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

2. Section Formula

If $P(x, y)$ divides the line joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then

$$x = \frac{mx_2 + nx_1}{m+n}; \quad y = \frac{my_2 + ny_1}{m+n}$$

If $\frac{m}{n}$ is positive, the division is internal, but if $\frac{m}{n}$ is negative, the division is external.

Note: *If P divides AB internally in the ratio m : n and Q divides AB externally in the ratio m : n, then P and Q are said to be harmonic conjugate to each other w.r.t. AB.*

Mathematically; $\frac{\mathbf{2}}{\mathbf{AB}} = \frac{\mathbf{1}}{\mathbf{AP}} + \frac{\mathbf{1}}{\mathbf{AQ}}$ *i.e., AP, AB and AQ are in H.P.*

3. Centroid and Incentre

If $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the coordinates of the centroid are: $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$

and the coordinates of the incentre are: $\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$

Note that incentre divides the angle bisectors in the ratio

$$(b + c) : a; \ (c + a) : b \text{ and } (a + b) : c.$$

REMEMBER:

(i) Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2 : 1.

(ii) In an isosceles triangle G, O, I and C lie on the same line.

4. Slope Formula

If θ is the angle at which a straight line is inclined to the positive direction of x-axis, and $0° \leq \theta < 180°, \theta \neq 90°$, then the slope of the line, denoted by m, is defined by $m = \tan\theta$. If θ is 90°, m does not exist, but the line is parallel to the y-axis.

If $\theta = 0$, then $m = 0$ and the line is parallel to the x-axis.

If $A(x_1, y_1)$ and $B(x_2, y_2)$, $x_1 \neq x_2$, are points on a straight line, then the slope m of the line is given by: $m = \left(\dfrac{y_1 - y_2}{x_1 - x_2}\right)$.

5. Condition of Collinearity of Three Points-(Slope Form)

Points $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ are collinear if $\left(\dfrac{y_1 - y_2}{x_1 - x_2}\right) = \left(\dfrac{y_2 - y_3}{x_2 - x_3}\right)$

6. Equation of a Straight line in Various Forms

(i) **Slope-intercept form:** $y = mx + c$ is the equation of a straight line whose slope is m and which makes an intercept c on the y-axis.

(ii) **Slope one point form:** $y - y_1 = m(x - x_1)$ is the equation of a straight line whose slope is m and which passes through the point (x_1, y_1)

(iii) **Parametric form:** The equation of the line in parametric form is given by $\dfrac{x - x_1}{\cos\theta} = \dfrac{y - y_1}{\sin\theta} = r$ (say). Where 'r' is the distance of any point (x, y) on the line from the fixed point (x_1, y_1) on the line. r is positive if the point (x, y) is on the right of (x_1, y_1) and negative if (x, y) lies on the left of (x_1, y_1).

(iv) **Two point form:** $y - y_1 = \dfrac{y_2 - y_1}{x_2 - x_1}(x - x_1)$ is the equation of a straight line which passes through the points (x_1, y_1) and (x_2, y_2).

(v) **Intercept form:** $\dfrac{x}{a} + \dfrac{y}{b} = 1$ is the equation of a straight line which makes intercepts a and b on OX and OY respectively.

(vi) **Perpendicular form:** $x\cos\alpha + y\sin\alpha = p$ is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes angle α with positive side of x-axis.

(vii) **General Form:** $ax + by + c = 0$ is the equation of a straight line in the general form.

7. Position of The Point $(x_1 + y_1)$ Relative to the Line $ax + by + c = 0$

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of $ax + by + c = 0$. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non-origin side of $ax + by + c = 0$.

8. The Ratio in Which A Given Line Divides the Line Segment Joining Two Points

Let the given line $ax + by + c = 0$ divide the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$, then $\frac{m}{n} = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$. If A and B are on the same side of the given line then $\frac{m}{n}$ is negative but if A and B are on opposite sides of the given line, then $\frac{m}{n}$ is positive.

9. Length of Perpendicular From a Point on A line

The length of perpendicular from $P(x_1, y_1)$ on $ax + by + c = 0$ is $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.

10. Angle Between Two Straight Lines in Terms of Their Slopes

If m_1 and m_2 are the slopes of two intersecting straight lines ($m_1 m_2 \neq -1$) and θ is the acute angle between them, then $\tan\theta \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$.

Note: *Let m_1, m_2, m_3 are the slopes of three lines $L_1 = 0; L_2 = 0; L_3 = 0$ where $m_1 > m_2 > m_3$ then the interior angles of the ΔABC found by these lines are given by,*

$\tan A = \frac{m_1 - m_2}{1 + m_1 m_2}$; $\tan B = \frac{m_2 - m_3}{1 + m_2 m_3}$ *and* $\tan C = \frac{m_3 - m_1}{1 + m_3 m_1}$

11. Parallel Lines

(i) When two straight lines are parallel their slopes are equal. Thus any line parallel to $ax + by + c = 0$ is of the type $ax + by + k = 0$. Where k is a parameter.

(ii) The distance between two parallel lines with equations $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$.

Note that the coefficients of x and y in both the equations must be same.

(iii) The area of the parallelogram $= \frac{p_1 p_2}{\sin\theta}$, where p_1 and p_2 are distances between two pairs of opposite sides and θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1 x + c_1, y = m_1 x + c_2$ and $y = m_2 x + d_1, y = m_2 x + d_2$ is given by $\left|\frac{(c_1 - c_2)(d_1 - d_2)}{m_1 - m_2}\right|$.

12. Perpendicular Lines

(i) When two lines of slopes m_1 and m_2 are at right angles, the product of their slopes is -1, *i.e.*, $m_1 m_2 = -1$. Thus any line perpendicular to $ax + by + c = 0$ is of the form $bx - ay + k = 0$, where k is any parameter.

(ii) Straight lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ are at right angles if and only if $aa' + bb' = 0$.

13. Equations of straight lines through (x_1, y_1) making angle α with $y = mx + c$ are

$(y - y_1) = \tan(\theta - \alpha)(x - x_1)$ and $(y - y_1) = \tan(\theta + \alpha)(x - x_1)$, where $\tan\theta = m$.

14. Condition of Concurrency

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$. **Alternatively:** If three constants A, B and C can be found such that $A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

15. Area of A Triangle

If $(x_i, y_i), i = 1,2,3$ are the vertices of a triangle, then its area is equal to $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, provided the vertices are considered in the counter clockwise sense. The above formula will give a (–) ve are a if the vertices $(x_i, y_i), i = 1,2,3$ are placed in the clockwise sense.

16. Condition of Collinearity of Three Points-(Area Form)

The points $(x_i, y_i), i = 1, 2, 3$ are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$.

17. The Equation of A Family of Straight Lines Passing Through The Points of Intersection of Two Given Lines

The equation of a family of lines passing through the point of intersection of $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ is given by $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrary real number.

> **Note:** *If* $u_1 = ax + by + c, u_2 = a'x + b'y + d, u_3 = ax + by + c', u_4 = a'x + b'y + d'$
> *then,* $u_1 = 0;\ u_2 = 0;\ u_3 = 0;\ u_4 = 0$ *form a parallelogram*
> $u_2u_3 - u_1u_4 = 0$ *represents the diagonal BD.*

Proof: Since it is the first degree equation in x and y therefore it is a straight line. Secondly point B satisfies the equation because the co-ordinates of B satisfy $u_2 = 0$ and $u_1 = 0$.

Similarly for the point D. Hence the result.

On the similar lines $u_1u_2 - u_3u_4 = 0$ represents the diagonal AC.

> **Note:** *The diagonal AC is also given by* $u_1 + \lambda u_4 = 0$ *and* $u_2 + \mu u_3 = 0$, *if the two equations are identical for some* λ *and* μ.
> *[For getting the values of* λ *and* μ *compare the coefficients of x,y and the constant terms.]*

18. Bisectors of The Angles Between Two Lines

(i) Equations of the bisectors of angles between the lines $ax + by + c = 0$ and $a'x + b'y + c' = 0$ $(ab' \neq a'b)$ are: $\frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$

(ii) To discriminate between the acute angle bisector and the obtuse angle bisector

If θ be the angle between one of the lines and one of the bisectors, find $\tan\theta$.

If $|\tan\theta| < 1$, then $2\theta < 90°$ so that this bisector is the acute angle bisector.

If $|\tan\theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(iii) To discriminate between the bisector of the angle containing the origin and that of the angle not containing the origin. Rewrite the equations, $ax + by + c = 0$ and $a'x + b'y + c' = 0$ such that the constant terms c, c' are positive. Then;

$\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = + \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle containing the origin and $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = - \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ gives the equation of the bisector of the angle not containing the origin.

(iv) To discriminate between acute angle bisector and obtuse angle bisector proceed as follows, write $ax + by + c = 0$ and $a'x + b'y + c' = 0$ such that constant terms are positive.

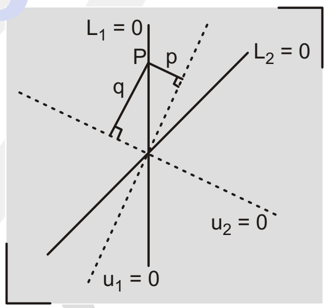
If $aa' + bb' < 0$, then the angle between the lines that contains the origin is acute and the equation of the bisector of this acute angle is $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = + \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$

Therefore $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = - \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ is the equation of other bisector.

If, however, $aa' + bb' > 0$, then the angle between the lines that contains the origin is obtuse and the equation of the bisector of this obtuse angle is:

$\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = + \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$; therefore $\dfrac{ax+by+c}{\sqrt{a^2+b^2}} = - \dfrac{a'x+b'y+c'}{\sqrt{a'^2+b'^2}}$ is the equation of other bisector.

(v) Another way of identifying an acute and obtuse angle bisector is as follows:



Let $L_1 = 0$ and $L_2 = 0$ are the given lines and $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0$ and $L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0$ and $u_2 = 0$ as shown. If,

$|p| < |q| \Rightarrow u_1$ is the acute angle bisector.

$|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector.

$|p| = |q| \Rightarrow$ the lines L_1 and L_2 are perpendicular.

Note: *Equation of straight lines passing through $P(x_1, y_1)$ and equally inclined with the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines and passing through the point P.*

19. A Pair of Straight Lines Through Origin

(i) A homogeneous equation of degree two of the type $ax^2 + 2hxy + by^2 = 0$ always represents a pair of straight lines passing through the origin and if:

(a) $h^2 > ab \Rightarrow$ lines are real and distinct.

(b) $h^2 = ab \Rightarrow$ lines are coincident.

(c) $h^2 < ab \Rightarrow$ lines are imaginary with real point of intersection *i.e.,* (0, 0)

(ii) If $y = m_1 x$ and $y = m_2 x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then; $m_1 + m_2 = -\frac{2h}{b}$ and $m_1 m_2 = \frac{a}{b}$.

(iii) If θ is the acute angle between the pair of straight lines represented by,

then; $\tan\theta = \left|\frac{2\sqrt{h^2 - ab}}{a + b}\right|$

The condition that these lines are:

(a) At right angles to each other is $a + b = 0$. *i.e.,* coefficient of x^2 + coefficient of $y^2 = 0$.

(b) Coincident is $h^2 = ab$.

(c) Equally inclined to the axis of x is $h = 0$. *i.e.,* coefficient of $xy = 0$.

Note: *A homogeneous equation of degree n represents n straight lines passing through origin.*

20. General Equation of Second Degree Representing A Pair Of Straight Lines

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if:

$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, *i.e.,* if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only.

21. The joint equation of a pair of straight lines joining origin to the points of intersection of the line given by

$$lx + my + n = 0 \qquad \text{....(i)}$$

and 2nd degree curve: $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ ...(ii)

is $ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx + my}{-n}\right) + 2fy\left(\frac{lx + my}{-n}\right) + c\left(\frac{lx + my}{-n}\right)^2 = 0$ ...(iii)

(iii) is obtained by homogenizing (ii) with the help of (i), by writing (i) in the form: $\left(\frac{lx + my}{-n}\right) = 1$.

22. The equation to the straight lines bisecting the angle between the straight lines,

$ax^2 + 2hxy + by^2 = 0$ is $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$.

23. The product of the perpendiculars, dropped from (x_1, y_1) to the pair of lines represented by the equation, $ax^2 + 2hxy + by^2 = 0$ is $\frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a - b)^2 + 4h^2}}$.

24. Any second degree curve through the four point of intersection of $f(xy) = 0$ and $xy = 0$ is given by $f(xy) + \lambda xy = 0$ where $f(xy) = 0$ is also a second degree curve.

25. Reflection of a Point About Line

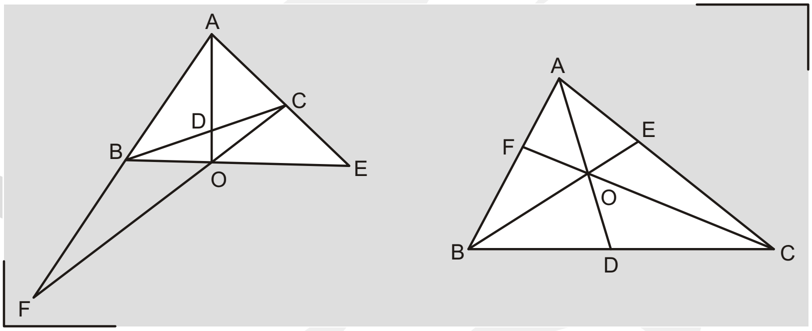
(i) Foot of the perpendicular from a point (x_1, y_1) on the line is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

(ii) The image of a point (x_1, y_1) about the line $ax + by + c = 0$

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

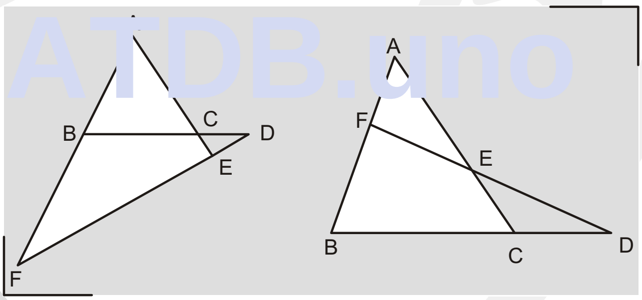
26. Cevas Theorem



If the lines joining a point O to the vertices of ΔABC meet the opposite sides in D, E, F respectively, then

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

27. Menelaus Theorem



If points D, E, F on the sides BC, CA and AB (suitably extended) of ΔABC are collinear then

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

Note: *Either all 3 points lie on the extended line segments or one lies on the extended line and the other two within the line segments.*

EXERCISE 1

Only One Choice is Correct:

1. If the straight lines joining the origin and the points of intersection of the curve $5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$ and $x + ky - 1 = 0$ are equally inclined to the x-axis then the value of k is :

(a) 1 (b) –1

(c) 2 (d) 3

2. Drawn from the origin are two mutually perpendicular straight lines forming an isosceles together with the straight line, $2x + y = a$. Then the area of triangle is :

(a) $\frac{a^2}{2}$ (b) $\frac{a^2}{3}$

(c) $\frac{a^2}{5}$ (d) None

3. Equation of bisector of the angle between two lines $3x - 4y + 12 = 0$ and $12x - 5y + 7 = 0$ which contains point $(-1, 4)$ in its region is :

(a) $21x + 27y - 121 = 0$ (b) $21x - 27y + 121 = 0$

(c) $21x + 27y + 191 = 0$ (d) $\frac{-3x + 4y - 12}{5} = \frac{12x - 5y + 7}{13}$

4. The point $(a^2, a + 1)$ lies in the angle between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin if :

(a) $a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$ (b) $a \in (-\infty, 3) \cup \left(\frac{1}{3}, 1\right)$

(c) $a \in \left(-3, \frac{1}{3}\right)$ (d) $a \in \left(\frac{1}{3}, \infty\right)$

5. A ray of light through $(2, 1)$ is reflected at a pint A on the y-axis and then passes through the point $(5, 3)$. Then co-ordinates of A are :

(a) $\left(0, \frac{11}{7}\right)$ (b) $\left(0, \frac{5}{11}\right)$

(c) $\left(0, \frac{11}{5}\right)$ (d) $\left(0, \frac{3}{5}\right)$

6. The combined equation of the pair of lines through $(3, -2)$ and parallel to the lines $x^2 - 4xy + 3y^2 = 0$ is :

(a) $x^2 - 4xy + 3y^2 - 14x - 24y - 45 = 0$ (b) $x^2 + 4xy + 3y^2 + 14x - 24y - 45 = 0$

(c) $x^2 - 4xy + 3y^2 - 14x + 24y + 45 = 0$ (d) $x^2 + 4xy - 3y^2 - 14x + 24y + 45 = 0$

7. If $(-2,6)$ is the image of the point $(4,2)$ with respect to the line $L=0$, then L is equal to :

(a) $3x-2y+11=0$ (b) $2x-3y+11=0$

(c) $3x-2y+5=0$ (d) $6x-4y+1=0$

8. A man starts from the point $P(-3,4)$ and reaches point $Q\,(0,1)$ touching x axis at R such that $PR+RQ$ is minimum, then the point R is :

(a) $(3/5,0)$ (b) $(-3/5,0)$

(c) $(-2/5,0)$ (d) $(-2,0)$

9. The equation of line segment AB is $y=x$. If A & B lie on same side of line mirror $2x-y=1$, then the equation of image of AB with respect to line mirror $2x-y=1$ is :

(a) $y=7x-5$ (b) $y=7x-6$

(c) $y=3x-7$ (d) $y=6x-5$

10. If $\dfrac{a}{\sqrt{bc}}-2=\sqrt{\dfrac{b}{c}}+\sqrt{\dfrac{c}{b}}$ where $a,b,c>0$, then family of lines $\sqrt{a}\,x+\sqrt{b}\,y+\sqrt{c}=0$ passes through the point:

(a) $(1,1)$ (b) $(1,-2)$

(c) $(-1,2)$ (d) $(-1,1)$

11. The perpendicular distance p_1, p_2, p_3 of points $(a^2,2a),(ab,a+b),(b^2,2b)$ respectively from straight line $x+y\tan\theta$ [illegible] $\tan$ [illegible] $\theta=$ [illegible] are in

(a) AP (b) [illegible]

(c) HP (d) AGP

12. ABC is a variable triangle such that A is $(1,2)$ B and C lie on $y=x+\lambda$ (where λ is variable), then locus of the orthocentre of triangle ABC is :

(a) $(x-1)^2+y^2=4$ (b) $x+y=3$

(c) $2x-y=0$ (d) $x+2y=0$

13. The line $2x+y=4$ meet x-axis at A and y-axis at B. The perpendicular bisector of AB meets the horizontal line through $(0,-1)$ at C. Let G be the centroid of ΔABC. The perpendicular distance from G to AB equals

(a) $\sqrt{5}$ (b) $\dfrac{\sqrt{5}}{3}$

(c) $2\sqrt{5}$ (d) $3\sqrt{5}$

14. Let ABC be a triangle. Let A be the point $(1,2)$, $y=x$ is the perpendicular bisector of AB and $x-2y+1=0$ is the angle bisector of angle C. If the equation of BC is given by $ax+by-5=0$, then the value of $a+b$ is :

(a) 1 (b) 2

(c) 3 (d) 4

15. $I(1,0)$ is the centre of in circle of triangle ABC, the equation of BI is $x-1=0$ and equation of CI is $x-y-1=0$, then angle BAC is :

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{3}$

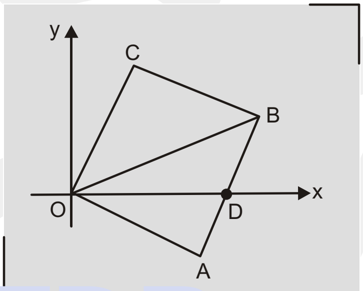
(c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

16. If the points where the lines $3x - 2y - 12 = 0$ and $x + ky + 3 = 0$ intersect both the coordinate axes are concyclic, then number of possible real values of k is :

(a) 1 (b) 2

(c) 3 (d) 4

17. In the figure shown, $OABC$ is a rectangle with dimensions $OA = 3$ units and $OC = 4$ units. If $AD = 1.5$ units then slope of diagonal OB will be :



(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{[illegible]}}{\sqrt{3}}$

(c) $\frac{1}{2}$ (d) $\frac{1}{3}$

18. In a ΔABC, the equations of right bisectors of sides AB and CA are $3x + 4y = 20$ and $8x + 6y = 65$ respectively. If the vertex A be $(10, 10)$, then the area of ΔABC will be :

(a) 14 (b) 21

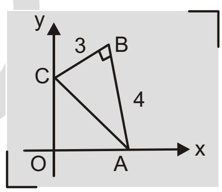
(c) 42 (d) 63

19. The least area of a quadrilateral with integral coordinates is :

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{3}{2}$ (d) 2

20. In the adjacent figure ΔABC is right angled at B. If $AB = 4$ and $BC = 3$ and side AC slides along the coordinate axes in such a way that 'B' always remains in the first quadrant, then B always lie on straight line :



(a) $y = x$ (b) $3y = 4x$

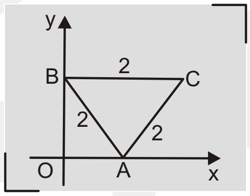
(c) $4y = 3x$ (d) $x + y = 0$

21. If the line $y = x$ is one of the angle bisector of the pair of lines $ax^2 + 2hxy + by^2 = 0$, then :

(a) $a + b = 0$ (b) $a - b = 0$

(c) $h = 0$ (d) $a + 2b = 0$

22. Adjacent figure represents a equilateral triangle ABC of side length 2 units. Locus of vertex C as the side AB slides along the coordinate axes is :



(a) $x^2 + y^2 - xy + 1 = 0$ (b) $x^2 + y^2 + xy\sqrt{3} = 1$

(c) $x^2 + y^2 = 1 + xy\sqrt{3}$ (d) $x^2 + y^2 - xy\sqrt{3} + 1 = 0$

23. Vertices of a variable triangle are $(3,4)$, $(5\cos\theta, 5\sin\theta)$ and $(5\sin\theta, -5\cos\theta)$ where $\theta \in R$, then locus of its orthocenter is :

(a) $(x + y - 1)^2 + (x - y - 7)^2 = 100$ (b) $(x + y - 7)^2 + (x - y - 1)^2 = 100$

(c) $(x + y - 7)^2 + (x + y - 1)^2 = 100$ (d) $(x + y - 7)^2 + (x - y + 1)^2 = 100$

24. Consider the triangle OAB where $O \equiv (0,0)$, $B \equiv (3,4)$. If orthocenter of triangle is $H(1,4)$, then coordinates of 'A' is

(a) $\left(0, \frac{15}{4}\right)$ (b) $\left(0, \frac{17}{4}\right)$

(c) $\left(0, \frac{21}{4}\right)$ (d) $\left(0, \frac{19}{4}\right)$

25. On the portion of the straight line, $x + 2y = 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co-ordinates :

(a) $(2,3)$ (b) $(3,2)$

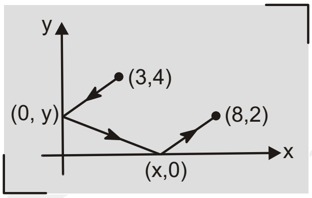
(c) $(3,3)$ (d) $(2,2)$

26. Through a point A on the x-axis a straight line is drawn parallel to y-axis so as to meet the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ in B and C. If $AB = BC$ then :

(a) $h^2 = 4ab$ (b) $8h^2 = 9ab$

(c) $9h^2 = 8ab$ (d) $4h^2 = ab$

27. Suppose that a ray of light leaves the point $(3,4)$, reflects off the y-axis towards the x-axis, reflects off the x-axis, and finally arrives at the point $(8,2)$. The value of x, is :



(a) $x = 4\frac{1}{2}$ (b) $x = 4\frac{1}{3}$

(c) $x = 4\frac{2}{3}$ (d) $x = 5\frac{1}{3}$

28. Given a triangle whose vertices are at $(0,0), (4,4)$ and $(10,0)$. A square is drawn in it such that its base is on the x-axis and its two corners are on the 2 sides of the triangle. The area of the square is equal to :

(a) $\frac{400}{49}$ (b) $\frac{400}{25}$

(c) $\frac{625}{16}$ (d) $\frac{625}{49}$

29. A, B and C are points in the xy-plane such that $A(1,2)$; $B(5,6)$ and $AC = 3BC$. Then :

(a) ABC is a unique triangle

(b) There can be only two such triangle

(c) No such triangle is possible

(d) There can be infinite number of such triangles.

30. A ray of light passing through the point $A(1,2)$ is reflected at a point B on the x-axis and then passes through $(5,3)$. Then the equation of AB is :

(a) $5x + 4y = 13$ (b) $5x - 4y = -3$

(c) $4x + 5y = 14$ (d) $4x - 5y = -6$

31. Vertices of a parallelogram $ABCD$ are $A(3,1), B(13,6), C(13,21)$ and $D(3,16)$. If a line passing through the origin divides the parallelogram into two congruent parts then the slope of the line is :

(a) $\frac{11}{12}$ (b) $\frac{11}{8}$

(c) $\frac{25}{8}$ (d) $\frac{13}{8}$

32. If the vertices P and Q of a triangle PQR are given by $(2,5)$ and $(4,-11)$ respectively, and the point R moves along the line N: $9x + 7y + 4 = 0$, then the locus of the centroid of the triangle PQR is a straight line parallel to :

(a) PQ (b) QR

(c) RP (d) N

33. In a triangle ABC, if $A(2,-1)$ and $7x-10y+1=0$ and $3x-2y+5=0$ are equations of an altitude and an angle bisector respectively drawn from B, then equation of BC is :

(a) $x+y+1=0$ (b) $5x+y+17=0$

(c) $4x+9y+30=0$ (d) $x-5y-7=0$

34. The image of the pair of lines represented by $ax^2+2hxy+by^2=0$ by the line mirror $y=0$ is :

(a) $ax^2-2hxy-by^2=0$

(b) $bx^2-2hxy+ay^2=0$

(c) $bx^2+2hxy+ay^2=0$

(d) $ax^2-2hxy+by^2=0$

35. In an isosceles right angled triangle, a straight line drawn from the mid-point of one of equal sides to the opposite angle. It divides the angle into two parts, θ and $(\pi/4-\theta)$. Then $\tan\theta$ and $\tan[(\pi/4)-\theta]$ are equal to :

(a) $\frac{1}{2},\frac{1}{3}$ (b) $\frac{1}{3},\frac{1}{4}$

(c) $\frac{1}{5},\frac{1}{6}$ (d) None of these

36. The line $(p+2q)x+(p-3q)y=p-q$ for different values of p and q, passes through the fixed point :

(a) $\left(\frac{3}{2},\frac{5}{2}\right)$ (b) $\left(\frac{2}{5},\frac{2}{5}\right)$

(c) $\left(\frac{3}{5},\frac{2}{5}\right)$ (d) $\left(\frac{2}{5},\frac{3}{5}\right)$

37. The orthocentre of a triangle whose vertices are $(0,0),(\sqrt{3},0)$ and $(0,\sqrt{6})$ is :

(a) $(2,1)$ (b) $(3,2)$

(c) $(4,1)$ (d) None of these

38. If the line $y=mx$ meets the lines $x+2y-1=0$ and $2x-y+3=0$ at the same point, then m is equal to :

(a) 1 (b) -1

(c) 2 (d) -2

39. The distance of any point (x,y) from the origin is defined as $d=\max\{|x|,|y|\}$, then the distance of the common point for the family of lines $x(1+\lambda)+\lambda y+2+\lambda=0$ (λ being parameter) from origin is :

(a) 1 (b) 2

(c) $\sqrt{5}$ (d) 0

40. Let $ax+by+c=0$ be a variable straight line, where a,b and c are 1^{st}, 3^{rd} and 7^{th} terms of some increasing A.P. Then the variable straight line always passes through a fixed point which lies on:

(a) $x^2 + y^2 = 13$ (b) $x^2 + y^2 = 5$

(c) $y^2 = 9x$ (d) $3x + 4y = 9$

41. Area of the triangle formed by the line $x + y = 3$ and angle bisector of the pair of straight lines $x^2 - y^2 + 2y - 1 = 0$ is :

(a) 2 sq. units (b) 4 sq. units

(c) 6 sq. units (d) 8 sq. units

42. The number of integral values of m, for which the x-coordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer is :

(a) 2 (b) 0

(c) 4 (d) 1

43. A line passes through $(1,0)$. The slope of the line,for which its intercept between $y = x - 2$ and $y = -x + 2$ subtends a right angle at the origin, is :

(a) $\pm 2/3$ (b) $\pm 3/2$

(c) ± 1 (d) None of these

44. A variable straight line passes through a fixed point (a, b) intersecting the coordinate axes at A & B. If 'O' is the origin, then the locus of centroid of triangle OAB is :

(a) $bx + ay - 3xy = 0$ (b) $bx + ay - 2xy = 0$

(c) $ax + by - 3xy = 0$ (d) $ax + by - 2xy = 0$

45. Two vertices of a triangle are $(5, -1)$ and $(-2, 3)$. If orthocenter of the triangle is origin, then the co-ordinates of third vertex is :

(a) $(4, 7)$ (b) $(3, 7)$

(c) $(-4, -7)$ (d) $(4, -7)$

46. The straight line $y = x - 2$ rotates about a point where it cuts the x-axis and becomes perpendicular to the straight line $ax + by + c = 0$. Then its equation is :

(a) $ax + by + 2a = 0$ (b) $ax - by - 2a = 0$

(c) $bx + ay - 2b = 0$ (d) $ay - bx + 2b = 0$

47. It is desired to construct a right angled triangle ABC ($\angle C = \pi/2$) in xy-plane so that its sides are parallel to co-ordinates axes and the medians through A and B lie on the lines $y = 3x + 1$ and $y = mx + 2$ respectively. The values of 'm' for which such a triangle is possible is/are :

(a) -12 (b) $3/4$

(c) $4/3$ (d) $1/12$

48. The medians AD and BE of a triangle ABC with vertices $A(0, b), B(0, 0)$ and $C(a, 0)$ are perpendicular to each other if :

(a) $b = \pm\sqrt{2}\, a$ (b) $a = \pm\sqrt{2}\, b$

(c) $b = \pm\sqrt{3}\, a$ (d) $a = \pm\sqrt{3}\, b$

49. The equations of the lines through $(-1, -1)$ and making angle $45°$ with the line $x + y = 0$ are given by :

(a) $x^2 - xy + x - y = 0$ (b) $xy - y^2 + x - y = 0$

(c) $xy + x + y = 0$ (d) $xy + x + y + 1 = 0$

50. The number of integral points inside the triangle made by the line $3x + 4y - 12 = 0$ with the coordinate axes which are equidistant from at least two sides is/are (an integral point is a point both of whose coordinates are integers):

(a) 1 (b) 2

(c) 3 (d) 4

51. If the lines $x + y + 1 = 0$; $4x + 3y + 4 = 0$ and $x + \alpha y + \beta = 0$, where $\alpha^2 + \beta^2 = 2$, are concurrent then:

(a) $\alpha = 1, \beta = -1$ (b) $\alpha = 1, \beta = \pm 1$

(c) $\alpha = -1, \beta = \pm 1$ (d) $\alpha = \pm 1, \beta = 1$

52. The straight line, $ax + by = 1$ makes with the curve $px^2 + 2axy + qy^2 = r$ a chord which subtends a right angle at the origin. Then:

(a) $r(a^2 + b^2) = p + q$ (b) $r(a^2 + p^2) = q + b$

(c) $r(b^2 + q^2) = p + a$ (d) none of these

53. Given the family of lines, $a(2x + y + 4) + b(x - 2y - 3) = 0$. Among the lines of the family, the number of lines situated at a distance of $\sqrt{10}$ from the point $M(2, -3))$ is:

(a) 0 (b) 1

(c) 2 (d) ∞

54. *m, n* are integer with $0 < n < m$. A is the point (m, n) on the cartesian plane. B is the reflection of A in the line $y = x$. C is the reflection of B in the y-axis, D is the reflection of C in the x-axis and E is the reflection of D in the y-axis. The area of the pentagon $ABCDE$ is:

(a) $2m(m + n)$ (b) $m(m + 3n)$

(c) $m(2m + 3n)$ (d) $2m(m + 3n)$

55. The area enclosed by the graphs of $|x + y| = 2$ and $|x| = 1$ is:

(a) 2 (b) 4

(c) 6 (d) 8

56. The ends of the base of an isosceles triangle are at (2, 0) and (0, 1) and the equation of one side is $x = 2$ then the orthocentre of the triangle is:

(a) $\left(\frac{3}{4}, \frac{3}{2}\right)$ (b) $\left(\frac{5}{4}, 1\right)$

(c) $\left(\frac{3}{4}, 1\right)$ (d) $\left(\frac{4}{3}, \frac{7}{12}\right)$

57. The equation of the pair of bisectors of the angles between two straight lines is, $12x^2 - 7xy - 12y^2 = 0$. If the equation of one line is $2y - x = 0$ then the equation of the other line is:

(a) $41x - 38y = 0$ (b) $11x + 2y = 0$

(c) $38x + 41y = 0$ (d) $11x - 2y = 0$

58. A piece of cheese is located at (12, 10) in a coordinate plane. A mouse is at (4,–2) and is running up the line $y = -5x + 18$. At the point (a, b), the mouse starts getting farther from the cheese rather than closer to it. The value of $(a + b)$ is:

(a) 6 (b) 10

(c) 18 (d) 14

59. The equations of L_1 and L_2 are $y = mx$ and $y = nx$, respectively. Suppose L_1 make twice as large of an angle with the horizontal (measured counterclockwise from the positive x-axis) as does L_2 and that L_1 has 4 times the slope of L_2. If L_1 is not horizontal, then the value of the product (mn) equals:

(a) $\frac{\sqrt{2}}{2}$ (b) $-\frac{\sqrt{2}}{2}$

(c) 2 (d) –2

60. If L is the line whose equation is $ax + by = c$. Let M be the reflection of L through the y-axis, and let N be the reflection of L through the x-axis. Which of the following must be true about M and N for all choices of a, b and c ?

(a) The x-intercepts of M and N are equal.

(b) The y-intercepts of M and N are equal.

(c) The slopes of M and N are equal.

(d) The slopes of M and N are reciprocal.

61. The line $x = c$ cuts the triangle with corners $(0, 0)$, (1, 1) and (9, 1) into two regions. For the area of the two regions to be the same c must be equal to :

(a) 5/2 (b) 3

(c) 7/2 (d) 3 or 15

62. The distance between the two parallel lines is 1 unit. A point 'A' is chosen to lie between the lines at a distance 'd' from one of them. Triangle ABC is equilateral with B on one line and C on the other parallel line. The length of the side of the equilateral triangle is:

(a) $\frac{2}{3}\sqrt{d^2 + d + 1}$ (b) $2\sqrt{\frac{d^2 - d + 1}{3}}$

(c) $2\sqrt{d^2 - d + 1}$ (d) $\sqrt{d^2 - d + 1}$

63. If m and b are real numbers and $mb > 0$, then the line whose equation is $y = mx + b$ cannot contain the point:

(a) (0, 2008) (b) (2008, 0)

(c) (0, – 2008) (d) (20, – 100)

64. Given A (0, 0) and $B(x, y)$ with $x \in (0,1)$ and $y > 0$. Let the slope of the line AB equals m_1. Point C lies on the line $x = 1$ such that the slope of BC equals m_2 where $0 < m_2 < m_1$. If the area of the triangle ABC can be expressed as $(m_1 - m_2) f(x)$, then the largest possible value of $f(x)$ is:

(a) 1 (b) 1/2

(c) 1/4 (d) 1/8

65. What is the y-intercept of the line that is parallel to $y = 3x$, and which bisects the area of a rectangle with corners at (0, 0) (4, 0), (4, 2) and (0, 2) ?

(a) (0, –7) (b) (0, –6)

(c) (0, –5) (d) (0, –4)

66. The vertex of right angle of a right angled triangle lies on the straight line $2x + y - 10 = 0$ and the two other vertices, at points (2, –3) and (4, 1) then the area of triangle in sq. units is:

(a) $\sqrt{10}$ (b) 3

(c) $\frac{33}{5}$ (d) 11

67. Given $A \equiv (1, 1)$ and AB is any line through it cutting the x-axis in B. If AC is perpendicular to AB and meets the y-axis in C, then the equation of locus of mid-point P of BC is:

(a) $x + y = 1$ (b) $x + y = 2$

(c) $x + y = 2xy$ (d) $2x + 2y = 1$

68. The number of possible straight lines, passing through (2, 3) and forming a triangle with coordinate axes, whose area is 12 sq. units, is:

(a) one (b) two

(c) three (d) four

69. Let $A \equiv (3,2)$ and $B \equiv (5,1)$. ABP is an equilateral triangle is constructed one the side of AB remote from the origin then the orthocentre of triangle ABP is:

(a) $\left(4 - \frac{1}{2}\sqrt{3}, \frac{3}{2} - \sqrt{3}\right)$ (b) $\left(4 + \frac{1}{2}\sqrt{3}, \frac{3}{2} + \sqrt{3}\right)$

(c) $\left(4 - \frac{1}{6}\sqrt{3}, \frac{3}{2} - \frac{1}{3}\sqrt{3}\right)$ (d) $\left(4 + \frac{1}{6}\sqrt{3}, \frac{3}{2} + \frac{1}{3}\sqrt{3}\right)$

70. If $P \equiv \left(\frac{1}{x_p}, p\right); Q = \left(\frac{1}{x_q}, q\right); R = \left(\frac{1}{x_r}, r\right)$

where $x_k \neq 0$, denotes the k^{th} terms of a H.P. for $k \in N$, then:

(a) ar. $(\Delta PQR) = \frac{p^2q^2r^2}{2} \sqrt{(p-q)^2 + (q-r)^2 + (r-p)^2}$

(b) ΔPQR is a right angled triangle

(c) the points P, Q, R are collinear

(d) None of these

ANSWERS

1.	(b)	**2.**	(c)	**3.**	(a)	**4.**	(a)	**5.**	(a)	**6.**	(c)	**7.**	(c)	**8.**	(b)	**9.**	(b)	**10.**	(d)
11.	(b)	**12.**	(b)	**13.**	(a)	**14.**	(b)	**15.**	(c)	**16.**	(b)	**17.**	(c)	**18.**	(c)	**19.**	(b)	**20.**	(b)
21.	(b)	**22.**	(c)	**23.**	(d)	**24.**	(d)	**25.**	(c)	**26.**	(b)	**27.**	(b)	**28.**	(a)	**29.**	(d)	**30.**	(a)
31.	(b)	**32.**	(d)	**33.**	(b)	**34.**	(d)	**35.**	(a)	**36.**	(d)	**37.**	(d)	**38.**	(b)	**39.**	(b)	**40.**	(a)
41.	(a)	**42.**	(a)	**43.**	(d)	**44.**	(a)	**45.**	(c)	**46.**	(d)	**47.**	(b)	**48.**	(b)	**49.**	(d)	**50.**	(a)
51.	(d)	**52.**	(a)	**53.**	(b)	**54.**	(b)	**55.**	(d)	**56.**	(b)	**57.**	(a)	**58.**	(b)	**59.**	(c)	**60.**	(c)
61.	(b)	**62.**	(b)	**63.**	(b)	**64.**	(d)	**65.**	(c)	**66.**	(b)	**67.**	(a)	**68.**	(c)	**69.**	(d)	**70.**	(c)

EXERCISE 2

One or More than One is/are Correct

1. Two sides of a triangle have the joint equation $(x-3y+2)(x+y-2)=0$, the third side which is variable always passes through the point $(-5,-1)$, then possible values of slope of third side such that origin is an interior point of triangle is/are:

(a) $\frac{-4}{3}$ (b) $\frac{-2}{3}$

(c) $\frac{-1}{3}$ (d) $\frac{1}{6}$

2. The equations of lines passing through point (2, 3) and having an intercept of length 2 units between the lines $2x+y=3$ and $2x+y=5$ are:

(a) $y=3$ (b) $x=2$

(c) $y=x+1$ (d) $4y+3x=18$

3. Two sides of a rhombus *ABCD* are parallel to lines $y=x+2$ and $y=7x+3$. If the diagonals of the rhombus intersect at point (1, 2) and the vertex *A* is on the *y*-axis is, then the possible coordinates of *A* are:

(a) $\left(0, \frac{5}{2}\right)$ (b) (0, 0)

(c) (0, 5) (d) (0, 3)

4. Possible values of θ for which the point $(\cos\theta, \sin\theta)$ lies inside the triangle formed by lines $x+y=2$; $x-y=1$ and $6x+2y=\sqrt{10}$ are:

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$

(c) $\frac{3\pi}{8}$ (d) $\frac{\pi}{2}$

5. Two equal sides of an isosceles triangle are given by the equations $7x-y+3=0$ and $x+y-3=0$ and its third side passes through the point, $(1,-10)$, then equations of the third side can be:

(a) $x-3y-31=0$ (b) $y-3x+13=0$

(c) $3x+y+7=0$ (d) $y-2x+12=0$

6. All the points lying inside the triangle formed by the points (1, 3), (5, 6), and (−1,2) satisfy:

(a) $3x+2y\geq 0$ (b) $2x+y+1\geq 0$

(c) $-2x+11\geq 0$ (d) $2x+3y-12\geq 0$

7. The bisectors of angle between the straight lines $y-b=\dfrac{2m}{1-m^2}(x-a)$ and $y-b=\dfrac{2m'}{1-m'^2}(x-a)$ are:

(a) $(y-b)(m+m')+(x-a)(1-mm')=0$

(b) $(y-b)(m+m')-(x-a)(1-mm')=0$

(c) $(y-b)(1-mm')+(x-a)(m+m')=0$

(d) $(y-b)(1-mm')-(x-a)(m+m')=0$

8. Straight lines $2x+y=5$ and $x-2y=3$ intersect at point A. Points B and C are chosen on these two lines such that $AB=AC$. Then the equation of a line BC passing through the point $(2,3)$ is:

(a) $3x-y-3=0$ (b) $x+3y-11=0$

(c) $3x+y-9=0$ (d) $x-3y+7=0$

9. The sides of a triangle are the straight lines $x+y=1, 7y=x$ and $\sqrt{3}y+x=0$. Then which of the following is an interior point of the triangle:

(a) circumcentre (b) centroid

(c) incentre (d) orthocentre

10. The x-coordinates of the vertices of a square of unit area are the roots of the equation $x^2-3|x|+2=0$ and the y-coordinates of the vertices are the roots of equation $y^2-3y+2=0$, then the possible vertices of the square is/are:

(a) (1, 1), (2, 1), (2, 2), (1, 2)

(b) (–1, 1), (–2, 1), (–2, 2), (–1, 2)

(c) (2, 1), (1, –1), (1, 2) (2, 2)

(d) (–2, 1), (–1, –1), (–1, 2), (–2, 2)

11. If one vertex of an equilateral triangle of side 'a' lies at origin and the other lies on the line $x-\sqrt{3}y=0$, then the coordinates of the third vertex are:

(a) $(0, a)$ (b) $\left(\dfrac{\sqrt{3}a}{2}, -\dfrac{a}{2}\right)$

(c) $(0, -a)$ (d) $\left(-\dfrac{\sqrt{3}a}{2}, \dfrac{a}{2}\right)$

12. Line $\dfrac{x}{a}+\dfrac{y}{b}=1$ cuts the coordinate axes at $A(a,0)$ and $B(0,b)$ and the line $\dfrac{x}{a'}+\dfrac{y}{b'}=-1$ at $A'(-a',0)$ and $B'(0,-b')$. If the points A,B,A',B' are concyclic then the orthocentre of the triangle ABA' is:

(a) $(0, 0)$ (b) $(0, b')$

(c) $\left(0, \dfrac{aa'}{b}\right)$ (d) $\left(0, \dfrac{bb'}{a}\right)$

13. The lines L_1 and L_2 denoted by $3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0$ intersect at the point P and have gradients m_1 and m_2 respectively. The acute angle between them is θ. Which of following relations hold good:

(a) $m_1 + m_2 = \frac{5}{4}$

(b) $m_1 m_2 = \frac{3}{8}$

(c) $\theta = \sin^{-1}\left(\frac{2}{5\sqrt{5}}\right)$

(d) sum of the abscissa and ordinate of point P is –1.

14. The area of triangle ABC is 20 cm^2. The coordinates of vertex A are (–5, 0) and B are (3, 0). The vertex C lies on the line $x - y = 2$. The coordinates of C are:

(a) (5, 3) (b) (–3, –5)
(c) (–5, –7) (d) (7, 5)

15. Let $B(1, -3)$ and $D(0, 4)$ represent two vertices of rhombus $ABCD$ in (x, y) plane, then coordinates of vertex A if $\angle BAD = 60°$ can be equal to:

(a) $\left(\frac{1-7\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}\right)$ (b) $\left(\frac{1+7\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right)$

(c) $\left(\frac{-1+7\sqrt{3}}{2}, \frac{-1+\sqrt{3}}{2}\right)$ (d) $\left(\frac{-1-7\sqrt{3}}{2}, \frac{-1-\sqrt{3}}{2}\right)$

16. Let $L_1: 3x + 4y = 1$ and $L_2: 5x - 12y + 2 = 0$ be two given lines. Let image of every point on L_1 with respect to a line L lies on L_2 then possible equation of L can be:

(a) $14x + 112y - 23 = 0$ (b) $64x - 8y - 3 = 0$
(c) $11x - 4y = 0$ (d) $52y - 45x = 7$

17. Let $A(1, 1)$ and $B(3, 3)$ be two fixed points and P be a variable point such that area of ΔPAB remains constant equal to 1 for all position of P, then locus of P is given by:

(a) $2y = 2x + 1$ (b) $2y = 2x - 1$
(c) $y = x + 1$ (d) $y = x - 1$

18. If one diagonal of a square is the portion of line $\frac{x}{a} + \frac{y}{b} = 1$ intercepted by the axes, then the extremities of the other diagonal of the square are:

(a) $\left(\frac{a+b}{2}, \frac{a+b}{2}\right)$ (b) $\left(\frac{a-b}{2}, \frac{a+b}{2}\right)$

(c) $\left(\frac{a-b}{2}, \frac{b-a}{2}\right)$ (d) $\left(\frac{a+b}{2}, \frac{b-a}{2}\right)$

19. Two straight lines $u = 0$ and $v = 0$ passes through the origin and the angle between them is $\tan^{-1}\left(\frac{7}{9}\right)$. If the ratio of slopes of $v = 0$ and $u = 0$ is $\frac{9}{2}$, then their equations are:

(a) $y = 3x$ and $3y = 2x$
(b) $2y = 3x$ and $3y = x$
(c) $y + 3x = 0$ and $3y + 2x = 0$
(d) $2y + 3x = 0$ and $3y + x = 0$

20. The points $A(0,0)$, $B(\cos\alpha, \sin\alpha)$ and $C(\cos\beta, \sin\beta)$ are the vertices of a right angled triangle if:

(a) $\sin\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{\sqrt{2}}$
(b) $\cos\left(\frac{\alpha-\beta}{2}\right) = -\frac{1}{\sqrt{2}}$
(c) $\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{\sqrt{2}}$
(d) $\sin\left(\frac{\alpha-\beta}{2}\right) = -\frac{1}{\sqrt{2}}$

21. $ABCD$ is rectangle with $A(-1,2)$, $B(3,7)$ and $AB : BC = 4 : 3$. If d is the distance of origin from the intersection point of diagonals of rectangle, then possible values of $[d]$ is/are (where $[\cdot]$ denote greatest integer function)

(a) 3
(b) 4
(c) 5
(d) 6

22. A straight line L drawn through the point $A(1,2)$ intersects the line $x + y = 4$ at a distance of $\frac{\sqrt{6}}{3}$ units from A. The angle made by L with positive direction of x-axis can be :

(a) $\frac{\pi}{12}$
(b) $\frac{\pi}{6}$
(c) $\frac{\pi}{3}$
(d) $\frac{5\pi}{12}$

23. Let x_1 and y_1 be the roots of $x^2 + 8x - 2009 = 0$; x_2 and y_2 be the roots of $3x^2 + 24x - 2010 = 0$ and x_3 and y_3 be the roots of $9x^2 + 72x - 2011 = 0$. The points $A(x_1, y_2)$ $B(x_2, y_2)$ and $C(x_3, y_3)$:

(a) can not lie on a circle
(b) form a triangle of area 2 sq. units
(c) form a right-angled triangle
(d) are collinear

ANSWERS

1.	(b, c, d)	**2.**	(b, d)	**3.**	(a, b)	**4.**	(a, b, c)	**5.**	(a, c)	**6.**	(a, b, c)
7.	(a, d)	**8.**	(a, b)	**9.**	(b, c)	**10.**	(a, b)	**11.**	(a, b, c, d)	**12.**	(b, c)
13.	(b, c, d)	**14.**	(b, d)	**15.**	(a, b)	**16.**	(a, b)	**17.**	(c, d)	**18.**	(a, c)
19.	(a, b, c, d)	**20.**	(a, b, c, d)	**21.**	(b, d)	**22.**	(a, d)	**23.**	(a, d)		

EXERCISE 3

Comprehension: (1)

The base of an isosceles triangle is equal to 4, the base angle is equal to 45°. A straight line cuts the extension of the base at a point *M* at the angle θ and bisects the lateral side of the triangle which is nearest to *M*.

1. The area of quadrilateral which the straight line cuts off from the given triangle is:

(a) $\frac{3+\tan\theta}{1+\tan\theta}$
(b) $\frac{3+2\tan\theta}{1+\tan\theta}$
(c) $\frac{3+\tan\theta}{1-\tan\theta}$
(d) $\frac{3+5\tan\theta}{1+\tan\theta}$

2. The possible range of values in which area of quadrilateral which straight line cuts off from the given triangle lie in:

(a) $\left(\frac{5}{2},\frac{7}{2}\right)$
(b) $(4,5)$
(c) $\left(4,\frac{9}{2}\right)$
(d) $(3,4)$

3. The length of portion of straight line inside the triangle may lie in the range:

(a) (2, 4)
(b) $\left(\frac{3}{2},\sqrt{3}\right)$
(c) $(\sqrt{2},2)$
(d) $(\sqrt{2},\sqrt{3})$

Comprehension: (2)

Let *ABCD* is a square with sides of unit length. Points *E* and *F* are taken on sides *AB* and *AD* respectively so that *AE* = *AF*. Let *P* be any point inside the square *ABCD*.

1. The maximum possible area of quadrilateral *CDFE* is :

(a) $\frac{1}{8}$
(b) $\frac{1}{4}$
(c) $\frac{3}{8}$
(d) $\frac{5}{8}$

2. The value of $(PA)^2-(PB)^2+(PC)^2-(PD)^2$ is equal to:

(a) 3
(b) 2
(c) 1
(d) 0

3. Let a line passing through point A divides the square $ABCD$ into two parts so that area of one portion is double the other, then the length of portion of line inside the square is:

(a) $\frac{\sqrt{10}}{3}$ (b) $\frac{\sqrt{11}}{3}$

(c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{13}}{3}$

Comprehension: (3)

Consider a trapezoid *ABCD*, one of whose non parallel sides *AB* which is 8cm long is perpendicular to the base. The base *BC* and *AD* of trapezoid are 6cm and 10cm in lengths respectively. Let L_1, L_2, L_3, L_4 represent the lines *AB*, *BC*, *CD* and *DA* respectively and $d(P, L)$ denote the perpendicular distance of point *P* from line *L*.

1. Find the area of region inside the trapezoid $ABCD$ in which the point Q can lie satisfying $d(Q, L_4) \le d(Q, L_3)$:

(a) $3(3\sqrt{5} + \sqrt{3})$ (b) $24(\sqrt{3} - 1)$

(c) $4(5 - \sqrt{5})$ (d) $25(\sqrt{5} - 1)$

2. Distance of the point R lying on line AD from vertex A so that perimeter of triangle RBC is minimum is:

(a) 2 (b) 3

(c) 4 (d) 5

3. The maximum possible area of rectangle inscribed in the trapezoid so that one of its sides lies on the larger base of trapezoid is:

(a) 36 (b) 54

(c) 42 (d) 48

Comprehension: (4)

Consider a variable line '*L*' which passes through the point of intersection '*P*' of the lines $3x + 4y - 12 = 0$ and $x + 2y - 5 = 0$ meeting the coordinate axes at points *A* and *B*.

1. Locus of the middle point of the segment AB has the equation:

(a) $3x + 4y = 4xy$ (b) $3x + 4y = 3xy$

(c) $4x + 3y = 4xy$ (d) $4x + 3y = 3xy$

2. Locus of the feet of the perpendicular from the origin on the variable line 'L' has the equation:

(a) $2(x^2 + y^2) - 3x - 4y = 0$ (b) $2(x^2 + y^2) - 4x - 3y = 0$

(c) $x^2 + y^2 - 2x - y = 0$ (d) $x^2 + y^2 - x - 2y = 0$

3. Locus of the centroid of the variable triangle OAB has the equation (where 'O' is the origin):

(a) $3x + 4y + 6xy = 0$ (b) $4x + 3y - 6xy = 0$

(c) $3x + 4y - 6xy = 0$ (d) $4x + 3y + 6xy = 0$

Comprehension: (5)

Consider 3 non-collinear points $A(9,3)$, $B(7,-1)$ and $C(1,-1)$. Let $P(a,b)$ be the centre and R is the radius of circle 'S' passing through points A, B, C. Also $H(\bar{x}, \bar{y})$ are the coordinates of the orthocentre of triangle ABC whose area be denoted by Δ.

1. If D, E and F are the middle points of BC, CA and AB respectively then the area of the triangle DEF is :

(a) 12 (b) 6

(c) 4 (d) 3

2. The value of $a + b + R$ equals:

(a) 3 (b) 12

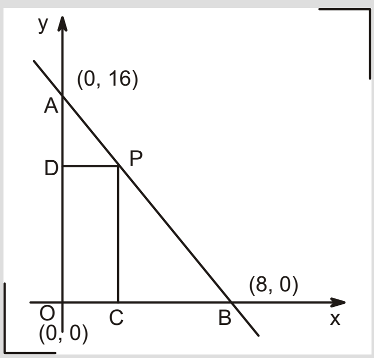
(c) 13 (d) none of these

3. The ordered pair $(\bar{x}, \bar{y})$ is:

(a) (9, 6) (b) (–9, 6)

(c) (9, –5) (d) (9, 5)

Comprehension: (6)



In the diagram, a line is drawn through the points $A(0,16)$ and $B(8,0)$. Point P is chosen in the first quadrant on the line through A and B. Points C and D are chosen on the x and y axis respectively, so that $PDOC$ is a rectangle.

1. Perpendicular distance of the line AB from the point (2, 2) is :

(a) $\sqrt{4}$ (b) $\sqrt{10}$

(c) $\sqrt{20}$ (d) $\sqrt{50}$

2. The sum of the coordinates of the point P if $PDOC$ is a square is:

(a) $\frac{32}{3}$ (b) $\frac{16}{3}$

(c) 16 (d) 11

3. Number of possible ordered pair(s) (x, y) of all positions of point P on AB so that area of the rectangle $PDOC$ is 30 sq. units is:

(a) 3 (b) 2

(c) 1 (d) 0

Comprehension: (7)

Let a and b be the lengths of the legs of a right triangle with following properties

(a) All 3 sides of the triangle are integers.

(b) The perimeter of the triangle is numerically equal to area of the triangle, it is given that a [illegible]

1. The number of ordered pairs (a, b) will be :

(a) 1 (b) 2

(c) 3 (d) 4

2. Maximum possible perimeter of the triangle is :

(a) 27 (b) 28

(c) 29 (d) 30

3. Minimum possible area of the triangle is :

(a) 24 (b) 25

(c) 26 (d) 27

Comprehension: (8)

Let $A \equiv (0,0)$, $B \equiv (5,0)$, $C \equiv (5,3)$ and $D \equiv (0,3)$ are the vertices of rectangle $ABCD$. If P is a variable point lying inside the rectangle $ABCD$ and $d(P, L)$ denote perpendicular distance of point P from line L.

1. If $d(P, AB) \le \min\{d(P, BC), d(P, CD), d(P, AD)\}$, then area of the region in which P lies is :

(a) $\frac{17}{4}$ (b) $\frac{19}{4}$

(c) $\frac{21}{4}$ (d) $\frac{23}{4}$

2. If $d(P,AB) \geq \max\{d(P,BC),\ d(P,CD),\ d(P,AD)\}$, then area of the region in which P lies is :

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) $\frac{1}{4}$

3. If $\left(d(P,AB)-\frac{3}{2}\right)^2 + d(P,AD)^2 \geq 1$, then area of region in which P lies is :

(a) $15-2\pi$ (b) $10-\frac{\pi}{2}$ (c) $15-\pi$ (d) $15-\frac{\pi}{2}$

Comprehension: (9)

The equation of an altitude of an equilateral triangle is $\sqrt{3}\,x + y = 2\sqrt{3}$ and one of its vertices is $(3,\sqrt{3})$ then

1. The possible number of triangles is :

(a) 1 (b) 2 (c) 3 (d) 4

2. Which of the following can't be the vertex of the triangle :

(a) $(0,0)$ (b) $(0,2\sqrt{3})$ (c) $(3,-\sqrt{3})$ (d) None of these

3. Which of the following can be the possible orthocentre of the triangle :

(a) $(1,\sqrt{3})$ (b) $(\ldots,\sqrt{3})$ [illegible] (c) $(0,2)$ [illegible] (d) None of these

Comprehension: (10)

Given point $A(6,30)$ and point $B(24,6)$, equation of line AB is $4x+3y=114$. Point $P(0,\lambda)$ is a point on y-axis such that $0<\lambda<38$ and point $Q(0,k)$ is a point on y-axis such that $k>38$.

1. For all positions of point P, angle APB is maximum when point P is :

(a) $(0,12)$ (b) $(0,15)$ (c) $(0,18)$ (d) $(0,21)$

2. The maximum value of angle APB is :

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{2\pi}{3}$ (d) $\frac{3\pi}{4}$

3. For all position of point Q, angle AQB is maximum when point Q is :

(a) $(0,54)$ (b) $(0,58)$ (c) $(0,60)$ (d) None of these

Comprehension: (11)

***OACB* is a square on *x-y* plane where *O* is the origin. A line through *A* intersects the diagonal *OC* at *D* internally, side *OB* at *E* internally and side *CB* at *F* externally. Given that $AD:DE = 4:3$, $AD = 5$ units and the square lies completely in first quadrant.**

1. The area of square will be :

(a) 36 (b) 42 (c) 49 (d) 82

2. The abscissa of F will be :

(a) $-\frac{8}{3}$ (b) $-\frac{7}{3}$ (c) $-\frac{5}{3}$ (d) $-\frac{4}{3}$

3. Let O' be the reflection of O along AD. The equation of circumcircle of $\Delta AO'E$ will be :

(a) $x^2 + y^2 - 7x - 21y = 0$ (b) $4(x^2 + y^2) - 7x - 21y = 0$

(c) $4(x^2 + y^2 - 7x) - 21y = 0$ (d) $x^2 + y^2 - 21x - 7y = 0$

Comprehension: (12)

A variable line 'L' is drawn through $O(0,0)$ to meet the lines $L_1 : y - x - 10 = 0$ and $L_2 : y - x - 20 = 0$ at points A and B respectively. A point P is taken on line 'L'.

1. If $\frac{2}{OP} = \frac{1}{OA} + \frac{1}{OB}$, then locus of P is :

(a) $3x + 3y = 40$ (b) $3x + 3y + 40 = 0$ (c) $3x - 3y = 40$ (d) $3y - 3x = 40$

2. If $OP^2 = (OA)(OB)$, then locus of P is :

(a) $(y-x)^2 = 100$ (b) $(y-x)^2 = 50$ (c) $(y-x)^2 = 200$ (d) $(y-x)^2 = 250$

3. If $\frac{1}{OP^2} = \frac{1}{(OA)^2} + \frac{1}{(OB)^2}$, then locus of P is :

(a) $(y-x)^2 = 80$ (b) $(y-x)^2 = 100$ (c) $(y-x)^2 = 144$ (d) $(y-x)^2 = 400$

Comprehension: (13)

P is an interior point of triangle ABC. AP, BP, CP when produced meet the sides at D, E, F respectively. If $BD = 2DC$ and $AE = 3EC$, then

1. $AP : PD =$

(a) 5 : 6 (b) 6 : 5 (c) 8 : 3 (d) 9 : 2

2. $BP : PE =$

(a) 5 : 6 (b) 6 : 5 (c) 8 : 3 (d) 7 : 4

3. $CP : PF =$

(a) 5 : 6 (b) 6 : 5 (c) 7 : 4 (d) 8 : 3

Comprehension: (14)

A ray of light travelling along the line OP (O being origin) is reflected by the line mirror $2x-3y+1=0$, the point of incidence being $P(1,1)$. The reflected ray, travelling along PQ is again reflected by the line mirror $2x-3y-1=0$, the point of incidence being Q, from Q ray move along QR, where R lies on the line $2x-3y+1=0$

1. The equation of QR is:

(a) $13x-13y=20$ (b) $13x-13y+20=0$ (c) $y=x-1$ (d) $13x-13y+17=0$

2. The ordinate of point R is:

(a) $\frac{73}{13}$ (b) $\frac{53}{13}$ (c) $\frac{23}{13}$ (d) 1

ANSWERS

Comprehension-1:	1.	(d)	2.	(d)	3.	(c)
Comprehension-2:	1.	(d)	2.	(d)	3.	(d)
Comprehension-3:	1.	(d)	2.	(b)	3.	(d)
Comprehension-4:	1.	(a)	2.	(b)	3.	(c)
Comprehension-5:	1.	(d)	2.	(b)	3.	(c)
Comprehension-6:	1.	(c)	2.	(a)	3.	(b)
Comprehension-7:	1.	(b)	2.	(d)	3.	(a)
Comprehension-8:	1.	(c)	2.	(d)	3.	(d)
Comprehension-9:	1.	(b)	2.	(d)	3.	(a)
Comprehension-10:	1.	(c)	2.	(b)	3.	(b)
Comprehension-11:	1.	(c)	2.	(b)	3.	(c)
Comprehension-12:	1.	(d)	2.	(c)	3.	(a)
Comprehension-13:	1.	(d)	2.	(c)	3.	(a)
Comprehension-14:	1.	(a)	2.	(b)		

EXERCISE 4

Assertion and Reason

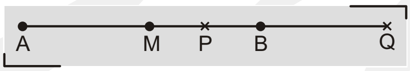
(a) Statement -1 is true , statement-2 is true and statement-2 is correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true and statement-2 is not the correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

1. A line segment AB is divided internally and externally in the same ratio at P and Q respectively and M is the midpoint of AB.



Statement-1: MP, MB, MQ are in G.P.

because

Statement-2: AP, AB and AQ are in H.P.

2. Given a ΔABC whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$; $C(x_3, y_3)$. Let there exists a point $P(a, b)$ such that $6a = 2x_1 + x_2 + 3x_3$; $6b = 2y_1 + y_2 + 3y_3$.

Statement-1: Area of triangle PBC must be less than area of ΔABC.

because

Statement-2: P lies inside the triangle ABC.

3. Let $A(x_1, y_1)$ and $B(x_2, y_2)$ are two fixed points in x-y plane. Let us construct a line passing through 'A' at a perpendicular distance 'P' from B in the same plane, then

Statement-1: It is possible that no such line exist.

because

Statement-2: If $P < AB$, then no lines can be drawn through A at perpendicular distance 'P' from B.

4. Let 'P' denote the perimeter of ΔABC. If M is a point in the interior of ΔABC, then

Statement-1: $MA + MB + MC < P$

because

Statement-2: $MA + MB < AC + BC$

5. Let $a_1x + b_1y + c_1 = 0$,$a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ represent three lines L_1, L_2 and L_3 respectively

Statement-1: If L_1, L_2, L_3 are concurrent, then

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

because

Statement-2: If $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$, then lines L_1, L_2, L_3 must be concurrent at point whose x-y coordinates are finite numbers.

6. Consider a pair of lines represented by $ax^2 + 2hxy + by^2 = 0$ where a, b, h are real numbers and $h^2 > ab$, then

Statement-1: If $a + b + 2h = 0$, then one line of the pair $ax^2 + 2hxy + by^2 = 0$ bisects the angle between coordinate axes in first and third quadrants.

because

Statement-2: If $ax + y(2h + a) = 0$ is a factor of $ax^2 + 2hxy + by^2 = 0$, then $b + 2h + a = 0$

7. Statement-1: If a, b, c are variables such that $3a + 2b + 4c = 0$, then family of lines given by $ax + by + c = 0$ passes through a fixed point $\left(\frac{3}{4}, \frac{1}{2}\right)$.

because

Statement-2: The equation $ax + by + c = 0$ will represent a family of lines passing through a fixed point if there exists a linear relation between a, b and c.

8. Statement-1: The area of triangle formed by points A (20, 22), B (21, 24) and C (22, 23) is same as the area of triangle formed by points P (0, 0), Q (1, 2), R (2, 1).

because

Statement-2: The area of triangle is invariant with respect to the translation of the coordinate axes.

9. Statement-1: The equation $2xy + 3x - 4y = 12$ does not represent a line pair.

because

Statement-2: A general equation of degree two in which coefficient of $x^2 = 0$ and coefficient of $y^2 = 0$ and coefficient of $xy \neq 0$ can not represent a line pair.

10. Let points A, B, C are represented by $(a\cos\theta_i, a\sin\theta_i), i = 1,2,3$; and $\cos(\theta_1 - \theta_2) + \cos(\theta_2 - \theta_3) + \cos(\theta_3 - \theta_1) = -\frac{3}{2}$; then

Statement-1: Orthocentre of ΔABC is at origin.

because

Statement-2: ΔABC is equilateral triangle.

ANSWERS

1.	(a)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(c)	**6.**	(a)	**7.**	(a)	**8.**	(a)	**9.**	(c)	**10.**	(a)

EXERCISE 5

Match the Columns:

1. Let $D(0,\sqrt{3})$, $E(1,0)$, $F(-1, 0)$ be the feet of perpendiculars dropped from vertices A, B, C to opposite sides BC, CA, AB respectively of triangle ABC

	Column-I		Column-II
(a)	The ratio of the inradius of ΔABC to the inradius of ΔDEF is	**(p)**	2
(b)	Let 'H' be the orthocentre of ΔABC, then the greatest integer which is less than or equal to square of the length AH is	**(q)**	3
(c)	The square of the sum of ordinates of points A, B and C is	**(r)**	4
(d)	The length of side AB of ΔABC is	**(s)**	5

2.

	Column-I		Column-II
(a)	If the lines $x + 2ay + a = 0$, $x + 3by + b = 0$, $x + 4cy + c = 0$, where $a, b, c, \in R$ are concurrent, then a,b,c are in	**(p)**	A.P.
(b)	The points with coordinates $(2a, 3a), (3b, 2b)$ (c, c) where a, b, c, $\in R$ are collinear, then a,b,c are in	**(q)**	G.P.
(c)	If lines $ax + 2y + 1 = 0$, $bx + 3y + 1 = 0$ and $cx + 4y + 1 = 0$ where $a,b,c \in R$ passes through the same point, then a,b,c are in	**(r)**	H.P.
(d)	Let a,b,c be distinct non- negative real numbers. If the lines $ax + ay + c = 0$, $x + 1 = 0$, $cx + cy + b = 0$ pass through the same point then a,c,b are in	**(s)**	neither A.P. nor G.P. nor H.P.

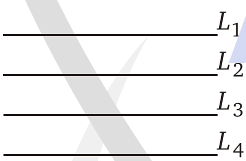
3.

	Column-I		Column-II
(a)	If a,b,c are in A.P., then lines $ax + by + c = 0$ are concurrent at	**(p)**	$(-4, -7)$
(b)	A point on the line $x + y = 4$ which lies at a unit distance from the line $4x + 3y = 10$ is	**(q)**	$(-7, 11)$
(c)	Orthocentre of triangle made by lines $x + y = 1$, $x - y + 3 = 0$, $2x + y = 7$ is	**(r)**	$(1, -2)$
(d)	Two vertices of a triangle are $(5, -1)$ and $(-2, 3)$. If orthocentre is the origin then coordinates of the third vertex are	**(s)**	$(-1, 2)$

4.

	Column-I		Column-II
(a)	The number of integral values of 'a' for which point (a, a^2) lies completely inside the triangle formed by lines $x=0$, $y=0$, $2y+x=3$.	**(p)**	0
(b)	The number of values of a of the form $\frac{K}{3}$ where $K \in I$ so that point (a, a^2) lies between the lines $x+y=2$ and $4x+4y-3=0$	**(q)**	1
(c)	The reflection of point $(t-1, 2t+2)$ in a line is $(2t+1, t)$ then the slope of line is	**(r)**	2
(d)	In a triangle ABC, the bisector of angles B and C lie along the lines $y=x$ and $y=0$. If A is (1, 2) then $\sqrt{10}\, d(A, BC)$ equals (where $d(A, BC)$ denotes the perpendicular distance of A from BC.)	**(s)**	4

5. Given four parallel lines L_1, L_2, L_3 and L_4 as shown in figure. Let d_{ij} denote the perpendicular distance between lines L_i and L_j $i, j \in \{1,2,3,4\}$. Let P be a point, sum of whose perpendicular distances from four lines is K, also $d_{12} < d_{23} < d_{34}$ Then the complete locus of point P.



	Column-I		Column-II
(a)	If $K = d_{12} + 2d_{23} + d_{34}$	**(p)**	Not possible
(b)	If $K = d_{12} + 2d_{23} + d_{34} + 2\alpha$, where $0 < \alpha < d_{12}$	**(q)**	Entire region between the lines L_2 and L_3
(c)	If $K = d_{12} + 2d_{23} + d_{34} + 2\alpha$ where $0 < \alpha < d_{34}$	**(r)**	Entire region between the lines L_1 and L_2
(d)	If $K < d_{12} + 2d_{23} + d_{34}$	**(s)**	Entire region between the lines L_1 and L_2 and between L_3 and L_4

6.

	Column-I		Column-II
(a)	If $P\left(1+\frac{t}{\sqrt{2}}, 2+\frac{t}{\sqrt{2}}\right)$ be any point on a line then value of t for which the point P lies between parallel lines $x+2y=1$ and $2x+4y=15$ is	**(p)**	(1, 2)
(b)	If the point $(2x_1 - x_2 + t(x_2 - x_1), 2y_1 - y_2 + t(y_2 - y_1))$ divides the join of (x_1, y_1) and (x_2, y_2) internally, then	**(q)**	$\left(-\frac{\sqrt{13}-1}{2}, -1\right) \cup \left(1, \frac{\sqrt{13}-1}{2}\right)$
(c)	If the point $(1, t)$ always remains in the interior of the triangle formed by the lines $y=x, y=0$ and $x+y=4$, then	**(r)**	$\left(\frac{-4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6}\right)$
(d)	Set of values of 't' for which the point $P(t, t^2-2)$ lies inside the triangle formed by lines $x+y=1$, $y=x+1$ and $y=-1$ is	**(s)**	(0, 1)

7. Vertex A of the ΔABC is at origin. The equation of medians through B and C are $15x-4y-240=0$ and $15x-52y+240=0$ respectively.

	Column-I		Column-II
(a)	The coordinates of incenter of ΔABC are	**(p)**	$\left(\frac{56}{3}, 10\right)$
(b)	The coordinates of centroid of ΔABC are	**(q)**	$(21, 12)$
(c)	The coordinates of excenter opposite to vertex C of ΔABC are	**(r)**	$(12, 21)$
(d)	The coordinates of orthocenter of ΔABC are	**(s)**	$(-4, 7)$
		(t)	$(0, 63)$

ANSWERS

1. a → p; b → s; c → q; d → r

2. a → r; b → s; c → p; d → q

3. a → r; b → q; c → s; d → p

4. a → p; b → r; c → q; d → s

5. a → q; b → r; c → s; d → p

6. a → r; b → p; c → s; d → q

7. a → q; b → p; c → s; d → t

EXERCISE 6

Subjective Problems

1. $P(3,1)$, $Q(6,5)$ and $R(x, y)$ are three points such that angle PRQ is right angle and the area of ΔPRQ is 7, then number of such points R is.
2. The number of integral values of a for which the point $P(a^2, a)$ lies in the region corresponding to the acute angle between the lines $2y = x$ and $4y = x$ is.
3. The number of integral values of b for which the origin and the point $(1,1)$ lie on the same side of straight line $a^2x + aby + 1 = 0$ for $a \in R - \{0\}$ is.
4. If the pair of lines $6x^2 - \alpha xy - 3y^2 - 24x + 3y + \beta = 0$ intersect on x-axis, then find the value of $20\alpha - \beta$.
5. If n_1 is the number of points on the line $3x + 4y = 5$ which is at distance of $1 + \sin^2\theta$ units from $(2,3)$ and n_2 denotes the number of points on the line $3x + 4y = 5$ which is at distance of $\sec^2\theta + 2\operatorname{cosec}^2\theta$ units from $(1,3)$, then find the sum of roots of equations $n_2x^2 - 6x + n_1 = 0$.
6. In a ΔABC, the vertex A is $(1,1)$ and orthocenter is $(2,4)$. If the sides AB and BC are members of the family of straight lines $ax + by + c = 0$. Where a, b, c are in A.P. then the coordinates of vertex C are (h,k). Find the value of $2h + 12k$.
7. Let P be any point on the line $x - y + 3 = 0$ and A be a fixed point $(3,4)$. If the family of lines given by the equation $(3\sec\theta + 5\operatorname{cosec}\theta)\,x + (7\sec\theta - 3\operatorname{cosec}\theta)\,y + 11(\sec\theta - \operatorname{cosec}\theta) = 0$ are concurrent at a point B for all permissible values of θ and maximum value of $|PA - PB| = 2\sqrt{2n}$ $(n \in N)$, then find the value of n.
8. There exists two ordered triplets (a_1, b_1, c_1) and (a_2, b_2, c_2) for (a,b,c) for which the equation $4x^2 - 4xy + ay^2 + bx + cy + 1 = 0$ represents a pair of identical straight lines in x-y plane. Find the value of $a_1 + b_1 + c_1 + a_2 + b_2 + c_2$.
9. Each side of a square is of length 4 units. The center of the square is at $(3,7)$ and one of the diagonals is parallel to the line $y = x$. If the vertices of the square be $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ and (x_4, y_4) then find the value of $\max(y_1, y_2, y_3, y_4) - \min(x_1, x_2, x_3, x_4)$.
10. The base of an isosceles triangle is the intercept made by the line $x + 2y = 4$ with the coordinate axes. If the equations of the equal sides be $x = 4$ and $y = mx + c$ then find the value of $8m + c$.
11. The slope of one of lines given by $ax^2 + 2hxy + by^2 = 0$ be the square of the slope of the other, if $ab(a + b) + \alpha abh + \beta h^3 = 0$, then $\alpha + \beta$ is equals.
12. The slopes of three sides of a triangle ABC are $-1, -2, 3$ respectively. If the orthocentre of triangle ABC is origin, then the locus of its centroid is $y = \frac{a}{b}x$ where a, b are relatively prime then $b - a$ is equal to.

13. The equation of a line through the mid point of the sides AB and AD of rhombus $ABCD$, whose one diagonal is $3x-4y+5=0$ and one vertex is $A(3,1)$ is $ax+by+c=0$. Find the absolute value of $(a+b+c)$ where a,b,c are integers expressed in lowest form.

14. If there a real value of λ for which the image of point $(\lambda,\lambda-1)$ by the line mirror $3x+y=6\lambda$ is the point (λ^2+1,λ)? Then find λ.

15. Straight line L_1 is parallel to the bisector of first and third quadrant, forms a triangle of area 2 square units with coordinate axis in second quadrant. Line L_2 passes through $(1,1)$ and has positive x and y intercepts. L_2 makes a triangle of minimum are with coordinate axes. The area of the triangle formed by L_1, L_2 and x-axis is of form $\frac{p}{q}$ where p and q are relatively natural numbers. Find $|p-q|$.

16. Consider two lines $L_1 \equiv x-y=0$ and $L_2 \equiv x+y=0$ and a moving point $P(x,y)$. Let $d(P,L_1), i=1,2$ represents the perpendicular distance of the point P from L_1. If point P moves in certain region R in such a way that $\sum_{i=1}^{2} d(P,L_1) \in [2,4]$. Let the area of region R is A, then find $\frac{A}{4}$.

17. In a ΔABC, $A \equiv (\alpha,\beta)$, $B(1,2)$, $C(2,3)$ and point A lies on line $y=2x+3$, where $\alpha,\beta \in I$. If the area of ΔABC be such that area of triangle lies in interval $[2,3)$. Find the number of all possible coordinates of A.

18. Consider ΔABC with A [illegible] m [illegible] $-m$ [illegible] $-1)$ $B(-$ [illegible] $l, l+$ [illegible] $)$ is su[illegible] a line of slope 2, drawn through centroid of ΔABC me[illegible]ts th[illegible] ci[illegible]um[illegible] [illegible] on [illegible] axis, th[illegible]n [illegible] the value of $l+m$.

19. A variable line L_1 cuts $y=3x+1$ and $y=-2x+3$ at points P_1 and P_2. If the locus of midpoints of P_1 and P_2 is line L_2 with undefined slope where slope of L_1 is constant. If slope of L_1 is $\frac{p}{q}$, where p,q are coprime natural number, then find $p+q$.

20. Let A,B,C lies on lines $y=x$, $y=2x$ and $y=3x$ respectively. Also AB passes through fixed point $(1,0)$, BC Passes through fixed point $(0,-1)$, then AC also passes through fixed point (h,k), find the value of $h+k$.

ANSWERS

1.	0	**2.**	1	**3.**	3	**4.**	6	**5.**	3	**6.**	14	**7.**	5	**8.**	2	**9.**	8	**10.**	8
11.	2	**12.**	7	**13.**	1	**14.**	2	**15.**	3	**16.**	6	**17.**	4	**18.**	0	**19.**	3	**20.**	0

EXERCISE 7

1. Let PQR be a right angled isosceles triangle, right angled at $P(2, 1)$. If the equation of the line QR is $2x + y = 3$, then the equation representing the pair of lines PQ and PR is: **[IIT-JEE 1999]**

(a) $3x^2 - 3y^2 + 8xy + 20x + 10y + 25 = 0$ (b) $3x^2 - 3y^2 + 8xy - 20x - 10y + 25 = 0$

(c) $3x^2 - 3y^2 + 8xy + 10x + 15y + 20 = 0$ (d) $3x^2 - 3y^2 - 8xy - 10x - 15y - 20 = 0$

2. The equation of two equal sides AB and AC of an isosceles triangle ABC are $x + y = 5$ and $7x - y = 3$ respectively. Find the equations of the side BC if the area of the triangle of ABC is 5 units. **[REE 1999]**

3. (A) The incentre of the triangle with vertices $(1, \sqrt{3})$, $(0, 0)$ and $(2, 0)$ is:

(a) $\left(1, \frac{\sqrt{3}}{2}\right)$ (b) $\left(\frac{2}{3}, \frac{1}{\sqrt{3}}\right)$ (c) $\left(\frac{2}{3}, \frac{\sqrt{3}}{2}\right)$ (d) $\left(1, \frac{1}{\sqrt{3}}\right)$

(B) Let PS be the median of the triangle with vertices, $P(2, 2)$, $Q(6, -1)$ and $R(7, 3)$. The equation of the line passing through $(1, -1)$ and parallel to PS is:

[IIT-JEE (Screening) 2000]

(a) $2x - 9y - 7 = 0$ (b) $2x - 9y - 11 = 0$

(c) $2x + 9y - 11 = 0$ (d) $2x + 9y + 7 = 0$

(C) For points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ of the co-ordinate plane, a new distance $d(P, Q)$ is defined by $d(P, Q) = |x_1 - x_2| + |y_1 - y_2|$. Let $O = (0, 0)$ and $A = (3, 2)$. Prove that the set of points in the first quadrant which are equidistant (with respect to the new distance) from O and A consists of the union of a line segment of finite length and an infinite ray. Sketch this set in a labelled diagram. **[IIT-JEE (Mains) 2000]**

4. Find the position of point $(4, 1)$ after it undergoes the following transformations successively.

(i) Reflection about the line, $y = x - 1$.

(ii) Translation by one unit along x-axis in the positive direction.

(iii) Rotation through an angle $\pi/4$ about the origin in the anticlockwise direction.

[REE (Mains) 2000]

5. (A) Area of the parallelogram formed by the lines $y = mx$, $y = mx + 1$, $y = nx$ and $y = nx + 1$ equals:

(a) $\frac{|m + n|}{(m - n)^2}$ (b) $\frac{2}{|m + n|}$ (c) $\frac{1}{|m + n|}$ (d) $\frac{1}{|m - n|}$

(B) The number of integer values of m, for which the x co-ordinate of the point of intersection of the lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is:

[IIT-JEE (Screening) 2001]

(a) 2 (b) 0 (c) 4 (d) 1

6. (A) Let $P=(-1,0), Q=(0,0)$ and $R=(3,3\sqrt{3})$ be three points. Then the equation of the bisector of the angle PQR is:

(a) $\frac{\sqrt{3}}{2}x+y=0$ (b) $x+\sqrt{3}y=0$ (c) $\sqrt{3}x+y=0$ (d) $x+\frac{\sqrt{3}}{2}y=0$

(B) A straight line through the origin O meets the parallel lines $4x+2y=9$ and $2x+y+6=0$ at points P and Q respectively. Then the point O divides the segment PQ in the ratio:

(a) $1:2$ (b) $3:4$ (c) $2:1$ (d) $4:3$

(C) The area bounded by the curves $y=|x|-1$ and $y=-|x|+1$ is:

[IIT-JEE (Screening) 2002]

(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) 4

(D) A straight line L through the origin meets the line $x+y=1$ and $x+y=3$ at P and Q respectively. Through P and Q two straight lines L_1 and L_2 are drawn, parallel to $2x-y=5$ and $3x+y=5$ respectively. Lines L_1 and L_2 intersect at R. Show that the locus of R, as L varies, is a straight line. **[IIT-JEE (Mains) 2002]**

(E) A straight line L with negative slope passes through the point $(8,2)$ and cuts the positive co-ordinates axes at points P and Q. Find the absolute minimum value of $OP+OQ$, as L varies, where O is the origin. **[IIT-JEE (Mains) 2002]**

7. The area bounded by the angle bisectors of the lines $x^2-y^2+2y=1$ and the line $x+y=3$, is:

[IIT-JEE (Screening) 2004]

(a) 2 (b) 3 (c) 4 (d) 6

8. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through $P(h,k)$ with the lines $y=x$ and $x+y=2$ is $4h^2$. Find the locus of the point P.

[IIT-JEE (Mains) 2005]

9. (A) Let $O(0,0), P(3,4), Q(6,0)$ be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are:

(a) $(4/3,3)$ (b) $(3,2/3)$ (c) $(3,4/3)$ (d) $(4/3,2/3)$

(B) Lines $L_1: y-x=0$ and $L_2: 2x+y=0$ intersect the line $L_3: y+2=0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement-1: The ratio $PR:RQ$ equals $2\sqrt{2}:\sqrt{5}$

because

Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true. **[IIT-JEE 2007]**

10. Consider the lines given by

$$L_1 = x + 3y - 5 = 0, \quad L_2 = 3x - ky - 1 = 0, \quad L_3 = 5x + 2y - 12 = 0$$

Match the statements/expressions in **Column-I** with the statements/expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in OMR. **[IIT-JEE 2008]**

	Column-I		Column-II
(a)	L_1, L_2, L_3 are concurrent, if	**(p)**	$k = -9$
(b)	One of L_1, L_2, L_3 is parallel to at least one of the other two, if	**(q)**	$k = -\frac{6}{5}$
(c)	L_1, L_2, L_3 form a triangle, if	**(r)**	$k = \frac{5}{6}$
(d)	L_1, L_2, L_3 do not form a triangle, if	**(s)**	$k = 5$

11. The locus of the orthocentre of the triangle formed by the lines $(1+p)x - py + p(1+p) = 0$, $(1+q)x - qy + q(1+q) = 0$, and $y = 0$, where $p \neq q$, is: **[IIT 2009]**

(a) a hyperbola (b) a parabola (c) an ellipse (d) a straight line

12. A straight line L through the point $(3, -2)$ is inclined at an angle $60°$ to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is: **[IIT 2011]**

(a) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$ (b) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(c) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$ (d) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

13. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as $(0, 1), (1, 1)$ and $(1, 0)$ is : **[IIT-JEE (Mains) 2013]**

(a) $1 - \sqrt{2}$ (b) $2 + \sqrt{2}$ (c) $2 - \sqrt{2}$ (d) $1 + \sqrt{2}$

14. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is : **[IIT-JEE (Mains) 2013]**

(a) $\sqrt{3}y = x - 1$ (b) $y = x + \sqrt{3}$ (c) $\sqrt{3}y = x - \sqrt{3}$ (d) $y = \sqrt{3}x - \sqrt{3}$

15. For $a > b > c > 0$, the distance between $(1, 1)$ and the point of intersection of the lines $ax + by + c = 0$ and $bx + ay + c = 0$ is less than $2\sqrt{2}$. Then : **[IIT-JEE (Advance) 2013]**

(a) $a + b - c > 0$ (b) $a - b + c < 0$ (c) $a - b + c > 0$ (d) $a + b - c < 0$

ANSWERS

1. b **2.** $x - 3y + 21 = 0, x - 3y + 1 = 0, 3x + y = 12, 3x + y = 2$

3. **(A)** d; **(B)** d **4.** $(4, 1) \to (2, 3) \to (3, 3) \to (0, 3\sqrt{2})$ **5.** **(A)** d; **(B)** a

6. **(A)** c; **(B)** b; **(C)** b; **(D)** $x - 3y + 5 = 0$; **(E)** 18 **7.** a **8.** $y = 2x + 1, y = -2x + 1$

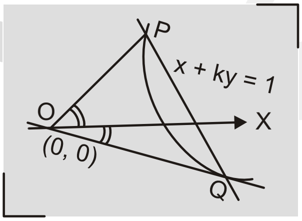
9. **(A)** c; **(B)** c **10.** (a) → s; (b) → p, q; (c) → r; (d) → p, q, s

11. d **12.** b **13.** c **14.** c **15.** a, c

SOLUTIONS 1

Only One Choice is Correct:

1. (b) Equation of pair OP and OQ is obtained by homogenising.



$\Rightarrow$ Equation of pair OP and OQ is

$$5x^2+12xy-6y^2+(4x-2y)(x+ky)$$

$$+3(x+ky)^2=0$$

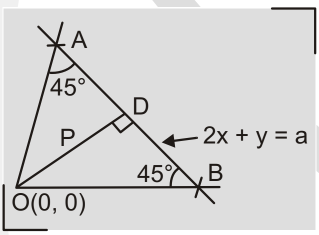
OP and OQ are equally inclined to x-axis.

Coefficient of $xy=0$

$\Rightarrow \quad 12+4k-2+6k=0$

$\Rightarrow \quad k=-1$

2. (c) $OD=AD=BD$



$$OD=P=\frac{|a|}{\sqrt{5}}$$

$$\text{Area of } \Delta OAB=\frac{1}{2}(2P)\,P=P^2$$

$$\text{Area}=\frac{a^2}{5}$$

3. (a)

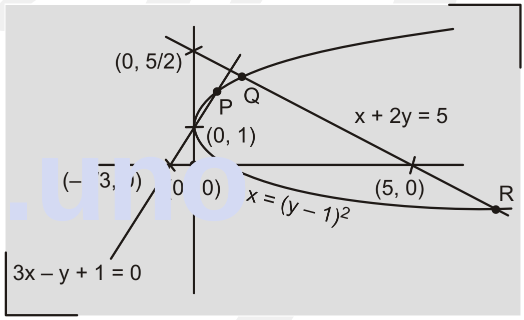
$\because [3(-1)-4(4)+12][12(-1)-5(4)+7]>0$

$\Rightarrow$ Equation of bisector containing $(-1,4)$ in its region is

$$\frac{3x-4y+12}{5}=\frac{12x-5y+7}{13}$$

$\Rightarrow \quad 21x+27y-121=0$

4. (a) Intersection of $x=(y-1)^2$ with lines are



$$(y-1)^2=\frac{y-1}{3}\Rightarrow y=1,\ y=\frac{4}{3}$$

$$\Rightarrow \quad P\equiv\left(\frac{1}{9},\frac{4}{3}\right)$$

$$(y-1)^2=5-2y\Rightarrow y^2=4\Rightarrow y=\pm 2$$

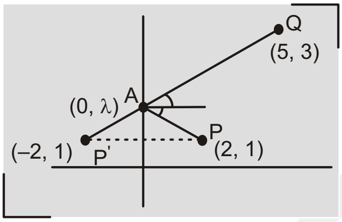
$$Q\equiv(1,2),\ R\equiv(9,-2)$$

$$\Rightarrow \quad a+1\in(-2,1)\cup\left(\frac{4}{3},2\right);$$

$$a\in(-3,0)\cup\left(\frac{1}{3},1\right)$$

5. (a) Equating slopes of $P'A$ and $P'Q$.

$$\frac{\lambda-1}{2}=\frac{3-1}{7}$$



$$\Rightarrow \qquad \lambda = \frac{11}{7} \Rightarrow A \equiv \left(0, \frac{11}{7}\right)$$

6. (c) $(x-3y)(x-y)=0$

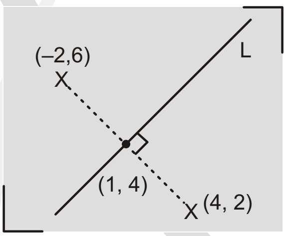
Equation of line parallel to line $x-3y=0$ and passing through $(3,-2)$ is $L_1 \equiv x-3y=9$

Similarly equation of line, parallel to line $x-y=0$ and passing through $(3,-2)$ is $L_2 \equiv x-y=5$

$\therefore$ Equation of pair L_1 and L_2 is $(x-3y-9)(x-y-5)=0$

$$\Rightarrow x^2-4xy+3y^2-14x+24y+45=0$$

7. (c) Equation of L is



$$y-4=\frac{3}{2}(x-1)$$

$$2y-8=3x-3$$

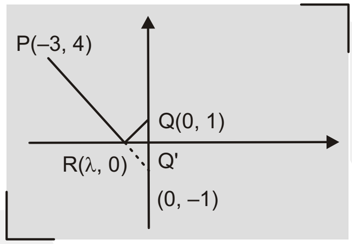
$$\Rightarrow 3x-2y+5=0$$

8. (b) $(PR+RQ)_{\min}=(PR+RQ')_{\min}=PQ'$

$$\Rightarrow \frac{0-4}{\lambda+3}=\frac{4+1}{-3-0}$$

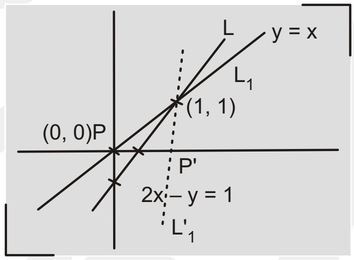
$$\Rightarrow \lambda=-\frac{3}{5}$$

$$\Rightarrow R \equiv \left(-\frac{3}{5},0\right)$$



9. (b) Image of $(0,0)$ w.r.t. L lies on L_1'

Image of $(0,0)$ w.r.t. L is



$$\frac{x-0}{2}=\frac{y-0}{-1}=-2\left(\frac{-1}{5}\right)$$

$$P' \equiv (x, y) = \left(\frac{4}{5}, \frac{-2}{5}\right)$$

Equation of L_1' which passes through $(1,1)$ and $\left(\frac{4}{5},\frac{-2}{5}\right)$ is

$$y-1=\frac{\frac{-2}{5}-1}{\frac{4}{5}-1}(x-1)=\frac{-7}{-1}(x-1)$$

$$y=7x-6$$

10. (d) $a-2\sqrt{bc}=b+c$

$$\Rightarrow (\sqrt{b}+\sqrt{c})^2-(\sqrt{a})^2=0$$

$$\Rightarrow \qquad \sqrt{b}+\sqrt{c}-\sqrt{a}=0$$

or $\qquad \sqrt{b}+\sqrt{c}+\sqrt{a}=0$ (rejected)

$$\sqrt{b}+\sqrt{c}-\sqrt{a}=0$$

$\Rightarrow$ $\sqrt{a}\,x+\sqrt{b}\,y+\sqrt{c}=0$ passes through fixed point $(-1,1)$.

11. (b) $p_1 = \dfrac{|a^2 + 2a\tan\theta + \tan^2\theta|}{|\sec\theta|}$

$$= \frac{(a+\tan\theta)^2}{|\sec\theta|}$$

$$p_3 = \frac{(b+\tan\theta)^2}{|\sec\theta|}$$

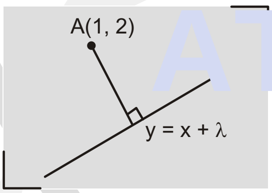
$$p_2 = \frac{|ab + (a+b)\tan\theta + \tan^2\theta|}{|\sec\theta|}$$

$$= \frac{|(a+\tan\theta)(b+\tan\theta)|}{|\sec\theta|}$$

$\Rightarrow \quad p_2^2 = p_1 p_3$

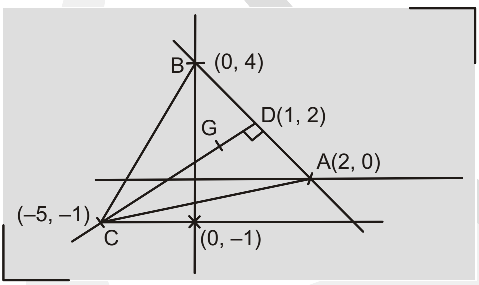
12. (b) Equation of BC varies

$\therefore$ Orthocentre will always lie on line perpendicular to $y = x + \lambda$ passing through $A(1,2)$.



$\Rightarrow$ Locus of orthocentre 'H' is $x + y = 3$.

13. (a) Slope of CD is $\dfrac{1}{2} \Rightarrow C \equiv (-5, -1)$

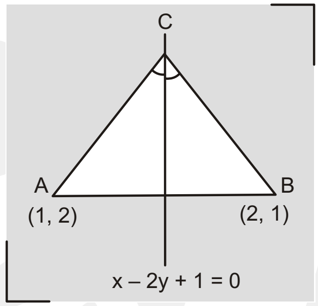


Perpendicular distance from G to AB

$= \dfrac{1}{3}$ (Perpendicular distance from C to AB)

$$= \frac{1}{3}\left(\frac{|-10-1-4|}{\sqrt{4+1}}\right) = \sqrt{5}$$

14. (b) Image of A say A' w.r.t. $x - 2y + 1 = 0$ lies on BC.



$$\frac{x-1}{1} = \frac{y-2}{-2} = -2\frac{(1-4+1)}{1+2^2} = \frac{4}{5}$$

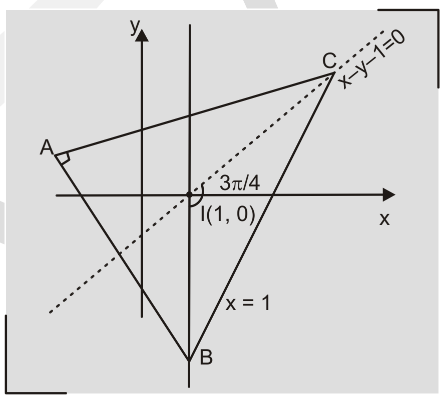
$$A' \equiv \left(\frac{9}{5}, \frac{2}{5}\right)$$

Equation of BC joining $A'\left(\dfrac{9}{5}, \dfrac{2}{5}\right)$ and $B(2,1)$ is

$$y - 1 = \frac{1 - \frac{2}{5}}{2 - \frac{9}{5}}(x-2) = \frac{3}{1}(x-2)$$

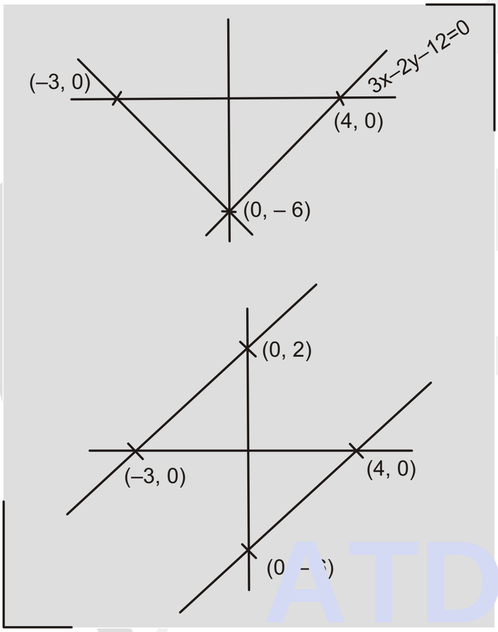
$3x - y - 5 = 0 \Rightarrow a + b = 3 - 1 = 2$

15. (c) $\angle BIC = \dfrac{3\pi}{4} = \dfrac{\pi}{2} + \dfrac{A}{2} \quad \Rightarrow \quad A = \dfrac{\pi}{2}$

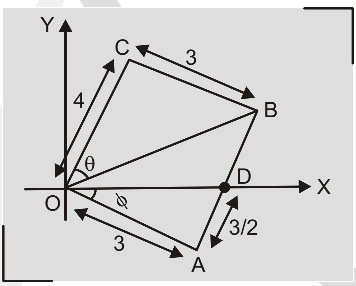


Hence, $\angle BAC = \frac{\pi}{2}$

16. (b) Possible cases are shown below



17. (c) $\tan\theta = \frac{3}{4}, \tan\phi = \frac{3}{2(3)} = \frac{1}{2}$



slope of $OB = \tan\left(\frac{\pi}{2} - (\theta + \phi)\right) = \cot(\theta + \phi)$

$$= \frac{1 - \frac{3}{4} \cdot \frac{1}{2}}{\frac{3}{4} + \frac{1}{2}} = \frac{1}{2}$$

18. (c) Let perpendicular bisector of AB is $3x + 4y - 20 = 0$ and perpendicular bisector of AC is $8x + 6y - 65 = 0$.

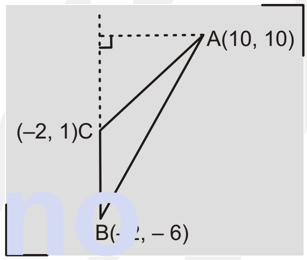
$\Rightarrow$ Image of A w.r.t. $3x + 4y - 20 = 0$ is B and image of A w.r.t. $8x + 6y - 65 = 0$ is C.

For B , $\frac{x - 10}{3} = \frac{y - 10}{4} = -2\left(\frac{30 + 40 - 20}{25}\right)$

$\Rightarrow \quad B \equiv (-2, -6)$

For C , $\frac{x - 10}{8} = \frac{y - 10}{6} = -2\left(\frac{80 + 60 - 65}{100}\right)$

$\Rightarrow \quad C \equiv (-2, 1)$

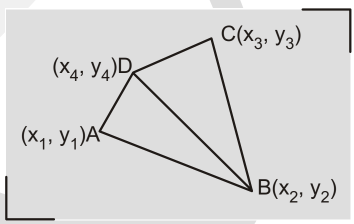


Area of $\Delta ABC = \frac{1}{2}(10 + 2)(1 + 6) = 42$

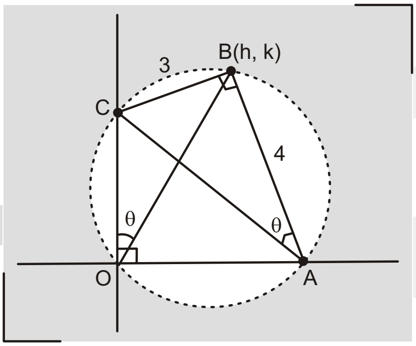
19. (b) Area of $ABCD$ = mod of

$$\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_4 & y_4 & 1 \end{vmatrix} + \text{mod of } \frac{1}{2}\begin{vmatrix} x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{vmatrix}$$

$$\geq \frac{1}{2}(1) + \frac{1}{2}(1) = 1$$



20. (b) $\tan\theta = \dfrac{3}{4}$ (from Fig.)



$\Rightarrow$ Slope of $OB = \tan\left(\dfrac{\pi}{2} - \theta\right) = \cot\theta$

$\Rightarrow$ Locus of (h, k) is $\dfrac{k}{h} = \dfrac{4}{3}$

$\Rightarrow$ $3y = 4x$

21. (b) Lines make complementary angles with X-axis $\Rightarrow m_1 m_2 = 1 \Rightarrow \dfrac{a}{b} = 1$

22. (c) $h = 2\cos\theta + 2\cos(120° - \theta)$

$= \cos\theta + \sqrt{3}\sin\theta$ ...(1)

$k = 0 + 2\sin(120° - \theta)$

$= \sqrt{3}\cos\theta + \sin\theta$ ....(2)

squarring and adding eqs. (1) and (2), we get

$$h^2 + k^2 = 4 + 4\sqrt{3}\sin\theta\cos\theta$$

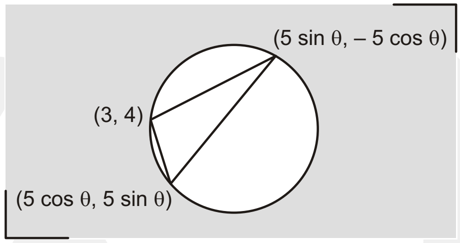
Multiplying eqs. (1) and (2), we get

$$hk = \sqrt{3} + 4\sin\theta\cos\theta$$

$\Rightarrow$ $(h^2 + k^2) - 4 = \sqrt{3}hk - 3$

$x^2 + y^2 = \sqrt{3}\,xy + 1$ is the required locus.

23. (d) By observation, it is clear that 3 vertices of triangle lie on circle $x^2 + y^2 = 25$.



$\therefore$ Centroid

$$G \equiv \left(\frac{5\sin\theta + 5\cos\theta + 3}{3}, \frac{5\sin\theta - 5\cos\theta + 4}{3}\right)$$

Circumcentre $O \equiv (0, 0)$

$\Rightarrow$ Orthocentre, $H \equiv (h, k)$

$$3\left(\frac{5\sin\theta + 5\cos\theta + 3}{3}\right) - 2(0) = h$$

$= 5\sin\theta + 5\cos\theta + 3$ ...(1)

$$3\left(\frac{5\sin\theta - 5\cos\theta + 4}{3}\right) - 2(0) = k$$

$= 5\sin\theta - 5\cos\theta + 4$ ...(2)

By eq. (1) + eq.(2),

$$h + k - 7 = 10\sin\theta$$

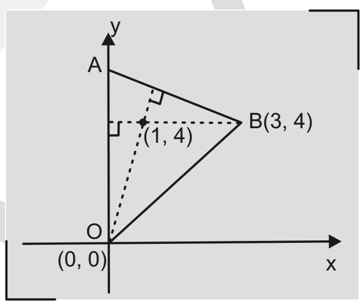
By eq. (1) – eq.(2),

$$h - k + 1 = 10\cos\theta$$

$\Rightarrow$ $(x + y - 7)^2 + (x - y + 1)^2 = 100$ is the required locus.

24. (d) $BH \perp OA \Rightarrow A$ lies on y-axis

equation of AH is :



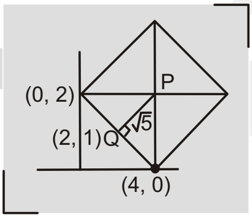
$$y - 4 = -\frac{3}{4}(x - 1)$$

Put $x = 0$,

$$\Rightarrow \quad y = 4 + \frac{3}{4} = \frac{19}{4}$$

$$\Rightarrow \quad A \equiv \left(0, \frac{19}{4}\right)$$

25. (c) Slope of $PQ = 2 = \tan\theta$



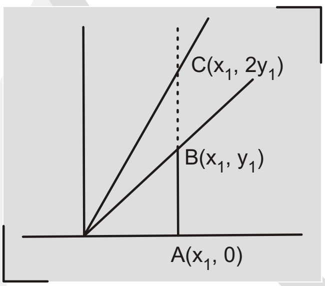
$$P \equiv \left(2 + \sqrt{5} \cdot \frac{1}{\sqrt{5}}, 1 + \sqrt{5} \cdot \frac{2}{\sqrt{5}}\right)$$

$$= (3, 3)$$

26. (b) Put $x = x_1$ in equation of pair of lines

$$\Rightarrow \quad by^2 + 2hx_1y + ax_1^2 = 0 \quad \begin{matrix} y_1 \\ 2y_1 \end{matrix}$$

$$3y_1 = -\frac{2hx_1}{b}, \; 2y_1^2 = \frac{ax_1^2}{b}$$

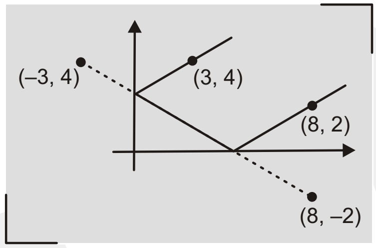


$$\Rightarrow \quad \frac{9y_1^2}{2y_1^2} = \frac{4h^2}{b^2} \frac{x_1^2 b}{ax_1^2}$$

$$\Rightarrow \quad \frac{9}{2} = \frac{4h^2}{ab} \quad \Rightarrow \quad 9ab = 8h^2$$

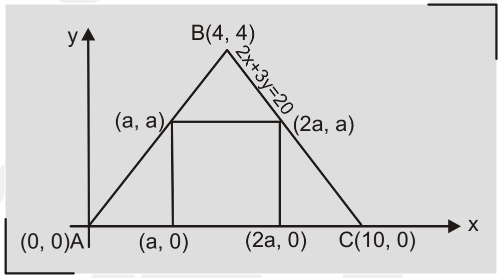
27. (b) $\frac{0+2}{x-8} = \frac{4+2}{-3-8} = -\frac{6}{11}$

$$x = \frac{13}{3}$$



28. (a) $(2a, a)$ lies on $2x + 3y = 20$

So $4a + 3a = 20$



$$\Rightarrow \quad a = \frac{20}{7}$$

$$\therefore \quad \text{Area} = \frac{400}{49}$$

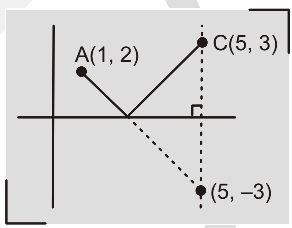
29. (d) Locus of C is a circle.

$\Rightarrow$ infinite ΔABC can be formed

30. (a) Reflection of C with x-axis $\equiv (5, -3)$

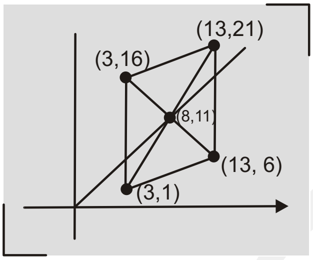
Equation of AB is $y - 2 = \frac{5}{-4}(x - 1)$

$5x + 4y = 13$

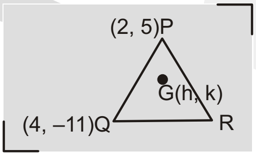


31. (b) Line passes through $(0, 0)$ and $(8, 11)$

$\Rightarrow$ its equation is $y = \frac{11}{8}x$



32. (d) $R \equiv (3h-6, 3k+6)$



R lies on $9x+7y+4=0$.

$\Rightarrow \quad 9(3h-6)+7(3k+6)+4=0$

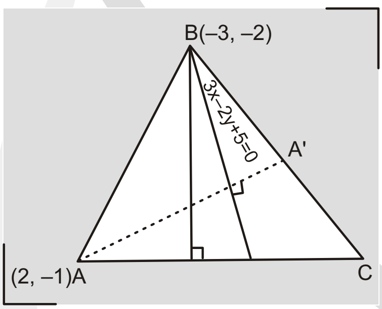
$\Rightarrow \quad (9h+7k)3-8=0$

Which is parallel to N.

33. (b) Image of $A(2,-1)$ w.r.t. $3x-2y+5=0$, A' is given by

$$\frac{x-2}{3}=\frac{y+1}{-2}=-2\cdot\frac{(6+2+5)}{13}=-2$$

$A' \equiv (-4,3)$



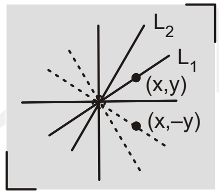
Equation of BC is

$$y-3=\frac{3+2}{-4+3}(x+4)$$

$$5x+y+17=0$$

34. (d) $ax^2+by^2+2hxy=0$

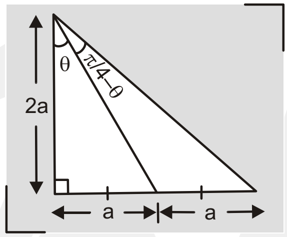
Put $x=X$, $y=-Y$, in the equation of pair L_1 and L_2.



$\Rightarrow \quad aX^2+b(-Y)^2+2h(X)(-Y)=0$

$\Rightarrow \quad aX^2+bY^2-2hXY=0$ is the required equation.

35. (a)



$$\tan\theta=\frac{a}{2a}=\frac{1}{2}$$

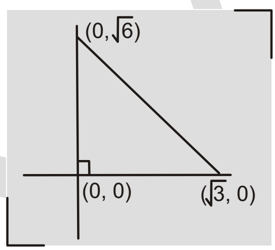
$$\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-1/2}{1+1/2}=\frac{1}{3}$$

36. (d) $p(x+y-1)+q(2x-3y+1)=0$

$\Rightarrow \quad (x+y-1)+\frac{q}{p}(2x-3y+1)=0$

Always passes through intersection of $x+y-1=0$ and $2x-3y+1=0$, which is $\left(\frac{2}{5},\frac{3}{5}\right)$.

37. (d) Orthocentre is $(0,0)$.



38. (b) $y-mx=0$, $x+2y-1=0$ and $2x-y+3=0$ are concurrent

$$\Rightarrow \begin{vmatrix} m & -1 & 0 \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 0 = 5m + 5 \Rightarrow m = -1$$

39. (b) $(x+2) + \lambda(x+y+1) = 0$

Fixed point, $P \equiv (-2, 1)$

Distance of P from origin $= 2$

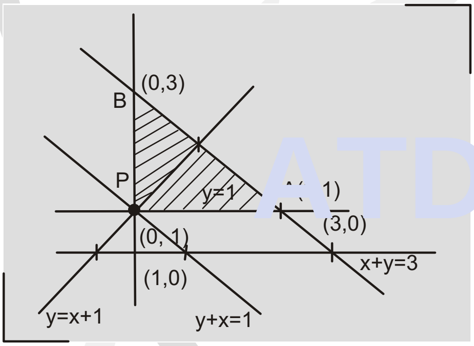
40. (a) $ax + (a+2d)y + (a+6d) = 0$

$\Rightarrow a(x+y+1) + 2d(y+3) = 0$

$\Rightarrow (x+y+1) + \dfrac{2d}{a}(y+3) = 0$

$\therefore$ Fixed point $\equiv (2, -3)$ which lies on $x^2 + y^2 = 13$

41. (a) PAB is the required triangle .



$\therefore$ Area of $\Delta PAB = \dfrac{1}{2} \times 2 \times 2 = 2$

42. (a) For point of intersection,

$3x + 4mx + 4 = 9$

$\Rightarrow x = \dfrac{5}{3+4m}$

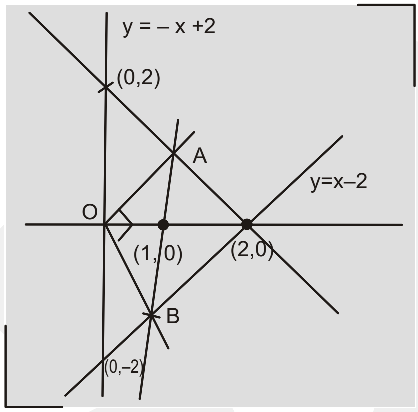
$3 + 4m = \pm 1, \pm 5$

$\Rightarrow m = -1/2, -1, 1/2, -2$

Number of integral values of $m = 2$

43. (d) Let the line be $y = m(x-1)$

Equation of pair of lines is $(y - x + 2)(y + x - 2) = 0$



$\Rightarrow y^2 - x^2 + 4x - 4 = 0$

Equation of pair OA and OB is obtained by homogenisation given by

$$y^2 - x^2 + 4x\left(\frac{mx - y}{m}\right) - 4\left(\frac{mx-y}{m}\right)^2 = 0$$

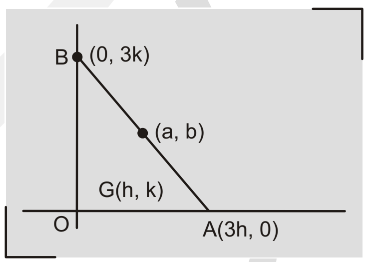
$\because$ $OA \perp OB$

$\therefore$ Coefficient of x^2 + Coefficient of $y^2 = 0$

$\Rightarrow 1 - 1 + 4 - 4 + \dfrac{4}{m^2} = 0 \Rightarrow m \to \infty$

$\therefore$ Line is given by $x = 1$.

44. (a) Equation of AB is $\dfrac{x}{h} + \dfrac{y}{k} = 3$



$\because$ AB passes through (a, b)

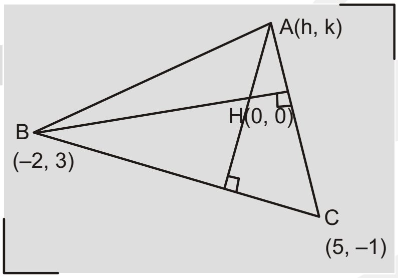
$\therefore \dfrac{a}{h} + \dfrac{b}{k} = 3$

$\Rightarrow bx + ay = 3xy$ is the required locus.

45. (c) Slope of $AH = \frac{k}{h} = \frac{7}{4}$

Slope of $AC = \frac{k+1}{h-5} = \frac{2}{3}$

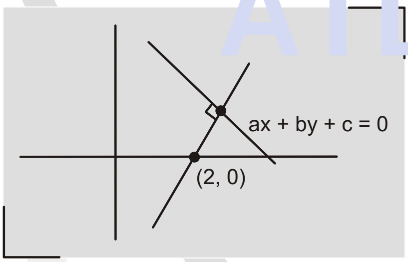
So, $3\left(\frac{7}{4}h + 1\right) = 2(h-5)$



$\Rightarrow \quad h = -4, k = -7$

$A = (-4, -7)$

46. (d) Equation of line after rotation becomes

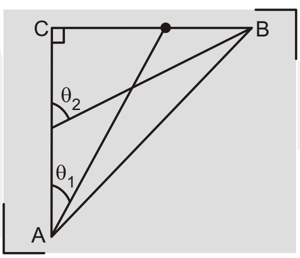


$bx - ay = c'$

$\because$ It passes through $(2,0)$. $\Rightarrow c' = 2b$

$\therefore$ Equation of line is $bx - ay = 2b$.

47. (b) $\tan\theta_1 = \frac{BC}{2AC}$



$\tan\theta_2 = \frac{BC}{(AC/2)} = \frac{2BC}{AC}$

$\frac{\tan\theta_1}{\tan\theta_2} = \frac{1}{4}$

Case-I : If $m < 3$,

$\tan\theta_1 = \frac{1}{3}$ and $\tan\theta_2 = \frac{1}{m}$

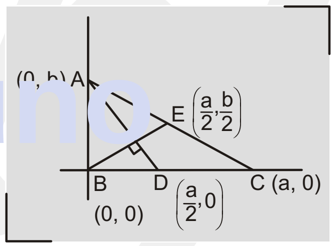
$\frac{m}{3} = \frac{1}{4} \Rightarrow m = \frac{3}{4}$

Case-II : If $m > 3$,

$\tan\theta_1 = \frac{1}{m}$ and $\tan\theta_2 = \frac{1}{3}$

$\Rightarrow \quad \frac{3}{m} = \frac{1}{4} \Rightarrow m = 12$

48. (b) $AD \perp BE$

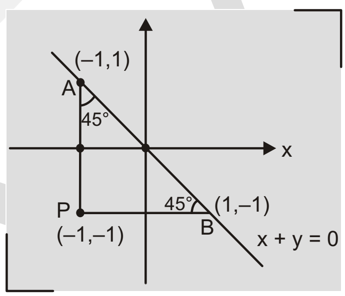


$\Rightarrow \left(\frac{b}{a}\right)\frac{-b}{(a/2)} = -1$

$\Rightarrow 2b^2 = a^2$

$\Rightarrow a = \pm\sqrt{2}\, b$

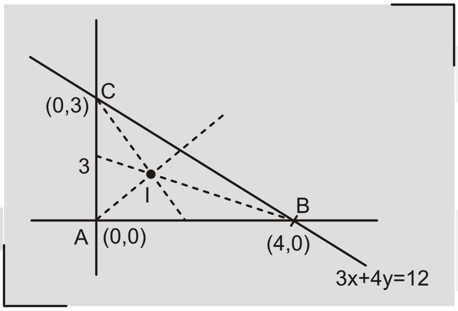
49. (d) Equation of pair PA and PB is



$(x+1)(y+1) = 0$

$\Rightarrow xy + x + y + 1 = 0$

50. (a) Point P must lie on at least one of the angle bisectors.



Incentre,

$$I \equiv \left(\frac{4(0)+3(4)+5(0)}{4+3+5}, \frac{4(3)+3(0)+5(0)}{4+3+5} \right)$$

$I \equiv (1,1)$

$P \equiv (1,1)$ only

51. (d) Lines are

$$x + y + 1 = 0, \; 4x + 3y + 4 = 0$$

and $\quad x + \alpha y + \beta = 0$

where, $\quad \alpha^2 + \beta^2 = 2$

$$\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

$1(3\beta - 4\alpha) - 1(4\beta - 4) + 1(4\alpha - 3)$

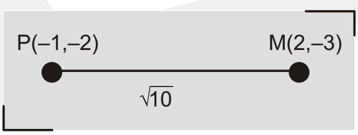
$= 3\beta - 4\alpha - 4\beta + 4 + 4\alpha - 3$

$= -\beta + 1 = 0 \quad \Rightarrow \quad \beta = 1$

$\therefore \quad \alpha = \pm 1$

52. (a) Homogeneous equation of the curve with line. Coefficient of x^2 + coefficient of $y^2 = 0$

53. (b) The point of intersection of the two lines are (–1,–2)

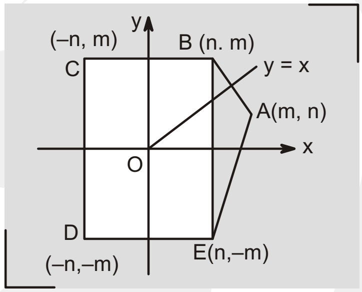


Distance $PM = \sqrt{10}$

Hence the required line is one which passes through (–1,–2) and is ⊥ to P.M.

$\Rightarrow B$

54 (b) Area of rectangle $BCDE = 4mn$



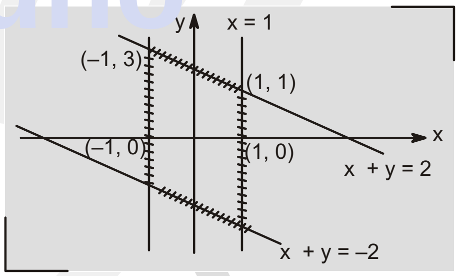
$$\text{Area of } \Delta ABE = \frac{2m(m-n)}{2}$$

$$= m^2 - mn$$

$\therefore$ area of pentagon $= 4mn + m^2 - mn$

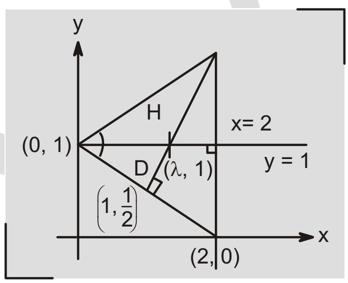
$$= m^2 + 3mn$$

55. (d) Figure is a parallelogram



$$\text{Area} = 2\left(\frac{1+3}{2} .2 \right) = 8$$

56. (b)



Slope of $HD = 2$

$$\Rightarrow \quad \frac{1-\frac{1}{2}}{\lambda-1} = 2 \quad \Rightarrow \quad \lambda = \frac{5}{4}$$

$$\therefore \quad H \equiv \left(\frac{5}{4}, 1\right)$$

57. (a) Image of point $(2,1)$ lying on $2y - x = 0$ w.r.t. $4x + 3y = 0$ is

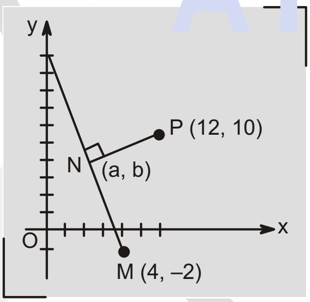
$$\frac{x-2}{4} = \frac{y-1}{3} = -2\frac{8+3}{25}$$

$$\Rightarrow \quad (x, y) \equiv \left(-\frac{38}{25}, -\frac{41}{25}\right)$$

$\therefore$ Other line passes through $(0,0)$ and $\left(-\frac{38}{25}, -\frac{41}{25}\right)$ and is given by

$$y = \frac{41}{38}x$$

58. (b) (a,b) is the foot of $\perp$ ar of $(12, 10)$ on the line $y + 5x = 18$



$$\Rightarrow \quad \frac{a-12}{5} = \frac{b-10}{1} = -\left(\frac{10+5(12)-18}{26}\right)$$

$$\Rightarrow \quad (a, b) = (2, 8)$$

59. (c) Let $m = \tan 2\theta$ and $n = \tan\theta$

and $m = 4n$ and $m \neq 0$,

$\therefore \quad \tan 2\theta \neq 0; \quad \therefore \quad \theta \neq 0$

$$\tan 2\theta = 4\tan\theta$$

$$\frac{2\tan\theta}{1-\tan^2\theta} = 4\tan\theta$$

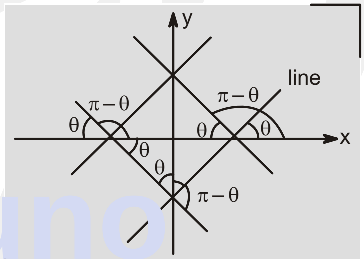
$$\therefore \quad 1 = 2 - 2\tan^2\theta$$

$$2\tan^2\theta = 1 \quad \Rightarrow \quad \tan^2\theta = 1/2$$

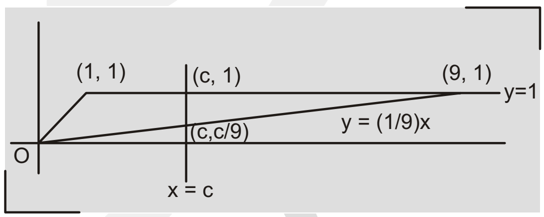
$$\therefore \quad mn = \tan 2\theta . \tan\theta = \frac{2\tan^2\theta}{1-\tan^2\theta}$$

$$= \frac{1}{1-(1/2)} = 2$$

60. (c) Reflecting a graph over the x-axis results in the line M whose equation is $ax - by = c$, while a reflection through the y-axis results in the line N whose equation is $-ax + by = c$. Both clearly have slope equal to a/b (from, say, the slope-intercept form of the equation.)



61. (b)

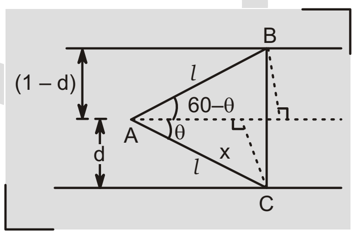


$$2 \times \frac{1}{2}\left(1 - \frac{c}{9}\right)(9-c) = \frac{1}{2} \times 8 \times 1$$

$$\Rightarrow \quad c = 3$$

or $\quad c = 15$ which is not possible

62. (b)



$$l\sin\theta = d \qquad \text{...(1)}$$

$$l\sin(60-\theta)=l\frac{\sqrt{3}}{2}\cos\theta-\frac{l}{2}\sin\theta$$

$$=1-d$$

$$\Rightarrow \quad l\cos\theta=\frac{2-d}{\sqrt{3}} \qquad ...(2)$$

Squaring and adding Eqns. (1) and (2)

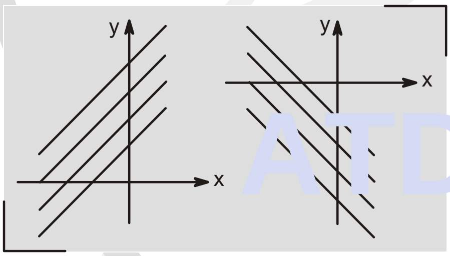
$$\Rightarrow \quad l^2=d^2+\frac{4+d^2-4d}{33}=\frac{4}{3}(d^2-d+1)$$

$$\Rightarrow \quad l=2\sqrt{\frac{d^2-d+1}{3}}$$

63. (b) $mb>0 \Rightarrow m>0$

and $b>0$ or $m<0$ and $b<0$

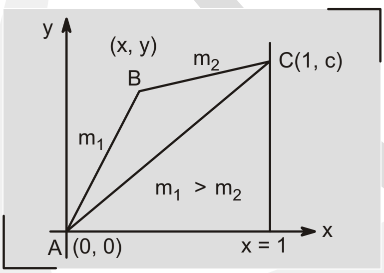
Hence possible lines are as shown



In both the cases x intercept cannot be +ve

$\Rightarrow$ (b)

64. (d) Let the coordinates of C be $(1, c)$



$$m_2=\frac{c-y}{1-x};\quad m_2=\frac{c-m_1x}{1-x}$$

$$m_2-m_2x=c-m_1x$$

$$(m_1-m_2)x=c-m_2$$

$$c=(m_1-m_2)x+m_2 \qquad ...(1)$$

now area of $\Delta ABC=\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ x & m_1x & 1\\ 1 & c & 1\end{vmatrix}$

$$=\frac{1}{2}[cx-m_1x]$$

$$=\frac{1}{2}|[((m_1-m_2)x+m_2)x-m_1x]|$$

$$=\frac{1}{2}|[(m_1-m_2)x^2+m_2x-m_1x]|$$

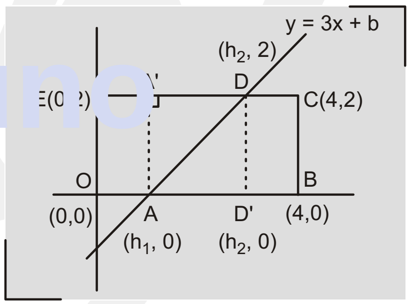
$$=\frac{1}{2}(m_1-m_2)(x-x^2)$$

$[\because x>x^2$ in $(0,1]$

Hence, $f(x)=\frac{1}{2}(x-x^2)$;

$f(x)]_{max}=\frac{1}{8}$ when $x=\frac{1}{2}$

65. (c)

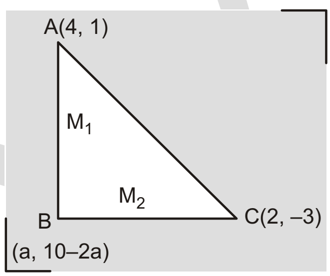


Line will pass through $(2,1)$

$\Rightarrow$ Eqn. of line is $y-3x=-5$

$\therefore$ y-intercept is $(0,-5)$

66. (b) $M_1M_2=-1$



$$\frac{9-2a}{a-4}\times\frac{13-2a}{a-2}=-1$$

$$117 - 26a - 18a + 4a^2 = -(a^2 - 6a + 8)$$

$$5a^2 - 50a + 125 = 0$$

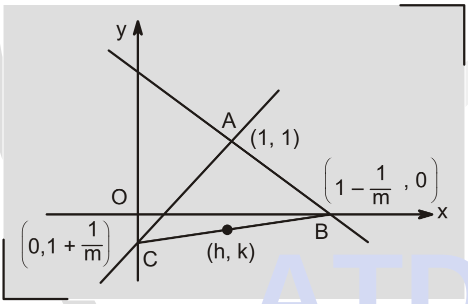
so B is (5, 0)

so area $= \frac{1}{2} AB \times AC = \frac{1}{2}\sqrt{2} \times 3\sqrt{2} = 3$

67. (a) $y - 1 = m(x - 1)$

$$y - 1 = -\frac{1}{m}(x - 1)$$

$$2h = 1 - \frac{1}{m}$$



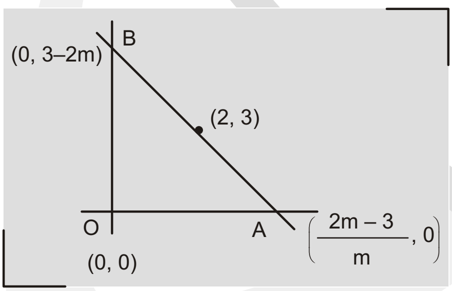
$$2k = 1 + \frac{1}{m}$$

locus is $\boxed{x + y = 1}$

68. (c) Equation of any line through (2, 3) is

$$y - 3 = m(x - 2)$$

$$y = mx - 2m + 3$$



with the help of the fig. area of $\Delta OAB = 12$

i.e., $\frac{1}{2}\left(\frac{2m - 3}{m}\right)(3 - 2m) = \pm 12$

taking + sign we get $(2m + 3)^2 = 0$

this gives one value of $m = -3/2$

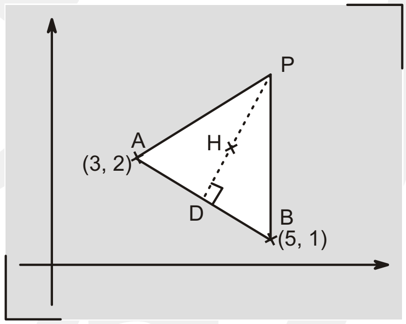
taking negative sign we get

$4m^2 - 36m + 9 = 0 (D > 0)$

quadratic in m gives 2 values of m

$\Rightarrow$ 3 st. lines are possible.

69. (d)



$$PD = AB \sin 60° = \sqrt{5}\, \frac{\sqrt{3}}{2}$$

$$HD = \frac{1}{3} \frac{\sqrt{5}\sqrt{3}}{2} = \frac{\sqrt{5}}{2\sqrt{3}}$$

Slope of $HD = 2$, $D \equiv \left(4, \frac{3}{2}\right)$

Using parametric, form

$$H \equiv \left(4 + \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{1}{\sqrt{5}}, \frac{3}{2} + \frac{\sqrt{5}}{2\sqrt{3}} \times \frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow H \equiv \left(4 + \frac{\sqrt{3}}{6}, \frac{3}{2} + \frac{\sqrt{3}}{3}\right)$$

70. (c) $\frac{1}{x_p} = a + (p - 1)d;$

$$\frac{1}{x_q} = a + (q - 1)d; \quad \frac{1}{x_r} = a + (r - 1)d$$

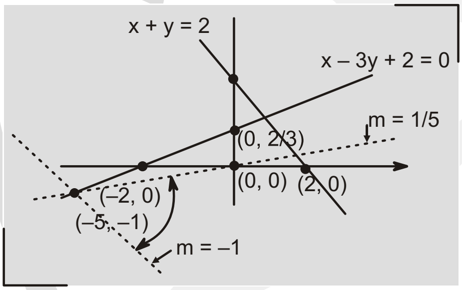
$$\Rightarrow \begin{vmatrix} a + (p-1)d & p & 1 \\ a + (q-1)d & q & 1 \\ a + (r-1)d & r & 1 \end{vmatrix} = 0$$

SOLUTIONS 2

One or More than One is/are Correct

1. (b, c, d)

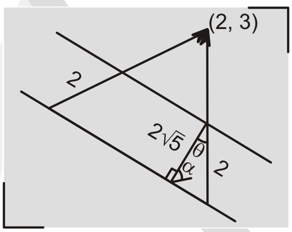
$m \in \left(-1, \frac{1}{5}\right)$ for origin to lie inside the triangle



2. (b, d)

Slope of line is $m = \tan(\alpha \pm \theta)$

$\tan\alpha = \frac{1}{2}.\ \tan\theta = 2$



$$m = \frac{\frac{1}{2} + 2}{1 - \frac{1}{2} \times 2} \to \infty$$

$$m = \frac{\frac{1}{2} - 2}{1 + \frac{1}{2} \times 2} = \frac{-3}{4}$$

eqn. of lines are

$x = 2$ and $y - 3 = \frac{-3}{4}(x - 2)$

$x = 2$ and $4y + 3x = 18$

3. (a, b)

The diagonal of rhombus is parallel to angle bisector of given lines

$$\frac{y - x - 2}{\sqrt{2}} = \pm\frac{y - 7x - 3}{5\sqrt{2}}$$

$\Rightarrow\ 4y + 2x - 7 = 0, 6y + 12x - 13 = 0$

$\therefore$ Diagonals are $2y + x = 5$

and $2x - y = 0$

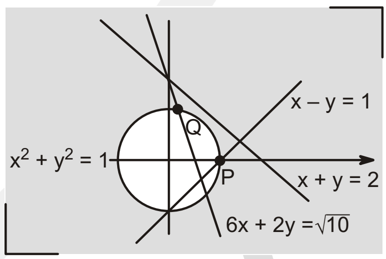
$\therefore$ Possible coordinates of A are $\left(0, \frac{5}{2}\right)$ or $(0, 0)$

4. (a, b, c, d)

For Q

$6\cos\theta + 2\sin\theta = \sqrt{10}$

$\tan^{-1} 3 = \alpha, \sin(\theta + \alpha) = \frac{1}{2}$



$$\theta + \alpha = \frac{5\pi}{6}$$

$$\theta = \frac{5\pi}{6} - \tan^{-1} 3$$

$$\therefore\ \theta \in \left(0, \frac{5\pi}{6} - \tan^{-1} 3\right)$$

5. (a, c)

3^{rd} side will be parallel to angle bisectors of given lines,

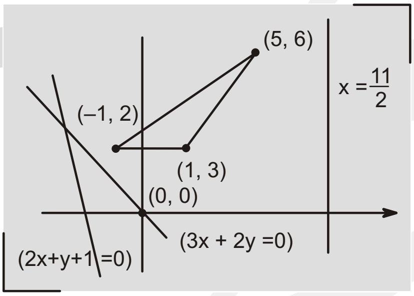
$$\frac{7x-y+3}{5\sqrt{2}} = \pm\frac{x+y-3}{\sqrt{2}}$$

$\Rightarrow \quad 2x - 6y = c$

$\therefore$ 3^{rd} side will be given by

$x - 3y = 31$ or $3x + y = -7$

6. (a, b, c)



7. (a, d)

$m = \tan\theta_1, m' = \tan\theta_2$

inclination of given lines are $2\theta_1$ and $2\theta_2$

$\therefore$ inclination of angle bisectors $\theta_1 + \theta_2$ or

$\theta_1 + \theta_2 + \frac{\pi}{2}$

$\therefore$ slopes will be $\frac{m+m'}{1-mm'}, \frac{mm'-1}{m+m'}$

Eqn. of bisectors are

$$y - b = \frac{(m+m')}{(1-mm')}(x-a),$$

$$y - b = \frac{(mm'-1)}{(m+m')}(x-a)$$

8. (a, b)

BC is parallel to angle bisector of lines

$2x + y = 5$ and $x - 2y = 3$

$$\frac{2x+y-5}{\sqrt{5}} = \pm\frac{x-2y-3}{\sqrt{5}} \Rightarrow x + 3y = c$$

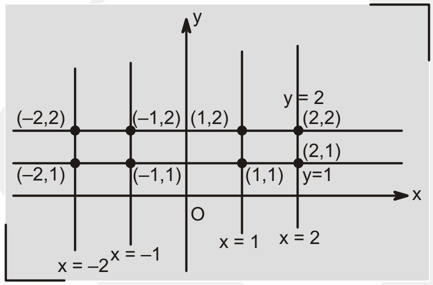
Eqn. of BC can be $x + 3y = 11$

or $3x - y = 3$

9. (b, c)

Triangle formed by lines is obtuse.

10. (a, b)

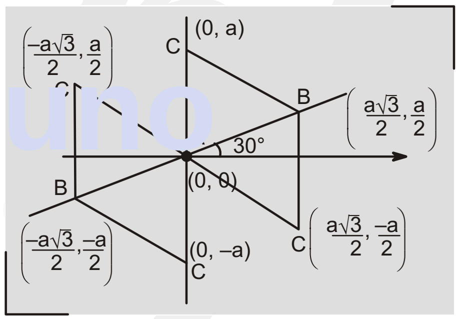


Possible squares have vertices

$(-2, 1), (-2, 2), (-1, 1), (-1, 2)$

or $(1, 1), (1, 2), (2, 1), (2, 2)$

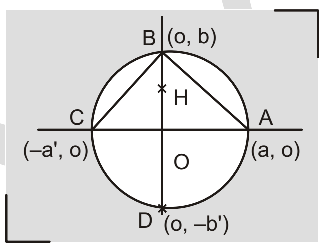
11. (a, b, c, d)



Possible ordinates of C are $(0, a)$, $(0, -a)$, $\left(\frac{a\sqrt{3}}{2}, \frac{-a}{2}\right), \left(\frac{-a\sqrt{3}}{2}, \frac{a}{2}\right)$

12. (b, c)

H is the image of D



$\Rightarrow \quad H \equiv (0, b')$

Applying power of 'O'

$$aa' = bb' \Rightarrow b' = \frac{aa'}{b}$$

13. (b, c)

$$(3x+4y+5)(x+2y+3)=0$$

$$m_1 = \frac{-3}{4}, m_2 = -\frac{1}{2} \Rightarrow m_1 m_2 = \frac{3}{8}$$

$$P \equiv (1,-2)$$

$$\tan\theta = \frac{-\frac{1}{2}+\frac{3}{4}}{1+\frac{3}{8}} = \frac{2}{11}$$

$$\Rightarrow \sin\theta = \frac{2}{5\sqrt{5}}$$

14. (b, d)

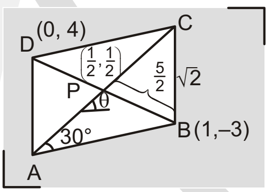
Let $C \equiv (\lambda, \lambda - 2)$

$$\frac{1}{2} \times 8 \times |\lambda - 2| = 20$$

$$\Rightarrow \lambda - 2 = \pm 5, \quad \lambda = 7, -3$$

$$C \equiv (7, 5), (-3, -5)$$

15. (a, b)



$$\tan\theta = \frac{1}{7}, \sin\theta = \frac{1}{5\sqrt{2}}, \cos = \frac{7}{5\sqrt{2}}$$

$$AP \equiv \frac{5}{2}\sqrt{2}\cot 30° = \frac{5}{2}\sqrt{2}\sqrt{3}$$

$$A \equiv \left(\frac{1}{2} - \frac{5\sqrt{2}\sqrt{3}}{2}\frac{7}{5\sqrt{2}}, \frac{1}{2} - \frac{5\sqrt{2}\sqrt{3}}{2}\frac{1}{5\sqrt{2}}\right)$$

or, $$\left(\frac{1}{2} + \frac{5\sqrt{2}\sqrt{3}}{2}\frac{7}{5\sqrt{2}}, \frac{1}{2} + \frac{5\sqrt{2}\sqrt{3}}{2}\frac{1}{5\sqrt{2}}\right)$$

$$\Rightarrow A = \left(\frac{1-7\sqrt{3}}{2}, \frac{1-\sqrt{3}}{2}\right)$$

or $$\left(\frac{1+7\sqrt{3}}{2}, \frac{1+\sqrt{3}}{2}\right)$$

16. (a, b)

L must be angle bisector of L_1 and L_2

$\therefore$ L is given by

$$\frac{3x+4y-1}{5} = \pm \frac{5x-12y+2}{13}$$

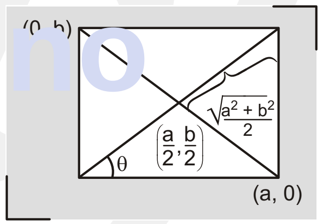
$$\Rightarrow 14x + 112y - 23 = 0,$$
$$64x - 8y - 3 = 0$$

17. (c, d)

P will be on line parallel to $y = x$ at a perpendicular distance of $\frac{1}{\sqrt{2}}$

locus of P will be $y - x = 1$ or $y - x = -1$

18. (a, c)



Other vertices are

$$\left(\frac{a}{2} + \frac{\sqrt{a^2+b^2}}{2}\frac{b}{\sqrt{a^2+b^2}}, \frac{b}{2} + \frac{\sqrt{a^2+b^2}}{2}\frac{a}{\sqrt{a^2+b^2}}\right)$$

and $$\left(\frac{a}{2} - \frac{\sqrt{a^2+b^2}}{2}\frac{b}{\sqrt{a^2+b^2}}, \frac{b}{2} - \frac{\sqrt{a^2+b^2}}{2}\frac{a}{\sqrt{a^2+b^2}}\right)$$

$$\Rightarrow \left(\frac{a+b}{2}, \frac{a+b}{2}\right) \text{ and } \left(\frac{a-b}{2}, \frac{b-a}{2}\right)$$

19. (a, b, c, d)

$$\frac{m_1}{m_2}=\frac{9}{2},\quad \frac{m_1-m_2}{1+m_1m_2}=\pm\frac{7}{9}$$

$$\Rightarrow \quad 9m_2^2-9m_2+2=0,$$

$$9m_2^2+9m_2+2=0$$

$$m_2=\frac{2}{3},\frac{1}{3}\,m_2=\frac{-2}{3},\frac{-1}{3}$$

$$m_2=\frac{2}{3},m_1=3$$

$$\Rightarrow \quad y=3x,\ 3y=2x$$

$$m_2=\frac{1}{3},m_1=\frac{3}{2}$$

$$\Rightarrow \quad 3y=x,\ 2y=3x$$

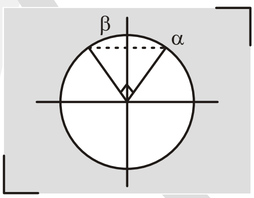
$$m_2=\frac{-2}{3},m_1=-3$$

$$\Rightarrow 3x+y=0,2x+3y=0$$

$$m_2=-\frac{1}{3},m_1=-\frac{3}{2}$$

$$\Rightarrow \quad x+3y=0,\ 3x+2y=0$$

20. (a, b, c, d)



$$\alpha-\beta=\pm\frac{\pi}{2},\pm\frac{3\pi}{2}$$

$$\frac{\alpha-\beta}{2}=\pm\frac{\pi}{4},\pm\frac{3\pi}{4}$$

$$\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}$$

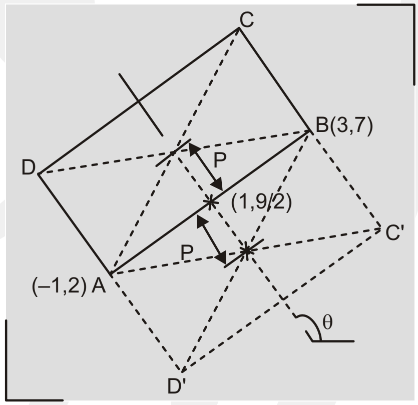
$$\sin\left(\frac{\alpha-\beta}{2}\right)=\pm\frac{1}{\sqrt{2}}$$

21. (b, d)

$$BC=\frac{3}{4}AB=\frac{3}{4}\sqrt{41}$$

$$P=\frac{1}{2}BC=\frac{3}{8}\sqrt{41}$$

$$\tan\theta=-\frac{4}{5}$$



Intersection point of diagonals can be

$$\left(1+\frac{3}{8}\sqrt{41}\left(-\frac{5}{\sqrt{41}}\right),\frac{9}{2}+\frac{3}{8}\sqrt{41}\left(\frac{4}{\sqrt{41}}\right)\right),$$

or $\left(1-\frac{3}{8}\sqrt{41}\left(-\frac{5}{\sqrt{41}}\right),\frac{9}{2}-\frac{3}{8}\sqrt{41}\left(\frac{4}{\sqrt{41}}\right)\right)$

i.e. $\left(-\frac{7}{8},6\right)$ or $\left(\frac{23}{8},3\right)$

$\therefore \quad d=\sqrt{\frac{49}{64}+36}$ or $\sqrt{\frac{529}{64}+9}$

Hence, $[d]=6$ and 4

22. (a, d)

$$\sin\theta=\frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{6}}{3}}=\frac{1}{2}\Rightarrow\theta=\frac{\pi}{6}$$

$\therefore$ Angle made by L with positive x-axis can be

$$= \frac{\pi}{4} - \frac{\pi}{6} \text{ and } \frac{\pi}{4} + \frac{\pi}{6}$$

$$= \frac{\pi}{12} \text{ and } \frac{5\pi}{12}$$

23. (a, d)

$$D = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$, we get,

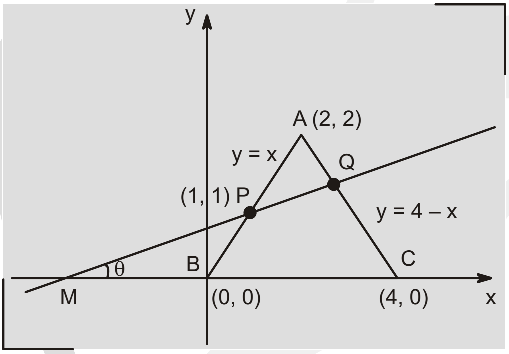
$$D = \begin{vmatrix} x_1 + y_1 & y_1 & 1 \\ x_2 + y_2 & y_2 & 1 \\ x_3 + y_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} -8 & y_1 & 1 \\ -8 & y_2 & 1 \\ -8 & y_3 & 1 \end{vmatrix} = 0$$

SOLUTIONS 3

Comprehension: (1)

Equation of line PM:

$y - 1 = \tan\theta(x - 1)$



Intersection point 'Q' of AC and MP

$4 - x - 1 = \tan\theta(x - 1)$

$$\Rightarrow \quad Q \equiv \left(\frac{3 + \tan\theta}{1 + \tan\theta}, \frac{1 + 3\tan\theta}{1 + \tan\theta}\right)$$

Area of ΔAPQ = modulus of

$$\frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ \dfrac{3 + \tan\theta}{1 + \tan\theta} & \dfrac{1 + 3\tan\theta}{1 + \tan\theta} & 1 \end{vmatrix} = \frac{1 - \tan\theta}{1 + \tan\theta}$$

1. (d) Area of quadrilateral $BPQC$,

$$A = \frac{1}{2} \times 4 \times 2 - \left(\frac{1 - \tan\theta}{1 + \tan\theta}\right)$$

$$= \frac{3 + 5\tan\theta}{1 + \tan\theta}$$

2. (d) $A = 5 - \dfrac{2}{1 + \tan\theta}$

$$\theta \in \left(0, \frac{\pi}{4}\right) \quad \Rightarrow 1 + \tan\theta \in (1, 2)$$

$\Rightarrow \quad A \in (3, 4)$

3. (c)

$$(PQ)^2 = \left(\frac{3 + \tan\theta}{1 + \tan\theta} - 1\right)^2 + \left(\frac{1 + 3\tan\theta}{1 + \tan\theta} - 1\right)^2$$

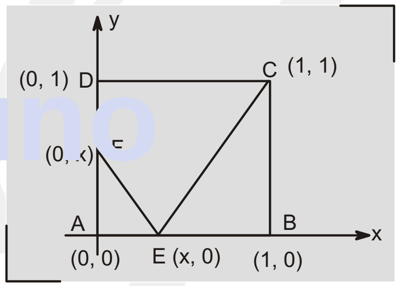
$$= \frac{4}{(\sin\theta + \cos\theta)^2}$$

$$= \frac{4}{1 + \sin 2\theta}, \qquad \theta \in \left(0, \frac{\pi}{4}\right)$$

$\Rightarrow \quad (PQ)^2 \in (2, 4) \Rightarrow PQ \in (\sqrt{2}, 2)$

Comprehension: (2)

1. (d)



Area of $CDFE$,

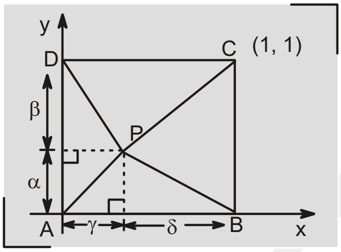
$$A(x) = 1 - \frac{1}{2}x^2 - \frac{1}{2}(1 - x)$$

$$= \frac{1 + x - x^2}{2}$$

$$A_{\max} = A\left(\frac{1}{2}\right) = \frac{1 + \frac{1}{2} - \frac{1}{4}}{2} = \frac{5}{8}$$

at $x = \dfrac{1}{2}$

2. (d)

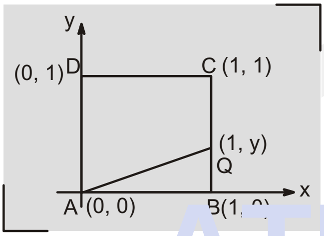


$$(PA)^2 - (PB)^2 + (PC)^2 - (PD)^2$$

$$= (\alpha^2 + \gamma^2) - (\alpha^2 + \delta^2) + (\beta^2 + \delta^2) - (\gamma^2 + \beta^2)$$

$$= 0$$

3. (d)



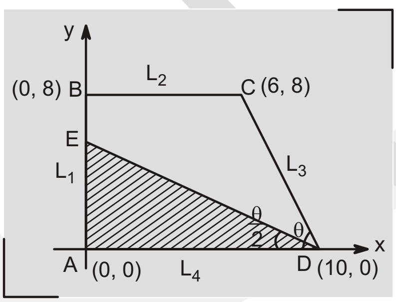
$$\frac{1}{2} y(1) = \frac{1}{3}(1)$$

$$\Rightarrow \quad y = \frac{2}{3}$$

$$\therefore \quad L_{AQ} = \sqrt{(1)^2 + \left(\frac{2}{3}\right)^2} = \frac{\sqrt{13}}{3}$$

Comprehension: (3)

1. (d)



$$\tan\theta = \frac{2\tan\frac{\theta}{2}}{1 - \tan^2\frac{\theta}{2}} = 2$$

$$\Rightarrow \quad \tan\frac{\theta}{2} = \frac{\sqrt{5} - 1}{2}$$

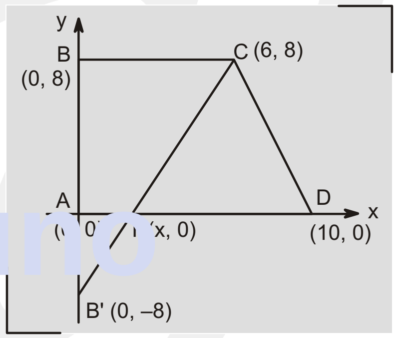
$$AE = 10\tan\frac{\theta}{2} = 5(\sqrt{5} - 1)$$

$\therefore$ Area of region required = shaded area

$$= \frac{1}{2} \times 10 \times 5(\sqrt{5} - 1)$$

$$= 25(\sqrt{5} - 1)$$

2. (b)



$$(\text{Perimeter of } \Delta RBC)_{min}$$

$$= (RB + RC)_{min} + BC$$

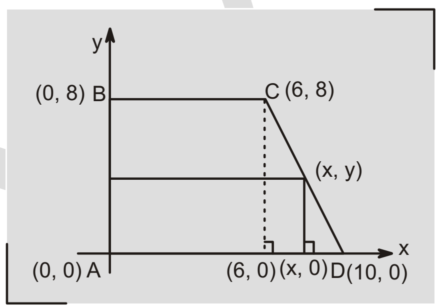
$$= (RB' + RC)_{min} + BC$$

$$= B'C + BC$$

$$\Rightarrow \quad \frac{8}{6 - x} = \frac{16}{6} \Rightarrow x = 3$$

$$\therefore \quad R \equiv (3, 0)$$

3. (d)



$$\frac{y}{8}=\frac{10-x}{10-6}\Rightarrow y=2(10-x)$$

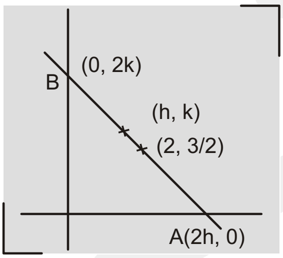
Area of rectangle, $A(x)=2x(10-x)$

$$x\in[6,10]$$

$$(A)_{\max}=A(6)=2\times 6\times(10-6)=48$$

Comprehension: (4)

1. (a) Intersection point of lines is $\left(2,\frac{3}{2}\right)$,



Equation, of AB is

$$\frac{x}{2h}+\frac{y}{2k}=1$$

put $\left(2,\frac{3}{2}\right)\Rightarrow\frac{1}{h}+\frac{3}{4k}=1$

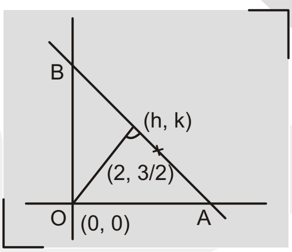
$\Rightarrow\quad 4k+3h=4hk$

$\Rightarrow\quad 3x+4y=4xy$

2. (b) Equation of AB is

$hx+ky=h^2+k^2$

Put $\left(2,\frac{3}{2}\right)$



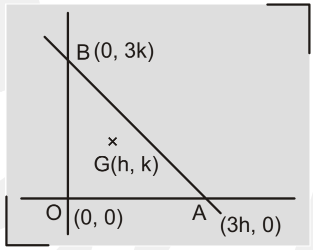
$\Rightarrow\quad 2h+\frac{3k}{2}=h^2+k^2$

$\Rightarrow\quad 2(x^2+y^2)-4x-3y=0$

3. (c) Equation of AB is

$$\frac{x}{3h}+\frac{y}{3k}=1$$

put $\left(2,\frac{3}{2}\right)$



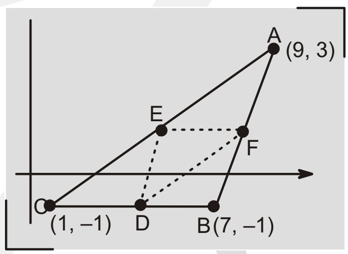
$\Rightarrow\quad \frac{2}{3h}+\frac{1}{2k}=1$

$4k+3h=6hk$

$\Rightarrow\quad 3x+4y-6xy=0$

Comprehension: (5)

1. (d) Area of $\Delta DEF=\frac{1}{4}$ area of ABC



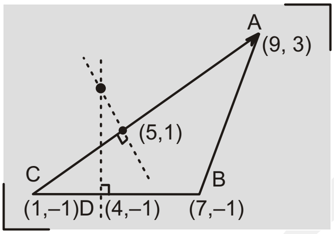
$$=\frac{1}{4}\times\frac{1}{2}\times 6\times 4=3$$

2. (b) Equation of perpendicular bisector of AC is

$$y-1=-2(x-5)$$

$\Rightarrow\quad y+2x=11$

perpendicular bisector of BC is



$$x = 4$$

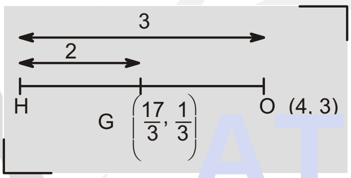
$\therefore$ Circumcentre $\equiv (4,3)$

$\Rightarrow \quad R = \sqrt{(4-1)^2 + (3+1)^2} = 5$

$\Rightarrow \quad a + b + R = 4 + 3 + 5 = 12$

3. (c) $O \equiv$ circumcentre, $G \equiv$ centroid

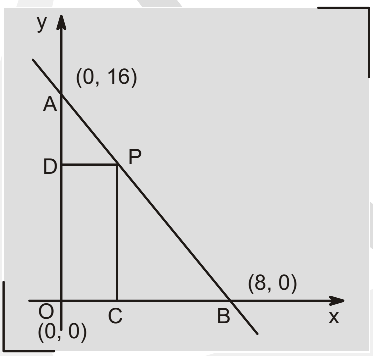
$$\Rightarrow \quad H \equiv \left(3\left(\frac{17}{3}\right) - 2(4), 3\left(\frac{1}{3}\right) - 2(3)\right)$$



$$H \equiv (9, -5)$$

Comprehension: (6)

1. (c) Equation of AB is

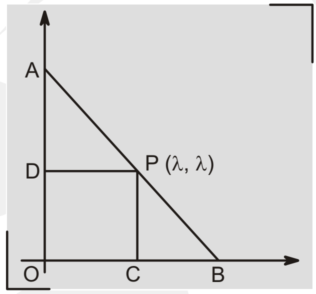


$$\frac{x}{8} + \frac{y}{16} = 1 \quad \Rightarrow 2x + y = 16$$

perpendicular distance of (2, 2) from AB

$$= \frac{|4 + 2 - 16|}{\sqrt{5}} = \frac{10}{\sqrt{5}} = 2\sqrt{5} = \sqrt{20}$$

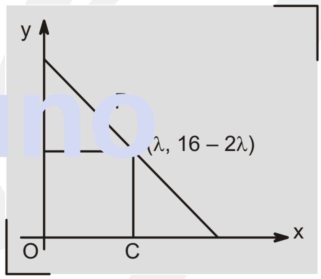
2. (a) Put (λ, λ) to eqn. of AB



$$\Rightarrow \quad 2\lambda + \lambda = 16 \quad \Rightarrow \lambda = \frac{16}{3}$$

Sum of coordinates of $P = 2\lambda = \dfrac{32}{3}$

3. (b)



Area of rectangle $PDOC$

$= \lambda(16 - 2\lambda) = 30$

$\Rightarrow \quad \lambda^2 - 8\lambda + 15 = 0$

$(\lambda - 3)(\lambda - 5) = 0$

$\Rightarrow \quad \lambda = 3, 5 \Rightarrow P \equiv (3, 10), (5, 6)$

Comprehension: (7)

$$a + b + \sqrt{a^2 + b^2} = \frac{1}{2}ab$$

$$(2a + 2b - ab)^2 = 4(a^2 + b^2)$$

$\Rightarrow \quad 8ab + a^2b^2 - 4ab(a + b) = 0$

$\Rightarrow \quad ab - 4a - 4b + 8 = 0$

$\Rightarrow \quad (a - 4)(b - 4) = 8 = 2 \times 4, \; 1 \times 8$

$\Rightarrow \quad (a, b) = (6, 8)$ or $(5, 12)$

1. (b) No. of ordered pairs $(a, b) = 2$

2. (d) For maximum perimeter,

$a = 5,\ b = 12 \Rightarrow$ Maximum perimeter

$= 5 + 12 + 13 = 30$

3. (a) For minimum area, $a = 6,\ b = 8$

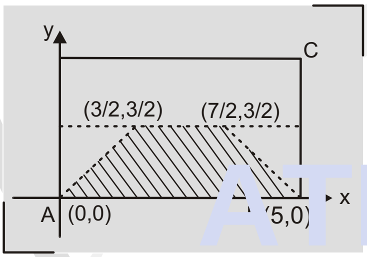
Minimum area $= \frac{1}{2} \times 6 \times 8 = 24$

Comprehension: (8)

1. (c) $d(P, AB) \le d(P, BC)$

$d(P, AB) \le d(P, CD)$

$d(P, AB) \le d(P, AD)$



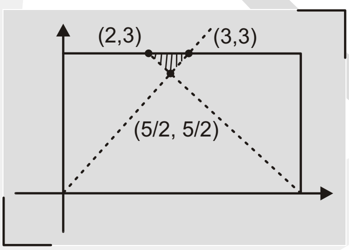
$\Rightarrow$ P lies in region as shown

Area $= \frac{1}{2} \times \frac{3}{2} \times (5 + 2) = \frac{21}{4}$

2. (d) $d(P, AB) \ge d(P, BC)$

$d(P, AB) \ge d(P, CD)$

$d(P, AB) \ge d(P, AD)$

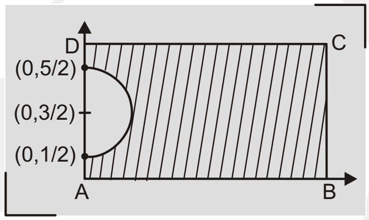


$\Rightarrow$ P lies in region as shown

Area $= \frac{1}{2} \times 1 \times \frac{1}{2} = \frac{1}{4}$

3. (d) $x^2 + \left(y - \frac{3}{2}\right)^2 \ge 1$

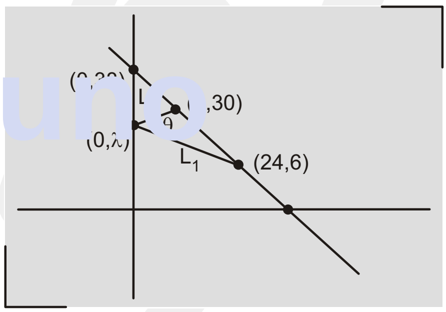
P lies in region as shown



Area $= 5 \times 3 - \frac{\pi(1)^2}{2} = 15 - \frac{\pi}{2}$

Comprehension: (10)

$$m_{L_1} = \frac{\lambda - 6}{-24}, m_{L_2} = \frac{\lambda - 30}{-6}$$



$$\tan\theta = \frac{m_{L_2} - m_{L_1}}{1 + m_{L_2} m_{L_1}} = \frac{\frac{30-\lambda}{6} - \frac{6-\lambda}{24}}{1 + \left(\frac{30-\lambda}{6}\right)\left(\frac{6-\lambda}{24}\right)}$$

$$= \frac{(38-\lambda)\,18}{(\lambda-18)^2}$$

$$\frac{d}{d\lambda}(\tan\theta) = \frac{18(\lambda-18)(\lambda-58)}{(\lambda-18)^4}$$

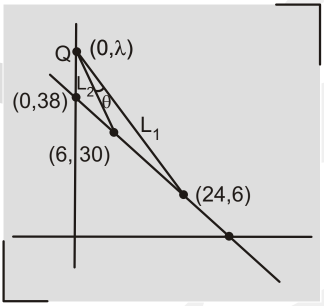
1. (c) $\tan\theta$ is maximum at $\lambda = 18$.

2. (b) $(\tan\theta)_{\max} \to \infty \Rightarrow \theta = \frac{\pi}{2}$

3. (b) $\tan\theta = \dfrac{m_{L_1} - m_{L_2}}{1 + m_{L_1} m_{L_2}} = \dfrac{(\lambda - 38)18}{(\lambda - 18)^2}$

$$\frac{d}{d\lambda}(\tan\theta) = \frac{18(18-\lambda)(\lambda-58)}{(\lambda-18)^4}$$

$\tan\theta$ is maximum at $\lambda = 58$



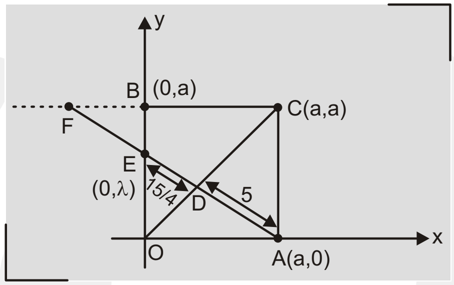
Comprehension: (11)

$\Delta ODE \sim \Delta CDA$

$\Rightarrow \dfrac{\lambda}{a} = \dfrac{15/4}{5} \Rightarrow \lambda = \dfrac{3}{4}a$

$\Rightarrow E \equiv \left(0, \dfrac{3}{4}a\right)$

Similarly $\Delta BFE \sim \Delta CFA$,



$\Rightarrow \dfrac{BF}{CF} = \dfrac{BE}{AC} = \dfrac{a/4}{a}$

$\Rightarrow BF = \dfrac{1}{4}(a + BF)$ or $BF = \dfrac{a}{3}$

$\Rightarrow F \equiv \left(-\dfrac{a}{3}, a\right)$

$$AE = \sqrt{a^2 + \left(\frac{3}{4}a\right)^2} = \frac{5}{4}a$$

$$= 5 + \frac{15}{4} = \frac{35}{4} \Rightarrow a = 7$$

1. (c) Area of square $= a^2 = 49$

2. (b) $F \equiv \left(-\dfrac{a}{3}, a\right) \equiv \left(-\dfrac{7}{3}, 7\right)$

3. (c) Circle circumscribing $\Delta AO'E$ has AE as diameter.

$\therefore$ Equation of circle is

$$x(x-7) + y\left(y - \frac{3}{4}(7)\right) = 0$$

$$\Rightarrow \quad 4(x^2 + y^2 - 7x) - 21y = 0$$

SOLUTIONS 4

Assertion and Reason

1. (A) P and Q are harmonic conjugates,

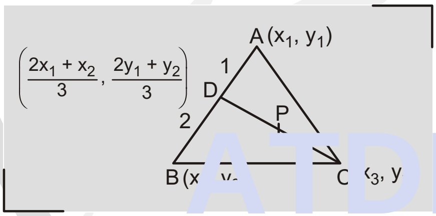
$\Rightarrow$ AP, AB, AQ are in H.P.

$$\Rightarrow \frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$$

$$\Rightarrow \frac{2}{2(BM)} = \frac{1}{BM + MP} + \frac{1}{BM + MQ}$$

$$\Rightarrow (BM)^2 = (PM)(QM)$$

2. (A)



$$P \equiv \left(\frac{\frac{2x_1 + x_2}{3} + x_3}{2}, \frac{\frac{2y_1 + y_2}{3} + y_3}{2} \right)$$

$$\equiv \left(\frac{2x_1 + x_2 + 3x_3}{6}, \frac{2y_1 + y_2 + 3y_3}{6} \right).$$

$\therefore$ P lies in side the ΔABC

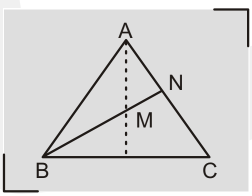
$\Rightarrow$ Area of $\Delta PBC <$ area of ΔABC

3. (C) If $P < AB$, 2 lines exist

$P = AB$, 1 line exist

$P > AB$, no lines exist.

4. (A)



$AC + BC = AN + NC + BC > AN + BN$

$= AN + BM + MN > AM + BM$

$\Rightarrow$ $AM + BM < AC + BC$ ...(1)

Similarly $BM + CM < AB + AC$ ...(2)

and $CM + AM < AB + BC$ ...(3)

Adding (1), (2) and (3)

$\Rightarrow$ $AM + BM + CM < AB + BC + CA = P$

5. (C) Let lines be

$$x + 2y + 3 = 0$$
$$x + 2y + 4 = 0$$
$$x + 2y + 5 = 0$$

$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 2 & 5 \end{vmatrix} = 0$, while lines are not concurrent.

6. (A) $\frac{y}{x} = m$, $bm^2 + 2hm + a = 0$

$\Rightarrow$ $a + b + 2h = 0$ $\Rightarrow$ $m = 1$ is a root.

If $by^2 + 2hxy + ax^2 = [ax + y(2h + a)]$
$(x - y)$

equating from both sides coefficient of y^2

$\Rightarrow$ $b = -(2h + a)$

7. (A) $3a + 2b + 4c = 0$ $\Rightarrow$ $\frac{3a}{4} + \frac{b}{2} + C = 0$

$\Rightarrow$ $ax + by + c = 0$ passes through fixed point $\left(\frac{3}{4}, \frac{1}{2}\right)$

Let $pa + qb + rc = 0$

$\Rightarrow$ $\frac{p}{r}a + \frac{q}{r}b + c = 0$

$\Rightarrow$ line $ax + by + c = 0$ passes through fixed point $\left(\frac{p}{r}, \frac{q}{r}\right)$

8. (A) Shift origin to (20, 22)

9. (C) $2xy + 3x - 4y - 6 = 0$

$(x - 2)(2y + 3) = 0$

10. (A) $[2(\cos\theta_1 \cos\theta_2 + \cos\theta_2 \cos\theta_3 + \cos\theta_3 \cos\theta_1) + \cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3]$

$+[2(\sin\theta_1 \sin\theta_2 + \sin\theta_2 \sin\theta_3 + \sin\theta_3 \sin\theta_1) + \sin^2\theta_1 + \sin^2\theta_2 + \sin^2\theta_3] = 0$

$\Rightarrow (\cos\theta_1 + \cos\theta_2 + \cos\theta_3)^2 + (\sin\theta_1 + \sin\theta_2 + \sin\theta_3)^2 = 0$

$\Rightarrow \cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

and $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 0$

$\Rightarrow$ Centroid and circumcentre of ΔABC are at origin.

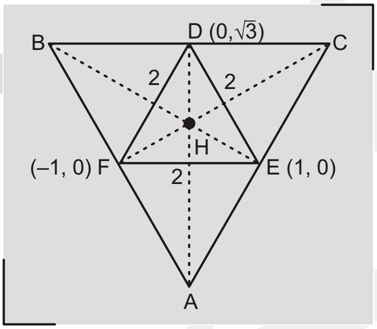
$\Rightarrow$ ΔABC is equilateral

$\therefore$ Orthocentre of ΔABC is also origin.

SOLUTIONS 5

Match the Columns:

1. a-p, b-s, c-q, d-r



Point A, B, C are excentres w.r.t. to ΔDEF,

$$\therefore \quad A \equiv \left(\frac{-2(0) + 2(1) + 2(-1)}{-2 + 2 + 2}, \frac{-2(\sqrt{3}) + 2(0) + 2(0)}{-2 + 2 + 2} \right)$$

$$B \equiv \left(\frac{-2(1) + 2(-1) + 2(0)}{-2 + 2 + 2}, \frac{-2(0) + 2(0) + 2(\sqrt{3})}{-2 + 2 + 2} \right) \equiv (-2, \sqrt{3})$$

$$C \equiv \left(\frac{-2(-1) + 2(1) + 2(0)}{-2 + 2 + 2}, \frac{-2(0) + 2(0) + 2(\sqrt{3})}{-2 + 2 + 2} \right) \equiv (2, \sqrt{3})$$

Orthocentre H of ΔABC is the incentre of ΔDEF

$$\Rightarrow \quad H \equiv \left(0, \frac{1}{\sqrt{3}}\right)$$

(a) In radius of ΔABC,

$$r_{ABC} = \frac{\Delta}{S} = \frac{\frac{\sqrt{3}}{4}(4)^2}{\left(\frac{4+4+4}{2}\right)}$$

$$= \frac{\sqrt{3}(4)}{6} = \frac{2}{\sqrt{3}}$$

In radius of ΔDEF,

$$r_{DEF} = \frac{\frac{\sqrt{3}}{4}(2)^2}{\left(\frac{2+2+2}{2}\right)} = \frac{1}{\sqrt{3}}.$$

$$\frac{r_{ABC}}{r_{DEF}} = 2$$

(b) $(AH)^2 = \left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)^2 = \frac{16}{3}$

$\Rightarrow \quad [(AH)^2] = 5$

(c) $(y_A + y_B + y_C)^2 = (\sqrt{3})^2 = 3$

(d) $\quad AB = 4$

2. a-r, b-s, c-p, d-q

(a) $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

$= -bc - 2a(c-b) + a(4c - 3b)$

$= -bc + 2ac - ab$

$\Rightarrow \quad \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$

(b) $\begin{vmatrix} 2a & 3a & 1 \\ 3b & 2b & 1 \\ c & c & 1 \end{vmatrix} = 0 = bc + ac - 5ab = 0$

(c) $\begin{vmatrix} a & 2 & 1 \\ b & 3 & 1 \\ c & 4 & 1 \end{vmatrix} = 0$

$= 4b - 3c - 4a + 2c + 3a - 2b$

$= 2b - a - c \Rightarrow \quad a + c = 2b$

(d) $\begin{vmatrix} a & a & c \\ c & c & b \\ 1 & 0 & 1 \end{vmatrix} = 0 = ab - c^2 + 0$

$\Rightarrow \quad c^2 = ab$

3. a-r, b-q, c-s, d-p

(a) $a + c = 2b \quad \Rightarrow \quad a + c - 2b = 0$

$\Rightarrow \quad ax + by + c = 0$ passes through fixed point (1, –2).

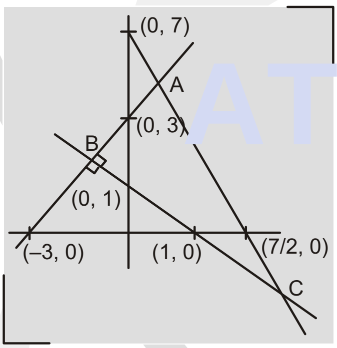
(b) Perpendicular distance of $P(\lambda, 4 - \lambda)$ from $4x + 3y = 10$

$= \dfrac{|4\lambda + 3(4 - \lambda) - 10|}{5} = 1$

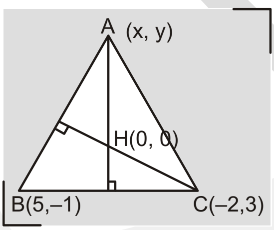
$\Rightarrow |\lambda + 2| = 5 \quad \Rightarrow \quad \lambda = -2 \pm 5 = 3, -7$

$P \equiv (3,\ 1), (-7, +11)$

(c) Point B $\equiv (-1, 2)$



(d) $\dfrac{y}{x} = \dfrac{7}{4}$



$\dfrac{y + 1}{x - 5} = \dfrac{2}{3}$

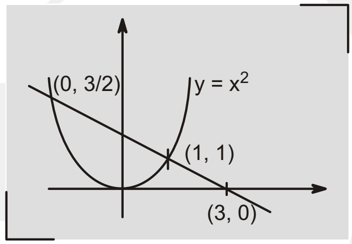
$\dfrac{21}{4} x = 2x - 13$

$x = -4, \quad y = -7$

4. a-p, b-r, c-q, d-s

(a) $2a^2 + a - 3 = 0$

$(2a + 3)(a - 1) = 0$

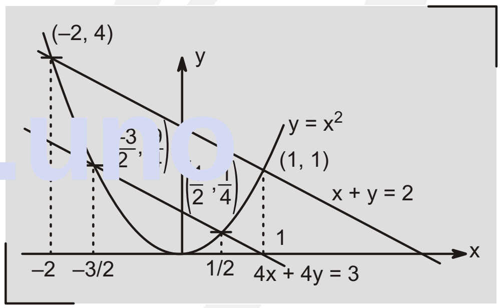


$\Rightarrow \quad a \in (0, 1)$

No. of integral values of $a = 0$

(b) $a^2 + a - 2 = 0$

$\Rightarrow \quad (a + 2)(a - 1) = 0 \quad \Rightarrow \quad a = -2, 1$



$4a^2 + 4a - 3 = 0$

$(2a - 1)(2a + 3) = 0$

$\Rightarrow \quad a = \dfrac{1}{2}, -\dfrac{3}{2}$

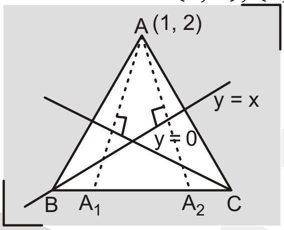
$\Rightarrow \quad a \in \left(-2, \dfrac{3}{-2}\right) \cup \left(\dfrac{1}{2}, 1\right)$

values of a of form $\dfrac{K}{3}$ are $\dfrac{-5}{3}, \dfrac{2}{3}$

(c) Slope of line joining $(t - 1, 2t + 2)$ and $(2t + 1, t)$ is $\dfrac{2t + 2 - t}{t - 1 - 2t - 1} = -1$

$\therefore$ slope of perpendicular bisectors of points is 1.

(d) Images of A w.r.t. $y=x$ and $y=0$ lies on BC which are (2, 1), (1, –2)



$\therefore$ Equation of BC is $y=3x-5$

Perpendicular distance of A from

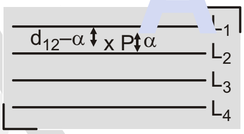
$$BC=\frac{|3-2-5|}{\sqrt{10}}$$

$$d(A,BC)=\frac{4}{\sqrt{10}}$$

$\Rightarrow$ $\sqrt{10}\,d(A,BC)=4$

5. a-q, b-r, c-s, d-p

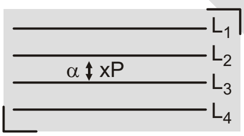
If P lies between L_1 and L_2, then



$$K=(d_{12}-\alpha)+\alpha+(d_{23}+\alpha)$$
$$+(d_{34}+d_{23}+\alpha)$$
$$=d_{12}+2d_{23}+d_{34}+2\alpha$$

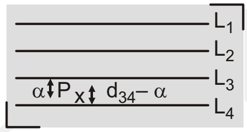
$\alpha\in(0,d_{12})$

If P lies between L_2 and L_3, then



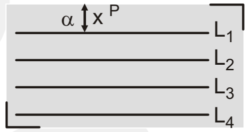
$$K=d_{12}+(d_{23}-\alpha)+(\alpha)+(d_{34}+\alpha)$$
$$+(d_{23}-\alpha)$$
$$=d_{12}+2d_{23}+d_{34}$$

If P lies between L_3 and L_4, then



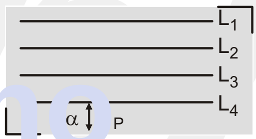
$$K=d_{12}+d_{23}+\alpha+d_{23}+\alpha+\alpha+d_{34}-\alpha$$
$$=d_{12}+2d_{23}+d_{34}+2\alpha \quad \alpha\in(0,d_{34})$$

If P lies above L_1



$$K=\alpha+(d_{12}+\alpha)+(d_{23}+d_{12}+\alpha)+$$
$$(d_{34}+d_{23}+d_{12}+\alpha)$$
$$K=2d_{12}+2d_{23}+d_{34}+4\alpha \quad , \quad \alpha>0$$

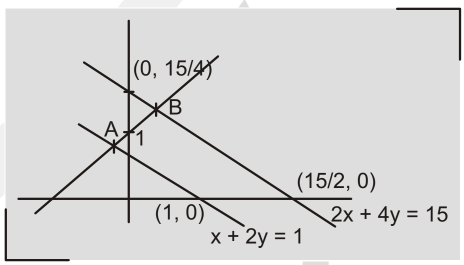
If P lies below L_4



$$K=(d_{12}+d_{23}+d_{34}+\alpha)+$$
$$(d_{23}+d_{34}+\alpha)+(d_{34}+\alpha)+\alpha$$
$$K=d_{12}+2d_{23}+2d_{34}+4\alpha \quad , \quad \alpha>0$$

6. a-r, b-p, c-s, d-q

(a) $y=x+1$



$$A\equiv\left(\frac{-1}{3},\frac{2}{3}\right),\ B\equiv\left(\frac{11}{6},\frac{17}{6}\right)$$

$$1+\frac{t}{\sqrt{2}}\in\left(\frac{-1}{3},\frac{11}{6}\right)$$

$$t \in \left(\frac{-4\sqrt{2}}{3}, \frac{5\sqrt{2}}{6} \right)$$

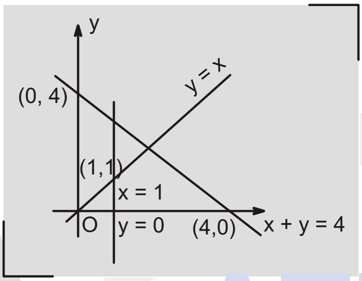
(b) $P((2-t)x_1 + (t-1)x_2, (2-t)y_1 + (t-1)y_2)$

divides (x_1, y_1), (x_2, y_2) internally in ratio $(t-1) : (2-t)$

$\therefore \ (t-1)(2-t) > 0 \quad \Rightarrow \quad t \in (1,2)$

(c) $t \in (0,1)$

(see from figure)

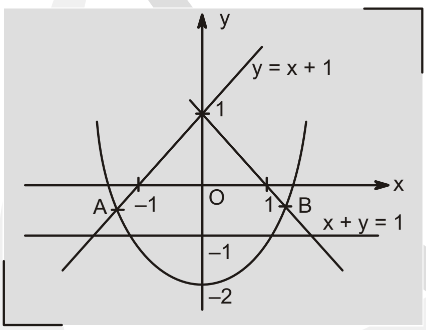


(d) for A:

$$x^2 - 2 = x + 1$$

$$x^2 - x - 3 = 0$$

$$x = \frac{1 \pm \sqrt{13}}{2}$$

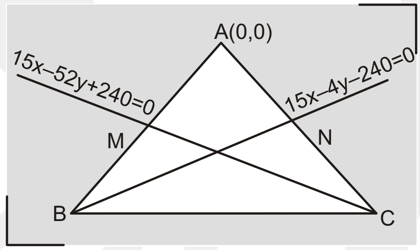


$$x = \frac{1-\sqrt{13}}{2}$$

similarly for B, $x = \dfrac{\sqrt{13}-1}{2}$

$$\therefore \ t \in \left(\frac{1-\sqrt{13}}{2}, -1 \right) \cup \left(1, \frac{\sqrt{13}-1}{2} \right)$$

7.



Let $B\left(x_1, \dfrac{15x_1 - 240}{4} \right)$

and $C\left(x_2, \dfrac{15x_2 + 240}{52} \right)$

Midpoints of AB and AC are $M\left(\dfrac{x_1}{2}, \dfrac{15x_1 - 240}{8} \right)$

and $N\left(\dfrac{x_2}{2}, \dfrac{15x_2 + 240}{104} \right)$

lie on $15x - 25y + 240 = 0$ and $15x - 4y - 240 = 0$ respectively

$\therefore \quad x_1 = 20$ and $x_2 = 36$

$\therefore \quad B(20,15)$ and $C(36,15)$

$\therefore \quad a = BC = 16, \ b = CA = 39$

and $c = AB = 25$

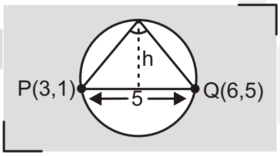
$\therefore \quad$ incentre $I(21,12)$, centroid $G\left(\dfrac{56}{3}, 10 \right)$,

excentre opposite to $C, I_3(-4,7)$ and orthocentre is $(0,63)$.

SOLUTIONS 6

Subjective Problems

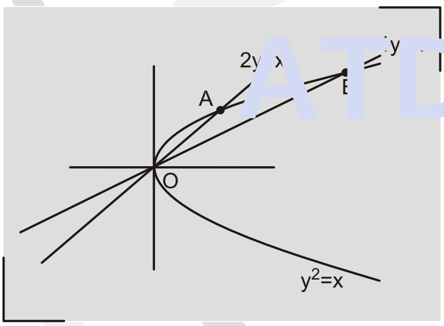
1. (0) $\Delta = \frac{1}{2} \times 5 \times h = 7 \Rightarrow h = \frac{14}{5}$



But h_{max} = Radius of circle with diameter $PQ = \frac{5}{2} < \frac{14}{5}$

$\Rightarrow$ No such triangle exists.

2. (1)



$2y = y^2$

$\Rightarrow \quad y = 0, 2 \Rightarrow A \equiv (4, 2)$

$4y = y^2$

$\Rightarrow \quad y = 0, 4 \Rightarrow B \equiv (16, 4)$

$\Rightarrow \quad a \in (2, 4)$

Number of integral values of $a = 1$

3. (3) $(a^2 + ab + 1)(1) > 0 \qquad \forall a \in R$

$\Rightarrow D < 0 \Rightarrow b^2 - 4 < 0 \qquad \Rightarrow b \in (-2, 2)$

4. (6) $6x^2 - \alpha xy - 3y^2 - 24x + 3y + \beta = 0$

$\therefore$ Intersection point of lines lies on x-axis.

Put $y = 0$,

$\Rightarrow 6x^2 - 24x + \beta = 0$ must have equal real roots

$\Rightarrow D = 0 \Rightarrow (24)^2 - 24\beta = 0 \quad \Rightarrow \beta = 24$

Apply condition of pair of lines,

i.e., $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

$$\Rightarrow 6(-3)\beta + (-\alpha)(-12)\frac{3}{2} - 6\left(\frac{3}{2}\right)^2 - (-3)(12)^2 - \beta\left(\frac{\alpha}{2}\right)^2 = 0$$

$$\Rightarrow -18(24) + 18\alpha - \frac{27}{2} + 3(144) - 6\alpha^2 = 0$$

$$\Rightarrow 6\alpha^2 - 18\alpha + \frac{27}{2} = 0$$

$$\Rightarrow 4\alpha^2 - 12\alpha + 9 = 0$$

$\alpha = 3/2 \qquad$ Hence, $\qquad 20\alpha - \beta = 6$

5. (3) Perpendicular distance of (2,3) from line $3x + 4y - 5 = 0$,

$$P_1 = \frac{|3(2) + 4(3) - 5|}{5} = \frac{13}{5}$$

$\because \quad 1 + \sin^2\theta < P_1 \ \forall \theta \in R \quad \Rightarrow \quad n_1 = 0$

Perpendicular distance of (1,3) from line $3x + 4y - 5 = 0$,

$$P_2 = \frac{|3 + 12 - 5|}{5} = 2$$

$\because \quad \sec^2\theta + 2\,\text{cosec}^2\theta \geq 3 > P_2 \quad \forall \theta \in R$

$\Rightarrow \quad n_2 = 2$

$\therefore$ Equation becomes, $2x^2 - 6x = 0$

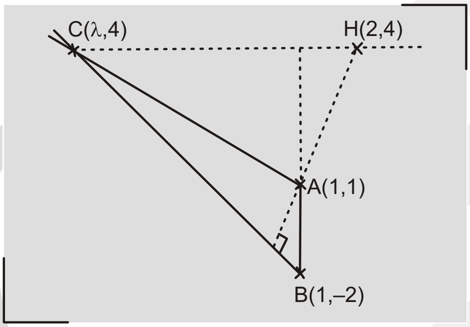
$\Rightarrow \quad x = 0, 3$

Hence, sum of roots $= 3$

6. (14) $a + c - 2b = 0$

$\Rightarrow$ $ax + by + c = 0$ passes through fixed point $(1, -2)$.

$\Rightarrow$ $B \equiv (1, -2)$



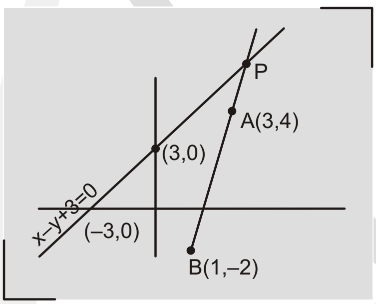
$$\text{Slope of } BC = -\frac{2-1}{4-1} = \frac{4+2}{\lambda - 1}$$

$$\lambda - 1 = -18 \Rightarrow \lambda = -17$$

$C \equiv (-17, 4) \equiv (h, k)$

$\therefore$ $2h + 12k = -34 + 48 = 14$

7. (5) $(3x + 7y + 11)\sec\theta + (5x - 3y - 11)$ $\operatorname{cosec}\theta = 0$ passes through intersection of $3x + 7y + 11 = 0$ and $5x - 3y - 11 = 0 \ \forall$ permissible values of θ given by $(1, -2)$



$\Rightarrow$ $B \equiv (1, -2)$

It is clear that $|PA - PB| \le AB$

$\Rightarrow$ $|PA - PB|_{max} = AB = \sqrt{6^2 + 2^2}$

$= 2\sqrt{10} \Rightarrow n = 5$

8. (2) By observation, directly equation must be

$(4x^2 - 4xy + y^2) + 2(2x - y) + 1 = 0$

or $4x^2 - 4xy + y^2 - 2(2x - y) + 1 = 0$

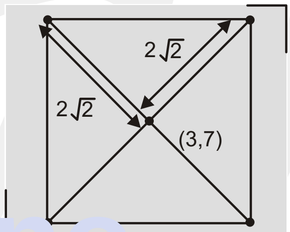
$(2x - y + 1)^2 = 0$ or $(2x - y - 1)^2 = 0$

$i.e.,$ $4x^2 - 4xy + y^2 + 4x - 2y + 1 = 0$

or $4x^2 - 4xy + y^2 - 4x + 2y + 1 = 0$

$\therefore$ $a_1 + b_1 + c_1 + a_2 + b_2 + c_2 = 1 + 4 - 2 + 1 - 4 + 2 = 2$

9. (8) Vertices of square are



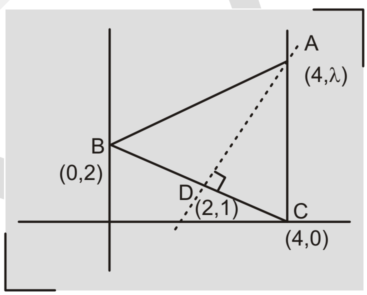
$$\left(3 + 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right), 7 \pm 2\sqrt{2}\,\frac{1}{\sqrt{2}}\right),$$

$$\left(3 + 2\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right), 7 \pm 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\right)$$

$i.e.,$ $(5,9)$, $(1,5)$, $(1,9)$, $(5,5)$

$\therefore$ $\max(y_1, y_2, y_3, y_4) - \min(x_1, x_2, x_3, x_4) = 9 - 1 = 8$

10. (8) Slope of $AD = \dfrac{\lambda - 1}{4 - 2} = 2 \Rightarrow \lambda = 5$



$\therefore$ Equation of AB is

$$y-2=\frac{5-2}{4-0}(x-0)$$

$$4y-8=3x$$

$$y=\frac{3}{4}x+2$$

$$\therefore \quad 8m+c=8\left(\frac{3}{4}+2\right)=8$$

11. (2) $a+2hm+bm^2=0 \begin{matrix} m_1 \\ m_1^2 \end{matrix}$

where $m=\frac{y}{x} \Rightarrow m_1+m_1^2=-\frac{2h}{b}; m_1^3=\frac{a}{b}$

$$\Rightarrow \quad -\frac{8h^3}{b^3}=(m_1+m_1^2)^3=m_1^3+m_1^6$$

$$+3m_1^3(m_1+m_1^2)$$

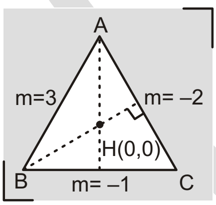
$$=\frac{a}{b}+\frac{a^2}{b^2}+\frac{3a}{b}\left(-\frac{2h}{b}\right)$$

$$\Rightarrow \quad -\frac{8h^3}{b^3}=\frac{ab+a^2-6ah}{b^2}$$

$$\Rightarrow \quad ab(a+b)-6abh+8h^3=0$$

$$\Rightarrow \quad \alpha+\beta=8-6=2$$

12. (7) Let slopes of BC, CA, AB be $-1, -2, 3$ respectively.



$\Rightarrow$ Slope of $AH=1 \Rightarrow A=(x_1, x_1)$

Slope of $BH=\frac{1}{2} \Rightarrow B=\left(x_2, \frac{1}{2}x_2\right)$

Slope of $CH=-\frac{1}{3} \Rightarrow C=\left(x_3, -\frac{x_3}{3}\right)$

$$\because \quad \text{Slope of } BC=\frac{\frac{x_2}{2}+\frac{x_3}{3}}{x_2-x_3}=\frac{3x_2+2x_3}{6(x_2-x_3)}$$

$$=-1 \Rightarrow 9x_2=4x_3$$

$$\because \quad \text{Slope of } CA=\frac{x_1+\frac{x_3}{3}}{x_1-x_3}=-2$$

$$=\frac{3x_1+x_3}{3(x_1-x_3)} \Rightarrow 9x_1=5x_3$$

$$G \equiv (h,k)=\left(\frac{x_1+x_2+x_3}{3}, \frac{x_1+\frac{x_2}{2}-\frac{x_3}{3}}{3}\right)$$

$$3h=x_3+\frac{5x_3}{9}+\frac{4x_3}{9}=2x_3 \qquad \text{....(1)}$$

$$3k=\frac{5x_3}{9}+\frac{2x_3}{9}-\frac{x_3}{3}=\frac{4x_3}{9} \qquad \text{....(2)}$$

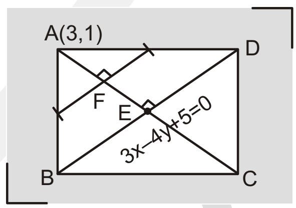
From eqs. (1) and (2), we get $\frac{k}{h}=\frac{2}{9} \Rightarrow y=\frac{2}{9}x$; $\quad b-a=9-2=7$

13. (1) Point E

$$\frac{x-3}{3}=\frac{y-1}{-4}=-\frac{9-4+5}{25}$$

$$=-\frac{2}{5} \Rightarrow E \equiv \left(\frac{9}{5}, \frac{13}{5}\right)$$

Line, L passing through midpoint of sides AB and AD passes through F.



$$F \equiv \left(\frac{3+\frac{9}{5}}{2}, \frac{\frac{13}{5}+1}{2}\right) \equiv \left(\frac{12}{5}, \frac{9}{5}\right)$$

Equation of line L is

$$3x-4y=\frac{36}{5}-\frac{36}{5}=0$$

$$\Rightarrow \quad 3x-4y=0 \Rightarrow |a+b+c|=|3-4|=1$$

14. (2) Image of $(\lambda, \lambda-1)$ w.r.t.

$3x + y - 6\lambda = 0$ is

$$\frac{x-\lambda}{3} = \frac{y-(\lambda-1)}{1}$$

$$= -2\left(\frac{3\lambda+\lambda-1-6\lambda}{9+1}\right) = \frac{2\lambda+1}{5}$$

$$(x, y) \equiv \left(\frac{11\lambda+3}{5}, \frac{7\lambda-4}{5}\right)$$

$$= (\lambda^2+1, \lambda) \Rightarrow \lambda = \frac{7\lambda-4}{5}$$

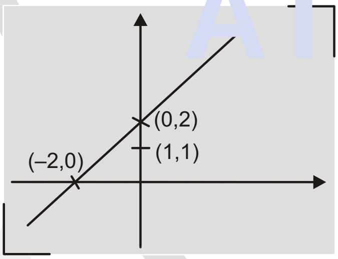
$\Rightarrow \quad \lambda = 2$

15. (3) $L_1 \equiv y = 2 + x$

$$L_2 = \frac{x}{a} + \frac{y}{b} = 1$$

Put $(1, 1)$,

$\Rightarrow \frac{1}{a} + \frac{1}{b} = 1$



$$\Delta = \frac{1}{2}ab \geq \frac{1}{2}\left[\frac{2}{\left(\frac{1}{a}+\frac{1}{b}\right)}\right]^2 = 2$$

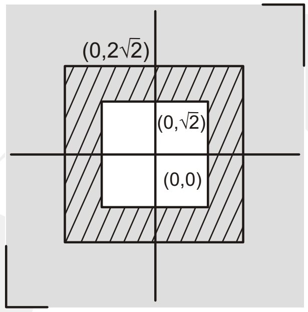
$\Delta \geq 2 \Rightarrow \Delta$ is minimum when $a = b = 2$

$\therefore \quad L_2 \equiv x + y = 2$

Area of $\Delta ABC = \frac{1}{2} \times 4 \times 2 = 4$

$\Rightarrow |p - q| = 4 - 1 = 3$

16. (6) $\frac{|x+y|}{\sqrt{2}} + \frac{|y-x|}{\sqrt{2}} \in [2, 4]$



If $\quad y \geq x$ and $y + x \geq 0$

$\Rightarrow \quad y \in [\sqrt{2}, 2\sqrt{2}]$

If $y \leq x$ and $y + x \leq 0$

$\Rightarrow \quad y \in [-2\sqrt{2}, -\sqrt{2}]$

If $y \geq x$ and $y + x \leq 0$

$\Rightarrow \quad x \in [-2\sqrt{2}, -\sqrt{2}]$

If $y \leq x$ and $y + x \geq 0$

$\Rightarrow \quad x \in [\sqrt{2}, 2\sqrt{2}]$

Area of region

$= (4\sqrt{2})^2 - (2\sqrt{2})^2 = 32 - 8 = 24$

17. (4) Area = Mod of

$$\frac{1}{2}\begin{vmatrix} \alpha & 2\alpha+3 & 1 \\ 1 & 2 & 1 \\ 2 & 3 & 1 \end{vmatrix} = \frac{1}{2}|-\alpha+2\alpha+3-1|$$

$\Rightarrow \frac{|\alpha+2|}{2} \in [2, 3)$

$\Rightarrow \alpha \in (-8, -6] \cup [2, 4)$

$\Rightarrow (\alpha, \beta) = (-7, -11), (-6, -9), (2, 7), (3, 9)$

Number of possible co-ordinates of $A = 4$

18. (0) $l + m = 0$

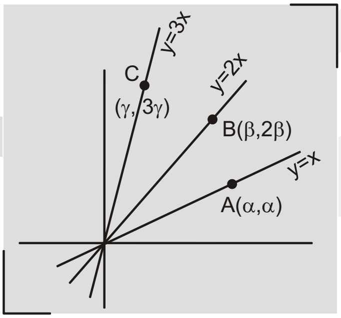
19. (3) $\frac{p}{q} = \frac{1}{2}; \qquad p + q = 3$

20. (0) Slope of $AB = \frac{\alpha}{\alpha-1} = \frac{2\beta}{\beta-1}$

$\Rightarrow \qquad \alpha = \frac{2\beta}{\beta+1}$

Slope of $BC = \dfrac{2\beta + 1}{\beta} = \dfrac{3\gamma + 1}{\gamma}$

$$\Rightarrow \quad \gamma = \frac{\beta}{1-\beta}$$



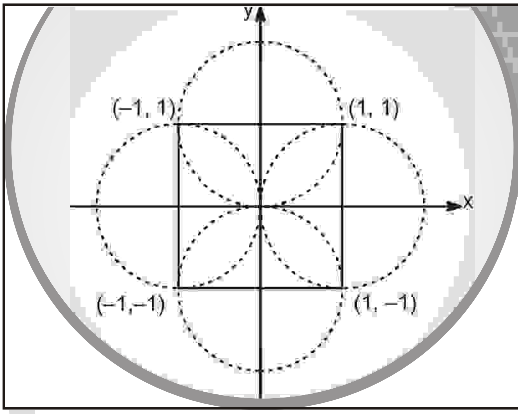
Equation of AC is

$$y - \frac{2\beta}{\beta+1} = \left(\frac{\frac{3\beta}{1-\beta} - \frac{2\beta}{\beta+1}}{\frac{\beta}{1-\beta} - \frac{2\beta}{\beta+1}}\right)\left(x - \frac{2\beta}{\beta+1}\right)$$

$$\Rightarrow \quad (x+y) - \beta(3y - 5x + 4) = 0$$

which passes through fixed point $\left(\frac{1}{2}, -\frac{1}{2}\right)$

$$\therefore \quad h + k = 0$$



2

CIRCLE

KEY CONCEPTS

1. EQUATION OF A CIRCLE IN VARIOUS FORMS

(a) The circle with centre (h,k) and radius 'r' has the equation;

$(x-h)^2+(y-k)^2=r^2$

(b) The general equation of a circle is $x^2+y^2+2gx+2fy+c=0$ with centre as:

$(-g,-f)$ and radius $=\sqrt{g^2+f^2-c}$.

Remember that every second degree equation in x and y in which coefficient of x^2 =coefficient of y^2 and there is no xy term always represents a circle.

If $g^2+f^2-c>0 \Rightarrow$ real circle.

$g^2+f^2-c=0 \Rightarrow$ point circle.

$g^2+f^2-c<0 \Rightarrow$ imaginary circle.

Note that the general equation of a circle contains three arbitrary constants g, f and c which corresponds to the fact that a unique circle passes through three non collinear points.

(c) The equation of a circle with (x_1,y_1) and (x_2,y_2) as its diameter is:

$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$.

Note that this will be the circle of least radius passing through (x_1,y_1) and (x_2,y_2).

2. INTERCEPTS MADE BY A CIRCLE ON THE AXES:

The intercepts made by the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ on the co-ordinate axes are $2\sqrt{g^2 - c}$ and $2\sqrt{f^2 - c}$ respectively.

Note: *If* $g^2 - c > 0 \Rightarrow$ *circle cuts the x-axis at two distinct points.*

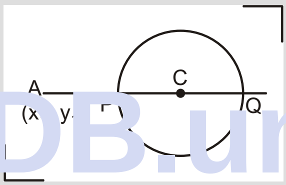
If $g^2 = c \quad \Rightarrow$ *circle touches the x-axis.*

If $g^2 < c \quad \Rightarrow$ *circle lies completely above or below the x-axis.*

3. POSITION OF A POINT w.r.t. A CIRCLE

The point (x_1, y_1) is inside, on or outside the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ according as $x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c \lesseqgtr 0$.

Note: *The greatest and the least distance of a point A from a circle with centre C and radius r is* $|AC + r|$ *and* $|AC - r|$ *respectively.*



4. LINE AND A CIRCLE

Let $L = 0$ be a line and $S = 0$ be a circle. If r is the radius of the circle and p is the length of the perpendicular from the centre on the line, then :

(i) $p > r \Leftrightarrow$ the line does not meet the circle *i.e.* passes out side the circle.

(ii) $p = r \Leftrightarrow$ the line touches the circle.

(iii) $p < r \Leftrightarrow$ the line is a secant of the circle.

(iv) $p = 0 \Rightarrow$ the line is a diameter of the circle.

5. PARAMETRIC EQUATION OF A CIRCLE

The parametric equation of $(x - h)^2 + (y - k)^2 = r^2$ are :

$x = h + r\cos\theta$; $y = k + r\sin\theta$; $-\pi < \theta \le \pi$ where (h, k) is the centre, r is the radius and θ is a parameter.

Note that equation of a straight line joining two point α and β on the circle $x^2 + y^2 = a^2$ is

$$x\cos\frac{\alpha + \beta}{2} + y\sin\frac{\alpha + \beta}{2} = a\cos\frac{\alpha - \beta}{2}.$$

6. TANGENT AND NORMAL

(a) The equation of the tangent to the circle $x^2 + y^2 = a^2$ at its point (x_1, y_1) is,

$xx_1 + yy_1 = a^2$. Hence equation of a tangent at $(a\cos\alpha, a\sin\alpha)$ is ;

$x\cos\alpha + y\sin\alpha = a$.The point of intersection of the tangents at the points $P(\alpha)$ and $Q(\beta)$ is

$$\frac{a\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, \frac{a\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}.$$

(b) The equation of the tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at its point (x_1, y_1) is $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

(c) $y = mx + c$ is always a tangent to the circle $x^2 + y^2 = a^2$ if $c^2 = a^2(1 + m^2)$ and the point of contact is $\left(-\frac{a^2m}{c}, \frac{a^2}{c}\right)$.

(d) If a line is normal / orthogonal to a circle then it must pass through the centre of the circle. Using this fact normal to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at (x_1, y_1) is $y - y_1 = \frac{y_1 + f}{x_1 + g}(x - x_1)$.

7. A FAMILY OF CIRCLES

(a) The equation of the family of circles passing through the points of intersection of two circles $s_1 = 0$ and $s_2 = 0$ is : $s_1 + ks_2 = 0\ (k \neq -1)$.

(b) The equation of the family of circles passing through the point of intersection of a circle $S = 0$ and a line $L = 0$ is given $S + kL = 0$.

(c) The equation of a family of circle passing through two given points (x_1, y_1) and (x_2, y_2) can be written in the from :

$$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) + K\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0 \text{ where } K \text{ is a parameter.}$$

(d) The equation of a family of circles touching a fixed line $y - y_1 = m(x - x_1)$ at the fixed point (x_1, y_1) is $(x - x_1)^2 + (y - y_1)^2 + K[y - y_1 - m(x - x_1)] = 0$, where K is a parameter.

In case the line through (x_1, y_1) is parallel to y-axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x - x_1)^2 + (y - y_1)^2 + K(x - x_1) = 0$

Also if line is parallel to x-axis the equation of the family of circles touching it at (x_1, y_1) becomes $(x - x_1)^2 + (y - y_1)^2 + K(y - y_1) = 0$.

(e) Equation of circle circumscribing a triangle whose sides are given by $L_1 = 0$; $L_2 = 0$ and $L_3 = 0$ is given by ; $L_1L_2 + \lambda L_2L_3 + \mu L_3L_1 = 0$ provided coefficient of $xy = 0$ and coefficient of $x^2 =$ coefficient of y^2.

(f) Equation of circle circumscribing a quadrilateral whose side in order are represented by the lines $L_1 = 0, L_2 = 0, L_3 = 0$ and $L_4 = 0$ is $L_1L_3 + \lambda L_2L_4 = 0$ provided coefficient of x^2 = coefficient of y^2 and coefficient of $xy = 0$.

8. LENGTH OF A TANGENT AND POWER OF A POINT

The length of a tangent from an external point (x_1, y_1) to the circle
$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ is given by $L = \sqrt{x_1^{\,2} + y_1^{\,2} + 2gx_1 + 2f_1y + c} = \sqrt{S_1}$.

Square of length of the tangent from the point P is also called **THE POWER OF POINT** w.r.t. a circle. Power of a point remains constant w.r.t. a circle.

Note that power of a point P is positive, negative or zero according as the point 'P' is outside, inside or on the circle respectively.

9. DIRECTOR CIRCLE:

The locus of the point of intersection of two perpendicular tangent is called the **DIRECTOR CIRCLES** of the given circle. The director circle of a circle is the concentric circle having radius equal to $\sqrt{2}$ times the original circle.

10. EQUATION OF THE CHORD WITH A GIVEN MIDDLE POINT

The equation of the chord of the circle $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ in terms of its mid point $M(x_1, y_1)$ is $y - y_1 = -\dfrac{x_1 + g}{y_1 + f}(x - x_1)$. This on simplification can be put in the form $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = x_1^{\,2} + y_1^{\,2} + 2gx_1 + 2fy_1 + c$ which is designated by $T = S_1$.

Note that the shortest chord of a circle passing through a point 'M' **inside the circle, is one chord whose middle point is M.**

11. CHORD OF CONTACT :

If two tangents PT_1 and PT_2 are drawn from $p(x_1, y_1)$ to the circle
$S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$, then the equation of the chord of contact T_1T_2 is :
$xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$.

REMEMBER:

(a) Chord of contact exists only if the point 'P' is not inside.

(b) Length of chord of contact $T_1T_2 = \dfrac{2LR}{\sqrt{R^2 + L^2}}$.

(c) Area of the triangle formed by the pair of the tangent and its chord of contact $= \dfrac{RL^3}{R^2 + L^2}$ Where R is the radius of the circle and L is the tangent from (x_1, y_1) on $S = 0$.

(d) Angle between the pair of tangents from $(x_1, y_1) = \tan^{-1}\left(\frac{2RL}{L^2 - R^2}\right)$ where R = radius ; L = length of tangent.

(e) Equation of the circle circumscribing the triangle PT_1T_2 is :
$(x - x_1)(x + g) + (y - y_1)(y + f) = 0.$

(f) The joint equation of a pair of tangents drawn from the point $A(x_1, y_1)$ to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ is : $SS_1 = T^2$.

Where $S \equiv x^2 + y^2 + 2gx + 2fy + c = 0$; $S_1 \equiv x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c$

$T \equiv xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c.$

12. POLE AND POLAR

(i) If through a point P in the plane of the circle, there be drawn any straight line to meet the circle in Q and R, the locus of the point of intersection of the tangents at Q and R *is* called the **Polar Of The Point *P*** ; also P is called the **Pole Of The Polar**.

(ii) The equation to the polar of a point $P(x_1, y_1)$ w.r.t. the circle $x^2 + y^2 = a^2$ is given by $\boldsymbol{xx_1 + yy_1 = a^2}$, and if the circle is general then the equation of the polar becomes $xx_1 + yy_1 + g(x + x_1) + f(y + y_1) + c = 0$. Note that if the point (x_1, y_1) be on the circle then the chord of contact, tangent and polar will be represented by the same equation.

(iii) Pole of a given line $Ax + By + c = 0$ w.r.t. any circle $x^2 + y^2 = a^2$ is $\left(-\frac{Aa^2}{c}, \frac{Ba^2}{c}\right)$.

(iv) If the polar of a point P pass through a point Q, then the polar of Q passes through P.

(v) Two lines L_1 and L_2 are conjugate of each other if pole of L_1 lies on L_2 and *vice versa*. Similarly two points P and Q are said to be conjugate of each other if the polar of P passes through Q and vice-versa.

13. COMMON TANGENTS TO TWO CIRCLE

(i) Where the two circle neither intersect nor touch each other , there are FOUR common tangents, two of them are transverse and the others are direct common tangents.

(ii) When they intersect there are two common tangents, both of them being direct.

(iii) When they touch each other :

(a) **EXTERNALLY:** There are three common tangents, two direct and one is the tangents at the point.

(b) **INTERNALLY:** Only one common tangents possible at their point of contact.

(iv) Length of an external common tangent and internal common tangent to the two circles is given by :$L_{ext} = \sqrt{d^2 - (r_1 - r_2)^2}$ and $L_{ext} = \sqrt{d^2 - (r_1 + r_2)^2}$.

Where d = distance between the centres of the two circles. r_1 and r_2 are the radii of the two circle.

(v) The direct common tangents meet at a point which divides the line joining centre of circles externally in the ratio of their radii.

Transverse common tangents meet at a point which divides the line joining centre of circles internally in the ratio of their radii.

14. RADICAL AXIS AND RADICAL CENTRE

The radical axis of two circles is the locus of points whose powers w.r.t. the two circles are equal. The equation of radical axis of the two circles $S_1 = 0$ and $S_2 = 0$ is given ;

$S_1 - S_2 = 0$ *i.e.*, $2(g_1 - g_2)x + 2(f_1 - f_2)y + (c_1 - c_2) = 0$.

Note That :

(a) If two circles intersect, then the radical axis is the common chord of the two circles.

(b) If two circles touch each other then the radical axis is the common tangent of the two circles at the common point of contact.

(c) Radical axis is always perpendicular to the line joining the centres of the two circles.

(d) Radical axis need not always pass through the mid point of the line joining the centres of the two circles.

(e) Radical axis bisects a common tangent between the two circles.

(f) The common point of intersection of the radical axes of there circles taken two at a time is called the radical centre of three circles

(g) A system of circles, every two which have the same radical axis is called coaxal system.

(h) Pairs of circles which do not have radical axis are concentric.

15. ORTHOGONALITY OF TWO CIRCLE

Two circles $S_1 = 0$ and $S_2 = 0$ are said to be orthogonal or said to intersect orthogonally if the tangents at their point of intersection include a right angle. The condition for two circles to be orthogonal is :

$2g_1g_2 + 2f_1f_2 = c_1 + c_2$.

Note:

(a) *Locus of the centre of a variable circle orthogonal to two fixed circles is the radical axis between the two fixed circles.*

(b) *If two circles are orthogonal, then the polar of a point 'P' on first circle w.r.t. the second circle passes through the point Q which is the other end of the diameter through P . Hence locus of a point which moves such that its polars w.r.t. the circles $S_1 = 0, S_2 = 0$ and $S_3 = 0$ are concurrent in a circle which is orthogonal to all the three circles.*

EXERCISE 1

Only One Choice is Correct:

1. Circles of radii 36 and 9 touch externally. The radius of the circle which touches the two circles externally and also their common tangent is:

(a) 4 (b) 5

(c) $\sqrt{17}$ (d) $\sqrt{18}$

2. $A(0, a)$ and $B(0, b), a, b > 0$ are two vertices of a triangle ABC where the vertex $C(x, 0)$ is variable. The value of x when $\angle ACB$ is maximum is:

(a) $\dfrac{a+b}{2}$ (b) $\sqrt{ab}$

(c) $\dfrac{2ab}{a+b}$ (d) $\dfrac{ab}{a+b}$

3. A circle passing through origin O cuts two straight lines $x - y = 0$ and $x + y = 0$ in points A and B respectively. If abscissae of A and B are roots of the equation $x^2 + ax + b = 0$, then the equation of the given circle is:

(a) $x^2 + y^2 + ax -$ [illegible] $y = 0$ (b) $x^2 + y^2 -$ [illegible] $\sqrt{4b - a^2} + yb = 0$

(c) $x^2 + y^2 + ax \pm y\sqrt{a^2 - 4b} = 0$ (d) $x^2 + y^2 - ax \pm y\sqrt{a^2 - 4b} = 0$

4. The minimum distance between the circle $x^2 + y^2 = 9$ and the curve $2x^2 + 10y^2 + 6xy = 1$ is:

(a) $2\sqrt{2}$ (b) 2

(c) $3 - \sqrt{2}$ (d) $3 - \dfrac{1}{\sqrt{11}}$

5. A pair of tangents are drawn from a point P to the circle $x^2 + y^2 = 1$. If the tangents make an intercept of 2 on the line $x = 1$, then locus of P is:

(a) straight line (b) pair of lines

(c) circle (d) parabola

6. Let $ABCD$ be a quadrilateral in which $AB || CD, AB \perp AD$ and $AB = 3CD$. The area of quadrilateral $ABCD$ is 4. The radius of a circle touching all the sides of quadrilateral is:

(a) $\sin\dfrac{\pi}{12}$ (b) $\sin\dfrac{\pi}{6}$

(c) $\sin\dfrac{\pi}{4}$ (d) $\sin\dfrac{\pi}{3}$

7. A circle with center (2, 2) touches the coordinate axes and a straight line AB where A and B lie on positive direction of coordinate axes such that the circle lies between origin and the line AB. If O be the origin then the locus of circumcenter of ΔOAB will be:

(a) $xy = x + y + \sqrt{x^2 + y^2}$ (b) $xy = x + y - \sqrt{x^2 + y^2}$

(c) $xy + x + y = \sqrt{x^2 + y^2}$ (d) $xy + x + y + \sqrt{x^2 + y^2} = 0$

8. The value of α for which the point $(\alpha, \alpha + 2)$ is an interior point of smaller segment of the curve $x^2 + y^2 - 4 = 0$ made by the chord of the curve whose equation is $3x + 4y + 12 = 0$

(a) $\left(-\infty, -\frac{20}{7}\right)$ (b) $(-2, 0)$

(c) $\left(-\infty, \frac{20}{7}\right)$ (d) $\alpha \in \phi$

9. The equation of circumcircle of an equilateral triangle is $x^2 + y^2 + 2gx + 2fy + c = 0$ and one vertex of triangle is $(1,1)$. The equation of incircle of triangle is:

(a) $4(x^2 + y^2) = g^2 + f^2$

(b) $4(x^2 + y^2) + 8gx + 8fy = (1 - g)(1 + 3g) + (1 - f)(1 + 3f)$

(c) $4(x^2 + y^2) + 8gx + 8fy = g^2 + f^2$

(d) $4(x^2 + y^2) + 8gx + 8fy = (2 - g)(1 + 3g) + (2 - f)(2 + 3f)$

10. The equation of the smallest circle passing through the points of intersection of the line $x + y = 1$ and the circle $x^2 + y^2 = 9$ is:

(a) $x^2 + y^2 + x + y - 8 = 0$

(b) $x^2 + y^2 - x - y - 8 = 0$

(c) $x^2 + y^2 - x + y - 8 = 0$

(d) $x^2 + y^2 - x - y + 8 = 0$

11. Let C be a circle $x^2 + y^2 = 1$. The line l intersects C at the point $(-1, 0)$ and the point P. Suppose that the slope of the line l is a rational number m. Number of choices for m for which both the coordinates of P are rational, is:

(a) 3 (b) 4

(c) 5 (d) infinitely many

12. The line $2x - y + 1 = 0$ is tangent to the circle at the point (2, 5) and the centre of the circles lies on $x - 2y = 4$. The radius of the circle is:

(a) $3\sqrt{5}$ (b) $5\sqrt{3}$

(c) $2\sqrt{5}$ (d) $5\sqrt{2}$

13. One circle has a radius of 5 and its center at (0, 5). A second circle has a radius of 12 and its centre at (12, 0). The length of a radius of a third circle which passes through the center of the second circle and both points of intersection of the first 2 circles, is equal to:

(a) 13/2 (b) 15/2

(c) 17/2 (d) none

14. In the xy-plane, the length of the shortest path from (0, 0) to (12, 16) that does not go inside the circle $(x-6)^2+(y-8)^2=25$ is:

(a) $10\sqrt{3}$ (b) $10\sqrt{5}$

(c) $10\sqrt{3}+\dfrac{5\pi}{3}$ (d) $10+5\pi$

15. Four unit circles pass through the origin and have their centres on the coordinate axes. The area of the quadrilateral whose vertices are the points of intersection (in pairs) of the circles, is:

(a) 1sq. unit

(b) $2\sqrt{2}$ sq. units

(c) 4 sq. units

(d) can not be uniquely determined, insufficient data

16. Consider 3 non-collinear points A, B, C with coordinates (0, 6), (5, 5) and (–1, 1) respectively. Equation of a line tangent to the circle circumscribing the triangle ABC and passing through the origin is:

(a) $2x-3y=0$ (b) $3x+2y=0$

(c) $3x-2y=0$ (d) $2x+3y=0$

17. A circle is inscribed in an equilateral triangle with side lengths 6 unit. Another circle is drawn inside the triangle (but outside the first circle) tangent to the first circle and two of the sides of the triangle. The radius of the smaller circle is:

(a) $1/\sqrt{3}$ (b) $2/3$

(c) $1/2$ (d) 1

18. A square $OABC$ is formed by line pairs $xy=0$ and $xy+1=x+y$ where 'O' is the origin. A circle with centre C_1 inside the square is drawn to touch the line pair $xy=0$ and another circle with centre C_2 and radius twice that of C_1, is drawn to touch the circle C_1 and the other line pair. The radius of the circle with centre C_1 is:

(a) $\dfrac{\sqrt{2}}{\sqrt{3}(\sqrt{2}+1)}$ (b) $\dfrac{2\sqrt{2}}{3(\sqrt{2}+1)}$

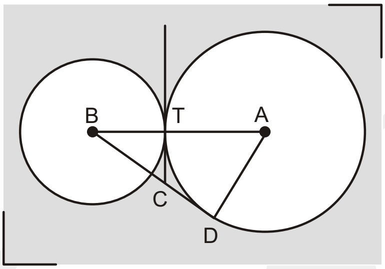
(c) $\dfrac{\sqrt{2}}{3(\sqrt{2}+1)}$ (d) $\dfrac{\sqrt{2}+1}{3\sqrt{2}}$

19. The shortest distance from the line $3x+4y=25$ to the circle $x^2+y^2=6x-8y$ is equal to:

(a) 7/5 (b) 9/5

(c) 11/5 (d) 32/5

20. Two circles with centres at A and B, touch at T. BD is the tangent at D and TC is a common tangent. AT has length 3 and BT has length 2. The length of CD is:



(a) 4/3
(b) 3/2
(c) 5/3
(d) 7/4

21. Triangle ABC is right angled at A. The circle with centre A and radius AB cuts BC and AC internally at D and E respectively. If $BD = 20$ and $DC = 16$ then the length AC equals:

(a) $6\sqrt{21}$
(b) $6\sqrt{26}$
(c) 30
(d) 32

22. From the point $A(0, 3)$ on the circle $x^2 + 4x + (y-3)^2 = 0$ a chord AB is drawn and extended to a point M such that $AM = 2AB$. The equation of the locus of M is:

(a) $x^2 + 8x + y^2 = 0$
(b) $x^2 + 8x + (y-3)^2 = 0$
(c) $(x-3)^2 + 8x + y^2 = 0$
(d) $x^2 + 8x + 8y^2 = 0$

23. If $x = 3$ is the chord of contact of the circle $x^2 + y^2 = 81$ then the equation of the corresponding pair of tangents, is:

(a) $x^2 - 8y^2 + 54x + 729 = 0$
(b) $x^2 - 8y^2 - 54x + 729 = 0$
(c) $x^2 - 8y^2 - 54x - 729 = 0$
(d) $x^2 - 8y^2 = 729$

24. The locus of the mid points of the chords of the circle $x^2 + y^2 - ax - by = 0$ which subtend a right angle at $(a/2, b/2)$ is:

(a) $ax + by = 0$
(b) $ax + by = a^2 + b^2$
(c) $x^2 + y^2 - ax - by + \dfrac{a^2 + b^2}{8} = 0$
(d) $x^2 + y^2 - ax - by - \dfrac{a^2 + b^2}{8} = 0$

25. From $(3, 4)$ chords are drawn to the circle $x^2 + y^2 - 4x = 0$. The locus of the mid points of the chords is:

(a) $x^2 + y^2 - 5x - 4y + 6 = 0$
(b) $x^2 + y^2 + 5x - 4y + 6 = 0$
(c) $x^2 + y^2 - 5x + 4y + 6 = 0$
(d) $x^2 + y^2 - 5x - 4y - 6 = 0$

26. The centre of the smallest circle touching the circles $x^2 + y^2 - 2y - 3 = 0$ and $x^2 + y^2 - 8x - 18y + 93 = 0$ is:

(a) (3, 2)
(b) (4, 4)
(c) (2, 7)
(d) (2, 5)

27. In a right triangle ABC, right angled at A, on the leg AC as diameter, a semicircle is described. The chord joining A with the point of intersection D of the hypotenuse and the semicircle, then the length AC equals to:

(a) $\dfrac{AB \cdot AD}{\sqrt{AB^2 + AD^2}}$ (b) $\dfrac{AB \cdot AD}{AB + AD}$

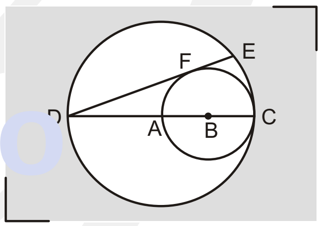
(c) $\sqrt{AB \cdot AD}$ (d) $\dfrac{AB \cdot AD}{\sqrt{AB^2 - AD^2}}$

28. If the circle $C_1 : x^2 + y^2 = 16$ intersects another circle C_2 of radius 5 in such a manner that the common chord is of maximum length and has a slope equal to 3/4, then the coordinates of the centre of C_2 are:

(a) $\left(\pm\dfrac{9}{5}, \pm\dfrac{12}{5}\right)$ (b) $\left(\pm\dfrac{9}{5}, \mp\dfrac{12}{5}\right)$

(c) $\left(\pm\dfrac{12}{5}, \pm\dfrac{9}{5}\right)$ (d) $\left(\pm\dfrac{12}{5}, \mp\dfrac{9}{5}\right)$

29. In the diagram, DC is a diameter of the large circle centered at A, and AC is a diameter of the smaller circle centered at B. If DE is tangent to the smaller circle at F and $DC = 12$ then the length of DE is:



(a) $8\sqrt{2}$ (b) 1[illegible]

(c) $9\sqrt{2}$ (d) $10\sqrt{2}$

30. Let C be the circle of radius unity centred at the origin. If two positive numbers x_1 and x_2 are such that the line passing through $(x_1, -1)$ and $(x_2, 1)$ is tangent to C then:

(a) $x_1 x_2 = 1$ (b) $x_1 x_2 = -1$

(c) $x_1 + x_2 = 1$ (d) $4x_1 x_2 = 1$

31. The distance between the chords of contact of tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ from the origin and the point (g, f) is:

(a) $\sqrt{g^2 + f^2}$ (b) $\dfrac{\sqrt{g^2 + f^2 - c}}{2}$

(c) $\dfrac{g^2 + f^2 - c}{2\sqrt{g^2 + f^2}}$ (d) $\dfrac{\sqrt{g^2 + f^2 + c}}{2\sqrt{g^2 + f^2}}$

32. The locus of the centers of the circles which cut the circles $x^2 + y^2 + 4x - 6y + 9 = 0$ and $x^2 + y^2 - 5x + 4y - 2 = 0$ orthogonally is:

(a) $9x + 10y - 7 = 0$ (b) $x - y + 2 = 0$

(c) $9x - 10y + 11 = 0$ (d) $9x + 10y + 7 = 0$

33. In a circle with centre 'O' PA and PB are two chords. PC is the chord that bisects the angle APB. The tangent to the circle at C is drawn meeting PA and PB extended at Q and R respectively. If $QC = 3, QA = 2$ and $RC = 4$, then length of RB equals:

(a) 2 (b) 8/3
(c) 10/3 (d) 11/3

34. Suppose that two circles C_1 and C_2 in a plane have no points in common. Then

(a) there is no line tangent to both C_1 and C_2.
(b) there are exactly four lines tangent to both C_1 and C_2.
(c) there are no lines tangent to both C_1 and C_2 or there are exactly two lines tangent to both C_1 and C_2.
(d) there are no lines tangent to both C_1 and C_2 or there are exactly four lines tangent to both C_1 and C_2.

35. If two chords of the circle $x^2 + y^2 - ax - by = 0$, drawn from the point (a, b) is divided by the x-axis in the ratio 2 : 1 then:

(a) $a^2 > 3b^2$ (b) $a^2 < 3b^2$
(c) $a^2 > 4b^2$ (d) $a^2 < 4b^2$

36. The angle at which the circles $(x-1)^2 + y^2 = 10$ and $x^2 + (y-2)^2 = 5$ intersect is:

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

37. The locus of the point of intersection of the tangent to the circle $x^2 + y^2 = a^2$, which include an angle of 45° is the curve $(x^2 + y^2)^2 = \lambda a^2(x^2 + y^2 - a^2)$. The value of λ is:

(a) 2 (b) 4
(c) 8 (d) 16

38. P is a point (a, b) in the first quadrant. If the two circles which pass through P and touch both the coordinate axes cut at right angles, then:

(a) $a^2 - 6ab + b^2 = 0$ (b) $a^2 + 2ab - b^2 = 0$
(c) $a^2 - 4ab + b^2 = 0$ (d) $a^2 - 8ab + b^2 = 0$

39. A circle of radius unity is centred at origin. Two particles start moving at the same time from the point (1, 0) and move around the circle in opposite direction. One of the particle moves counter clockwise with constant speed v and the other moves clockwise with constant speed $3v$. After leaving (1, 0), the two particles meet first at a point P, and continue until they meet next at point Q. The coordinates of the point Q are:

(a) (1, 0) (b) (0, 1)
(c) (0, –1) (d) (–1,0)

40. Three concentric circles of which the biggest is $x^2 + y^2 = 1$, have their radii in A.P. If the line $y = x + 1$ cuts all the circles in real and distinct points. The interval in which the common difference of the A.P. will lie is:

(a) $\left(0, \frac{1}{4}\right)$ (b) $\left(0, \frac{1}{2\sqrt{2}}\right)$

(c) $\left(0, \frac{2-\sqrt{2}}{4}\right)$ (d) none

41. A circle is inscribed into a rhombus *ABCD* with one angle $60°$. The distance from the centre of the circle to the nearest vertex is equal to 1. If *P* is any point of the circle, then $|PA|^2 + |PB|^2 + |PC|^2 + |PD|^2$ is equal to:

(a) 12 (b) 11

(c) 9 (d) none

42. The value of '*c*' for which the set, $\{(x, y) \mid x^2 + y^2 + 2x \le 1\} \cap \{(x, y) \mid x - y + c \ge 0\}$ contains only one point in common is:

(a) $(-\infty, -1] \cup [3, \infty)$ (b) $\{-1, 3\}$

(c) $\{-3\}$ (d) $\{-1\}$

43. A tangent at a point on the circle $x^2 + y^2 = a^2$ intersects a concentric circle *C* at two points *P* and *Q*. The tangents to the circle *C* at *P* and *Q* meet at a point on the circle $x^2 + y^2 = b^2$ then the equation of circle '*C*' is:

(a) $x^2 + y^2 = ab$ (b) $x^2 + y^2 = (a - b)^2$

(c) $x^2 + y^2 = (a + b)^2$ (d) $x^2 + y^2 = a^2 + b^2$

44. Tangents are drawn to the circle $x^2 + y^2 = 1$ at the points where it is met by the circles, $x^2 + y^2 - (\lambda + 6)x + (8 - 2\lambda)y - 3 = 0$. λ being the variable. The locus of the point of intersection of these tangents is:

(a) $2x - y + 10 = 0$ (b) $x + 2y - 10 = 0$

(c) $x - 2y + 10 = 0$ (d) $2x + y - 10 = 0$

45. *ABCD* is a square of unit area. A circle is tangent to two sides of *ABCD* and passes through exactly one of its vertices. The radius of the circle is:

(a) $2 - \sqrt{2}$ (b) $\sqrt{2} - 1$

(c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

46. A pair of tangents are drawn to a unit circle with centre at the origin and these tangents intersect at *A* enclosing an angle of $60°$. The area enclosed by these tangents and the arc of the circle is:

(a) $\frac{2}{\sqrt{3}} - \frac{\pi}{6}$ (b) $\sqrt{3} - \frac{\pi}{3}$

(c) $\frac{\pi}{3}-\frac{\sqrt{3}}{6}$ (d) $\sqrt{3}\left(1-\frac{\pi}{6}\right)$

47. A straight line with slope 2 and y-intercept 5 touches the circle, $x^2+y^2+16x+12y+c=0$ at a point Q. Then the coordinates of Q are:

(a) $(-6, 11)$ (b) $(-9, -13)$

(c) $(-10, -15)$ (c) $(-6, -7)$

48. A variable circle is drawn to touch the x-axis at the origin. The locus of the pole of the straight line $lx+my+n=0$ w.r.t the variable circle has the equation:

(a) $x(my-n)-ly^2=0$ (b) $x(my+n)-ly^2=0$

(c) $x(my-n)+ly^2=0$ (d) none of these

49. A foot of the normal from the point (4, 3) to a circle is (2, 1) and a diameter of the circle has the equation $2x-y-2=0$. Then the equation of the circle is:

(a) $x^2+y^2-4y+2=0$ (b) $x^2+y^2-4y+1=0$

(c) $x^2+y^2-2x-1=0$ (d) $x^2+y^2-2x+1=0$

50. AB is a diameter of a circle. CD is a chord parallel to AB and $2CD = AB$. The tangent at B meets the line AC produced at E then AE is equal to:

(a) AB (b) $\sqrt{2}AB$

(c) $2\sqrt{2}AB$ (d) $2AB$

51. Points P and Q are 3 units apart. A circle centre at P with a radius of 3 units intersects a circle centre d at Q with a radius $\sqrt{3}$ units at point A and B. The area of the quadrilateral $APBQ$ is:

(a) $\sqrt{99}$ (b) $\frac{\sqrt{99}}{2}$

(c) $\sqrt{\frac{99}{2}}$ (d) $\sqrt{\frac{99}{16}}$

52. The equation of a line inclined at an angle $\frac{\pi}{4}$ to the axis X, such that the two circles $x^2+y^2=4, x^2+y^2-10x-14y+65=0$ intercept equal lengths on it, is:

(a) $2x-2y-3=0$ (b) $2x-2y+3=0$

(c) $x-y+6=0$ (d) $x-y-6=0$

53. The equation of the circle symmetric to the circle $x^2+y^2-2x-4y+4=0$ about the line $x-y=3$ is:

(a) $x^2+y^2-10x+4y+28=0$ (b) $x^2+y^2+6x+8=0$

(c) $x^2+y^2-14x-2y+49=0$ (d) $x^2+y^2+8x+2y+16=0$

54. Consider the circles, $x^2+y^2=25$ and $x^2+y^2=9$. From the point A (0, 5) two segments are drawn touching the inner circle at the points B and C while intersecting the outer circle at the points D and E. If 'O' is the centre of both the circles then the length of the segment OF that is perpendicular to DE, is:

(a) 7/5 (b) 7/2 (c) 5/2 (d) 3

55. The locus of the center of the circles such that the point (2, 3) is the mid point of the chord $5x + 2y = 16$ is:

(a) $2x - 5y + 11 = 0$ (b) $2x + 5y - 11 = 0$ (c) $2x + 5y + 11 = 0$ (d) none of these

56. If $\left(a, \frac{1}{a}\right), \left(b, \frac{1}{b}\right), \left(c, \frac{1}{c}\right)$ and $\left(d, \frac{1}{d}\right)$ are four distinct points on a circle of radius 4 units then, $abcd$ is equal to:

(a) 4 (b) 1/4 (c) 1 (d) 16

57. A circle of constant radius 'a' passes through origin 'O' and cuts the axes of coordinates in points P and Q, then the equation of the locus of the foot of perpendicular from O to PQ is:

(a) $(x^2 + y^2)\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 4a^2$ (b) $(x^2 + y^2)^2\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = a^2$

(c) $(x^2 + y^2)^2\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = 4a^2$ (d) $(x^2 + y^2)\left(\frac{1}{x^2} + \frac{1}{y^2}\right) = a^2$

58. If a circle of constant radius 3k passes through the origin 'O' and meets coordinate axes at A and B then the locus of the centroid of the triangle OAB is:

(a) $x^2 + y^2 = (2k)^2$ (b) $x^2 + y^2 = (3k)^2$

(c) $x^2 + y^2 = (4k)^2$ (d) $x^2 + y^2 = (6k)^2$

59. Tangents are drawn from (4, 4) to the circle $x^2 + y^2 - 2x - 2y - 7 = 0$ to meet the circle at A and B. The length of the chord AB is:

(a) $2\sqrt{3}$ (b) $3\sqrt{2}$ (c) $2\sqrt{6}$ (d) $6\sqrt{2}$

60. Tangents are drawn from any point on the circle $x^2 + y^2 = R^2$ to the circle $x^2 + y^2 = r^2$. If the line joining the points of intersection of these tangents with the first circle also touch the second, then R equals:

(a) $\sqrt{2}r$ (b) $2r$ (c) $\frac{2r}{2-\sqrt{3}}$ (d) $\frac{4r}{3-\sqrt{5}}$

61. The complete set of real values of a for which at least one tangent to the parabola $y^2 = 4ax$ becomes normal to the circle $x^2 + y^2 - 2ax - 4ay + 3a^2 = 0$ is:

(a) [1, 2] (b) $[\sqrt{2}, 3]$ (c) R (d) $R - \{0\}$

62. A ray of light incident at the point $(-2, -1)$ gets reflected from the tangent at $(0, -1)$ to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle. The equation of the line along which the incident ray moves, is:

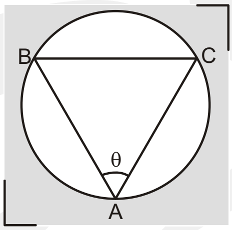
(a) $4x - 3y + 11 = 0$ (b) $4x + 3y + 11 = 0$ (c) $3x + 4y + 11 = 0$ (d) $4x + 3y + 7 = 0$

63. If from a moving point P on $x^2+y^2=4$ tangents PA and PB are drawn to $x^2+y^2=a^2$, then the locus of the circumcentre of triangle PAB, is $(0<|a|<2)$:

(a) $x^2+y^2=a^2+4$ (b) $x^2+y^2=4-a^2$

(c) $x^2+y^2=1$ (d) $x^2+y^2=2$

64. Let B and C be two fixed points of a given circle and A a variable point lying on major arc of this circle as shown in figure. The locus of feet of perpendiculars dropped from the midpoint of AB on AC is:



(a) Circle such that BC subtends angle $\tan^{-1}(\tan\theta)$ in its circumference

(b) Circle such that BC subtends angle $\tan^{-1}(2\tan\theta)$ in its circumference

(c) Circle such that BC subtends angle $2\tan^{-1}(\tan\theta)$ in its circumference

(d) Circle such that BC subtends angle $\tan^{-1}(3\tan\theta)$ in its circumference

65. For any real k, the circle $x^2+y^2+2kx+2ky-8=0$ passes through two fixed points A and B. Locus of the point of intersection of the tangents to the circle at A and B is:

(a) $x^2=4y$ (b) $y^2=4x$ (c) $x=y$ (d) $x+y=0$

66. If a line having y-intercept 'c' makes a chord of length 'a' to the circle $x^2+y^2=a^2$, then:

(a) $4c^2\ge 3a^2$ (b) $4c^2<3a^2$ (c) $4c^2>5a^2$ (d) $c^2>3a^2$

ANSWERS

1.	(a)	**2.**	(b)	**3.**	(c)	**4.**	(b)	**5.**	(d)	**6.**	(d)	**7.**	(a)	**8.**	(d)	**9.**	(b)	**10.**	(b)
11.	(d)	**12.**	(a)	**13.**	(a)	**14.**	(c)	**15.**	(c)	**16.**	(d)	**17.**	(a)	**18.**	(c)	**19.**	(a)	**20.**	(b)
21.	(b)	**22.**	(b)	**23.**	(b)	**24.**	(c)	**25.**	(a)	**26.**	(d)	**27.**	(d)	**28.**	(b)	**29.**	(a)	**30.**	(a)
31.	(c)	**32.**	(c)	**33.**	(b)	**34.**	(d)	**35.**	(a)	**36.**	(b)	**37.**	(c)	**38.**	(c)	**39.**	(d)	**40.**	(c)
41.	(b)	**42.**	(d)	**43.**	(a)	**44.**	(a)	**45.**	(a)	**46.**	(b)	**47.**	(d)	**48.**	(a)	**49.**	(c)	**50.**	(d)
51.	(b)	**52.**	(a)	**53.**	(a)	**54.**	(a)	**55.**	(a)	**56.**	(c)	**57.**	(c)	**58.**	(a)	**59.**	(b)	**60.**	(b)
61.	(d)	**62.**	(b)	**63.**	(c)	**64.**	(b)	**65.**	(c)	**66.**	(a)								

EXERCISE 2

One or More than One is/are Correct

1. If a circle passes through the point $\left(3, \sqrt{\frac{7}{2}}\right)$ and touches $x + y = 1$ and $x - y = 1$, then the centre of the circle is at:

(a) $(4,0)$ (b) $(4,2)$

(c) $(6,0)$ (d) $(7,0)$

2. Equations of four circles are $(x \pm a)^2 + (y \pm a)^2 = a^2$, then:

(a) The radius of the greatest circle touching all the four circles is $(\sqrt{2} + 1)\,a$

(b) The radius of the smallest circle touching all the four circles is $(\sqrt{2} - 1)\,a$

(c) Area of region enclosed by four given circles is $(4 - \pi)\,a^2$ sq. units

(d) The centres of four circles are the vertices of a square

3. The equation of the circle which touches the axes of coordinates and the line $\frac{x}{3} + \frac{y}{4} = 1$ and whose centre lies in the first quadrant is $x^2 + y^2 - 2rx - 2ry + r^2 = 0$, then r can be equal to:

(a) 1 (b) 2 (c) 3 (d) 6

4. If $(a,0)$ is a point on a diameter of the circle $x^2 + y^2 = 4$, then $x^2 - 4x - a^2 = 0$ must have:

(a) exactly one real root in $\left[-\frac{9}{10}, \frac{1}{10}\right]$

(b) exactly one real root in $\left[4, \frac{49}{10}\right]$

(c) exactly one real root in $[0,2]$

(d) two distinct real roots in $[-1,5]$

5. The locus of points of intersection of the tangents to $x^2 + y^2 = a^2$ at the extremeties of a chord of circle $x^2 + y^2 = a^2$ which touches the circle $x^2 + y^2 - 2ax = 0$ is/are:

(a) $y^2 = a(a - 2x)$ (b) $x^2 = a(a - 2y)$

(c) $x^2 + y^2 = (x - a)^2$ (d) $x^2 + y^2 = (y - a)^2$

6. The tangent drawn from the origin to the circle $x^2 + y^2 + 2gx + 2fy + f^2 = 0$ are perpendicular, if:

(a) $g = f$ (b) $g = -f$

(c) $g = 2f$ (d) $2g = f$

7. Two chords are drawn from the point $P(h, k)$ on the circle $x^2 + y^2 = hx + ky$. If the y-axis divides both the chords in the ratio 2 : 3, then which of the following may be correct ?

(a) $k^2 > 15h^2$ (b) $15k^2 > h^2$

(c) $h^2 = 15k^2$ (d) $k^2 > 5h^2$

8. The equation(s) of the tangent at the point (0, 0) to the circle, making intercepts of lengths $2a$ and $2b$ units on the coordinates axes, is/are:

(a) $ax + by = 0$ (b) $ax - by = 0$

(c) $x = y$ (d) $bx + ay = 0$

9. Consider the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. Let O be the centre of the circle and tangent at $A(7,3)$ and $B(5,1)$ meet at C. Let $S = 0$ represents family of circles passing through A and B, then:

(a) Area of quadrilateral $OACB = 4$

(b) the radical axis for the family of circles $S = 0$ is $x + y = 10$

(c) the smallest possible circle of the family $S = 0$ is $x^2 + y^2 - 12x - 4y + 38 = 0$

(d) the coordinates of point C are $(7,1)$

10. Let x, y be real variable satisfying the $x^2 + y^2 + 8x - 10y - 40 = 0$. Let $a = \max(\sqrt{(x+2)^2 + (y-3)^2})$ and $b = \min(\sqrt{(x+2)^2 + (y-3)^2})$ then

(a) $a + b = 18$ (b) $a + b = 4\sqrt{2}$

(c) $a - b = 4\sqrt{2}$ (d) $ab = 73$

11. Coordinates of the centre of a circle, whose radius is 2 unit and which touches the line pair $x^2 - y^2 - 2x + 1 = 0$ are:

(a) (4, 0) (b) $(1 + 2\sqrt{2}, 0)$

(c) (4, 1) (d) $(1, 2\sqrt{2})$

12. Point M moved on the circle $(x-4)^2 + (y-8)^2 = 20$. Then it broke away from it and moving along a tangent to the circle, cuts the x-axis at the point $(-2, 0)$. The coordinates of a point on the circle at which the moving point broke away is:

(a) $\left(-\frac{3}{5}, \frac{46}{5}\right)$ (b) $\left(-\frac{2}{5}, \frac{44}{5}\right)$

(c) (6, 4) (d) (3, 5)

13. If the area of the quadrilateral formed by the tangents from the origin to the circle $x^2 + y^2 + 6x - 10y + c = 0$ and the radii corresponding to the points of contact is 15, then a value of c is:

(a) 9 (b) 4

(c) 5 (d) 25

14. If $4a^2 - 5b^2 + 6a + 1 = 0$ and the line $ax + by + 1 = 0$ touches a fixed circle, then:

(a) centre of circle is at (3, 0) (b) the radius of circle is $\sqrt{5}$

(c) the radius of circle is $\sqrt{3}$ (d) the circle passes through $(1, 1)$

15. Let $P(1, 2\sqrt{2})$ is a point on circle $x^2 + y^2 = 9$. Locate the points on the given circle, which are at 2 units distance from point P.

(a) $(-1, 2\sqrt{2})$ (b) $(2\sqrt{2}, 1)$

(c) $\left(\frac{23}{9}, \frac{10\sqrt{2}}{9}\right)$ (d) $(3, 0)$

16. AC is diameter of circle. AB is a tangent. BC meets the circle again at D. $AC = 1$, $AB = a$, $CD = b$, then:

(a) $ab > 1$ (b) $ab < 1$

(c) $\frac{b}{a} > \frac{1}{a^2 + \frac{1}{2}}$ (d) $\frac{b}{a} < \frac{1}{a^2 + \frac{1}{2}}$

17. Equation of line that touches the curves $|y| = x^2$ and $x^2 + (y-2)^2 = 4$ where $x \neq 0$ is:

(a) $y = 4\sqrt{5}x + 20$ (b) $y = 4\sqrt{3}x - 12$

(c) $y = -4\sqrt{5}x + 20$ (d) $y = -4\sqrt{5}x - 20$

18. Let l_1, l_2 and l_3 are the lengths of the tangents drawn from a variable point P to the circle $x^2 + y^2 = a^2$; $x^2 + y^2 = 2ax$ and $x^2 + y^2 = 2ay$ respectively. These lengths satisfy the relation $l_1^4 = l_2^2 l_3^2 + a^4$. Then the locus of P can be:

(a) Line (b) Circle

(c) Parabola (d) Hyperbola

ANSWERS

1.	(a, c)	**2.**	(a, b, c, d)	**3.**	(a, d)	**4.**	(a, b, d)	**5.**	(a, c)	**6.**	(a, b)
7.	(a, b, d)	**8.**	(a, b, d)	**9.**	(a, c, d)	**10.**	(a, c, d)	**11.**	(b, d)	**12.**	(b, c)
13.	(a, d)	**14.**	(a, b, d)	**15.**	(a, c)	**16.**	(b, c)	**17.**	(a, b, c)	**18.**	(a, b)

EXERCISE 3

Comprehension: (1)

An altitude BD and a bisector BE are drawn in the triangle ABC from the vertex B. It is known that the length of side $AC = 1$, and the magnitudes of the angles BEC, ABD, ABE, BAC form an arithmetic progression.

1. The area of circle circumscribing ΔABC is:

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$ (d) π

2. Let 'O' be the circumcentre of ΔABC, the radius of circle inscribed in ΔBOC is :

(a) $\frac{1}{8\sqrt{3}}$ (b) $\frac{1}{4\sqrt{3}}$

(c) $\frac{1}{2\sqrt{3}}$ (d) $\frac{1}{2}$

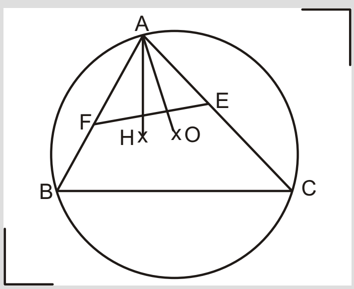
3. Let B' be the image of point B with respect to side AC of ΔABC, then the length BB' is equal to :

(a) $\frac{\sqrt{3}}{4}$ (b) $\frac{\sqrt{2}}{4}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$

Comprehension: (2)

Triangle ABC has circumcentre 'O' and orthocentre 'H'. Points E and F are chosen on sides AC and AB respectively such that $AE = AO$ and $AF = AH$ as shown in figure. Let 'R' be the radius of circle circumscribing the triangle ABC



1. The area of quadrilateral *BFEC* is equal to:

(a) $\frac{R^2}{4}(\sin B + \sin C)$

(b) $\frac{R^2}{8}\sin A \cos(B - C)$

(c) $\frac{R^2}{2}\sin 2A$

(d) $\frac{R^2}{2}(\sin 2B + \sin 2C)$

2. The length of segment *EF* is equal to:

(a) $R \cos A$

(b) R

(c) $R \sin 2A$

(d) $2R \cos A$

3. Let $\angle AFE = \frac{\pi}{3}$, then the sum of squares of the 3 altitudes of triangle *AFE* is equal to:

(a) $\frac{9R^2}{4}$

(b) $3R^2$

(c) $\frac{3R^2}{2}$

(d) $\frac{8R^2}{3}$

Comprehension: (3)

Let S_1 and S_2 be two fixed externally tangent circles with radius 2 and 3 respectively. Let S_3 be a variable circle internally tangent to both S_1 and S_2 at points A and B respectively. The tangents to S_3 at A and B meet at T, and $TA = 4$.

1. The radius of circle S_3 is equal to:

(a) 2

(b) 4

(c) 6

(d) 8

2. The area of circle circumscribing ΔTAB is:

(a) 10π

(b) 20π

(c) 40π

(d) 80π

3. Let C_1, C_2, C_3 be centres of circles S_1, S_2, S_3 respectively then which of the following must be true:

(a) $C_3C_1 + C_3C_2 = 5$

(b) $C_3C_1 - C_3C_2 = 3$

(c) $C_3C_1 + C_3C_2 = 3$

(d) $C_3C_1 - C_3C_2 = 1$

Comprehension: (4)

Let A, B, C, D lie on a line such that $AB = BC = CD = 1$. The points A and C are also joined by a semicircle with AC as diameter and P is a variable point on this semicircle such that $\angle PBD = \theta$, $0 \le \theta \le \pi$. Let R is the region bounded by arc AP, the straight line PD and line AD.

1. The maximum possible area of region R is:

(a) $\dfrac{2\pi - \sqrt{3}}{12}$ (b) $\dfrac{2\pi + 3\sqrt{3}}{6}$

(c) $\dfrac{(2\pi + 6\sqrt{3})}{12}$ (d) $\dfrac{2\pi + 3\sqrt{3}}{12}$

2. Let 'L' is the perimeter of region 'R', then L is equal to:

(a) $3 - \theta + \sqrt{5 - 4\cos\theta}$ (b) $3 + \pi + \sqrt{5 - 4\cos\theta}$

(c) $3 + \pi - \theta + \sqrt{5 - 4\cos\theta}$ (d) $3 + \pi + \theta + \sqrt{5 + 4\cos\theta}$

3. The non negative difference of greatest and least values of L is:

(a) $3 - \sqrt{3} + \dfrac{\pi}{3}$ (b) $\sqrt{3} - 3 + \dfrac{\pi}{3}$

(c) $\sqrt{3} - 3 + \dfrac{2\pi}{3}$ (d) $\pi - 2$

Comprehension: (5)

Let $P(a, b)$ be a variable point satisfying $4 \le a^2 + b^2 \le 9$ and $b^2 - 4ab + a^2 \le 0$. Let R be the complete equation represented in x-y plane in which P can lie.

1. Area of region R is:

(a) $\dfrac{2\pi}{3}$ (b) π

(c) $\dfrac{4\pi}{3}$ (d) $\dfrac{5\pi}{3}$

2. Minimum value of $|a + b|$ for all positions of P lying in region R is:

(a $\sqrt{3}$ (b) $2\sqrt{3}$

(c) $\sqrt{6}$ (d) $2\sqrt{6}$

3. Let A and B be two points in first quadrant lying in region R, then maximum possible distance between them is:

(a) 2 (b) $\sqrt{5}$

(c) $\sqrt{6}$ (d) $\sqrt{7}$

Comprehension: (6)

Let $f(x, y) = 0$ be the equation of a circle such that $f(0, y) = 0$ has equal real roots and $f(x, 0) = 0$ has two distinct real roots. Let $g(x, y) = 0$ be the locus of points 'P' from where tangents to circle $f(x, y) = 0$ make angle $\pi/3$ between them and $g(x, y) = x^2 + y^2 - 5x - 4y + c, c \in R$.

1. Let Q be a point from where tangents drawn to circle $g(x, y) = 0$ are mutually perpendicular.If A, B are the points of contact of tangent drawn from Q to circle $g(x, y) = 0$, then area of triangle QAB is

(a) $\frac{25}{12}$ (b) $\frac{25}{8}$

(c) $\frac{25}{4}$ (d) $\frac{25}{2}$

2. The area of region bounded by circle $f(x, y) = 0$ with x-axis in the first quadrant is

(a) $3 + \frac{25}{8}\left(\pi - \tan^{-1}\frac{1}{2}\right)$ (b) $3 + \frac{25}{4}\tan^{-1}\left(\frac{24}{11}\right)$

(c) $3 + \frac{25}{8}\left(2\pi - \tan^{-1}\frac{3}{4}\right)$ (d) $3 + \frac{25}{8}\left(2\pi - \tan^{-1}\left(\frac{24}{7}\right)\right)$

3. The number of points with positive integral coordinates satisfying $f(x, y) > 0, g(x, y) < 0; y > 3$ and $x < 6$ is

(a) 7 (b) 8

(c) 10 (d) 11

Comprehension (7)

Consider two circles S_1 and S_2 (externally touching) having centres at points A and B whose radii are 1 and 2 respectively. A tangent to circle S_1 from point B intersects the circle S_1 at point C.D is chosen on circle S_2 so that AC is parallel to BD and the two segments BC and AD do not intersect. Segment AD intersect the circle S_1 at E. The line through B and E intersects the circle S_1 at another point F.

1. The length of segment EF is:

(a) $2\sqrt{2}$ (b) $\frac{2\sqrt{2}}{3}$

(c) $\frac{2\sqrt{3}}{3}$ (d) $\sqrt{3}$

2. The area of triangle ABD is:

(a) 2 (b) $\sqrt{3}$

(c) $2\sqrt{2}$ (d) $\sqrt{5}$

3. The length of the segment DE is:

(a) $\sqrt{3}$ (b) $2\sqrt{2}$

(c) 2 (d) 3

Comprehension: (8)

In an acute triangle *ABC*, point *H* is the intersection point of altitude *CE* to *AB* and altitude *BD* to *AC*. A circles with *DE* as its diameter intersects *AB* and *AC* at points *F* and *G* respectively. If $BC = 25$, $BD = 20$ and $BE = 7$.

1. The sum of the length of all the sides of ΔABC is:

(a) 65 (b) 70

(c) 75 (d) 80

2. Area of the circle *S* is:

(a) 100π (b) 196π

(c) $\frac{225\pi}{4}$ (d) 400π

3. Let *FG* and *AH* intersect at point *K*, then the length of $AK =$

(a) $\frac{192}{25}$ (b) $\frac{216}{25}$

(c) $\frac{225}{24}$ (d) 9

Comprehension: (9)

The circle '*S*' touches the sides *AB* and *AD* of the rectangle *ABCD* and cuts the side *DC* at single point *F* and the side *BC* at a single point *E*. If $|AB| = 32, |AD| = 40$ and $|BE| = 1$.

1. The angle between pair of tangents drawn from the point *D* to the circle '*S*' is:

(a) $\pi - \tan^{-1}\left(\frac{25}{8}\right)$ (b) $\pi - \tan^{-1}\left(\frac{15}{7}\right)$

(c) $\pi - \tan^{-1}\left(\frac{15}{8}\right)$ (d) $\frac{\pi}{2} - \tan^{-1}\left(\frac{5}{3}\right)$

2. The area of trapezoid *AFCB* is:

(a) 960 (b) 1020

(c) 1140 (d) 1180

3. The radius of circle is:

(a) 22 (b) 23

(c) 25 (d) 27

Comprehension: (10)

A point $P(\alpha, \beta)$ is called rational point if both α and β are rational numbers and if both α and β are integers, then point $P(\alpha,\beta)$ is called lattice point.

1. Line $x + y = n (n \in N)$ cuts coordinates axes at A and B. If O is origin then number of lattice points within the triangle AOB is:

(a) $\frac{n^2 - 4n + 3}{2}$ (b) $\frac{n^2 + n}{2}$

(c) $\frac{n^2 - n}{2}$ (d) $\frac{n^2 - 3n + 2}{2}$

2. The number of rational points that could be on a circle, whose centre is $(2, \sqrt{3})$ and radius is $2\sqrt{3}$ units, is :

(a) 1 (b) 2 (c) 3 (d) Infinite

3. Number of lattice points on the circumference of circle $x^2 + y^2 = 25$ is:

(a) 14 (b) 8 (c) 12 (d) 10

Comprehension: (11)

Consider a right triangle ABC right angled at vertex B with $AB = 3$ and $BC = 4$. A circle 'S' touching the side BC is drawn intersecting the sides AB at points D and E and the side AC at points F and G respectively. Also D and F are the midpoints of sides AB and AC respectively, then:

1. Radius of circle S is:

(a) 1 (b) $\frac{5}{4}$

(c) $\frac{7}{6}$ (d) $\frac{13}{12}$

2. The length of portion FG of hypotenuse AC is:

(a) $\frac{5}{2}$ (b) $\frac{13}{10}$

(c) $\frac{11}{10}$ (d) 1

3. The length of chord EF of circle S is:

(a) $\frac{17}{6}$ (b) $\frac{5}{2}$

(c) $\frac{13}{6}$ (d) $\frac{11}{6}$

Comprehension: (12)

Given a line segment $AB, A \equiv (0,0)$ and $B(a,0)$. Three circles S_1, S_2, S_3, of radius R are centred at the end points and the midpoint of the line segment AB. A fourth circles S_4 is drawn touching the 3 given circles.

1. If $0 < R < \frac{a}{4}$, then sum of all possible distinct values of radius of S_4 is:

(a) $\frac{a^2}{16R}$ (b) $\frac{a^2}{7R}$ (c) $\frac{3a^2}{16R}$ (d) $\frac{a^2}{4R}$

2. If $0 < R < \frac{a}{4}$, then number of possible circles S_4 is:

(a) 2 (b) 4 (c) 6 (d) 8

3. If $\frac{a}{4} < R < \frac{a}{2}$, then radius of circle S_4 is:

(a) $\frac{a^2}{16R}$ (b) $\frac{a^2}{8R}$

(c) $\frac{3a^2}{16R}$ (d) $\frac{a}{4R}$

Comprehension: (13)

Let $A \equiv (0,0), B \equiv (4,0)$ and on segment AB is given a point M. On the same side of AB squares $AMCD$ and $BMFE$ are constructed above AB. The circumcircles S_1 and S_2 of two squares $AMCD$ and $BMFE$ respectively whose centres are P and Q, intersect in M and another point N.

1. The point of intersection of the lines FA and BC is:

(a) N (b) outside S_1 but inside S_2

(c) outside S_2 but inside S_1 (d) inside S_1 and S_2 both

2. For all positions of M varying along the segment AB, the lines MN passes through the fixed point $R(a,b)$, then $a+b=$

(a) 0 (b) 1 (c) $\sqrt{3}$ (d) 2

3. The locus of midpoints of all segments PQ as M varies along the segment AB is:

(a) line segment $(x,1), x \in [1,3]$ (b) line segment $(x,1), x \in [2,4]$

(c) line segment $(x,1), x \in [0,3]$ (d) line segment $(x,1), x \in [0,4]$

Comprehension: (14)

Let C be a circle of radius r with centre at O, let P be a point outside C and D be a point on C. A line through P intersects C at Q and R, S is the midpoint of QR.

1. For different choices of lines through P, what is the curve on which S lies:

(a) a straight line (b) an arc of circle with P as centre

(c) an arc of circle with PS as diameter (d) an arc of circle with OP diameter

2. Let P is situated at a distance 'd' from centre O, then which of the following does not equal the product $(PQ)(PR)$.

(where T is a point on C and PT is tangent to C)

(a) $d^2 - r^2$ (b) $(PT)^2$ (c) $(PS)^2 - (QS)(RS)$ (d) $(PS)^2$

3. Let ABC be an equilateral triangle inscribed in C. If α, β, γ denote the distances of D from vertices A, B, C respectively, what is the value of product $(\beta + \gamma - \alpha)(\gamma + \alpha - \beta)(\alpha + \beta - \gamma)$:

(a) 0 (b) $\dfrac{\alpha\beta\gamma}{8}$

(c) $\dfrac{\alpha^3 + \beta^3 + \gamma^3 - 3\alpha\beta\gamma}{6}$ (d) $\alpha + \beta + \gamma$

Comprehension: (15)

Given a line segment AB, $A \equiv (0,0)$ and $B(a,0)$. Three circles S_1, S_2, S_3 of radius R are centred at the end points and the midpoint of the line segment AB. A fourth circle S_4 is drawn touching the 3 given circles.

1. If $0 < R < \dfrac{a}{4}$, then sum of all possible distinct values of radius of S_4 is:

(a) $\dfrac{a^2}{16R}$ (b) $\dfrac{a^2}{7R}$ (c) $\dfrac{3a^2}{16R}$ (d) $\dfrac{a^2}{4R}$

2. If $0 < R < \dfrac{a}{4}$, then number of possible circle S_4 is:

(a) 2 (b) 4 (c) 6 (d) 8

3. If $\dfrac{a}{4} < R < \dfrac{a}{2}$, then radius of circle S_4 is:

(a) $\dfrac{a^2}{16R}$ (b) $\dfrac{a^2}{8R}$ (c) $\dfrac{3a^2}{16R}$ (d) $\dfrac{a^2}{4R}$

Comprehension: (16)

The line $y = ax + b$ intersects the curve $C: x^2 + y^2 + 6x - 10y + 1 = 0$ at the points A and B. If the lines segment AB subtends a right angle at origin then the locus of the point (a, b) is the curve $g(x, y) = 0$.

1. The equation of curve $g(x, y) = 0$ is:

(a) $x^2 + 2y^2 - 6xy + 10y + 1 = 0$
(b) $x^2 + 2y^2 - 6xy - 10y + 1 = 0$
(c) $x^2 - 2y^2 - 6xy + 10y + 1 = 0$
(d) $x^2 - 2y^2 - 6xy - 10y + 1 = 0$

2. The slope of tangent to the curve $g(x, y) = 0$ at the point where the line $y = 1$ intersects it in first quadrant is:

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

3. The equation of chord of the curve C whose middle point is (1, 2) is:

(a) $4x - 3y + 2 = 0$ (b) $4x + 3y - 10 = 0$ (c) $3x + 4y - 11 = 0$ (d) $3x - 4y + 5 = 0$

ANSWERS

Comprehension-1:	1.	(b)	2.	(b)	3.	(d)
Comprehension-2:	1.	(d)	2.	(b)	3.	(a)
Comprehension-3:	1.	(d)	2.	(b)	3.	(d)
Comprehension-4:	1.	(b)	2.	(c)	3.	(d)
Comprehension-5:	1.	(d)	2.	(c)	3.	(d)
Comprehension-6:	1.	(d)	2.	(d)	3.	(d)
Comprehension-7:	1.	(c)	2.	(c)	3.	(c)
Comprehension-8:	1.	(d)	2.	(c)	3.	(b)
Comprehension-9:	1.	(c)	2.	(d)	3.	(c)
Comprehension-10:	1.	(d)	2.	(b)	3.	(c)
Comprehension-11:	1.	(d)	2.	(c)	3.	(c)
Comprehension-12:	1.	(c)	2.	(c)	3.	(a)
Comprehension-13:	1.	(a)	2.	(a)	3.	(a)
Comprehension-14:	1.	(d)	2.	(d)	3.	(a)
Comprehension-15:	1.	(c)	2.	(c)	3.	(a)
Comprehension-16:	1.	(b)	2.	(d)	3.	(a)

EXERCISE 4

Assertion and Reason

(A) Statement-1 is true, Statement-2 is true and Statement-2 is correct explanation for Statement-1

(B) Statement-1 is true, Statement-2 is true and Statement-2 is not correct explanation for Statement-1

(C) Statement-1 is true, Statement-2 is false

(D) Statement-1 is false, Statement-2 is true

1. Consider 3 equal circles of radius r_1 within a circle of radius r_2 each to touch the other two and the given circle.

Statement-1: $\frac{r_1}{r_2} = \frac{\sqrt{3}}{\sqrt{3}+1}$

because

Statement-2: Incentre of triangle formed by joining centres of 3 equal circles is same as centre of given circle.

2. Let a circle be inscribed in the quadrant of a circle of diameter 4, then

Statement-1: The radius of inscribed circle is the positive root of the equation $r^2 + 4r - 4 = 0$

because

Statement-2: Distance between their centres = 2 (radius of circle inscribed).

3. Let A and B are two points both lying within a given circle S, and 'P' be a point on circumference of 'S' at which AB subtends the greatest angle.

Statement-1: If $A \equiv (1, 1), B \equiv (1, -1)$ and equation of S is $x^2 + y^2 = 4$ then P will be $(2,0)$

because

Statement-2: 'P' will be the point where a circle passing through A and B touches the circle S.

4. Let PQ be fixed chord in a circle 'S' and AB is any diameter, then

Statement-1: If PQ is represented by equation $y = -1$, S represented by $x^2 + y^2 = 4$ and A, B lie on same side of PQ then the sum of the perpendiculars let fall from A and B on PQ is equal to 4.

because

Statement-2: The sum of the perpendiculars let fall from A and B on PQ is same for all position of AB.

5. If two equal chords AB and CD of a circle intersect at point P,

Statement-1: if ACB and CBD are minor segments of AB and CD. $AP = 2$ and $PB = 7$, then $PC = 7$ and $PD = 2$.

because

Statement-2: $AP = PD$ and $PB = PC$.

6. AB is a fixed chord of a circle and XY is any chord having its middle point Z on AB, then

Statement-1: XY is greatest if XY coincides with AB

because

Statement-2: XY is greatest if Z becomes the middle point of AB.

7. Let the bisector of $\angle A$ of ΔABC meets BC in D and the circumcircle of ΔABC in E, then

Statement-1: AD is less than the G..M. of AB and AC

because

Statement-2: ΔABD is similar to ΔAEC.

8. Let $ABCD$ is a rectangle, the diagonal AC and BD intersect at O. A straight line through B intersects DC produced at E and DA produced at F such that $OE = OF$, then

Statement-1: $(CE)(DE) = (AF)(DF)$

because

Statement-2: If a line through any point P intersects the circle with centre at 'O' and radius 'r' at Q and R then $(PQ)(PR) = (OP)^2 + r^2$.

9. Let $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of a triangle ABC.

Statement-1: If angle C is obtuse then the quantity $(x_3 - x_1)(x_3 - x_2) + (y_3 - y_1)(y_3 - y_2)$ is negative.

because

Statement-2: Diameter of a circle subtends obtuse angle at any point lying inside the semicircle.

10. Let C be a circle with centre 'O' and HK is the chord of contact of tangents drawn from a point A. OA intersects the circle 'C' at P and Q and B is the midpoint of HK, then

Statement-1: AB is the Harmonic mean of AP and AQ

because

Statement-2: AK is the Geometric mean of AB and AO and OA is the arithmetic mean of AP and AQ.

11. Let the diagonals of a convex quadrilateral $ABCD$ intersect at point P and a, b, c, d denote the length of sides AB, BC, CD and DA respectively, then

Statement-1: Diagonals of quadrilateral $ABCD$ are perpendicular if $a^2 + c^2 = b^2 + d^2$

because

Statement-2: $a^2 + c^2 \geq b^2 + d^2 \geq (AP)^2 + (BP)^2 + (CP)^2 + (DP)^2$

or $b^2 + d^2 \geq a^2 + c^2 \geq (AP)^2 + (BP)^2 + (CP)^2 + (DP)^2$

ANSWERS

1.	(d)	**2.**	(c)	**3.**	(a)	**4.**	(c)	**5.**	(c)	**6.**	(d)	**7.**	(a)	**8.**	(c)	**9.**	(a)	**10.**	(a)
11.	(c)																		

EXERCISE 5

Match the Columns:

1. Let E, F, G, H be 4 distinct points inside square $ABCD$ whose area is 1 square units such that $\angle EDC = \angle ECD = \angle HDA = \angle HAD = \angle GAB = \angle GBA = \angle FBC = \angle FCB = 15°$

	Column-I		Column-II
(a)	If area of quadrilateral $EFGH$ is equal to $a - \sqrt{b}$ where $a, b \in N$, then $a + b =$	**(p)**	1
(b)	Let $\angle AEB = \dfrac{\pi}{k}$; then $k =$	**(q)**	3
(c)	The radius of circle circumscribing the ΔAHD is equal to	**(r)**	5
(d)	Let the lengths of perpendiculars from vertices G, A, H to opposite sides of triangle AHG be h_1, h_2, h_3 respectively. Let $\dfrac{1}{h_1^2} + \dfrac{1}{h_2^2} + \dfrac{1}{h_3^2} = a + \sqrt{b}$ where $a, b \in N$, then $\dfrac{b}{a} =$	**(s)**	6

2. Let $A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)$ be 3 distinct points lying on circle $S: x^2 + y^2 = 1$, such that $x_1x_2 + y_1y_2 + x_2x_3 + y_2y_3 + x_3x_1 + y_3y_1 = -\dfrac{3}{2}$

	Column-I		Column-II
(a)	Let P be any arbitrary point lying on S, then $(PA)^2 + (PB)^2 + (PC)^2 =$	**(p)**	3
(b)	Let the perpendicular dropped from point 'A' to BC meets S at Q and $\angle OBQ = \dfrac{\pi}{k}$, where '$O$' is origin, then $k =$	**(q)**	4
(c)	Let R be the point lying on line $x + y = 2$, at the minimum distance from S and the square of maximum distance of R from S is $a + b\sqrt{b}$, then $a + b =$	**(r)**	5
(d)	Let I and G represent incenter and centroid of ΔABC respectively, then $IA + IB + IC + GA + GB + GC =$	**(s)**	6

3. In the triangle ABC, the angle bisector AK is perpendicular to the median BM and $\angle ABC = 120°$,

	Column-I		Column-II
(a)	The value of ratio $\frac{BC}{AB}$ is equal to	**(p)**	$\frac{3\sqrt{3}}{32}(\sqrt{13}-1)$
(b)	The value of ratio of radius of the circle circumscribing the triangle ABC to the side length AB is equal to	**(q)**	$\frac{\sqrt{13}-1}{2}$
(c)	The ratio of the area of ΔABC to the area of the circle circumscribing ΔABC is equal to	**(r)**	$\frac{1}{2}$
(d)	The value of ratio of the sides AB to AC is equal to	**(s)**	$\frac{2}{\sqrt{3}}$

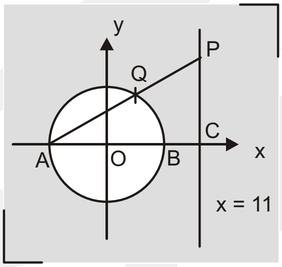
4. There are two circles in a parallelogram. One of them of radius 3, is inscribed in the parallelogram, and the other touches two sides of the parallelogram and the first circle. The distance between the points of tangency which lie on the same side of the parallelogram is equal to 3.

	Column-I		Column-II
(a)	The radius of the other circle is	**(p)**	$\frac{75}{2}$
(b)	Area of the parallelogram is equal to	**(q)**	75
(c)	Let d_1, d_2 denote the lengths of the diagonals of parallelogram, then the product $d_1 d_2$ is equal to	**(r)**	$5\sqrt{31}$
(d)	Let d_1, d_2 be the diagonals of the parallelogram then the value of $d_1 + d_2$ is equal to	**(s)**	$\frac{3}{4}$

5. In the parallelogram $ABCD$ with angle $A = 60°$, the bisector of angle B is drawn which cuts the side CD at a point E. A circle S_1 of radius 'r' is inscribed in the ΔECB. Another circle 'S_2' is inscribed in the trapezoid $ABED$.

	Column-I		Column-II
(a)	The value of radius of S_2 is	**(p)**	$2\sqrt{3}\,r$
(b)	The value of distance between the centres of S_1 and S_2 is	**(q)**	$\frac{\sqrt{3}}{2}r$
(c)	The value of the length of internal common tangent of S_1 and S_2 is	**(r)**	$\sqrt{7}\,r$
(d)	The value of the length CE is	**(s)**	$\frac{3}{2}r$

6. In the given figure, the circle $x^2 + y^2 = 25$ intersects x-axis at points A and B. The line $x = 11$ intersects x-axis, at point C. Point P moves along the line $x = 11$ above the x-axis and AP intersects the circle at Q.



	Column-I		Column-II
(a)	The coordinates of point P if the ΔAOB has the maximum area is	**(p)**	(11, 0)
(b)	The coordinates of point P if Q is the middle point of AP is	**(q)**	(11, 8)
(c)	The co-ordinates of P if the area of ΔAQB is $\left(\frac{1}{4}\right)^{th}$ of the area of ΔAPC is	**(r)**	(11, 12)
(d)	The co-ordinates of P if $\lvert AP - BP \rvert$ is maximum	**(s)**	(11, 16)

7.

	Column-I		Column-II
(a)	Two intersecting circles	**(p)**	have a common tangent
(b)	Two circles touching each other	**(q)**	have a common normal
(c)	Two non concentric circles, one strictly inside the other	**(r)**	do not have a common normal
(d)	Two concentric circles of different radii	**(s)**	do not have a radical axis

8. Match the following : Let C and C_1 be circles of radii 1 and $r > 1$ respectively touching the coordinate axes, Column-II gives values of r for the conditions in Column-I .

	Column-I		Column-II
(a)	C passes through the centre of C_1	**(p)**	3
(b)	C and C_1 touch each other	**(q)**	$\frac{2+\sqrt{2}}{2}$
(c)	C and C_1 are orthogonal	**(r)**	$2+\sqrt{3}$
(d)	C and C_1 have longest common chord	**(s)**	$3+2\sqrt{2}$

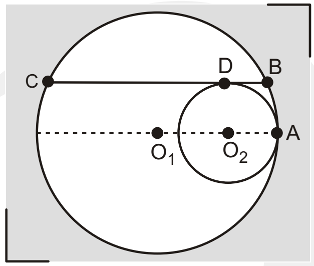
9. Triangles ABC are described on a given base BC and of a given vertical angle α.

	Column-I		Column-II
(a)	The locus of orthocentre of ΔABC is	**(p)**	Part of circle such that BC subtend angle t $\pi-\alpha$ its circumference
(b)	The locus of incentre of ΔABC is	**(q)**	Part of circle such that BC subtend angle $\frac{\pi}{2}+\frac{\alpha}{2}$ at its circumference
(c)	The locus of the excentre corresponding to vertex opposite to base of ΔABC is	**(r)**	Part of circle such that BC subtend angle $\frac{\pi}{2}-\frac{\alpha}{2}$ at its circumference
(d)	The locus of centroid of ΔABC is:	**(s)**	Part of circle such that PQ subtend angle α at its circumference where P, Q are points of trisection of segment BC.

10. Match the column:

	Column-I		Column-II
(a)	Line L is the radical axis of the circles $S_1 \equiv x^2+y^2-2x-2y-7=0$ and $S_2 \equiv x^2+y^2+6x-8y=0$. If (x_1, y_1) and (x_2, y_2) denote the coordinates of the extremeties of the diameter of S_2 which is perpendicular to L, then $\frac{1}{5}(x_1^2+x_2^2+y_1^2+y_2^2)$ is equal to	**(p)**	13
(b)	The pair of lines represented by $x^2+y^2+3xy+4x+y-1=0$ intersect at P. If Q and R are the point of intersection of the pair of lines with the x-axis and the area of the ΔPQR is Δ, then $\Delta^2 =$	**(q)**	20
(c)	If the coordinates of radical centre of circles $x^2+y^2-25=0$; $2x^2+2y^2-4x+6y-7=0$; $x^2+y^2-x-2y-9=0$ is (α, β) then, $2(\alpha+\beta)$ is equal to	**(r)**	25
(d)	Let m_1 and m_2 are the slopes of the tangents drawn to circle $x^2+y^2-4x-8y-5=0$ from the point $P(-1,-2)$, and $\lvert m_1+m_2 \rvert = p/q$ where p and q are relatively prime natural numbers, then $p+q$ is equal to	**(s)**	27
		(t)	29

11. Let two circles S_1 and S_2 having centres O_1 and O_2 have radius R and r respectively $(r < R)$ touching each other internally at a point A. A tangent to smaller circle at point D intersects the larger one at points B and C as shown. Let AB and AC intersects the circles S_2 at points L and K respectively.



	Column-I		Column-II
(a)	$\frac{AK}{AC} =$	**(p)**	$\frac{\sqrt{R}}{\sqrt{R} + \sqrt{R-r}}$
(b)	$\frac{AC}{CD} =$	**(q)**	$\frac{R}{r}$
(c)	$\frac{AL}{AB} =$	**(r)**	$\frac{r}{R-r}$
(d)	$\frac{AO}{AD} =$	**(s)**	$\frac{r}{R}$
	(where O is incentre of ΔABC)	**(t)**	$\sqrt{\frac{R}{R-r}}$

12. Let $S = \{(x, y) : x^2 + y^2 - 6x - 8y + 21 \le 0\}$

Match the Column-I with Column-II

	Column-I		Column-II
(a)	$\max\left\{\frac{12x}{7} - \frac{5y}{7}; (x, y) \in S\right\}$	**(p)**	3
(b)	$\min\left\{\frac{1}{2}(x^2 + y^2 + 1) + (x - y); (x, y) \in S\right\}$	**(q)**	4
(c)	$\max\left\{\frac{3x}{7} + \frac{4y}{7}; (x, y) \in S\right\}$	**(r)**	5
(d)	$\min\left\{\frac{\sqrt{3}y + \lvert x-3\rvert}{\lvert x-3\rvert}; (x, y) \in S\right\}$	**(s)**	6
		(t)	7

ANSWERS

1. $a \to r;\ b \to q;\ c \to p;\ d \to s$

2. $a \to s;\ b \to p;\ c \to r;\ d \to s$

3. $a \to q;\ b \to s;\ c \to p;\ d \to r$

4. $a \to s;\ b \to p;\ c \to q;\ d \to r$

5. $a \to s;\ b \to r;\ c \to q;\ d \to p$

6. $a \to s;\ b \to q;\ c \to r;\ d \to p$

7. $a \to p,q;\ b \to p,q;\ c \to q;\ d \to q,s$

8. $a \to q;\ b \to s;\ c \to r;\ d \to p$

9. $a \to p;\ b \to q;\ c \to r;\ d \to s$

10. $a \to q;\ b \to q;\ c \to t;\ d \to p$

11. $a \to s;\ b \to t;\ c \to s;\ d \to p$

12. $a \to s;\ b \to q;\ c \to r;\ d \to q$

EXERCISE 6

Subjective Problems

1. If the circle C_1 touches x-axis and the line $y = x\tan\theta, \theta \in \left(0, \frac{\pi}{2}\right)$ in first quadrant and circle C_2 touches the line $y = x\tan\theta$, y-axis and circle C_1 in such a way that ratio of radius of C_1 to radius of C_2 is 2 : 1, then value of $\tan\frac{\theta}{2} = \frac{\sqrt{a} - b}{c}$ where a, b, c are relatively prime natural numbers find $a + b + c$.

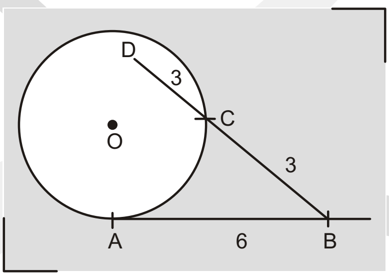
2. A point D is taken on the side AC of an acute triangle ABC, such that $AD = 1, DC = 2$ and BD is an altitude of ΔABC. A circle of radius 2, which passes through points A and D, touches at point D a circle circumscribed about the ΔBDC. The area of ΔABC is A, then $\frac{A^2}{15} =$.

3. How many ordered pair of integers (a, b) satisfy all the following inequalities $a^2 + b^2 < 16, a^2 + b^2 < 8a, a^2 + b^2 < 8b$?

4. Circle S_1 is centered at $(0,3)$ with radius 1. Circle S_2 is externally tangent to circle S_1 and also tangent to x-axis. If the locus of the centre of the variable circle S_2 can be expressed as $y = 1 + \frac{x^2}{\lambda}$. Find λ.

5. Let two parallel lines L_1 and L_2 with positive slope are tangent to the circle $C_1 : x^2 + y^2 - 2x - 16y + 64 = 0$. If L_1 is also tangent to the circle $C_2 : x^2 + y^2 - 2x + 2y - 2 = 0$ and equation of L_2 is $a\sqrt{a}x - by + c - a\sqrt{a} = 0$ where $a, b, c \in N$, then find the value of $\frac{a + b + c}{2}$.

6. In the figure AB is tangent at A to circle with centre O; point D is interior to circle and DB intersects the circle at C. If $BC = DC = 3, OD = 2$ and $AB = 6$, then find the radius of the circle.



7. The lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to the same circle whose radius is r, then $4r$ is equal to.

8. There exists two circles passing through $(-1, 1)$ and touching the lines $x + y = 2$ and $x - y = 2$ whose radii are r_1 and r_2, and $r_1 + r_2 = a\sqrt{2}$, then a is equal to.

9. Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose the tangents at the points. $B(1, 7)$ and $D(4, -2)$ on the circle meet at C. The area of the quadrilateral $ABCD$ is N, then $\frac{N}{15} =$.

10. If radii of the smallest and the largest circle passing through $(\sqrt{3}, \sqrt{2})$ and touching the circle $x^2 + y^2 - 2\sqrt{2}y - 2 = 0$ are r_1 and r_2 respectively, then find the mean of r_1 and r_2.

11. Given a convex quadrilateral $ABCD$ circumscribed about a circle of diameter 1. Inside $ABCD$, there is a point M such that $|MA|^2 + |MB|^2 + |MC|^2 + |MD|^2 = 2$. Find the area of $ABCD$.

12. Tow circle of radii 8 and 4 touch each other externally at a point A. Through a point B taken on the larger circle a straight line is drawn touching the smaller circle at C. Find BC if $AB = \sqrt{6}$.

13. Given a rectangle $ABCD$ where $|AB| = 2a, |BC| = a\sqrt{2}$. On the side AB, as on diameter, a semicircle is constructed externally. Let M be an arbitrary point on the semicircle, the line MD intersect AB at N, and MC at L. If $|AL|^2 + |BN|^2 = \lambda a^2$, then $\lambda =$

14. The distance between centres of two circles is equal to 1. Let x be the side of a rhombus two opposite vertices of which lie on one circle, and the other two on the other if the radii of the circles are 6 and 3, then $[x] =$ (where [.] denote greates integer function)

15. Circle of radii 4 and 2 touch each other internally. The side of equilateral triangle one vertex of which coincides with the point of tangency and the other two lying on the two given circle is:

16. Two circles of radius 8 are placed inside a semi-circle of radius 25. The two circles are each tangent to the diameter and to the semicircle. If the distance between the centers of the two circles is λ, then the value of $\frac{60}{\lambda}$.

ANSWERS

1.	22	**2.**	9	**3.**	6	**4.**	8	**5.**	7	**6.**	4	**7.**	3	**8.**	6	**9.**	5	**10.**	1
11.	1	**12.**	3	**13.**	4	**14.**	6	**15.**	4	**16.**	2								

EXERCISE 7

1. (A) If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$ (where $p, q \neq 0$) are bisected by the x-axis, then:

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$ (c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

(B) Let L_1 be a straight line through the origin and L_2 be the straight line $x + y = 1$. If the intercepts made by the circle $x^2 + y^2 - x + 3y = 0$ on L_1 and L_2 are equal, then which of the following equations can represent L_1 ? **[IIT-JEE 1999]**

(a) $x + y = 0$ (b) $x - y = 0$ (c) $x + 7y = 0$ (d) $x - 7y = 0$

2. (A) The triangle PQR is inscribed in the circle, $x^2 + y^2 = 25$. If Q and R have co-ordinates $(3, 4)$ and $(-4, 3)$ respectively, then $\angle QPR$ is equal to:

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{6}$

(B) If the circles, $x^2 + y^2 + 2x + 2ky + 6 = 0$ and $x^2 + y^2 + 2ky + k = 0$ intersect orthogonally, then 'k' is: **[IIT-JEE (Screening) 2000]**

(a) 2 or $-\frac{3}{2}$ (b) -2 or $-\frac{3}{2}$ (c) 2 or $\frac{3}{2}$ (d) -2 or $\frac{3}{2}$

3. (A) Extremities of a diagonal of a rectangle are $(0, 0)$ and $(4, 3)$. Find the equation of the tangents to the circumcircle of a rectangle which are parallel to this diagonal.

(B) Find the point on the straight line, $y = 2x + 11$ which is nearest to the circle,

$$16(x^2 + y^2) + 32x - 8y - 50 = 0$$

(C) A circle of radius 2 units rolls on the outerside of the circle, $x^2 + y^2 + 4x = 0$, touching it externally. Find the locus of the centre of this outer circle. Also find the equations of the common tangents of the two circles when the line joining the centres of the two circles is inclined at an angle of $60°$ with x-axis. **[REE (Mains) 2000]**

4. (A) Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle then $2r$ equals:

[IIT-JEE (Screening) 2001]

(a) $\sqrt{PQ \cdot RS}$ (b) $\frac{PQ + RS}{2}$

(c) $\frac{2PQ \cdot RS}{PQ + RS}$ (d) $\sqrt{\frac{(PQ)^2 + (RS)^2}{2}}$

(B) Let $2x^2 + y^2 - 3xy = 0$ be the equation of a pair of tangents drawn from the origin 'O' to a circle of radius 3 with centre in the first quadrant. If A is one of the points of contact, find the length of OA. **[IIT-JEE (Mains) 2001]**

5. (A) Find the equation of the circle which passes through the points of intersection of circles $x^2 + y^2 - 2x - 6y + 6 = 0$ and $x^2 + y^2 + 2x - 6y + 6 = 0$ and intersects the circle $x^2 + y^2 + 4x + 6y + 4 = 0$ orthogonally. **[REE (Mains) 2001]**

(B) Tangents TP and TQ are drawn from a point T to the circle $x^2 + y^2 = a^2$. If the point T lies on the line $px + qy = r$, find the locus of centre of the circumcircle of triangle TPQ. **[REE (Mains) 2001]**

6. (A) If the tangent at the point P on the circle $x^2 + y^2 + 6x + 6y = 2$ meets the straight line $5x - 2y + 6 = 0$ at a point Q on the y-axis, then the length of PQ is:

(a) 4 (b) $2\sqrt{5}$ (c) 5 (d) $3\sqrt{5}$

(B) If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1 + m^2}$ is a common tangent to $x^2 + y^2 = b^2$ and $(x - a)^2 + y^2 = b^2$ is: **[IIT-JEE (Screening) 2002]**

(a) $\frac{2b}{\sqrt{a^2 - 4b^2}}$ (b) $\frac{\sqrt{a^2 - 4b^2}}{2b}$ (c) $\frac{2b}{a - 2b}$ (d) $\frac{b}{a - 2b}$

7. The radius of the circle, having centre at $(2, 1)$, whose one of the chord is a diameter of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ **[IIT-JEE (Screening) 2004]**

(a) 1 (b) 2 (c) 3 (d) $\sqrt{3}$

8. Line $2x + 3y + 1 = 0$ is a tangent to a circle at $(1, -1)$. This circle is orthogonal to a circle which is drawn having diameter as a line segment with end points $(0, -1)$ and $(-2, 3)$. Find equation of circle. **[IIT-JEE 2004]**

9. A circle is given by $x^2 + (y - 1)^2 = 1$, another circle C touches it externally and also the x-axis, then the locus of its centre is: **[IIT-JEE (Screening) 2005]**

(a) $\{(x, y) : x^2 = 4y\} \cup \{(x, y) : y \le 0\}$

(b) $\{(x, y) : x^2 + (y - 1)^2 = 4\} \cup \{(x, y) : y \le 0\}$

(c) $\{(x, y) : x^2 = y\} \cup \{(0, y) : y \le 0\}$

(d) $\{(x, y) : x^2 = 4y\} \cup \{(0, y) : y \le 0\}$

10. (A) Let $ABCD$ be a quadrilateral with area 18, with side AB parallel to the side CD and $AB = 2CD$. Let AD be perpendicular to AB and CD. If a circle is drawn inside the quadrilateral $ABCD$ touching all the sides, then its radius is:

(a) 3 (b) 2 (c) 3/2 (d) 1

(B) Tangents are drawn from the point $(17, 7)$ to the circle $x^2 + y^2 = 169$.

Statement-1: The tangents are mutually perpendicular.

because

Statement-2: The locus of the points from which mutually perpendicular tangents can be drawn to the given circle is $x^2 + y^2 = 338$. **[IIT-JEE 2007]**

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

11. (A) Consider the two curves

$C_1 : y^2 = 4x;\quad C_2 : x^2 + y^2 - 6x + 1 = 0$. Then:

(a) C_1 and C_2 touch each other only at one point

(b) C_1 and C_2 touch each other exactly at two points

(c) C_1 and C_2 intersect (but do not touch) at exactly two points

(d) C_1 and C_2 neither intersect nor touch each other

(B) Consider, $L_1 : 2x + 3y + P - 3 = 0;\quad L_2 : 2x + 3y + P + 3 = 0$,

where P is a real number and $C : x^2 + y^2 + 6x - 10y + 30 = 0$.

Statement-1: If line L_1 is a diameter of circle C, then line L_2 is not always a diameter of circle C.

because

Statement-2: If line L_1 is a diameter of circle C, then line L_2 is not a chord of circle C.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

(C) Comprehension

A circle C of radius 1 is inscribed in an equilateral triangle PQR. The points of contact of C with the sides PQ, QR, RP are D, E, F respectively. The line PQ is given by the equation $\sqrt{3}x + y - 6 = 0$ and the point D is $\left(\frac{3\sqrt{3}}{2}, \frac{3}{2}\right)$.

Further, it is given that the origin and the centre of C are on the same side of the line PQ. **[IIT-JEE 2008]**

(i) The equation of circle C is:

(a) $(x - 2\sqrt{3})^2 + (y - 1)^2 = 1$

(b) $(x - 2\sqrt{3})^2 + \left(y + \frac{1}{2}\right)^2 = 1$

(c) $(x - \sqrt{3})^2 + (y + 1)^2 = 1$

(d) $(x - \sqrt{3})^2 + (y - 1)^2 = 1$

(ii) Points E and F are given by:

(a) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), (\sqrt{3}, 0)$ (b) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right), (\sqrt{3}, 0)$

(c) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ (d) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right), \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(iii) Equations of the sides RP, RQ are:

(a) $y = \frac{2}{\sqrt{3}}x + 1, y = -\frac{2}{\sqrt{3}}x - 1$ (b) $y = \frac{1}{\sqrt{3}}x, y = 0$

(c) $y = \frac{\sqrt{3}}{2}x + 1, y = -\frac{\sqrt{3}}{2}x - 1$ (d) $y = \sqrt{3}x, y = 0$

12. Tangents drawn from the point $P(1, 8)$ to the circle $x^2 + y^2 - 6x - 4y - 11 = 0$ touch the circle at the points A and B. The equation of the circumcircle of the triangle PAB is: **[IIT 2009]**

(a) $x^2 + y^2 + 4x - 6x + 19 = 0$ (b) $x^2 + y^2 - 10y + 19 = 0$

(c) $x^2 + y^2 - 2x + 6y - 29 = 0$ (d) $x^2 + y^2 - 6x - 4y + 19 = 0$

13. The centres of two circles C_1 and C_2 each of unit radius are at a distance of 6 units from each other. Let P be the mid-point of the line segment joining the centres of C_1 and C_2 and C be a circle touching circles C_1 and C_2 externally. If a common tangent to C_1 and C passing through P is also a common tangent to C_2 and C, then the radius of the circle C is: **[IIT 2009]**

14. The circle passing through the point $(-1, 0)$ and touching the y-axis at $(0, 2)$ also passes through the point: **[IIT 2011]**

(a) $\left(-\frac{3}{2}, 0\right)$ (b) $\left(-\frac{5}{2}, 2\right)$ (c) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (d) $(-4, 0)$

15. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{4}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is: **[IIT 2011]**

16. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is: **[IIT-JEE 2012]**

(a) $20(x^2 + y^2) - 36x + 45y = 0$ (b) $20(x^2 + y^2) + 36x - 45y = 0$

(c) $36(x^2 + y^2) - 20x + 45y = 0$ (d) $36(x^2 + y^2) + 20x - 45y = 0$

Paragraph for question Nos. 17 to 18

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3}, 1)$. A straight line L, perpendicular to PT is a tangent to the circle $(x - 3)^2 + y^2 = 1$.

[IIT-JEE 2012]

17. A common tangent of the two circles is:

(a) $x = 4$ (b) $y = 2$ (c) $x + \sqrt{3}y = -1$ (d) $x + 2\sqrt{2}y = 6$

18. A possible equation of L is:

(a) $x - \sqrt{3}y = +1$ (b) $x + \sqrt{3}y = 1$

(c) $x - \sqrt{3}y = -1$ (d) $x + \sqrt{3}y = 5$

19. The circle passing through $(1, -2)$ and touching the axis of x at $(3, 0)$ also passes through the point: **[IIT-JEE (Mains) 2013]**

(a) $(-2, 5)$ (b) $(-5, 2)$

(c) $(2, -5)$ (d) $(5, -2)$

20. Circle(s) touching x-axis at a distance 3 from the origin and having an intercept of length $2\sqrt{7}$ on y-axis is(are): **[IIT-JEE (Advance) 2013]**

(a) $x^2 + y^2 - 6x + 8y + 9 = 0$ (b) $x^2 + y^2 - 6x + 7y + 9 = 0$

(c) $x^2 + y^2 - 6x - 8y + 9 = 0$ (d) $x^2 + y^2 - 6x - 7y + 9 = 0$

ANSWERS

1. **(A)** d; **(B)** b, c **2.** **(A)** c; **(B)** a

3. **(A)** $6x - 8y + 25 = 0$ and $6x - 8y - 25 = 0$; **(B)** $(-9/2, 2)$

(C) $x^2 + y^2 + 4x - 12 = 0, T_1 : \sqrt{3}x - y + 2\sqrt{3} + 4 = 0, T_2 : \sqrt{3}x - y + 2\sqrt{3} - 4 = 0$ (D.C.T.)

$T_3 : x + \sqrt{3}y - 2 = 0, T_4 : x + \sqrt{3}y + 6 = 0$ (T.C.T.)

4. **(A)** a; **(B)** $OA = 3(3 + \sqrt{10})$

5. **(A)** $x^2 + y^2 + 14x - 6y + 6 = 0$; **(B)** $2px + 2qy = r$

6. **(A)** c; **(B)** a

7. c **8.** $2x^2 + 2y^2 - 10x - 5y + 1 = 0$ **9.** d

10. **(A)** b; **(B)** a **11.** **(A)** b; **(B)** c; **(C)** **(i)** d; **(ii)** a; **(iii)** d

12. b **13.** 8 **14.** d **15.** 2

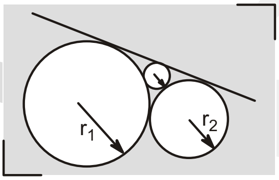
16. a **17.** d **18.** a **19.** d

20. a, c

SOLUTIONS 1

Only One Choice is Correct:

1. (a)

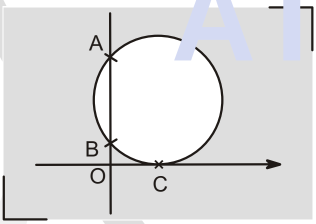


$$2\sqrt{rr_1} + 2\sqrt{rr_2} = 2\sqrt{r_1 r_2}$$

$\Rightarrow$ $\sqrt{r}(6+3) = 6 \times 3$

$\Rightarrow$ $r = 4$

2. (b) For $\angle ACB$ to be maximum, circle passing through A, B will touch x-axis at C.

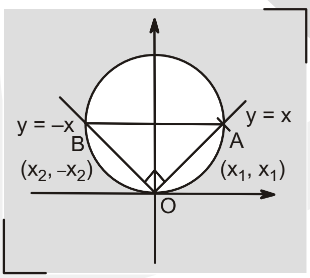


$$OC^2 = (OA)(OB)$$

$$x^2 = ab$$

$$x = \sqrt{ab}$$

3. (c)



$$(x - x_1)(x - x_2) + (y - x_1)(y + x_2) = 0$$

$$x^2 + y^2 - (x_1 + x_2)x + (x_2 - x_1)y = 0$$

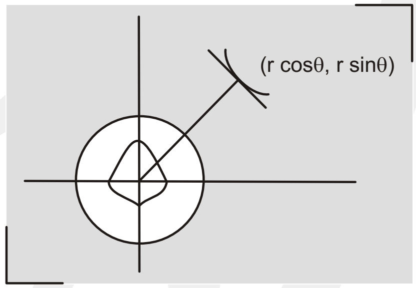
$$x^2 + y^2 + ax \pm (\sqrt{a^2 - 4b})\, y = 0$$

4. (b)

$$r^2(2\cos^2\theta + 10\sin^2\theta + 6\sin\theta\cos\theta) = 1$$

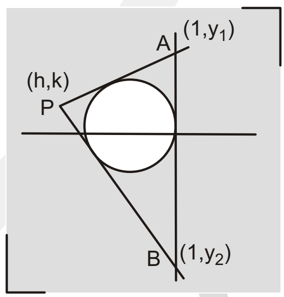
$$r^2 = \frac{1}{3\sin 2\theta - 4\cos 2\theta + 6} \le \frac{1}{6-5} = 1$$

$$r \le 1$$



Minimum distance between curves = [illegible] − [illegible] = [illegible]

5. (d) Equation of pair of tangents PA and PB is



$$(xh + yk - 1)^2 = (x^2 + y^2 - 1)(h^2 + k^2 - 1)$$

Put $x = 1$, $(h-1)^2 + 2ky(h-1) = y^2(h^2 - 1)$

$$y^2(h+1) - 2ky - (h-1) = 0 \quad \left< {y_1 \atop y_2} \right.$$

$$AB = |y_1 - y_2| = 2$$

$$\Rightarrow \quad 4 = \frac{4k^2}{(h+1)^2} + \frac{4(h-1)}{(h+1)}$$

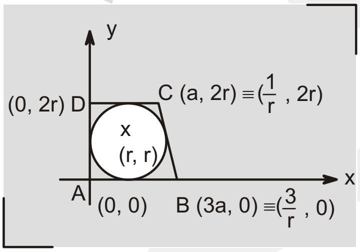
$$(h+1) = k^2 + (h^2 - 1) \Rightarrow k^2 = 2(h+1)$$

$$y^2 = 2(x+1)$$

6. (d) Area of trapezium

$$ABCD = \frac{1}{2}(a+3a)(2r) = 4$$

$$\Rightarrow \quad ar = 1$$



Equation of BC is $y = -r^2\left(x - \frac{3}{r}\right)$

$$y + r^2x - 3r = 0$$

$\because$ BC is tangent to circle

$$\Rightarrow \quad \frac{|r + r^3 - 3r|}{\sqrt{1+r^4}} = r$$

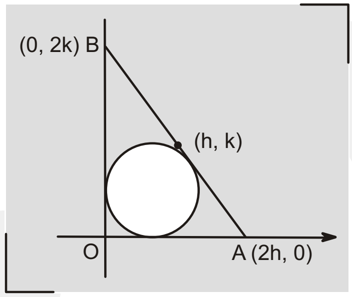
$$\Rightarrow \quad r^4 + 4 - 4r^2 = 1 + r^4$$

$$\Rightarrow \quad r = \frac{\sqrt{3}}{2}$$

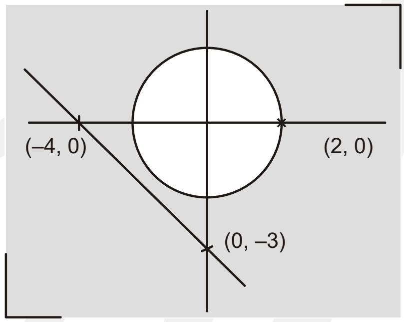
7. (a) $r = \frac{\Delta}{s} = \dfrac{\frac{1}{2}\times(2h)(2k)}{\frac{1}{2}(2h + 2k + 2\sqrt{h^2+k^2})}$

$$2 = \frac{2hk}{h + k + \sqrt{h^2+k^2}}$$

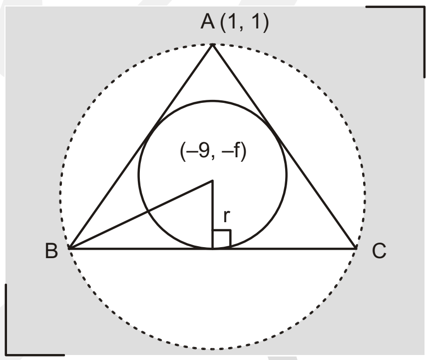
$\Rightarrow$ Locus of $xy = x + y + \sqrt{x^2+y^2}$



8. (d) Line does not intersect the circle



9. (b) $r = R\cos 60° = \frac{R}{2}$



Equation of incircle is

$$x^2 + y^2 + 2gx + 2fy + g^2 + f^2$$

$$= \frac{g^2 + f^2 - c}{4} = \frac{(g+1)^2}{4} + \frac{(f+1)^2}{4}$$

$$R^2 = (g+1)^2 + (f+1)^2$$

$= g^2 + f^2 + 2g + 2f + 2 = g^2 + f^2 - c$

$\therefore$ Equation of incircle becomes

$4(x^2 + y^2) + 8gx + 8fy$

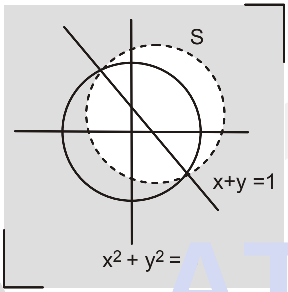
$= 2(g + f) - 3(g^2 + f^2) + 2$

$= (2g - 3g^2 + 1) + (2f - 3f^2 + 1)$

$= (1 - g)(1 + 3g) + (1 - f)(1 + 3f)$

10. (b) $S \equiv x^2 + y^2 - 9 + \lambda(x + y - 1)$

Centre $= \left(\frac{-\lambda}{2}, \frac{-\lambda}{2}\right)$



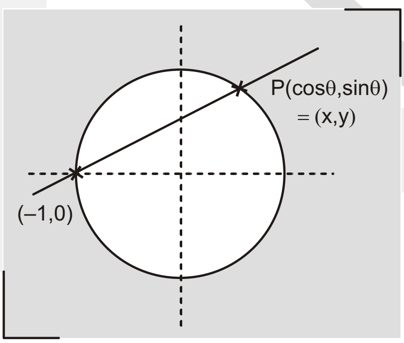
Centre lies on $x + y = 1$

$\Rightarrow \quad -\lambda = 1$

$S \equiv x^2 + y^2 - x - y - 8 = 0$

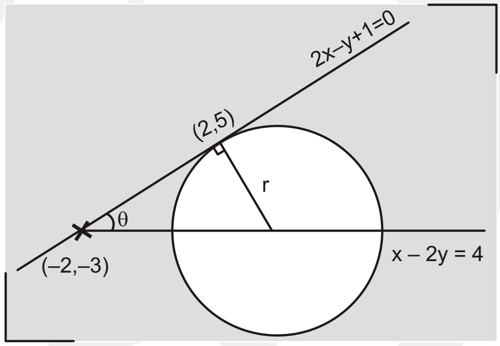
11. (d) $m = \dfrac{\sin\theta}{\cos\theta + 1} = \tan\left(\dfrac{\theta}{2}\right)$

$$P(x, y) = \left(\frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}, \frac{2\tan\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}}\right)$$



$\therefore \quad x, y \in Q$, if $m \in Q \Rightarrow$ Infinite points are possible.

12. (a)

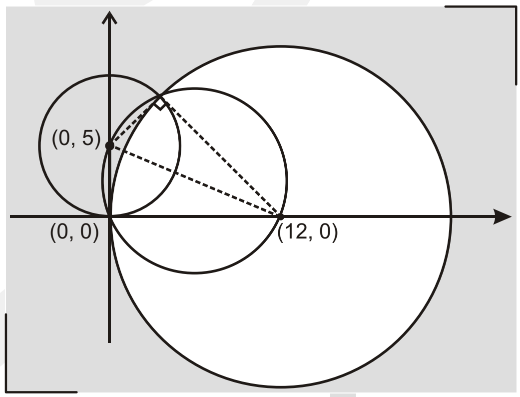


$$\tan\theta = \frac{2 - \frac{1}{2}}{1 + 2\left(\frac{1}{2}\right)} = \frac{3}{4}$$

$$\frac{r}{\sqrt{8^2 + 4^2}} = \tan\theta = \frac{3}{4} = \frac{r}{4\sqrt{5}}$$

$\Rightarrow \quad r = 3\sqrt{5}$

13. (a)



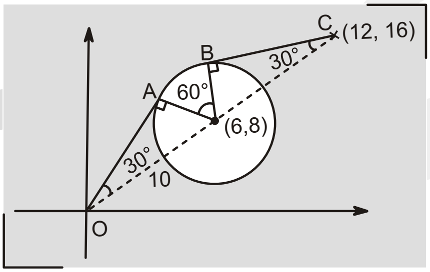
From figure it is clear that circle has AB as diameter

$\Rightarrow \quad r = \dfrac{13}{2}$

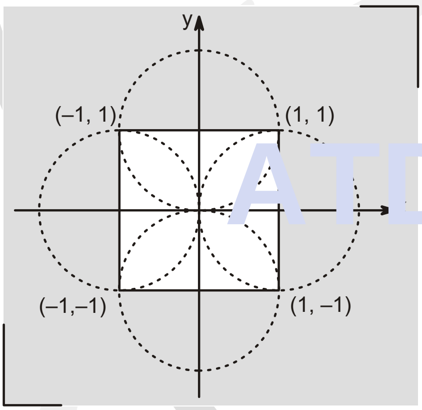
14. (c) *OABC* in the shortest path

$$OA + \overline{AB} + BC = 10\cos 30° + \frac{5\pi}{3} + 10\cos 30°$$

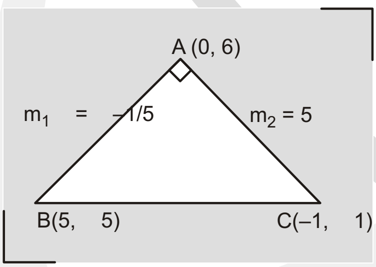
$$= 10\sqrt{3} + \frac{5\pi}{3}$$



15. (c) Ar. (square of sides 2) = 4 sq. units



16. (d) Note that the *ABC* is right angled at *A*



$\therefore$ equation of circle

$$(x+1)(x-5) + (y-1)(y-5) = 0$$

$$x^2 + y^2 - 4x - 6y - 5 + 5 = 0$$

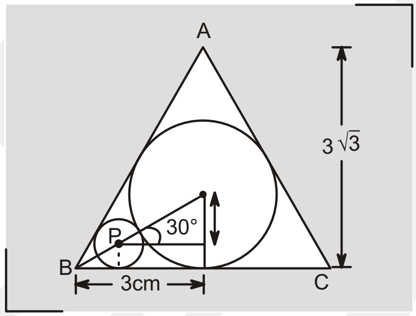
$$x^2 + y^2 - 4x - 6y = 0$$

Hence the circle passes through the origin

$\therefore$ tangent at (0, 0) is $4x + 6y = 0$

$$2x + 3y = 0$$

17. (a)



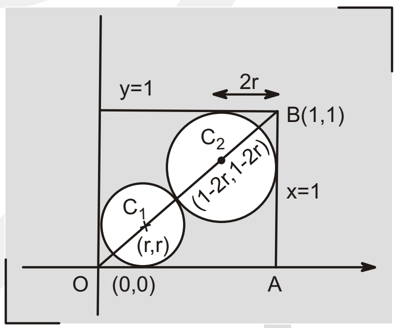
$$x = \sqrt{3} - r = PB$$

now $\dfrac{\sqrt{3} - r}{\sqrt{3} + r} = \sin 30° = \dfrac{1}{2}$

$$\frac{r}{\sqrt{3}} = \frac{2-1}{2+1} = \frac{1}{3}$$

$$r = \frac{1}{\sqrt{3}}$$

18. (c) C_1, C_2 lie on $y = x$ line



$$OB = OC_1 + C_1C_2 + C_2B$$

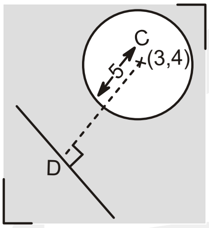
$$\sqrt{2} = r\sqrt{2} + 3r + 2r\sqrt{2}$$

$$r = \frac{\sqrt{2}}{3(\sqrt{2}+1)}$$

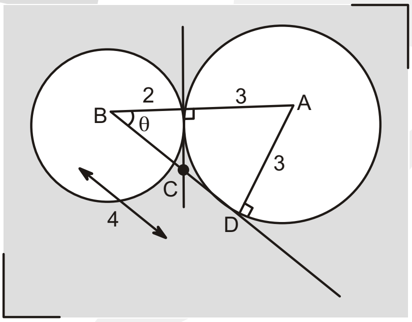
19. (a) Shortest distance $= CD - r$

$$= \frac{|9 - 16 - 25|}{5} - 5$$

$$= \frac{32}{5} - 5 = \frac{7}{5}$$



20. (b)



$$BC = \frac{2}{\cos\theta} = \frac{2}{\frac{4}{5}} = \frac{5}{2}$$

$$CD = 4 - \frac{5}{2} = \frac{3}{2}$$

21. (b) $(AC)^2 + r^2 = (36)^2$ ...(1)

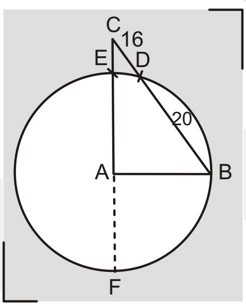
$(CE)(CF) = (CD)(BC)$

$\Rightarrow (AC - r)(AC + r) = 16 \times 36$ ...(2)

On adding (1) and (2), we get

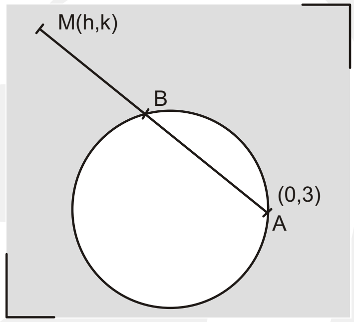
$\Rightarrow 2(AC)^2 = 36 \times 52$

$\Rightarrow AC = 6\sqrt{26}$



22. (b) $B \equiv \left(\frac{h}{2}, \frac{k+3}{2}\right)$

Put B to equation of circle



$$\Rightarrow \left(\frac{h}{2}\right)^2 + 4\left(\frac{h}{2}\right) + \left(\frac{k+3}{2} - 3\right)^2 = 0$$

Required locus is

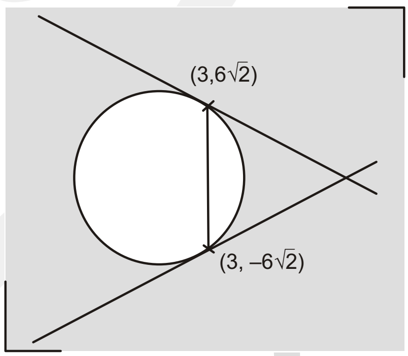
$x^2 + 8x + (y-3)^2 = 0$

23. (b) Equation of pair of tangents is

$(3x + 6\sqrt{2}y - 81)(3x - 6\sqrt{2}y - 81) = 0$

$(x - 27)^2 - 8y^2 = 0$

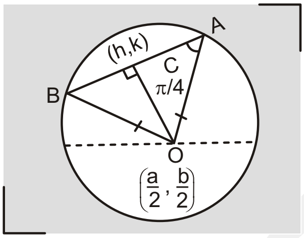
$x^2 - 54x - 8y^2 + 729 = 0$



24. (c) $OA^2 = AC^2 + OC^2$

$$\frac{a^2}{4} + \frac{b^2}{4} = 2\left[\left(h - \frac{a}{2}\right)^2 + \left(k - \frac{b}{2}\right)^2\right]$$

$$\Rightarrow h^2 + k^2 - ah - bk + \frac{a^2}{8} + \frac{b^2}{8} = 0$$

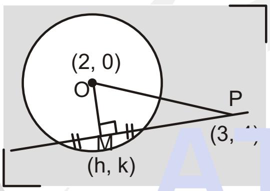


Required Locus is

$$x^2+y^2-ax-by+\frac{a^2+b^2}{8}=0$$

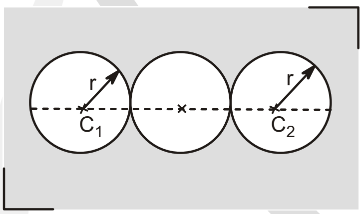
25. (a) Locus is Arc of the circle with OP as diameter intercepted by the given circle. *i.e.*,

$$(x-2)(x-3)+y(y-4)=0$$



26. (d) Centre will be mid point of C_1 and C_2

$$\equiv\left(\frac{0+4}{2},\frac{1+9}{2}\right)=(2,5)$$

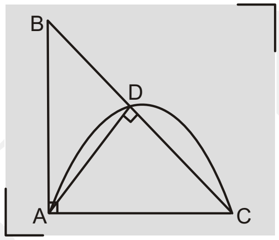


27. (d) $(AC^2)=(AD)^2+(CD)^2$

$$(AB)^2=(BD)(BC)=BD(BD+DC)$$

$$\Rightarrow (BD)(DC)=(AB)^2-(BD)^2=(AD)^2$$

$$\Rightarrow DC=\frac{AD^2}{BD}=\frac{(AD)^2}{\sqrt{(AB)^2-(AD)^2}}$$



$$\therefore\ (AC)^2=(AD)^2+\frac{(AD)^4}{(AB^2)-(AD)^2}$$

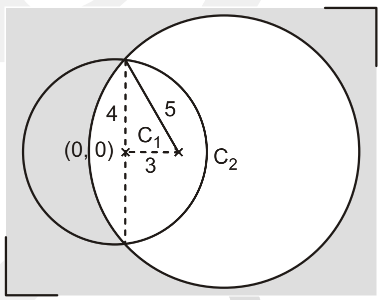
$$=\frac{(AB)^2(AD)^2}{(AB)^2-(AD)^2}$$

$$AC=\frac{(AB)(AD)}{\sqrt{(AB)^2-(AD)^2}}$$

28. (b) Slope of $C_1C_2=-\frac{4}{3}=\tan\theta$

$$\Rightarrow C_2\equiv\left(0+3\left(-\frac{3}{5}\right),0+3\left(\frac{4}{5}\right)\right)$$

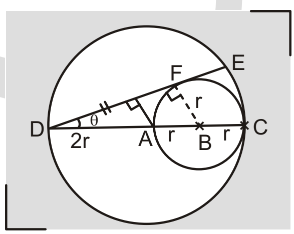
or $\equiv\left(0-3\left(-\frac{3}{5}\right),0-3\left(\frac{4}{5}\right)\right)$



$$C_2\equiv\left(\frac{9}{5},-\frac{12}{5}\right)\text{ or }\left(-\frac{9}{5},\frac{12}{5}\right)$$

$$4r=12\Rightarrow r=3$$

29. (a)

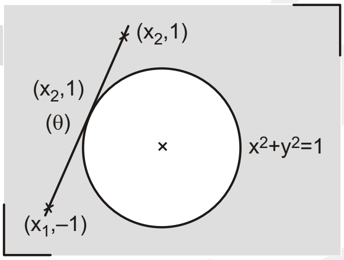


$$\sin\theta = \frac{r}{3r} = \frac{1}{3}$$

$$DE = 2(2r\cos\theta) = 4\times 3\left(\frac{2\sqrt{2}}{3}\right)$$

$$DE = 8\sqrt{2}$$

30. (a) Let the equation of tangent is



$x\cos\theta + y\sin\theta = 1$

$(x_1, -1)$ lies on tangent

$\Rightarrow \quad x_1\cos\theta - \sin\theta = 1$

$x_1\cos\theta = 1 + \sin\theta \quad \ldots(1)$

now $(x_2, 1)$ lies on tangent

$\Rightarrow \quad x_2\cos\theta = 1 - \sin\theta \quad \ldots(2)$

$(1)\times(2), \quad x_1x_2\cos^2\theta = 1 - \sin^2\theta = \cos^2\theta$

$\Rightarrow \quad x_1x_2 = 1$

31. (c) Equation of chord of contact of the pair of tangent from (0, 0) and (g, f) are

$gx + fy + c = 0 \quad \ldots(1)$

and $\quad gx + fy + \dfrac{g^2 + f^2 + c}{2} = 0 \quad \ldots(2)$

These lines are parallel

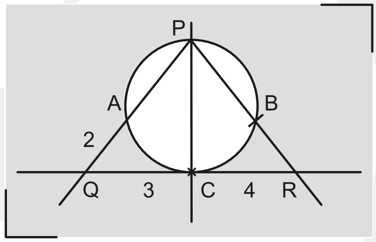
Hence distance $= \left|\dfrac{c - \dfrac{g^2 + f^2 + c}{2}}{\sqrt{g^2 + f^2}}\right|$

32. (c) Locus of centres will be the radical axis of two given circles given by

$$9x - 10y + 11 = 0$$

33. (b) $(PQ)(AQ) = (QC)^2$

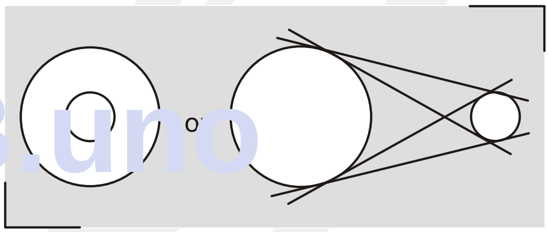
$\Rightarrow \quad PQ = \dfrac{9}{2}$



$\dfrac{QC}{RC} = \dfrac{PQ}{PR} \Rightarrow PR = 6$

$(RC)^2 = (RB)(RP) \Rightarrow RB = \dfrac{8}{3}$

34. (d)



35. (a) Put $\left(h, -\dfrac{b}{2}\right)$ to equation of circle

$h^2 + \dfrac{b^2}{4} - ah + \dfrac{b^2}{2} = 0$

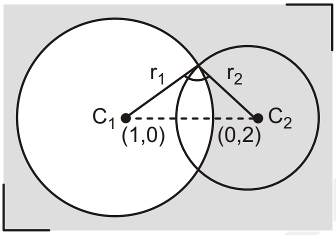
$h^2 - ah + \dfrac{3b^2}{4} = 0$ must get two distinct real roots

$\Rightarrow \quad D > 0 \Rightarrow a^2 - 3b^2 > 0$

36. (b) $\cos\theta = \dfrac{r_1^2 + r_2^2 - (C_1C_2)^2}{2r_1r_2}$

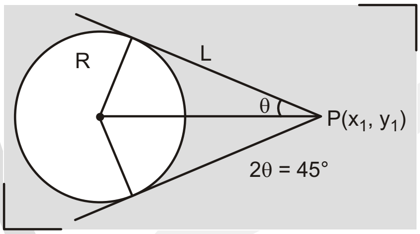
$= \dfrac{10 + 5 - 5}{2\cdot\sqrt{10}.\sqrt{5}} = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta = \dfrac{\pi}{4}$



37. (c) $\tan\theta = \frac{R}{L}$ where $2\theta = 45°$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$



$$1 = \frac{2(R/L)}{1-(R^2/L^2)} = \frac{2RL}{L^2 - R^2}$$

$$= \frac{2a\sqrt{x_1^2 + y_1^2 - a^2}}{x_1^2 + y_1^2 - a^2 - a^2}$$

$$(x^2 + y^2 - 2a^2)^2 = 4a^2(x^2 + y^2 - a^2)$$

$$(x^2 + y^2)^2 + 4a^4 - 4a^2(x^2 + y^2)$$

$$= 4a^2(x^2 + y^2 - a^2)$$

$$(x^2 + y^2)^2 + 8a^4 = 8a^2(x^2 + y^2)$$

$$(x^2 + y^2)^2 = 8a^2(x^2 + y^2 - a^2)$$

$\Rightarrow \quad \lambda = 8$

38. (c) Let the circle be

$$(x-r)^2 + (y-r)^2 = r^2$$

$$x^2 + y^2 - 2rx - 2ry + r^2 = 0 \begin{cases} r_1 \\ r_2 \end{cases}$$

Orthogonality

$$\Rightarrow \quad 2r_1r_2 + 2r_1r_2 = r_1^2 + r_2^2$$

$$\Rightarrow \quad 6r_1r_2 = (r_1 + r_2)^2 \qquad \ldots(1)$$

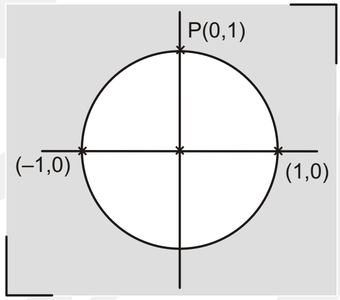
Circle passes through (a, b)

$$\Rightarrow \quad r^2 - 2(a+b)r + (a^2 + b^2) = 0$$

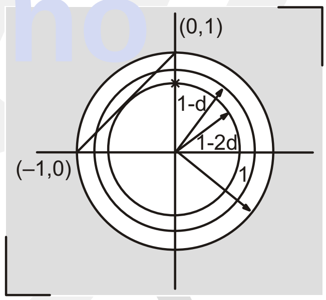
From (1), $6(a^2 + b^2) = 4(a+b)^2$

$$\Rightarrow \quad a^2 + b^2 - 4ab = 0$$

39. (d)



40. (c) For max common difference, the smallest circle just touches the line



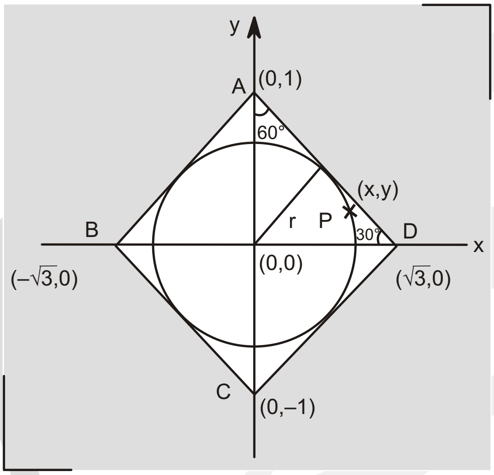
$$\Rightarrow \quad 1 - 2d = \frac{1}{\sqrt{2}}$$

$$d = \frac{\sqrt{2}-1}{2\sqrt{2}}$$

$$\therefore \quad d \in \left(0, \frac{\sqrt{2}-1}{2\sqrt{2}}\right)$$

$$i.e., \quad \left(0, \frac{2-\sqrt{2}}{4}\right)$$

41. (b) $r = \sqrt{3}\sin 30^\circ = \dfrac{\sqrt{3}}{2}$



$(PA)^2 + (PB)^2 + (PC)^2 + (PD)^2$

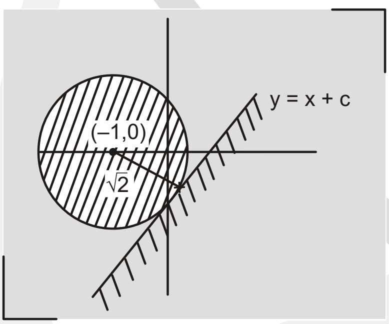
$= (x-\sqrt{3})^2 + y^2 + x^2 + (y-1)^2$

$+(x+\sqrt{3})^2 + y^2 + x^2 + (y+1)^2$

$= 4(x^2 + y^2 + 2)$

$= 4\left(\dfrac{3}{4} + 2\right) = 11$

42. (d) Line is tangent to circle as shown



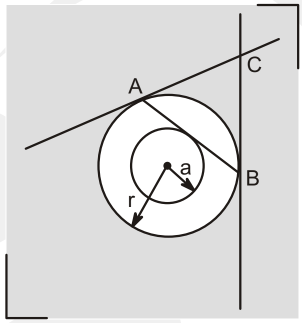
$\Rightarrow \dfrac{1-c}{\sqrt{2}} = \sqrt{2}$

$[\because (-1,0)$ lies above the line $y - x - c = 0]$

$\Rightarrow c = -1$

43. (a) Equation of AB is $xh + yk = r^2$

AB is tangent to $x^2 + y^2 = a^2$



$\Rightarrow \dfrac{r^2}{\sqrt{h^2+k^2}} = a$

$\Rightarrow r^2 = ab \qquad [\because h^2 + k^2 = b^2]$

$\Rightarrow$ Equation of circle is $x^2 + y^2 = ab$

44. (a) Equation of AB is

$xh + yk = 1 \qquad \ldots(1)$

$AB \equiv S_1 - S_2 = 0$

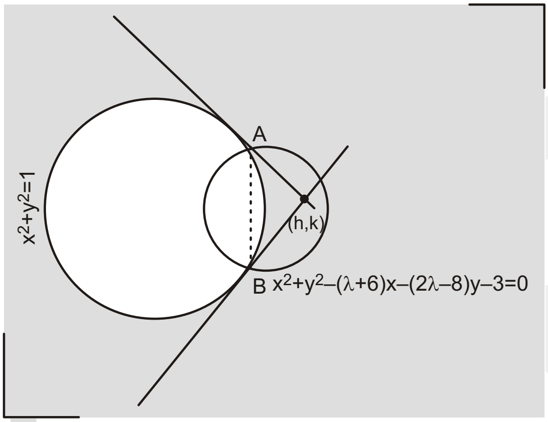
$\Rightarrow (\lambda+6)x + (2\lambda-8)y + 2 = 0 \qquad \ldots(2)$

(1) & (2) are identical

$\Rightarrow \dfrac{h}{\lambda+6} = \dfrac{k}{2\lambda-8} = \dfrac{-1}{2}$

$\Rightarrow \dfrac{2h}{2\lambda+12} = \dfrac{k}{2\lambda-8} = \dfrac{2h-k}{20}$

$= -\dfrac{1}{2}\left(\text{as } \dfrac{a}{b} = \dfrac{c}{b} = \dfrac{a-c}{b-d}\right)$

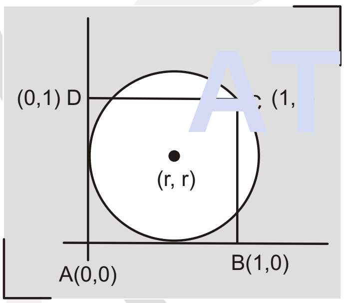


$\Rightarrow \quad 2x - y = -10$

$\Rightarrow \quad 2x - y + 10 = 0$

45. (a) Equation of circle is

$x^2 + y^2 - 2rx - 2ry + r^2 = 0$



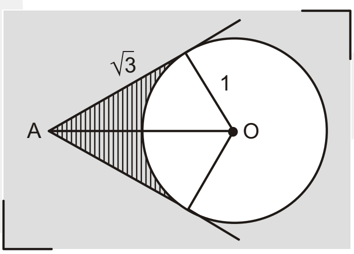
Put $(1, 1)$

$\Rightarrow \quad r^2 - 4r + 2 = 0,\ r = 2 \pm \sqrt{2}$

$\Rightarrow \quad r = 2 - \sqrt{2} \qquad (\because r < 1)$

46. (b) $r = 1;\ L = \sqrt{3}$

area of quadrilateral $= \sqrt{3}$

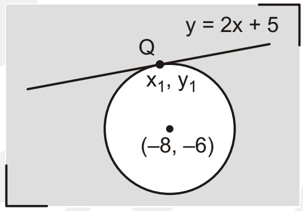


area of sector $= \dfrac{1}{2} \cdot 1 \cdot \dfrac{2\pi}{3} = \dfrac{\pi}{3}$

shaded region $= \sqrt{3} - \dfrac{\pi}{2}$

47. (d) $y_1 = 2x_1 + 5 \quad]$

and $\dfrac{(y_1 + 6)}{x_1 + 8} \times 2 = -1$

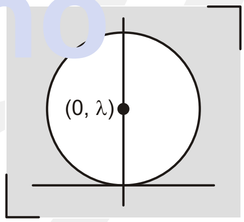


$\Rightarrow \quad x_1 = -6 \quad$ and $\quad y_1 = -7$

48. (a) Equation of $\odot$ is

$x^2 + (y - \lambda)^2 = \lambda^2$

$x^2 + y^2 - 2\lambda y = 0 \qquad \ldots(1)$



Let the pole be (h, k). It's polar w.r.t. (1) is

$xh + ky - \lambda(k + y) = 0$

$xh + (k - \lambda)y - \lambda k = 0 \qquad \ldots(2)$

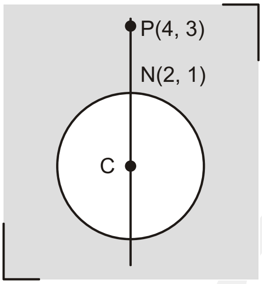
compare it with $lx + my + n = 0$ and eliminate λ.

49. (c) Diameter of the circle is given as

$2x - y - 2 = 0 \qquad \ldots(1)$

slope of PN: $\dfrac{3 - 1}{4 - 2} = 1$

equation of normal through PN is



$$y-1=(x-2)$$

$$x-y-1=0 \qquad \ldots(2)$$

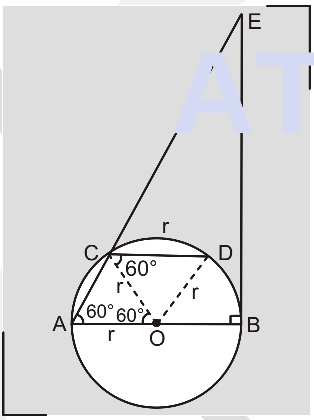
solving (1) and (2), centre is (1, 0)

Hence equation of the circle is

$$(x-1)^2+y^2=(2-1)^2+1$$

$$x^2+y^2-2x-1=0$$

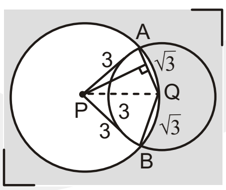
50. (d)



From figure it is clear that

$$\Rightarrow \quad AE=\frac{AB}{\cos 60°}=2(AB)$$

51. (b) Area of $APBQ=2$ (area ΔAPQ)



$$=2\left(\frac{1}{2}\times\sqrt{3}\times\sqrt{3^2-\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

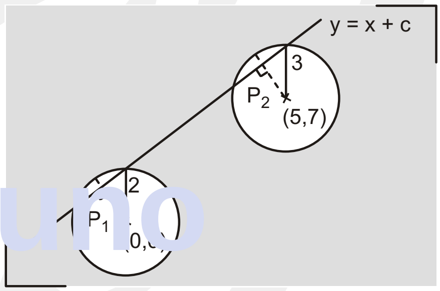
$$=\sqrt{3}\cdot\frac{\sqrt{33}}{2}=\frac{\sqrt{99}}{2}$$

52. (a) $r_1^2-P_1^2=r_2^2-P_2^2$

$$\Rightarrow \quad 4-\frac{C^2}{2}=9-\frac{(2-C)^2}{2}$$

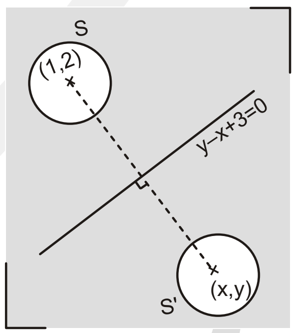
$$\Rightarrow \quad C=-\frac{3}{2}$$

$$\Rightarrow \quad \text{equation of line } y=x-\frac{3}{2}$$



53. (a) $\dfrac{x-1}{-1}=\dfrac{y-2}{1}=-2\dfrac{(2-1+3)}{2}$

$$\Rightarrow \quad (x,y)=(5,-2)$$



Eqn. of S' is $(x-5)^2+(4+2)^2=1$

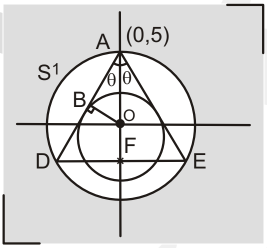
$$x^2+y^2-10x+4y+28=0$$

54. (a) $\sin\theta = \frac{3}{5}$

Slope of AB, AE are

$$\tan\left(\frac{\pi}{2}-\theta\right), \tan\left(\frac{\pi}{2}+\theta\right) = \frac{4}{3}, -\frac{4}{3}$$

Equation of OB is $y = -\frac{3}{4}x$



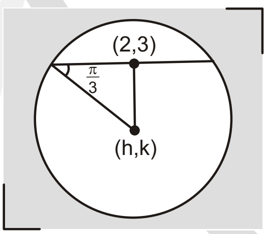
$\Rightarrow \quad 4y + 3x = 0$

D is image of A w.r.t. OB

$$\frac{x-0}{3} = \frac{y-5}{4} = -2\,\frac{4(5)}{25} = -\frac{8}{5}$$

$$D \equiv \left(\frac{-24}{5}, \frac{-7}{5}\right) \Rightarrow OF = \frac{7}{5}$$

55. (a)



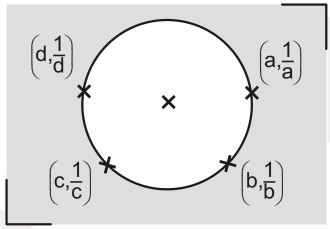
$$\frac{k-3}{h-2} = \frac{2}{5}$$

$$5k = 2h + 11$$

$$2x - 5y + 11 = 0$$

56. (c) Let circle is

$$x^2 + y^2 + 2gx + 2fy + c = 0$$



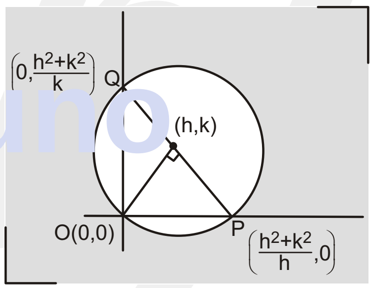
Put $\left(t, \frac{1}{t}\right)$

$$t^4 + 2gt^3 + ct^2 + 2ft + 1 = 0 \quad \text{(roots } a, b, c, d\text{)}$$

$$\Rightarrow \quad abcd = 1$$

57. (c) Equation of PQ is

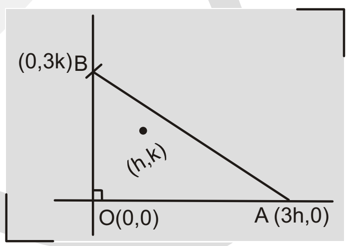
$$hx + ky = h^2 + k^2 \left(0, \frac{h^2+k^2}{k}\right)$$



$$PQ^2 = 4a^2$$

$$(h^2+k^2)^2\left(\frac{1}{h^2} + \frac{1}{k^2}\right) = 4a^2$$

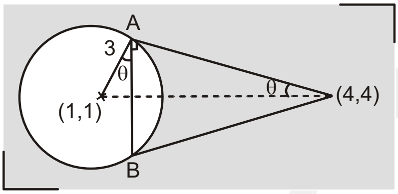
58. (a)



$$(3h)^2 + (3k)^2 = (6k)^2$$

$$h^2 + k^2 = 4k^2$$

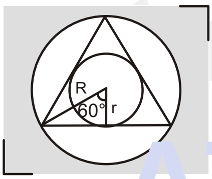
59. (b) $\sin\theta = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$



$\theta = \frac{\pi}{4}$

$AB = 2r\cos\theta = 2(3)\frac{1}{\sqrt{2}} = 3\sqrt{2}$

60. (b) $r = R\cos 60° = \frac{R}{2}$



61. (d) Let equation of tangent to $y^2 = 4ax$ be

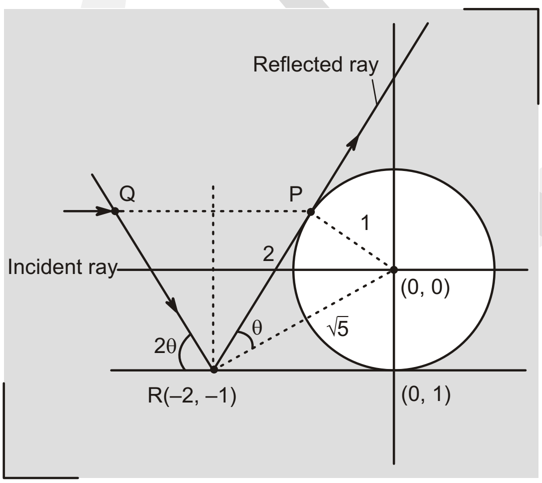
$ty = x + at^2$

If passes through $(a, 2a)$

$\Rightarrow$ $2at = a + at^2$ $\Rightarrow$ $a(t-1)^2 = 0$

$\Rightarrow$ $a \in R - \{0\}$

62. (b)



$\tan 2\theta = \frac{2\left(\frac{1}{2}\right)}{1-\frac{1}{4}} = \frac{4}{3}$

Slope of $QR = -\frac{4}{3}$

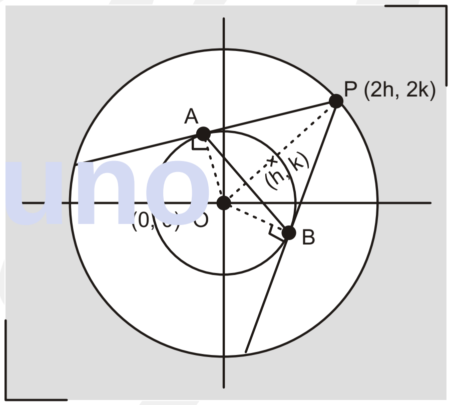
$\therefore$ Equation of incident ray QR is

$y + 1 = -\frac{4}{30}(x +$

$3y + 4x + 11 = 0$

63. (c) Circumcircle of ΔPAB has OP as diameter

If circumcentre $\equiv (h, k)$



$\Rightarrow$ $P \equiv (2h, 2k)$

P lies on $x^2 + y^2 = 4$

$\Rightarrow$ $(2h)^2 + (2k)^2 = 4$

$\therefore$ Locus of circumcentre is $x^2 + y^2 = 1$.

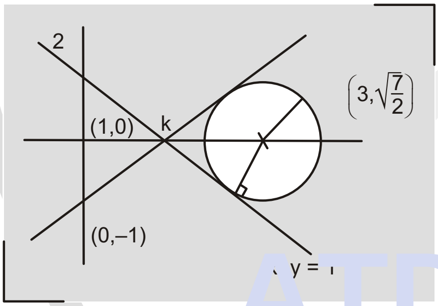
SOLUTIONS 2

One or More Than One Correct

1. (a, c)

$$(x-3)^2+\left(y-\sqrt{\frac{7}{2}}\right)^2=\left(\frac{x+y-1}{\sqrt{2}}\right)^2$$

$$=\left(\frac{x-y-1}{\sqrt{2}}\right)^2$$



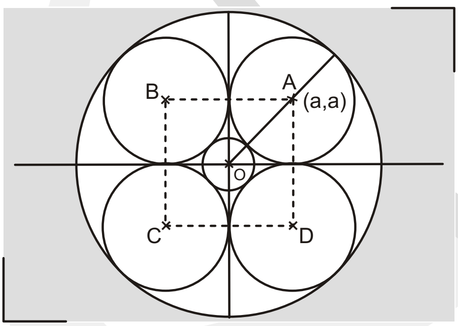
From $x+y-1=\pm(x-y-1)$

$\Rightarrow$ $y=0$ or $x=1$ (rejected)

$$(x-3)^2+\frac{7}{2}=\frac{(x-1)^2}{2}$$

$\Rightarrow$ $x=4,6$

2. (a, b, c, d)



$OA=\sqrt{2}a$

Largest radius $=\sqrt{2}a+a$

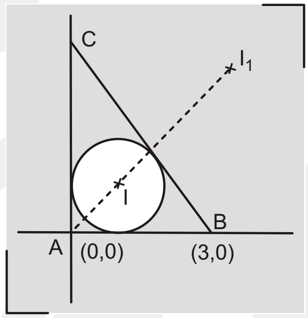
Smallest radius $=\sqrt{2}a-a$

Area enclosed by circles

$$=(2a)^2-4\left(\frac{\pi}{4}a^2\right)=(4-\pi)a^2$$

3. (a, d)

Circles will be in circle and excentral circle of $\Delta\, ABC$ as shown

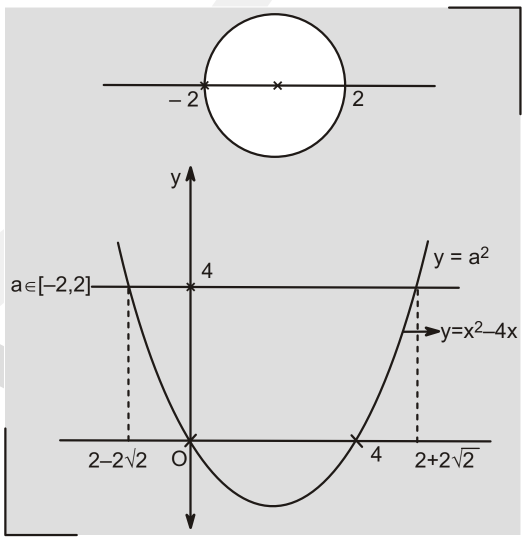


$$I\equiv\left(\frac{4(3)}{4+3+5},\frac{3(4)}{4+3+5}\right)\equiv(1,1)$$

$$I_1\equiv\left(\frac{4(3)}{-5+4+3},\frac{4(3)}{-5+4+3}\right)\equiv(6,6)$$

$r=1,6$

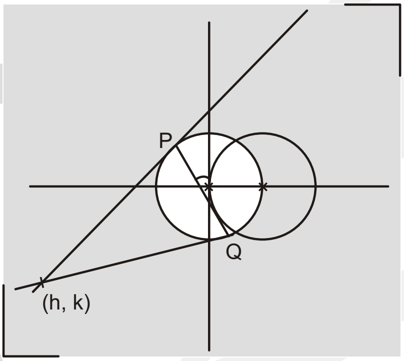
4. (a, b, d)



Exactly one root in $[2-2\sqrt{2},0]\cup[4,2+2\sqrt{2}]$

5. (a, c)

PQ is chord of contact of (h,k) w.r.t. $x^2+y^2=a^2$



$\Rightarrow$ Equation of *PQ* is $xh+yk=a^2$

$\because$ *PQ* is tangent to $(x-a)^2+y^2=a^2$

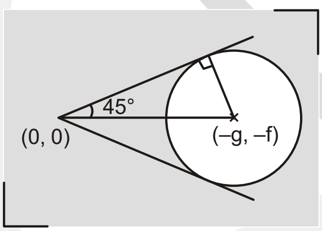
$\Rightarrow$ $\dfrac{|ah-a^2|}{\sqrt{h^2+k^2}}=a$

$\Rightarrow$ $(h-a)^2=h^2+k^2$

$\therefore$ Locus is $(x-a)^2=x^2+y^2$

$i.e., y^2=a(a-2x)$

6. (a, b)

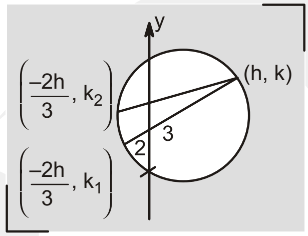


$$\sqrt{g^2+f^2}=\frac{\sqrt{g^2+f^2-f^2}}{\sin 45°}$$

$\Rightarrow$ $g^2+f^2=2(g)^2$

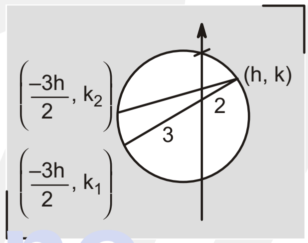
$$g^2=f^2\Rightarrow g=\pm f$$

7. (a, b, d)



Put $x=\dfrac{-2h}{3}, y^2-ky+\dfrac{10h^2}{9}=0$

$D>0 \quad\Rightarrow\quad k^2-\dfrac{40}{9}h^2>0$



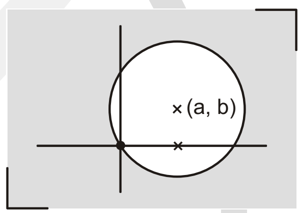
put $x=-\dfrac{3h}{2}$

$\Rightarrow$ $y^2-ky+\dfrac{15h^2}{4}=0$

$D>0 \quad\Rightarrow\quad k^2-15h^2>0$

$k^2>15h^2$

8. (a, b, d)



Eqn. of circle is

$(x\pm a^2)+(y\pm b)^2=a^2+b^2$

or $(x\pm b)^2+(y\pm a)^2=a^2+b^2$

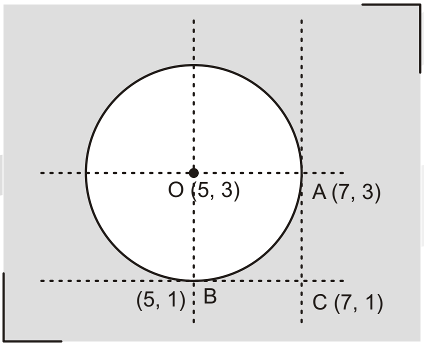
$x^2+y^2\pm 2ax\pm 2by=0$

or $x^2+y^2\pm 2bx\pm 2ay=0$

Eqn. of tangent origin is

$ax \pm by = 0$ or $bx \pm ay = 0$

9. (a, c, d)



Area of square $OACB = 2 \times 2 = 4$

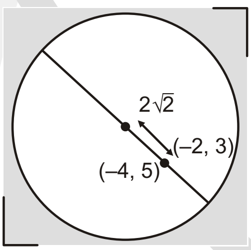
Radical axis of the family $S = 0$ is line AB whose equation $x = y + 4$

Smallest possible circle of family $S = 0$ has AB as diameter given by

$(x-5)(x-7) + (y-1)(y-3) = 0$

$\Rightarrow \quad x^2 + y^2 - 12x - 4y + 38 = 0$

10. (a, c, d)



$a = 9 + 2\sqrt{2}$

$b = 9 - 2\sqrt{2}$

$ab = 81 - 8 = 73$

$a + b = 18$

$a - b = 4\sqrt{2}$

11. (b, d)

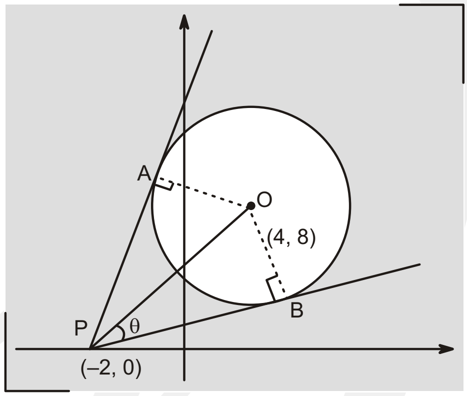
Centre of circles which touches the lines

$x + y = 1$ and $x - y = 1$

are $(1 \pm 2\sqrt{2}, 0)$ and

$(1, \pm 2\sqrt{2})$

12. (b, c)



$\sin\theta = \dfrac{2\sqrt{5}}{10} = \dfrac{1}{\sqrt{5}}$

Slope of PA and PB are $\tan(\alpha \pm \theta)$

where $\tan\alpha = \dfrac{8}{6} = \dfrac{4}{3}$

$= \dfrac{\frac{4}{3} + \frac{1}{2}}{1 - \frac{4}{3}\cdot\frac{1}{2}}, \dfrac{\frac{4}{3} - \frac{1}{2}}{1 + \frac{4}{3}\cdot\frac{1}{2}}$

$= \dfrac{11}{2}, \dfrac{5}{10}$

$\therefore A, B \equiv \left(4 + 2\sqrt{5}\left(\dfrac{-11}{5\sqrt{5}}\right), 8 + 2\sqrt{5}\left(\dfrac{2}{5\sqrt{5}}\right)\right),$

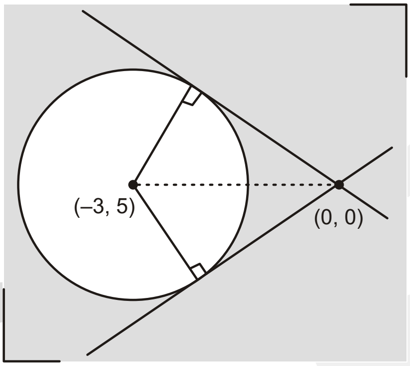
$\left(4 - 2\sqrt{5}\left(\dfrac{-1}{\sqrt{5}}\right), 8 - 2\sqrt{5}\left(\dfrac{2}{\sqrt{5}}\right)\right) \equiv \left(\dfrac{-2}{5}, \dfrac{44}{5}\right)$

$= (6, 4)$

13. (a, d)

Area of quadrilateral

$15 = \sqrt{34 - C}\,\sqrt{C}$



$$C^2 - 34C + 225 = 0$$
$$(C - 25)(C - 9) = 0$$
$$C = 9, 25$$

14. (a, b, d)

$$9a^2 + 6a + 1 = 5(a^2 + b^2)$$

$\Rightarrow \quad \dfrac{3a+1}{\sqrt{a^2+b^2}} = \sqrt{5}$

$\Rightarrow$ ⊥ar distance of (3, 0)

from $ax + by + 1 = 0$ is $\sqrt{5}$

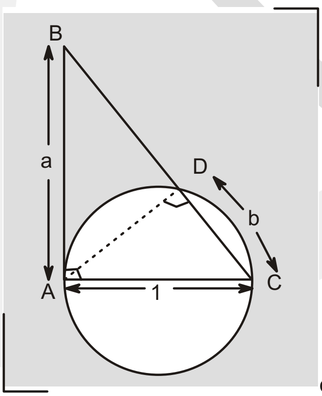
$\Rightarrow$ Centre of circle $= (3, 0)$

radius $= \sqrt{5}$

16. (b, c)

$$\tan C = \frac{AD}{b} = \frac{a}{1}$$

$\Rightarrow \quad AD = ab < AC = 1 \quad \Rightarrow \quad ab < 1$



c

$$(AC)^2 = (AD)^2 + (CD)^2 = a^2b^2 + b^2 = 1$$

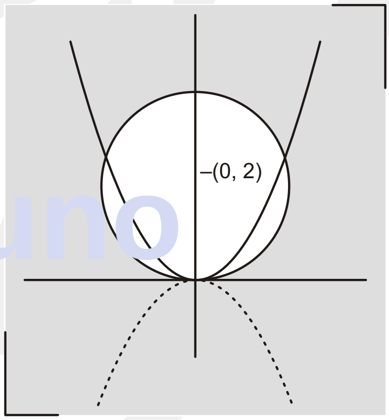
$\Rightarrow \quad b^2 = \dfrac{1}{a^2+1}$

$\Rightarrow \quad \dfrac{b^2}{a^2} = \dfrac{1}{a^4 + a^2} > \dfrac{1}{a^4 + a^2 + \frac{1}{4}}$

$$= \frac{1}{\left(a^2 + \frac{1}{2}\right)^2}$$

$\Rightarrow \quad \dfrac{b}{a} > \dfrac{1}{a^2 + \frac{1}{2}}$

17. (a, b, c)



Eqn. of tangent to $y = x^2$ is

$$tx = y + \frac{1}{4}t^2$$

⊥ar from centre $(0, 2) = 2$

$\Rightarrow \quad \dfrac{\left|\frac{1}{4}t^2 + 2\right|}{\sqrt{1+t^2}} = 2$

$\Rightarrow \quad (t^2 + 8)^2 = 64(1 + t^2)$

$\Rightarrow \quad t^4 - 48t^2 = 0$

$\Rightarrow \quad t = 0, \pm 4\sqrt{3}$

Eqn. of tangent is $\pm 4\sqrt{3}x = y + 12$

Eqn. of tangent to $y = -x^2$ is

$$tx = y - \frac{1}{4}t^2$$

$\perp$ar from $(0,2) = 2$

$$\Rightarrow \quad \frac{\left(\frac{1}{4}t^2 - 2\right)}{\sqrt{1+t^2}} = 2$$

$$\Rightarrow \quad t^4 - 80t^2 = 0$$

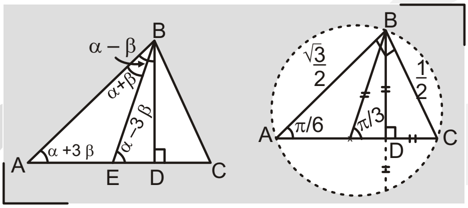
$$\Rightarrow \quad t = \pm 4\sqrt{5}$$

$\therefore$ Equation of tangents are

$$\pm 4\sqrt{5}x = y - 20$$

SOLUTIONS 3

Comprehension: (1)



Also, $\alpha - 3\beta = (\alpha + 3\beta) + (\alpha + \beta)$

[using exterior angle theorem]

$\Rightarrow \quad a = -7\beta$

From $\Delta ABD, \alpha - \beta + \alpha + 3\beta = \frac{\pi}{2}$

$\therefore \quad \beta = -\frac{\pi}{24}, \alpha = \frac{7\pi}{24}$

$\therefore \quad \angle B = 2(\alpha + \beta) = \frac{\pi}{2},$

$\angle A = \frac{\pi}{6}, \angle C = \frac{\pi}{3}$

1. (b) Area of circle circumscribing

$\Delta ABC = \pi\left(\frac{1}{2}\right)^2 = \frac{\pi}{4}$

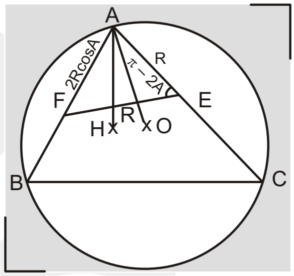
2. (b) ΔBOC is equilateral

$$\Rightarrow \quad r = \frac{\frac{\sqrt{3}}{4}\left(\frac{1}{2}\right)^2}{\frac{1}{2}\left(\frac{3}{2}\right)} = \frac{1}{4\sqrt{3}}$$

3. (d) $BD = \frac{\sqrt{3}}{2}\sin\frac{\pi}{6} = \frac{\sqrt{3}}{4}$

$\therefore \quad BB' = 2BD = \frac{\sqrt{3}}{2}$

Comprehension: (2)



1. (d) Area of quadrilateral

$BFEC = \Delta ABC - \Delta AFE$

$= \Delta AOB + \Delta BOC + \Delta COA - \Delta AFE$

$= \frac{1}{2}R^2(\sin 2C + \sin 2A + \sin 2B$

$-\sin(\pi - 2A))$

$= \frac{1}{2}R^2(\sin 2B + \sin 2C)$

2. (b) $AE = R; \quad AF = 2R\cos A$

$\Rightarrow \quad EF = R$

3. (a) If $\angle AFE = \frac{\pi}{3}$

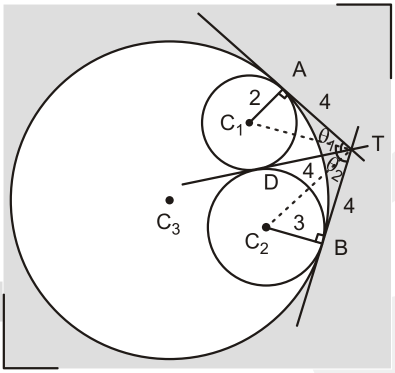
$\Rightarrow \quad \Delta AFE$ is equilateral

sum of squares of altitudes

$= 3\left(\frac{R\sqrt{3}}{2}\right)^2 = \frac{9}{4}R^2$

Comprehension: (3)

AT and BT are radical axis to C_3 and C_1 and C_3 & C_2 respectively.



$\therefore$ T is radical centre $\Rightarrow$ radical axis of C_1 and C_2 *i.e.,* common tangent passes through T.

$TA = TB = TD = 4$

$$\tan\frac{\theta_1}{2} = \frac{2}{4} = \frac{1}{2}, \tan\frac{\theta_2}{2} = \frac{3}{4},$$

$$(\angle ATD = \theta_1, \angle BTD = \theta_2)$$

1. (d) $r_3 = TA \tan\dfrac{\theta_1 + \theta_2}{2} = 4\left(\dfrac{\frac{1}{2} + \frac{3}{4}}{1 - \frac{1}{2}\frac{3}{4}}\right) = 8$

2. (b) Circumcircle of ΔTAB will pass through C_3 has TC_3 as diameter

$$\Rightarrow \quad \text{Area} = \pi\left(\frac{TC_3}{2}\right)^2,$$

$$TC_3 = \frac{TA}{\cos\left(\frac{\theta_1 + \theta_2}{2}\right)} = \frac{4}{1/\sqrt{5}} = 4\sqrt{5}$$

$$\text{Area} = \pi(2\sqrt{5})^2 = 20\pi$$

3. (d) $C_3C_1 = r_3 - r_1$

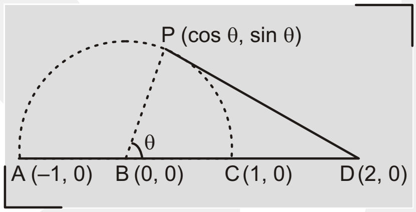
$C_3C_2 = r_3 - r_2$

$\Rightarrow \quad C_3C_1 - C_3C_2 = r_2 - r_1 = 1$

Comprehension: (4)

1. (b) Area $= \dfrac{1}{2}(1)^2(\pi - \theta) + \dfrac{1}{2}(2)\sin\theta$

$$= \frac{\pi}{2} + \sin\theta - \frac{\theta}{2}$$

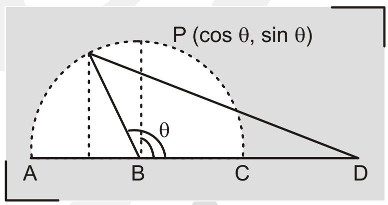


$$A = \frac{\pi}{2} + \sin\theta - \frac{\theta}{2}$$

$$\frac{dA}{d\theta} = \cos\theta - \frac{1}{2}, \quad \frac{+ \mid -}{\theta = \pi/3}$$

A is max. for $\theta = \dfrac{\pi}{3}$

$$\Rightarrow A_{max} = \frac{\pi}{3} + \frac{\sqrt{3}}{2} = \frac{2\pi + 3\sqrt{3}}{6}$$



$$A = \frac{1}{2}\sin\theta(2) + \frac{1}{2}(\pi - \theta)$$

2. (c) $L = 3 + (\pi - \theta) + \sqrt{(2 - \cos\theta)^2 + \sin^2\theta}$

$$= 3 + (\pi - \theta) + \sqrt{5 - 4\cos\theta}$$

3. (c) $\dfrac{dL}{d\theta} = -1 + \dfrac{4\sin\theta}{2\sqrt{5 - 4\cos\theta}} = 0$

$\Rightarrow \quad 4\sin^2\theta = 5 - 4\cos\theta$

$\Rightarrow \quad 4\cos^2\theta - 4\cos\theta + 1 = 0$

$\Rightarrow \quad \cos\theta = \dfrac{1}{2} \quad \Rightarrow \quad \theta = \dfrac{\pi}{3}$

$L(0) = 3 + \pi + \sqrt{1} = 4 + \pi$

$$L(\pi) = 3 + \sqrt{5+4} = 6$$

$$L(\pi/3) = 3 + \frac{2\pi}{3} + \sqrt{5 - \frac{4}{2}} = 3 + \sqrt{3} + \frac{2\pi}{3}$$

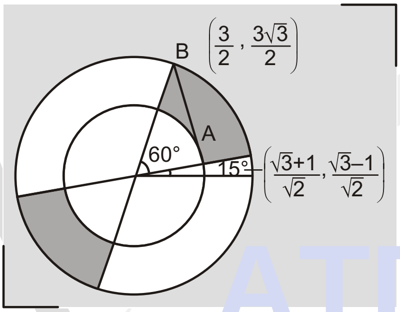
$$L_{max} = 4 + \pi$$

$$L_{min} = 6$$

$$\Rightarrow \quad L_{max} - L_{min} = \pi - 2$$

Comprehension: (5)

Let $\frac{b}{a} = \tan\theta$



$$\left(\frac{b}{a}\right)^2 - \frac{4b}{a} + 1 \le 0$$

$$\Rightarrow \quad \tan\theta \in [2-\sqrt{3}, 2+\sqrt{3}]$$

$$\Rightarrow \quad \theta \in [15°, 75°]$$

1. (d) Area $= \frac{\pi}{3}(9-4) = \frac{5\pi}{3}$

2. (c) $|r(\cos\theta + \sin\theta)| = r\sqrt{2}|\sin(\theta + 45°)|$

$$\ge 2\sqrt{2}\sin(15° + 45°)$$

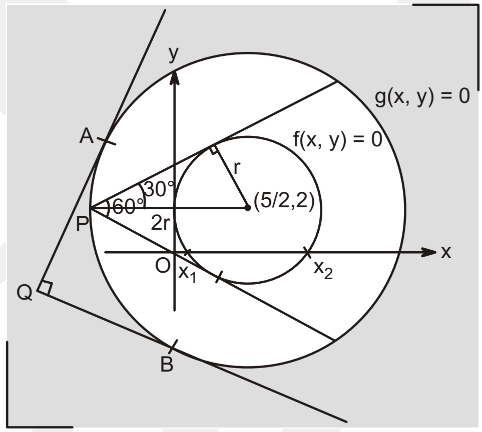
$$\text{minimum} = 2\sqrt{2}\,\frac{\sqrt{3}}{2} = \sqrt{6}$$

3. (d) $(AB) = \sqrt{9 + 4 - 2(3)(2)\cos 60°}$

$$= \sqrt{7}$$

Comprehension: (6)

$g(x, y) = 0$ represent a circle concentric with circle $f(x, y) = 0$ with radius twice the radius of $f(x, y) = 0$, centre $= \left(\frac{5}{2}, 2\right)$



$$f(x, y) = x^2 + y^2 - 5x - 4y + 4 = 0$$

$$g(x, y) = x^2 + y^2 - 5x - 4y - \frac{59}{4} = 0$$

1. (d) Area of $\Delta QAB = \frac{1}{2} \times 5 \times 5 = \frac{25}{2}$

2. (d) $\theta = \tan^{-1}\frac{3}{4}$

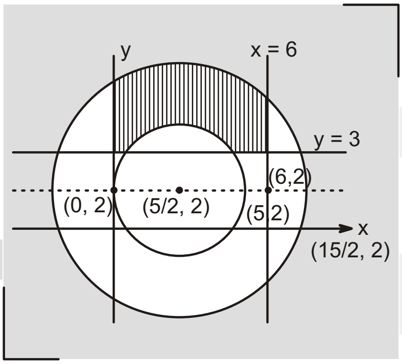
$$2\theta = \tan^{-1}\left(\frac{2\left(\frac{3}{4}\right)}{1 - \frac{9}{16}}\right) = \tan^{-1}\left(\frac{24}{7}\right)$$

Area of region inside $f(x, y) = 0$ above x-axis

$$= \frac{1}{2}\left(\frac{5}{2}\right)^2\left(2\pi - \tan^{-1}\left(\frac{24}{7}\right)\right) + \frac{1}{2} \times 3 \times 2$$

$$= 3 + \frac{25}{8}\left(2\pi - \tan^{-1}\left(\frac{24}{7}\right)\right)$$

3. (d) Points satisfying the conditions are

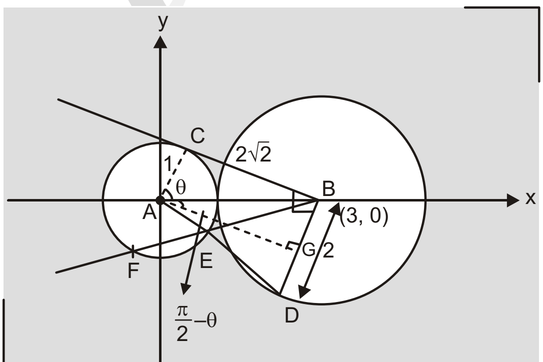


(1, 5) (1, 6), (2, 5), (2, 6) (3, 5), (3, 6)
(4, 5), (4, 6), (5, 4), (5, 5), (5, 6).

Comprehension: (7)

1. (c) 2. (c) 3. (c)

$$\tan\theta = 2\sqrt{2}$$
$$AG = 2\sqrt{2}$$
$$m_{BD} = 2\sqrt{2}$$



$$D \equiv \left(3 - 2\left(\frac{1}{3}\right), 0 - 2\left(\frac{2\sqrt{2}}{3}\right)\right)$$
$$\equiv \left(\frac{7}{3}, \frac{-4\sqrt{2}}{3}\right)$$

$$m_{AE} = -\tan(\pi - 2\theta) = \tan 2\theta$$
$$= \frac{2(2\sqrt{2})}{1-8} = -\frac{4\sqrt{2}}{7}$$

$$E \equiv \left(0 - 1\left(-\frac{7}{9}\right), 0 - 1\left(\frac{4\sqrt{2}}{9}\right)\right)$$
$$\equiv \left(\frac{7}{9}, \frac{-4\sqrt{2}}{9}\right)$$

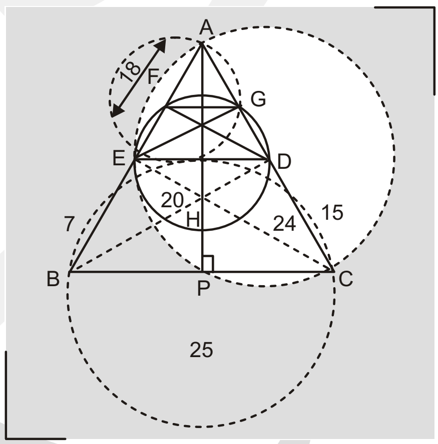
Eqn. of BE is $y = \frac{\sqrt{2}}{5}(x-3)$, Solving intersection of line BE with circle S_1,

$$x^2 + \frac{2}{25}(x-3)^2 = 1$$
$$\Rightarrow 27x^2 - 12x - 7 = 0$$
$$\Rightarrow (9x-7)(3x+1) = 0$$
$$\Rightarrow F \equiv \left(-\frac{1}{3}, \frac{-2\sqrt{2}}{3}\right)$$

Comprehension: (8)

1. (d) 2. (c) 3. (b)

$$\sin C = \frac{4}{5}, \sin B = \frac{24}{25}$$



$$\tan A = -\tan(B+C) = -\frac{\frac{24}{7} + \frac{4}{3}}{1 - \frac{24}{7} \times \frac{4}{3}}$$

$$= \frac{4}{3} = \tan C$$

$\Rightarrow \quad A = C$

$\Rightarrow \quad BC = AB = 25$

$\Rightarrow \quad AC = 2(CD) = 30$

$AB + BC + CA = 80$

$AP = CE = 24$

$DE = AD = DC = 15$

Area of $S = \frac{\pi(15)^2}{4} = \frac{225\pi}{4}$

($AEPC$ is cyclic quadrilateral with AC as diameter)

$\because \quad EFGD$ is cyclic

$\Rightarrow \quad \angle AFG = \angle GDE$

$\because \quad BEDC$ is cyclic $\Rightarrow \angle GDE = \angle ABC$

$\Rightarrow \angle AFG = \angle ABC \quad \Rightarrow FG || BC$

$\therefore \quad \Delta AFG \sim \Delta ABC$

$\Rightarrow \quad \frac{AK}{AP} = \frac{AF}{AB}$

$AF = \frac{1}{2} AE = 9 \quad \because AD = DE$

$\therefore \quad AK = \frac{9 \times 24}{25} = \frac{216}{25}$

Comprehension: (9)

1. (c) 2. (d) 3. (c)

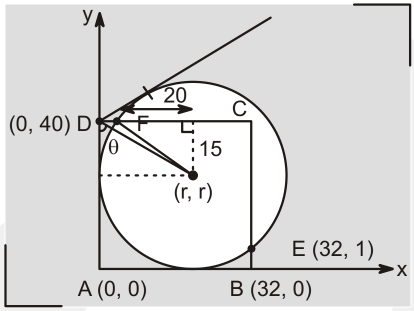
Equation of circle is

$x^2 + y^2 - 2rx - 2ry + r^2 = 0$

Put (32, 1)

$\Rightarrow \quad r^2 - 66r + 1025 = 0$

$\Rightarrow \quad r = 25, \; r = 41, r = 41$, is rejected



$$\theta = \tan^{-1}\left(\frac{25}{15}\right)$$

$$\Rightarrow \quad 2\theta = 2\tan^{-1}\left(\frac{5}{3}\right) = \pi - \tan^{-1}\left(\frac{15}{8}\right)$$

Area of

$$AFCB = \frac{1}{2}(40)(32+27) = 1180$$

Comprehension: (10)

1. (d) $\alpha + \beta < n \Rightarrow \alpha + \beta \le n - 1$

$1 \le \alpha \le n-1, 1 \le \beta \le n-1, \alpha, \beta \in 1$

No. of points (α, β)

$$= {}^{n-1-2+2}C_2 = {}^{n-1}C_2 = \frac{(n-1)(n-2)}{2}$$

$$= \frac{n^2 - 3n + 2}{2}$$

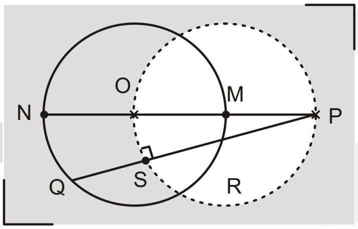
2. (b) Number of rational points is 2 given by $(5, 0)$ and $(-1, 0)$.

3. (c) Lattice points are $(\pm 5, 0), (0, \pm 5)$, $(\pm 3, \pm 4), (\pm 4, \pm 3)$

$= 2 + 2 + 4 + 4 = 12$

Comprehension: (14)

1. (d) Locus of S is a part of circle with OP as diameter passing inside the circle C.



2. (d) $(PR)(PQ) = (NP)(MP) = (d + r)(d - r)$

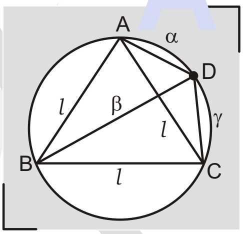
$$= d^2 - r^2$$

$$= (PS - SR)(PS + SQ) = PS^2 - SQ^2$$

$$(\because SQ = SR)$$

$$= (PS)^2 - (SQ)(SR)$$

3. (a) Using Ptolemy's theorem



$$(BD)(AC) = (AB)(CD) + (BC)(AD)$$

$$\beta l = l\gamma + \alpha l$$

$$\Rightarrow \beta = \gamma + \alpha$$

Comprehension: (16)

1. (b) By homogenisation,

$$x^2 + y^2 + 2(3x - 5y)\left(\frac{y - ax}{b}\right) + \left(\frac{y - ax}{b}\right)^2 = 0$$

As the angle subtended at origin $= \pi/2$

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow 1 + 1 - \frac{6a}{b} - \frac{10}{b} + \frac{1}{b^2} + \frac{a^2}{b^2} = 0$$

$$\Rightarrow a^2 + 2b^2 - 6ab - 10b + 1 = 0$$

$\therefore$ locus of (a, b) will be

$$g(x, y) = x^2 + 2y^2 - 6xy - 10y + 1 = 0$$

2. (d) $y = 1$ cuts the curve $g(x, y)$ at $x^2 - 6x - 7 = 0$

$$\Rightarrow x = 7, -1$$

$\therefore$ in first quadrant at $(7, 1)$

$$\therefore \frac{dy}{dx} = -\left(\frac{x - 3y}{2y - 3x - 5}\right)\Bigg|_{(7,1)} = \frac{-4}{-24} = \frac{1}{6}$$

3. (a) Equation of the chord with middle point (1, 2) will be $T = S_1$

$$\Rightarrow x \cdot 1 + y \cdot 2 + 3(x + 1) - 5(y + 2) + 1$$

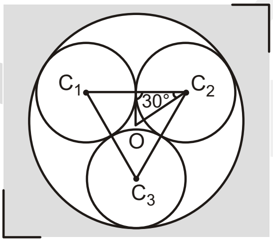
$$= 1^2 + 2^2 + 6 \cdot 1 - 10 \cdot 2 + 1$$

$$\Rightarrow 4x - 3y + 2 = 0$$

SOLUTIONS 4

Assertion and Reason

1. (D) $\Delta C_1 C_2 C_3$ is equilateral



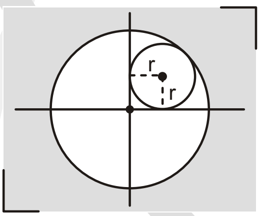
$$OC_2 = r_2 - r_1$$

$$r_1 = (r_2 - r_1)\cos 30°$$

$$\Rightarrow \frac{r_2}{r_1} = \frac{2}{\sqrt{3}} + 1 = \frac{2+\sqrt{3}}{\sqrt{3}}$$

$$\frac{r_1}{r_2} = \frac{\sqrt{3}}{2+\sqrt{3}}$$

2. (C)

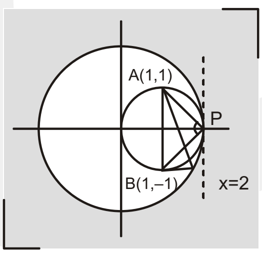


$$C_1C_2 = r\sqrt{2}$$

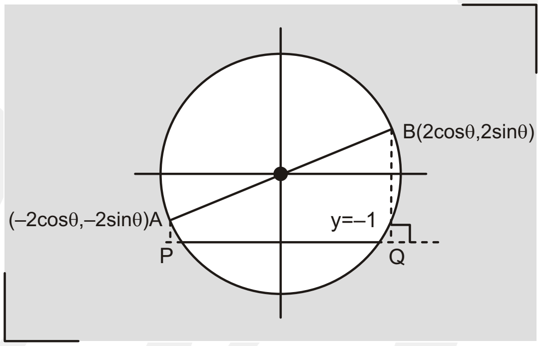
$$2 - r = r\sqrt{2}$$

$$r^2 + 4r - 4 = 0$$

3. (A)



4. (C)



$$P_1 + P_2 = |2\sin\theta + 1| + |-2\sin\theta + 1| = 2$$

If A, B lie on opposite sides of PQ, then

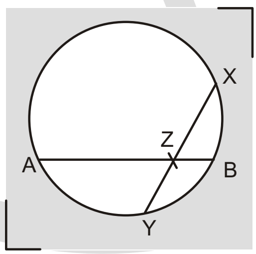
$$P_1 + P_2 = \text{constant}$$

5. (C) If ACB and CBD are minor segments of AB and CD then $AP = PD, PB = PC$

If ACB is minor and CBD is major segment of AB and CD respectively then, $PB = PD$ and $AP = PC$.

6. (D) $(XZ)(YZ) = (XZ)^2$

$$= (AZ)(ZB) \le \left(\frac{AZ + ZB}{2}\right)^2$$

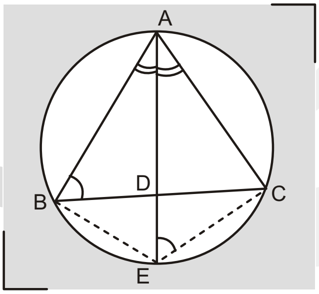


$$\Rightarrow XZ \le \frac{AB}{2}$$

$$\Rightarrow XY \le AB$$

7. (A) $\because \quad \Delta ABD \sim \Delta AEC$

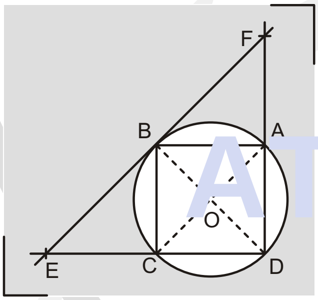
$$\Rightarrow \quad \frac{AB}{AE} = \frac{AD}{AC}$$



$$\Rightarrow (AD)(AE) = (AB)(AC) > (AD)^2$$

$$\therefore \quad AD < \sqrt{(AB)(AC)}$$

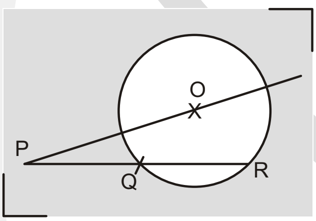
8. (C)



$$(CE)(DE) = (OE)^2 - r^2$$

$$(FA)(FD) = (OF)^2 - r^2$$

$$\because \quad OE = OF$$



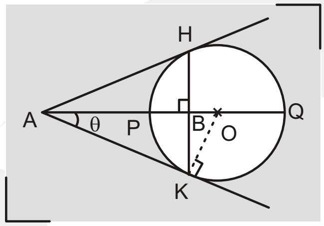
$$\Rightarrow \quad (CE)(DE) = (FA)(FD)$$

$$\because \quad (PQ)(PR) = (OP + r)(OP - r)$$

$$= (OP)^2 - r^2$$

9. (A) Use diametrical form of circle with $A(x_1, y_1)$ and $B(x_2, y_2)$ as diameter.

10. (A)



$$\frac{AP + AQ}{2} = \frac{(OA - r) + (OA + r)}{2} = OA$$

$$\cos\theta = \frac{AK}{OA} = \frac{AB}{AK}$$

$$\Rightarrow \quad (AK)^2 = (OA)(AB)$$

Also $(AK)^2 = (AP)(AQ)$

$$\therefore \quad (OA)(AB) = (AP)(AQ)$$

$$\Rightarrow \quad \left(\frac{AP + AQ}{2}\right) AB = (AP)(AQ)$$

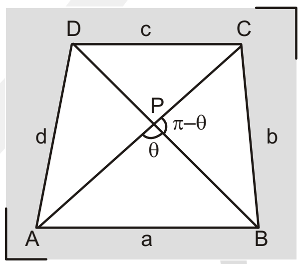
$$\Rightarrow \quad AB = \frac{2(AP)(AQ)}{(AP + AQ)}$$

11. (C) S-1 is true, S-2 is false

Case-I If $\theta \in [\pi/2, \pi)$

$$a^2 = (PA)^2 + (PB)^2 - 2(PA)(PB)\cos\theta$$

$$\geq (PA)^2 + (PB)^2$$



$$b^2 \leq (PB)^2 + (PC)^2$$

$$c^2 \geq (PC^2) + (PD)^2$$

$$d^2 \leq (PD)^2 + (PA)^2$$

$$\Rightarrow \quad a^2 + c^2 \geq (PA)^2 + (PB)^2 + (PC)^2$$

$$+ (PD)^2 \geq b^2 + d^2$$

Case-II If $\theta \in \left[0, \frac{\pi}{2}\right]$

$$\Rightarrow \quad a^2 \le (PA)^2 + (PB)^2,$$

$$b^2 \ge (PB)^2 + (PC)^2,$$

$$c^2 \le (PC)^2 + (PD)^2,$$

$$d^2 \ge (PD)^2 + (PA)^2$$

$$\Rightarrow \quad a^2 + c^2 \le (PA)^2 + (PB)^2 + (PC)^2 + (PD)^2 \le b^2 + d^2$$

If $a^2 + c^2 = b^2 + d^2$

$$\Rightarrow \quad a^2 = (PA)^2 + (PB)^2,$$

$$b^2 = (PB)^2 + (PC)^2,$$

$$c^2 = (PC)^2 + (PD)^2,$$

$$d^2 = (PD)^2 + (PA)^2$$

$$\Rightarrow \quad \theta = \frac{\pi}{2}$$

SOLUTIONS 5

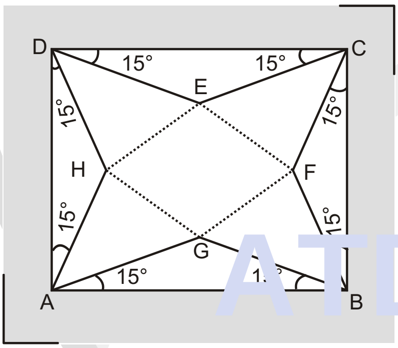
Match the Columns:

1. a → r; b → q; c → p; d → s

(a) $FH = 1 - 2\left(\frac{1}{2}\tan 15°\right) = \sqrt{3} - 1$

Area, $A = \frac{1}{2} d_1 d_2 = \frac{1}{2}(\sqrt{3} - 1)^2 = 2 - \sqrt{3}.$

$a + b = 5$



(b) $\angle AEB = \frac{\pi}{3}$

(c) $2R = \frac{AD}{\sin(150°)} = \frac{1}{(1/2)} = 2$

$\Rightarrow \quad R = 1$

(d) $h_1 = a\sin 60° = \frac{1/2}{\cos 15°}\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{3}}{\sqrt{2}(\sqrt{3}+1)}$

$\frac{1}{h_1^2} + \frac{1}{h_2^2} + \frac{1}{h_3^2} = 3\left(\frac{2(4+2\sqrt{3})}{3}\right)$

$= 8 + 4\sqrt{3} = 8 + \sqrt{48} \Rightarrow \frac{b}{a} = 6$

2. a → s; b → p; c → r; d → s

$(2x_1x_2 + 2x_2x_3 + 2x_3x_1 + x_1^2 + x_2^2 + x_3^2) +$

$(y_1^2 + y_2^2 + y_3^2 + 2y_1y_2 + 2y_2y_3 + 2y_3y_1) = 0$

$\Rightarrow \quad (x_1 + x_2 + x_3)^2 + (y_1 + y_2 + y_3)^2 = 0$

$\Rightarrow \quad x_1 + x_2 + x_3 = 0$

and $y_1 + y_2 + y_3 = 0$

$\Rightarrow$ Circumcenter and centroid of ΔABC coincide

$\Rightarrow$ ΔABC is equilateral

(a)

$(PA)^2 + (PB)^2 + (PC)^2$

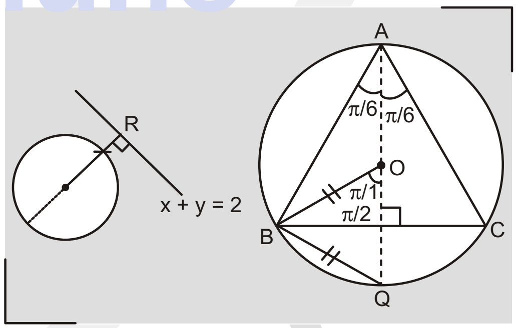
$= (x - x_1)^2 + (y - y_1)^2 + (x - x_2)^2$

$+ (y - y_2)^2$

$+ (x - x_3)^2 + (y - y_3)^2$

$= 3(x^2 + y^2 + 1) = 6$

(b) $\angle OBQ = \frac{\pi}{3}$, $k = 3$



(c) Max. distance of R from S,

$d = \frac{2}{\sqrt{2}} + 1 = \sqrt{2} + 1$

$\Rightarrow \quad d^2 = 3 + 2\sqrt{2}$

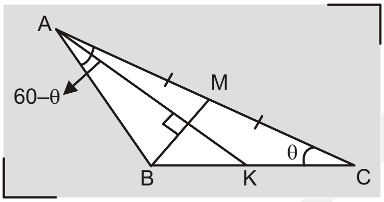
$a + b = 3 + 2 = 5.$

(d) I and G coincide with origin

$IA = IB = IC = GA = GB = GC = 1$

$IA + IB + IC + GA + GB + GC = 6.$

3. a → q; b → s; c → p; d → r



ΔAMB is isosceles

$\because$ angle bisector of $\angle A$ is $\perp$ to MB

$$\frac{c}{\sin\theta} = \frac{2c}{\sin 60^\circ} \Rightarrow \sin\theta = \frac{\sqrt{3}}{4}$$

$[\because AB = c, AC = 2c]$

$$a^2 = c^2 + 4c^2 - 4c^2\cos(60°–\theta)$$

$$= 5c^2 - 4c^2\left(\frac{\sqrt{13}}{4}\frac{1}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{4}\right)$$

$$a = \frac{\sqrt{7-\sqrt{13}}}{\sqrt{2}}c$$

$$= \frac{\sqrt{14-2\sqrt{13}}}{2}c = \frac{\sqrt{13}-1}{2}c$$

(a) $\frac{a}{c} = \frac{\sqrt{13}-1}{2}$

(b) $R = \frac{abc}{4\Delta} = \frac{ac(2c)}{2ac\frac{\sqrt{3}}{2}} = \frac{2c}{\sqrt{3}}$

Alt. $2R = \frac{c}{\sin\theta} \Rightarrow \frac{R}{c} = \frac{1}{\frac{2\sqrt{3}}{4}} = \frac{2}{\sqrt{3}}$

(c) $\frac{\Delta}{\pi R^2} = \frac{\frac{1}{2}\left(ac\frac{\sqrt{3}}{2}\right)}{\pi\frac{4}{3}c^2}$

$$= \frac{3\sqrt{3}}{16}\frac{a}{c} = \frac{3\sqrt{3}}{32}(\sqrt{13}-1)$$

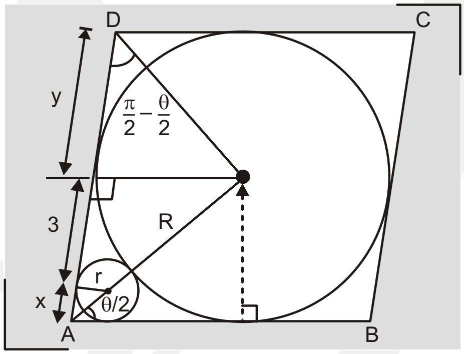
(d) $AB = c, AC = 2c, \frac{AB}{AC} = \frac{1}{2}.$

4. a → s; b → p; c → q; d → r

$$2\sqrt{Rr} = 3 \Rightarrow r = \frac{9}{4\times 3} = \frac{3}{4}$$

$$\frac{r}{R} = \frac{x}{x+3} = \frac{1}{4} \Rightarrow x = 1$$

$$\tan\frac{\theta}{2} = \frac{3}{4}.$$



$$y = R\cot\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$$= 3\tan\frac{\theta}{2} = \frac{9}{4}$$

$ABCD$ is a rhombus with side

$$= 4 + \frac{9}{4} = \frac{25}{4}.$$

Area of $ABCD$

$$= \frac{25}{4}\times\frac{25}{4}\sin\theta = \frac{25}{4}\times\frac{25}{4}\frac{2\left(\frac{3}{4}\right)}{1+\frac{9}{16}}$$

$$A = \frac{75}{2} = \frac{1}{2}d_1 d_2$$

$$\Rightarrow d_1 d_2 = 75$$

$$\frac{d_1^2}{4} + \frac{d_2^2}{4} = \frac{625}{16} = \frac{(d_1+d_2)^2 - 2d_1 d_2}{16}$$

$$\Rightarrow d_1 + d_2 = \sqrt{775} = 5\sqrt{31}$$

5. a → s; b → r; c → q; d → p

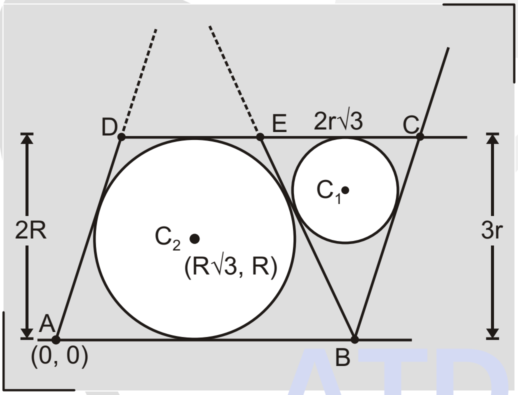
$\perp$ar distance between AB and CD

$$2R = 3r \quad \Rightarrow \quad R = \frac{3r}{2}$$

$$C_1 = (2R\sqrt{3}, 2r) = (3\sqrt{3}r, 2r)$$

$$C_2 = (R\sqrt{3}, R) = \left(\frac{3\sqrt{3}r}{2}, \frac{3r}{2}\right)$$

$$C_1C_2 = \sqrt{\frac{r^2}{4} + \frac{27r^2}{4}} = r\sqrt{7}$$



Length of common tangent

$$= \sqrt{7r^2 - (R+r)^2}$$

$$= \sqrt{7r^2 - \frac{25r^2}{4}} = \frac{\sqrt{3}}{2}r.$$

6. a → s; b → q; c → r; d → p

(a) Q lies on y-axis *i.e.*, OQ = radius

$$\frac{y_P - 0}{11+5} = 1$$

$$\Rightarrow \quad y_P = 16$$

(b) $$x_Q = \frac{11-5}{2} = 3, y_Q$$

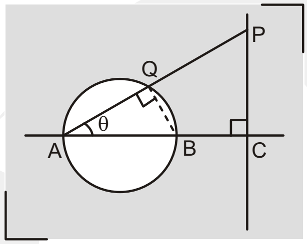
$$= \sqrt{25-9} = 4$$

$$\frac{y_P + Q}{2} = 4$$

$$\Rightarrow \quad y_P = 8$$

(c) Let $\angle QAB = \theta$

$$\frac{1}{4}\frac{1}{2}(16)(16\tan\theta) = \frac{1}{2} \times (10\cos\theta)(10\sin\theta)$$



$$\cos^2\theta = \frac{16 \times 16}{4 \times 10 \times 10} = \frac{16}{25}$$

$$\Rightarrow \quad \cos\theta = \frac{4}{5}$$

$$y_P = 16\left(\frac{3}{4}\right) = 12.$$

(d) $y_P = 0$.

7. a → p, q; b → p, q; c → q; d → q, s

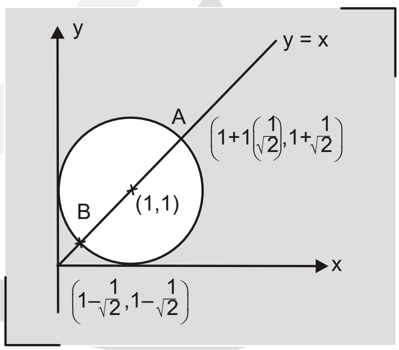
8. a → q; b → s; c → r; d → p

(a) Using parametric form,

$$A \equiv \left(1 + \frac{1}{\sqrt{2}}, 1 + \frac{1}{\sqrt{2}}\right),$$

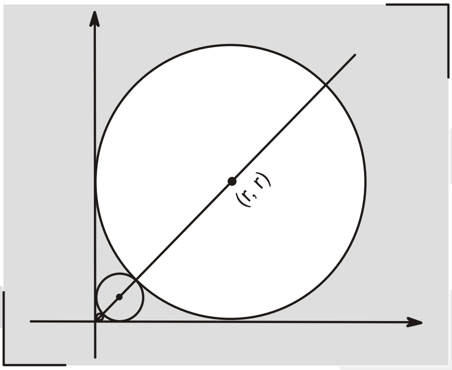
$$B \equiv \left(1 - \frac{1}{\sqrt{2}}, 1 - \frac{1}{\sqrt{2}}\right),$$

$$r = 1 + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}+1}{\sqrt{2}} = \frac{2+\sqrt{2}}{2}$$



(b) $$C_1C_2 = r_1 + r_2$$

$$\Rightarrow \quad 2(r-1)^2 = (r+1)^2$$



$\Rightarrow \quad r^2 - 6r + 1 = 0$

$\Rightarrow \quad r = 3 + 2\sqrt{2}$

(c) $C \equiv x^2 + y^2 - 2x - 2y + 1 = 0$

$C_1 \equiv x^2 + y^2 - 2rx - 2ry + r^2 = 0$

orthogonality

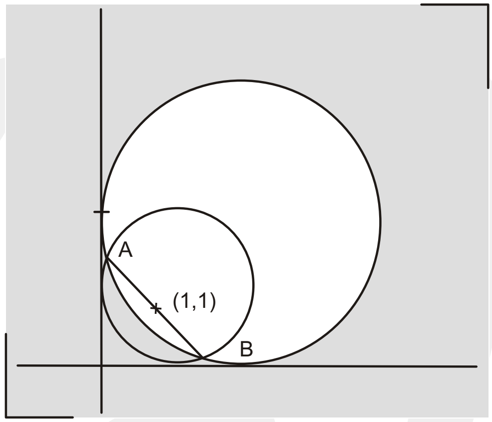
$\Rightarrow \quad 2r(1) + 2r(1) = r^2 + 1$

$\Rightarrow \quad r^2 - 4r + 1 = 0$

$\Rightarrow \quad r = 2 \pm \sqrt{3}$

$\Rightarrow \quad r = 2 + \sqrt{3}$

(d) Equation of common chord AB is



$(x^2 + y^2 - 2x - 2y + 1) - (x^2 + y^2$

$-2rx - 2ry + r^2) = 0$

$\Rightarrow \quad (2r - 2)x + (2r - 2)y + (1 - r^2) = 0$

Put $(1, 1)$

$\Rightarrow \quad 4r - 4 + 1 - r^2 = 0$

$\Rightarrow \quad r^2 - 4r + 3 = 0 \Rightarrow (r-1)(r-3) = 0$

$\Rightarrow \quad r = 3$

9. a → p; b → q; c → r; d → s

10. a → q; b → q; c → t; d → p

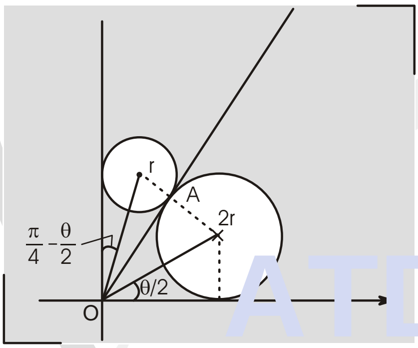
SOLUTIONS 6

Subjective Problems

1. (22)

$$OA = r\cot\left(\frac{\pi}{4} - \frac{\theta}{2}\right) = 2r\cot\frac{\theta}{2}$$

Let, $\tan\frac{\theta}{2} = t$



$$\Rightarrow \frac{1+t}{1-t} = \frac{2}{t}$$

$$\Rightarrow t = \frac{-3 \pm \sqrt{17}}{2}$$

$$\therefore \tan\frac{\theta}{2} = \frac{\sqrt{17} - 3}{2}$$

$$\Rightarrow a + b + c = 17 + 3 + 2 = 22$$

2. (9)

$$\cos\alpha = \frac{1/2}{2} = \frac{1}{4} \text{ [from } \Delta OED]$$

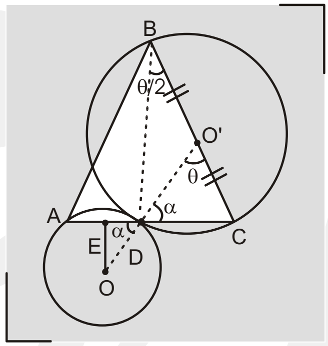
$$\theta = \pi - 2\alpha$$

$$BD = 2\cot\frac{\theta}{2} = 2\cot\left(\frac{\pi}{2} - \alpha\right) = 2\tan\alpha$$

[from ΔBDC]

$$= 2\sqrt{15}$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} \times (AC)(BD)$



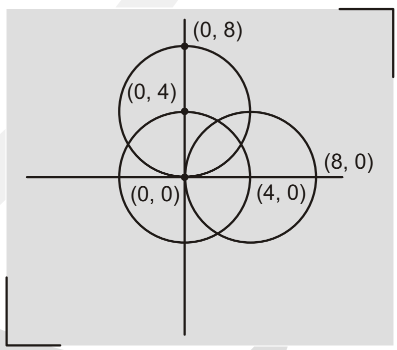
$$= \frac{1}{2} \times 3 \times 2\sqrt{15} = 3\sqrt{15} = \sqrt{135}$$

$$\frac{A^2}{1} = 9$$

3. (6)

Points satisfying the inequalities are common to all 3 circles given by (1, 1), (1, 2); (2, 1), (2, 2), (2, 3), (3, 2)

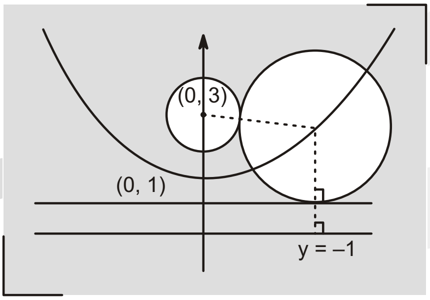
number of ordered pairs $(a, b) = 6$



4. (8) Locus is parabola with directrix $y = -1$ and focus (0,3) given by $x^2 = 4(2)(y - 1)$

$$\frac{x^2}{8}+1=y$$

$\Rightarrow \quad \lambda = 8$

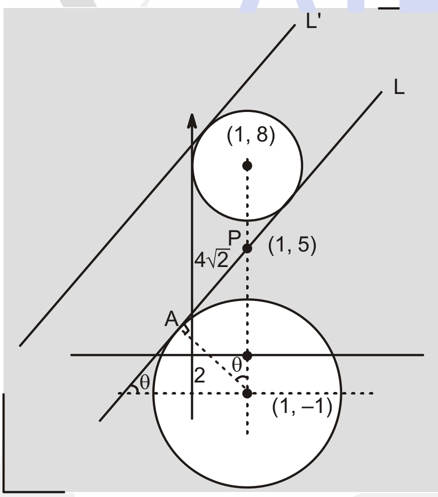


5. (7)

Equation of transverse common tangent L with positive slope is $y-5=2\sqrt{2}(x-1)$

$$[\because \text{slope} = \tan\theta = \frac{4\sqrt{2}}{2} = 2\sqrt{2}]$$

$$L \equiv 2\sqrt{2}x - y + (5-2\sqrt{2}) = 0$$



Let equation of tangent L' which is parallel to L is $2\sqrt{2}x - y = C$

$$\Rightarrow \left|\frac{2\sqrt{2}(1)-8-C}{\sqrt{8+1}}\right| = 1$$

$$2\sqrt{2}-8-C=\pm 3$$

$$\Rightarrow C = 2\sqrt{2}-11, 2\sqrt{2}-5$$

$\therefore$ equation of L' is

$$2\sqrt{2}x - y + 11 - 2\sqrt{2} = 0$$

$$\Rightarrow \frac{a+b+c}{2} = \frac{2+1+11}{2} = 7$$

6. (4)

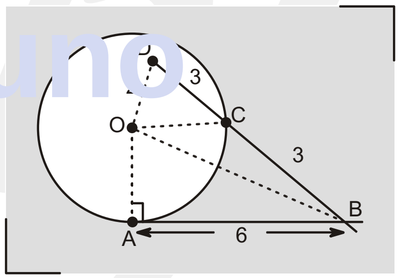
$$OC^2 = r^2 = \frac{2(OD^2+OB^2)-BD^2}{4}$$

[Length of median dropped from 'O' to BD in ΔOBD]

$$r^2 = \frac{2(4+(OB)^2)-6^2}{4} \quad \text{...(1)}$$

Also,

$$(OB)^2 = (OA)^2 + (AB)^2 = r^2 + 36 \quad \text{...(2)}$$



From (1) and (2), we get

$$\therefore \quad 4r^2 = 2(r^2+36) - 28$$

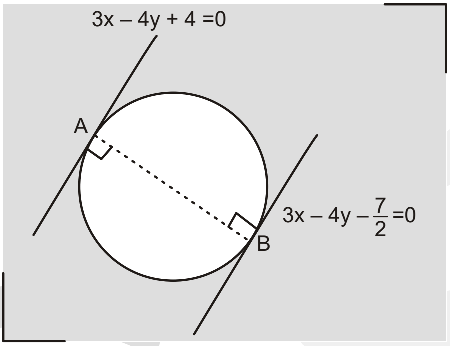
$$r^2 = 22 \Rightarrow r = \sqrt{22}$$

$$\Rightarrow [r] = 4$$

7. (3)

$2r =$ perpendicular distance between two parallel tangents

$$\Rightarrow 2r = \left|\frac{4+\frac{7}{2}}{5}\right| = \frac{15}{2\times 5} = \frac{3}{2}$$



$$r=\frac{3}{4} \quad \Rightarrow \quad 4r=3$$

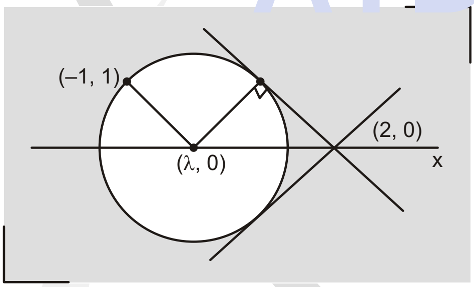
8. (6)

$$r^2=(\lambda+1)^2+1=\left(\frac{|\lambda-2|}{\sqrt{2}}\right)^2$$

$$=\frac{(\lambda-2)^2}{2}$$

$$\lambda^2+8\lambda=0$$

$$\Rightarrow \quad \lambda=0,-8$$



$\Rightarrow$ Equation of circle are

$$x^2+y^2=2$$

and $(x+8)^2+y^2=50$

$i.e., x^2+y^2-2=0$

and $x^2+y^2+16x+14=0$

$$r_1+r_2=\sqrt{2}+\sqrt{50}=6\sqrt{2}$$

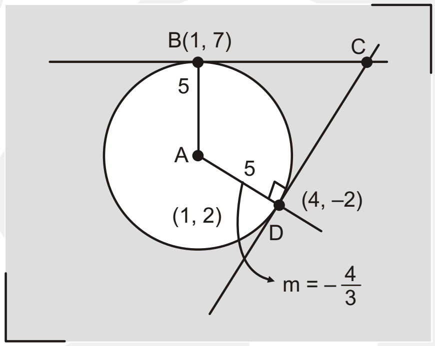
9. (5)

Equation of CD is

$$y+2=\frac{3}{4}(x-4)$$

$$\Rightarrow \quad 4y-3x+20=0$$

Put $y=7$



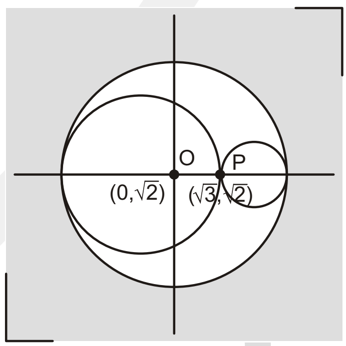
$\Rightarrow$ $C\equiv(16,7)$

Area of quadrilateral $ABCD$,

$$N=2\left(\frac{1}{2}\times5\times15\right)=75$$

$$\Rightarrow \quad \frac{N}{15}=5$$

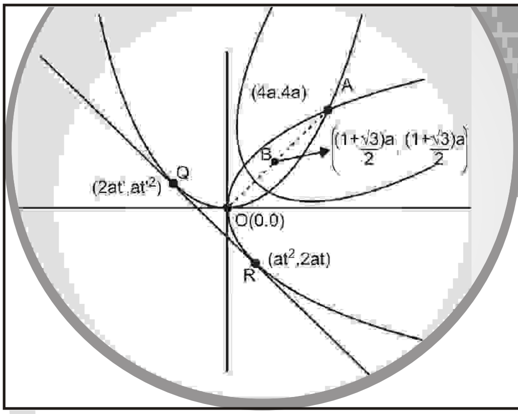
10. 1) Maximum and minimum distance of point $P(\sqrt{3},\sqrt{2})$ from circle is $2+\sqrt{3}$ and $2-\sqrt{3}$



$\therefore$ Largest and smallest circles passing through P must have $\sqrt{3}+2$ and $2-\sqrt{3}$ as diameters.

$$\Rightarrow \quad 2(r_1+r_2)=\sqrt{3}+2+2-\sqrt{3}=4$$

$$\Rightarrow \quad \frac{r_1+r_2}{2}=1$$



3

PARABOLA

KEY CONCEPTS

1. CONIC SECTIONS

A conic section, or conic is the locus of a point which moves in a plane so that its distance from a fixed point is in a constant ratio to its perpendicular distance from a fixed straight line.

The fixed point is called the Focus.

The fixed straight line is called the Directrix.

The constant ratio is called the Eccentricity denoted by e.

The line passing through the focus and perpendicular to the directrix is called the Axis.

A point of intersection of a conic with its axis is called a Vertex.

2. GENERAL EQUATION OF A CONIC: FOCAL DIRECTRIX PROPERTY

The general equation of a conic with focus (p, q) and directrix $lx + my + n = 0$ is :

$(l^2 + m^2)[(x-p)^2 + (y-q)^2] = e^2(lx + my + n)^2 \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$

3. DISTINGUISHING BETWEEN THE CONIC

The nature of the conic section depends upon the position of the focus S w.r.t. the directrix and also upon the value of the eccentricity e. Two different cases arise.

Case (I) : When The Focus Lies On The Directrix.

In this case $D \equiv abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ and the general equation of a conic represents a pair of straight lines if :

$e > 1$ the lines will be real and distinct intersecting at S.

$e = 1$ the lines will coincident.

$e < 1$ the lines will be imaginary.

Case (II) : When The Focus Does Not Lie On Directrix.

a parabola	an ellipse	a hyperbola	rectangular hyperbola
$e=1; D\neq 0$,	$0<e<1; D\neq 0$;	$e>1; D\neq 0$;	$e>1; D\neq 0$
$h^2=ab$	$h^2<ab$	$h^2>ab$	$h^2>ab; a+b=0$

4. PARABOLA : DEFINITION

A parabola is the locus of a point which moves in a plane, such that its distance from a fixed point (focus) is equal to its perpendicular distance from a fixed straight line (directrix).

Standard equation of a parabola is $y^2=4ax$. For this parabola:

(i) Vertex is $(0,0)$ (ii) Focus is $(a,0)$ (iii) Axis is $y=0$ (iv) Directrix is $x+a=0$

Focal Distance

The distance of a point on the parabola from the focus is celled the **F**ocal **D**istance **O**f **T**he **P**oint.

Focal Chord

A chord of the parabola, which passes through the focus is called a **F**ocal **C**hord.

Double or Dinate

A chord of the parabola perpendicular to the axis of the symmetry is called a **D**ouble **O**rdinate.

Latus Rectum

A double ordinate passing through the focus or a focal chord perpendicular to the axis of parabola is called the Latus Rectum. For $y^2=4ax$.

Length of the latus rectum $=4a$.

ends of the latus rectum are $L(a,2a)$ and $L'(a,-2a)$.

Note that: (i) Perpendicular distance from focus on directrix = half the latus rectum.

(ii) Vertex is middle point of the focus and the point of intersection of directrix and axis.

(iii Two parabolas are laid to be equal if they have the same latus rectum.

Four standard forms of the parabola are $y^2=4ax$; $y^2=-4ax$; $x^2=4ay$; $x^2=-4ay$

5. POSITION OF A POINT RELATIVE TO PARABOLA

The point $(x_1\ y_1)$ lies outside, on or inside the parabola $y^2=4ax$ according as the expression $y_1^2-4ax_1$ is positive, zero or negative.

6. LINE AND A PARABOLA

The line $y=mx+c$ meets the parabola $y^2=4ax$ in two points real, coincident or imaginary according as $a \lesseqgtr cm \Rightarrow$ condition of tangency is, $c=\dfrac{a}{m}$.

7. Length of the chord intercepted by the parabola on the line $y=mx+c$ is: $\left(\dfrac{4}{m^2}\right)\sqrt{a(1+m^2)(a-mc)}$.

Note: *length of the focal chord making an angle α with the x-axis is* $4a\,\text{Cosec}^2\alpha$.

8. PARAMETRIC REPRESENTATION

The simplest and the best form of representing the co-ordinates of a point on the parabola is $(at^2, 2at)$.

The equations $x = at^2$ and $y = 2at$ together represents the parabola $y^2 = 4ax, t$ being the parameter. The equation of a chord joining t_1 and t_2 is $2x - (t_1 + t_2)y + 2\,at_1t_2 = 0$.

Note: *If the chord joining* t_1, t_2 and t_3, t_4 *pass through a point* $(c, 0)$ *on the axis, then* $t_1t_2 = t_3t_4 = -c/a$.

9. TANGENTS TO THE PARABOLA $y^2 = 4ax$

(i) $yy_1 = 2a(x + x_1)$ at the point (x_1, y_1) ; **(ii)** $y = mx + \frac{a}{m}(m \neq 0)$ at $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

(iii) $ty = x + at^2$ at $(at^2, 2at)$.

Note: *Point of intersection of the tangents at the point* t_1 *and* t_2 *is* $[at_1t_2, a(t_1 + t_2)]$.

10. NORMALS TO THE PARABOLA $y^2 = 4ax$

(i) $y - y_1 = -\frac{y_1}{2a}(x - x_1)$ at (x_1, y_1);

(ii) $y = mx - 2am - am^3$ at $(am^2 - 2am)$

(iii) $y + tx = 2at + at^3$ at $(at^2, 2at)$

Note: *Point of intersection of normals at* t_1 *and* t_2 *are,* $a(t_1^2 + t_2^2 + t_1t_2 + 2); -at_1t_2(t_1 + t_2)$.

11. THREE VERY IMPORTANT RESULTS :

(a) If t_1 and t_2 are the ends of a focal chord of the parabola $y^2 = 4ax$ then $t_1t_2 = -1$. Hence the co-ordinates at the extremities of a focal chord can be taken as $(at^2, 2at)$ and $\left(\frac{a}{t^2}, -\frac{2a}{t}\right)$.

(b) If the normals to the parabola $y^2 = 4ax$ at the point t_1, meets the parabola again at the point t_2, then $t_2 = -\left(t_1 + \frac{2}{t_1}\right)$.

(c) If the normals to the parabola $y^2 = 4ax$ at the points t_1 and t_2 intersect again on the parabola at the point 't_3' then $t_1t_2 = 2; t_3 = -(t_1 + t_2)$ and the line joining t_1 and t_2 passes through a fixed point $(-2a, 0)$.

General Note:

(i) Length of subtangent at any point $P(x, y)$ on the parabola $y^2 = 4ax$ equals twice the abscissa of the point P. Note that the subtangent is bisected at the vertex.

(ii) Length of subnormal is constant for all points on the parabola and is equal to the semi latus rectum.

(iii) If a family of straight lines can be represented by an equation $\lambda^2 P + \lambda Q + R = 0$ where λ is a parameter and P, Q, R are linear functions of x and y then the family of lines will be tangent to the curve $Q^2 = 4PR$

12. The equation to the pair of tangents which can be drawn from any point $(x_1 y_1)$ to the parabola $y^2 = 4ax$ is given by : $SS_1 = T^2$ where

$S \equiv y^2 - 4ax$; $S_1 = y_1^2 - 4ax_1$; $T \equiv y\,y_1 - 2a(x + x_1)$.

13. DIRECTOR CIRCLE

Locus of the point of intersection of the perpendicular tangents to the parabola $y^2 = 4ax$ is called the **D**irector **C**ircle. It's equation is $x + a = 0$ which is parabola's own directrix.

14. CHORD OF CONTACT

Equation to the chord of contact of tangents drawn from a point $P(x_1, y_1)$ is $yy_1 = 2a(x + x_1)$. Remember that the area of the triangle formed by the tangents from the point (x_1, y_1) and the chord of contact is $(y_1^2 - 4ax_1)^{3/2} \div 2a$. Also note that the chord of contact exists only if the point P is not inside.

15. POLAR AND POLE

(i) Equation of the Polar of the point $P(x_1, y_1)$ w.r.t. the parabola $y^2 = 4ax$ is.

$$y\,y_1 = 2a(x + x_1)$$

(ii) The pole of the line $lx + my + n = 0$ w.r.t. the parabola $y^2 = 4ax$ is $\left(\frac{n}{1}, -\frac{2am}{1}\right)$.

Note: *(i) The polar of the focus of the parabola is the directrix.*

(ii) When the point (x_1, y_1) lies without the parabola the equation of its polar is the same as the equation to the chord of contact of tangents drawn from (x_1, y_1) when (x_1, y_1) is on the parabola the polar is the same as the tangent at the point.

(iii) If the polar of a point P passes through the point Q, then the polar of Q goes through P.

(iv) Two straight lines are said to be conjugated to each other w.r.t. a parabola when the pole of one lies on the other.

(v) Polar of a given point P w.r.t. any Conic is the locus of the harmonic conjugate of P w.r.t. the two points is which any line through P cuts the conic.

16. CHORD WITH A GIVEN MIDDLE POINT

Equation of the chord of the parabola $y^2 = 4ax$ whose middle point is

(x_1, y_1) is $y - y_1 = \dfrac{2a}{y_1}(x - x_1)$. This reduced to $T = S_1$

where $T \equiv y\,y_1 - 2a(x + x_1)$ and $S_1 \equiv y_1^2 - 4ax_1$.

17. DIAMETER

The locus of the middle points of a system of parallel chords of a Parabola is called a Diameter. Equation to the diameter of a parabola is $y = 2a/m$, where $m =$ slope of parallel chords.

Note:

(i) The tangent at the extremity of a diameter of a parabola is parallel to the system of chords it bisects.

(ii) The tangent at the ends of any chords of a parabola meet on the diameter which bisects the chord.

(iii) A line segment from a point P on the parabola and parallel to the system of parallel chords is called the ordinate to the diameter bisecting the system of parallel chords and the chords are called its double ordinate.

18. IMPORTANT HIGHLIGHTS

(a) If the tangent and normal at any point 'P' of the parabola intersect the axis at T and G then $ST = SG = SP$ where 'S' is the focus. In other words the tangent and the normal at a point P on the parabola are the bisectors of the angle between the focal radius and the perpendicular from P on the directrix. From this we conclude that all rays emanating from S will become parallel to the axis of the parabola after reflection.

(b) The portion of a tangent to a parabola cut off between the directrix and the curve subtends a right angle at the focus.

(c) The tangents at the extremities of a focal chord intersect at right angles on the **directrix**, and hence a circle on any focal chord as diameter touches the directrix. Also a circle on any focal radii of a point $P(at^2, 2at)$ as diameter touches the tangent at the vertex and intercepts a chord of length $a\sqrt{1+t^2}$ on a normal at the point P.

(d) Any tangent to a parabola and the perpendicular on it from the focus meet on the tangent at the vertex.

(e) If the tangents at P and Q meet in T, then :

(i) TP and TQ subtend equal angles at the focus S.

(ii) $ST^2 = SP.SQ$ and The triangles SPT and STQ are similar.

(f) Tangents and Normals at the extremities of the latus rectum of a parabola $y^2 = 4ax$ constitute a square, their points of intersection being $(-a, 0)$ and $(3a, 0)$.

(g) Semi latus rectum of the parabola $y^2 = 4ax$, is the harmonic mean between segments of any focal chord of the parabola is ; $2a = \frac{2bc}{b+c}$ $i.e., \frac{1}{b} + \frac{1}{c} = \frac{1}{a}$.

(h) The circle circumscribing the triangle formed by any three tangents to a parabola passes through the focus.

(i) The orthocentre of any triangle formed by three tangents to a parabola $y^2 = 4ax$ lies on the directrix and has the co-ordinates $-a, a(t_1 + t_2 + t_3 + t_1 t_2 t_3)$.

(j) The area of the triangle formed by three points on a parabola is twice the area of the triangle formed by the tangents at these points.

(k) If normal drawn to a parabola passes through a point $P(h, k)$ then

$k = mh - 2am - am^3$ $i.e., am^3 + m(2a - h) + k = 0$.

Then gives $m_1 + m_2 + m_3 = 0$; $m_1 m_2 + m_2 m_3 + m_3 m_1 = \frac{2a - h}{a}$; $m_1 m_2 m_3 = -\frac{k}{a}$.

where $m_1, m_2,$ and m_3 are the slopes of the three concurrent normals. Note that the algebraic sum of the:

- slopes of the three concurrent normals is zero.
- ordinates of the three co-normal points on the parabola is zero.
- Centroid of the Δ formed by three co-normal points lies on the x-axis.

(l) A circle circumscribing the triangle formed by three co-normal points passes through the vertex of the parabola and its equation is, $2(x^2 + y^2) - 2(h + 2a)x - ky = 0$, where (h, k) is the point of concurrence of three normals.

EXERCISE 1

Only One Choice is Correct:

1. Parabolas $(y-\alpha)^2 = 4a(x-\beta)$ and $(y-\alpha)^2 = 4a'(x-\beta')$ will have a common normal (other than the normal passing through vertex of parabola) if :

(a) $\frac{2(a-a')}{\beta'-\beta} < 1$ (b) $\frac{2(a-a')}{\beta-\beta'} < 1$

(c) $\frac{2(a'-a)}{\beta+\beta'} < 1$ (d) $\frac{2(a'-a)}{\beta+\beta'} > 1$

2. If the line $x+y-1=0$ is a tangent to a parabola with focus $(1,2)$ at A and intersects the directrix at B and tangent at vertex at C respectively, then $AC \cdot BC$ is equal to :

(a) 2 (b) 1

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

3. If the segment intercepted by the parabola $y^2 = 4ax$ with the line $lx + my + n = 0$ subtends a right angle at vertex then :

(a) $al + n = 0$ (b) $4am + n = 0$

(c) $4al + n = 0$ (d) $4am - n = 0$

4. If the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$ cuts the parabola again at $(aT^2, 2aT)$, then complete set of values of T satisfies :

(a) $T^2 \ge 8$ (b) $T \in (-\infty, -8) \cup (8, \infty)$

(c) $-2 \le T \le 2$ (d) $T^2 < 8$

5. If the line $y - \sqrt{3}\,x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, and if $P \equiv (\sqrt{3}, 0)$, then $PA \cdot PB$ is equal to:

(a) $\frac{2(\sqrt{3}+2)}{3}$ (b) $\frac{4\sqrt{3}}{2}$

(c) $\frac{4(2-\sqrt{3})}{3}$ (d) $\frac{4(\sqrt{3}+2)}{3}$

6. $Px + 2y = 1$ is normal to parabola $y^2 = 4ax$ for :

(a) no value of P (b) exactly one value of P

(c) exactly two values of P (d) exactly three values of P

7. The vertex of a parabola is at $(3,2)$ and its directrix is the line $x - y + 1 = 0$. Then the equation of its latus rectum is :

(a) $x - y = 2$ (b) $x - y = 3$

(c) $x - y = 1$ (d) $x + y = 2$

8. Locus of point of intersection of normals drawn at end points of focal chord of a parabola $y^2 = 4ax$ is :

(a) $y^2 = a(x - a)$ (b) $y^2 = a(x - 2a)$

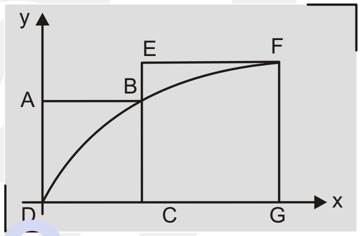
(c) $y^2 = a(x - 3a)$ (d) $y^2 = a(x - 4a)$

9. Two mutually perpendicular tangents of the parabola $y^2 = 4ax$ meet its axis in P_1 and P_2. If S is the focus of the parabola then $\frac{1}{SP_1} + \frac{1}{SP_2}$ is equal to :

(a) $\frac{4}{a}$ (b) $\frac{2}{a}$

(c) $\frac{1}{a}$ (d) $\frac{1}{4a}$

10. $ABCD$ and $EFGC$ are squares and the curve $y = k\sqrt{x}$ passes through the origin D and the points B and F. The ratio $\frac{FG}{BC}$ is :



(a) $\frac{\sqrt{5}+1}{2}$ (b) $\frac{\sqrt{3}+1}{2}$

(c) $\frac{\sqrt{5}+1}{4}$ (d) $\frac{\sqrt{3}+1}{4}$

11. The points of contact Q and R of tangent from the point $P(2,3)$ on the parabola $y^2 = 4x$ are :

(a) $(9,6)$ and $(1,2)$ (b) $(1,2)$ and $(4,4)$

(c) $(4,4)$ and $(9,6)$ (d) $(9,6)$ and $(1/4, 1)$

12. A tangent is drawn to the parabola $y^2 = 4x$ at the point 'P' whose abscissa lies in the interval $[1, 4]$. The maximum possible area of the triangle formed by the tangent at 'P', ordinate of the point 'P' and the x-axis is equal to :

(a) 8 (b) 16

(c) 24 (d) 32

13. The set of points (x, y) whose distance from the line $y = 2x + 2$ is the same as the distance from $(2,0)$ is a parabola. This parabola is congruent to the parabola in standard form $y = Kx^2$ for some K which is equal to :

(a) $\frac{\sqrt{5}}{12}$ (b) $\frac{\sqrt{5}}{4}$

(c) $\frac{4}{\sqrt{5}}$ (d) $\frac{12}{\sqrt{5}}$

14. If the normal to a parabola $y^2 = 4ax$ at P meets the curve again in Q and if PQ and the normal at Q makes angles α and β respectively with the x-axis then $\tan\alpha(\tan\alpha + \tan\beta)$ has the value equal to :

(a) 0 (b) –2
(c) –1/2 (d) –1

15. Let A and B be two points on a parabola $y^2 = x$ with vertex V such that VA is perpendicular to VB and θ is the angle between the chord VA and the axis of the parabola. The value of $\frac{|VA|}{|VB|}$ is :

(a) $\tan\theta$ (b) $\tan^3\theta$
(c) $\cot^2\theta$ (d) $\cot^3\theta$

16. A parabola $y = ax^2 + bx + c$ crosses the x-axis at $(\alpha, 0)\ (\beta, 0)$ both to the right of the origin. A circle also passes through these two points. The length of a tangent from the origin to the circle is :

(a) $\sqrt{\frac{bc}{a}}$ (b) ac^2
(c) $\frac{b}{a}$ (d) $\sqrt{\frac{c}{a}}$

17. C is the centre of the circle with centre $(0, 1)$ and radius unity. P is the parabola $y = ax^2$. The set of values of 'a' for which they meet at a point other than the origin, is :

(a) $a > 0$ (b) $a \in \left(0, \frac{1}{2}\right)$
(c) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (d) $\left(\frac{1}{2}, \infty\right)$

18. Through the vertex O of the parabola, $y^2 = 4ax$ two chords OP and OQ are drawn and the circles on OP and OQ as diameters intersect in R. If θ_1, θ_2 and ϕ are the angles made with the axis by the tangents at P and Q on the parabola and by OR, then the value of, $\cot\theta_1 + \cot\theta_2$ is equal to:

(a) $-2\tan\phi$ (b) $-2\tan(\pi - \phi)$
(c) 0 (d) $2\cot\phi$

19. Tangents are drawn from the points on the line $x - y + 3 = 0$ to parabola $y^2 = 8x$. Then the variable chords of contact pass through a fixed point whose co-ordinates are :

(a) $(3, 2)$ (b) $(2, 4)$ (c) $(3, 4)$ (d) $(4, 1)$

20. The latus rectum of a parabola whose focal chord PSQ is such that $SP = 3$ and $SQ = 2$ is given by:

(a) 24/5 (b) 12/5
(c) 6/5 (d) None of these

21. The equation of the other normal to the parabola $y^2 = 4ax$ which passes through the intersection of those at $(4a, -4a)$ and $(9a, -6a)$ is :

(a) $5x - y + 115a = 0$ (b) $5x + y - 135a = 0$
(c) $5x - y - 115a = 0$ (d) $5x + y + 115 = 0$

22. A common tangent is drawn to the circle $x^2 + y^2 = c^2$ and the parabola $y^2 = 4ax$. If the angle which this tangent makes with the axis of x is $\pi/4$ then the relationship between a and c $(a, c > 0)$ is:

(a) $a = \sqrt{2}\, c$ (b) $c = a\sqrt{2}$

(c) $a = 2c$ (d) $c = 2a$

23. The locus of the foot of the perpendiculars drawn from the vertex on a variable tangent to the parabola $y^2 = 4ax$ is :

(a) $x(x^2 + y^2) + ay^2 = 0$ (b) $y(x^2 + y^2) + ax^2 = 0$

(c) $x(x^2 - y^2) + ay^2 = 0$ (d) None of these

24. The triangle PQR of area 'A' is inscribed in the parabola $y^2 = 4ax$ such that the vertex P lies at the vertex of the parabola and the base QR is a focal chord. The modulus of the difference of the ordinates of the points Q and R is :

(a) $\frac{A}{2a}$ (b) $\frac{A}{a}$

(c) $\frac{2A}{a}$ (d) $\frac{4A}{a}$

25. The normal chord of a parabola $y^2 = 4ax$ at the point whose ordinate is equal to the abscissa, then angle subtended by normal chord at the focus is :

(a) $\frac{\pi}{4}$ (b) $\tan^{-1}\sqrt{2}$

(c) $\tan^{-1} 2$ (d) $\frac{\pi}{2}$

26. Length of the intercept on the normal at the point $P(at^2, 2at)$ of the parabola $y^2 = 4ax$ made by the circle described on the focal distance of the point P as diameter is :

(a) $a\sqrt{2 + t^2}$ (b) $\frac{a}{2}\sqrt{1 + t^2}$

(c) $2a\sqrt{1 + t^2}$ (d) $a\sqrt{1 + t^2}$

27. In a parabola $y^2 = 4ax$ the angle θ that the latus rectum subtends at the vertex of the parabola is:

(a) dependent on the length of the latus rectum

(b) independent of the latus rectum and lies between $\frac{5\pi}{6}$ and π

(c) independent of the latus rectum and lies between $\frac{3\pi}{4}$ and $\frac{5\pi}{6}$

(d) independent of the latus rectum and lies between $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$

28. The distance between a tangent to the parabola $y^2 = 4Ax\,(A > 0)$ and the parallel normal with gradient 1 is :

(a) $4A$ (b) $2\sqrt{2}A$

(c) $2A$ (d) $\sqrt{2}A$

29. Tangents are drawn from the point $(-1, 2)$ on the parabola $y^2 = 4x$. The length, these tangents will intercept on the line $x = 2$ is :

(a) 6 (b) $6\sqrt{2}$

(c) $2\sqrt{6}$ (d) none of these

30. The locus of the middle points of chords of the parabola $y^2 = 4x$, which are of constant length '$2l$' is :

(a) $(4x + y^2)(y^2 - 4) = 4l^2$ (b) $(4y + x^2)(x^2 - 4) = 4l^2$

(c) $(4y - x^2)(x^2 + 4) = 4l^2$ (d) $(4x - y^2)(y^2 + 4) = 4l^2$

31. In a square matrix A of order $3, a_{ii} = m_i + i$ where $i = 1, 2, 3$ and m_i's are the slopes (in increasing order of their absolute value) of the 3 normals concurrent at the point $(9, -6)$ to the parabola $y^2 = 4x$. Rest all other entries of the matrix are one. The value of det. (A) is equal to :

(a) 37 (b) –6

(c) –4 (d) –9

32. A circle C passes though the points of intersection of the parabola $y + 1 = (x - 4)^2$ and the x-axis. The length of tangent from origin to C is :

(a) 8 (b) 15

(c) $\sqrt{8}$ (d) $\sqrt{15}$

33. For the parabola $y^2 + 4x - 4y = 4$, the straight line $x - y + 3 = 0$ is :

(a) focal chord (b) normal chord

(c) both focal chord and normal chord (d) none of these

ANSWERS

1.	(a)	**2.**	(a)	**3.**	(c)	**4.**	(a)	**5.**	(d)	**6.**	(b)	**7.**	(b)	**8.**	(c)	**9.**	(c)	**10.**	(a)
11.	(b)	**12.**	(b)	**13.**	(a)	**14.**	(b)	**15.**	(d)	**16.**	(d)	**17.**	(d)	**18.**	(a)	**19.**	(c)	**20.**	(a)
21.	(b)	**22.**	(a)	**23.**	(a)	**24.**	(c)	**25.**	(d)	**26.**	(d)	**27.**	(d)	**28.**	(b)	**29.**	(b)	**30.**	(d)
31.	(c)	**32.**	(d)	**33.**	(b)														

EXERCISE 2

One or More than One is/are Correct

1. If two distinct chords of a parabola $y^2 = 4ax$ passing through the point $(a, 2a)$ are bisected by line $x + y = 1$, then the length of the latus rectum can not be :

(a) 2 (b) 4

(c) 5 (d) 7

2. A is a point on the parabola $y^2 = 4ax$. The normal at A cuts the parabola again at point B. If AB subtends a right angle at the vertex of the parabola, then the slope of AB is :

(a) 2 (b) $\sqrt{2}$

(c) $-\sqrt{2}$ (d) None of these

3. Three normals are drawn from the point $(c, 0)$ to the curve $y^2 = x$. One normal is x-axis and the other two normals are perpendicular, then :

(a) $c > \frac{1}{2}$ (b) $0 < c < \frac{1}{2}$

(c) $c = \frac{3}{4}$ (d) $c = \frac{1}{2}$

4. Suppose that a normal drawn at a point $P(at^2, 2at)$ to parabola $y^2 = 4ax$ meets it again at Q. If the length of PQ is minimum, then :

(a) $t = \pm\sqrt{2}$ (b) $t = \pm\sqrt{3}$

(c) $PQ = 6\sqrt{3}$ (d) Q is $(8a, \pm 4\sqrt{2}\,a)$

5. P is a point on the parabola $y^2 = 4x$ and Q is a point on the line $2x + y + 4 = 0$. If the line $x - y + 1 = 0$ is the perpendicular bisector of PQ, then the co-ordinates of P can be :

(a) $(1, -2)$ (b) $(4, 4)$

(c) $(9, -6)$ (d) $(16, 8)$

6. If the tangents to the parabola $y^2 = 4ax$ at (x_1, y_1) and (x_2, y_2) meet at (x_3, y_3), then :

(a) x_1, x_3, x_2 are in A.P. (b) x_1, x_3, x_2 are in G.P.

(c) y_1, y_3, y_2 are in G.P. (d) y_1, y_3, y_2 are in A.P.

7. A variable chord PQ of the parabola $y^2 = 4ax$ is drawn parallel to the line $y = x$. If the parameter of the points P and Q on the parabola be t_1 and t_2 respectively, then :

(a) $t_1 + t_2 = 2$

(b) $t_1 t_2 = \frac{2}{a}$

(c) locus of point of intersection of tangents at P and Q is $y = 2a$

(d) locus of point of intersection of normals at P and Q is $2x - y = 12a$

8. If P_1P_2 and Q_1Q_2, two focal chords of a parabola are at right angles, then :

(a) area of the quadrilateral $P_1Q_1P_2Q_2$ is minimum when the chords are inclined at an angle $\frac{\pi}{4}$ to the axis of the parabola

(b) minimum area is twice the area of the square on the latus rectum of the parabola

(c) minimum area of $P_1Q_1P_2Q_2$ cannot be found

(d) minimum area is thrice the area of the square on the latus rectum of the parabola

9. Let there be two parabolas with the same axis, focus of each being exterior to the other and the latus recta being $4a$ and $4b$. The locus of the middle points of the intercepts between the parabolas made on the lines parallel to the common axis is a :

(a) straight line if $a = b$ (b) parabola if $a \neq b$

(c) parabola $\forall\ a, b \in R$ (d) none of these

10. Let $y^2 = 4ax$ be a parabola and $x^2 - y^2 = a^2$ be a hyperbola. Then number of common tangents is :

(a) 2 for $a < 0$ (b) 1 for $a < 0$

(c) 2 for $a > 0$ (d) 1 for $a > 0$

11. The chord AB of the parabola $y^2 = 4ax$ cuts the axis of the parabola at C. If $A \equiv (at_1^2, 2at_1)$ and $B \equiv (at_2^2, 2at_2)$ and $AC : AB = 1:3$, then :

(a) $t_2 = 2t_1$ (b) $t_2 + 2t_1 = 0$

(c) $t_1 + 2t_2 = 0$ (d) $6t_1^2 = t_2(t_1 + 2t_2)$

12. A line L passing through the focus of the parabola $y^2 = 4(x-1)$ intersects the parabola in two distinct points. If 'm' be the slope of the line L, then :

(a) $m \in (-\infty, 0)$ (b) $m \in [0, \infty)$

(c) $m \in (0, \infty)$ (d) none of these

13. Consider a circle with its centre lying on the focus of the parabola $y^2 = 2px$ such that it touches the directrix of the parabola. Then a point of intersection of the circle and the parabola is :

(a) $\left(\frac{p}{2}, p\right)$ (b) $\left(\frac{p}{2}, -p\right)$

(c) $\left(\frac{-p}{2}, p\right)$ (d) $\left(\frac{-p}{2}, \frac{-p}{2}\right)$

14. Variable circle is described to pass through point (1,0) and tangent to the curve $y = \tan(\tan^{-1} x)$. The locus of the centre of the circle is a parabola whose :

(a) length of the latus rectum is $2\sqrt{2}$

(b) axis of symmetry has the equation $x + y = 1$

(c) vertex has the co-ordinates $(3/4, 1/4)$

(d) none of these

15. The range of α for which the points $(\alpha, 2+\alpha)$ and $\left(\frac{3}{2}\alpha, \alpha^2\right)$ lie on opposite sides of the line $2x + 3y = 6$ can lie in intervals :

(a) $(-\infty, -2)$ (b) $(-2, 0)$
(c) $(0, 1)$ (d) $(2, 4)$

16. If the point $\left(\sin\theta, \frac{1}{\sqrt{2}}\right)$ lies exterior to both the parabolas $y^2 = |x|$, then θ can belong to :

(a) $\left(0, \frac{\pi}{6}\right)$ (b) $\left(-\frac{\pi}{6}, 0\right)$
(c) $\left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$ (d) $\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

17. Equation of a common tangent to the circle, $x^2 + y^2 = 50$ and the parabola, $y^2 = 40x$ can be :

(a) $x + y - 10 = 0$ (b) $x - y + 10 = 0$
(c) $x + y + 10 = 0$ (d) $x - y - 10 = 0$

18. Let $y^2 = 4ax$ be a parabola and $x^2 + y^2 + 2\ bx = 0$ be a circle. If parabola and circle touch each other externally then :

(a) $a > 0,\ b > 0$ (b) $a > 0,\ b < 0$
(c) $a < 0,\ b > 0$ (d) $a < 0,\ b < 0$

19. If from the vertex of a parabola $y^2 = 4ax$ a pair of chords be drawn at right angles to one another and with these chords as adjacent sides a rectangle be made, then the locus of the further angle of the rectangle is :

(a) an equal parabola (b) a parabola with focus at $(8a, 0)$
(c) a parabola with directrix as $x - 7a = 0$ (d) not a parabola

20. Through a point $P(-2, 0)$, tangents PQ and PR are drawn to the parabola $y^2 = 8x$. Two circles each passing through the focus of the parabola and one touching parabola at Q and other at R are drawn. Which of the following point(s) with respect to the triangle PQR lie(s) on the common chord of the two circles ?

(a) centroid (b) orthocentre
(c) incentre (d) circumcentre

21. If two distinct chords of parabola $y^2 = 4ax (a > 0)$ passing through $(a, 2a)$ are bisected by the line $x + y = 1$; then the length of the latus rectum can be:

(a) 1 (b) 4 (c) 3 (d) 2

ANSWERS

1.	(b, c, d)	**2.**	(b, c)	**3.**	(a, c)	**4.**	(a, c, d)	**5.**	(a, c)	**6.**	(b, d)
7.	(a, c, d)	**8.**	(a, b)	**9.**	(a, b)	**10.**	(a, c)	**11.**	(b, d)	**12.**	(a, c)
13.	(a, b)	**14.**	(b, c)	**15.**	(a, c)	**16.**	(a, b, c)	**17.**	(b, c)	**18.**	(a, d)
19.	(a, c)	**20.**	(a, b, c, d)	**21.**	(a, c, d)						

EXERCISE 3

Comprehension: (1)

Let from a point $A(h, k)$, 3 distinct normals can be drawn to parabola $y^2 = 4ax$ and the feet of these normals on parabola be points $P(at_1^2, 2at_1)$, $Q(at_2^2, 2at_2)$ and $R(at_3^2, 2at_3)$

1. The controid of ΔPAQ has co-ordinates :

(a) $\left(\frac{2}{3}(h-2a), 0\right)$ (b) $\left(\frac{2}{3}(h-3a), 0\right)$

(c) $\left(\frac{2}{3}(2h-a), 0\right)$ (d) $\left(\frac{2}{3}(h-a), 0\right)$

2. If tangents at P and Q to parabola $y^2 = 4ax$ meet on line $x = -a$, then t_1, t_2 are the roots of the equation :

(a) $x^2 - t_3x + 1 = 0$ (b) $x^2 + t_3x + 1 = 0$

(c) $x^2 - t_3x - 1 = 0$ (d) $x^2 + t_3x - 1 = 0$

3. Let the point A varies such that the point P and Q are the ends of a focal chord then locus of point A is :

(a) $y^2 = a(x-2a)$ (b) $y^2 = a(x-a)$

(c) $y^2 = a(x-3a)$ (d) $y^2 = 3a(x-a)$

Comprehension: (2)

A tangent is drawn at any point P on the parabola $y^2 = 8x$ and on it is taken a point $Q(\alpha, \beta)$ from which pair of tangents QA and QB are drawn to circle $x^2 + y^2 = 4$.

1. The locus of point of concurrency of the chord of contact AB of the circle $x^2 + y^2 = 4$ is :

(a) $y^2 - 2x = 0$ (b) $y^2 - x^2 = 4$

(c) $y^2 + 2x = 0$ (d) $y^2 - 2x^2 = 4$

2. The points from which perpendicular tangents can be drawn both to the given circle and the parabola is :

(a) $(4, \pm\sqrt{3})$ (b) $(-1, \sqrt{2})$

(c) $(-\sqrt{2}, -\sqrt{2})$ (d) $(-2, \pm 2)$

3. The locus of circumcentre of ΔAQB if $P \equiv (8,8)$ is :

(a) $x - 2y + 4 = 0$ (b) $x + 2y - 4 = 0$

(c) $x - 2y - 4 = 0$ (d) $x + 2y + 4 = 0$

Comprehension: (3)

Let the two parabolas $y^2 = 4ax$ and $x^2 = 4ay, a > 0$ intersect at O and A(O being origin). Parabola P whose directrix is the common tangent to the two parabolas and whose focus is the point which divides OA internally in the ratio $(1+\sqrt{3}):(7-\sqrt{3})$

1. The equation of the common tangent to $y^2 = 4ax$ and $x^2 = 4ay$ is :

(a) $x + y + a = 0$ (b) $x + y - a = 0$

(c) $x - y + a = 0$ (d) $x - y - a = 0$

2. The equation of the Parabola P is :

(a) $(x-y)^2 = (2+\sqrt{3})a(x+y-(1+\sqrt{3})a)$

(b) $(x-y)^2 = (2+\sqrt{3})\,a(2x+2y-(2+\sqrt{3})\,a)$

(c) $(x-y)^2 = (2+\sqrt{3})\,a(2x+2y-(1+\sqrt{3})\,a)$

(d) $(x-y)^2 = (2-\sqrt{3})a(x+y-(1+\sqrt{3})a)$

3. Extremities of latus rectum of P are :

(a) $\left(\frac{a}{2}, \frac{(3+2\sqrt{3})a}{2}\right), \left(\frac{(3+2\sqrt{3})a}{2}, \frac{a}{2}\right)$ (b) $\left(-\frac{a}{2}, \frac{(3-\sqrt{3})a}{2}\right), \left(\frac{(3-\sqrt{3})a}{2}, -\frac{a}{2}\right)$

(c) $\left(\frac{a}{2}, \frac{(3-\sqrt{3})a}{2}\right), \left(\frac{(3-\sqrt{3})a}{2}, \frac{a}{2}\right)$ (d) $\left(-\frac{a}{2}, \frac{(3+2\sqrt{3})a}{2}\right), \left(\frac{(3+2\sqrt{3})a}{2}, -\frac{a}{2}\right)$

Comprehension: (4)

$y = f(x)$ is a parabola of the form $f(x) = x^2 + bx + 1$, b is a constant. The tangent line is drawn at the point where $f(x)$ cuts y-axis, also touches $x^2 + y^2 = r^2\,(r > 0)$. It is also given that at least one tangent can be drawn from point P to $y = f(x)$ where P is a point at which $y = |x - \alpha|$ is non differentiable $\forall\, \alpha \in R$.

1. For maximum value of b, the area of circle is :

(a) $\frac{\pi}{10}$ (b) $\frac{\pi}{5}$

(c) π (d) 5π

2. $\lim\limits_{b \to 0} \dfrac{\sqrt{r_{\text{max.}} - r}}{\sin b} =$

(a) $\dfrac{1}{\sqrt{2}}$ (b) $-\dfrac{1}{\sqrt{2}}$

(c) $\dfrac{1}{2}$ (d) Not exist

3. Locus of vertex of parabola is :

(a) $y = 1 - x^2,\ x \in [-1,1], y \in [-1,0]$ (b) $y = 1 - x^2, x \in [-2,2], y \in [0,1]$

(c) $y = 1 - x^2, x \in [-2,2], y \in [-3,1]$ (d) $y = 1 - x^2, x \in [-1,1], y \in [0,1]$

Comprehension: (5)

The limiting value of expression $\dfrac{4x^2 + 2y^2 - 6xy}{6x^2 + \sqrt{2}y - 8xy}$ is A as point (x, y) on curve $x^2 + y^2 = 1$ approaches the position $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$ where A is such that $(5A, 0)$ is a point as focus of parabola S having axis parallel to x-axis, vertex at origin.

1. The two common tangents can be drawn to both circle and parabola from external point whose co-ordinates are :

(a) $\left(\dfrac{-4}{\sqrt{15} - 1}, 0\right)$ (b) $\left(\dfrac{-4}{\sqrt{17} + 1}, 0\right)$

(c) $\left(\dfrac{-4}{\sqrt{17} - 1}, 0\right)$ (d) $\left(\dfrac{-4}{\sqrt{15} + 1}, 0\right)$

2. Locus of midpoints of chords of parabola, which subtend a right angle at vertex of parabola is :

(a) $y^2 - 4x + 32 = 0$ (b) $y^2 + 4x - 32 = 0$

(c) $y^2 - 32x + 4 = 0$ (d) $y^2 + 32x - 4 = 0$

3. Position of point $\left(\dfrac{1}{5}, 5A\right)$ with respect to circle is :

(a) inside (b) on circle

(c) outside (d) none of these

Comprehension: (6)

Let a tangent to parabola $y^2 = 4ax$ at point $P(at^2, 2at)$, $t \neq 0$ intersects its directrix at point Q. Let 'S' represents the focus of parabola $y^2 = 4ax$ and C represents the circle circumscribing the triangle PQS.

1. The angle between the parabola $y^2 = 4ax$ and the circle C at point P is :

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$

(c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$

2. If normal to parabola $y^2 = 4ax$ at point P intersects the line joining Q and S at R, then $\frac{(PS)(QR)}{(PQ)(PR)}$ is equal to :

(a) 4 (b) 3

(c) 2 (d) 1

3. Area of circle C is :

(a) $\frac{\pi a^2(1+t^2)^3}{8t^2}$ (b) $\frac{\pi a^2(1+t^2)^3}{4t^4}$

(c) $\frac{\pi a^2(1+t^2)^3}{4t^2}$ (d) $\frac{\pi a^2(1-t^2)^3}{4t^2}$

Comprehension (7)

Consider one side AB of a square ABCD, (read in order) on the line $y = 2x - 17$, and the other two vertices C, D on the parabola $y = x^2$.

1. Minimum intercept of the line CD on y-axis, is :

(a) 3 (b) 4

(c) 2 (d) 6

2. Maximum possible area of the square $ABCD$ can be :

(a) 980 (b) 1160

(c) 1280 (d) 1520

3. The area enclosed by the line CD with minimum y-intercept and the parabola $y = x^2$ is :

(a) $\frac{15}{3}$ (b) $\frac{14}{3}$

(c) $\frac{22}{3}$ (d) $\frac{32}{3}$

Comprehension: (8)

The function f satisfies $f(x) + f(2x + y) + 5xy = f(3x - y) + 2x^2 + 1$ for all real numbers x, y. Let a chord to parabola $x^2 = 4y$, normals to parabola at ends of which satisfy the relation, $m_1 m_2 = -2$ where m_1, m_2 represent slope of normals, passes through a fixed point 'P' on axis of parabola. Let $y = g(x)$ represent line passing through point P.

1. The value of $f(10)$ is equal to :

(a) -61 (b) -49 (c) -21 (d) -10

2. The minimum area bounded by $y = g(x)$ & $y = f(x)$ is :

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

3. The tangent to $y = f(x)$ at $x = 0$ has slope equal to :

(a) -1 (b) 0 (c) 1 (d) 2

4. Let $y = g(x)$ intersects $y = f(x)$ at two distinct points A, B, then the slope of $g(x)$ if length of segment AB is 4 units is :

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

Comprehension: (9)

Let $ABCD$ be a square of side length 2 units. C_2 is the circle through vertices A, B, C, D and C_1 is the circle touching all the sides of the square $ABCD$. L is a line through A.

1. If P is a point on C_1 and Q in another point on C_2, then $\dfrac{PA^2 + PB^2 + PC^2 + PD^2}{QA^2 + QB^2 + QC^2 + QD^2}$ is equal to:

(a) 0.75 (b) 1.25

(c) 1 (d) 0.5

2. A circle touches the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is:

(a) ellipse (b) hyperbola

(c) parabola (d) parts of straight line

3. A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1 T_2 T_3$ is:

(a) 1/2 sq. units (b) 2/3 sq. units

(c) 1 sq. units (d) 2 sq. units

Comprehension: (10)

Let curve S_1 be the locus of a point $P(h, k)$ which moves in xy plane such that it always satisfy the relation min $\{x^2 + (h-k)x + (1-h-k)\}$ = max $\{-x^2 + (h+k)x - (1+h+k)\}$. Let S_2 is a curved mirror passing through (8, 6) having the property that all light says emerging from origin, after getting reflected from the mirror becomes parallel to x-axis. Also the area of region bounded between y-axis and S_2 is 8/3.

1. The area of smaller region bounded between S_1 and S_2 is equal to:

(a) 2π (b) $\pi - \frac{8}{3}$ (c) $\pi + \frac{8}{3}$ (d) $2\pi - \frac{8}{3}$

2. If the circle $(x-4)^2 + y^2 = r^2$ internally touches the curve S_2, then $r =$

(a) 5 (b) 4 (c) 3 (d) 2

3. The ratio in which the curve $y = \left[\sin^2 \frac{x}{4} + \cos \frac{x}{4}\right]$, where [.] denote greatest integer function divides the curve S_1 is:

(a) $4\pi + 3\sqrt{3} : 8\pi - 3\sqrt{3}$ (b) $4\pi - 3\sqrt{3} : 8\pi + 3\sqrt{3}$

(c) 1 : 1 (d) $4\pi - \sqrt{3} : 8\pi + \sqrt{3}$

ANSWERS

Comprehension-1:	1.	(a)	2.	(d)	3.	(c)		
Comprehension-2:	1.	(c)	2.	(d)	3.	(a)		
Comprehension-3:	1.	(a)	2.	(c)	3.	(d)		
Comprehension-4:	1.	(b)	2.	(d)	3.	(d)		
Comprehension-5:	1.	(c)	2.	(a)	3.	(c)		
Comprehension-6:	1.	(d)	2.	(d)	3.	(c)		
Comprehension-7:	1.	(a)	2.	(c)	3.	(d)		
Comprehension-8:	1.	(b)	2.	(c)	3.	(b)	4.	(a)
Comprehension-9:	1.	(a)	2.	(c)	3.	(c)		
Comprehension-10:	1.	(d)	2.	(b)	3.	(b)		

EXERCISE 4

Match the Columns:

1. The locus of the middle point of the chords of parabola $y^2 = 4x$ which

	Column-I		Column-II
(a)	are normal to parabola is	**(p)**	$y^4 + 4(1-x)y^2 + 4(1-4x) = 0$
(b)	subtend a constant angle $\frac{\pi}{4}$ at the vertex is	**(q)**	$y^4 + (4-2x)y^2 + 8 = 0$
(c)	are of given length 2	**(r)**	$y^2 = 2(x+2)$
(d)	are such that normals at their extremities meet on same parabola.	**(s)**	$y^4 - 4xy^2 + 4x^2 + 32y^2 - 96x + 64 = 0$

2. Normals of parabola $y^2 = 4x$ at P and Q meets at $R(x_2, 0)$ and tangent at P and Q meets at $T(x_1, 0)$

	Column-I		Column-II
(a)	If $x_2 = 3$, then area of quadrilateral $PTQR$ is	**(p)**	3/2
(b)	If length of tangent PT is $4\sqrt{5}$, then $x_2 =$	**(q)**	6
(c)	The possible values of x_2 so that 3 distinct normals can be drawn to the parabola from point R is/are.	**(r)**	8
(d)	If $x_2 = 4$ and area of circle circumscribing ΔPQR is $k\pi$, then k is equal to	**(s)**	9

3. If $y = x + 1$ is axis of parabola, $y + x = 4$ is tangent of same parabola at its vertex and $y = 2x + 3$ is one of its tangent, then

	Column-I		Column-II
(a)	If equation of directrix of parabola is $ax + by - 29 = 0$, then $a + b =$	**(p)**	9
(b)	If length of latus rectum of parabola is $\frac{a\sqrt{2}}{b}$ where a and b are relatively prime natural numbers, then $a + b =$	**(q)**	18
(c)	Let extremities of latus rectum are (a_1, b_1) and (a_2, b_2), then $[a_1 + b_1 + a_2 + b_2] =$ (where $[\cdot]$ denote greatest integer function)	**(r)**	23
(d)	If equation of parabola is $a(x - y + 1)^2 = b(x + y - 4)$ where a and b are relatively prime natural numbers then $a + b =$	**(s)**	37

4.

	Column-I		Column-II
(a)	The point (8,8) is one extremity of focal chord of parabola $y^2 = 8x$. The length of this focal chord is	**(p)**	1
(b)	The equation $(26x - 1)^2 + (26y - 3)^2 = k(5x - 12y + 1)^2$ will represent a parabola if k is	**(q)**	4/3
(c)	The length of common chord of curves $y^2 = 4(x + 1)$ and $4x^2 + 9y^2 = 36$ is	**(r)**	4
(d)	A focal chord of parabola $y^2 = 4ax$ is of length $4a$. The angle subtended by it at the vertex of the parabola is θ then $\lvert\tan\theta\rvert$ is equal to	**(s)**	25/2

5.

	Column-I		Column-II
(a)	The equation of tangent of the ellipse $\frac{x^2}{2}+y^2=1$ which cuts off equal lengths of intercepts on coordinate axis is $y=\pm x+a$, then a can be equal to	**(p)**	$-\sqrt{3}$
(b)	The normal $y=mx-2am-am^3$ to the parabola $y^2=4ax$ subtends a right angle at the vertex then m can be equal to	**(q)**	$-\sqrt{2}$
(c)	The equation of the common tangent to parabola $y^2=4x$ and $x^2=4y$ is $x+y+\frac{k}{\sqrt{3}}=0$, then k is equal to	**(r)**	$\sqrt{2}$
(d)	An equation of common tangent to parabola $y^2=8x$ and the hyperbola $3x^2-y^2=3$ is $2x+\frac{ky}{\sqrt{2}}+1=0$, then k can be equal to	**(s)**	$\sqrt{3}$

6.

	Column-I		Column-II
(a)	The normal chord at a point $(t^2, 2t)$ on the parabola $y^2=4x$ subtends a right angle at the vertex, then t^2 is	**(p)**	4
(b)	The area of the triangle inscribed in the curve $y^2=4x$, whose vertices are $(1,2),(4,4),(16,8)$ is	**(q)**	2
(c)	The number of distinct normal possible from $\left(\frac{11}{4},\frac{1}{4}\right)$ to the parabola $y^2=4x$ is	**(r)**	3
(d)	The normal at $(a,2a)$ on $y^2=4ax$ meets the curve again at $(at^2, 2at)$, then the value of $\lvert t-1\rvert$ is	**(s)**	6

7. Normals are drawn from point $(4,1)$ to the parabola $y^2 = 4x$. The tangents at the feet of normals to the parabola $y^2 = 4x$ form a triangle ABC.

	Column-I		Column-II
(a)	The distance of focus of parabola $y^2 = 4x$ from centroid of ΔABC is	**(p)**	$\frac{5}{3}$
(b)	The distance of focus of parabola $y^2 = 4x$ from orthocentre of ΔABC is	**(q)**	$\frac{\sqrt{10}}{2}$
(c)	The distance of focus of parabola $y^2 = 4x$ from circumcentre of ΔABC is	**(r)**	$\frac{\sqrt{7}}{2}$
(d)	Area of ΔABC is	**(s)**	$\frac{\sqrt{5}}{2}$
		(t)	$\sqrt{5}$

ANSWERS

1. a → q; b → s; c → p; d → r

2. a → r; b → q; c → q,r,s; d → s

3. a → q; b → r; c → p; d → s

4. a → s; b → r; c → r; d → q

5. a → p,s; b → q,r; c → s; d → q,r

6. a → q; b → s; c → q; d → p

7. a → p; b → t; c → q; d → s

EXERCISE 5

Subjective Problems

1. If $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) be three points on parabola $y^2 = 4ax$ and the normals at these points meet in a point, then $\dfrac{x_1 - x_2}{y_3} + \dfrac{x_2 - x_3}{y_1} + \dfrac{x_3 - x_1}{y_2}$ is equal to

2. A parabola of latus rectum l, touches a fixed equal parabola, the axes of two curves being parallel. The locus of the vertex of moving curve is a parabola of latus rectum kl, then k is equal to.

3. If the normals at the points where the straight line $lx + my = 1$ meet the parabola $y^2 = 4ax$, meet on the normal at the point $\left(k\dfrac{am^2}{l^2}, \dfrac{kam}{l}\right)$ of parabola $y^2 = 4ax$, then k is equal to

4. Let PG is the normal at point P to a parabola cuts its axis in G and is produced to Q so that $GQ = 1/2\, PG$. The other normals which pass through Q intersect at an angle of π/k, then $k =$

5. The locus of the point of intersection of two tangents to the parabola $y^2 = 4ax$ which with the tangent at the vertex form a triangle of constant area c^2, is the curve $x^2(y^2 - 4ax) = \lambda c^4$, then $\lambda =$

6. Find the ratio of area of triangle formed by three points on a parabola to the area of triangle formed by tangents at these points.

7. 'O' is the vertex of parabola $y^2 = 4x$ and L is the upper end of latus rectum. If LH is drawn perpendicular to OL meeting x-axis in H, then the length of double ordinate through H is $\sqrt{N}$, then $N =$

8. The radius of circle which passes through the focus of parabola $x^2 = 4y$ and touches it at point $(6, 9)$ is $k\sqrt{10}$, then $k =$

9. From the point $(-1, 2)$ tangent lines are drawn to the parabola $y^2 = 4x$. If area of triangle formed by the chord of contact and the tangents is $N\sqrt{2}$, then $N =$

10. Let $K(c, 0)$ be the point which has the property that if any chord PQ of the parabola $y^2 = 4x$ be drawn through it, then $\dfrac{1}{(PK)^2} + \dfrac{1}{(QK)^2}$ is the same for all positions of the chord, then $c =$

11. Tangent are drawn at those point on the parabola $y^2 = 16x$ whose ordinate are in the ratio 4 : 1. If the locus of point of intersection of these tangents is $y^2 = kx$, then $[k/3]$ is:
([.] denote greatest integer function)

12. If the focus of parabola $y^2 + 8 = 4x$ coincides with one of the foci of ellipse $3x^2 + by^2 - 12x = 0$, then the reciprocal of eccentricity of ellipse is:

13. In the above problem, $b =$

ANSWERS

1.	0	**2.**	2	**3.**	4	**4.**	2	**5.**	4	**6.**	2	**7.**	80	**8.**	5	**9.**	8	**10.**	2
11.	8	**12.**	2	**13.**	4														

EXERCISE 6

1. Find the equations of the common tangents of the circle $x^2 + y^2 - 6y + 4 = 0$ and the parabola $y^2 = x$. **[REE 1999]**

2. (A) If the line $x - 1 = 0$ is the directrix of the parabola $y^2 - kx + 8 = 0$ then one of the values of 'k' is:

(a) 1/8 (b) 8 (c) 4 (d) 1/4

(B) If $x + y = k$ is normal to $y^2 = 12x$, then 'k' is: **[IIT-JEE (Screening) 2000]**

(a) 3 (b) 9 (c) –9 (d) –3

3. Find the locus of the points of intersection of tangents drawn at the ends of all normal chords of the parabola $y^2 = 8(x - 1)$. **[REE 2001]**

4. (A) The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the x-axis is: **[IIT-JEE (Screening) 2001]**

(a) $\sqrt{3}y = 3x + 1$ (b) $\sqrt{3}y = -(x + 3)$

(c) $\sqrt{3}y = x + 3$ (d) $\sqrt{3}y = -(3x + 1)$

(B) The equation of the directrix of the parabola, $y^2 + 4y + 4x + 2 = 0$ is:

[IIT-JEE (Screening) 2001]

(a) $x = -1$ (b) $x = 1$ (c) $x = -3/2$ (d) $x = 3/2$

5. The locus of the mid-point of the line segment joining the focus to a moving point on the parabola $y^2 = 4ax$ is another parabola with directrix: **[IIT-JEE (Screening) 2002]**

(a) $x = -a$ (b) $x = -a/2$ (c) $x = 0$ (d) $x = a/2$

6. The equation of the common tangent to the curves $y^2 = 8x$ and $xy = -1$ is:

[IIT-JEE (Screening) 2002]

(a) $3y = 9x + 2$ (b) $y = 2x + 1$ (c) $2y = x + 8$ (d) $y = x + 2$

7. (A) The slope of the focal chords of the parabola $y^2 = 16x$ which are tangents to the circle $(x - 6)^2 + y^2 = 2$ are: **[IIT-JEE (Screening) 2003]**

(a) ± 2 (b) $-1/2, 2$ (c) ± 1 (d) $-2, 1/2$

(B) Normals are drawn from the point 'P' with slopes m_1, m_2, m_3 to the parabola $y^2 = 4x$. If locus of P with $m_1 m_2 = \alpha$ is a part of the parabola itself then find α. **[IIT-JEE 2003]**

8. The angle between the tangents drawn from the points (1, 4) to the parabola $y^2 = 4x$ is:

[IIT-JEE (Screening) 2004]

(a) $\pi/2$ (b) $\pi/3$ (c) $\pi/4$ (d) $\pi/6$

9. Let P be a point on the parabola $y^2 - 2y - 4x + 5 = 0$, such that the tangent on the parabola at P intersects the directrix at point Q. Let R be the point that divides the line segment PQ externally in the ratio $\frac{1}{2} : 1$. Find the locus of R. **[IIT-JEE 2004]**

10. (A) The axis of parabola is along the line $y = x$ and the distance of vertex from origin is $\sqrt{2}$ and that of origin from its focus is $2\sqrt{2}$. If vertex and focus both lie in the 1st quadrant, then the equation of the parabola is: **[IIT-JEE 2006]**

(a) $(x+y)^2 = (x-y-2)$ (b) $(x-y)^2 = (x+y-2)$

(c) $(x-y)^2 = 4(x+y-2)$ (d) $(x-y)^2 = 8(x+y-2)$

(B) The equations of common tangents to the parabola $y = x^2$ and $y = -(x-2)^2$ is/are: **[IIT-JEE 2006]**

(a) $y = 4(x-1)$ (b) $y = 0$ (c) $y = -4(x-1)$ (d) $y = -30x - 50$

(C) Normals are drawn at points P, Q and R lying on the parabola $y^2 = 4x$ which intersect at $(3, 0)$. Then: **[IIT-JEE 2006]**

	Column-I		Column-II
(i)	Area of ΔPQR	**(a)**	2
(ii)	Radius of circumcircle of ΔPQR	**(b)**	5/2
(iii)	Centroid of ΔPQR	**(c)**	(5/2, 0)
(iv)	Circumcentre of ΔPQR	**(d)**	(2/3, 0)

11. Statement-1: The curve $y = \frac{-x^2}{2} + x + 1$ is symmetric with respect to the line $x = 1$.

because

Statement-2: A parabola is symmetric about its axis. **[IIT-JEE 2007]**

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

12. Comprehension

Consider the circle $x^2 + y^2 = 9$ and the parabola $y^2 = 8x$. They intersect at P and Q in the first and the fourth quadrants, respectively. Tangents to the circle at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S. [IIT-JEE 2007]

(A) The ratio of the areas of the triangles PQS and PQR is:

(a) $1:\sqrt{2}$ (b) $1:2$ (c) $1:4$ (d) $1:8$

(B) The radius of the circumcircle of the triangle PRS is:

(a) 5 (b) $3\sqrt{3}$ (c) $3\sqrt{2}$ (d) $2\sqrt{3}$

(C) The radius of the incircle of the triangle PQR is:

(a) 4 (b) 3 (c) 8/3 (d) 2

13. Let $P(x_1, y_1)$ and $Q(x_2, y_2)$, $y_1 < 0, y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are: **[IIT-JEE 2008]**

(a) $x^2 + 2\sqrt{3}y = 3 + \sqrt{3}$ (b) $x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$

(c) $x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$ (d) $x^2 - 2\sqrt{3}y = 3 - \sqrt{3}$

14. The tangent PT and the normal PN to the parabola $y^2 = 4ax$ at a point P on it meet its axis at points T and N, respectively. The locus of the centroid of the triangle PTN is a parabola whose: **[IIT-JEE 2009]**

(a) vertex is $\left(\frac{2a}{3}, 0\right)$ (b) directrix is $x = 0$ (c) latus rectum is $\frac{2a}{3}$ (d) focus is $(a, 0)$

15. Let A and B be two distinct points on the parabola $y^2 = 4x$. If the axis of the parabola touches a circle of radius r having AB as its diameter, then the slope of the line joining A and B can be: **[IIT-JEE 2010]**

(a) $-\frac{1}{r}$ (b) $\frac{1}{r}$ (c) $\frac{2}{r}$ (d) $-\frac{2}{r}$

16. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from $(0, 0)$ to (x, y) in the ratio $1:3$. Then the locus of P is: **[IIT-JEE 2011]**

(a) $x^2 = y$ (b) $y^2 = 2x$ (c) $y^2 = x$ (d) $x^2 = 2y$

17. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point $(9, 6)$, then L is given by: **[IIT-JEE 2011]**

(a) $y - x + 3 = 0$ (b) $y + 3x - 33 = 0$ (c) $y + x - 15 = 0$ (d) $y - 2x + 12 = 0$

18. Consider the parabola $y^2 = 8x$. Let Δ_1 be the area of the triangle formed by the end points of its latus rectum and the point $P\left(\frac{1}{2}, 2\right)$ on the parabola, and Δ_2 be the area of the triangle formed by drawing tangents at P and at the end points of the latus rectum. Then $\frac{\Delta_1}{\Delta_2}$ is: **[IIT-JEE 2011]**

19. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is: **[IIT-JEE 2012]**

20. Given : A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$. **[IIT-JEE (Mains) 2013]**

Statement-1: An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

because

Statement-2: If the line, $y = mx + \frac{\sqrt{5}}{m} (m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$.

(a) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for statement-1.

(b) Statement-1 is true, statement-2 is true; statement-2 is NOT a correct explanation for statement-1.

(c) Statement-1 is true, statement-2 is false.

(d) Statement-1 is false, statement-2 is true.

21. Comprehension

Let PQ be a focal chord of the parabola $y^2 = 4ax$. The tangents to the parabola at P and Q meet at a point lying on the line $y = 2x + a$, $a > 0$.

[IIT-JEE (Advance) 2013]

(A) If chord PQ subtends an angle θ at the vertex of $y^2 = 4ax$, then $\tan\theta =$

(a) $\frac{2}{3}\sqrt{7}$ (b) $\frac{-2}{3}\sqrt{7}$ (c) $\frac{2}{3}\sqrt{5}$ (d) $\frac{-2}{3}\sqrt{5}$

(B) Length of chord PQ is:

(a) $7a$ (b) $5a$ (c) $2a$ (d) $3a$

22. A line $L: y = mx + 3$ meets y-axis at $E(0, 3)$ and the arc of the parabola $y^2 = 16x, 0 \le y \le 6$ at the point $F(x_0, y_0)$. The tangent to the parabola at $F(x_0, y_0)$ intersects the y-axis at $G(0, y_1)$. The slope m of the line L is chosen such that the area of the triangle EFG has a local maximum. Match Column-I with Column-II and select the correct answer using the code given below the columns : **[IIT-JEE (Advance) 2013]**

	Column-I		Column-II
(a)	$m =$	**(p)**	1/2
(b)	Maximum area of ΔEFG is	**(q)**	4
(c)	$y_0 =$	**(r)**	2
(d)	$y_1 =$	**(s)**	1

ANSWERS

1. $x - 2y + 1 = 0$; $y = mx + \frac{1}{4m}$ where $m = \frac{-5 \pm \sqrt{30}}{10}$

2. **(A)** c; **(B)** b

2. $(x+3)y^2 + 32 = 0$

4. **(A)** c; **(B)** d

5. c

6. d

7. **(A)** c; **(B)** $\alpha = 2$

8. b

9. $2(y-1)^2(x-2) = (3x-4)^2$

10. **(A)** d; **(B)** a, b; **(C)** (i) a, (ii) b, (iii) d, (iv) c

11. a

12. **(A)** c; **(B)** b; **(C)** d

13. b, c

14. a, d

15. c, d

16. c

17. a, b, d

18. 2

19. 4

20. b

21. **(A)** d; **(B)** b

22. $a \to s$; $b \to p$; $c \to q$; $d \to r$

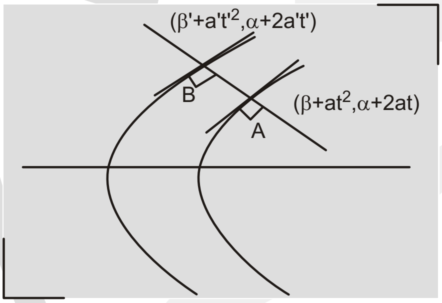
SOLUTIONS 1

Only One Choice is Correct:

1. (a) Eqn. of AB is

$$y-\alpha=-t(x-\beta)+2at+at^3 \qquad ...(1)$$

or $y-\alpha=-t'(x-\beta')+2a't'+a't'^3 \qquad ...(2)$



(1) and (2) are identical

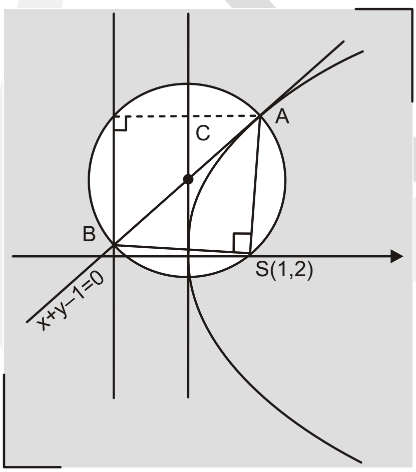
$\Rightarrow \quad t=t'$ and $2at+at^3+\beta t+\alpha$

$$=2a't+a't^3+t\beta'+\alpha$$

$$\Rightarrow \quad t^2=\frac{(\beta'-\beta)+2(a'-a)}{(a-a')} \qquad ...(3)$$

$$\Rightarrow \quad \frac{\beta'-\beta}{a-a'}>2 \quad \Rightarrow \quad \frac{2(a-a')}{(\beta'-\beta)}<1$$

2. (a)

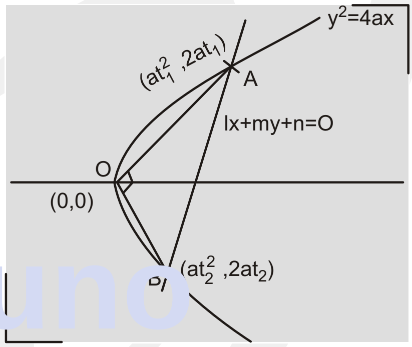


Using power of C

$$(BC)(AC)=(CS)^2=\left(\frac{1+2-1}{\sqrt{2}}\right)^2=2$$

3. (c) $OA \perp OB$

$$\Rightarrow \quad \frac{2}{t_1}\frac{2}{t_2}=-1 \quad \Rightarrow \quad t_1t_2=-4$$



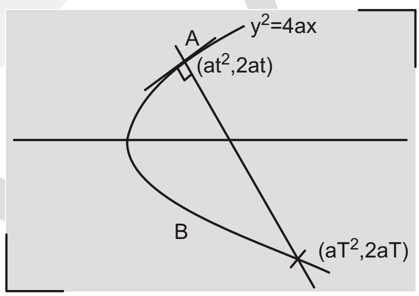
Put $(at^2, 2at)$ in eqn. of AB

$$\Rightarrow \quad lat^2+2mat+n=0 \begin{smallmatrix} t_1 \\ t_2 \end{smallmatrix}$$

$$t_1t_2=-4=\frac{n}{la}$$

$$\Rightarrow \quad n+4la=0$$

4. (a) $T=-t-\dfrac{2}{t}$



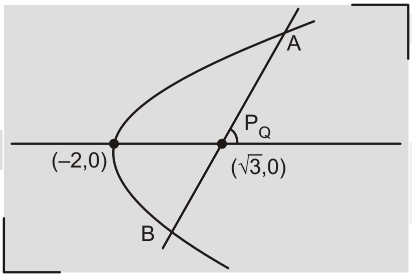
$|T|\geq 2\sqrt{2}$ (applying A.M. $\geq$ G.M.)

$T^2\geq 8$

5. (d) $PA = r_1,\ PB = -r_2$

Put $(\sqrt{3} + r\cos\theta,\ r\sin\theta)$ to $y^2 = x + 2$

$\Rightarrow \quad r^2 \sin^2\theta - r\cos\theta - (\sqrt{3}+2) = 0$ ⟨ r_1, r_2



$$(PA)(PB) = -r_1 r_2 = \frac{\sqrt{3}+2}{\sin^2\theta}$$

$$= (\sqrt{3}+2)(1+\cot^2\theta)$$

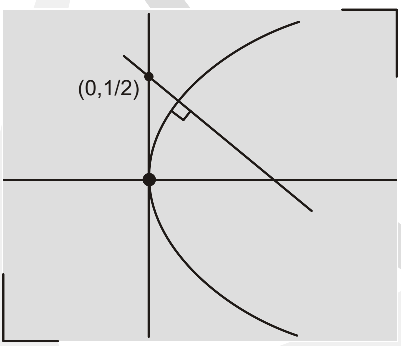
$$= (\sqrt{3}+2)\left(1+\frac{1}{3}\right) \quad [\because\ \tan\theta = \sqrt{3}]$$

$$(PA)(PB) = \frac{4}{3}(2+\sqrt{3})$$

6. (b) Equation of normal to $y^2 = 4ax$

$y = -tx + 2at + at^3$

Put $\left(0, \frac{1}{2}\right)$



$$2at + at^3 = \frac{1}{2}$$

$$f(t) = 2at^3 + 4at - 1$$

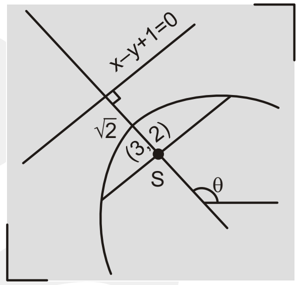
$$f'(t) = 6at^2 + 4a = 2a(3t^2+2) \neq 0$$

$\Rightarrow \quad f(t)$ can have only one real root.

7. (b) $\tan\theta = -1$

$$\text{focus, } S \equiv \left(3 - \sqrt{2}\left(-\frac{1}{\sqrt{2}}\right), 2 - \sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\right)$$

$S \equiv (4, 1)$



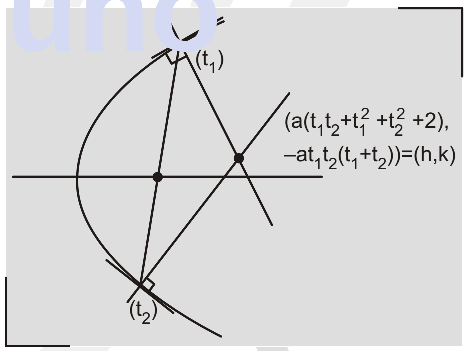
Eqn. of $LR \equiv x - y = 4 - 1 = 3$

$x - y = 3$

8. (c) $t_1 t_2 = -1$

$\Rightarrow \quad k = a(t_1 + t_2)$

$h = a(t_1^2 + t_2^2 + 1)$



$$\frac{k^2}{a^2} = \left(\frac{h}{a} - 1\right) + 2(-1) = \frac{h}{a} - 3$$

$\Rightarrow \quad y^2 = a(x - 3a)$

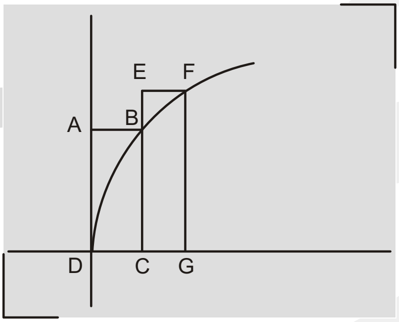
9. (c)

$$\frac{1}{a+at_1^2} + \frac{1}{a+at_2^2} = \frac{1}{a+at_1^2} + \frac{1}{a+a\left(-\frac{1}{t_1}\right)^2}$$

$$= \frac{1}{a}$$

10. (a) $y^2 = k^2 x \Rightarrow y^2 = 4\left(\frac{k^2}{4}\right)x$

$$B \equiv \left(\frac{k^2}{4}t_1^2, \frac{k^2}{2}t_1\right), F\left(\frac{k^2}{4}t_2^2, \frac{k^2}{2}t_2\right)$$



$$\frac{k^2}{4}t_1^2 = \frac{k^2}{2}t_1 \Rightarrow t_1 = 2$$

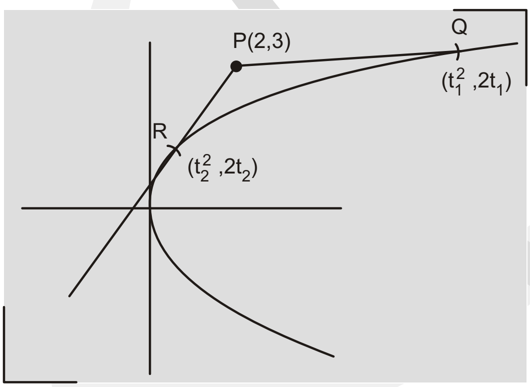
$$\frac{k^2}{2}t_2 = \frac{k^2}{4}(t_2^2 - t_1^2)$$

$$2t_2 = t_2^2 - 4 \Rightarrow t_2^2 - 2t_2 - 4 = 0$$

$\Rightarrow \quad t_2 = 1 + \sqrt{5}$

$$\frac{FG}{BC} = \frac{t_2}{t_1} = \frac{1+\sqrt{5}}{2}$$

11. (b) Equation of tangent at $(t^2, 2t)$



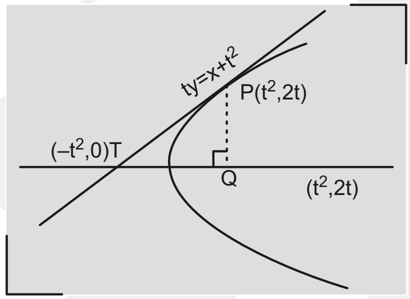
$yt = x + t^2$

Put $(2,3) \Rightarrow t^2 - 3t + 2 = 0$

$t = 1, 2 \Rightarrow Q, R \equiv (1,2)$ and $(4,4)$

12. (b) Area of $\Delta PTQ = \left|\frac{1}{2} \times (2t^2)(2t)\right|$

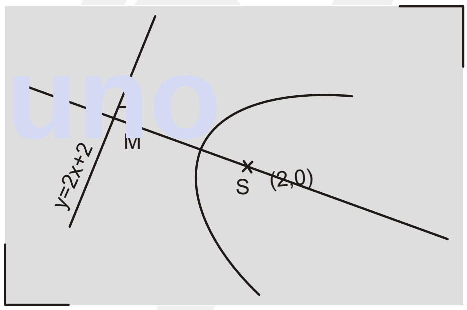
$\Delta = 2|t^3|$



$t^2 \in [1,4] \Rightarrow t \in [-2,-1] \cup [1,2]$

$\Delta_{max} = 16$ for $t = \pm 2$

13. (a) $x^2 = \frac{1}{k}y$



Length of $LR = 2(SM)$

$$= 2\frac{|2(2) + 2 - 0|}{\sqrt{2^2 + 1}} = \frac{12}{\sqrt{5}}$$

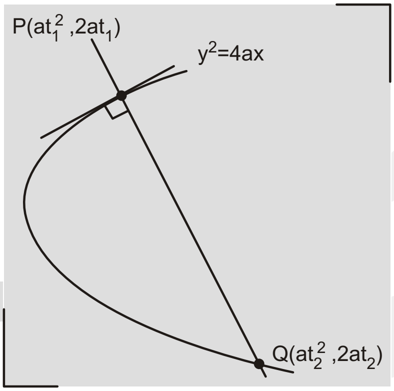
$$\Rightarrow \frac{1}{k} = \frac{12}{\sqrt{5}} \Rightarrow k = \frac{\sqrt{5}}{12}$$

14. (b) $t_2 = -t_1 - \frac{2}{t_1}$

$(t_1 + t_2)t_1 = -2$

$-t_1 = \tan\alpha, -t_2 = \tan\beta$

$\Rightarrow \tan\alpha(\tan\alpha + \tan\beta) = -2$



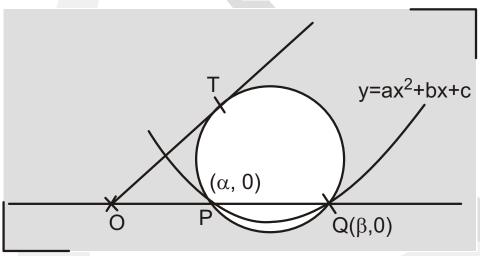
15. (d) $VA \perp VB \Rightarrow \frac{2}{t_1}\frac{2}{t_2} = -1 \Rightarrow t_1 t_2 = -4$

$$\left|\frac{VA}{VB}\right| = \frac{\sqrt{\frac{1}{4}t_1^2 + \frac{1}{16}t_1^4}}{\sqrt{\frac{1}{4}t_2^2 + \frac{1}{16}t_2^4}} = \frac{|t_1|\sqrt{4+t_1^2}}{|t_2|\sqrt{4+t_2^2}}$$

$$\frac{|t_1|\sqrt{4+t_1^2}}{\left|-\frac{4}{t_1}\right|\sqrt{4+\frac{16}{t_1^2}}} = \frac{|t_1^3|}{8} = \frac{8\cot^3\theta}{8} = \cot^3\theta$$

$$\left[\because \tan\theta = \frac{2}{t_1}\right]$$

16. (d) $ax^2 + bx + c = 0 \begin{smallmatrix}\alpha \\ \beta\end{smallmatrix} \Rightarrow \alpha\beta = \frac{c}{a}$



$$(OT)^2 = (OP)(OQ) = \alpha\beta = \frac{c}{a}$$

$$\Rightarrow \quad OT = \sqrt{\frac{c}{a}}$$

17. (d) Put $x^2 = \frac{y}{a}$

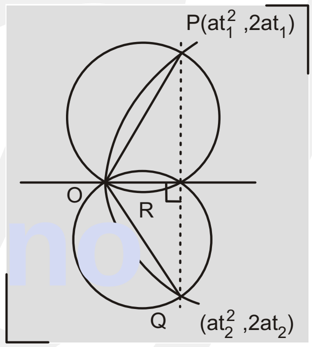
$$\Rightarrow y^2 + \left(\frac{1}{a} - 2\right)y = 0$$

$$\Rightarrow y = 0, \quad y = 2 - \frac{1}{a}$$

$$2 > 2 - \frac{1}{a} > 0 \Rightarrow a > \frac{1}{2}$$

18. (a) $\angle ORP = \frac{\pi}{2} = \angle ORQ$

slope of $PQ = \frac{2}{t_1 + t_2}$



$$\Rightarrow \text{slope of } OR = -\left(\frac{t_1 + t_2}{2}\right) = \tan\phi$$

$$\Rightarrow \frac{1}{t_1} = \tan\theta_1 \quad \frac{1}{t_2} = \tan\theta_2$$

$$\Rightarrow \tan\phi = -\left(\frac{\cot\theta_1 + \cot\theta_2}{2}\right)$$

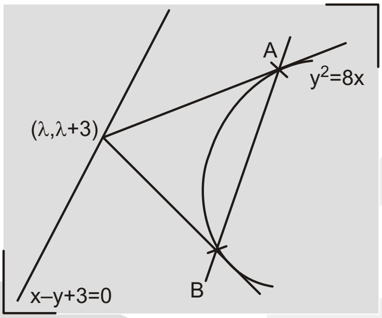
$$\Rightarrow \cot\theta_1 + \cot\theta_2 = -2\tan\phi$$

19. (c) Equation of AB is

$y(\lambda + 3) = 4(x + \lambda)$

$\Rightarrow (3y - 4x) + \lambda(y - 4) = 0$

represents family of lines passing through $(3, 4)$



20. (a) SP, semi latus rectum, SQ are in H.P.

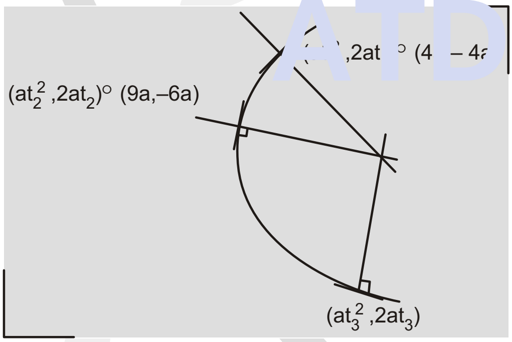
$$\Rightarrow \frac{2}{2a} = \frac{1}{SP} + \frac{1}{SQ} = \frac{1}{3} + \frac{1}{2} = \frac{5}{6}$$

$$\Rightarrow a = \frac{6}{5} \quad \Rightarrow \text{Length of } LR = 4a = \frac{24}{5}$$

21. (b) $t_1 = -2,\ t_2 = -3$

$$t_1 + t_2 + t_3 = 0$$

$$\Rightarrow t_3 = 5$$



Equation of normal at $(at_3^2, 2at_3)$

$$y = -t_3 x + 2at_3 + at_3^3$$

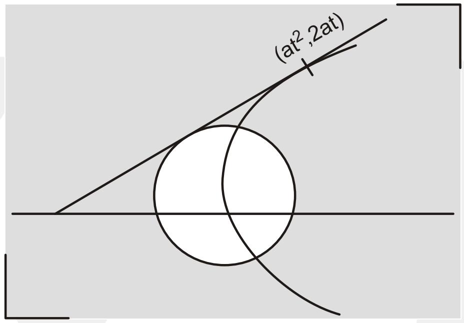
$$y = -5x + 10a + 125a$$

$$5x + y - 135a = 0$$

22. (a) Equation of tangent is

$$yt = x + at^2$$

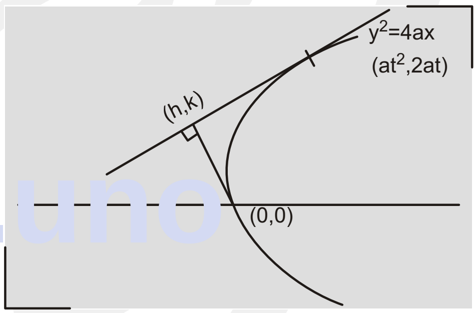
$$\text{slope} = \frac{1}{t} = \tan\frac{\pi}{4} = 1 \Rightarrow t = 1$$



$$\Rightarrow y = x + a$$

$$\Rightarrow c = \frac{|0 + a - 0|}{\sqrt{2}} = \frac{a}{\sqrt{2}} \Rightarrow a = \sqrt{2}c$$

23. (a) Equation of tangent 'L' is



$$yt - x = at^2 \qquad \ldots(1)$$

Also L is given by $hx + ky = h^2 + k^2 \qquad \ldots(2)$

(1) and (2) are identical

$$\Rightarrow \frac{t}{k} = -\frac{1}{h} = \frac{at^2}{h^2 + k^2} \Rightarrow t = -\frac{k}{h}$$

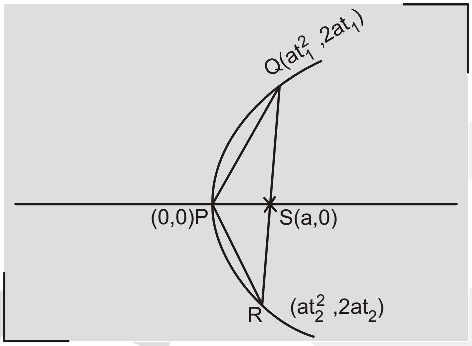
$$\therefore \quad at = \frac{h^2 + k^2}{k} = a\left(-\frac{k}{h}\right)$$

$$\Rightarrow x(x^2 + y^2) + ay^2 = 0$$

24. (c) Area = modulus of $\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \end{vmatrix}$

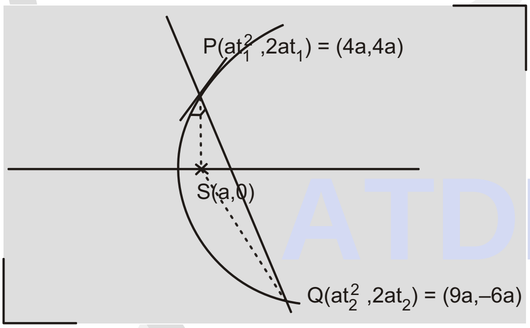
$$= |a^2 t_1 t_2 (t_1 - t_2)|$$

$$A = |-a^2(t_1 - t_2)|$$



$$|2a(t_1 - t_2)| = \frac{2A}{a}$$

25. (d) $at_1^2 = 2at_1 \Rightarrow t_1 = 2$



$$t_2 = -t_1 - \frac{2}{t_1} = -3$$

$$m_{PS} = \frac{4a - 0}{4a - a} = \frac{4}{3}$$

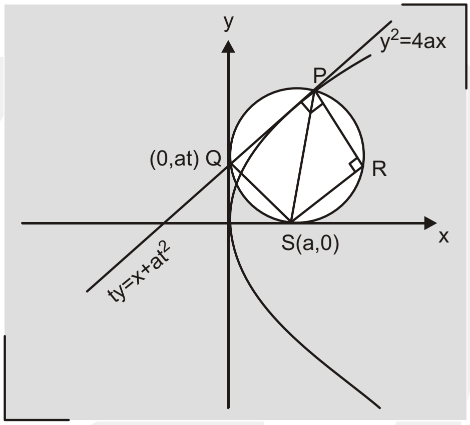
$$m_{QS} = \frac{-6a - 0}{9a - a} = -\frac{3}{4}$$

$$m_{PS} m_{QS} = -1 \quad \Rightarrow PS \perp SQ$$

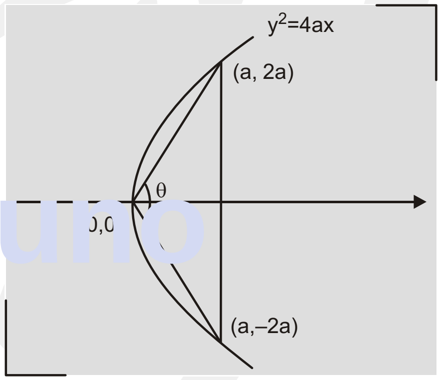
26. (d) $PRSQ$ is a rectangle

$$\Rightarrow \quad PR = SQ = \sqrt{a^2 + a^2 t^2}$$

$$PR = a\sqrt{1 + t^2}$$



27. (d) $\theta = 2\tan^{-1} 2$



$$\sqrt{3} < 2 < \sqrt{2} + 1$$

$$\Rightarrow \quad \frac{\pi}{3} < \tan^{-1} 2 < \frac{3\pi}{8}$$

$$\Rightarrow \quad \frac{2\pi}{3} < \theta < \frac{3\pi}{4}$$

28. (b) $\frac{1}{t_1} = 1 \quad \Rightarrow t_1 = 1$

Equation of tangent at A is

$$y t_1 = x + at_1^2$$

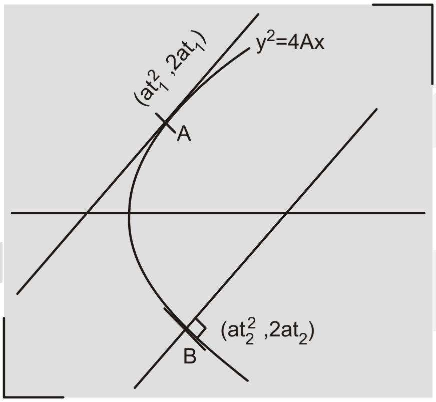
$$\Rightarrow \quad y = x + a \qquad \ldots(1)$$

$$-t_2 = 1 \Rightarrow t_2 = -1$$

Equation of normal at B is

$$y = -t_2 x + 2at_2 + at_2^3$$

$\Rightarrow \quad y = x - a - 2a \Rightarrow y = x - 3a \qquad \ldots(2)$



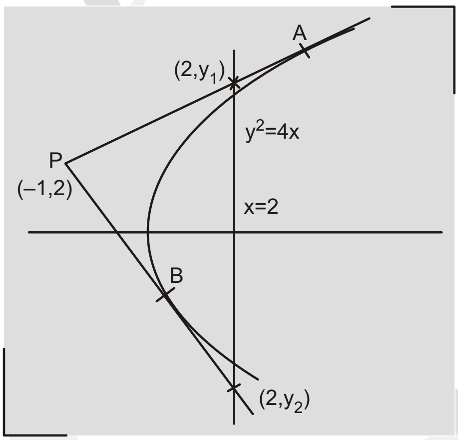
$\perp$ar distance between lines (1) and (2) is

$$\frac{(a+3a)}{\sqrt{1^2+1^2}} = \frac{4a}{\sqrt{2}} = 2\sqrt{2}\,a$$

29. (b) Equation of pair PA and PB is

$$(y(2) - 2(x-1))^2 = (y^2 - 4x)(4+4)$$

Put $\quad x = 2$



$\Rightarrow \quad (y-1)^2 = 2(y^2 - 8)$

$\Rightarrow \quad y^2 + 2y - 17 = 0$

$$(y_1 - y_2)^2 = (-2)^2 - 4(-17) = 72$$

$$\Rightarrow |y_1 - y_2| = 6\sqrt{2}$$

30. (d) Put $(h + r\cos\theta,\ k + r\sin\theta)$ to $y^2 = 4x$

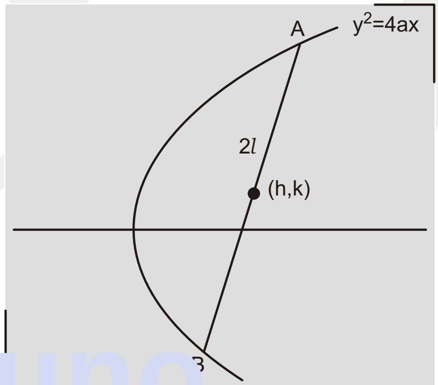
$\Rightarrow \quad k^2 + r^2\sin^2\theta + 2kr\sin\theta = 4h + 4r\cos\theta$

$\Rightarrow \quad r^2\sin^2\theta + (2k\sin\theta - 4\cos\theta)r$

$$+(k^2 - 4h) = 0 \begin{array}{l} \nearrow l \\ \searrow -l \end{array}$$

$$l + (-l) = 2k\sin\theta - 4\cos\theta = 0$$

$\Rightarrow \quad \tan\theta = \dfrac{2}{k}$



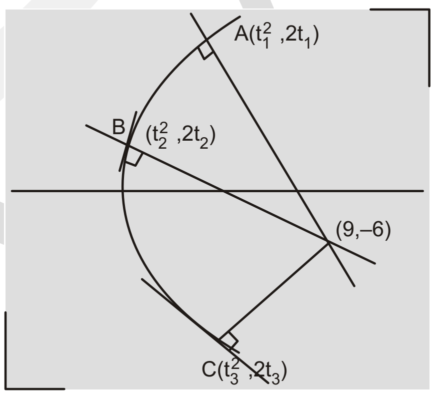
$$l(-l) = \frac{k^2 - 4h}{\sin^2\theta} = (k^2 - 4h)(1 + \cot^2\theta)$$

$\Rightarrow \quad (4x - y^2)\left(1 + \dfrac{y^2}{4}\right) = l^2$

$\Rightarrow \quad (4x - y^2)(4 + y^2) = 4l^2$

31. (c) Equation of normal at $(t^2, 2t)$ is

$$y = -tx + 2t + t^3$$



Put $\quad (9, -6)$

$\Rightarrow t^3 - 7t + 6 = 0 \Rightarrow (t-1)(t-2)(t+3) = 0$

$\Rightarrow t = 1, 2 - 3$

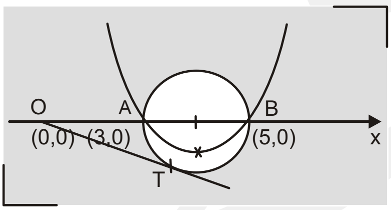
$a_{11} = m_1 + 1 = -1 + 1 = 0$

$a_{22} = m_2 + 2 = -2 + 2 = 0$

$a_{33} = m_3 + 3 = 3 + 3 = 6$

$$|A| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 6 \end{vmatrix} = -4$$

32. (d)



$(OT)^2 = (OA)(OB)$

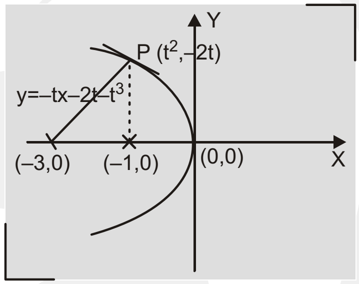
$= 3 \times 5 = 15$

$\Rightarrow \quad OT = \sqrt{15}$

33. (b) $(y-2)^2 = -4(x-2)$

$y - 2 = Y, x - 2 = X$

$Y^2 = -4X$



Line is $X - Y = -3$

$\Rightarrow \quad Y = X + 3 \qquad ...(1)$

$Y = -tX - 2t - t^3 \qquad ...(2)$

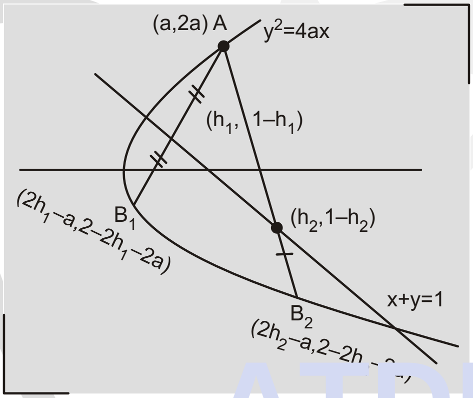
(1) and (2) are identical for $t = -1$

SOLUTIONS 2

One or More than One is/are Correct

1. (b, c, d)

Put $(2h-a, 2-2h-2a)$ in the equation of parabola



$\Rightarrow \quad 4(1-h-a)^2 = 4a(2h-a)$

$\Rightarrow \quad h^2 - 2h + (2a^2 - 2a + 1) = 0 \begin{matrix} h_1 \\ h_2 \end{matrix}$

For two distinct real roots

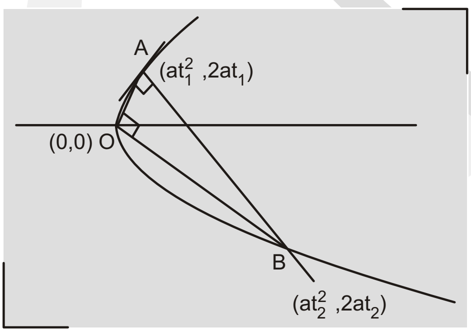
$D > 0 \quad \Rightarrow \quad 4 - 4(2a^2 - 2a + 1) > 0$

$\Rightarrow \quad a(a-1) < 0$

$\Rightarrow \quad a \in (0, 1)$

$\Rightarrow \quad$ Length of Latus rectum $\in (0, 4)$

2. (b, c)



$$\frac{2}{t_1}\frac{2}{t_2} = -1 \quad \Rightarrow \quad t_1 t_2 = -4$$

$$t_2 = -t_1 - \frac{2}{t_1} \quad \Rightarrow \quad \frac{-4}{t_1} = -t_1 - \frac{2}{t_1}$$

$$t_1 = \pm\sqrt{2}$$

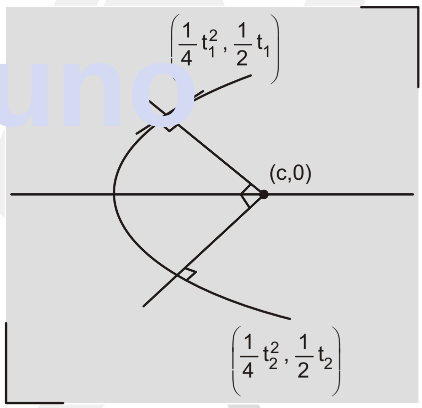
$\therefore \quad$ slope of $AB = -t_1 = \mp\sqrt{2}$

3. (a, c)

Equation of normal at $\left(\frac{1}{4}t^2, \frac{1}{2}t\right)$ is

$$y = -tx + \frac{1}{2}t + \frac{1}{4}t^3$$

Put $(c, 0)$



$\Rightarrow \quad \frac{1}{4}t^3 + \left(\frac{1}{2} - c\right)t = 0$

$\Rightarrow \quad t^2 - (4c-2) = 0 \begin{matrix} t_1 \\ t_2 \end{matrix}$

$\Rightarrow \quad 4c - 2 > 0 \Rightarrow c > \frac{1}{2}$ and $t_1 t_2 = -1$

$\Rightarrow \quad 4c - 2 = 1 \quad \Rightarrow \quad c = 3/4$

4. (a, c, d)

$$(PQ)^2 = a^2[(t^2 - t'^2)^2 + 4(t - t')^2]$$

$$= a^2[(t + t')^2 - 4tt'][(t + t')^2 + 4]$$

$$t' = -t - \frac{2}{t}$$

$$= a^2\left[\frac{4}{t^2} + 4(t^2 + 2)\right]\left[\frac{4}{t^2} + 4\right]$$

$$= 16a^2\left(t^2 + \frac{1}{t^2} + 2\right)\left(\frac{1}{t^2} + 1\right)$$

$$= 16a^2\left[3 + t^2 + \frac{1}{t^4} + \frac{3}{t^2}\right]$$

$$PQ^2 = 16a^2(3 + x + \frac{1}{x^2} + \frac{3}{x}) \qquad x > 0$$

$$\frac{d}{dx}(PQ^2) = 16a^2\left(1 - \frac{2}{x^3} - \frac{3}{x^2}\right)$$

$$= 16a^2\frac{(x^3 - 3x - 2)}{x^3}$$

$$= \frac{16a^2(x+1)^2(x-2)}{x^3}$$

PQ^2 is minimum at $t^2 = 2$

$$(PQ)_{\min} = 4a\sqrt{3 + 2 + \frac{1}{4} + \frac{3}{2}} = 6\sqrt{3}\,a$$

$$t' = -\sqrt{2} - \frac{2}{\sqrt{2}} = -2\sqrt{2}$$

$\Rightarrow \quad Q \equiv (8a, -4\sqrt{2}\,a)$

or $\quad t' = \sqrt{2} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$

$\Rightarrow \quad Q \equiv (8a, 4\sqrt{2}\,a)$

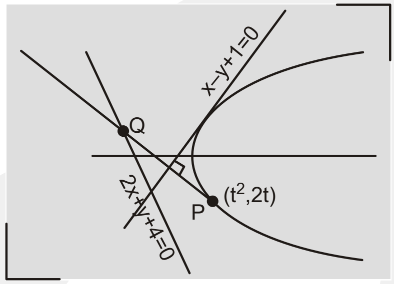
5. (a, c)

Q is image of P w.r.t. $x - y + 1 = 0$

$$\frac{x - t^2}{1} = \frac{y - 2t}{-1} = -2\left(\frac{t^2 - 2t + 1}{1^2 + 1^2}\right)$$

$$Q \equiv (x, y) = (2t - 1, t^2 + 1)$$

Put Q to equation $2x + y + 4 = 0$

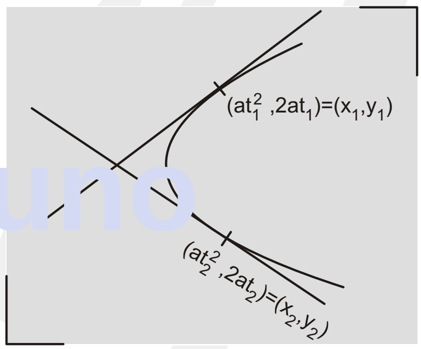


$\Rightarrow \quad 2(2t - 1) + (t^2 + 1) + 4 = 0$

$(t + 3)(t + 1) = 0 \quad \Rightarrow t = -1, -3$

$\Rightarrow \quad P \equiv (1, -2), (9, -6)$

6. (b, d)



$$(at_1^2)(at_2^2) = (at_1t_2)^2$$

$\Rightarrow \quad x_1x_2 = x_3^2$

$$\frac{2at_1 + 2at_2}{2} = a(t_1 + t_2)$$

$\Rightarrow \quad \frac{y_1 + y_2}{2} = y_3$

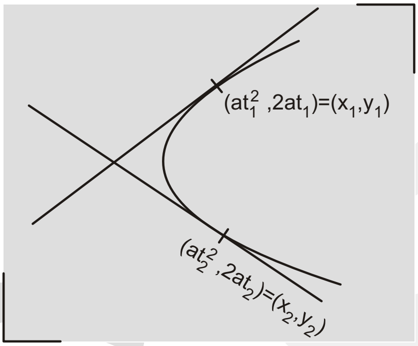
7. (a, c, d)

$$\text{slope} = \frac{2}{t_1 + t_2} = 1 \Rightarrow t_1 + t_2 = 2$$

locus of R is

$$k = a(t_1 + t_2) = 2a$$

$\Rightarrow \quad y = 2a$



locus of S is

$$h = a((t_1+t_2)^2 - t_1t_2 + 2) = a(6 - t_1 t_2)$$

$$k = -a t_1 t_2 (t_1+t_2) = -2a t_1 t_2$$

$$\therefore \quad 2h - k = 12a$$

$$2x - y = 12a$$

8. (a, b)

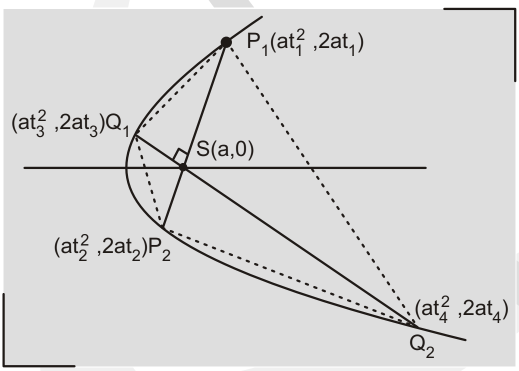
$$P_1P_2 = (a + at_1^2) + (a + at_2^2)$$

$$= a[(t_1+t_2)^2 + 4] \quad \because t_1 t_2 = -1$$

$$Q_1Q_2 = a[t_3+t_4)^2 + 4]$$

$$\frac{2}{(t_1+t_2)}\frac{2}{(t_3+t_4)} = -1$$

$$\Rightarrow \quad |(t_1+t_2)(t_3+t_4)| = 4$$



Area of $P_1Q_1P_2Q_2$, $A = \frac{1}{2}|\vec{d_1} \times \vec{d_2}|$

$$= \frac{1}{2}d_1d_2 = \frac{1}{2}a^2[(t_1+t_2)^2+4][t_3+t_4)^2+4]$$

$$A \geq \frac{1}{2}a^2 2\sqrt{4(t_1+t_2)^2}\,2\sqrt{4(t_3+t_4)^2}$$

$$= 8a^2|t_1+t_2|\,|t_3+t_4| = 32a^2$$

$A_{min} = 2(4a)^2$ at

$$(t_1+t_2)^2 = 4 = (t_3+t_4)^2$$

$\Rightarrow \quad t_1+t_2 = 2, t_3+t_4 = -2$

or $\quad t_1+t_2 = -2,$

$t_1+t_2 = -2, \; t_3+t_4 = 2$

Slope of $P_1P_2 = 1$, slope of $Q_1Q_2 = -1$

or slope of $P_1P_2 = -1$, slope of $Q_1Q_2 = 1$

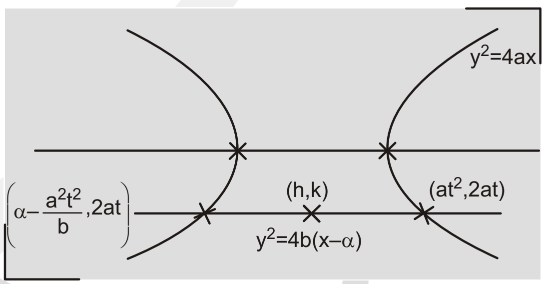
9. (a, b)

$$k = 2at$$

$$2h = at^2 + \alpha - \frac{a^2t^2}{b}$$

$$2h = \left(a - \frac{a^2}{b}\right)\frac{k^2}{4a^2} + \alpha$$

$$y^2\left(\frac{b-a}{4ab}\right)\frac{k^2}{4a^2} + \alpha = 2x$$

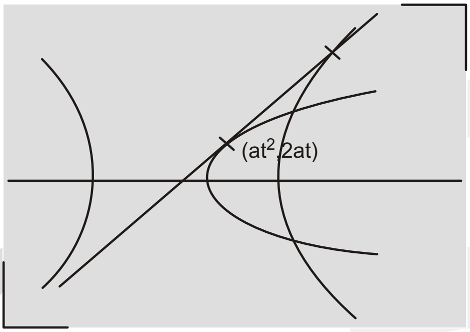


If $a = b$, $\quad 2x = \alpha$, $\quad$ straight line

If

$a \neq b$, $y^2(b-a) + 4ab\alpha = 8abx$, Parabola

10. (a, c)



Equation of tangent to $y^2 = 4ax$ is

$$yt = x + at^2 \Rightarrow y = \frac{1}{t}x + at \qquad ...(1)$$

$\Rightarrow \quad a^2t^2 = a^2\frac{1}{t^2} - a^2$

for (1) to represent tangent to $x^2 - y^2 = a^2$

$\Rightarrow \quad t^4 + t^2 - 1 = 0$

$\Rightarrow \quad t^2 = \frac{\sqrt{5}-1}{2} \Rightarrow t = \pm\sqrt{\frac{\sqrt{5}-1}{2}}$

$\Rightarrow$ There exists two tangents $\forall\, a \in R - \{0\}$

11. (b, d)

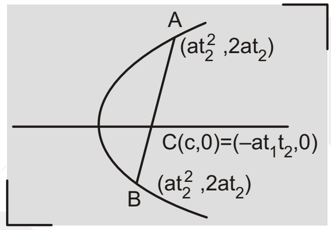
$$t_1 t_2 = -\frac{c}{a} \Rightarrow c = -at_1 t_2$$

$$\frac{AC^2}{CB^2} = \frac{(at_1^2 + at_1t_2)^2 + 4a^2t_1^2}{(at_2^2 + at_1t_2)^2 + 4a^2t_2^2} = \frac{t_1^2}{t_2^2}$$

$$\frac{CB}{AC} = \pm\frac{t_2}{t_1} \Rightarrow \frac{CB}{AC} + 1 = \pm\frac{t_2}{t_1} + 1$$

$\Rightarrow \quad \frac{AB}{AC} = \frac{t_2 + t_1}{t_1} = 3$

or $\quad \frac{AB}{AC} = \frac{t_1 - t_2}{t_1} = 3$



$\Rightarrow \quad t_2 + t_1 = 3t_1$ or $t_1 - t_2 = 3t_1$

$\Rightarrow \quad 2t_1 - t_2 = 0$ or $2t_1 + t_2 = 0$

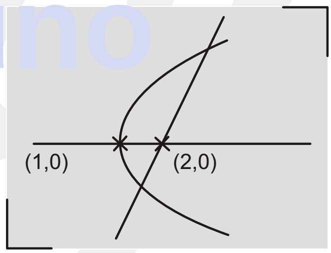
[$\because$ if $\frac{CB}{AC} = -\frac{t_2}{t_1} \Rightarrow t_1$ and t_2 are of opp. sign.]

$$6t_1^2 - 2t_2^2 - t_1t_2 = 0 = 6t_1^2 + 3t_1t_2 - 4t_1t_2 - 2t_2^2$$

$\Rightarrow \quad (3t_1 - 2t_2)(2t_1 + t_2) = 0$

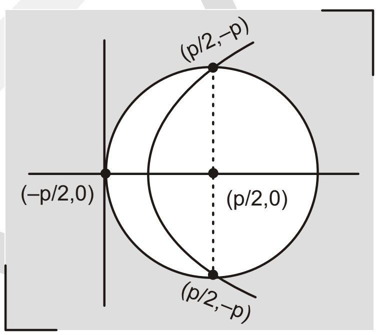
12. (a, c)

$m \neq 0$

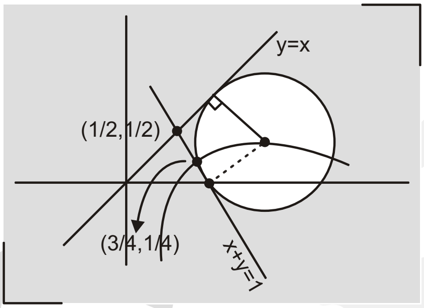


13. (a, b)

AB is latus rectum of parabola

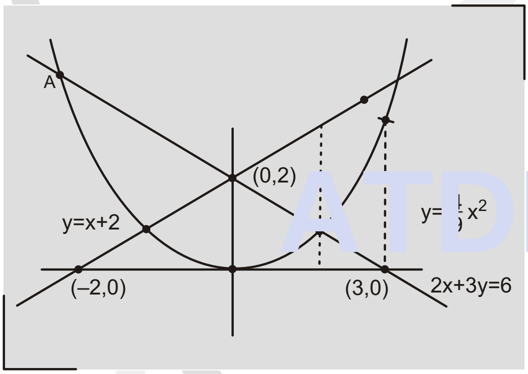


14. (b, c)



$$4a = 2\left(\frac{|0-1|}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$$

15. (a, c)



For point A, B

$$2x + 3\left(\frac{4}{9}x^2\right) = 6$$

$$2x^2 + 3x - 9 = 0$$

$$2x^2 + 6x - 3x - 9 = 0 = (2x-3)(x+3) = 0$$

$$\Rightarrow \quad A \equiv (-3, 4) \text{ and } B \equiv \left(\frac{3}{2}, 1\right)$$

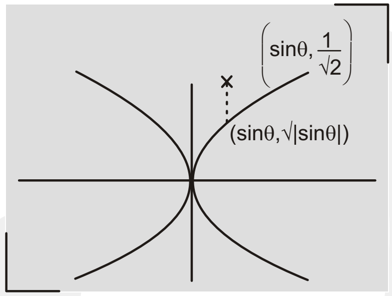
$$\therefore \quad \frac{3}{2}\alpha = -3 \Rightarrow \alpha = -2,$$

$$\frac{3}{2}\alpha = \frac{3}{2} \Rightarrow \alpha = 1$$

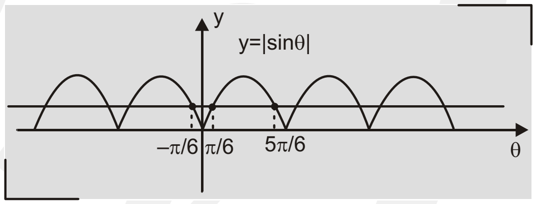
$$\therefore \quad \alpha \in (-\infty, -2) \cup (0, 1)$$

16. (a, b, c)

$$\sqrt{|\sin\theta|} < \frac{1}{\sqrt{2}}$$



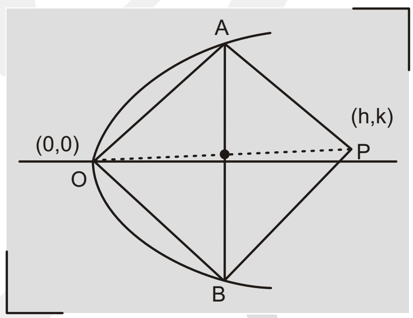
$$\Rightarrow \quad |\sin\theta| < \frac{1}{2}$$



$$\theta \in \left(n\pi - \frac{\pi}{6}, n\pi + \frac{\pi}{6}\right), n \in I$$

19. (a, c)

Equation of AB is



$$y\left(\frac{k}{2}\right) - 2a\left(x + \frac{h}{2}\right) = \frac{k^2}{4} - 2ah$$

$$2yk - 8ax = k^2 - 4ah$$

Homogenising, we get

$$(k^2 - 4ah)y^2 - 4ax(2yk - 8ax) = 0$$

coefficient of x^2 + coefficient of $y^2 = 0$

$$\Rightarrow \quad k^2 - 4ah + 32a^2 = 0$$

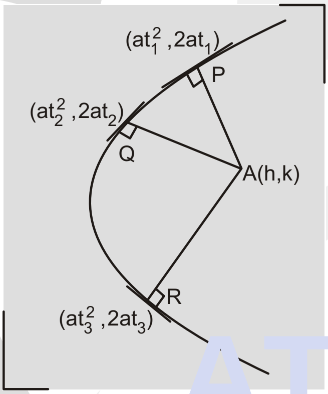
$$\Rightarrow \quad y^2 = 4a(x - 8a)$$

SOLUTIONS 3

Comprehension: (1)

1. (a) $y = -tx + 2at + at^3$ is normal to $y^2 = 4ax$ at $(at^2, 2at)$

Put (h, k)



$\Rightarrow at^3 + (2a - h)t - k = 0 \begin{matrix} t_1 \\ t_2 \\ t_3 \end{matrix}$

$t_1 + t_2 + t_3 = 0,$

$t_1t_2 + t_2t_3 + t_3t_1 = \frac{2a - h}{a}$

$t_1^2 + t_2^2 + t_3^2 = (\Sigma t_1)^2 - 2\Sigma t_1t_2$

$= 0 - \frac{2(2a - h)}{a}$

$\therefore$ Centroid

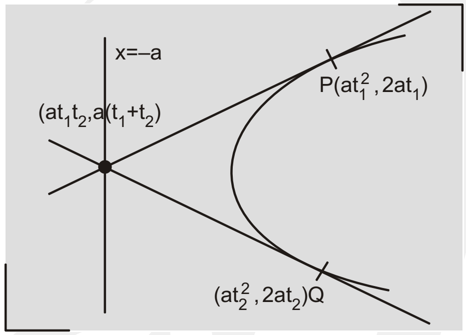
$\equiv \left(\frac{a}{3}(t_1^2 + t_2^2 + t_3^2), \frac{2a}{3}(t_1 + t_2 + t_3)\right)$

$\equiv \left(\frac{a}{3}\frac{2}{a}(h - 2a), 0\right)$

Centroid $\equiv \left(\frac{2}{3}(h - 2a), 0\right)$

2. (d) Intersection point

$\equiv (at_1t_2, a(t_1 + t_2)) \equiv (-a, -at_3)$



Quadratic whose roots are t_1, t_2 is

$\Rightarrow x^2 - (t_1 + t_2)x + t_1t_2 = 0$

$\Rightarrow x^2 + t_3x - 1 = 0$

3. (c) $at^3 + (2a - h)t - k = 0 \begin{matrix} t_1 \\ t_2 \\ t_3 \end{matrix}$

$t_1t_2 = -1 \Rightarrow t_1t_2t_3 = -t_3 = -t_3 = \frac{k}{a}$

$\Rightarrow t_3 = -\frac{k}{a}$

Put t_3 to the cubic

$a\left(-\frac{k^3}{a^3}\right) - \frac{k}{a}(2a - h) - k = 0$

$\frac{k^2}{a^2} + 2 - \frac{h}{a} + 1 = 0$

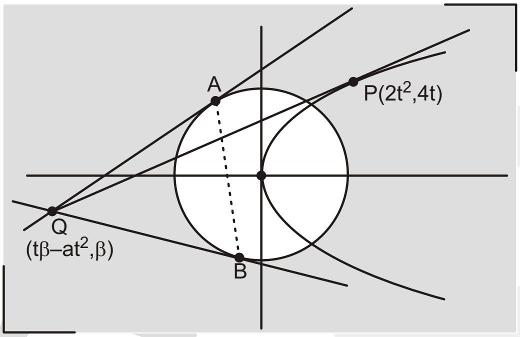
$k^2 = a(h - 3a)$

$\Rightarrow y^2 = a(x - 3a)$

Comprehension: (2)

1. (c) Equation of tangent PQ is $ty = x + 2t^2$

Equation of COC of Q w.r.t. $x^2 + y^2 = 4$ is $x(t\beta - 2t^2) + y\beta = 4$



$$(tx + y)\beta - (4 + 2t^2 x) = 0$$

$\therefore$ COC's are concurrent at $\left(-\dfrac{2}{t^2}, \dfrac{2}{t}\right)$

$\therefore$ $h = -\dfrac{2}{t^2}$

$k = \dfrac{2}{t} \Rightarrow k^2 = \dfrac{4}{t^2}$

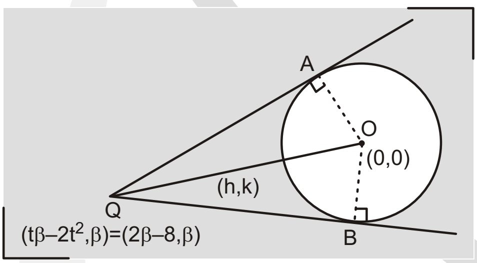
$\Rightarrow \dfrac{k^2}{h} = -2 \Rightarrow y^2 = -2x$

2. (d) Point lies on $x = -2$

and $x^2 + y^2 = 2(4) = 8$

$\therefore$ $y^2 = 4 \Rightarrow y = \pm 2$

3. (a) Circumcentre is midpoint of OQ



$2h = 2\beta - 8$

$2k = \beta$

$\Rightarrow 4k - 2h = 8$

$2y = x + 4$

Comprehension: (3)

1. (a), 2. (c), 3. (d)

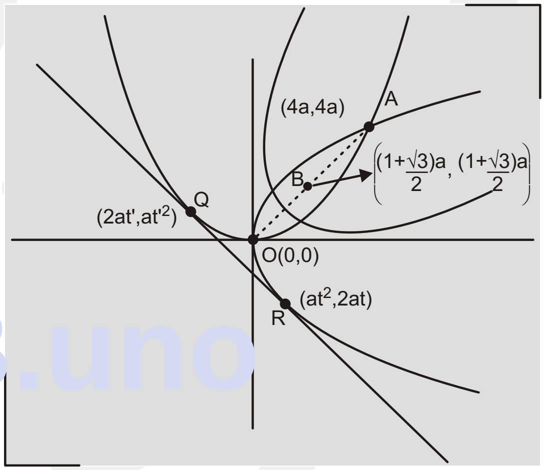
Eqn. of tangent at Q

$xt' - y = at'^2$ ...(1)

Eqn. of tangent at P

$yt - x = at^2$ ...(2)

(1) and (2) are identical



$\Rightarrow -t = -\dfrac{1}{t'} = \dfrac{at^2}{at'^2}$

$\Rightarrow t^4 = -t \Rightarrow t = -1$ ($t = 0$ rejected)

$\therefore$ Common tangent is $x + y + a = 0$

LR of P is

$$2\left(\frac{\dfrac{(1+\sqrt{3})a}{2} + \dfrac{(1+\sqrt{3})a}{2} + a}{\sqrt{2}}\right) = \sqrt{2}(2+\sqrt{3})a$$

Eqn. of parabola P is

$$\frac{(y-x)^2}{2} = \sqrt{2}(2+\sqrt{3})a$$

$$\left(\frac{\left(x+y-\frac{(1+\sqrt{3})a}{2}\right)}{\sqrt{2}}\right)$$

$$(y-x)^2=(2+\sqrt{3})\,a\,(2x+2y-(1+\sqrt{3})a)$$

Extremities of *L.R.* of *P* is

$$\left(\frac{(1+\sqrt{3})}{2}a-\frac{1}{\sqrt{2}}\frac{(2+\sqrt{3})a}{\sqrt{2}},\frac{(1+\sqrt{3})a}{2}+\frac{1}{2}\frac{(2+\sqrt{3})a}{\sqrt{2}}\right)$$

$$\left(\frac{(1+\sqrt{3}}{2}a+\frac{1}{\sqrt{2}}\frac{(2+\sqrt{3})a}{\sqrt{2}},\frac{(1+\sqrt{3})a}{2}-\frac{1}{\sqrt{2}}\frac{(2+\sqrt{3})a}{\sqrt{2}}\right)$$

$$\equiv\left(-\frac{a}{2},\frac{(3+2\sqrt{3})a}{2}\right),\left(\frac{(3+2\sqrt{3})a}{2},-\frac{a}{2}\right)$$

Comprehension: (4)

$$y=x^2+bx+1$$

For tangent from all points $(\alpha, 0)$ on x-axis

$\therefore\quad x^2+bx+1\ge 0\ \forall\ x\in R$

$\Rightarrow\quad D\le 0 \Rightarrow\quad b^2-4\le 0$

$\Rightarrow\quad b\in[-2,2]$

$\therefore\quad b_{max}=2$

Eqn. of tangent at (0, 1) to $y=x^2+bx+1$

is $y+1=bx+2\Rightarrow y=bx+1$ ...(1)

$\because$ Eqn. (1) represents tangent to

$$x^2+y^2=r^2$$

$$\Rightarrow\qquad r=\frac{1}{\sqrt{1+b^2}}$$

1. (b) for $b=2$, $r=\frac{1}{\sqrt{5}}$

$$A=\frac{\pi}{5}$$

2. (d) $r=\frac{1}{\sqrt{1+b^2}}$, $r_{max}=1$

$$\lim_{b\to 0}\frac{\sqrt{1-\frac{1}{\sqrt{1+b^2}}}}{\sin b}=\lim_{b\to 0}\frac{\sqrt{(1+b^2)^{1/2}-1}}{(1+b^2)^{1/4}\sin b}$$

$$=\lim_{b\to 0}\frac{|b|}{(\sqrt{1+b^2}+1)^{1/2}(1+b^2)^{1/4}\sin b}$$

$LHL=-\frac{1}{\sqrt{2}}$, $RHL=\frac{1}{\sqrt{2}}\Rightarrow$ Limit does not exist

3. (d) Vertex $\equiv\left(-\frac{b}{2},1-\frac{b^2}{4}\right)=(h,k)$,

$$k=1-h^2,\quad y=1-x^2,$$

$$-\frac{b}{2}\in[-1,1]\Rightarrow\frac{b^2}{4}\in[0,1]$$

$$\Rightarrow\ 1-\frac{b^2}{4}\in[0,1]$$

$$\Rightarrow\ x\in[-1,1],\quad y\in[0,1]$$

Comprehension: (5)

$$(x,y)=(\cos\theta,\sin\theta)$$

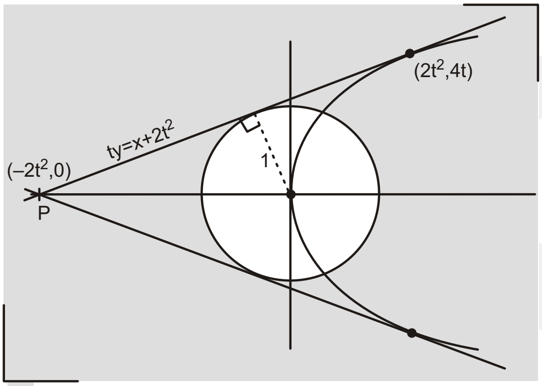
$$A=\lim_{\theta\to\frac{\pi}{4}}\frac{4\cos^2\theta+2\sin^2\theta-6\sin\theta\cos\theta}{6\cos^2\theta+\sqrt{2}\sin\theta-8\sin\theta\cos\theta}$$

$$=\lim_{\theta\to\frac{\pi}{4}}\frac{(4\cos\theta+2\sin\theta)(\cos\theta-\sin\theta)}{6\cos\theta(\cos\theta-\sin\theta)+\sqrt{2}\sin\theta(1-\sqrt{2}\cos\theta)}$$

$$=\lim_{\theta\to\frac{\pi}{4}}\frac{(4\cos\theta-2\sin\theta)(\cos\theta-\sin\theta)}{6\cos\theta+\sqrt{2}\sin\theta\frac{(1-\sqrt{2}\cos\theta)}{(\cos\theta-\sin\theta)}}$$

$$=\lim_{\theta\to\frac{\pi}{4}}\frac{(4\cos\theta-2\sin\theta)}{6\cos\theta-\sqrt{2}\sin\theta\frac{(\cos\theta+\sin\theta)}{(1+\sqrt{2}\cos\theta)}}=\frac{2}{5}$$

1. (c) Perpendicular from $(0,0)=1$



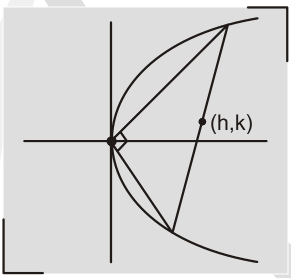
$$\Rightarrow \quad \frac{2t^2}{\sqrt{t^2+1}}=1$$

$$\Rightarrow \quad 4t^4=t^2+1$$

$$\Rightarrow \quad t^2=\frac{1+\sqrt{17}}{8}$$

$$P\equiv\left(-\frac{\sqrt{17}+1}{4},0\right)=\left(-\frac{4}{\sqrt{17}-1},0\right)$$

2. (a) Equation of chord whose midpoint is (h,k) is



$$yk-4(x+h)=k^2-8h$$

$$yk-4x=k^2-4h$$

Homogenising,

$$(k^2-4h)\,y^2=8x\,(yk-4x)$$

$$\Rightarrow \quad (k^2-4h)\,y^2+32x^2-8kxy=0$$

coefficient of x^2 + coefficient of $y^2=0$

$$\Rightarrow \quad k^2-4h+32=0$$

Hence, $y^2-4x+32=0$

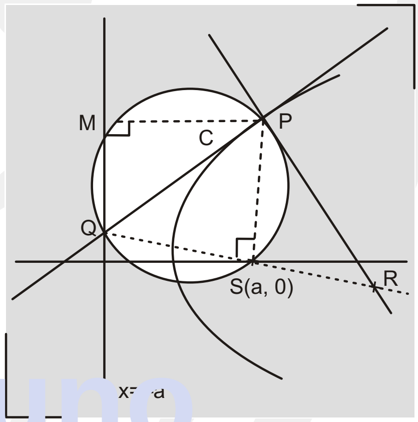
3. (c) Obvious

Comprehension: (6)

1. (d) PQ is diameter of circle

$\Rightarrow$ tangent of parabola is normal to C

$\therefore$ Circle & parabola are orthogonal



2. (d) Area of

$$\Delta PQR=\frac{1}{2}(PS)(QR)=\frac{1}{2}(PQ)(PR)$$

$$\Rightarrow \quad \frac{(PS)(QR)}{(PQ)(PR)}=1$$

3. (c) $(PQ)^2$

$$=(at^2+a)^2+\left(2at-\frac{at^2-a}{t}\right)^2$$

$$=a^2\left[(t^2+1)^2+\frac{(t^2+1)^2}{t^2}\right]=\frac{(t^2+1)^3a^2}{t^2}$$

$$\text{Area of } C=\frac{\pi}{4}(PQ)^2=\frac{\pi}{4}\frac{(t^2+1)^3}{t^2}a^2$$

Comprehension: (7)

1. (a), 2. (c), 3. (d)

Equation of CD is

$$x(t_1+t_2)=2y+\frac{1}{2}t_1t_2.,$$

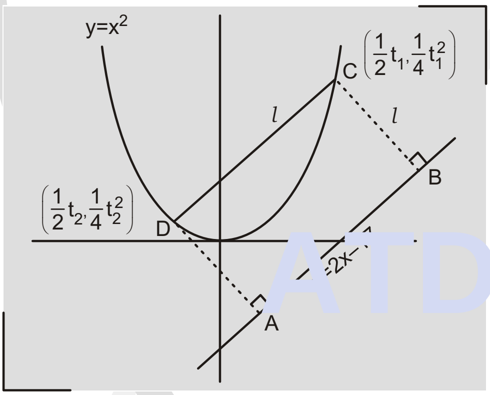
$$y \text{ intercept} = -\frac{1}{4}t_1 t_2$$

$$\text{slope} = \frac{t_1 + t_2}{2} = 2 \Rightarrow t_1 + t_2 = 4$$

$$l = \frac{\frac{1}{4}t^2 - 2\left(\frac{1}{2}t\right) + 17}{\sqrt{5}}$$

$$\Rightarrow \quad t^2 - 4t + (-4l\sqrt{5} + 68) = 0 \begin{matrix} \nearrow t_1 \\ \searrow t_2 \end{matrix} \quad ...(1)$$

$$\text{Also } l^2 = \frac{1}{4}(t_1 - t_2)^2 + \frac{1}{16}(t_1^2 - t_2^2)^2$$



$$\Rightarrow l^2 = [(t_1 + t_2)^2 - 4t_1 t_2]\left[\frac{1}{4} + \frac{1}{16}(t_1 + t_2)^2\right]$$

$$\Rightarrow l^2 = [16 - 4(-4l\sqrt{5} + 68)]\left[\frac{1}{4} + \frac{1}{16} \times 16\right]$$

$$= 5[4l\sqrt{5} - 64]$$

$$\Rightarrow l^2 - 20\sqrt{5}\,l + 320 = (l - 4\sqrt{5})(l - 16\sqrt{5}) = 0$$

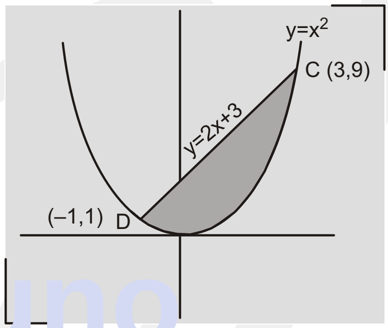
$$l = 4\sqrt{5},\ 16\sqrt{5}$$

$$\Rightarrow \quad A_{max} = l_{max}^2 = (16\sqrt{5})^2 = 1280$$

From (1),

$$t^2 - 4t - 12 = 0 \begin{matrix} \nearrow t_1 \\ \searrow t_2 \end{matrix}, \ \text{If } l = 4\sqrt{5}$$

$$\Rightarrow \quad y \text{ intercept of } CD \text{ is } y = -\frac{1}{4}t_1 t_2 = 3$$



$$\Rightarrow \quad t_1 = 6,\ t_2 = -2$$

$$C \equiv (3,9), D(-1,1)$$

Area of shaded region

$$= \int_{-1}^{3} (2x + 3 - x^2)\, dx = \frac{32}{3}$$

SOLUTIONS 4

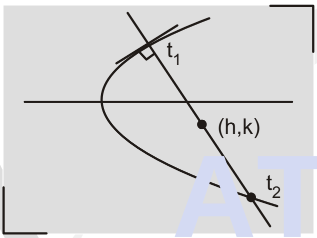
Match the Columns:

1. a-q; b-s; c-p; d-r

(a) $t_2 = -t_1 - \dfrac{2}{t_1}$

$$2(t_1 + t_2) = 2x = -\frac{4}{t_1}$$

$$\Rightarrow \quad t_1 = \frac{-2}{k},\ t_2 = k + \frac{2}{k}$$

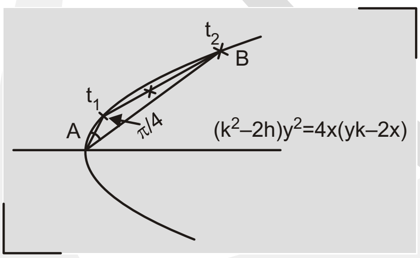


$$2h = (t_1^2 + t_2^2) = \frac{4}{k^2} + \left(k^2 + \frac{4}{k^2} + 4\right)$$

$$= \frac{8}{k^2} + 4 + k^2$$

$$y^4 + (4 - 2x)y^2 + 8 = 0$$

(b)



Eqn. of AB is $T = S_1 \Rightarrow yk - 2x = k^2 - 2h$

Homogenise

$$\Rightarrow \quad (k^2 - 2h)\,y^2 = 4x(yk - 2x)$$

$$\Rightarrow \quad 8x^2 + (k^2 - 2h)\,y^2 - 4kxy = 0$$

$$\Rightarrow (8 + k^2 - 2h)^2 = 4[(2k)^2 - 8(k^2 - 2h)]$$

$$\Rightarrow \quad (y^2 - 2x + 8)^2 - 16(4x - y^2) = 0$$

$$\Rightarrow \quad y^4 - 4xy^2 + 4x^2 + 32y^2 - 96x + 64 = 0$$

(c) Put $(h + r\cos\theta,\ k + r\sin\theta)$ in equation of parabola

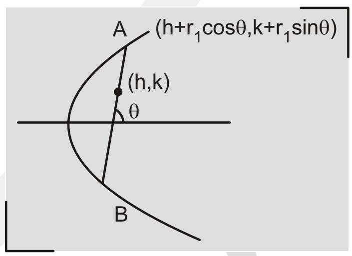
$$k^2 + r^2\sin^2\theta + 2kr\sin\theta - 4(h + r\cos\theta) = 0$$

$$r^2\sin^2\theta + (2k\sin\theta - 4\cos\theta)\,r + k^2 - 4h = 0 \begin{matrix} 1 \\ -1 \end{matrix}$$

Sum of roots $= 0$

$$\Rightarrow \quad 2k\sin\theta - 4\cos\theta = 0$$

$$\Rightarrow \quad \tan\theta = \frac{2}{k}$$



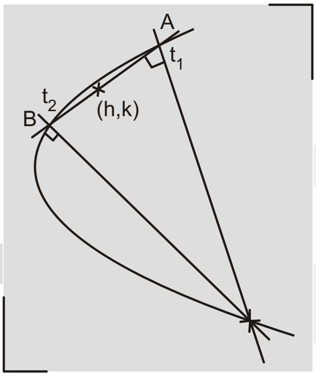
Product of roots $= -1$

$$\Rightarrow \quad -1 = (k^2 - 4h)\left(1 + \frac{k^2}{4}\right)$$

$$-4 = (y^2 + 4)(y^2 - 4x)$$

$$\Rightarrow \quad y^4 + 4(1 - x)\,y^2 + 4(1 - 4x) = 0$$

(d) Equation of AB is



$$yk - 2x = k^2 - 2h$$

Put $(t^2, 2t)$

$$2t^2 - 2kt + (k^2 - 2h) = 0$$

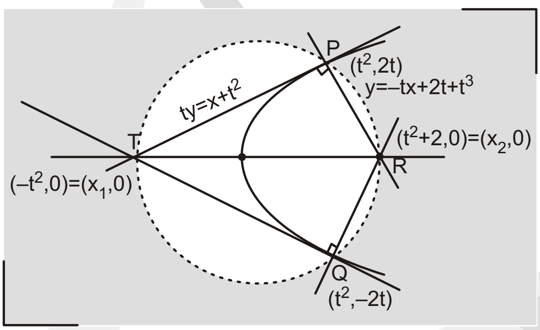
$$t_1 t_2 = 2 \Rightarrow \frac{k^2 - 2h}{2} = 2$$

$$y^2 = 2(x+2)$$

2. a- r; b-q; c-q, r, s; d-s

(a) $t^2 + 2 = 3 \Rightarrow t = \pm 1$

Area of $PTQR = 2\left(\frac{1}{2}(3+1)2\right) = 8$



(b) $PT = \sqrt{4t^4 + 4t^2} = 4t\sqrt{1+t^2}$

$$4t^2(1+t^2) = 80$$

$$(t^2+5)(t^2-4) = 0$$

$$t = \pm 2 \Rightarrow x_2 = 6$$

(c) $x_2 = t^2 + 2 > 2$

(d) $x_2 = 4 \Rightarrow t^2 = 2$

TR is diameter, $TR = 6$

Area of circle $= \frac{\pi}{4}(TR)^2 = 9\pi$

3. a-q; b-r; c-p; d-s

$$O = \left(\frac{3}{2}, \frac{5}{2}\right), \; Q = \left(\frac{1}{3}, \frac{11}{3}\right)$$

Feet of perpendicular from focus on any tangent lies on tangent at vertex

$\Rightarrow$ focus $S \equiv \left(\frac{17}{9}, \frac{26}{9}\right)$

foot of directrix

$$\equiv \left(2\left(\frac{3}{2}\right) - \frac{17}{9}, 2\left(\frac{5}{2}\right) - \frac{26}{9}\right) \equiv \left(\frac{10}{9}, \frac{19}{9}\right)$$

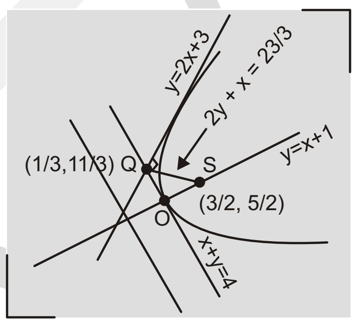
Equation of directrix is

$$x + y = \frac{29}{9} \Rightarrow 9x + 9y - 29 = 0$$

Length of latus rectum

$$= 4\sqrt{\left(\frac{17}{9} - \frac{3}{2}\right)^2 + \left(\frac{19}{9} - \frac{5}{2}\right)^2}$$

Length of $LR = \frac{14\sqrt{2}}{9}$ (Here, LR = latus rectum)



Extremities of

$$LR \equiv \left(\frac{17}{9} + \frac{7\sqrt{2}}{9}\left(\cos\frac{3\pi}{4}\right), \frac{26}{9} + \frac{7\sqrt{2}}{9}\left(\sin\frac{3\pi}{4}\right)\right)$$

$$\left(\frac{17}{9}-\frac{7\sqrt{2}}{9}\left(\cos\frac{3\pi}{4}\right),\frac{26}{9}-\frac{7\sqrt{2}}{9}\left(\sin\frac{3\pi}{4}\right)\right)$$

Extremities of

$$LR \equiv \left(\frac{10}{9},\frac{33}{9}\right) \text{ and } \left(\frac{24}{9},\frac{19}{9}\right)$$

Equation of parabola is

$$\left(\frac{y-x-1}{\sqrt{2}}\right)^2=\frac{14}{9}\sqrt{2}\,\frac{|y+x-4|}{\sqrt{2}}$$

$$\Rightarrow\quad 9(y-x-1)^2=28(y+x-4)$$

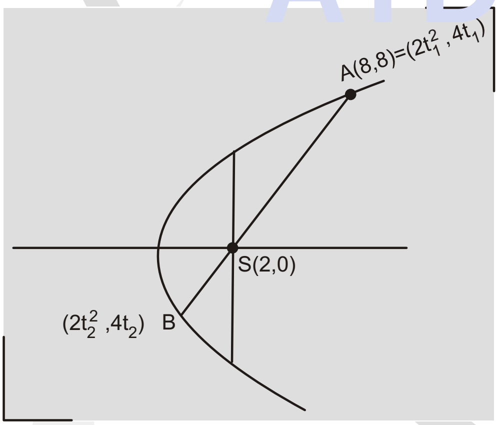
4. a-s; b-r; c-r; d-q

(a) $t_1=2$

$$\text{Slope of } AB=\frac{2}{t_1+t_2}=\frac{8}{6}=\frac{4}{3}$$

$$\Rightarrow\quad t_1+t_2=\frac{3}{2}$$

$$t_2=-\frac{1}{2}$$



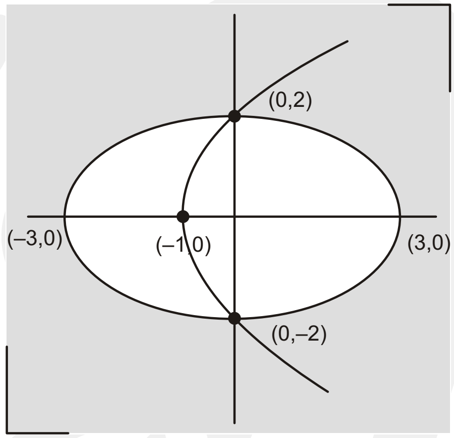
$$AB=AS+SB=(2+2t_1^2)+(2+2t_2^2)$$

$$=4+2\left(4+\frac{1}{4}\right)=\frac{25}{2},\quad AB=\frac{25}{2}$$

(b) $\left(x-\frac{1}{26}\right)^2+\left(y-\frac{3}{26}\right)^2=\frac{k}{4}\left(\frac{5x-12y+1}{13}\right)^2$

$$\Rightarrow\quad \frac{k}{4}=1\Rightarrow k=4$$

(c)

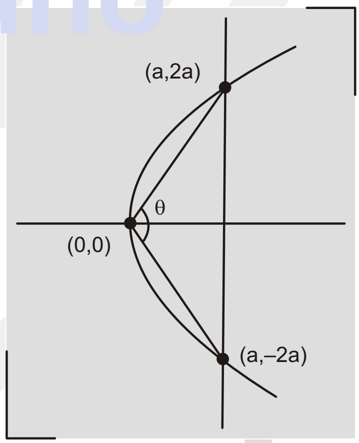


$$4x^2+9\times4(x+1)=36$$

$$\Rightarrow\quad x^2+9x=0\Rightarrow x=0,-9$$

Length of common chord = 4

(d) $\tan\frac{\theta}{2}=2$



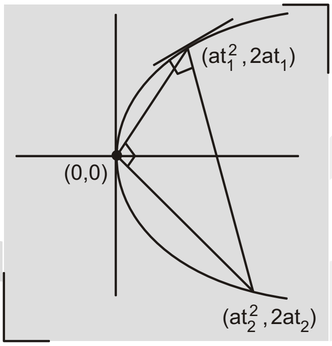
$$|\tan\theta|=\left|\frac{2(2)}{1-(2)^2}\right|=\frac{4}{3}$$

5. a-p,s ; b-q,r ; c-s ; d-q,r

(a) For line to become tangent to ellipse

$$a^2=2+1\Rightarrow a=\pm\sqrt{3}$$

(b)



$$\frac{2}{t_1}\frac{2}{t_2} = -1,\ t_1 t_2 = -4$$

$$t_2 = -t_1 - \frac{2}{t_1} = -\frac{4}{t_1}$$

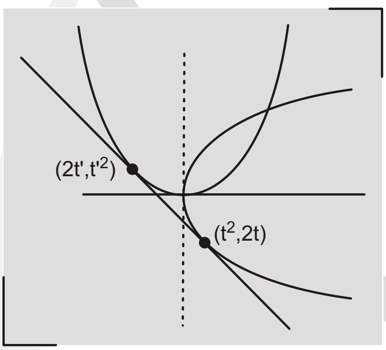
$$m = -t_1 = \pm\sqrt{2}$$

(c) $ty = x + t^2 \quad \Rightarrow \quad ty - x = t^2 \quad ...(1)$

$t'x = y + t'^2 \quad \Rightarrow \quad t'x - y = t'^2 \quad ...(2)$

(1) and (2) are identical

$$\Rightarrow \quad -t = -\frac{1}{t'} = \frac{t^2}{t'^2} = t^4$$



$t^3 = -1 \qquad \Rightarrow \quad t = -1$

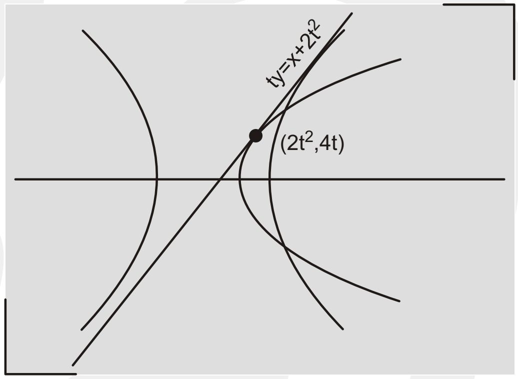
$-y - x = 1$

$x + y + 1 = 0 \ \Rightarrow\ k = \sqrt{3}$

(d) $ty = x + 2t^2$

$$y = \frac{1}{t}x + 2t$$

For line to become tangent to hyperbola



$$4t^2 = \frac{1}{t^2} - 3 \quad \Rightarrow \quad 4t^4 + 3t^2 - 1 = 0$$

$$\Rightarrow \quad (4t^2 - 1)(t^2 + 1) = 0$$

$$t = \pm\frac{1}{2}$$

$$\pm\frac{1}{2}y = x + \frac{1}{2} \quad \Rightarrow \quad 2x + 1 = \pm y$$

$$2x \pm y + 1 = 0$$

6. a-q; b-s; c-q; d-p

(a) $t^2 = 2$

(b) $\Delta = |(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|$

$= 1 \times 2 \times 3 = 6$

$(\because t_1 = 1, t_2 = 2, t_3 = 4)$

(c) Put $\left(\frac{11}{4}, \frac{1}{4}\right)$ to equation

$$y = -tx + 2t + t^3$$

$$\frac{1}{4} = -\frac{11}{4}t + 2t + t^3$$

$$\Rightarrow \quad 4t^3 - 3t - 1 = 0$$

$$\Rightarrow \quad (t - 1)(2t + 1)^2 = 0$$

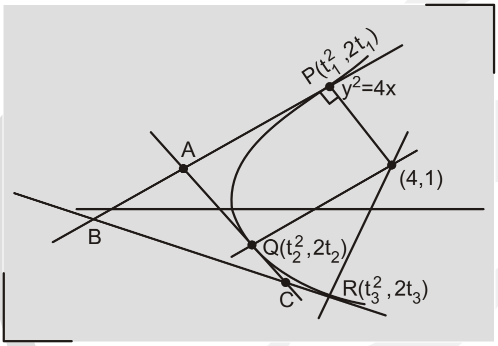
2 distinct normals are drawn through $\left(\frac{11}{4}, \frac{1}{4}\right)$

7. a-p; b-t; c-q; d-s

Equation of normal at $(t^2, 2t)$ is

$$y + tx = 2t + t^3$$

Put $(4,1)$



$\Rightarrow \quad t^3 - 2t - 1 = 0$

$\Rightarrow \quad t = -1, \dfrac{1-\sqrt{5}}{2}, \dfrac{1+\sqrt{5}}{2}$

$\therefore \quad t_1 = \dfrac{1+\sqrt{5}}{2}, t_2 = \dfrac{1-\sqrt{5}}{2}, t_3 = -1$

$A \equiv (t_1 t_2, t_1 + t_2) \equiv (-1, 1)$

$B \equiv (t_1 t_3, t_1 + t_3) \equiv \left(\dfrac{-1-\sqrt{5}}{2}, \dfrac{\sqrt{5}-1}{2}\right)$

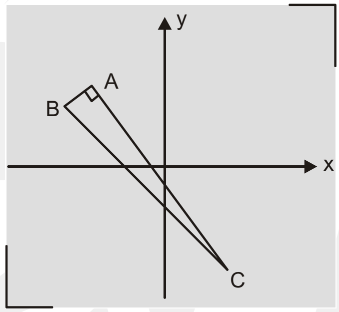
$C \equiv (t_2 t_3, t_2 + t_3) \equiv \left(\dfrac{\sqrt{5}-1}{2}, \dfrac{-\sqrt{5}-1}{2}\right)$

Centroid of $\Delta ABC, G \equiv \left(\dfrac{-2}{3}, 0\right)$

Orthocentre of ΔABC, $H \equiv (-1, 1)$

Circumcentre of ΔABC,

$O \equiv \left(\dfrac{-1}{2}, \dfrac{-1}{2}\right) \equiv$ midpoint of BC



Focus $S \equiv (1, 0)$

(a) $SG = \dfrac{5}{3}$

(b) $SH = \sqrt{2^2 + 1^2} = \sqrt{5}$

(c) $SO = \sqrt{\left(\dfrac{3}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2} = \dfrac{\sqrt{10}}{2}$

(d) Area of

$$\Delta ABC = \frac{1}{2}(AB)(AC) = \frac{1}{2}\frac{\sqrt{20+8\sqrt{5}}}{2}\frac{\sqrt{20-8\sqrt{5}}}{2} = \frac{\sqrt{5}}{2}$$

SOLUTIONS 5

Subjective Problems

1. (0) $\dfrac{x_1 - x_2}{y_3} + \dfrac{x_2 - x_3}{y_1} + \dfrac{x_3 - x_1}{y_2}$

$$= \frac{a(t_1^2 - t_2^2)}{2at_3} + \frac{a(t_2^2 - t_3^2)}{2at_1} + \frac{a(t_3^2 - t_1^2)}{2at_2}$$

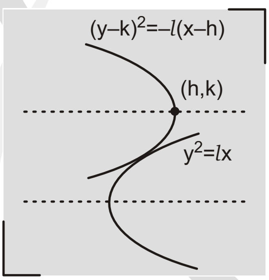
$$= \frac{(t_1 - t_2)(-t_3)}{2t_3} + \frac{(t_2 - t_3)(-t_1)}{2t_1} + \frac{(t_3 - t_1)(-t_2)}{2t_2}$$

$[\because t_1 + t_2 + t_3 = 0]$

$$= -\frac{1}{2}[t_1 - t_2 + t_2 - t_3 + t_3 - t_1]$$

$= 0$

2. (2) For intersection of two parabolas



$(y - k)^2 = lh - y^2$

$\Rightarrow 2y^2 - 2ky + k^2 - lh = 0$

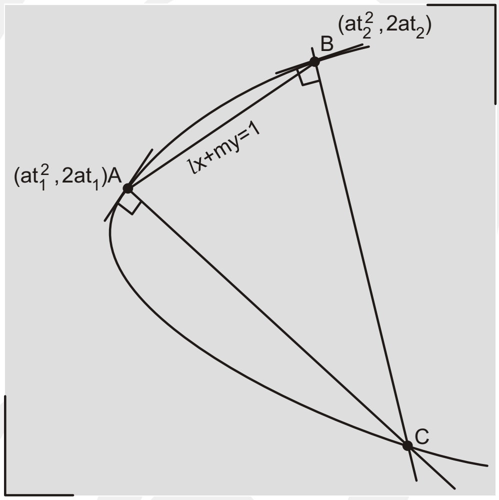
$D = 0$ ($\because$ curves touches)

$\Rightarrow 4k^2 - 8(k^2 - hl) = 0$

$\Rightarrow k^2 = 2lh \Rightarrow y^2 = 2lx$

3. (4) Put $(at^2, 2at)$ to equation of AB *i.e.*

$lx + my = 1$



$\Rightarrow lat^2 + 2mat - 1 = 0$ (roots t_1, t_2)

$\Rightarrow t_1 + t_2 = -\dfrac{2m}{l}$

$\because t_1 + t_2 + t_3 = 0$

$[\because A, B, C$ are co-normal points$]$

$\Rightarrow t_3 = \dfrac{2m}{l}$

$\therefore C \equiv (at_3^2, 2at_3) = \left(\dfrac{4m^2a}{l^2}, \dfrac{4am}{l}\right)$

$\Rightarrow k = 4$

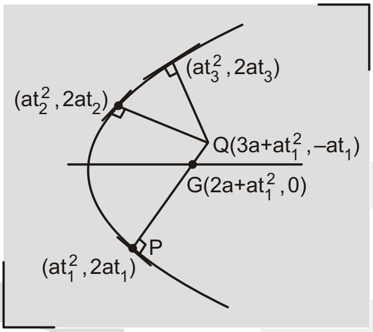
4. (2) $Q \equiv (a(t_2^2 + t_3^2 + 2 + t_2t_3),$

$-at_2t_3(t_2 + t_3))$

$-at_2t_3(t_2 + t_3) = -at_1$

$\Rightarrow -at_2t_3(-t_1) = -at_1$

$[\because t_1, t_2, t_3$ are co-normal points$]$



$\Rightarrow \quad t_2 t_3 = -1$

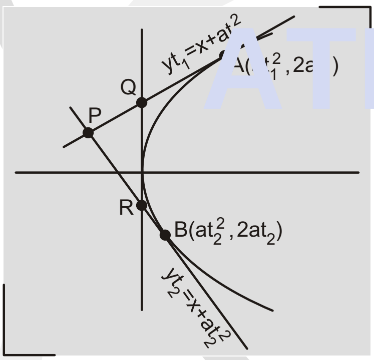
$\therefore \quad (-t_2)(-t_3) = -1$

$\Rightarrow$ normals at t_2 and t_3 are perpendicular.

5. (4) $Q \equiv (0, at_1), R \equiv (0, at_2)$

$P \equiv (at_1 t_2, a(t_1 + t_2)) = (h, k)$

Perpendicular through P to QR bisects QR



$$\text{Area of } \Delta PQR = \frac{1}{2} \times |a(t_1 - t_2)\, at_1 t_2|$$

$$c^2 = \frac{a^2}{2}|t_1 t_2 (t_1 - t_2)|$$

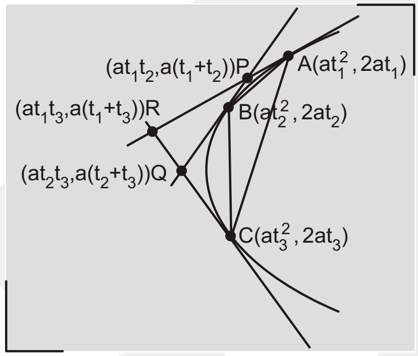
$$\Rightarrow \quad c^4 = \frac{a^4}{4}\, t_1^2 t_2^2 [(t_1 + t_2)^2 - 4t_1 t_2]$$

$$= \frac{a^4}{4} \frac{h^2}{a^2}\left[\left(\frac{k}{a}\right)^2 - 4\frac{h}{a}\right]$$

$$4c^4 = h^2 (k^2 - 4ah)$$

$$\Rightarrow \quad x^2 (y^2 - 4ax) = 4c^4 \quad \Rightarrow \lambda = 4$$

6. (2)

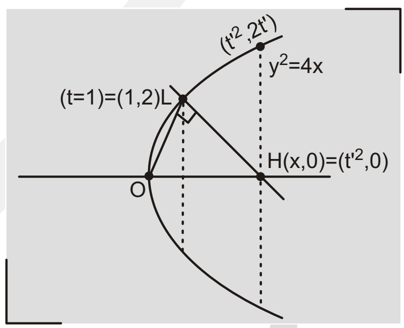


$$\frac{\Delta ABC}{\Delta PQR} = \frac{\left|\frac{1}{2}\begin{vmatrix} at_1^2 & 2at_1 & 1 \\ at_2^2 & 2at_2 & 1 \\ at_3^2 & 2at_3 & 1 \end{vmatrix}\right|}{\left|\frac{1}{2}\begin{vmatrix} at_1 t_2 & a(t_1 + t_2) & 1 \\ at_2 t_3 & a(t_3 + t_2) & 1 \\ at_3 t_1 & a(t_3 + t_1) & 1 \end{vmatrix}\right|}$$

$$= \frac{a^2|(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)|}{\frac{1}{2}a^2|(t_1 - t_2)(t_3 - t_2)(t_3 - t_1)|} = 2$$

7. (80) $\dfrac{0-2}{x-1} = -\dfrac{1}{2} \Rightarrow x - 1 = 4 \Rightarrow x = 5$

$t'^2 = 5 \Rightarrow t' = \sqrt{5}$



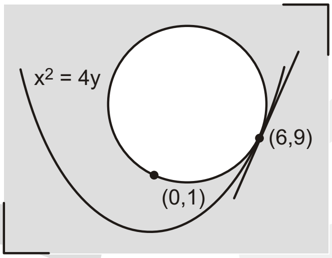
Length of double ordinate

$= 4t' = 4\sqrt{5} = \sqrt{80}$

$N = 80$

8. (5) Equation of tangent to parabola at P is

$2(y+9)=6x \Rightarrow y+9=3x$



Equation of circle is

$(x-6)^2+(y-9)^2+\lambda(y-3x+9)=0$

Put $(0,1)$

$\Rightarrow$ $36+64+\lambda(10)=0$

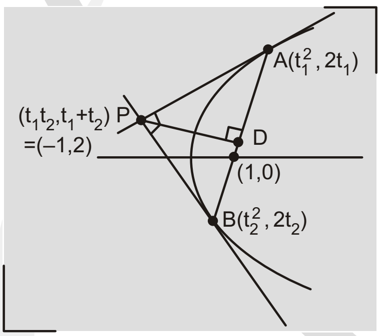
$\Rightarrow$ $\lambda=-10$

$\therefore$ Equation of circle is

$x^2+y^2+18x-28y+27=0$

$\therefore$ radius $=\sqrt{9^2+(14)^2-27}=5\sqrt{10}$

9. (8) Equation of AB is



$2y=2(x-1)$

$\Rightarrow$ $y=x-1$

$PD=\frac{|-1-1-2|}{\sqrt{2}}=2\sqrt{2}$

$AB=(1+t_1^2)+(1+t_2^2)=2+t_1^2+t_2^2$

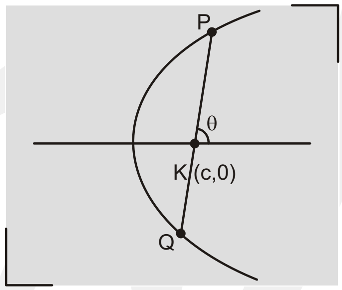
$=(t_1+t_2)^2+4=4+4=8$

Area of $\Delta PAB=\frac{1}{2}\times 2\sqrt{2}\times 8=8\sqrt{2}$ $\quad N=8$

10. (2) Put $(c+r\cos\theta, r\sin\theta)$ to the equation $y^2=4x$

$\Rightarrow$ $r^2\sin^2\theta-4\cos\theta\, r-4c=0 \begin{cases} r_1 \\ r_2 \end{cases}$

$r_1=PK, r_2=-QK$



$\frac{1}{(PK)^2}+\frac{1}{(QK)^2}=\frac{1}{r_1^2}+\frac{1}{r_2^2}$

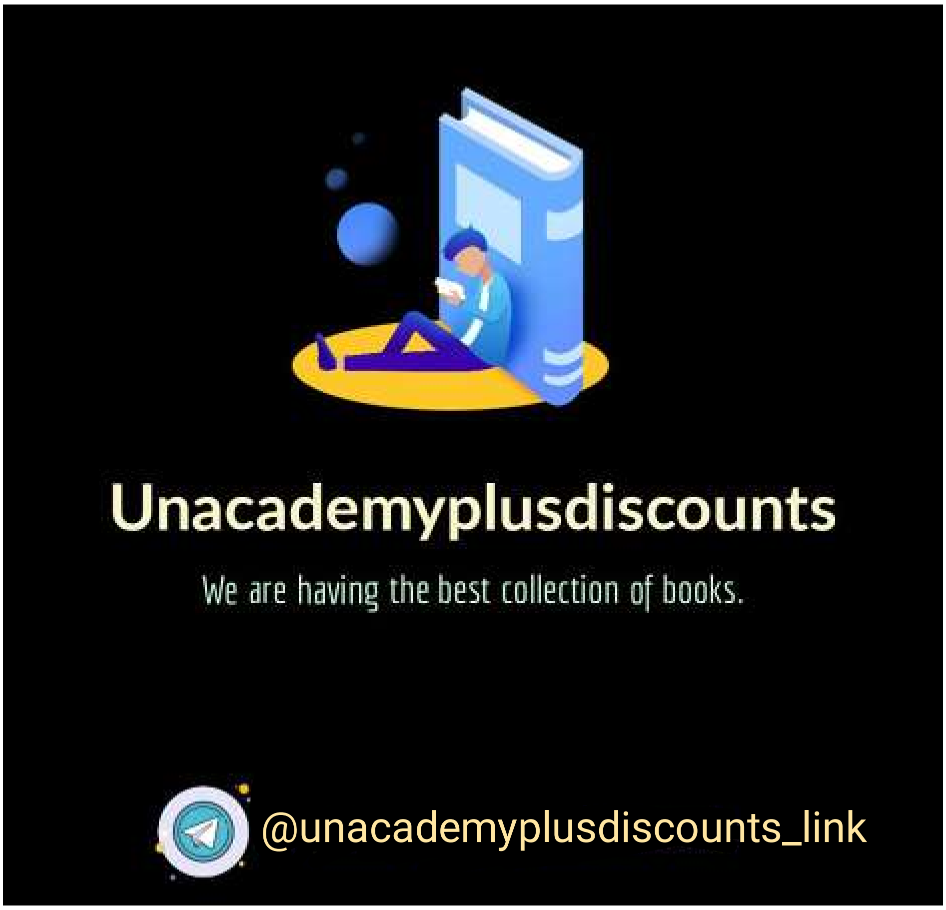
$=\frac{(r_1+r_2)^2-2r_1r_2}{(r_1r_2)^2}$

$=\frac{\frac{16\cos^2\theta}{\sin^4\theta}+\frac{8c}{\sin^2\theta}}{\frac{16c^2}{\sin^4\theta}}$

$=\frac{16\cos^2\theta+8c\sin^2\theta}{16c^2}$

$8c=16$ for $\frac{1}{(PK)^2}+\frac{1}{(QK)^2}$ to be independent of θ.

$\Rightarrow$ $c=2$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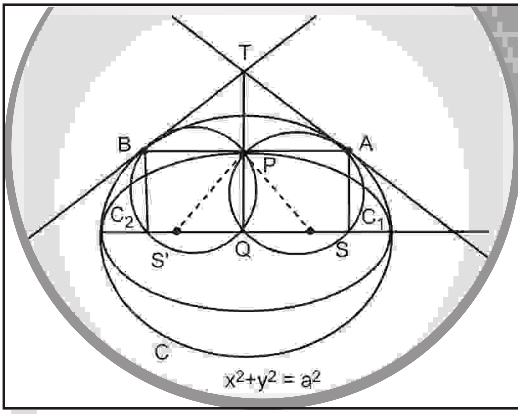


4 ELLIPSE

KEY CONCEPTS

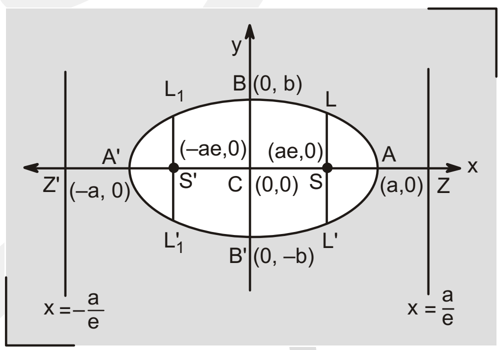
1. STANDARD EQUATION AND DEFINITIONS

Standard equation of an ellipse referred to its principal axes along the co-ordinate axes is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Where $a > b$ and $b^2 = a^2(1-e^2) \quad \Rightarrow a^2 - b^2 = a^2e^2$.

Where $e =$ eccentricity $(0 < e < 1)$.

FOCI : $S \equiv (ae, 0)$ and $S' \equiv (-ae, 0)$.



Equations of Directrixes

$$x = \frac{a}{e} \text{ and } x = -\frac{a}{e}.$$

Vertices :

$$A' \equiv (-a, 0) \text{ and } A \equiv (a, 0).$$

Major Axis :

The line segment $A'A$ in which the foci S' and S lie is of length $2a$ and is called the **major axis** $(a > b)$ of the ellipse. Point of intersection of major axis with directrix is called **the foot of the directrix (z).**

Minor Axis :

The y-axis intersects the ellipse in the points $B' \equiv (0, -b)$ and $B \equiv (0, b)$. The line segment $B'B$ of length $2b(b < a)$ is called the **minor axis** of the ellipse.

Principal Axis :

The major and minor axis together are called **principal axis** of the ellipse.

Centre :

The point which bisects every chord of the conic drawn through it is called the **centre** of the conic. $C \equiv (0,0)$ the origin is the centre of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Diameter :

A chord of the conic which passes through the centre is called a **diameter** of the conic.

FOCAL CHORD : A chord which passes through a focus is called a **focal chord**.

Double Ordinate :

A chord perpendicular to the major axis is called a **double ordinate.**

Latus Rectum :

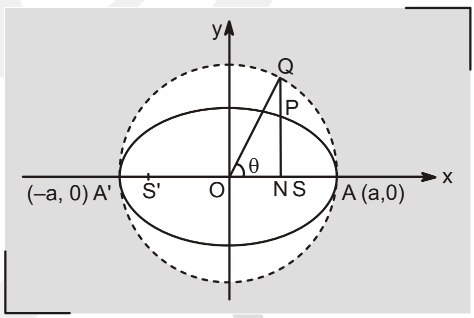
The focal chord perpendicular to the major axis is called the **latus rectum.** Length of latus rectum $(\boldsymbol{L\,L'}) = \frac{2b^2}{a} = \frac{(\textit{minor axis})^2}{\textit{major axis}} = 2a(1-e^2) = 2e$ (distance from focus to the corresponding directrix)

2. POSITION OF A POINT w.r.t. AN ELLIPSE

The point $P(x_1, y_1)$ lies outside, inside or on the ellipse according as; $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 >< \text{ or } = 0$.

3. AUXILIARY CIRCLE/ECCENTRIC ANGLE

A circle described on major axis as diameter is called the **auxiliary circle.**



Let Q be a point on the auxiliary circle $x^2+y^2=a^2$ such that QP produced is perpendicular to the x-axis then P and Q are called as the CORRESPONDING POINTS on the ellipse and the auxiliary circle respectively 'θ' is called the ECCENTRIC ANGLE of the point P on the ellipse $(0 \le \theta < 2\pi)$.

Note that $\frac{\ell(PN)}{\ell(QN)} = \frac{b}{a} = \frac{\textit{Semi minor axis}}{\textit{Semi major axis}}$

Hence "If from each point of a circle perpendiculars are drawn upon a fixed diameter then the locus of the points dividing these perpendiculars in a given ratio an ellipse of which the given circle is the auxiliary circle".

4. PARAMETRIC REPRESENTATION

The equations $x = a\cos\theta$ and $y = b\sin\theta$ together represent the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

Where θ is a parameter. Note that if $P(\theta) \equiv (a\cos\theta, b\sin\theta)$ is on the ellipse then ;
$Q(\theta) \equiv (a\cos\theta, a\sin\theta)$ is on the auxiliary circle.

5. LINE AND AN ELLIPSE

The line $y = mx + c$ meets the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in two points real, coincident or imaginary according as c^2 is $< = \text{or} >$ $a^2m^2 + b^2$.

Hence $y = mx + c$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ if $c^2 = a^2m^2 + b^2$.

The equation to the chord of the ellipse joining two points with eccentric angles α and β is given by $\frac{x}{a}\cos\frac{\alpha+\beta}{2} + \frac{y}{b}\sin\frac{\alpha+\beta}{2} = \cos\frac{\alpha-\beta}{2}$.

6. TANGENTS

(i) $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$ is tangent to the ellipse at (x_1, y_1).

(ii) $y = mx \pm \sqrt{a^2m^2 + b^2}$ is tangent to the ellipse for all values of m.

Note that there are two tangents to the ellipse having the same m, *i.e.*, there are two tangents parallel to any given direction.

(iii) $\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$ is tangent to the ellipse at the point $(a\cos\theta, b\sin\theta)$.

(iv) The eccentric angles of points of contact of two parallel tangents differ by π. Conversely if the difference between the eccentric angles of two points is π then the tangents at these points are parallel.

(v) Point of intersection of the tangents at the point α and β is $a\dfrac{\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}, b\dfrac{\sin\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}$.

7. NORMALS

(i) Equation of the normal at (x_1, y_1) is $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2 = a^2e^2$.

(ii) Equation of the normal at the point $(a\cos\theta, b\sin\theta)$ is ; $ax\sec\theta - by\operatorname{cosec}\theta = (a^2 - b^2)$.

(iii) Equation of a normal in terms of its slope 'm' is $y = mx - \frac{(a^2 - b^2)m}{\sqrt{a^2 + b^2m^2}}$.

8. DIRECTOR CIRCLE

Locus of the point of intersection of the tangents which meet at right angles is called the **director circle**. The equation to this locus is $x^2 + y^2 = a^2 + b^2$ *i.e.*, a circle whose centre is the centre of the ellipse and whose radius is the length of the line joining the ends of the major and minor axis.

9. CHORD OF CONTACT

Equation of chord of contact of tangents drawn from a point $P(x_1, y_1)$ to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $T = 0$, where $T = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1$.

10. CHORD WITH A GIVEN MIDDLE POINT

Equation of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, whose middle point is (x_1, y_1) is $T = S_1$, where

$S_1 = \frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - 1, T = \frac{xx_1}{a^2} + \frac{yy_1}{b^2} - 1.$

11. DIAMETER

The locus of the middle points of a system of parallel chords with slope 'm' of an ellipse is a straight line passing through the centre of the ellipse, called its diameter and has the equation $y = -\frac{b^2}{a^2 m} x$.

12. IMPORTANT HIGHLIGHTS

Referring to an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

H-1 If P be any point on the ellipse with S and S' as its foci then $\ell(SP) + \ell(S'P) = 2a$.

H-2 The product of the length's of the perpendicular segments from the foci on any tangent to the ellipse is b^2 and the feet of these perpendiculars Y, Y' lie on its auxiliary circle. The tangents at these feet to the auxiliary circle meet on the ordinate of P and that the locus of their point of intersection is a similar ellipse as that of the original one. Also the lines joining centre to the feet of the perpendicular Y and focus to the point of contact of tangent are parallel.

H-3 If the normal at any point P on the ellipse with centre C meet the major and minor axes in G & g respectively, and if CF be perpendicular upon this normal, then

(i) $PF.PG = b^2$ **(ii)** $PF.Pg = a^2$ **(iii)** $PG.Pg = SP.S'P$ **(iv)** $CG.CT = CS^2$

(v) locus of the mid point of Gg is another ellipse having the same eccentricity as that of the original ellipse. [where S and S' are the focii of the ellipse and T is the point where tangent at P meet the major axis.]

H-4 The tangent and normal at a point P on the ellipse bisect the external and internal angles between the focal distances of P. This refers to the well known reflection property of the ellipse which states that rays from one focus are reflected through other focus and vice-versa. Hence we can deduce that the straight lines joining each focus to the foot of the perpendicular from the other focus upon the tangent at any point P meet on the normal PG and bisects it where G is the point where normal at P meets the major axis.

H-5 The portion of the tangent to an ellipse between the point of contact and the directrix subtends a right angle at the corresponding focus.

H-6 The circle on any focal distance as diameter touches the auxiliary circle.

H-7 Perpendiculars from the centre upon all chords which join the ends of any perpendicular diameters of the ellipse are of constant length.

H-8 If the tangent at the point P of a standard ellipse meets the axis in T and t and CY is the perpendicular on it from the centre then,

(i) $T.t.PY = a^2 - b^2$ and (ii) least value of Tt is $a + b$.

EXERCISE 1

Only One Choice is Correct:

1. If $\frac{x^2}{f(4a)} + \frac{y^2}{f(a^2-5)} = 1$ represents an ellipse with major axis as y-axis and f is a decreasing function, such that $f(x) > 0.\ \forall x \in R$, then complete set of values of a is :

(a) $(-\infty, 1)$ (b) $(-\infty, -1) \cup (5, \infty)$

(c) $(-1, 4)$ (d) $(-1, 5)$

2. If the inclination of a diameter PP' of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is θ and $(PP')^2$ is the A.M. of squares of the lengths of major and minor axes, then $\tan\theta$ is equal to :

(a) $\pm\frac{b}{a}$ (b) $\pm\frac{a}{b}$

(c) ± 1 (d) $\pm\sqrt{3}$

3. Ellipses are described with line segement AB as the fixed major axis. The locus of an end of a latus rectum is :

(a) straight line (b) parabola

(c) circle (d) ellipse

4. A series of ellipses $E_1, E_2, \ldots\ldots E_n$ are drawn such that E_n touches E_{n-1} at the extremeties of the major axis of E_{n-1} and the foci of E_n coincide with the extremeties of minor axis of E_{n-1}. If ecentricity of the ellipses is independent of n, then the value of ecentricity is :

(a) $\frac{\sqrt{5}}{3}$ (b) $\frac{\sqrt{5}+1}{4}$

(c) $\frac{\sqrt{5}-1}{2}$ (d) $\frac{\sqrt{5}-1}{3}$

5. If base of a triangle is the major axis of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and third vertex moves on the ellipse, then maximum area of triangle will be:

(a) 6 (b) 72

(c) 12 (d) none of these

6. The angle between ellipse $\frac{x^2}{4} + y^2 = 1$ and circle $x^2 + y^2 = 2$ is θ, then $\tan\theta$ is equal to:

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$

(c) $\frac{1}{2\sqrt{2}}$ (d) none of these

7. The locus of point of intersection of perpendicular tangents of ellipse $\frac{(x-1)^2}{16}+\frac{(y-1)^2}{9}=1$ is:

(a) $x^2+y^2=25$
(b) $x^2+y^2+2x+2y-23=0$
(c) $x^2+y^2-2x-2y-23=0$
(d) none of these

8. If normal at any point P to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 (a>b)$ meet the axes at M and N so that $\frac{PM}{PN}=\frac{2}{3}$, then the value of eccentricity is:

(a) $\frac{1}{\sqrt{2}}$
(b) $\frac{\sqrt{2}}{\sqrt{3}}$
(c) $\frac{1}{\sqrt{3}}$
(d) none of these

9. If the line joining foci subtends an angle of 90° at an extremity of minor axis then the eccentricity of the ellipse is:

(a) $\frac{1}{\sqrt{3}}$
(b) $\frac{1}{\sqrt{2}}$
(c) $\frac{1}{2}$
(d) none of these

10. The length of sides of square which can be made by four perpendicular tangents to the ellipse $\frac{x^2}{7}+\frac{2y^2}{11}=1$ is:

(a) 1
(b) 5
(c) 6
(d) none of these

11. The eccentricity of the ellipse which meets the straight line $\frac{x}{7}+\frac{y}{2}=1$ on the axis of x and the straight line $\frac{x}{3}-\frac{y}{5}=1$ on the axis of y and whose axis lie along the axes of coordinates is:

(a) $\frac{3\sqrt{2}}{7}$
(b) $\frac{2\sqrt{6}}{7}$
(c) $\frac{2\sqrt{3}}{7}$
(d) none of these

12. The point on the ellipse $x^2+2y^2=6$ closest to the line $x+y=7$:

(a) $(1,2)$
(b) $(2,1)$
(c) $(3,2)$
(d) none of these

13. If straight line $\frac{ax}{3}+\frac{by}{4}=c$ is a normal to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 (a>b)$, then a^2-b^2 is equal to:

(a) $4c$ (b) $5c$

(c) $6c$ (d) none of these

14. If S and S' are the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and P is any point on it, then difference of maximum and minimum of $SP.S'P$ is equal to:

(a) 16 (b) 9

(c) 15 (d) 25

15. Point 'O' is the centre of the ellipse with major axis AB and minor axis CD. Point F is one focus of the ellipse. If $OF=6$ and the diameter of the inscribed circle of triangle OCF is 2, then the product $(AB)(CD)$ is equal to :

(a) 65 (b) 52

(c) 78 (d) none of these

16. The angle between the tangents drawn from the point $(\sqrt{7},1)$ to the ellipse $3x^2+5y^2=15$ is :

(a) $\pi/6$ (b) $\pi/4$

(c) $\pi/3$ (d) $\pi/2$

17. Which one of the following is the common tangent to the ellipses, $\frac{x^2}{a^2+b^2}+\frac{y^2}{b^2}=1$ and $\frac{x^2}{a^2}+\frac{y^2}{a^2+b^2}=1$?

(a) $ay=bx+\sqrt{a^4-a^2b^2+b^4}$ (b) $by=ax-\sqrt{a^4+a^2b^2+b^4}$

(c) $ay=bx-\sqrt{a^4+a^2b^2+b^4}$ (d) $by=ax+\sqrt{a^4-a^2b^2+b^4}$

18. The normal at a variable point P on an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of eccentricity e meets the axes of the ellipse in Q and R then the locus of the mid-point of QR is a conic with an eccentricity e' such that:

(a) e' is independent of e (b) $e'=1$

(c) $e'=e$ (d) $e'=1/e$

19. The latus rectum of a conic section is the width of the function through the focus. The positive difference between the lengths of the latus rectum of $3y=x^2+4x-9$ and $x^2+4y^2-6x+16y=24$ is :

(a) $\frac{1}{2}$ (b) 2

(c) $\frac{3}{2}$ (d) $\frac{5}{2}$

20. $x-2y+4=0$ is a common tangent to $y^2=4x$ and $\frac{x^2}{4}+\frac{y^2}{b^2}=1$. Then the value of b and the other common tangent are given by :

(a) $b=3$; $x+2y+4=0$ (b) $b=\sqrt{3}$; $x+2y+4=0$

(c) $b=\sqrt{3}$; $x+2y-4=0$ (d) $b=\sqrt{3}$; $x-2y-4=0$

21. The area of the rectangle formed by the perpendiculars from the centre of the standard ellipse to the tangent and normal at its point whose eccentric angle is $\pi/4$, is :

(a) $\frac{(a^2-b^2)ab}{a^2+b^2}$ (b) $\frac{(a^2-b^2)}{(a^2+b^2)ab}$

(c) $\frac{(a^2-b^2)}{ab(a^2+b^2)}$ (d) $\frac{a^2+b^2}{(a^2-b^2)ab}$

22. The point of intersection of the tangents at the point P on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and its corresponding point Q on the auxiliary circle meet on the line :

(a) $x=a/e$ (b) $x=0$

(c) $y=0$ (d) none of these

23. An ellipse having foci at $(3,3)$ and $(-4,4)$ and passing through the origin has eccentricity equal to:

(a) $\frac{3}{7}$ (b) $\frac{2}{7}$

(c) $\frac{5}{7}$ (d) $\frac{3}{5}$

24. C is the centre of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and A and B are two points on the ellipse such $\angle ACD=90°$. Then $\frac{1}{CA^2}+\frac{1}{CB^2}=$:

(a) $\frac{25}{144}$ (b) $\frac{144}{25}$

(c) $\frac{7}{12}$ (d) $\frac{12}{7}$

25. Consider the particle travelling clockwise on the elliptical path $\frac{x^2}{100}+\frac{y^2}{25}=1$. The particle leaves the orbit at the point $(-8,3)$ and travels in a straight line tangent to the ellipse. At what point will the particle cross the y-axis ?

(a) $\left(0,\frac{25}{3}\right)$ (b) $\left(0,-\frac{25}{3}\right)$

(c) $(0,9)$ (d) $\left(0,\frac{7}{3}\right)$

26. A bar of length 20 units moves with its ends on two fixed straight lines at right angles. A point P marked on the bar at a distance of 8 units from one end describes a conic whose eccentricity is:

(a) $\frac{5}{9}$ (b) $\frac{\sqrt{2}}{3}$

(c) $\frac{4}{9}$ (d) $\frac{\sqrt{5}}{3}$

27. P and Q are two points on the upper half of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. The centre of the ellipse is at the origin 'O' and PQ is parallel to the x-axis such that the triangle OPQ has the maximum possible area. A point is randomly selected from inside of the upper half of the ellipse. The probability that is lies outside the triangle, is :

(a) $\frac{\pi-1}{\pi}$ (b) $\frac{2\pi-1}{2\pi}$ (c) $\frac{\pi-1}{2\pi}$ (d) $\frac{\pi-1}{4\pi}$

28. Which of the following is an equation of the ellipse with centre $(-2,1)$, major axis running from $(-2,6)$ to $(-2,-4)$ and focus at $(-2,5)$?

(a) $\frac{(x-2)^2}{25}+\frac{(y+1)^2}{16}=1$ (b) $\frac{(x+2)^2}{25}+\frac{(y-1)^2}{9}=1$

(c) $\frac{(x-2)^2}{9}+\frac{(y+1)^2}{25}=1$ (d) $\frac{(x+2)^2}{9}+\frac{(y-1)^2}{25}=1$

29. If $P(\theta)$ and $Q\left(\frac{\pi}{2}+\theta\right)$ are two points on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then locus of midpoint of PQ is :

(a) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{2}$ (b) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2$

(c) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=4$ (d) $\frac{x^2}{a^2}+\frac{y^2}{b^2}=8$

30. An ellipse has OB as semi minor axis, F and F' its foci and the angle FBF' is a right angle, then the eccentricity of ellipse is :

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{\sqrt{2}}$

31. The length of the latus rectum of the ellipse $9(2x+3y-1)^2+16(3x-2y+5)^2=25$ is:

(a) $\frac{3\sqrt{117}}{104}$ (b) $\frac{15\sqrt{13}}{13}$ (c) $\frac{5\sqrt{117}}{104}$ (d) $\frac{5\sqrt{13}}{104}$

32. Let F_1, F_2 are foci of the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$; P is a point on the ellipse such that $\frac{PF_1}{PF_2}=2$. Then the area of ΔPF_1F_2 is:

(a) $4\sqrt{2}$ (b) $4\sqrt{5}$ (c) $4/\sqrt{5}$ (d) 4

ANSWERS

1.	(d)	**2.**	(a)	**3.**	(b)	**4.**	(c)	**5.**	(c)	**6.**	(b)	**7.**	(c)	**8.**	(c)	**9.**	(b)	**10.**	(b)
11.	(b)	**12.**	(b)	**13.**	(b)	**14.**	(b)	**15.**	(a)	**16.**	(d)	**17.**	(b)	**18.**	(c)	**19.**	(a)	**20.**	(b)
21.	(a)	**22.**	(c)	**23.**	(c)	**24.**	(a)	**25.**	(a)	**26.**	(d)	**27.**	(a)	**28.**	(d)	**29.**	(a)	**30.**	(d)
31.	(c)	**32.**	(d)																

EXERCISE 2

One or More than One is/are Correct

1. $\dfrac{x^2}{2}+\dfrac{y^2}{1}=1$ is an ellipse with foci S_1 and S_2. Rectangle S_1PS_2Q is completed (where P and Q are on the ellipse):
(a) number of such pair P, Q is one
(b) area of rectangle S_1PS_2Q is equal to 2 sq. units
(c) there will be infinite such pairs P, Q
(d) rectangle S_1PS_2Q is a square

2. Line through $P(a, 2)$ meets the ellipse $\dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ at A and D and meets the coordinate axes at B and C so that PA, PB, PC, PD are in G.P., then possible values of a can be :
(a) 5 (b) 8 (c) 10 (d) –7

3. If ellipses $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ be described having the same major axis, but a variable minor axis, then for all values of b, the tangents at the ends of their latus rectum can pass through :
(a) $(0, a)$ (b) $(0, -a)$
(c) $(0, 2a)$ (d) $(0, -2a)$

4. If the tangent at the point $\left(4\cos\theta, \dfrac{16\sin\theta}{\sqrt{11}}\right)$ to ellipse $16x^2+11y^2=256$ is also tangent to circle $x^2+y^2-2x=15$, then θ equals :
(a) $\dfrac{\pi}{3}$ (b) $\dfrac{2\pi}{3}$ (c) $\dfrac{5\pi}{6}$ (d) $\dfrac{5\pi}{3}$

5. For a point P on ellipse the circles with PS and PS' where S, S' represent foci of ellipse as diameter intersect the auxiliary circle of ellipse at A, A_1 and B, B_1 respectively, then which of the following is/are correct ?
(a) A and A_1 coincide, B and B_1 coincide
(b) Segment AB is tangent to ellipse at P
(c) Tangents at A and B on auxiliary circles are perpendicular.
(d) SA and $S'B$ are parallel

6. Equation of tangents drawn from $(2, 3)$ to the ellipse $\dfrac{x^2}{16}+\dfrac{y^2}{9}=1$ is/are:
(a) $x+y+5=0$ (b) $x+y-5=0$
(c) $y+3=0$ (d) $y-3=0$

7. $4(x-2y+1)^2+9(2x+y+2)^2=25$ represents ellipse whose:

(a) centre is $(-1,0)$
(b) eccentricity is $\frac{\sqrt{5}}{3}$
(c) length of major axis is $\frac{\sqrt{5}}{2}$
(d) equation of major axis is $2x+y+2=0$

8. The eccentric angle of point on the ellipse $x^2+3y^2=6$ at a distance 2 unit from origin is:

(a) $\frac{\pi}{4}$
(b) $\frac{3\pi}{4}$
(c) $\frac{5\pi}{4}$
(d) $\frac{7\pi}{4}$

9. For the ellipse $5x^2+9y^2+10x-36y-4=0$ which of the following are true :

(a) centre is $(1,2)$
(b) one of the foci is $(-3,2)$
(c) length of latus rectum is $\frac{10}{3}$
(d) eccentricity is $\frac{2}{3}$

10. If the tangent to the ellipse $x^2+4y^2=16$ at the point $P(\theta)$ is a normal to circle $x^2+y^2-8x-4y=0$ then θ equals:

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) 0
(d) $-\frac{\pi}{4}$

11. If the focus, centre and eccentricity of an ellipse are respectively $(3,4)(2,3)$ and $\frac{1}{2}$, then its equation is given by:

(a) $\frac{(x+y-5)^2}{16}+\frac{(x-y+1)^2}{12}=1$
(b) $\frac{(x-y+1)^2}{16}+\frac{(x+y-5)^2}{12}=1$
(c) $\sqrt{(x-3)^2+(y-4)^2}+\sqrt{(x-2)^2+(y-1)^2}=4\sqrt{2}$
(d) $\sqrt{(x-3)^2+(y-4)^2}+\sqrt{(x-1)^2+(y-2)^2}=4\sqrt{2}$

12. The locus of incentre of ΔPF_1F_2 where P is a variable point lying on ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with F_1, F_2 as foci and eccentricity e is a conic whose:

(a) equation is $\frac{x^2}{a^2e^2}+\frac{y^2(1+e)^2}{b^2e^2}=1$
(b) equation is $\frac{x^2}{a^2e^2}+\frac{y^2(1-e)^2}{b^2e^2}=1$
(c) eccentricity is $\sqrt{\frac{2e}{1+e}}$
(d) eccentricity is $\sqrt{\frac{2e}{1-e}}$

13. A parallelogram circumscribes the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and two of its opposite angular points lie on straight lines $x^2=\lambda^2$, $\lambda\neq 0$, the locus of the other two vertices is:

(a) ellipse if $\lambda>3$

(b) hyperbola if $\lambda\in(0,3)$

(c) circle if $\lambda=\frac{9}{\sqrt{5}}$

(d) ellipse if $\lambda\in(0,3)$

14. On the ellipse $4x^2+9y^2=1$, the points at which the tangents are parallel to line $8x=9y$ are :

(a) $\left(\frac{2}{5},\frac{1}{5}\right)$

(b) $\left(-\frac{2}{5},\frac{1}{5}\right)$

(c) $\left(-\frac{2}{5},\frac{-1}{5}\right)$

(d) $\left(\frac{2}{5},\frac{-1}{5}\right)$

15. Let $P(x_1,y_1)$ and $Q(x_2,y_2), y_1<0,\ y_2<0$ be the end points the latus rectum of ellipse $x^2+4y^2=4$. The equations of parabolas with latus rectum PQ are :

(a) $x^2+2\sqrt{3}y=3+\sqrt{3}$

(b) $x^2-2\sqrt{3}y=3+\sqrt{3}$

(c) $x^2+2\sqrt{3}y=3-\sqrt{3}$

(d) $x^2-2\sqrt{3}y=3-\sqrt{3}$

16. If the ellipse $x^2+k^2y^2=k^2a^2$ is confocal with the hyperbola $x^2-y^2=a^2 (k>1)$. Then which of the following statement(s) is/are correct?

(a) Ratio of eccentricities of ellipse and hyperbola is $\frac{1}{\sqrt{3}}$

(b) Ratio of major axis of ellipse and transverse axis of hyperbola is $\sqrt{3}$

(c) Ratio of minor axis of ellipse and conjugate axis of hyperbola is $\sqrt{3}$

(d) Ratio of length lf latus rectum of ellipse and hyperbola is $\frac{1}{\sqrt{3}}$

ANSWERS

1.	(a, b, d)	**2.**	(b, c, d)	**3.**	(a, b)	**4.**	(a, d)	**5.**	(a, b, d)	**6.**	(b, d)
7.	(a, b, d)	**8.**	(a, b, c, d)	**9.**	(b, c, d)	**10.**	(a, c)	**11.**	(a, d)	**12.**	(a, c)
13.	(a, b, c)	**14.**	(b, d)	**15.**	(b, c)	**16.**	(a, b, d)				

EXERCISE 3

Comprehension: (1)

Consider an ellipse $S_1 \equiv \frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$. Let $S_2 = 0$ be a parabola on the right of the y-axis confocal with $S_1 = 0$ having vertex at the centre of $S_1 = 0$. Let P be the point of intersection of the parabola and the directrix of ellipse in the first quadrant and $L = 0$ is the directrix of the parabola.

1. The ordinate of the point from which the pair of tangents drawn to both the parabola and the ellipse are separately at right angles is:

(a) $a\sqrt{1-e^2}$ (b) $\sqrt{2}a\sqrt{1-e^2}$

(c) $2a\sqrt{1-e^2}$ (d) $a\sqrt{2-e^2}$

2 Pair of tangents are drawn from any point on $L = 0$ to $S_1 = 0$ and $S_2 = 0$. Then the locus of point of intersection of their chord of contact is:

(a) $x = ae$ (b) $x = \frac{a}{e}$

(c) $x = \frac{a\sqrt{1-e^2}}{e}$ (d) $x = ae\sqrt{1-e^2}$

3. If the tangent at point Q on $S_1 = 0$ and the line joining the points P to the focus of $S_1 = 0$ intersect at the auxiliary circle of $S_1 = 0$, then eccentric angle of the point Q (Q lies in first quadrant) is:

(a) $\tan^{-1}\left(\frac{e}{\sqrt{1+e^2}}\right)$ (b) $\tan^{-1}\left(\frac{2e}{\sqrt{1-e^2}}\right)$

(c) $\tan^{-1}\left(\frac{\sqrt{2}e}{\sqrt{1+e^2}}\right)$ (d) $\tan^{-1}\left(\frac{e}{\sqrt{1-e^2}}\right)$

Comprehension: (2)

An ellipse has major axis of length 4 and minor axis of length 2. It is slipping between the coordinate axes in the first quadrant while maintaining contact with both x-axis and y-axis.

1. The locus of the centre of the ellipse is :

(a) $x + y = 3$ (b) $(y-1)^2 = 4(x-2)$

(c) $x^2 + y^2 = 5$ (d) $x^2 + y^2 = 20$

2. The locus of foci of the ellipse is:

(a) $x^2 + y^2 = 10$ (b) $x^4 + y^4 - x^2y^2 = 1$

(c) $x^2 + y^2(x^2 + y^2) = 4$ (d) $x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} = 16$

3. When the ordinate of centre of ellipse is 1, a light ray passing through origin and centre of ellipse strikes the tangent at vertex of the parabola $y^2 + 4x - 10y - 15 = 0$. The reflected ray cuts y-axis at the point :

(a) (0,8) (b) (0,10) (c) (0,12) (d) (0,16)

Comprehension: (3)

Let $A\left(\frac{1}{2},0\right)$, $B\left(\frac{3}{2},0\right)$, $C\left(\frac{5}{2},0\right)$ be the given points and P be a point satisfying max $\{PA + PB, PB + PC\} < 2$

1. All points P are points common to:

(a) two ellipse (b) two hyperbolas

(c) a circle and an ellipse (d) a circle and an hyperbola

2. The locus of P is symmetric about

(a) origin (b) the line $y = x$ (c) y-axis (d) x-axis

3. The area of region of the point P is:

(a) $\sqrt{2}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$ (b) $\sqrt{3}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$

(c) $2\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$ (d) $3\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$

Comprehension: (4)

A tangent is drawn at any point $P(4\cos\theta, 3\sin\theta)$ on the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and on it is taken a point $Q(\alpha, \beta)$ from which pair of tangents QA and QB are drawn to the circle $x^2 + y^2 = 12$.

1. The locus of the point of concurrency of chord of contact AB of the circle $x^2 + y^2 = 12$ is:

(a) $\frac{x^2}{9} + \frac{y^2}{16} = 1$ (b) $\frac{x^2}{12} + \frac{y^2}{4} = 1$

(c) $\frac{x^2}{12} + \frac{y^2}{16} = 1$ (d) $\frac{x^2}{6} + \frac{y^2}{8} = 1$

2. The locus of the circumcentre of ΔQAB if $\theta = \frac{\pi}{4}$ is:

(a) $\frac{x}{4} + \frac{y}{3} = \sqrt{3}$

(b) $\frac{x}{4} + \frac{y}{3} = \frac{1}{\sqrt{2}}$

(c) $x^2 + y^2 = 6$

(d) $\frac{x}{3} + \frac{y}{4} = \frac{1}{\sqrt{2}}$

3. The number of points on the curve given by locus of circumcentre in problem 2 from which perpendicular tangents can be drawn to the parabola $x^2 + 4y = 0$ is:

(a) 1

(b) 2

(c) 3

(d) 4

Comprehension: (5)

Let an ellipse having major axis and minor axis parallel to *x*-axis and *y*-axis respectively. Its two foci *S* and *S′* are (2, 1), (4, 1) and a line $x + y = 9$ is a tangent to this ellipse at point *P*.

1. Eccentricity of the ellipse is:

(a) $\frac{1}{2\sqrt{3}}$

(b) $\frac{1}{\sqrt{13}}$

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

2. Length of major axis is:

(a) $\sqrt{13}$

(b) $2\sqrt{11}$

(c) $2\sqrt{13}$

(d) $4\sqrt{3}$

ANSWERS

Comprehension-1:	1.	(b)	2.	(b)	3.	(b)
Comprehension-2:	1.	(c)	2.	(d)	3.	(b)
Comprehension-3:	1.	(a)	2.	(d)	3.	(b)
Comprehension-4:	1.	(a)	2.	(b)	3.	(a)
Comprehension-5:	1.	(b)	2.	(c)		

EXERCISE 4

Match the Columns:

1. Match the column :

	Column-I		Column-II
(a)	A stick of length 10 meter slides on co-ordinate axes, then locus of a point dividing this stick reckoning from x-axis in the ratio 6 : 4 is a curve whose eccentricity is e, then $9e$ is equal to	**(p)**	$\sqrt{6}$
(b)	AA' is major axis of an ellipse $3x^2+2y^2+6x-4y-1=0$ and P is variable point on it then greatest value of area of a triangle APA' is	**(q)**	$2\sqrt{7}$
(c)	Distance between foci of the curve represented by the equation $x=1+4\cos\theta,\ y=2+3\sin\theta$ is	**(r)**	$\frac{128}{3}$
(d)	Tangents are drawn to the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ at end points of latus rectum. The area of quadrilateral so formed is	**(s)**	$3\sqrt{5}$

2. Match the locus of the middle points of chords of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ as given in Column-I to the equations given in Column-II.

	Column-I		Column-II
(a)	Which subtend a right angle at centre of ellipse	**(p)**	$x^2+y^2=(a^2+b^2)\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2$
(b)	Whose length is constant $=2$	**(q)**	$\frac{\left(x-\frac{a}{2}\right)^2}{a^2}+\frac{y^2}{b^2}=\frac{1}{4}$
(c)	The tangents at the ends of which intersect at right angles	**(r)**	$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right)=\frac{x^2}{a^4}+\frac{y^2}{b^4}$
(d)	Which passes through the $(a,0)$	**(s)**	$\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}-1\right)\left(1+\frac{b^4x^2}{a^4y^2}\right)+\left(\frac{1}{a^2}+\frac{b^2x^2}{a^4y^2}\right)=0$

3. Match the column :

	Column-I		Column-II
(a)	The radius of circle passing through both the foci of ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and having centre at $(0,3)$ is r, then $r=$	**(p)**	2
(b)	If the length of the major axis of an ellipse is 3 times the length of its minor axis then eccentricity, $e=$	**(q)**	4
(c)	Let $\theta_1, \theta_2, \theta_3, \theta_4$ be eccentric angles of four concyclic points of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $\theta_1+\theta_2+\theta_3+\theta_4=k\pi$, then k can be equal to	**(r)**	$\frac{2\sqrt{2}}{3}$
		(s)	6

4. Match the column :

	Column-I		Column-II
(a)	The number of rational points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is	**(p)**	Infinite
(b)	The number of integral points on the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ is	**(q)**	4
(c)	The number of rational points on the ellipse $\frac{x^2}{3}+y^2=1$	**(r)**	2
(d)	The number of integral points on the ellipse $\frac{x^2}{3}+y^2=1$	**(s)**	0

ANSWERS

1. $a \to s;\ b \to p;\ c \to q;\ d \to r$

2. $a \to r;\ b \to s;\ c \to p;\ d \to q$

3. $a \to q;\ b \to r;\ c \to p, q, s$

4. $a \to p;\ b \to q;\ c \to p;\ d \to r$

EXERCISE 5

Subjective Problems

1. An isosceles ΔABC is inscribed in the circle $x^2 + y^2 = 16$, $A = (0, 4)$. From points A, B and C three ordinates are drawn which cut the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ at the points P, Q and R respectively such that A and P are on the opposite sides, Q and B are on the same side and C and R are on the same side of the major axis. If ΔPQR is right angled at P and θ is the smallest angle of the ΔABC, then find the value of $16\tan^2\theta + 8\tan\theta + 9$.

2. Rectangle $ABCD$ has area 200. An ellipse with area 200π passes through A and C and has foci at B and D. Let perimeter of the rectangle $ABCD$ is P, then $\frac{P}{10} =$

3. Variable pairs of chords at right angles and drawn through any point P (with eccentric angle $\pi/4$) on the ellipse $\frac{x^2}{4} + y^2 = 1$, to meet the ellipse at two points say A and B. If the line joining A and B passes through a fixed point $Q(a, b)$ such that $a^2 + b^2$ has the value equal to $\frac{m}{n}$, where m, n are relatively prime positive integers, find $\left[\frac{m+n}{3}\right]$ where [.] denote greatest integer function.

4. A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$ at P and Q. Let angle between tangents at P and Q of the ellipse $x^2 + 2y^2 = 6$ is $\frac{\pi}{k}$, then $k =$

5. An ellipse has foci at $(9, 20)$ and $(49, 55)$ in the x-y plane and is tangent to x-axis. Let the length of its major axis is L, then $\left[\frac{L}{11}\right] =$ (where $[\cdot] =$ G.I.F)

6. Common tangents are drawn to parabola $y^2 = 4x$ and the ellipse $3x^2 + 8y^2 = 48$ touching the parabola at A and B and ellipse at C and D. If the area of quadrilateral of $ABCD$ is $55\sqrt{N}$, then $N =$

7. If the equation of the curve on reflection of the ellipse $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$ about the line $x - y - 2 = 0$ is $16x^2 + 9y^2 + k_1 x - 36y + k_2 = 0$, then $\frac{k_1 + k_2}{22}$ is ________

8. If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is normal to the circle $x^2 + y^2 + 4x + 1 = 0$, then find the maximum value of ab.

9. A normal is drawn to the ellipse $\frac{x^2}{(a^2+2a+2)^2}+\frac{y^2}{(a^2+1)^2}=1$ whose centre is at O. If maximum radius of the circle, centered at the origin and touching the normal, is 5 then find the positive value of 'a'.

10. A circle concentric to the ellipse $\frac{4x^2}{289}+\frac{y^2}{\lambda^2}=1\left(\lambda<\frac{17}{2}\right)$ passes through foci S_1, S_2 and cuts ellipse at point P. If area of ΔPS_1S_2 is 30 sq. units then find $\frac{S_1S_2}{13}$.

11. Let an ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ and a parabola $x^2=2y$ intersecting each other above x-axis at P and Q. Tangents to ellipse at P and Q intersect y-axis at R and tangents to parabola at P and Q intersect y-axis at S, then the area of quadrilateral $PRQS$ is $\lambda\sqrt{3}$, then $[\lambda]=$

(where [.] denote greatest integer function)

12. In above problem if $\mu=\frac{\text{area of }\Delta PQR}{\text{area of }\Delta PQS}$, then $2\mu=$

ANSWERS

1.	24	**2.**	8	**3.**	6	**4.**	2	**5.**	7	**6.**	2	**7.**	6	**8.**	4	**9.**	2	**10.**	1
11.	7	**12.**	3																

EXERCISE 6

1. (A) If x_1, x_2, x_3 as well as y_1, y_2, y_3 are in G.P. with the same common ratio, then the points $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3):

(a) lie on a straight line (b) lie on an ellipse

(c) lie on a circle (d) are vertices of a triangle

(B) On the ellipse, $4x^2 + 9y^2 = 1$, the points at which the tangents are parallel to the line $8x = 9y$ are:

(a) $\left(\frac{2}{5}, \frac{1}{5}\right)$ (b) $\left(-\frac{2}{5}, \frac{1}{5}\right)$ (c) $\left(-\frac{2}{5}, -\frac{1}{5}\right)$ (d) $\left(\frac{2}{5}, -\frac{1}{5}\right)$

(C) Consider the family of circles, $x^2 + y^2 = r^2, 2 < r < 5$. If in the first quadrant, the common tangent to a circle of the family and the ellipse $4x^2 + 25y^2 = 100$ meets the co-ordinate axes at A and B, then find the equation of the locus of the mid-point of AB. **[IIT-JEE 1999]**

2. Find the equation of the largest circle with centre $(1, 0)$ that can be inscribed in the ellipse $x^2 + 4y^2 = 16$. **[REE 1999]**

3. Let ABC be an equilateral triangle inscribed in the circle $x^2 + y^2 = a^2$. Suppose perpendiculars from A, B, C to the major axis of the ellipse, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ meet the ellipse respectively at P, Q, R so that P, Q, R lie on the same side of the major axis as A, B, C respectively. Prove that the normals to the ellipse drawn at the points P, Q and R are concurrent. **[IIT-JEE 2000]**

4. Let C_1 and C_2 be two circles with C_2 lying inside C_1. A circle C lying inside C_1 touches C_1 internally and C_2 externally. Identify the locus of the centre of C. **[IIT-JEE 2001]**

5. Find the condition so that the line $px + qy = r$ intersects the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in points whose eccentric angles differ by $\frac{\pi}{4}$. **[REE 2001]**

6. Prove that, in an ellipse, the perpendicular from a focus upon any tangent and the line joining the centre of the ellipse to the point of contact meet on the corresponding directrix. **[IIT-JEE 2002]**

7. (A) The area of the quadrilateral formed by the tangents at the ends of the latus rectum of the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ is:

(a) $9\sqrt{3}$ sq. units (b) $27\sqrt{3}$ sq. units (c) 27 sq. units (d) none of these

(B) The value of θ for which the sum of intercept on the axis by the tangent at the point $(3\sqrt{3}\cos\theta, \sin\theta), 0 < \theta < \pi/2$ on the ellipse $\frac{x^2}{27} + y^2 = 1$ is least, is:

[IIT-JEE (Screening) 2003]

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{8}$

8. The locus of the middle point of the intercept of the tangents drawn from an external point to the ellipse $x^2+2y^2=2$, between the coordinates axes, is: **[IIT-JEE (Screening) 2004]**

(a) $\frac{1}{x^2}+\frac{1}{2y^2}=1$ (b) $\frac{1}{4x^2}+\frac{1}{2y^2}=1$ (c) $\frac{1}{2x^2}+\frac{1}{4y^2}=1$ (d) $\frac{1}{2x^2}+\frac{1}{y^2}=1$

9. (A) The minimum area of triangle formed by the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and coordinate axes is: **[IIT-JEE (Screening) 2005]**

(a) ab sq. units (b) $\frac{a^2+b^2}{2}$ sq. units

(c) $\frac{(a+b)^2}{2}$ sq. units (d) $\frac{a^2+ab+b^2}{3}$ sq. units

(B) Find the equation of common tangent in 1st quadrant to the circle $x^2+y^2=16$ and the ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$. Also find the length of the intercept of the tangent between the coordinate axes. **[IIT-JEE (Mains) 2005]**

10. The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2+9y^2=9$ meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin O is: **[IIT-JEE 2009]**

(a) $\frac{31}{10}$ (b) $\frac{29}{10}$ (c) $\frac{21}{10}$ (d) $\frac{27}{10}$

11. The normal at the point P on the ellipse $x^2+4y^2=16$ meets the x-axis at Q. If M is the mid-point of the line segment PQ, then the locus of M intersects the latus rectums of the given ellipse at the points: **[IIT-JEE 2009]**

(a) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{2}{7}\right)$ (b) $\left(\pm\frac{3\sqrt{5}}{2}, \pm\frac{\sqrt{19}}{4}\right)$ (c) $\left(\pm 2\sqrt{3}, \pm\frac{1}{7}\right)$ (d) $\left(\pm 2\sqrt{3}, \pm\frac{4\sqrt{3}}{7}\right)$

Paragraph for questions 12 to 14

Tangents are drawn from the point $P(3, 4)$ to the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ touching the ellipse at points A and B. **[IIT-JEE 2010]**

12. The coordinates of A and B are:

(a) $(3,0)$ and $(0,2)$ (b) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

(c) $\left(-\frac{8}{5}, \frac{2\sqrt{161}}{15}\right)$ and $(0,2)$ (d) $(3,0)$ and $\left(-\frac{9}{5}, \frac{8}{5}\right)$

13. The orthocentre of the triangle PAB is:

(a) $\left(5, \frac{8}{7}\right)$ (b) $\left(\frac{7}{5}, \frac{25}{8}\right)$ (c) $\left(\frac{11}{5}, \frac{8}{5}\right)$ (d) $\left(\frac{8}{25}, \frac{7}{25}\right)$

14. The equation of the locus of the point whose distances from the point P and the line AB are equal, is:

(a) $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$ (b) $x^2 + 9y^2 + 6xy - 54x + 62y - 241 = 0$

(c) $9x^2 + 9y^2 - 6xy - 54x - 62y - 241 = 0$ (d) $x^2 + y^2 - 2xy + 27x + 31y - 120 = 0$

15. The ellipse $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point $(0, 4)$ circumscribes the rectangle R. The eccentricity of the ellipse E_2 is: **[IIT-JEE 2012]**

(a) $\frac{\sqrt{2}}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

16. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at $(0, 3)$ is : **[IIT-JEE (Mains) 2013]**

(a) $x^2 + y^2 - 6y + 5 = 0$ (b) $x^2 + y^2 - 6y - 7 = 0$

(c) $x^2 + y^2 - 6y +$ [illegible] (d) $x^2 + y^2 - 6y - 5 = 0$

17. A vertical line passing through the point $(h, 0)$ intersects the ellipse $\frac{x^2}{4} + \frac{y^2}{3} = 1$ at the points P and Q. Let the tangents to the ellipse at P and Q meet at the point R. If $\Delta(h) =$ area of the triangle PQR, $\Delta_1 = \max_{1/2 \le h \le 1} \Delta(h)$ and $\Delta_2 = \min_{1/2 \le h \le 1} \Delta(h)$, then $\frac{8}{\sqrt{5}}\Delta_1 - 8\Delta_2 =$

[IIT-JEE (Advance) 2013]

ANSWERS

1. **(A)** a; **(B)** b, d; **(C)** $25y^2 + 4x^2 = 4x^2y^2$ **2.** $(x-1)^2 + y^2 = \frac{11}{3}$

4. Locus is an ellipse with foci as the centres of the circles C_1 and C_2.

5. $a^2p^2 + b^2q^2 = r^2 \sec^2 \frac{\pi}{8} = (4 - 2\sqrt{2})r^2$ **7.** **(A)** c; **(B)** a; **8.** c

9. **(A)** a; **(B)** $AB = \frac{14}{\sqrt{3}}$ **10.** d **11.** c **12.** d

13. c **14.** a **15.** c **16.** b

17. 9

SOLUTIONS 1

Only One Choice is Correct:

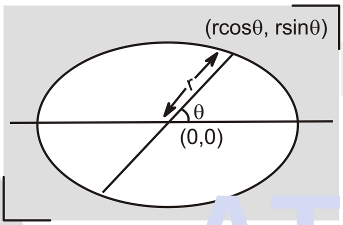
1. (d) $0 < f(4a) < f(a^2 - 5)$

$\Rightarrow \quad 4a > a^2 - 5$

$\Rightarrow \quad (a-5)(a+1) < 0$

$\Rightarrow \quad a \in (-1, 5)$

2. (a) $r^2\left(\dfrac{\cos^2\theta}{a^2} + \dfrac{\sin^2\theta}{b^2}\right) = 1$



$$2(2r)^2 = (2a)^2 + (2b)^2$$

$\Rightarrow \quad 2r^2 = a^2 + b^2$

$\Rightarrow \quad (a^2 + b^2)\left(\dfrac{\cos^2\theta}{a^2} + \dfrac{\sin^2\theta}{b^2}\right) = 2$

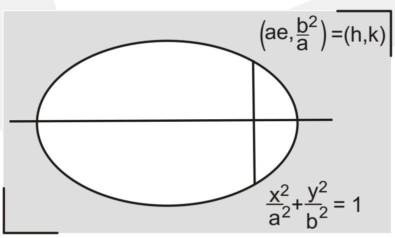
$\Rightarrow \quad \dfrac{b^2}{a^2}\cos^2\theta + \dfrac{a^2}{b^2}\sin^2\theta$

$$= 1 = \sin^2\theta + \cos^2\theta$$

$\Rightarrow \quad \left(\dfrac{b^2 - a^2}{a^2}\right)\cos^2\theta = \left(1 - \dfrac{a^2}{b^2}\right)\sin^2\theta$

$\Rightarrow \quad \tan\theta = \pm\dfrac{b}{a}$

3. (b)



$$h = ae$$

$$k = \frac{b^2}{a} = a(1 - e^2)$$

$$= a\left(1 - \frac{h^2}{a^2}\right)$$

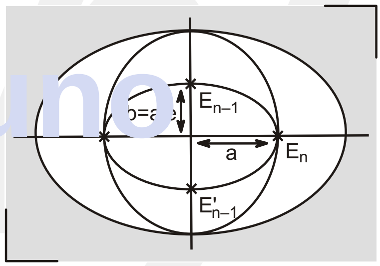
$$ay = (a^2 - x^2)$$

$$x^2 = a(a - y) \rightarrow \text{Parabola}$$

4. (c) $b' = a,\ a'e = b$

$$b'^2 = a'^2 - a'^2 e^2$$

$$a^2 = \frac{b^2}{e^2} - b^2$$



$$e^2 = \frac{b^2}{a^2}(1 - e^2) = (1 - e^2)^2$$

$$1 - e^2 = \pm e$$

$$e^2 + e - 1 = 0$$

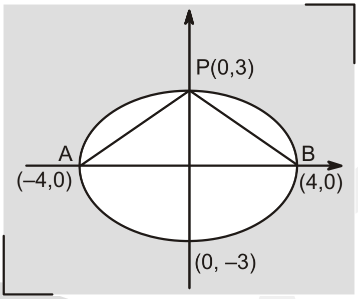
or $\quad e^2 - e - 1 = 0$

$\Rightarrow \quad e = \dfrac{-1 \pm \sqrt{5}}{2}$

or $\quad \dfrac{1 \pm \sqrt{5}}{2}$

$\Rightarrow \quad e = \dfrac{\sqrt{5} - 1}{2}$

5. (c) For area to become maximum altitude should be maximum



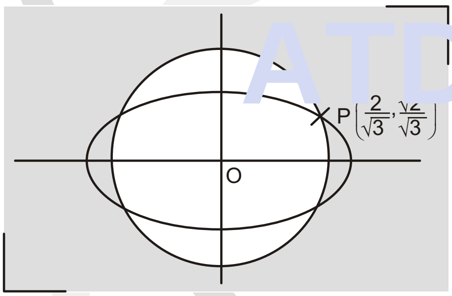
$$\Rightarrow \quad P(0,3) \quad \text{or} \quad P(0,-3)$$

$$(\Delta PAB)_{\max} = \frac{1}{2} \times 8 \times 3 = 12$$

6. (b) Equation of normal to ellipse at P is

$$\frac{4x}{2/\sqrt{3}} - \frac{y}{\sqrt{2}/\sqrt{3}} = 3$$

$$\Rightarrow \quad 2x - \frac{y}{\sqrt{2}} = \sqrt{3}$$



$\Rightarrow$ slope of normal to ellipse at $P = m_1 = 2\sqrt{2}$

Slope of normal to circle at

$$P = m_2 = \frac{\frac{\sqrt{2}}{\sqrt{3}} - 0}{\frac{2}{\sqrt{3}} - 0} = \frac{1}{\sqrt{2}}$$

$$\tan\theta = \frac{2\sqrt{2} - \frac{1}{\sqrt{2}}}{1 + 2\sqrt{2}\frac{1}{\sqrt{2}}} = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

7. (c) Locus is director circle given by

$$(x-1)^2 + (y-1)^2 = 16 + 9$$

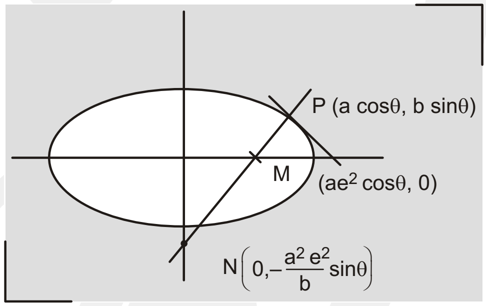
$$\Rightarrow \quad x^2 + y^2 - 2x - 2y - 23 = 0$$

8. (c) $(PM)^2 = a^2\cos^2\theta(1-e^2)^2 + b^2\sin^2\theta$

$$= b^2(1 - e^2\cos^2\theta)$$

$$(PN)^2 = a^2\cos^2\theta + \frac{\sin^2\theta}{b^2}(b^2 + a^2e^2)^2$$

$$= \frac{a^4}{b^2}(1 - e^2\cos^2\theta)$$



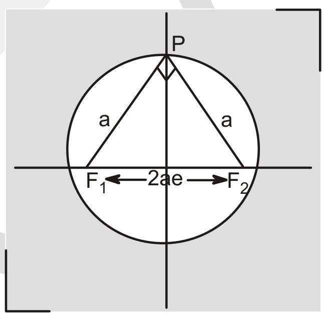
$$\left(\frac{PM}{PN}\right)^2 = \frac{b^4}{a^4} = \frac{4}{9}$$

$$\Rightarrow \quad \frac{b^2}{a^2} = \frac{2}{3} = 1 - e^2$$

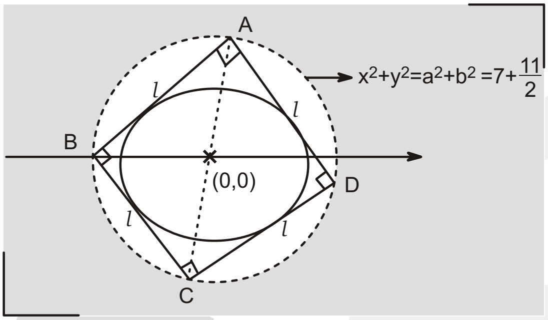
$$e = \frac{1}{\sqrt{3}}$$

9. (b) $a^2 + a^2 = (2ae)^2$

$$\Rightarrow \quad e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$$



10. (b) $l^2 + l^2 = (AC)^2 = \left(2\sqrt{a^2 + b^2}\right)^2$



$$2l^2 = 4\left(7+\frac{11}{2}\right)$$

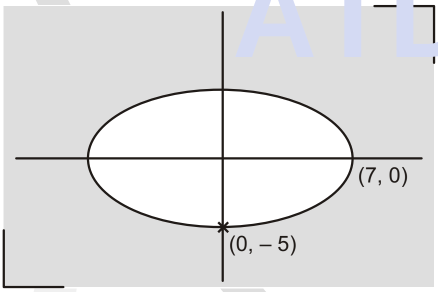
$$l^2 = 14 + 11 = 25$$

$$\Rightarrow \quad l = 5$$

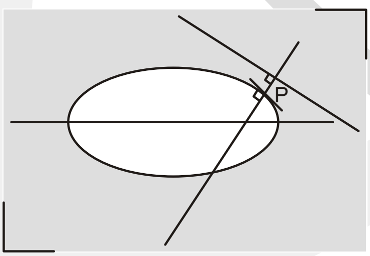
11. (b) $b^2 = a^2(1-e^2)$

$$25 = 49(1-e^2)$$

$$e = \frac{2\sqrt{6}}{7}$$



12. (b) Shortest distance is along common normal



Equation of normal at P is

$$\sqrt{6}x\sec\theta - \sqrt{3}y\,\text{cosec}\,\theta = 3$$

$$\text{slope} = \sqrt{2}\tan\theta = 1$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad P \equiv \left(\sqrt{6}\,\frac{\sqrt{2}}{\sqrt{3}}, \sqrt{3}\,\frac{1}{\sqrt{3}}\right) \equiv (2,1)$$

13. (b) Let $\frac{ax}{3} + \frac{by}{4} = c$ is normal to ellipse at $(a\cos\theta, b\sin\theta)$ Equation of normal to ellipse at $(a\cos\theta, b\sin\theta)$ is

$$ax\sec\theta - by\,\text{cosec}\,\theta = a^2 - b^2 \qquad \ldots(1)$$

$$\frac{ax}{3} + \frac{by}{4} = c \qquad \ldots(2)$$

(1) and (2) are identical

$$\Rightarrow \quad 3\sec\theta = -4\,\text{cosec}\,\theta = \frac{a^2-b^2}{c}$$

$$\Rightarrow \quad \cos\theta = \frac{3c}{a^2-b^2}, \sin\theta = \frac{-4c}{a^2-b^2}$$

$$\Rightarrow \quad \left(\frac{3c}{a^2-b^2}\right)^2 + \left(\frac{-4c}{a^2-b^2}\right)^2 = 1$$

$$\Rightarrow \quad a^2 - b^2 = 5c$$

14. (b) $(SP)(S'P) = a(1-e\cos\theta)a(1+e\cos\theta)$

$$= a^2 - a^2e^2\cos^2\theta$$

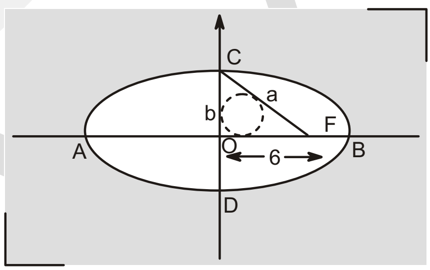
$$= 25 - 9\cos^2\theta$$

$$\max = 25 - 9(0) = 25$$

$$\min = 25 - 9(1) = 16$$

$$\max - \min = 25 - 16 = 9$$

15. (a)



$$a^2 = b^2 + 6^2 \qquad \ldots(1)$$

$$r = \frac{\Delta}{S} = \frac{\frac{1}{2} \times 6 \times b}{\frac{1}{2}(a + b + 6)}$$

$$= \frac{6b}{a + b + 6} = 1$$

$\Rightarrow \quad 5b = a + 6 \qquad \ldots(2)$

From eqn. (1) and (2)

$$(5b - 6)^2 = b^2 + 36$$

$\Rightarrow \quad 24b^2 - 60b = 0$

$\Rightarrow \quad b = \frac{5}{2}$

$$a = \frac{25}{2} - 6 = \frac{13}{2}$$

$$(AB)(CD) = (2a)(2b) = 4 \times \frac{13}{2} \times \frac{5}{2} = 65$$

16. (d) $\frac{x^2}{5} + \frac{y^2}{3} = 1$

Equation of director circle is

$$x^2 + y^2 = 8 \qquad \ldots(1)$$

$(\sqrt{7}, 1)$ satisfy the equation (1)

$\Rightarrow$ Angle between tangents is $\frac{\pi}{2}$

Let any tangent to the ellipse is $y = mx \pm \sqrt{a^2m^2 + b^2}$ which is passing through $(\sqrt{7}, 1)$

$$(1 - \sqrt{7}\,m)^2 = 5m^2 + 3$$

$\Rightarrow \quad m^2 - \sqrt{7}m - 1 = 0$

If m_1, m_2 are roots $\Rightarrow m_1 m_2 = -1$

$\Rightarrow$ two tangents are perpendicular to each other.

17. (b) Equation of tangent to $\frac{x^2}{a^2 + b^2} + \frac{y^2}{b^2} = 1$ is

$$y = mx \pm \sqrt{(a^2 + b^2)m^2 + b^2} \qquad \ldots(1)$$

$\because$ (1) is also tangent to $\frac{x^2}{a^2} + \frac{y^2}{a^2 + b^2} = 1$

$\Rightarrow (a^2 + b^2)m^2 + b^2 = a^2m^2 + (a^2 + b^2)$

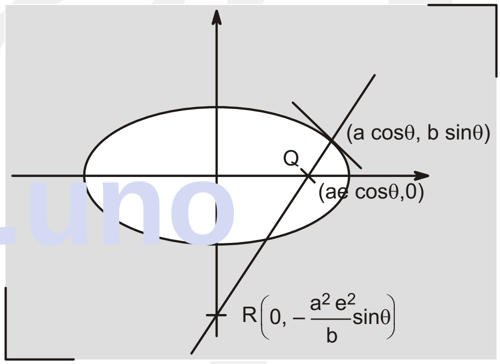
$\Rightarrow \quad m = \pm \frac{a}{b}$

Equation of common tangents are

$by = ax \pm \sqrt{a^2(a^2 + b^2) + b^4}$

18. (c) $2h = ae^2 \cos\theta$

$$2k = \frac{-a^2 e^2 \sin\theta}{b}$$



$\Rightarrow \quad \frac{4h^2}{(ae^2)^2} + \frac{4k^2}{\left(a^2 \frac{e^2}{b}\right)^2} = 1$

$$\frac{x^2}{\left(\frac{ae^2}{2}\right)^2} + \frac{y^2}{\left(\frac{a^2e^2}{2b}\right)^2} = 1$$

$$\frac{a^2e^4}{4} = \frac{a^4e^4}{4b^2}(1 - (e')^2)$$

$\Rightarrow \quad b^2 = a^2(1 - (e')^2)$

$\Rightarrow \quad e = e'$

19. (a) $(x + 2)^2 = 3\left(y + \frac{13}{3}\right)$

$\Rightarrow$ Length of latus rectum of parabola, $L_1 = 3$

$$\frac{(x-3)^2}{49} + \frac{(y+2)^2}{(49/4)} = 1$$

Lengths of latus rectum of ellipse,

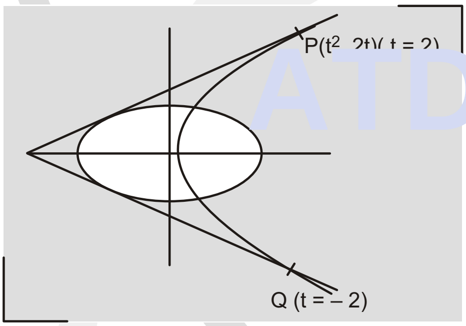
$$L_2 = \frac{2b^2}{a} = \frac{2\left(\frac{49}{4}\right)}{7} = 7/2$$

$$|L_1 - L_2| = |3 - \frac{7}{2}| = 1/2$$

20. (b) Equation of tangent at P is

$$y = \frac{x}{2} + 2 \qquad \text{...(1)}$$

For (1) to be tangent to $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$



$$\Rightarrow (2)^2 = 4\left(\frac{1}{2}\right)^2 + b^2$$

$$\Rightarrow \quad b^2 = 3 \qquad \Rightarrow b = \pm\sqrt{3}$$

Equation of tangent at Q is

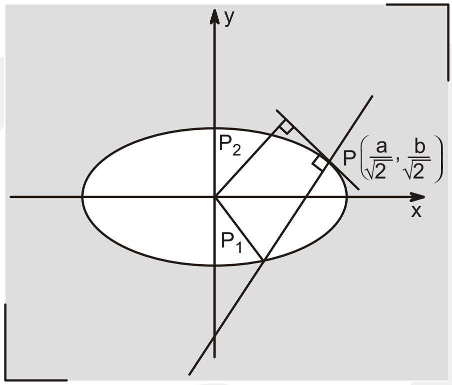
$$-2y = x + 4 \quad \Rightarrow \quad 2y + x + 4 = 0$$

21. (a) Equation of normal at P,

$$ax\sqrt{2} - by\sqrt{2} = a^2 - b^2$$

$$P_1 = \frac{a^2 - b^2}{\sqrt{2(a^2+b^2)}}$$

Equation of tangent at P is



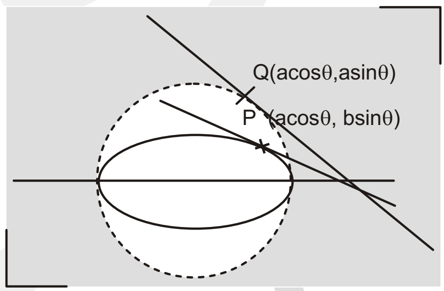
$$\frac{x}{\sqrt{2}a} + \frac{y}{\sqrt{2}b} = 1$$

$$P_2 = \frac{1}{\sqrt{\frac{1}{2a^2} + \frac{1}{2b^2}}} = \frac{\sqrt{2}\,ab}{\sqrt{a^2+b^2}}$$

$$P_1P_2 = \frac{ab(a^2-b^2)}{(a^2+b^2)}$$

22. (c) Equation of tangent at P is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1 \qquad \text{...(1)}$$



Equation of tangent at Q is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{a} = 1 \qquad \text{...(2)}$$

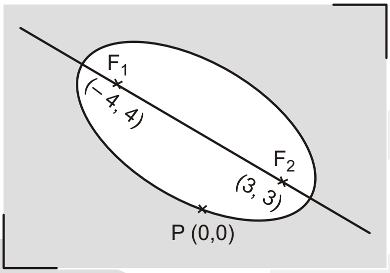
(1) – (2)

$$\Rightarrow \qquad y\sin\theta\left(\frac{1}{b} - \frac{1}{a}\right) = 0$$

$$\Rightarrow \qquad y = 0$$

$\therefore$ Point of intersection lie on line $y = 0$

23. (c) $PF_1 + PF_2 = 2a$



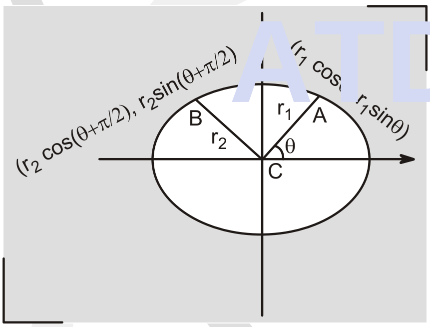
$\Rightarrow \quad 3\sqrt{2} + 4\sqrt{2} = 2a$ [Here $P = (0,0)$]

$\Rightarrow \quad 7\sqrt{2} = 2a$

$$2ae = \sqrt{7^2 + 1^2} = 5\sqrt{2}$$

$$\Rightarrow \quad e = \frac{5\sqrt{2}}{7\sqrt{2}} = \frac{5}{7}$$

24. (a) Put $(r\cos\theta, r\sin\theta)$ to the equation of ellipse

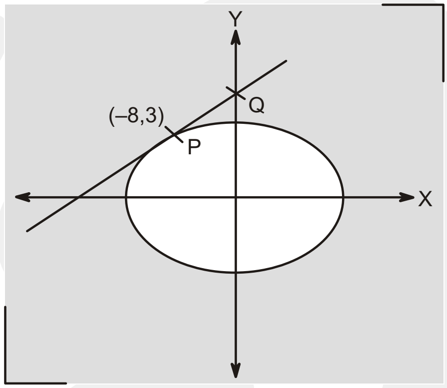


$$\Rightarrow \quad r^2\left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right) = 1$$

$$\Rightarrow \quad \frac{1}{r_1^2} + \frac{1}{r_2^2} = \left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right) + \left(\frac{\cos^2\left(\theta + \frac{\pi}{2}\right)}{a^2} + \frac{\sin^2\left(\theta + \frac{\pi}{2}\right)}{b^2}\right)$$

$$= \frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{16} + \frac{1}{9} = \frac{25}{144}$$

25. (a) Equation of tangent at $(-8,3)$ is



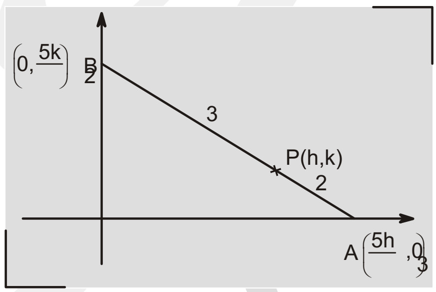
$$\frac{x(-8)}{100} + \frac{y(3)}{25} = 1$$

Put $\quad x = 0$

$$\Rightarrow \quad y = \frac{25}{3}$$

$$\Rightarrow \quad Q = \left(0, \frac{25}{3}\right)$$

26. (d)



$$(AB)^2 = \left(\frac{5h}{3}\right)^2 + \left(\frac{5k}{2}\right)^2 = (20)^2$$

$$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{4} = 16$$

$$\Rightarrow \quad \frac{x^2}{144} + \frac{y^2}{64} = 1$$

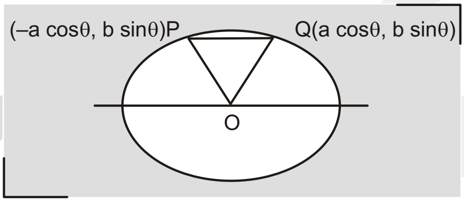
$$64 = 144(1 - e^2)$$

$$\Rightarrow \quad e^2 = 1 - \frac{4}{9} = \frac{5}{9} \quad \Rightarrow \quad e = \frac{\sqrt{5}}{3}$$

27. (a) Area of

$$\Delta OPQ = \frac{1}{2}(2a\cos\theta)(b\sin\theta)$$

$$= \frac{ab}{2}\sin 2\theta \le \frac{ab}{2}$$

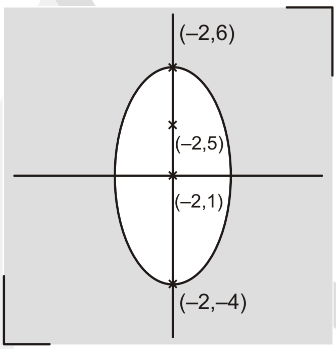


$$\text{Probability} = \frac{\frac{\pi ab}{2} - \frac{ab}{2}}{\frac{\pi ab}{2}} = \frac{\pi - 1}{\pi}$$

28 (d) $\frac{(x+2)^2}{a^2} + \frac{(y-1)^2}{b^2} = 1$

$be = 4 = 5e$

$\Rightarrow \quad a^2 = 5^2 - (5e)^2 = 25 - 16 = 9$

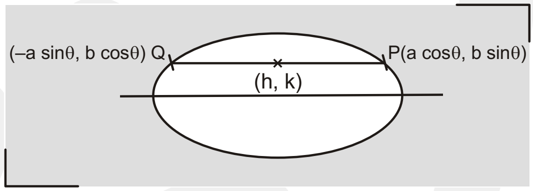


Equation of ellipse is

$$\frac{(x+2)^2}{9} + \frac{(y-1)^2}{25} = 1$$

29. (a) $2h = a(\cos\theta - \sin\theta)$

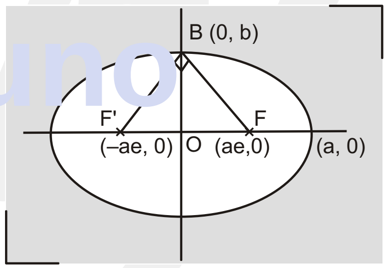
$2k = b(\cos\theta + \sin\theta)$



$$\Rightarrow \left(\frac{2h}{a}\right)^2 + \left(\frac{2k}{b}\right)^2 = 2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$$

30. (d) (Slope of BF') (slope of BF) $= -1$



$$\Rightarrow \left(-\frac{b}{ae}\right)\left(\frac{b}{ae}\right) = -1$$

$$\Rightarrow b^2 = a^2e^2$$

$$\Rightarrow a^2(1-e^2) = a^2e^2$$

$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

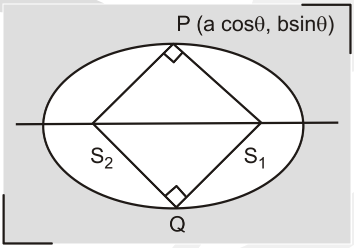
SOLUTIONS 2

One or More than One is/are Correct

1. (a, b, d)

$$(S_1S_2)^2 = (S_1P)^2 + (S_2P)^2$$

$$(2ae)^2 = a^2[(1-e\cos\theta)^2 + (1+e\cos\theta)^2]$$



$$2e^2 = 1 + e^2\cos^2\theta \quad ...(1)$$

$\because \quad b^2 = a^2(1-e^2)$

$\Rightarrow \quad 1 = 2(1-e^2) \Rightarrow e^2 = \frac{1}{2}$

$\therefore$ From eqn. (1), we have

$$\cos\theta = 0 \quad \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

Area of

$S_1PS_2Q = (S_1P)(S_2P) = a^2 = 2$

$[\because S_1P = S_2P = a]$

2. (b, c, d)

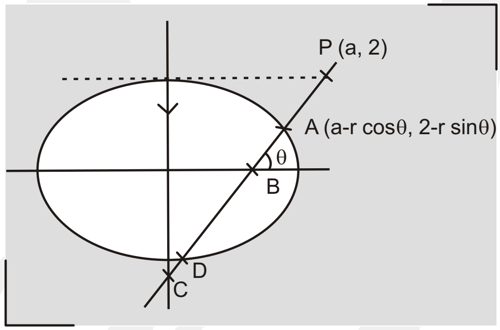
Put $(a - r\cos\theta, 2 - r\sin\theta)$ to eqn. of ellipse.

$$\Rightarrow \frac{a^2 + r^2\cos^2\theta - 2ar\cos\theta}{9} + \frac{4 + r^2\sin^2\theta - 4r\sin\theta}{4} - 1 = 0$$

$$\Rightarrow (PA)(PD) = \frac{\frac{a^2}{9}}{\frac{\cos^2\theta}{9} + \frac{\sin^2\theta}{4}}$$

$$= \frac{4a^2}{4\cos^2\theta + 9\sin^2\theta}$$

Put $(a - r\cos\theta, 2 - r\sin\theta)$ to equation $xy = 0$



$$\Rightarrow 2a + r^2\sin\theta\cos\theta - (a\sin\theta + 2\cos\theta)r = 0$$

$$\Rightarrow (PB)(PC) = \frac{2a}{\sin\theta\cos\theta}$$

$$PA \cdot PD = PB \cdot PC$$

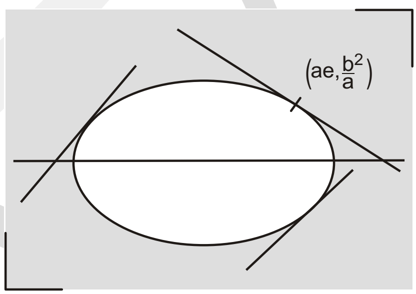
$$\Rightarrow \frac{4a^2}{4\cos^2\theta + 9\sin^2\theta} = \frac{2a}{\sin\theta.\cos\theta}$$

$$\Rightarrow a = \frac{4\cot\theta + 9\tan\theta}{2}$$

$$\Rightarrow |a| \geq \sqrt{4 \times 9} = 6$$

$$\Rightarrow a \in (-\infty, -6] \cup [6, \infty)$$

3. (a, b)



Tangents at extremities of L.R. are

$$\frac{xe}{a} + \frac{y}{a} = 1$$

or $\frac{xe}{a} - \frac{y}{a} = 1$

$xe + (y - a) = 0$

or $xe - (y + a) = 0$

Fixed points are $(0, a)$ or $(0, -a)$

4. (a, d)

Equation of tangent is

$$\frac{x}{4}\cos\theta + \frac{\sqrt{11}y\sin\theta}{16} = 1$$

$4\cos\theta\, x + \sqrt{11}\sin\theta\, y = 16$ which touches the circle $(x-1)^2 + y^2 = 16$

$$\Rightarrow \frac{|4\cos\theta - 16|}{\sqrt{16\cos^2\theta + 11\sin^2\theta}} = 4$$

$$\Rightarrow (\cos\theta - 4)^2 = 16\cos^2\theta + 11\sin^2\theta$$

$$= \cos^2\theta - 8\cos\theta + 16$$

$$\Rightarrow 4\cos^2\theta + 8\cos\theta - 5 = 0$$

$$= 4\cos^2\theta - 2\cos\theta + 10\cos\theta - 5$$

$$\Rightarrow (2\cos\theta + 5)(2\cos\theta - 1) = 0$$

$$\Rightarrow \cos\theta = \frac{1}{2}$$

5. (a, b, d)

$$\angle SQP = \angle S'QP = \frac{\pi}{2}$$

$\Rightarrow$ S', Q, S are collinear

T is radical centre of C, C_1 and C_2.

Equation of circle C_2

$$C_2 : (x - a\cos\theta)(x + ae) + (y - b\sin\theta)y = 0$$

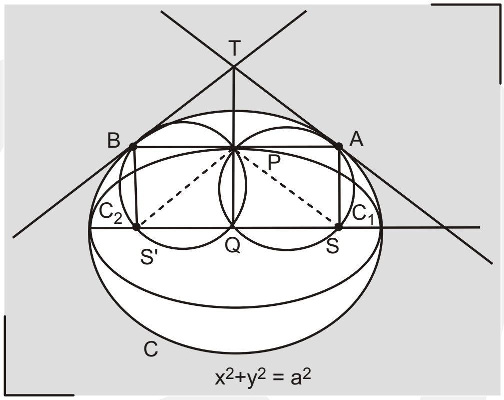
$$C : x^2 + y^2 - a^2 = 0$$

Equation of tangent to C_2 and C is

$$a(\cos\theta - e)x + b\sin\theta y + a^2(e\cos\theta - 1) = 0$$

Put $x = a\cos\theta$

$$a^2(\cos^2\theta - e\cos\theta) + b\sin\theta y + a^2(e\cos\theta - 1) = 0$$



$$\Rightarrow b\sin\theta y = a^2\sin^2\theta$$

$$\Rightarrow y = \frac{a^2}{b}\sin\theta$$

$$y = \left(a\cos\theta, \frac{a^2}{b}\sin\theta\right)$$

Equation of chord of contact of T w.r.t. $x^2 + y^2 = a^2$

$$x\cos\theta + y\frac{a}{b}\sin\theta = a$$

$\Rightarrow \frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$ which is same as equation of tangent to ellipse at P.

$\because$ SA and $S'B$ both are $\perp$ to AB

$\Rightarrow$ $S'B \,||\, SA$

6. (b, d)

Equation of tangent is

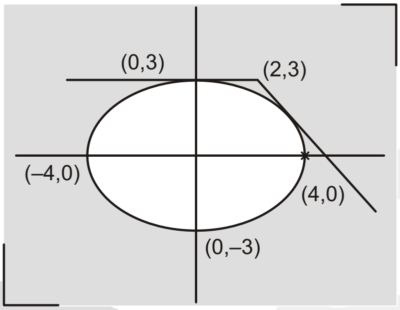
$$y = mx \pm \sqrt{16m^2 + 9}$$

Put $(2, 3)$

$$\Rightarrow (3 - 2m)^2 = 16m^2 + 9$$

$$\Rightarrow 12m^2 + 12m = 0$$

$$\Rightarrow m = 0, -1$$



$\therefore$ Tangents are $y = 3,\ y - 3 = -1(x - 2)$

$y = 3$ and $y + x = 5$

7. (a, b, d)

$$\frac{\left(\frac{x - 2y + 1}{\sqrt{5}}\right)^2}{(5/4)} + \frac{\left(\frac{2x + y + 2}{\sqrt{5}}\right)^2}{(5/9)} = 1$$

$$b^2 = a^2(1 - e)^2$$

$$\Rightarrow \quad \frac{5}{9} = \frac{5}{4}(1 - e^2)$$

$$\Rightarrow \quad e = \frac{\sqrt{5}}{3}$$

Intersection of major axis which is $2x + y + 2 = 0$ and minor axis which is $x - 2y + 1 = 0$ is $(-1, 0)$

$\Rightarrow$ centre $\equiv (-1, 0)$

Length of major axis $= 2a = \sqrt{5}$

8. (a, b, c, d)

$P \equiv (\sqrt{6}\cos\theta, \sqrt{2}\sin\theta)$

$6\cos^2\theta + 2\sin^2\theta = 4$

$\Rightarrow \quad 4\cos^2\theta = 2$

$\Rightarrow \quad \cos\theta = \pm\frac{1}{\sqrt{2}}$

Hence, $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

9. (b, c, d)

$$\frac{(x + 1)^2}{9} + \frac{(y - 2)^2}{5} = 1$$

centre $\equiv (-1, 2)$

$$5 = 9(1 - e^2) \Rightarrow e = \frac{2}{3}$$

Foci $\equiv \left(-1 + 3\left(\frac{2}{3}\right), 2 + 0\right)$

and $\left(-1 - 3\left(\frac{2}{3}\right), 2 + 0\right)$

Foci $\equiv (1, 2)$ and $(-3, 2)$

Length of L.R. $= \frac{2b^2}{a} = 2\left(\frac{5}{3}\right)$

$\Rightarrow$ Length of L.R. $= \frac{10}{3}$

10. (a, c)

$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$

Equation of tangent at $P(\theta)$ is

$$\frac{x\cos\theta}{4} + \frac{y\sin\theta}{2} = 1$$

tangent passes through $(4, 2)$

$\Rightarrow \quad \cos\theta + \sin\theta = 1$

$\Rightarrow \quad \sin\left(\theta + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\Rightarrow \quad \theta + \frac{\pi}{4} = \frac{\pi}{4}, \frac{3\pi}{4}$

$\Rightarrow \quad \theta = 0, \frac{\pi}{2}$

11. (a, d)

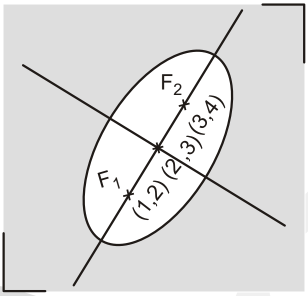
Equation of major axis is $y = x + 1$

Equation of minor axis is $x + y = 5$

$ae = \sqrt{(4 - 3)^2 + (3 - 2)^2}$

$= \sqrt{2} \Rightarrow a = 2\sqrt{2}$

$b^2 = a^2 - a^2e^2 = 8 - 2 = 6 \Rightarrow b = \sqrt{6}$



Equation of ellipse is

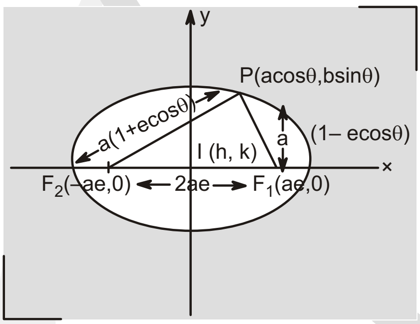
$$\frac{\left(\frac{x+y-5}{\sqrt{2}}\right)^2}{(2\sqrt{2})^2}+\frac{\left(\frac{y-x-1}{\sqrt{2}}\right)^2}{(\sqrt{6})^2}=1$$

Also its equation is

$$\sqrt{(x-3)^2+(y-4)^2}+\sqrt{(x-1)^2+(y-2)^2}=2(2\sqrt{2})$$

12. (a, c)

$$h=\frac{a^2e(1+e\cos\theta)-a^2e(1-e\cos\theta)+2a^2e\cos\theta}{2a+2ae}$$



$$\Rightarrow\quad h=ae\cos\theta$$

$$k=\frac{2ae\,b\sin\theta}{2a+2ae}=\frac{eb\sin\theta}{(1+e)}$$

$$\therefore\quad \frac{h^2}{(ae)^2}+\frac{k^2}{\left(\frac{be}{1+e}\right)^2}=1$$

Equation of locus of incentre is

$$\frac{x^2}{(ae)^2}+\frac{y^2}{\left(\frac{be}{1+e}\right)^2}=1$$

$$\therefore\quad \frac{b^2e^2}{(1+e)^2}=a^2e^2(1-e')^2$$

$$\Rightarrow\quad \frac{a^2e^2(a-e^2)}{(1+e)^2}=a^2e^2(1-e')^2$$

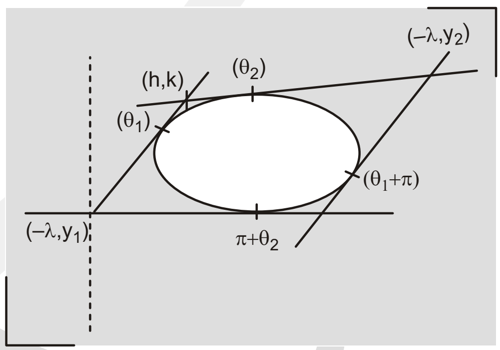
$$\Rightarrow\quad e'^2=1-\frac{1-e}{1+e}=\frac{2e}{1+e}$$

$$\Rightarrow\quad e'=\sqrt{\frac{2e}{1+e}}$$

13. (a, b, c)

$$h=\frac{2\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \quad \ldots(1)$$

$$k=\frac{2\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\theta_1-\theta_2}{2}\right)} \quad \ldots(2)$$



$$\Rightarrow\quad \frac{h^2}{9}+\frac{k^2}{4}=\frac{1}{\cos^2\left(\frac{\theta_1-\theta_2}{2}\right)}$$

$$\text{Also } -\lambda = \frac{3\cos\left(\frac{\pi}{2}+\frac{\theta_1+\theta_2}{2}\right)}{\cos\left(\frac{\pi}{2}+\frac{\theta_2-\theta_1}{2}\right)}$$

$$= \frac{3\sin\left(\frac{\theta_1+\theta_2}{2}\right)}{\sin\left(\frac{\theta_2-\theta_1}{2}\right)} \qquad \ldots(3)$$

Dividing (2) by (3) we get,

$$\tan\left(\frac{\theta_2-\theta_1}{2}\right) = \frac{3}{2}\frac{k}{\lambda}$$

$$\frac{h^2}{9}+\frac{k^2}{4} = \sec^2\left(\frac{\theta_2-\theta_1}{2}\right)$$

$$\therefore \quad \frac{h^2}{9}+\frac{k^2}{4} = 1+\frac{9}{4}\frac{k^2}{\lambda^2}$$

$$\frac{x^2}{9}+\frac{y^2}{\left(4\frac{\lambda^2}{\lambda^2-9}\right)} = 1$$

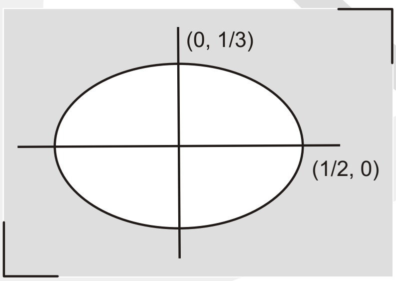
14. (b, d)

Equation of tangents are

$$y = \frac{8}{9}x \pm \sqrt{\frac{1}{4}\left(\frac{8}{9}\right)^2+\frac{1}{9}}$$

$$= \frac{8}{9}x \pm \frac{5}{9}$$

$$9y = 8x+5, \qquad 9y = 8x-5$$



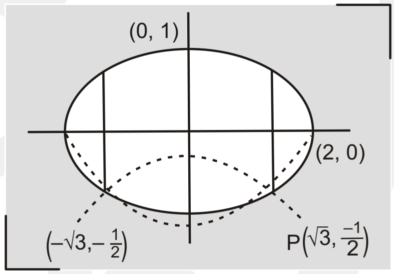
Tangent at (x_1, y_1) is $4xx_1 + 9yy_1 = 1$

$$\Rightarrow \quad \frac{9y_1}{9} = \frac{4x_1}{-8} = \frac{1}{5}$$

$$\text{or} \quad \frac{9y_1}{9} = \frac{4x_1}{-8} = -\frac{1}{5}$$

$$(x_1, y_1) = \left(-\frac{2}{5}, \frac{1}{5}\right) \text{ or } \left(\frac{2}{5}, -\frac{1}{5}\right)$$

15. (b, c)



$$1 = 4(1-e^2) \Rightarrow e = \frac{\sqrt{3}}{2}$$

Length of latus rectum of parabola

$$= 4a = 2\sqrt{3}$$

$$\Rightarrow \quad a = \frac{\sqrt{3}}{2}$$

Equation of parabolas are

$$2\sqrt{3}\left(y-\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\right)\right) = x^2$$

$$\text{or} \quad -2\sqrt{3}\left(y-\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\right)\right) = x^2$$

$$2\sqrt{3}y + (1+\sqrt{3})\sqrt{3} = x^2$$

$$\text{or} \quad -2\sqrt{3}y - (1-\sqrt{3})\sqrt{3} = x^2$$

$$x^2 - 2\sqrt{3}y = \sqrt{3}+3$$

$$\text{or} \quad x^2 + 2\sqrt{3}y = 3-\sqrt{3}$$

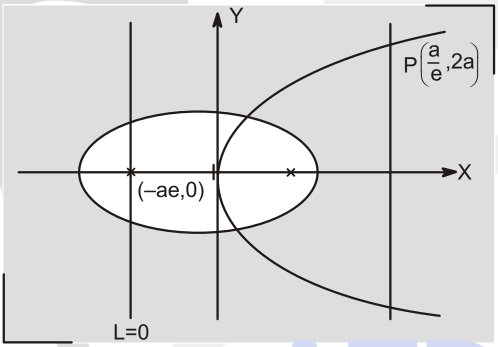
SOLUTIONS 3

Comprehension: (1)

1. (b) $L \equiv x = -ae$

$x^2 + y^2 = a^2 + b^2 = a^2 + a^2(1-e^2)$

where $x = -ae$



$\Rightarrow \quad a^2e^2 + y^2 = a^2 + a^2(1-e^2)$

$y^2 = 2a^2(1-e^2)$

$y = \sqrt{2}a\sqrt{1-e^2}$

2. (b) Chord of contacts of any point $(-ae, \lambda)$ w.r.t. ellipse and parabola respectively are

$-\frac{xe}{a} + \frac{y\lambda}{a^2(1-e^2)} = 1$

$y\lambda - ae(1-e^2)x = a^2(1-e^2) \qquad \ldots(1)$

and $y\lambda - 2ae(x - ae) = 0$

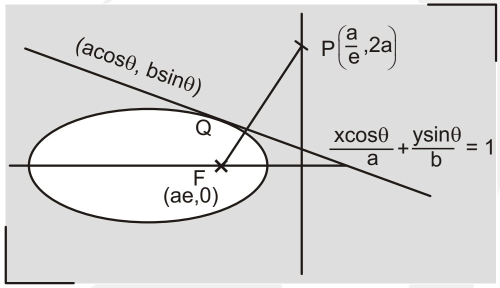
$y\lambda - 2aex = -2a^2e^2 \qquad \ldots(2)$

By eqns. (1) – (2)

$\Rightarrow \quad -ae(1+e^2)\,x = -a^2(1+e^2)$

$\Rightarrow \quad x = \frac{a}{e}$

3. (b) $\therefore$ Foot of perpendicular from focus of ellipse to any tangent to it lies on auxiliary circle



$\Rightarrow \quad FP \perp$ tangent at Q

$\frac{2a - 0}{\frac{a}{e} - ae} = \frac{a\sin\theta}{b\cos\theta} = \frac{a}{b}\tan\theta$

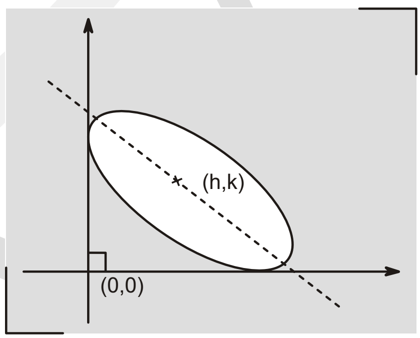
$\Rightarrow \quad \tan\theta = \frac{b}{a}\frac{2e}{1-e^2}$

$= \frac{a\left(\sqrt{1-e^2}\right)2e}{a(1-e^2)}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{2e}{\sqrt{1-e^2}}\right)$

Comprehension: (2)

1. (c) Origin lies on director circle of ellipse



$\Rightarrow \quad (0-h)^2 + (0-k)^2 = a^2 + b^2$

$= 2^2 + 1^2 = 5$

$$\Rightarrow \quad x^2 + y^2 = 5$$

2. (d) $x_1 x_2 = b^2 = 1$

$$y_1 y_2 = b^2 = 1$$

$$F_1 \equiv (x_1, y_1),\ F_2 \equiv \left(\frac{1}{x_1}, \frac{1}{y_1}\right)$$

$$\Rightarrow F_1 \equiv (h, k),\ F_2 = \left(\frac{1}{h}, \frac{1}{k}\right)$$

$$\left(h - \frac{1}{h}\right)^2 + \left(k - \frac{1}{k}\right)^2 = (2ae)^2$$

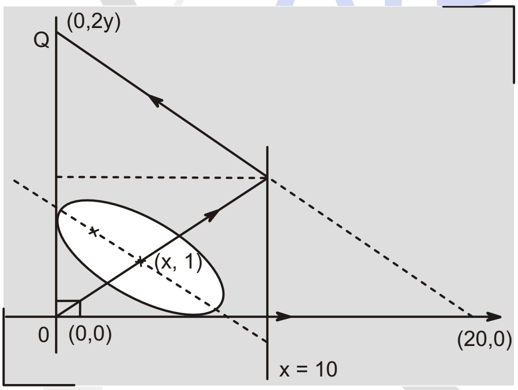
$$= (2\sqrt{3})^2 = 12$$

$$(x^2 + y^2)\left(1 + \frac{1}{x^2 y^2}\right) = 16$$

3. (b) $x^2 + 1^2 = 5 \Rightarrow x = 2$

$$\frac{y}{10} = \frac{1}{x} = \frac{1}{2} \Rightarrow y = 5$$

$$Q \equiv (0, 10)$$



Comprehension: (3)

1. (a), 2. (d), 3. (b)

$PA + PB < 2$ and $PB + PC < 2$

$PA + PB < 2$

Region inside ellipse with foci A and B

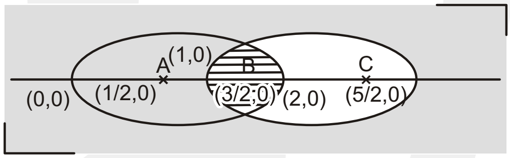
$$2a = 2,\ a = 1$$

$$2ae = \frac{3}{2} - \frac{1}{2} = 1$$

$$b^2 = a^2 - a^2e^2 = 1 - \frac{1}{4} = \frac{3}{4}$$

Equation of ellipse is

$$\frac{(x-1)^2}{1} + \frac{y^2}{3/4} = 1$$



$PB + PC < 2 \Rightarrow P$ lies inside ellipse with foci B and C

Whose equation is $\dfrac{(x-2)^2}{1} + \dfrac{y^2}{3/4} = 1$

Locus of P is shown by shaded region which is symmetric about x-axis.

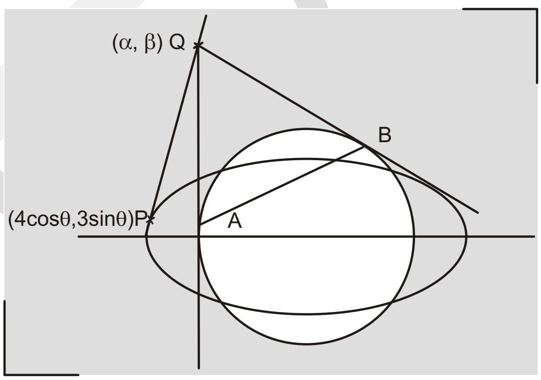
Area of region

$$= 4\int_1^{3/2} \frac{\sqrt{3}}{2}\sqrt{1-(x-2)^2}\,dx$$

$$= \sqrt{3}\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)$$

Comprehension: (4)

1. (a) Equation of chord of contact AB is



$$\alpha x + \beta y = 12 \qquad \ldots(1)$$

Equation of tangent to ellipse at P is

$$\frac{x\cos\theta}{4}+\frac{y\sin\theta}{3}=1$$

$\because$ (α,β) lies on it

$\Rightarrow$ $3\alpha\cos\theta+4\beta\sin\theta=12$ ...(2)

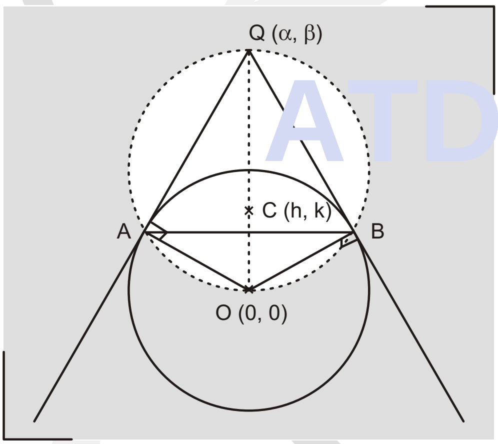
From eqn. (1) and (2)

lines $\alpha x+\beta y=12$ are concurrent at points $(3\cos\theta, 4\sin\theta)$

$\therefore$ Locus of points of concurrency is

$$\frac{x^2}{9}+\frac{y^2}{16}=1$$

2. (b) Equation of tangent to ellipse at $P\left(\frac{4}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$ is,



$$\frac{x}{4\sqrt{2}}+\frac{y}{3\sqrt{2}}=1$$

$\because$ (α,β) lies on it

$\Rightarrow$ $3\alpha+4\beta=12\sqrt{2}$ ...(1)

From figure it is clear that OQ is diameter of circumcircle of ΔQAB

$\Rightarrow$ $2h=\alpha,\quad 2k=\beta$

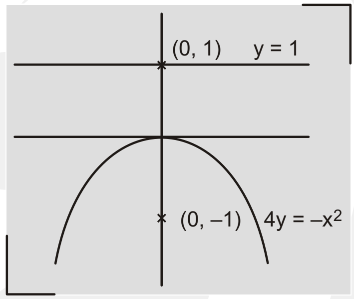
$\therefore$ From (1) we get

$3(2h)+4(2k)=12\sqrt{2}$

$\therefore$ The required. locus is $3x+4y=6\sqrt{2}$

3. (a) $\perp$ar tangents are drawn to $y=-\frac{x^2}{4}$

It means points lie on directrix of parabola *i.e.*, $y=1$



$\therefore$ Points required are the points of intersection of lines

$\frac{x}{4}+\frac{y}{3}=\frac{1}{\sqrt{2}}$ and $y=1$

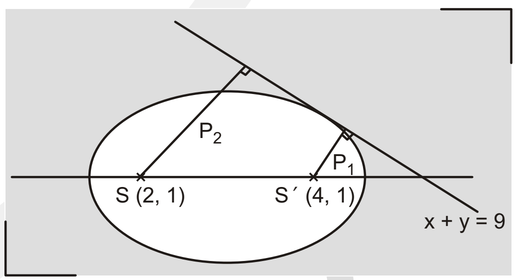
$\therefore$ No. of points required = 1.

Comprehension: (5)

1.(b), 2 (c)

$2ae=2 \quad \Rightarrow ae=1$ ...(1)

$P_1P_2=b^2=\frac{4}{\sqrt{2}}\frac{6}{\sqrt{2}}=12$



$b^2=12$ ...(2)

$b^2=a^2-a^2e^2=a^2-1$

$\Rightarrow$ $12=a^2-1 \Rightarrow a=\sqrt{13}$

$\therefore$ Length of major axis $=2a=2\sqrt{13}=\sqrt{52}$

$ae=1$ (from ...(1)

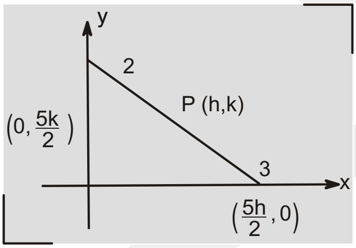
$\Rightarrow$ $e=\frac{1}{\sqrt{13}}$

SOLUTIONS 4

Match the Columns:

1. $a \to s$; $b \to p$; $c \to q$; $d \to r$

(a) $\left(\dfrac{5h}{2}\right)^2 + \left(\dfrac{5k}{3}\right)^2 = (10)^2$



$\Rightarrow \quad \dfrac{x^2}{(4)^2} + \dfrac{y^2}{(6)^2} = 1$

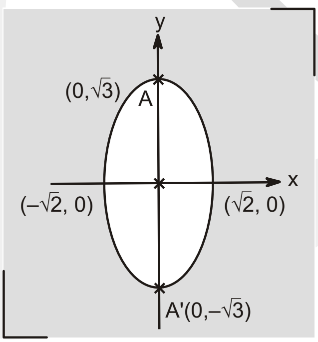
$\Rightarrow \quad 16 = 36(1 - e^2)$

$e^2 = \dfrac{5}{9} \Rightarrow 9e = 3\sqrt{5}$

(b) $\dfrac{(x+1)^2}{2} + \dfrac{(y-1)^2}{3} = 1$

$x + 1 = x,\ y - 1 = y$

$(\Delta APA')_{max} = \dfrac{1}{2} \times \sqrt{2} \times 2\sqrt{3} = \sqrt{6}$



(c) $\dfrac{(x-1)^2}{16} + \dfrac{(y-2)^2}{9} = 1$

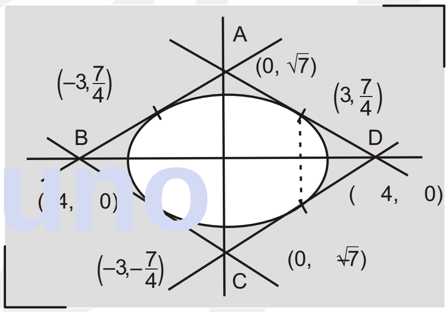
$a^2e^2 = a^2 - b^2 = 16 - 9 = 7$

$\Rightarrow \quad 2ae = 2\sqrt{7}$

(d) $\dfrac{x^2}{16} + \dfrac{y^2}{7} = 1$

$16e^2 = 16 - 7 = 9$

$e = \dfrac{3}{4}$



Extremities of $LR \equiv \left(\pm 3, \pm \dfrac{7}{4}\right)$

Equation of tangent to ellipse at point $\left(3, \dfrac{7}{4}\right)$

$\dfrac{3x}{16} + \dfrac{y}{4} = 1$

$\Rightarrow \quad 3x + 4y = 16$

$\Rightarrow \quad A \equiv (0, 4), B \equiv \left(-\dfrac{16}{3}, 0\right),$

$C \equiv (0, -4),\ D \equiv \left(\dfrac{16}{3}, 0\right)$

Area of rhombus

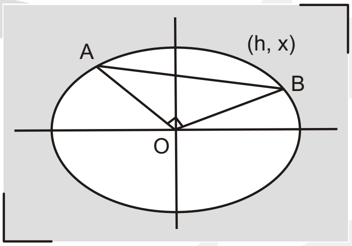
$ABCD = \dfrac{1}{2} \times \dfrac{32}{3} \times 8 = \dfrac{128}{3}$

2. $\mathbf{a \to r\,;\ b \to s\,;\ c \to p;\ d \to q}$

Let (h, k) be the midpoint of chord, then equation of chord is

$$\frac{xh}{a^2} + \frac{yk}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2}$$

(a) Homogeniring we get equation of pair *OA* and *OB*



$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)\left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)^2 = \left(\frac{xh}{a^2} + \frac{yk}{b^2}\right)^2$$

Pair of lines OA and OB are perpendicular

$\Rightarrow$ Coefficient of x^2 + Coefficient of $y^2 = 0$

$$\Rightarrow \left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)^2 \left(\frac{1}{a^2} + \frac{1}{b^2}\right) - \left(\frac{h^2}{a^4} + \frac{k^2}{b^4}\right) = 0$$

Locus is

$$\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2 \left(\frac{1}{a^2} + \frac{1}{b^2}\right) = \frac{x^2}{a^4} + \frac{y^2}{b^4}$$

(b) Put $(h + r\cos\theta,\ k + r\sin\theta)$ to equation of ellipse

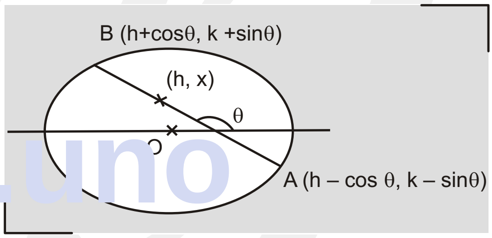
$$\frac{h^2 + r^2\cos^2\theta + (2h\cos\theta)r}{a^2} + \frac{k^2 + r^2\sin^2\theta + (2k\sin\theta)r}{b^2} = 1$$

$$\Rightarrow r^2\left(\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}\right) + \left(\frac{2h\cos\theta}{a^2} + \frac{2k\sin\theta}{b^2}\right)r + \left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right) = 0 \begin{matrix} l \\ -l \end{matrix}$$

$$\Rightarrow \frac{2h\cos\theta}{a^2} + \frac{2k\sin\theta}{b^2} = 0$$

$$\Rightarrow \tan\theta = -\frac{b^2 h}{a^2 k}$$

and
$$\frac{\left(\frac{h^2}{a^2} + \frac{k^2}{b^2} - 1\right)}{\frac{\cos^2\theta}{a^2} + \frac{\sin^2\theta}{b^2}} = 1(-1)$$



$$\left(\frac{1}{a^2} + \frac{1}{b^2}\left(\frac{b^4x^2}{a^4y^2}\right)\right) = \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right)\left(1 + \frac{b^4x^2}{a^4y^2}\right)$$

is the required locus.

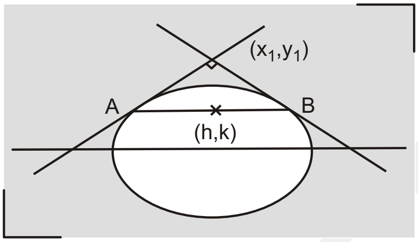
(c) Equation of *AB* is

$$\frac{xh}{a^2} + \frac{yk}{b^2} = \frac{h^2}{a^2} + \frac{k^2}{b^2} \qquad ...(1)$$

Also *AB* is chord of contact of (x_1, y_1) w.r.t. ellipse

$\Rightarrow$ Equation of *AB* is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1 \qquad ...(2)$$



$\therefore$ (1) and (2) are identical

$$\Rightarrow \quad \frac{x_1}{h} = \frac{y_1}{k} = \frac{1}{\frac{h^2}{a^2} + \frac{k^2}{b^2}}$$

(x_1, y_1) lies on director circle $x^2 + y^2 = a^2 + b^2$ of ellipse

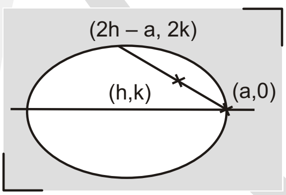
$$\Rightarrow \quad \frac{h^2 + k^2}{\left(\frac{h^2}{a^2} + \frac{k^2}{b^2}\right)} = a^2 + b^2$$

$$\Rightarrow x^2 + y^2 = (a^2 + b^2)\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2$$ is

the required locus.

(d) Put $(2h - a, 2k)$ to equation of ellipse

$$\Rightarrow \quad \frac{(2h-a)^2}{a^2} + \frac{(2k)^2}{b^2} = 1$$



$\dfrac{\left(x - \frac{a}{2}\right)^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{1}{4}$ is the required locus.

3. a → q ; b → r ; c → p, q, s

(a) $9 = 16(1 - e^2)$

$4e \pm \sqrt{7}$

$r = \sqrt{9 + 7} = 4$

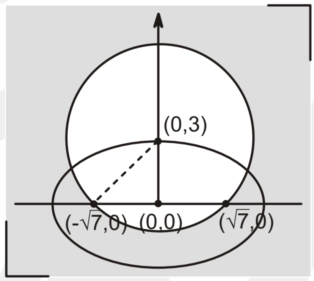
(b) $2a = 3(2b)$

$\Rightarrow \quad a^2 = 9a^2(1 - e^2)$

$\Rightarrow \quad e^2 = \dfrac{8}{9} \Rightarrow e = \dfrac{2\sqrt{2}}{3}$

(c) Let four points lie on circle $x^2 + y^2 + 2gx + 2fy + c = 0$

Put $(a\cos\theta, b\sin\theta)$ to equation of circle



$$a^2\cos^2\theta + b^2\sin^2\theta + 2ga\cos\theta + 2fb\sin\theta + c = 0$$

Let $t = \tan\dfrac{\theta}{2}$, then

$$a^2\left(\frac{1-t^2}{1+t^2}\right)^2 + b^2\frac{4t^2}{(1+t^2)^2} + 2ga\frac{(1-t^2)}{(1+t^2)} + \frac{4fbt}{1+t^2} + c = 0$$

$$a^2(1 + t^4 - 2t^2) + 4b^2t^2 + 2ga(1 - t^4) + 4fb(t + t^3) + c(1 + t^4 + 2t^2) = 0$$

$$(a + c - 2ga)t^4 + 4bft^3 + (4b^2 + 2c - 2a)t^2 + 4fbt + (a^2 + 2ga + c) = 0$$

$$\tan\left(\frac{\theta_1 + \theta_2 + \theta_3 + \theta_4}{2}\right) = \frac{s_1 - s_3}{1 - s_2 + s_4} = 0$$

$\Rightarrow \quad \theta_1 + \theta_2 + \theta_3 + \theta_4 = 2n\pi, n \in I$

4. a → p ; b → q ; c → p ; d → r

(a)

$$P(3\cos\theta,\ 2\sin\theta) \equiv \left(\frac{3(1-\tan^2\theta)}{\left(1+\tan^2\frac{\theta}{2}\right)}, \frac{4\tan\frac{\theta}{2}}{\left(1+\tan^2\frac{\theta}{2}\right)} \right)$$

$\tan\frac{\theta}{2}$ can take infinite rational values

(b) $P(\pm 3, 0), (0, \pm 2)$

(c) $P(\sqrt{3}\cos\theta,\ \sin\theta)$

$$\frac{\tan\theta}{\sqrt{3}} = k \in Q$$

$$\Rightarrow \quad P\left(\frac{\sqrt{3}}{\sqrt{1+3k^2}}, \frac{\sqrt{3}\,k}{\sqrt{1+3k^2}} \right)$$

If $\quad \dfrac{\sqrt{1+3k^2}}{\sqrt{3}} = \lambda \qquad \lambda \in Q$

$$\lambda^2 - k^2 = \frac{1}{3}$$

$$(\lambda - k)(\lambda + k) = \frac{1}{3}$$

Which can be expressed as product of two rationals in infinite ways

say for example $\lambda - k = \frac{1}{2}$;

$$\lambda + k = \frac{2}{3}$$

$$\Rightarrow \quad \lambda = \frac{7}{12},\ k = \frac{1}{12}$$

(d) $\quad P \equiv (0, \pm 1)$

SOLUTIONS 5

Subjective Problems

1. (24) $Q \equiv (-4\cos\phi, 3\sin\phi)$,

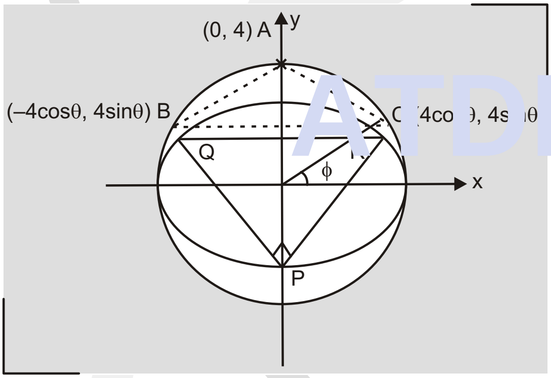
$R \equiv (+4\cos\phi, 3\sin\phi)\,(0,4)$

$PQ \perp PR$

$$\Rightarrow \frac{3(\sin\phi+1)}{4\cos\phi}\,\frac{3(\sin\phi+1)}{-4\cos\phi} = -1$$

$$\Rightarrow 25\sin^2\phi + 18\sin\phi - 7 = 0$$

$$\Rightarrow \sin\phi = \frac{7}{25} \quad (\sin\phi = -1 \text{ rejected})$$



$$\tan\frac{\phi}{2} = \sqrt{\left(1-\frac{24}{25}\right)\Big/\left(1+\frac{24}{25}\right)} = \frac{1}{7}$$

$$\Rightarrow \theta = \angle ABC = \frac{1}{2}\left(\frac{\pi}{2}-\phi\right)$$

$$\tan\theta = \frac{1-\tan\frac{\phi}{2}}{1+\tan\frac{\phi}{2}} = \frac{1-\frac{1}{7}}{1+\frac{1}{7}} = \frac{3}{4}$$

$$\Rightarrow 16\tan^2\theta + 8\tan\theta + 9$$

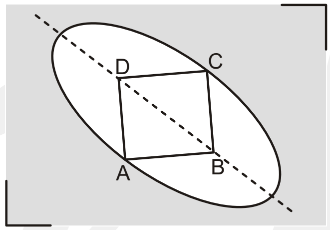
$$= 16\times\frac{9}{16} + 8\times\frac{3}{4} + 9 = 24$$

2. (8) $(AB)(AD) = 200$

$AB + AD = 2a$

$BD^2 = AB^2 + AD^2 = (2ae)^2$

$(AB+AD)^2 = AB^2 + AD^2 + 2(AB)(AD)$



$$\Rightarrow 4a^2 = 4a^2e^2 + 400$$

$$\Rightarrow b = 10$$

$$\Rightarrow a^2(1-e^2) = b^2 = 100$$

$$\Rightarrow a = 20$$

$$P = 2(AB+AD) = 4a = 80$$

$$\frac{P}{10} = 8$$

3. (6) Let $X = x - \sqrt{2}$, $Y = y - \frac{1}{\sqrt{2}}$

$\Rightarrow$ Equation of ellipse becomes

$$\frac{x^2}{4} + y^2 = 1$$

$$\Rightarrow \frac{X^2}{4} + Y^2 + \frac{X}{\sqrt{2}} + \sqrt{2}\,Y = 0$$

$$\frac{(X+\sqrt{2})^2}{4} + \left(Y + \frac{1}{\sqrt{2}}\right)^2 = 1$$

$$\Rightarrow \frac{X^2}{4} + Y^2 + \frac{1}{\sqrt{2}}X + \sqrt{2}\,Y = 0$$

Let equation of AB be $aX + bY = 1$

Homogeniring, we get

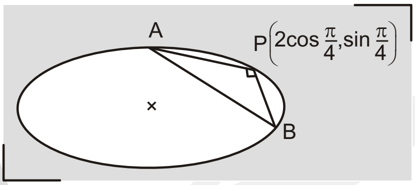
$$\frac{X^2}{4}+Y^2+\left(\frac{X}{\sqrt{2}}+\sqrt{2}\,Y\right)(aX+bY)=0$$

$\because$ $AP \perp AB \Rightarrow$ coeff. of X^2 + coeff. of $Y^2=0$

$$\Rightarrow \quad \frac{a}{\sqrt{2}}+\sqrt{2}b+\frac{5}{4}=0$$

$$\Rightarrow \quad -\frac{4}{5\sqrt{2}}a-\frac{4\sqrt{2}}{5}b=0$$

$\Rightarrow$ AB passes through fixed point



$\Rightarrow$

$$\left(-\frac{4}{5\sqrt{2}},\frac{-4\sqrt{2}}{5}\right)\equiv\left(-\frac{4}{5\sqrt{2}}+\sqrt{2},\frac{-4\sqrt{2}}{5}+\frac{1}{\sqrt{2}}\right)$$

$$=\left(\frac{6}{5\sqrt{2}},\frac{-3}{5\sqrt{2}}\right)$$

$$\Rightarrow \quad a=\frac{6}{5\sqrt{2}}, \qquad b=\frac{-3}{5\sqrt{2}}$$

$$a^2+b^2=\frac{36+9}{50}=\frac{9}{10}=\frac{m}{n}$$

$$\therefore \quad \left[\frac{m+n}{3}\right]=\left[\frac{19}{3}\right]=6$$

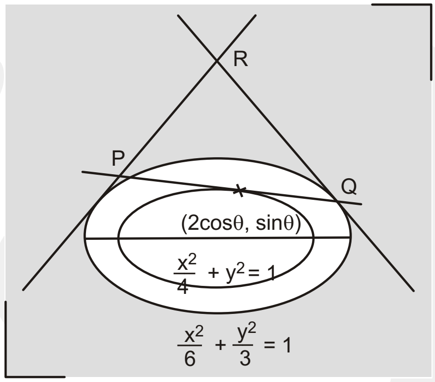
4. (2) Equation of tangent to $\frac{x^2}{4}+y^2=1$ at point $(2\cos\theta,\sin\theta)$ is

$$\frac{x\cos\theta}{2}+y\sin\theta=1 \qquad \text{...(1)}$$

Equation of chord of contact of $R(h,k)$

w.r.t. $\frac{x^2}{6}+\frac{y^2}{3}=1$ is

$$\frac{hx}{6}+\frac{ky}{3}=1 \qquad \text{...(2)}$$



$\because$ Eq. (1) and (2) are identical

$$\Rightarrow \quad \frac{h/3}{\cos\theta}=\frac{k/3}{\sin\theta}=1$$

$$\Rightarrow \quad h^2+k^2=9=6+3$$

$\Rightarrow$ (h,k) lies on director circle of ellipse

$$\frac{x^2}{6}+\frac{y^2}{3}=1$$

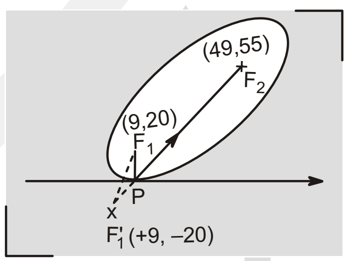
= [illegible] = [illegible]

5. (7) $2a=PF_1+PF_2=PF_1'+PF_2$

$$=F_1'F_2=\sqrt{40^2+(75)^2}=85$$

$$\Rightarrow \quad L=85$$

$$\therefore \quad \left[\frac{L}{11}\right]=7$$



6. (2) Equation of tangent at A is

$ty=x+t^2$

$$\because \quad y=\frac{1}{t}x+t$$

$\because$ It is tangent to ellipse also

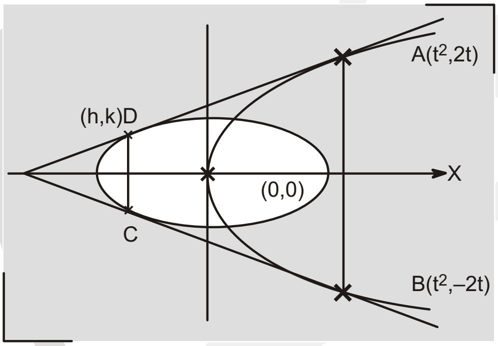
$$\Rightarrow \quad t^2 = \frac{16}{t^2} + 6$$

$$\Rightarrow \quad t^4 - 6t^2 - 16 = (t^2 - 8)(t^2 + 2) = 0$$

$$\Rightarrow \quad t = \pm 2\sqrt{2}$$

$\therefore \quad A, B = (8, 4\sqrt{2})$ and $(8, -4\sqrt{2})$

Let $D \equiv (h, k)$



Tangent to

$y^2 = 4x$ at A is $2\sqrt{2}y - x = 8$ ...(1)

Tangent to ellipse at B is

$3xh + 8ky = 48$ ...(2)

$\because$ (1) and (2) are identical

$$\Rightarrow \quad \frac{8k}{2\sqrt{2}} = -\frac{3h}{1} = \frac{48}{8} = 6$$

$$\Rightarrow \quad h = -2, \quad k = \frac{3}{\sqrt{2}}$$

$$\therefore \quad D, C \equiv \left(-2, \frac{3}{\sqrt{2}}\right) \text{ and } \left(-2, -\frac{3}{\sqrt{2}}\right)$$

Area of trapezium

$$ABCD = \frac{1}{2}(8\sqrt{2} + 3\sqrt{2})10$$

$$= 55\sqrt{2} \quad \Rightarrow \quad N = 2$$

7. (6) Image of (h, k) on ellipse about $x - y - 2 = 0$ is say (h', k')

$$\therefore \quad \frac{h' - h}{1} = \frac{k' - k}{-1}$$

$$= -2\frac{h - k - 2}{1 + 1} = -h + k + 2$$

$$\Rightarrow \quad h' = k + 2, k' = h - 2$$

$$(h, k) \equiv (k' + 2, h' - 2)$$

$$\because \quad \frac{(h-4)^2}{16} + \frac{(k-3)^2}{9} = 1$$

$$\Rightarrow \quad \frac{(k'+2-4)^2}{16} + \frac{(h'-2-3)^2}{9} = 1$$

$$\Rightarrow \quad 16h'^2 + 9k'^2 - 36k' - 160h' + 292 = 0$$

$\Rightarrow$ Equation of reflection of ellipse is

$$16x^2 + 9y^2 - 160x - 36y + 292 = 0$$

$$\therefore \quad \frac{k_1 + k_2}{22} = \frac{292 - 160}{22} = 6$$

8. (4) Equation of tangent of ellipse at point $(a\cos\theta, b\sin\theta)$ is $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

It passes through $(-2, 0)$

$$\Rightarrow \quad -\frac{2}{a}\cos\theta = 1$$

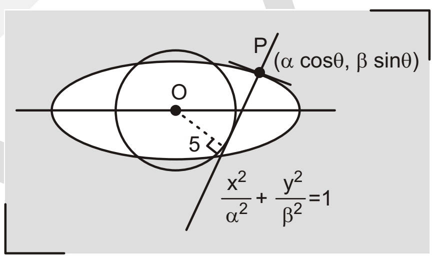
Also its slope $-\frac{b}{a}\cot\theta = 2$

$$\therefore \quad \sec\theta = -\frac{2}{a}, \tan\theta = -\frac{b}{2a}$$

$$\therefore \quad \frac{4}{a^2} = 1 + \frac{b^2}{4a^2}$$

$$\Rightarrow \quad 16 = 4a^2 + b^2 \geq 2(2ab) \quad \Rightarrow \quad ab \leq 4$$

9. (2) Equation of normal at P is $\alpha x \sec\theta - \beta y \operatorname{cosec}\theta = \alpha^2 - \beta^2$



$$\therefore \quad r = \frac{|\alpha^2 - \beta^2|}{\sqrt{\alpha^2\sec^2\theta + \beta^2\operatorname{cosec}^2\theta}}$$

$$= \frac{|\alpha^2 - \beta^2|}{\sqrt{\alpha^2 + \beta^2 + \alpha^2 \tan^2\theta + \beta^2 \cot^2\theta}}$$

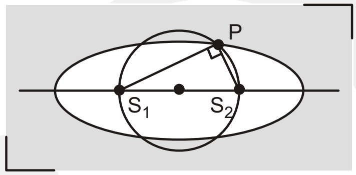
$$\le \frac{|\alpha^2 - \beta^2|}{\sqrt{\alpha^2 + \beta^2 + 2\alpha\beta}}$$

$$r_{max} = |\alpha - \beta| = a^2 + 2a + 2 - (a^2 + 1)$$

$$= 2a + 1$$

$\therefore \quad 5 = 2a + 1 \Rightarrow a = 2$

10. (1) area of $\Delta PS_1S_2 = \frac{1}{2}(PS_1)(PS_2) = 30$

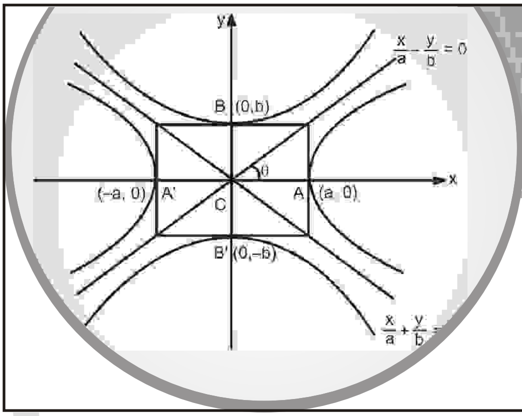


$\Rightarrow \quad (PS_1)(PS_2) = 60$

$PS_1 + S_2P = 2a = 17$

$(S_1S_2) = PS_1^2 + PS_2^2 = (17)^2 - 2(60) = 169$

$\therefore \quad \frac{S_1S_2}{13} = 1$



5

HYPERBOLA

KEY CONCEPTS

The Hyperbola is conic whose eccentricity is greater than unity. (e > 1).

1. STANDARD EQUATION AND DEFINITIONS

Standard equation of the hyperbola is

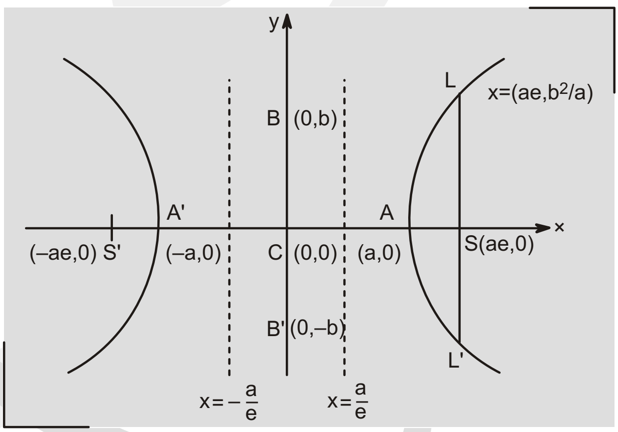
$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1.$$

Where $b^2 = a^2(e^2-1)$

or $a^2e^2 = a^2 + b^2,$

i.e., $e^2 = 1+\frac{b^2}{a^2}$

$$=1+\left(\frac{\text{C.A.}}{\text{T.A.}}\right)^2$$



Foci:

$$S \equiv (ae, 0)$$

and $S' \equiv (-ae, 0).$

Equations of Directrices:

$$x=\frac{a}{e} \quad \text{and} \quad x=-\frac{a}{e}.$$

Vertices:

$$A \equiv (a,0) \text{ and } A' \equiv (-a,0).$$

I (Latus rectum) $= \frac{2b^2}{a} = \frac{(\text{C.A.})^2}{\text{T.A.}} = 2a(e^2-1).$

Note: *$I(L.R.) = 2e$ (distance from focus to the corresponding directrix)*

TRANSVERSE AXIS: *The line segment A'A of length 2a in which the foci S' and S both lie is called the* ***T.A. of the hyperbola.***

CONJUGATE AXIS: *The line segment B'B between the two points $B' \equiv (0, -b)$ and $B \equiv (0, b)$ is called as the* ***C.A. of the hyperbola.***

The T.A. and the C.A. of the hyperbola are together called the Principal axes of the hyperbola.

2. FOCAL PROPERTY

The difference of the focal distances of any point on the hyperbola is constant and equal to transverse axis *i.e.*, $||PS| - |PS'|| = 2a$. The distance SS' = focal length.

3. CONJUGATE HYPERBOLA

Two hyperbolas such that transverse and conjugate axes of one hyperbola are respectively the conjugate and the transverse axes of the other are called **CONJUGATE HYPERBOLAS** of each other.

$$e.g.\ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

and $-\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are conjugate hyperbolas of each.

Note: ***That:-***

(a) *If e_1 and e_2 are the eccentricities of the hyperbola and its conjugate then $e_1^{-2} + e_2^{-2} = 1$.*

(b) *The foci of a hyperbola and its conjugate are concyclic and form the vertices of a square.*

(c) *Two hyperbolas are said to be similar if they have the same eccentricity.*

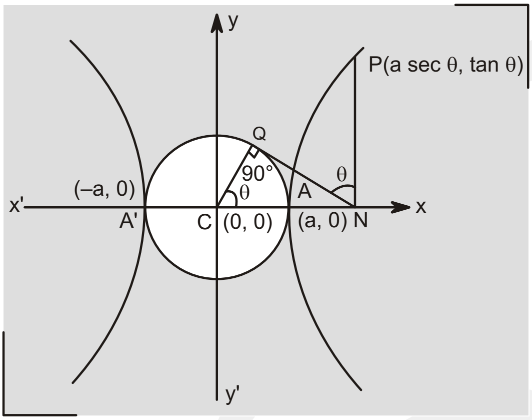
4. RECTANGULAR OR EQUILATERAL HYPERBOLA

The particular kind of hyperbola in which the lengths of the transverse and conjugate axis are equal is called an **EQUILATERAL HYPERBOLA.** Note that the eccentricity of the rectangular hyperbola is $\sqrt{2}$ and the length of its latus rectum is equal to its transverse or conjugate axis.

5. AUXILIARY CIRCLE

A circle drawn with centre C and T. A. as a diameter is called the **AUXILIARY CIRCLE** of the hyperbola. Equation of the auxiliary circle is $x^2 + y^2 = a^2$.

Note from the figure that P and Q are called the **"CORRESPONDING POINTS"** of the hyperbola and the auxiliary circle. 'θ' is called the eccentric angle of the point 'P' on the hyperbola. $(0 \le \theta < 2\pi)$.



Note: *The equations $x = a\sec\theta$ and $y = b\tan\theta$ together represents the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.*

where θ is a parameter. The parametric equations : $x = a\cos h\,\phi$, $y = b\sin h\,\phi$ also represents the same hyperbola.

General Note:

Since the fundamental equation to the hyperbola only differs from that to the ellipse in having $-b^2$ instead of b^2 it will be found that many propositions for the hyperbola are derived from those for the ellipse by simply changing the sign of b^2.

6. POSITION OF A POINT 'P' w.r.t. A HYPERBOLA

The quantity $\frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} = 1$ is positive, zero or negative according as the point (x_1, y_1) lies within, upon or without the curve.

7. LINE AND A HYPERBOLA

The straight line $y = mx + c$ is a secant, a tangent or passes outside the hyperbola $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ according as : $c^2 > = < a^2m^2 - b^2$.

8. TANGENTS AND NORMALS

Tangents:

(a) Equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point (x_1, y_1) is $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$.

Note: *In general two tangents can be drawn from an external point $(x_1\ y_1)$ to the hyperbola and they are $y - y_1 = m_1(x - x_1)$ and $y - y_1 = m_2(x - x_2)$, where m_1 and m_2 are roots of the equation $(x_1^2 - a^2)m^2 - 2x_1y_1m + y_1^2 + b^2 = 0$. If $D < 0$, then no tangent can be drawn from $(x_1,\ y_1)$ to the hyperbola.*

(b) Equation of the tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $(a\sec\theta,\ b\tan\theta)$ is $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$.

Note: *Point of intersection of the tangents at* θ_1 *and* θ_2 *is* $x=a\frac{\cos\frac{\theta_1-\theta_2}{2}}{\cos\frac{\theta_1+\theta_2}{2}},\ y=b\frac{\sin\frac{\theta_1+\theta_2}{2}}{\cos\frac{\theta_1+\theta_2}{2}}$.

(c) $y=mx\pm\sqrt{a^2m^2-b^2}$ can be taken as the tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

Note that there are two parallel tangents having the same slope m.

(d) Equation of a chord joining α and β

$$\frac{x}{a}\cos\frac{\alpha-\beta}{2}-\frac{y}{b}\sin\frac{\alpha+\beta}{2}=\cos\frac{\alpha+\beta}{2}$$

NORMALS:

(a) The equation of the normal to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ at the point $P(x_1, y_1)$ on it is $\frac{a^2x}{x_1}+\frac{b^2y}{y_1}=a^2+b^2=a^2e^2$.

(b) The equation of the normal at the point $P\,(a\sec\theta,\ b\tan\theta)$ on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{ax}{\sec\theta}+\frac{by}{\tan\theta}=a^2+b^2=a^2e^2$.

(c) Equation to the chord of contact, polar, chord with a given middle point, pair of tangents from an external point is to be interpreted as in ellipse.

9. DIRECTOR CIRCLE

The locus of the intersection of tangents which are at right angles is known as the **DIRECTOR CIRCLE** of the hyperbola. The equation to the director circle is :

$$x^2+y^2=a^2-b^2.$$

If $b^2<a^2$ this circle is real; if $b^2=a^2$ the radius of the circle is zero and it reduces to a point circle at the origin.

In this case the centre is the only point from which the tangents at right angles can be drawn to the curve.

If $b^2>a^2$, the radius of the circle is imaginary, so that there is no such circle and so no tangents at right angle can be drawn to the curve.

10. PAIR OF TANGENTS

The equation to the pair of tangents which can be drawn from any point (x_1, y_1) to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is given by: $SS_1 = T^2$ where:

$$S \equiv \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1; \quad S_1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1; \quad T \equiv \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$$

11. CHORD OF CONTACT

Equation to the chord of contact of tangents drawn from a point $P(x_1, y_1)$ to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

$$T = 0, \text{ where } T = \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$$

12. CHORD WITH A GIVEN MIDDLE POINT

Equation of the chord of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ whose middle point is (x_1, y_1) is $T = S_1$, where $S_1 = \frac{x_1^2}{a^2} - \frac{y_1^2}{b^2} - 1$; $T = \frac{xx_1}{a^2} - \frac{yy_1}{b^2} - 1$

13. DIAMETER

The locus of the middle of a system of parallel chords with slope 'm' of hyperbola is called its diameter. It is a straight line passing through the centre of the hyperbola and has the equation $y = -\frac{b^2}{a^2 m}x$.

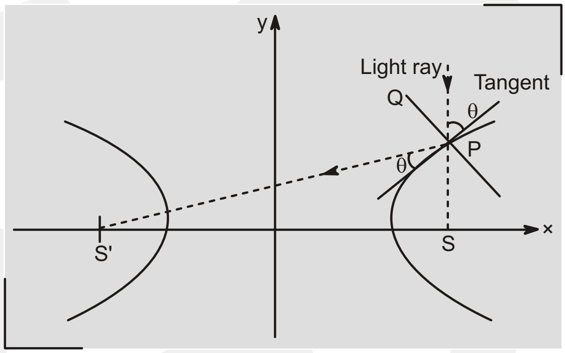
Note: *All diameters of the hyperbola pass through its centre.*

14. HIGHLIGHTS ON TANGENT AND NORMAL

H-1 Locus of the feet of the perpendicular drawn from focus of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ upon any tangent is its auxiliary circle *i.e.*, $x^2 + y^2 = a^2$ and the product of the feet of these perpendiculars is b^2. (semi C.A.)^2

H-2 The portion of the tangent between the point of contact and the directrix subtends a right angle at the corresponding focus.

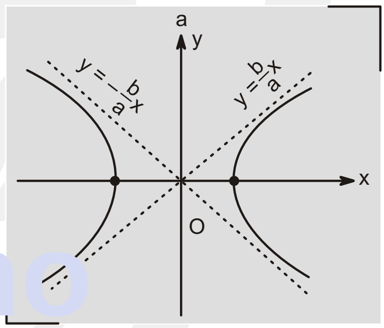
H-3 The tangent and normal at any point of a hyperbola bisect the angle between the focal radii. This spells the reflection property of the hyperbola as **"An incoming light ray"** aimed towards one focus is reflected from the outer surface of the hyperbola towards the other focus. It follows that if an ellipse and a hyperbola have the same foci, they cut at right angles at any of their common point. Note that the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{a^2-k^2}-\frac{y^2}{k^2-b^2}=1(a>k>b>0)\,X$ are confocal and therefore orthogonal.



H-4 The foci of the hyperbola and the points P and Q in which any tangent meets the tangents at the vertices are concyclic with PQ as diameter of the circle.

15. ASYMPTOTES

Definition: If the length of the perpendicular let fall from a point on a hyperbola to a straight line tends to zero as the point on the hyperbola moves to infinity along the hyperbola, then the straight line is called **asymptote** of the hyperbola.



Equations of asymptote:

$$\frac{x}{a}+\frac{y}{b}=0 \text{ and } \frac{x}{a}-\frac{y}{b}=0$$

Pair of asymptotes: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

Note: *(i) A hyperbola and its conjugate have the same asymptote.*

(ii) The equation of the pair of asymptotes differs from the equations of hyperbola. (or conjugate hyperbola) by the constant term only.

(iii) The asymptotes pass through the centre of the hyperbola and are equally inclined to the transverse axis of the hyperbola. Hence the bisectors of the angles between the asymptotes are the principle axes of the hyperbola.

(iv) The asymptotes of a hyperbola are the diagonals of the rectangle formed by the lines drawn through the extremities of each axis parallel to the other axis.

(v) A simple method to find the co-ordinates of the centre of the hyperbola expressed as a general equation of degree 2 should be remembered as:

Let $f(x, y)=0$ represents a hyperbola.

Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. Then the point of intersection of $\frac{\partial f}{\partial x}=0$ and $\frac{\partial f}{\partial y}=0$ gives the centre of the hyperbola.

Remark:

(i) *No tangent to the hyperbola can be drawn from its centre.*

(ii) *Only one tangent to the hyperbola can be drawn from a point lies on its asymptotes other than centre*

(iii) *Two tangents can be drawn to the hyperbola from any of its external points which does not lie at its asymptotes.*

16. HIGHLIGHTS ON ASYMPTOTES

H-1 If from any point on the asymptote a straight line be drawn perpendicular to the transverse axis, the product of the segments of this line, intercepted between the point and the curve is always equal to the square of the semi conjugate axis.

H-2 Perpendicular from the foci on either asymptote meet if in the same points as the corresponding directrix and the common points of intersection lie on the auxiliary circle.

H-3 The tangent at any point P on a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ with centre C, meets the asymptotes in Q and R and cuts off a $\Delta\ CQR$ of constant area equal to ab from the asymptotes and the portion of the tangent intercepted between the asymptote is bisected at the point of contact. This implies that locus of the center of the circle circumscribing the $\Delta\ CQR$ in case of a rectangular hyperbola is the hyperbola itself and for a standard hyperbola the locus would be the curve, $4(a^2x^2-b^2y^2)=(a^2+b^2)^2$.

H-4 If the angle between the asymptotes of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is 2θ then $e=\sec\theta$.

17. RECTANGULAR HYPERBOLA

Rectangular hyperbola referred to its asymptotes as axis of coordinates.

(a) Equation is $xy=c^2$ with parametric representation $x=ct$, $y=c/t$, $t\in R-\{0\}$.

(b) Equation of a chord joining the points $P(t_1)$ *and* $Q(t_2)$ is $x+t_1t_2y=c(t_1+t_2)$ with slope $m=-\frac{1}{t_1t_2}$.

(c) Equation of the tangent at $P(x_1, y_1)$ is $\frac{x}{x_1}+\frac{y}{y_1}=2$ and at $P(t)$ is $\frac{x}{t}+ty=2c$.

(d) Equation of normal : $y-\frac{c}{t}=t^2(x-ct)$.

(e) Chord with a given middle point as (h,k) is $kx+hy=2hk$.

EXERCISE 1

Only One Choice is Correct:

1. If $f(x) = x^3 + \alpha x^2 + \beta x + \gamma$, where α, β, γ are rational numbers and two roots of $f(x) = 0$ are eccentricities of a parabola and a rectangular hyperbola, then $\alpha + \beta + \gamma$ is equal to :

(a) -1 (b) 0

(c) 1 (d) 2

2. The number of points on rectangular hyperbola $xy = c^2$ from which two tangents drawn to ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $0 < b < a < c$ are $\perp$ar to each other is :

(a) 0 (b) 1

(c) 2 (d) Infinite

3. If the tangent at the point $P(h, k)$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ cuts the circle $x^2 + y^2 = a^2$ at the points $Q(x_1, y_1)$ and $R(x_2, y_2)$ then $\frac{1}{y_1} + \frac{1}{y_2}$ is equal to:

(a) $\frac{2}{k}$ (b) $\frac{1}{k}$

(c) $\frac{a}{k}$ (d) $\frac{b}{k}$

4. The values of 'm' for which a line with slope m is common tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 (a \neq b)$ and parabola $y^2 = 4ax$ can lie in interval:

(a) $(0, 1)$ (b) $(-\infty, -1) \cup (1, \infty) - \left\{\pm\sqrt{\frac{1+\sqrt{5}}{2}}\right\}$

(c) $(-1, 0)$ (d) none of these

5. Let S_1 and S_2 be the foci of a rectangular hyperbola, which has the centre at Q, then for any point P on the hyperbola $S_1P \cdot S_2P$ equals to:

(a) $S_1S_2^2$ (b) QS_1^2

(c) QP^2 (d) $4QP^2$

6. If the portion of the asymptote between centre and the tangent at the vertex of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ in the third quadrant is cut by the line $y + \lambda(x + a) = 0$; λ being parameter, then:

(a) $\lambda \in R^+$ (b) $\lambda \in R^-$

(c) $\lambda \in (0, 1)$ (d) $[0, \infty)$

7. A circle cuts two perpendicular lines so that each intercept has given length. The locus of the centre of the circle is a conic whose eccentricity is:

(a) 1 (b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$ (d) none of these

8. Let F_1, F_2 are the foci of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and F_3, F_4 are the foci of its conjugate hyperbola. If e_H and e_C are their eccentricities respectively then the statement which holds true is:

(a) Their equations of the asymptotes are different.

(b) $e_H > e_C$

(c) Area of the quadrilateral formed by their foci is 50 sq. units.

(d) Their auxillary circles will have the same equation.

9. The parametric equations for the conic section $x^2 - 8x - 4y^2 - 16y - 4 = 0$ are:

(a) $x = -4 + 2\sec\theta,\ y = 2 + \tan\theta$

(b) $x = 4 + 2\tan\theta,\ y = -2 + \sec\theta$

(c) $x = -4 + 2\tan\theta,\ y = 2 + \sec\theta$

(d) $x = 4 + 2\sec\theta,\ y = -2 + \tan\theta$

10. The locus of a point in the Argand plane that moves satisfying the equation,
$|z - 1 + i| - |z - 2 - i| = 3$ is:

(a) a circle with radius 3 and centre at $z = 3/2$

(b) an ellipse with its foci at $1 - i$ and $2 + i$ and major axis $= 3$

(c) a hyperbola with its foci at $1 - i$ and $2 + i$ and its transverse axis $= 3$

(d) none of the above

11. If the eccentricity of the hyperbola $x^2 - y^2\sec^2\alpha = 5$ is $\sqrt{3}$ times the eccentricity of the ellipse $x^2\sec^2\alpha + y^2 = 25$, then a value of α is :

(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ (d) $\pi/2$

12. The locus of the foot of the perpendicular from the centre of the hyperbola $xy = c^2$ on a variable tangent is:

(a) $(x^2 - y^2)^2 = 4c^2xy$ (b) $(x^2 + y^2)^2 = 2c^2xy$

(c) $(x^2 + y^2) = 4c^2xy$ (d) $(x^2 + y^2)^2 = 4c^2xy$

13. Let the major axis of a standard ellipse equals the transverse axis of a standard hyperbola and their director circles have radius equal to $2R$ and R respectively. If e_1 and e_2 are the eccentricities of the ellipse and hyperbola then the correct relation is:

(a) $4e_1^2 - e_2^2 = 6$ (b) $e_1^2 - 4e_2^2 = 2$

(c) $4e_2^2 - e_1^2 = 6$ (d) $2e_1^3 - e_2^2 = 4$

14. A conic passes through the point $(2, 4)$ and is such that the segment of any of its tangents at any point contained between the co-ordinate is bisected at the point of tangency. Then the foci of the conic are:

(a) $(2\sqrt{2}, 0)$ and $(-2\sqrt{2}, 0)$ (b) $(2\sqrt{2}, 2\sqrt{2})$ and $(-2\sqrt{2}, -2\sqrt{2})$

(c) $(4, 4)$ and $(-4, -4)$ (d) $(4\sqrt{2}, 4\sqrt{2})$ and $(-4\sqrt{2}, -4\sqrt{2})$

15. Locus of the point of intersection of the tangents at the points with eccentric angles ϕ and $\frac{\pi}{2} - \phi$ on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is :

(a) $x = a$ (b) $y = b$

(c) $x = b$ (d) $y = ab$

16. With one focus of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$ as the centre, a circle is drawn which is tangent to the hyperbola with no part of the circle being outside the hyperbola. The radius of the circle is:

(a) less than 2 (b) 2

(c) $\frac{11}{3}$ (d) none of these

17. AB is double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that ΔAOB (where 'O' is the origin) is an equilateral triangle, then the eccentricity e of the hyperbola satisfies :

(a) $e > \sqrt{3}$ (b) $1 < e < \frac{2}{\sqrt{3}}$

(c) $e = \frac{2}{\sqrt{3}}$ (d) $e > \frac{2}{\sqrt{3}}$

18. The asymptote of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ form with any tangent to the hyperbola a triangle whose area is $a^2 \tan\lambda$ in magnitude then its eccentricity is :

(a) $\sec\lambda$ (b) $\text{cosec}\,\lambda$

(c) $\sec^2\lambda$ (d) $\text{cosec}^2\lambda$

19. Locus of the middle point of the parallel chords with gradient m of the rectangular hyperbola $xy = c^2$ is :

(a) $y + mx = 0$ (b) $y - mx = 0$

(c) $my - x = 0$ (d) $my + x = 0$

20. The equation to the chord joining two points (x_1, y_1) and (x_2, y_2) on the rectangular hyperbola $xy = c^2$ is :

(a) $\frac{x}{x_1 + x_2} + \frac{y}{y_1 + y_2} = 1$ (b) $\frac{x}{x_1 - x_2} + \frac{y}{y_1 - y_2} = 1$

(c) $\frac{x}{y_1 + y_2} + \frac{y}{y_1 - y_2} = 1$

(d) $\frac{x}{y_1 - y_2} + \frac{y}{x_1 - x_2} = 1$

21. Number of common tangent with finite slope to the curves $xy = c^2$ and $y^2 = 4ax$ is :

(a) 0

(b) 1

(c) 2

(d) 4

22. If the normal to the rectangular hyperbola $xy = c^2$ at the point 't' then $t^3 t_1$ has the value equal to :

(a) 1

(b) –1

(c) 0

(d) none of these

ANSWERS

1.	(a)	**2.**	(a)	**3.**	(a)	**4.**	(b)	**5.**	(c)	**6.**	(a)	**7.**	(c)	**8.**	(c)	**9.**	(d)	**10.**	(c)
11.	(b)	**12.**	(d)	**13.**	(c)	**14.**	(c)	**15.**	(b)	**16.**	(b)	**17.**	(d)	**18.**	(a)	**19.**	(a)	**20.**	(a)
21.	(b)	**22.**	(b)																

EXERCISE 2

One or More than One is/are Correct

1. The normal at one extremity of latus rectum (in 1st quadrant) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b > 0$ meets the rectangular hyperbola $xy = 9$ at points P and Q, then:

(a) If P is $\left(6, \frac{3}{2}\right) \Rightarrow Q$ is $\left(-\frac{3\sqrt{2}}{2}, -3\sqrt{2}\right)$

(b) Eccentricity of hyperbola is $\sqrt{2}$

(c) If P is $\left(6, \frac{3}{2}\right) \Rightarrow Q$ is $\left(-\frac{3e}{2}, -\frac{6}{e}\right)$ where e is eccentricity of the given ellipse

(d) If O is origin, then product of slopes of OP and OQ is positive

2. The equation(s) to common tangent(s) to the two hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ is/are:

(a) $y = x + \sqrt{a^2 - b^2}$ (b) $y = x - \sqrt{a^2 - b^2}$

(c) $y = -x + \sqrt{a^2 - b^2}$ (d) $y = -x - \sqrt{a^2 - b^2}$

3. Straight line $Ax + By + D = 0$ would be tangent to $xy = c^2$, if:

(a) $A > 0, B > 0$ (b) $A < 0, B < 0$

(c) $A > 0, B < 0$ (d) $A < 0, B > 0$

4. If the normal to the rectangular hyperbola $x^2 - y^2 = 4$ at a point P meets the coordinates axes in Q and R and O is the centre of the hyperbola. Then:

(a) $OP = PQ$ (b) $OP = PR$ (c) $PQ = PR$ (d) $QR = 2\,OP$

5. For the hyperbola $\frac{x^2}{9} - \frac{y^2}{3} = 1$ the incorrect statement is :

(a) the acute angle between its asymptotes is 60°

(b) its eccentricity is 4/3

(c) length of the latus rectum is 2

(d) product of the perpendicular distances from any point on the hyperbola on its asymptotes is less than the length of its latus rectum.

6. The tangent to the hyperbola, $x^2 - 3y^2 = 3$ at the point $(\sqrt{3}, 0)$ when associated with two asymptotes constitutes :

(a) scalene triangle (b) an equilateral triangle

(c) a triangle whose area is $\sqrt{3}$ sq. units (d) a right isosceles triangle

7. Which of the following equations in parametric form can represent a hyperbola, where 't' is a parameter ?

(a) $x = \frac{a}{2}\left(t + \frac{1}{t}\right)$ and $y = \frac{b}{2}\left(t - \frac{1}{t}\right)$ (b) $\frac{tx}{a} - \frac{y}{b} + t = 0$ and $\frac{x}{a} + \frac{ty}{b} - 1 = 0$

(c) $x = e^t + e^{-t}$ and $y = e^t - e^{-t}$ (d) $x^2 - 6 = 2\cos t$ and $y^2 + 2 = 4\cos^2 \frac{t}{2}$

8. The differential equation $\frac{dx}{dy} = \frac{3y}{2x}$ represents a family of hyperbolas (except when it represents a pair of lines) with eccentricity :

(a) $\sqrt{\frac{3}{5}}$ (b) $\sqrt{\frac{5}{3}}$ (c) $\sqrt{\frac{2}{5}}$ (d) $\sqrt{\frac{5}{2}}$

9. Given ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$, if the ordinate of one of the points of intersection is produced to cut asymptote at P, then which of the following is true ?

(a) they have the same foci

(b) square of the ordinate of point of intersection is $\frac{63}{25}$.

(c) sum of the square of coordinates of P is 16

(d) P lies on the auxiliary circle formed by ellipse.

10. Solutions of the differential equation $(1 - x^2)\frac{dy}{dx} + xy = ax$ where $a \in R$, is:

(a) a conic which is an ellipse or a hyperbola with principal axes parallel to co-ordinate axes.

(b) centre of the conic is $(0, a)$

(c) length of one of the principal axes is 1.

(d) length of one of the principal axes is equal to 2.

11. If $(a\sec\theta,\ b\tan\theta)$ and $(a\sec\phi,\ b\tan\phi)$ are the ends of a focal chord of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan\frac{\phi}{2}\tan\frac{\phi}{2}$ can be equal to :

(a) $\frac{e-1}{e+1}$ (b) $\frac{1-e}{1+e}$ (c) $\frac{1+e}{1-e}$ (d) $\frac{e+1}{e-1}$

ANSWERS

1.	(b, c, d)	**2.**	(a, b, c, d)	**3.**	(a, b)	**4.**	(a, b, c, d)	**5.**	(b, d)	**6.**	(b, c)
7.	(a, c, d)	**8.**	(b, d)	**9.**	(a, b, c, d)	**10.**	(a, b, d)	**11.**	(b, c)		

EXERCISE 3

Comprehension: (1)

Consider an ellipse $\frac{x^2}{36}+\frac{y^2}{18}=1$. There is a hyperbola whose one asymptote is major axis of given ellipse. If eccentricity of given ellipse and hyperbola are reciprocal to each other, both have same centre and both touch each other in first and third quadrant.

1. Focus of hyperbola is equal to :

(a) $\left(\frac{3}{2},\frac{3}{2}\right)$ (b) $\left(\frac{3}{\sqrt{2}},\frac{3}{\sqrt{2}}\right)$

(c) $(3\sqrt{2},3\sqrt{2})$ (d) $[3(2^{3/4}), 3(2^{3/4})]$

2. No. of points in x-y plane from where $\perp$ tangents can be drawn to hyperbola:

(a) 0 (b) 1

(c) infinite (d) none of these

3. The equation of common tangent to given ellipse and hyperbola in first quadrant is :

(a) $\frac{x}{\sqrt{2}}+y=3$ (b) $\frac{x}{\sqrt{2}}+y=6$

(c) $x+y\sqrt{2}=3\sqrt{2}$ (d) $x+y\sqrt{2}=6$

Comprehension: (2)

Consider a hyperbola 'H' whose centre is at origin and line $x+y=2$ touches it at point $(1,1)$. The tangent $x+y=2$ intersects the asymptotes of H at point A and B such that length of segment $AB=6\sqrt{2}$.

1. Equation of pair of directrices of H is :

(a) $x^2+y^2+2xy-1=0$ (b) $5x^2+5y^2+10xy-4=0$

(c) $5x^2+5y^2+10xy-2=0$ (d) $5x^2+5y^2+10xy-6=0$

2. Equation of the tangent to H at point $\left(-1,\frac{7}{2}\right)$ on it is :

(a) $3x+2y=2$ (b) $3x+2y=4$

(c) $4x+2y=9$ (d) $6x+4y=7$

3. Equation of H w.r.t. $x-y$ system referring to transverse axis as x-axis and conjugate axis as y-axis respectively is:

(a) $x^2 - \dfrac{y^2}{18} = 1$ (b) $\dfrac{x^2}{2} - \dfrac{y^2}{18} = 1$

(c) $\dfrac{x^2}{18} - \dfrac{y^2}{2} = 1$ (d) $\dfrac{x^2}{18} - y^2 = 1$

Comprehension: (3)

If the normal to the hyperbola $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ at any point $P(a \sec\theta, b \tan\theta)$ meets the transverse and conjugate axes in G and g respectively and if 'F' is the foot of perpendicular to the normal at P from the centre 'C', then:

1. The value of $(PG)^2$ is :

(a) $\dfrac{b^2}{a^2}(b^2 \operatorname{cosec}^2\theta + a^2 \cot^2\theta)$ (b) $\dfrac{a^2}{b^2}(b^2 \operatorname{cosec}^2\theta + a^2 \cot^2\theta)$

(c) $\dfrac{b^2}{a^2}(b^2 \sec^2\theta + a^2 \tan^2\theta)$ (d) $\dfrac{b^2}{a^2}(b^2 \tan^2\theta + a^2 \sec^2\theta)$

2. The value of $(PF)^2$ is :

(a) $\dfrac{a^2b^2}{b^2 \sec^2\theta + a^2 \tan^2\theta}$ (b) $\dfrac{a^2b^2}{b^2 \tan^2\theta + a^2 \sec^2\theta}$

(c) $\dfrac{a^2b^2}{b^2 \operatorname{cosec}^2\theta + a^2 \cot^2\theta}$ (d) $\dfrac{a^2b^2}{b^2 \cot^2\theta + a^2 \operatorname{cosec}^2\theta}$

3. The value of $|PF||PG|$ is equal to:

(a) $b^2(\sec^2\theta + \tan^2\theta)$ (b) a^2

(c) b^2 (d) $b^2 \sec^2\theta + a^2 \tan^2\theta$

Comprehension: (4)

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ intersect at the points A and B.

1. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is:

(a) $2x - \sqrt{5y} - 20 = 0$ (b) $2x - \sqrt{5} + 4 = 0$

(c) $3x - 4y + 8 = 0$ (d) $4x - 3y + 4 = 0$

2. Equation of the circle with AB as its diameter is:

(a) $x^2 + y^2 - 12x + 24 = 0$ (b) $x^2 + y^2 + 12x + 24 = 0$

(c) $x^2 + y^2 + 24x - 12 = 0$ (d) $x^2 + y^2 - 24x - 12 = 0$

Comprehension: (5)

If P_1, P_2, P_3 are three points on the hyperbola $xy = c^2$ with abscissa x_1, x_2, x_3 then :

1. Area of triangle $P_1P_2P_3$ is :

(a) $\left|\frac{c^2}{2}\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{x_1x_2x_3}\right|$ (b) $\left|\frac{c^2}{2}\frac{(x_1 - x_3)(x_1 + x_2)(x_2 + x_3)}{x_1x_2x_3}\right|$

(c) $\left|\frac{c^2}{2}(x_1 + x_2)(x_2 - x_3)(x_3 - x_1)x_1x_2x_3\right|$ (d) $\left|\frac{c^2}{2}(x_1x_2x_3)(x_1 + x_2 + x_3)\right|$

2. Area of triangle formed by tangents at P_1, P_2, P_3 :

(a) $\left|2c^2(x_2 - x_3)(x_3 - x_1)(x_1 - x_2)\right|$

(b) $\left|2c^2\frac{(x_2 - x_3)(x_3 - x_1)(x_1 - x_2)}{(x_2 + x_3)(x_3 + x_1)(x_1 + x_2)}\right|$

(c) $\left|2c^2\frac{(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)}{x_1x_2x_3}\right|$

(d) $\left|2c^2(x_1x_2x_3)(x_1 - x_2)(x_2 - x_3)(x_3 - x_1)\right|$

ANSWERS

Comprehension-1:	1.	(d)	2.	(a)	3.	(b)
Comprehension-2:	1.	(c)	2.	(b)	3.	(b)
Comprehension-3:	1.	(c)	2.	(a)	3.	(c)
Comprehension-4:	1.	(b)	2.	(a)		
Comprehension-5:	1.	(a)	2.	(b)		

EXERCISE 4

Match the Columns:

1. Let the foci of the hyperbola $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$ be the vertices of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the foci of the ellipse be the vertices of the hyperbola. Let the eccentricities of the ellipse and hyperbola be e_1 and e_2 respectively.

	Column-I		Column-II
(a)	$\frac{b}{B}$	**(p)**	1
(b)	e_1+e_2 can not be equal to	**(q)**	2
(c)	If the angle between the asymptotes of the hyperbola is $\frac{2\pi}{3}$, then $2e_1 \le$	**(r)**	3
(d)	If $e_1=\frac{1}{\sqrt{2}}$ and (x, y) is point of intersection of ellipse and hyperbola, then $\frac{x^2}{y^2}=$	**(s)**	4

2. Match the column:

	Column-I		Column-II
(a)	The angle between asymptotes of the hyperbola $5x^2-2\sqrt{7}\,xy-y^2-2x+1=0$ is $\frac{\pi}{k}$, then $k=$	**(p)**	2
(b)	Portion of asymptotes of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ between centre and the tangent at vertex in first quadrant is cut by line $y+\tan\theta(x-a)=0$, where $\theta=\pi/k$, then k can be equal to	**(q)**	3
(c)	Let the double ordinate PP' of the hyperbola $x^2-\frac{y^2}{4}=1$ cut the asymptotes in Q and Q' lies on same side of x-axis as that of P and P' respectively then $\lvert PQ\rvert\lvert PQ'\rvert=$	**(r)**	4
(d)	If the tangent and normal to hyperbola $x^2-y^2=4$ at a point cut off intercepts $(a_1, 0)$, $(a_2, 0)$ on the x-axis and $(0, b_1)$, $(0, b_2)$ on the y-axis respectively then the value of $a_1a_2+b_1b_2$	**(s)**	6
		(t)	0

3. The tangent to a curve at a point $P(x, y)$ meets the x-axis at T and y-axis at S while the normal at P meets the x-axis at N and y-axis at M, O is the origin. Match the locus of point P satisfying the condition in column-I to the curve given in column-II:

	Column-I		Column-II
(a)	$TP = PS$	**(p)**	Straight line
(b)	$NM = NP$	**(q)**	Circle
(c)	$TP = OP$	**(r)**	Ellipse
(d)	$NP = OP$	**(s)**	Hyperbola

4. Match the column:

	Column-I		Column-II
(a)	If eccentricity of conjugate hyperbola of the given hyperbola $\left\| \sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2} \right\| = 3$ is e' then value of $8e'$ is	**(p)**	8
(b)	If area of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ inscribed in a square of side length $5\sqrt{2}$ is A then $\frac{A}{\pi}$ equal to	**(q)**	12
(c)	Any chord of the conic $x^2 + y^2 + xy = 1$ passing through origin is bisected at a point (p, q) then $(p + q + 12)$ equals	**(r)**	10
(d)	Length of the shortest chord of the parabola $y^2 = 4x + 8$ which belong to the family of lines $(1+\lambda)y + (\lambda - 1)x + 2(1-\lambda) = 0$, is	**(s)**	7
		(t)	9

ANSWERS

1. $a \to p$; $b \to p, q$; $c \to p, q, r, s$; $d \to s$

2. $a \to q$; $b \to q, r, s$; $c \to r$; $d \to t$

3. $a \to s$; $b \to r$; $c \to p, s$; $d \to q, s$

4. $a \to r$; $b \to q$; $c \to q$; $d \to p$

EXERCISE 5

Subjective Problems

1. The graphs of $x^2+y^2+6x-24y+72=0$ and $x^2-y^2+6x+16y-46=0$, intersect at 4 points. Compute the sum of distances of the 4 points from point $(-3,2)$.

2. The tangents to hyperbola $x^2-9y^2=9$ are drawn from point $(3,2)$, then the area of triangle that these tangents form with their chord of contact is:

3. A transversal cut the same branch of a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ in P, P' and the asymptotes in Q, Q', then find $(PQ+PQ')-(P'Q'+P'Q)$:

4. If two points P and Q on the hyperbola $9x^2-5y^2=1$ whose centre is C be such that CP is perpendicular to CQ, then find $\frac{1}{CP^2}+\frac{1}{CQ^2}$.

5. A normal to the hyperbola $\frac{x^2}{4}-\frac{y^2}{1}=1$, has equal intercepts on positive x and y-axes. If this normal touches the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, then find $\frac{9}{25}(a^2+b^2)$.

ANSWERS

1.	40	**2.**	8	**3.**	0	**4.**	4	**5.**	3

EXERCISE 6

1. (A) The curve described parametrically by, $x = t^2 + t + 1,\ y = t^2 - t + 1$ represents:

(a) a parabola (b) an ellipse

(c) a hyperbola (d) a pair of straight lines

(B) Let $P\ (a\sec\theta, b\tan\theta)$ and $Q\ (a\sec\phi, b\tan\phi)$, where $\theta + \phi = \frac{\pi}{2}$, be two points on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If (h, k) is the point of intersection of the normals at P and Q, then k is equal to:

(a) $\frac{a^2 + b^2}{a}$ (b) $-\left(\frac{a^2 + b^2}{a}\right)$ (c) $\frac{a^2 + b^2}{b}$ (d) $-\left(\frac{a^2 + b^2}{b}\right)$

(C) If $x = 9$ is the chord of contact of the hyperbola $x^2 - y^2 = 9$, then the equation of the corresponding pair of tangents, is: **[IIT-JEE 1999]**

(a) $9x^2 - 8y^2 + 18x - 9 = 0$ (b) $9x^2 - 8y^2 - 18x + 9 = 0$

(c) $9x^2 - 8y^2 - 18x - 9 = 0$ (d) $9x^2 - 8y^2 + 18x + 9 = 0$

2. The equation of the common tangent to the curve $y^2 = 8x$ and $xy = -1$ is:

[IIT-JEE (Screening) 2002]

(a) $3y = 9x + 2$ (b) $y = 2x + 1$ (c) $2y = x + 8$ (d) $y = x + 2$

3. Given the family of hyperbolas $\frac{x^2}{\cos^2\alpha} - \frac{y^2}{\sin^2\alpha} = 1$ for $\alpha \in (0, \pi/2)$ which of the following does not change with varying α? **[IIT-JEE (screening) 2003]**

(a) abscissa of foci (b) eccentricity

(c) equations of directrices (d) abscissa of vertices

4. The line $2x + \sqrt{6}y = 2$ is a tangent to the curve $x^2 - 2y^2 = 4$. The point of contact is:

[IIT-JEE (Screening) 2004]

(a) $(4, -\sqrt{6})$ (b) $(7, -2\sqrt{6})$ (c) $(2, 3)$ (d) $(\sqrt{6}, 1)$

5. Tangents are drawn from any point on the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ to the circle $x^2 + y^2 = 9$. Find the locus of mid-point of the chord of contact. **[IIT-JEE (Mains) 2005]**

6. If a hyperbola passes through the focus of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axis coincides with the major and minor axis of the ellipse, and product of their eccentricities is 1, then: **[IIT-JEE 2006]**

(a) equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{16}=1$ (b) equation of hyperbola is $\frac{x^2}{9}-\frac{y^2}{25}=1$

(c) focus of hyperbola is $(5, 0)$ (d) focus of hyperbola is $(5\sqrt{3}, 0)$

7. Comprehension: (3 questions)

Let *ABCD* be a square of side length 2 units. C_2 is the circle through vertices *A*, *B*, *C*, *D* and C_1 is the circle touching all the sides of the square *ABCD*. *L* is a line through *A*. [IIT-JEE 2006]

(A) If P is a point on C_1 and Q in another point on C_2, then $\frac{PA^2+PB^2+PC^2+PD^2}{QA^2+QB^2+QC^2+QD^2}$ is equal to:

(a) 0.75 (b) 1.25 (c) 1 (d) 0.5

(B) A circle touches the line L and the circle C_1 externally such that both the circles are on the same side of the line, then the locus of centre of the circle is:

(a) ellipse (b) hyperbola

(c) parabola (d) parts of straight line

(C) A line M through A is drawn parallel to BD. Point S moves such that its distances from the line BD and the vertex A are equal. If locus of S cuts M at T_2 and T_3 and AC at T_1, then area of $\Delta T_1T_2T_3$ is:

(a) 1/2 sq. units (b) 2/3 sq. units (c) 1 sq. units (d) 2 sq. units

8. (A) A hyperbola, having the transverse axis of length $2\sin\theta$, is confocal with the ellipse $3x^2+4y^2=12$. Then its equation is: **[IIT-JEE 2007]**

(a) $x^2\operatorname{cosec}^2\theta - y^2\sec^2\theta = 1$ (b) $x^2\sec^2\theta - y^2\operatorname{cosec}^2\theta = 1$

(c) $x^2\sin^2\theta - y^2\cos^2\theta = 1$ (d) $x^2\cos^2\theta - y^2\sin^2\theta = 1$

(B) Match the statements in **Column-I** with the properties in **Column-II. [IIT-JEE 2007]**

	Column-I		Column-II
(a)	Two intersecting circles	**(p)**	have a common tangent
(b)	Two mutually external circles	**(q)**	have a common normal
(c)	Two circles, one strictly inside the other	**(r)**	do not have a common tangent
(d)	Two branches of a hyperbola	**(s)**	do not have a common normal

9. (A) Let a and b be non-zero real numbers. Then, the equation $(ax^2+by^2+c)(x^2-5xy+6y^2)=0$ represents

(a) four straight lines, when $c=0$ and a, b are of the same sign.

(b) two straight lines and a circle, when $a=b$, and c is of sign opposite to that of a.

(c) two straight lines and a hyperbola, when a and b are of the same sign and c is of sign opposite to that of a.

(d) a circle and an ellipse, when a and b are of the same sign and c is of sign opposite to that of a.

(B) Consider a branch of the hyperbola, $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the focus of the hyperbola nearest to the point A, then the area of the triangle ABC is: **[IIT-JEE 2008]**

(a) $1-\sqrt{\frac{2}{3}}$ (b) $\sqrt{\frac{3}{2}}-1$ (c) $1+\sqrt{\frac{2}{3}}$ (d) $\sqrt{\frac{3}{2}}+1$

10. Match the conics in **Column-I** with the statements/expressions in **Column-II.**

[IIT-JEE 2009]

	Column-I		Column-II
(a)	Circle	**(p)**	The locus of the point (h, k) for which the line $hx + ky = 1$ touches the circle $x^2 + y^2 = 4$
(b)	Parabola	**(q)**	Points z in the complex plane satisfying $\lvert z+2\rvert - \lvert z-2\rvert = \pm 3$
(c)	Ellipse	**(r)**	Points of the conic have parametric representation $x = \sqrt{3}\left(\frac{1-t^2}{1+t^2}\right), y = \frac{2t}{1+t^2}$
(d)	Hyperbola	**(s)**	The eccentricity of the conic lies in the interval $1 \le x < \infty$
		(t)	Points z in the complex plane satisfying $\text{Re}(z+1)^2 = \lvert z\rvert^2 + 1$

11. An ellipse intersects the hyperbola $2x^2 - 2y^2 = 1$ orthogonally. The eccentricity of the ellipse is reciprocal of that of the hyperbola. If the axes of the ellipse are along the coordinate axes, then: **[IIT-JEE 2009]**

(a) Equation of ellipse is $x^2 + 2y^2 = 2$ (b) The foci of ellipse are $(\pm 1, 0)$

(c) Equation of ellipse is $x^2 + 2y^2 = 4$ (d) The foci of ellipse are $(\pm\sqrt{2}, 0)$

Paragraph for Questions 12 to 13

The circle $x^2 + y^2 - 8x = 0$ and hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$ intersect at the points A and B. **[IIT-JEE 2010]**

12. Equation of a common tangent with positive slope to the circle as well as to the hyperbola is:

(a) $2x-\sqrt{5}y-20=0$ (b) $2x-\sqrt{5}y+4=0$

(c) $3x-4y+8=0$ (d) $4x-3y+4=0$

13. Equation of the circle with AB as its diameter is:

(a) $x^2+y^2-12x+24=0$ (b) $x^2+y^2+12x+24=0$

(c) $x^2+y^2+24x-12=0$ (d) $x^2+y^2-24x-12=0$

14. The line $2x+y=1$ is tangent to the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If this line passes through the point of intersection of the nearest directrix and the x-axis, then the eccentricity of the hyperbola is: **[IIT-JEE 2010]**

15. Let $P(6,3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the x-axis at $(9,0)$, then the eccentricity of the hyperbola is: **[IIT-JEE 2011]**

(a) $\sqrt{\frac{5}{2}}$ (b) $\sqrt{\frac{3}{2}}$ (c) $\sqrt{2}$ (d) $\sqrt{3}$

16. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be reciprocal to that of the ellipse $x^2+4y^2=4$. If the hyperbola passes through a focus of the ellipse, then: **[IIT-JEE 2011]**

(a) the equation of the hyperbola is $\frac{x^2}{3}-\frac{y^2}{2}=1$

(b) a focus of the hyperbola is $(2,0)$

(c) th eccentricity of the hyperbola is $\sqrt{\frac{5}{3}}$

(d) the equation of the hyperbola is $x^2-3y^2=3$

17. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x-y=1$. The points of contact of the tangents on the hyperbola are: **[IIT-JEE 2012]**

(a) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (b) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$

(c) $(3\sqrt{3},-2\sqrt{2})$ (d) $(-3\sqrt{3},2\sqrt{2})$

ANSWERS

1. **(A)** a; **(B)** d; **(C)** b
2. d
3. a
4. a
5. $\frac{x^2}{9} - \frac{y^2}{4} = \left(\frac{x^2 + y^2}{9}\right)^2$
6. a, c
7. **(A)** a; **(B)** c; **(C)** c
8. **(A)** a; **(B)** (a) p, q; (b) p, q; (c) q, r; (d) q, r
9. **(A)** b; **(B)** b
10. (a) $\to$ p; (b) $\to$ s, t; (c) $\to$ r; (d) q, s
11. a, b
12. b
13. a
14. 2
15. b
16. b, d
17. a, b

SOLUTIONS 1

Only One Choice is Correct:

1. (a)

$$(x-\sqrt{2})(x-1)(x+\sqrt{2}) = (x^2-2)(x-1)$$
$$= x^3 - x^2 - 2x + 2$$
$$\alpha + \beta + \gamma = -1 - 2 + 2 = -1$$

2. (a) Point of hyperbola $\left(ct, \frac{c}{t}\right)$ lie on director circle $x^2 + y^2 = a^2 + b^2$ of ellipse

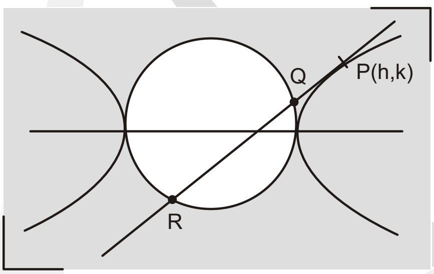
$\Rightarrow$ $a^2 + b^2 = c^2\left(t^2 + \frac{1}{t^2}\right) \geq 2c^2$ which is impossible

$\because$ $a^2 + b^2 < c^2 + c^2 = 2c^2$

$(\because b < a < c)$

3. (a) Equation of tangent at P is $\frac{xh}{a^2} - \frac{yk}{b^2} = 1$

Put $x = \left(1 + \frac{yk}{b^2}\right)\frac{a^2}{h}$ in equation $x^2 + y^2 = a^2$



$$\Rightarrow \left(1 + \frac{y^2k^2}{b^4} + \frac{2yk}{b^2}\right)\frac{a^4}{h^2} + y^2 = a^2$$

$$\Rightarrow y^2\left(\frac{k^2a^4}{b^4h^2} + 1\right) + \frac{2ka^4}{b^2h^2}y + \left(\frac{a^4}{h^2} - a^2\right) = 0 \genfrac{}{}{0pt}{}{\nearrow y_1}{\searrow y_2}$$

$$\Rightarrow \frac{1}{y_1} + \frac{1}{y_2} = \frac{\frac{-2ka^4}{b^2h^2}}{\frac{a^4 - a^2h^2}{h^2}}$$

$$= -\frac{2ka^2}{b^2(a^2-h^2)} = -\frac{2k}{b^2\left(1 - \frac{h^2}{a^2}\right)}$$

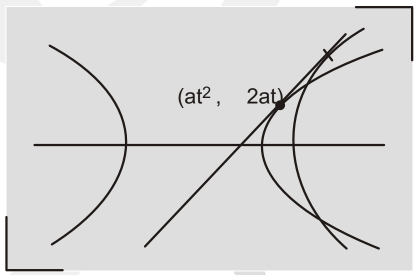
$$= -\frac{2k}{b^2\left(-\frac{k^2}{b^2}\right)} = \frac{2}{k}$$

4. (b) Equation of tangent to hyperbola is

$$y = mx \pm \sqrt{a^2m^2 - b^2}$$

$\because$ It touches parabola also

Equation of tangent to parabola.



$$\Rightarrow a^2m^2 - b^2 = \frac{a^2}{m^2}$$

$$\Rightarrow a^2m^4 - b^2m^2 - a^2 = 0$$

$$m^2 = \frac{b^2 \pm \sqrt{b^4 + 4a^4}}{2a^2}$$

$$= \frac{1}{2}\left[\frac{b^2}{a^2} \pm \sqrt{\frac{b^4}{a^4} + 4}\right]$$

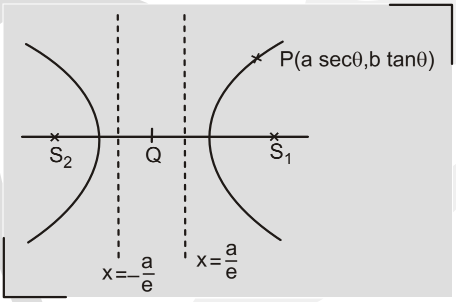
Clearly $\frac{b^2}{a^2} \neq 1, m^2 > 1$

$\Rightarrow \quad m \in (-\infty, -1) \cup (1, \infty)$

and $m^2 \neq (1+\sqrt{5})/2$

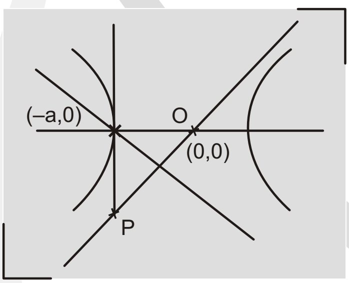
5. (c) $(S_1P)(S_2P)$

$$= e\left(a\sec\theta - \frac{a}{e}\right) e\left(a\sec\theta + \frac{a}{e}\right)$$



$$= a^2(e^2\sec^2\theta - 1)$$
$$= a^2(2\sec^2\theta - 1)$$
$$= a^2(\sec^2\theta + \tan^2\theta)$$
$$= a^2\sec^2\theta + b^2\tan^2\theta$$
$$= QP^2$$

6. (a)



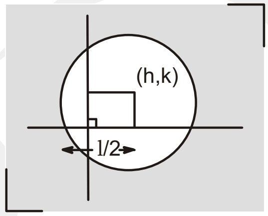
$y = -\lambda(x+a)$

cut the segment OP if

$-\lambda < 0 \Rightarrow \lambda > 0$

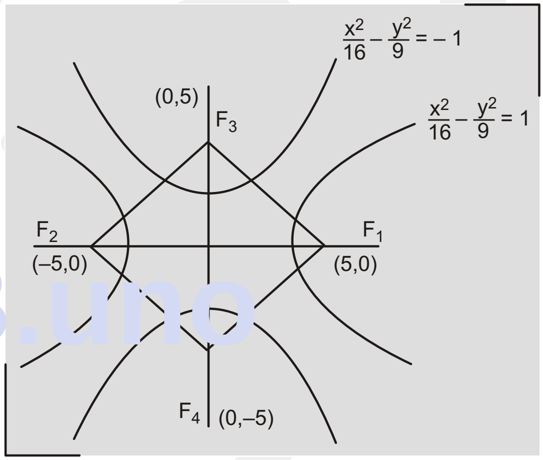
7. (c) $r^2 = \frac{l_1^2}{4} + k^2 = \frac{l_2^2}{4} + h^2$

$\Rightarrow \quad x^2 - y^2 = \frac{l_1^2 - l_2^2}{4}$



Which is rectangular hyperbola

8. (c)



$$9 = 16(e_H^2 - 1) \Rightarrow e_H = \frac{5}{4}$$

$F_1, F_2 \equiv (\pm 5, 0)$,

$$16 = 19(e_C^2 - 1) \Rightarrow e_C = \frac{5}{3}$$

$F_3, F_4 \equiv (0, \pm 5)$

Area of $F_1F_3F_2F_4 = \frac{1}{2} \times 10 \times 10 = 50$

9. (d) $(x-4)^2 - 4(y+2)^2 = 4$

$$\frac{(x-4)^2}{4} - (y+2)^2 = 1$$

$(x, y) \equiv (4 + 2\sec\theta, -2 + \tan\theta)$

10. (c) Obvious

11. (b) $\frac{x^2}{1} - \frac{y^2}{\cos^2\alpha} = 5;$

$\cos^2\alpha + 1 = e_1^2$

$\frac{x^2}{\cos^2\alpha} + \frac{y^2}{1} = 25$

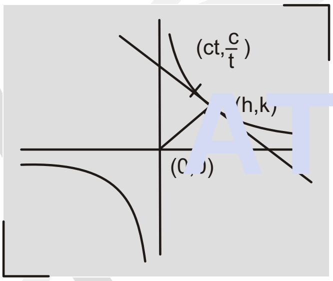
$1(1 - e_2^2) = \cos^2\alpha \Rightarrow e_2^2 = \sin^2\alpha$

$e_1^2 = 3e_2^2 \Rightarrow \cos^2\alpha + 1$

$= 3 - 3\cos^2\alpha \Rightarrow \cos^2\alpha = \frac{1}{2}$

$\Rightarrow \cos\alpha = \frac{1}{\sqrt{2}}$

12. (d) Equation of tangent is



$hx + xy = h^2 + k^2$ ...(1)

Also tangent at P is

$\frac{x}{t} + yt = 2c$ ...(2)

$\because$ (1) and (2) are identical

$\Rightarrow ht = \frac{k}{t} = \frac{h^2 + k^2}{2c}$

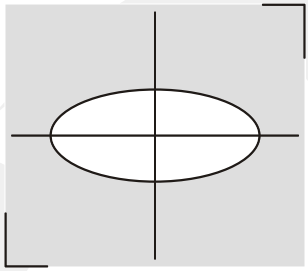
$\Rightarrow \frac{h^2 + k^2}{2ch} = \frac{2ck}{h^2 + k^2}$

$\Rightarrow (x^2 + y^2)^2 = 4c^2xy$

13. (c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad \frac{x^2}{a^2} - \frac{y^2}{B^2} = 1$

$a^2 + b^2 = 4R^2 = a^2(2 - e_1^2)$

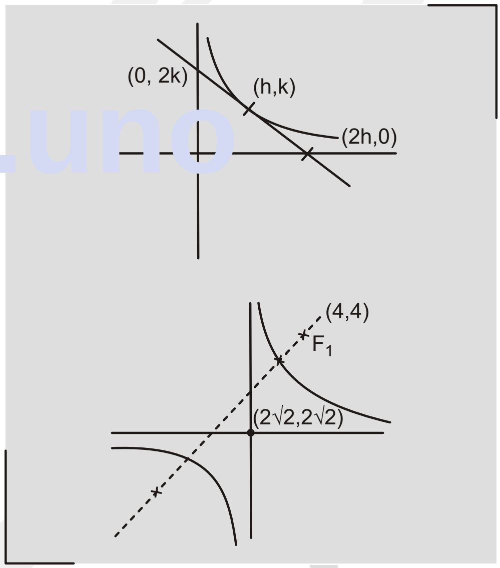
$a^2 - B^2 = R^2 = a^2(2 - e_2^2)$



$4 = \frac{2 - e_1^2}{2 - e_2^2}$

$\Rightarrow 4e_2^2 - e_1^2 = 6$

14. (c) $\left(\frac{dy}{dx}\right)_{(h,k)} = -\frac{k}{h}$



$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow \ln(xy) = c$

$\Rightarrow$ Put $(2, 4)$; $c = \ln 8$

$\Rightarrow xy = 8$

$F_1; F_2 \equiv (4, 4), (-4, -4)$

15. (b) Tangent at ϕ is

$$\frac{x\sec\phi}{a}-\frac{y\tan\phi}{b}=1 \qquad \ldots(1)$$

Tangent at $\frac{\pi}{2}-\phi$ is

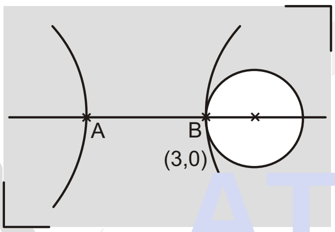
$$\frac{x\operatorname{cosec}\phi}{a}-\frac{y\cot\phi}{b}=1 \qquad \ldots(2)$$

$(1)\times\operatorname{cosec}\phi-(2)\times\sec\phi$

$$\Rightarrow \frac{y}{b}\left(\frac{1}{\sin\phi}-\frac{1}{\cos\phi}\right)=\frac{1}{\sin\phi}-\frac{1}{\cos\phi}$$

$$\Rightarrow y=b$$

16. (b)



$$16=9(e^2-1)$$

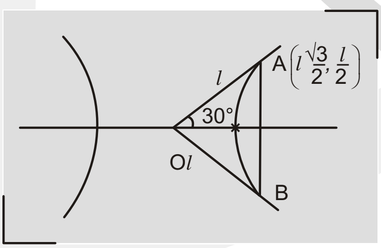
$$e=\frac{5}{3}$$

$F_1=(5,0)$

Circle can be drawn touching hyperbola at point A or B only

$\therefore$ Radius $=2$

17. (d) Put $\left(l\frac{\sqrt{3}}{2},\frac{l}{2}\right)$ to equation of hyperbola



$$\Rightarrow l^2\left(\frac{3}{4a^2}-\frac{1}{4b^2}\right)=1$$

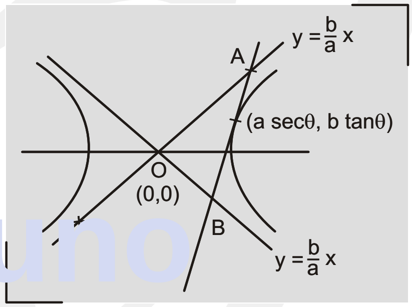
$$\Rightarrow \frac{1}{l}=\frac{3}{4a^2}-\frac{1}{4b^2}>0$$

$$\Rightarrow \frac{b^2}{a^2}>\frac{1}{3}\Rightarrow e^2-1>\frac{1}{3}\Rightarrow e^2>\frac{4}{3}$$

$$\Rightarrow e>\frac{2}{\sqrt{3}}$$

18. (a) equation of tangent is

$$\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$$



Solving point of intersection of tangent with asymptotes we get,

$A\equiv a(\sec\theta+\tan\theta),\ b(\sec\theta+\tan\theta)$

$B\equiv a(\sec\theta-\tan\theta),\ -b(\sec\theta-\tan\theta)$

Area of

$$\Delta OAB=\left|\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ a(\sec\theta+\tan\theta) & b(\sec\theta+\tan\theta) & 1\\ a(\sec\theta-\tan\theta) & -b(\sec\theta-\tan\theta) & 1\end{vmatrix}\right|$$

$$=ab$$

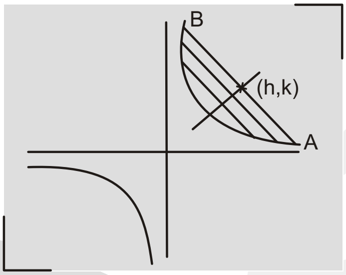
$$=ab=a^2\tan\lambda$$

$$\Rightarrow \frac{b^2}{a^2}=\tan^2\lambda$$

$$\Rightarrow e^2-1=\tan^2\lambda$$

$$\Rightarrow e=\sec\lambda$$

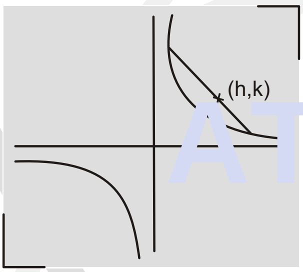
19. (a) Equation of AB is



$$\frac{x}{h}+\frac{y}{k}=2$$

$$m=-\frac{k}{h} \qquad \therefore \qquad y=-mx$$

20. (a) Equation of chord whose mid-point is (h,k) is



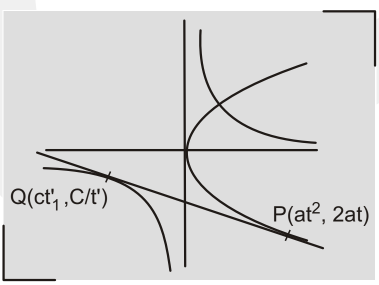
$$\frac{xk+yh}{2}=c^2=hk$$

$$\frac{x}{h}+\frac{y}{k}=2$$

Put $h=\frac{x_1+x_2}{2}$, $k=\frac{y_1+y_2}{2}$

$$\Rightarrow \quad \frac{x}{x_1+x_2}+\frac{y}{y_1+y_2}=1$$

21. (b) Equation of PQ



$$yt-x=at^2 \qquad \text{...(1)}$$

$$\frac{x}{t'}+yt'=2c \qquad \text{...(2)}$$

$\because$ (1) and (2) are identical

$$\Rightarrow \quad -t'=\frac{t}{t'}=\frac{at^2}{2c}$$

$$\Rightarrow \quad \frac{a(t')^4}{2c}=-t'$$

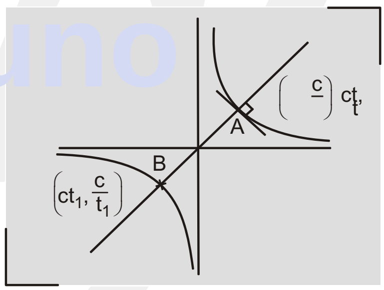
$$\Rightarrow \quad (t')^3=-\frac{2c}{a}$$

$\Rightarrow$ Only one real value of t' exist.

22. (b) Slope of normal at A

$$=-\frac{1}{\frac{dy}{dx}}=\frac{x^2}{c^2}=t^2$$

Slope of chord AB



$$=\frac{c\left(\frac{1}{t}-\frac{1}{t_1}\right)}{c(t-t_1)}=\frac{-1}{tt_1}$$

$$\Rightarrow \quad t^2=-\frac{1}{tt_1}$$

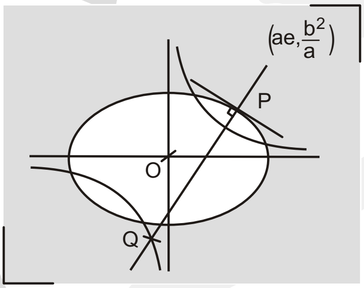
$$\Rightarrow \quad t^3t_1=-1$$

SOLUTIONS 2

One or More than One is/are Correct

1. (b, c, d)

Equation of normal at $\left(ae, \frac{b^2}{a}\right)$



$\frac{a}{e}x - ay = a^2e^2$

$x - ey = ae^3$

Put $y = \frac{9}{x}, x - 9\frac{e}{x} = ae^3$

$\Rightarrow x^2 - ae^3x - 9e = 0$ (roots x_1, x_2)

If $x_1 = 6,\ x_1x_2 = -9e,$

$x_2 = -\frac{3e}{2}$

$y_2 = \frac{9}{x_2} = -\frac{6}{e}$

$\Rightarrow$ (slope of OP) (slope of QQ)

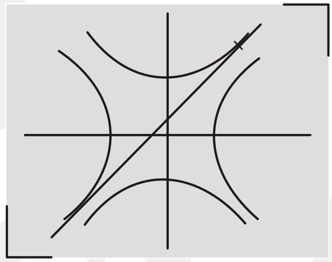
$= \left(\frac{y_1}{x_1}\right)\left(\frac{y_2}{x_2}\right) = \frac{9}{x_1^2}\frac{9}{x_2^2} = \frac{81}{x_1^2x_2^2} > 0.$

2. (a, b, c, d)

Equation of tangent of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$y = mx \pm \sqrt{a^2m^2 - b^2}$ ...(1)

$\because$ (1) is tangent to $\frac{x^2}{(-b^2)} - \frac{y^2}{(-a^2)} = 1$



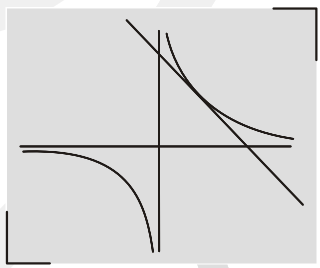
$\Rightarrow a^2m^2 - b^2 = -b^2m^2 - (-a^2)$

$\Rightarrow m^2 = 1$

$\therefore$ Equation of common tangents are $y = \pm x \pm \sqrt{a^2 - b^2}$

3. (a, b)

$y = \frac{c^2}{x}$ is decreasing function



$\Rightarrow$ slope $= -\frac{A}{B} < 0$

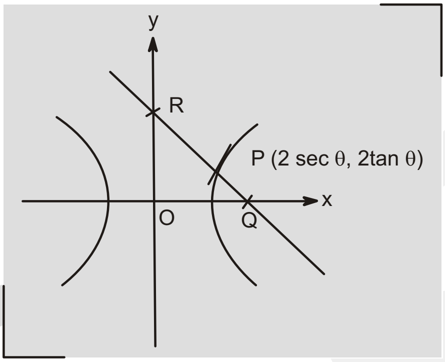
$\Rightarrow \frac{A}{B} > 0$

$\Rightarrow A > 0, B > 0$ or $A < 0, B < 0$

4. (a, b, c, d)

Equation of normal at P

$\frac{x}{\sec\theta} + \frac{y}{\tan\theta} = 4$



$\Rightarrow \quad Q \equiv (4\sec\theta,\ 0),\ R \equiv (0,\ 4\tan\theta)$

$\therefore \quad OP = PQ = PR = 2\sqrt{\sec^2\theta + \tan^2\theta}$

$QR = 4\sqrt{\sec^2\theta + \tan^2\theta}$

5. (b, d)

$$\frac{x^2}{9} - \frac{y^2}{3} = 1;\ 3 = 9(e^2 - 1)$$

$$\Rightarrow \quad e = \frac{2}{\sqrt{3}}$$

Equation of pair of asymptotes is

$$\frac{x^2}{9} = \frac{y^2}{3} \qquad y = \pm\frac{x}{\sqrt{3}}$$

$\Rightarrow$ Angle between asymptotes $= \dfrac{\pi}{3}$

Length of L.R. $= \dfrac{2b^2}{a} = 2$

Product of perpendicular distance of any point $P(3\sec\theta,\ \sqrt{3}\tan\theta)$ from asymptotes

$$P_1P_2 = \frac{|3\sec\theta - 3\tan\theta|}{2}\frac{|3\sec\theta + 3\tan\theta|}{2}$$

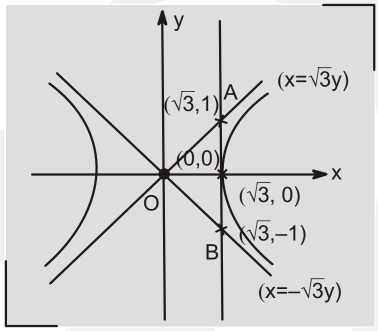
$$= \frac{9}{4} > 2$$

6. (b, c)

In $\Delta\ AOB$,

$OA = 2 = OB = AB$

Area of $\Delta\ AOB = \dfrac{\sqrt{3}}{4}(2)^2 = \sqrt{3}$



7. (a, c, d)

(a)

$$\left(\frac{x}{a}\right)^2 - \left(\frac{y}{b}\right)^2 = \frac{1}{4}\left[\left(t + \frac{1}{t}\right)^2 - \left(t - \frac{1}{t}\right)^2\right] = 1$$

(c) $x^2 - y^2 = (e^t + e^{-t})^2 - (e^t - e^{-t})^2 = 4$

(d) $x^2 - 6 = 2\cos t$

$y^2 \ldots 2 = 2(1 + \cos t)$

$\Rightarrow \quad x^2 - 6 = y^2$

$\Rightarrow \quad x^2 - y^2 = 6$

8. (b, d)

$$\int 2x\,dx = \int 3y\,dy$$

$$\Rightarrow \quad x^2 = \frac{3y^2}{2} + c$$

$$\frac{x^2}{c} - \frac{y^2}{(2c/3)} = 1$$

If $c < 0,\ c + \dfrac{2c}{3} = \dfrac{2c}{3}e^2$

$$\Rightarrow \quad e = \sqrt{\frac{5}{2}}$$

If $c > 0,\ c + \dfrac{2c}{3} = ce^2$

$$\Rightarrow \quad e = \sqrt{\frac{5}{3}}$$

9. (a, b, c, d)

For ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$

and hyperbola $\frac{x^2}{(144/25)}-\frac{y^2}{(81/25)}=1$

$$16-7=16e_1^2$$

and $\frac{144}{25}+\frac{81}{25}=\frac{144}{25}e_2^2$

$$\frac{3}{4}=e_1$$

$$e_2=\frac{15}{12}=\frac{5}{4}$$

$$e_2=\frac{5}{4}$$

$$\text{Foci}\equiv(\pm 3,0)$$

For point of intersection of hyperbola and ellipse

$$\frac{x^2}{144}+\frac{y^2}{63}=\frac{1}{9}$$

$$\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$$

$$\Rightarrow y^2\left(\frac{1}{63}+\frac{1}{81}\right)=\frac{1}{9}-\frac{1}{25}=\frac{16}{9\times 25}$$

$$\Rightarrow y^2\left(\frac{9+7}{81\times 7}\right)=\frac{16}{9\times 25}$$

$$\Rightarrow y=\pm\frac{3\sqrt{7}}{5}$$

$$x^2=\left(1-\frac{63}{25\times 7}\right)16=\frac{16\times 16}{25}$$

$$x=\pm\frac{16}{5}$$

Point of intersection $\equiv\left(\pm\frac{16}{5},\pm\frac{3\sqrt{7}}{5}\right)$

Equation of asymptote $y=\pm\frac{3}{4}x$

Point, $P\equiv\left(\pm\frac{16}{5},\pm\frac{12}{5}\right)$

Sum of square of coordinates of $P=\frac{256+144}{25}=16$

$$x^2+y^2=16=a^2$$

$\Rightarrow$ P lies on auxiliary circle.

10. (a, b, d)

$$-\int\frac{dy}{y-a}=\int\frac{x}{1-x^2}dx$$

$$\Rightarrow -\ln|y-a|=-\frac{1}{2}\ln|1-x^2|-\ln c^2$$

$$\Rightarrow (y-a)^2=c^2|1-x^2|$$

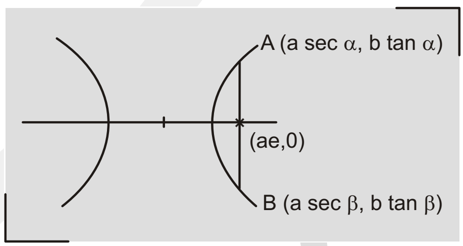
$$\frac{(y-a)^2}{c^2}+x^2=1$$

or $x^2-\frac{(y-a)^2}{c^2}=1$

11. (b, c)

Equation of chord AB is $\frac{x}{a}\cos\frac{\alpha-\beta}{2}-\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\frac{\alpha+\beta}{2}$

Put $(\pm ae,0)$ in above equation



$$\Rightarrow \frac{\cos\frac{\alpha+\beta}{2}}{\cos\frac{\alpha-\beta}{2}}=\frac{e}{1}\text{ or }\frac{-e}{1}$$

Applying componendo and dividendo,

$$\tan\frac{\alpha}{2}\tan\frac{\beta}{2}=\frac{1-e}{1+e}\text{ or }\frac{1+e}{1-e}$$

SOLUTIONS 3

Comprehension: (1)

1. (d) 2. (a) 3. (b)

$$\frac{x^2}{36}+\frac{y^2}{18}=1$$

$$18=36(1-e^2)$$

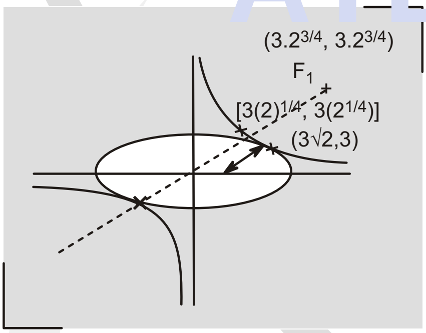
$\Rightarrow \quad e=\frac{1}{\sqrt{2}}$

$\therefore$ Eccentricity of hyperbola, $e'=\sqrt{2}$

$\Rightarrow$ Hyperbola is rectangular with coordinate axes as principal axes drawn in 1^{st} and 3^{rd} quadrant

$\Rightarrow$ Equation of hyperbola is $xy=c^2$

Hyperbola to ellipse



$\Rightarrow \quad \frac{x^2}{36}+\frac{c^4}{18x^2}=1$

$\Rightarrow \quad x^4-36x^2+2c^4=0$

$\Rightarrow \quad D=0$

$\Rightarrow \quad (36)^2-8c^4=0 \Rightarrow c^2=9\sqrt{2}$

$\therefore$ Hyperbola is $xy=9\sqrt{2}$

Equation of common tangent to ellipse and hyperbola is

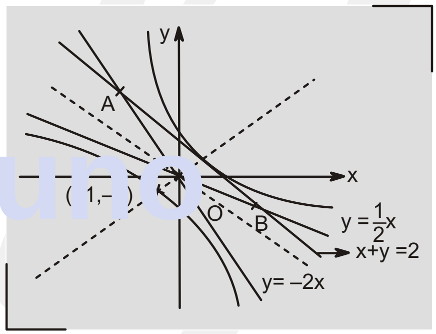
$$\frac{x}{36}(3\sqrt{2})+\frac{y(3)}{18}=1$$

$$\frac{x}{6\sqrt{2}}+\frac{y}{6}=1$$

Comprehension: (2)

1. (c) 2. (b) 3. (b)

$\therefore$ Portion of tangent to hyperbola intercepted between asymptotes is bisected at its point of contact.



$\Rightarrow \quad PA=PB=3\sqrt{2}$

$\Rightarrow \quad A\equiv\left(1+3\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right),1+3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\right)$

$A\equiv(-2,4)$

$B\equiv\left(1-3\sqrt{2}\left(-\frac{1}{\sqrt{2}}\right),1-3\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\equiv(4,-2)\right)$

$B\equiv(4,-2)$

Equation of asymptotes of H are $y=-\frac{1}{2}x$ and $y=-2x$ (The equations are OA and OB)

Equation of pair of asymptotes of H is $(2y+x)(y+2x)=0$

$\Rightarrow \quad 2x^2+5xy+2y^2=0$

Equation of hyperbola 'H' is $2x^2 + 5xy + 2y^2 = c$

$(1, 1)$ lies on $H \Rightarrow c = 9$

$\therefore$ Equation of H is $2x^2 + 5xy + 2y^2 = 9$

Equation of tangent to H at $\left(-1, \frac{7}{2}\right)$ is

$$2x(-1) + \frac{5}{2}\left(x\left(\frac{7}{2}\right) + y(-1)\right) + 2y\left(\frac{7}{2}\right) = 9$$

$\Rightarrow$ $3x + 2y = 4$

Equation of w.r.t. $y = x$ as x-axis and $y = -x$ as y-axis $\frac{X - Y}{\sqrt{2}} = x$, $\frac{X + Y}{\sqrt{2}} = y$

$$\Rightarrow 2\left(\frac{(x-y)^2 + (x+y)^2}{2}\right) + 5\left(\frac{x^2 - y^2}{2}\right) = 9$$

$$\frac{x^2}{2} - \frac{y^2}{18} = 1$$

$2(e^2 - 1) = 18 \Rightarrow e = \sqrt{10}$

Foot of directrices are

$$\left(0 \pm \frac{\sqrt{2}}{\sqrt{10}}\frac{1}{\sqrt{2}}, 0 \pm \frac{\sqrt{2}}{\sqrt{10}\sqrt{2}}\right)$$

$$= \left(\frac{1}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right), \left(-\frac{1}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right)$$

Equation of directrices are

$$x + y = \frac{2}{\sqrt{10}},\ x + y = -\frac{2}{\sqrt{10}}$$

Equation of pair of directrices is

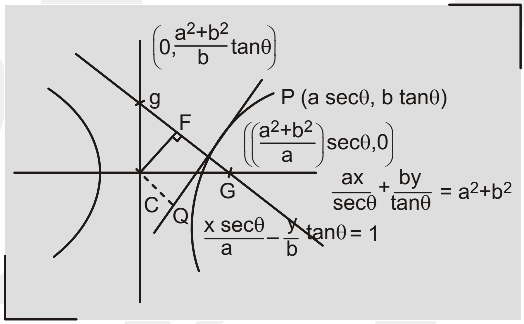
$$\left(x + y - \frac{2}{\sqrt{10}}\right)\left(x + y + \frac{2}{\sqrt{10}}\right) = 0$$

$$x^2 + y^2 + 2xy = \frac{4}{10} = \frac{2}{5}$$

$$5x^2 + 5y^2 + 10xy - 2 = 0$$

Comprehension: (3)

1. (c) 2. (a) 3. (c)



$$PG^2 = \left(\frac{a^2 - (a^2 - b^2)}{a}\right)^2 \sec^2\theta + b^2\tan^2\theta$$

$$\Rightarrow PG^2 = \frac{b^4}{a^2}\sec^2\theta + b^2\tan^2\theta$$

$$= \frac{b^2}{a^2}(b^2\sec^2\theta + a^2\tan^2\theta)$$

$$PG^2 = \frac{b^2}{a^2}(b^2\sec^2\theta + a^2\tan^2\theta)$$

$$PF^2 = C\theta^2 = \frac{1}{\frac{\sec^2\theta}{a^2} + \frac{\tan^2\theta}{b^2}}$$

$$= \frac{a^2 b^2}{a^2\tan^2\theta + b^2\sec^2\theta}$$

$$PF^2 = \frac{a^2 b^2}{b^2\sec^2\theta + a^2\tan^2\theta}$$

Equation of circle 'S' with FG as diameter is

$S: x(x - ae^2\sec\theta) + y^2 = 0$

$|PF||PG| = |\text{Power of } P \text{ w.r.t. circle } 'S'|$

$= |a^2\sec^2\theta(1 - e^2) + b^2\tan^2\theta|$

$|b^2(\tan^2\theta - \sec^2\theta)| = b^2$

$|PF|\,|PG| = b^2$

Comprehension: (4)

1. (b) 2. (a)

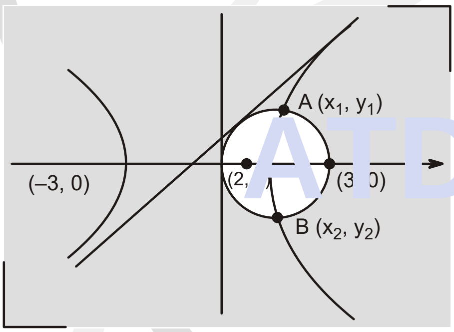
$$x^2 - 8x + \left(\frac{x^2}{9} - 1\right)4 = 0$$

$\Rightarrow \quad 13x^2 - 7x - 36 = 0$

$\Rightarrow \quad (13x + 6)(x - 6) = 0$

$\Rightarrow \quad A \equiv (6, 2\sqrt{3}), B \equiv (6, -2\sqrt{3})$

Let tangent to $\frac{x^2}{9} - \frac{y^2}{4} = 1$ be



$y = \text{mx} \pm \sqrt{9\text{m}^2 - 4}$ ...(1)

$\because$ (1) is tangent to circle

$$\Rightarrow \quad \frac{4m \pm \sqrt{9m^2 - 4}}{\sqrt{1 + m^2}} = 4$$

$\Rightarrow \quad 16m^2 + (9m^2 - 4) \pm 8\sqrt{9m^2 - 4}$

$= 16 + 16m^2$

$\Rightarrow \quad 495m^4 + 104m^2 - 400 = 0$

$$\Rightarrow \quad m^2 = \frac{-104 + \sqrt{(104)\quad + 1600(495)}}{2(495)}$$

$$\Rightarrow \quad m \quad = \frac{4}{5} \quad \Rightarrow \quad m = \frac{2}{\sqrt{5}}$$

1. (b) Equation of tangent is

$$y = \frac{2x}{\sqrt{5}} + \frac{4}{5}$$

$\Rightarrow \quad \sqrt{5}y = 2x + 4$

2. (a) Equation of circle is

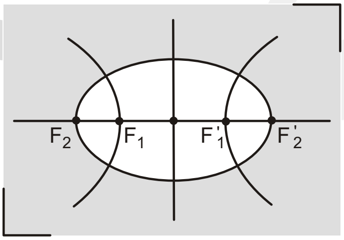
$(x - 6)^2 + (y + 2\sqrt{3})(y - 2\sqrt{3}) = 0$

$\Rightarrow \quad x^2 + y^2 - 12x + 4 = 0$

SOLUTIONS 4

Match the Columns:

1. a – p; b – p, q ; c – p, q, r, s; d – s



$$A = ae_1$$

$$a = Ae_2$$

$\Rightarrow \quad Aa = aAe_1 e_2$

$\Rightarrow \quad e_1 e_2 = 1$

$$b^2 = a^2 - a^2 e_1^2 = a^2 - A^2$$

$$B^2 = A^2 e_2^2 - A^2 = a^2 - A^2$$

$\Rightarrow \quad b = B$

$$e_1 + e_2 = e_1 + \frac{1}{e_1} > 2$$

$\because \quad e_1 \neq 1$

Angle between asymptotes

$$= 2\tan^{-1}\frac{B}{A} = \frac{2\pi}{3}$$

$\Rightarrow \quad \dfrac{B}{A} = \sqrt{3}$

$\Rightarrow \quad e_2^2 - 1 = 3 \Rightarrow e_2 = 2$

$\Rightarrow \quad e_1 = \dfrac{1}{2}$

For point of intersection of ellipse and hyperbola.

$$\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1 \qquad \ldots(1)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \ldots(2)$$

Solving (1) and (2), we get

$$x^2\left(\frac{1}{a^2B^2} + \frac{1}{A^2b^2}\right) = \frac{1}{B^2} + \frac{1}{b^2}$$

$$y^2\left(\frac{1}{a^2B^2} + \frac{1}{A^2b^2}\right) = \frac{1}{A^2} - \frac{1}{a^2}$$

$\Rightarrow \quad \dfrac{x^2}{y^2} = \dfrac{(b^2 + B^2)A^2a^2}{(a^2 - A^2)B^2b^2}$

$$b^2 + B^2 = 2(a^2 - A^2) \quad ; \quad B = b$$

$$e_1 = \frac{1}{\sqrt{2}} \quad \text{and} \quad e_2 = \sqrt{2}$$

$\Rightarrow \quad A = \dfrac{a}{\sqrt{2}}$

$\Rightarrow \quad a^2 = 2A^2$

$\therefore \quad \dfrac{x^2}{y^2} = \dfrac{2A^2(2A^2)}{B^4} = \dfrac{4A^4}{B^4}$

$$= \frac{4}{(e_2^2 - 1)^2} = 4$$

$\Rightarrow \quad \dfrac{x^2}{y^2} = 4$

2. a – q; b – q, r, s; c – r ; d – t

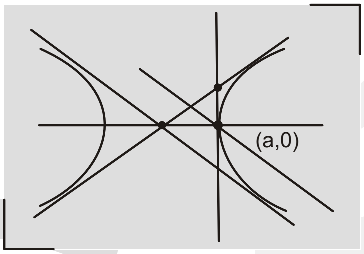
(a) Equation of pair of asymptotes is

$$5x^2 - 2\sqrt{7}\,xy - y^2 - 2x + C = 0$$

$$\tan\theta = \frac{2\sqrt{7 - 5(-1)}}{5 - 1} = \sqrt{3}$$

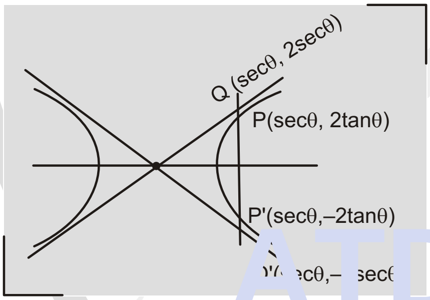
$\Rightarrow \quad \theta = \dfrac{\pi}{3}$

(b) From fig. it is clear that $\theta \in \left(0, \frac{\pi}{2}\right)$



(c) $|PQ||PQ'| = 2(\sec\theta - \tan\theta)$

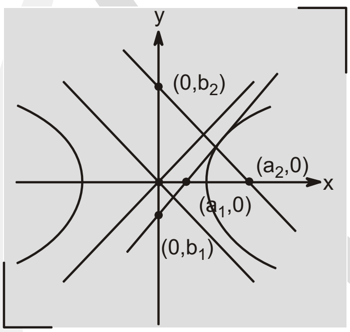
$2(\sec\theta + \tan\theta)$



(d) Equation of tangent is

$x\sec\theta - y\tan\theta = 2$

$\therefore \quad a_1 = 2\cos\theta,\ b_1 = -2\cot\theta$



Equation of normal is

$$\frac{x}{\sec\theta} + \frac{y}{\tan\theta} = 4$$

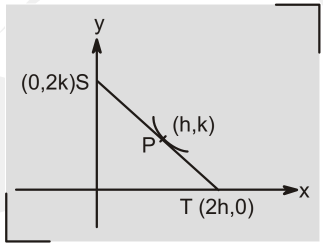
$\therefore \quad a_2 = 4\sec\theta,\ b_2 = 4\tan\theta$

$\therefore \quad a_1a_2 + b_1b_2 = 8 - 8 = 0$

$a_1a_2 + b_1b_2 = 0$

3. a → s ; b → r ; c → p, s ; d → q, s

(a) $\left(\frac{dy}{dx}\right)_{(h,k)} = -\frac{k}{h}$

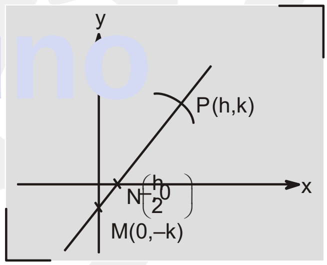


$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x}$

$\Rightarrow \quad \ln(xy) = c$

$\Rightarrow \quad xy = \lambda \Rightarrow$ Hyperbola

(b) $\left(\frac{dy}{dx}\right)_{(h,k)} = -\frac{h}{2k}$

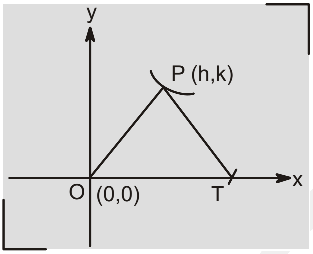


$\Rightarrow \quad \frac{dy}{dx} = -\frac{x}{2y}$

$\Rightarrow \quad y^2 + \frac{x^2}{2} = \lambda \quad \Rightarrow$ Ellipse

(c) $(TP)^2 = y^2 + \left(\frac{y}{\left(\frac{dy}{dx}\right)}\right)^2$

$\Rightarrow \quad y^2 + \left(\frac{y}{\left(\frac{dy}{dx}\right)}\right)^2 = x^2 + y^2$

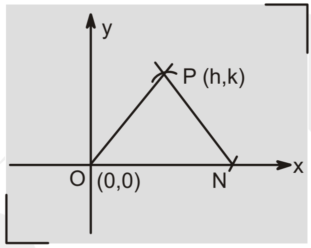


$$\Rightarrow \quad \frac{dy}{dx} = \pm \frac{y}{x}$$

$$\Rightarrow \quad xy = \lambda \quad \text{or} \quad y = \lambda x$$

$\Rightarrow$ hyperbola or straight line

(d) $x^2 + y^2 = y^2 + \left(y \dfrac{dy}{dx} \right)^2$



$$\Rightarrow \quad \frac{dy}{dx} = \pm \frac{x}{y}$$

$$y^2 - x^2 = \lambda$$

or $\quad y^2 + x^2 = \lambda$

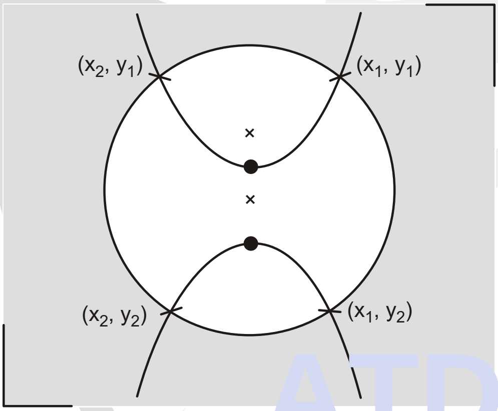
$\Rightarrow$ hyperbola or circle

SOLUTIONS 5

Subjective Problems

1. (40) $(x+3)^2+(y-12)^2=81$

$$(x+3)^2-(y-8)^2=-9$$



Let (h,k) be intersection point, then

$(h+3)^2+(k-12)^2=81$ ...(1)

$(h+3)^2-(k-8)^2=-9$ ...(2)

(1) + (2)

$\Rightarrow$ $2h^2+12h-8k+26=0$

$h^2+6h-4k+13=0$ ....(3)

(1) – (2)

$\Rightarrow$ $2k^2-40k+118=0$

$k^2-20k+59=0$

$\Rightarrow$ $y^2-20k+59=0 \genfrac{}{}{0pt}{}{\nearrow y_1}{\searrow y_2}$ ...(4)

$\therefore$ $y_1+y_2=20$, $y_1>0$, $y_2>0$

Distance of (h,k) from $(-3,\ 2)$

$d=\sqrt{(h+3)^2+(k-2)^2}=k$

$\therefore$ Sum of distances of all intersection points

$=2(y_1+y_2)=2(20)=40$

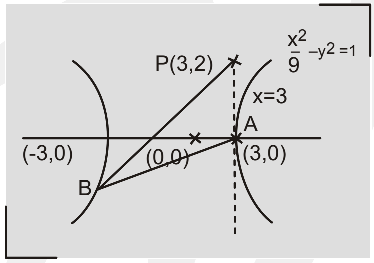
2. (8) Equation of tangent through P is

$y=mx\pm\sqrt{9m^2-1}$ ....*

Put $(3,2)$ in (*) we get,

$9m^2+4-12m=9m^2-1;$

$$m=\frac{5}{12},\ m\to\infty$$



Equation of PB is

$5x-12y+9=0$ ...(1)

Let B is (h,k), then

Eqn. of PB is

$xh-9yk-0=0$ ...(2)

(1) and (2) are identical

$\Rightarrow$ $\frac{h}{5}=\frac{9k}{12}=-1$

$(h,k)=\left(-5,-\frac{12}{9}\right)$

Area of $\Delta\, PAB=\frac{1}{2}\times 2\times(8)$

3. (0) Equation of pair of asymptotes is

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$

For intersection points of transversal and pair of asymptotes

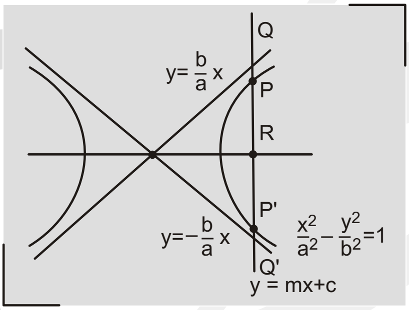
$\frac{x^2}{a^2}-\frac{(mx+c)^2}{b^2}=0 \genfrac{}{}{0pt}{}{\nearrow x_1}{\searrow x_2}$ ...(1)

For intersection points of transversal and hyperbola,

$$\frac{x^2}{a^2} - \frac{(mx+c)^2}{b^2} = 1 \begin{matrix} x_3 \\ x_4 \end{matrix} \quad \text{...(2)}$$

$\Rightarrow \quad x_1 + x_2 = x_3 + x_4$

$\Rightarrow \quad y_1 + y_2 = y_3 + y_4$



$\therefore$ Midpoints of PP' and QQ' are same (say R)

$\therefore \quad RQ = RQ'$ ...(1)

$RP = RP'$ ...(2)

$(1) - (2) \Rightarrow PQ = P'Q'$

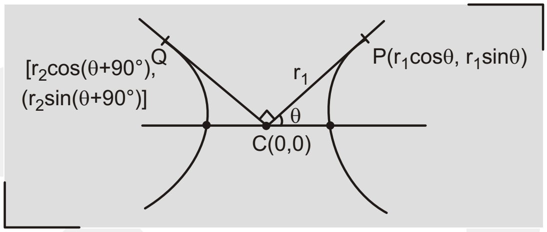
$PQ + PP' = P'Q' + PP'$

$\Rightarrow P'Q = PQ'$

$\therefore \quad (PQ - P'Q') + (PQ' + P'Q) = 0$

4. (4) $\dfrac{1}{CP^2} + \dfrac{1}{CQ^2} = \dfrac{1}{r_1^2} + \dfrac{1}{r_2^2}$

P lies on $9x^2 - 5y^2 = 1$



$\Rightarrow \quad 9r_1^2\cos^2\theta - 5r_1^2\sin^2\theta = 1$

$\Rightarrow \quad \dfrac{1}{r_1^2} = 9\cos^2\theta - 5\sin^2\theta$

Q lies on $9x^2 - 5y^2 = 1$

$\Rightarrow \quad 9r_2^2\sin^2\theta - 5r_2^2\cos^2\theta = 1$

$\Rightarrow \quad \dfrac{1}{r_2^2} = 9\sin^2\theta - 5\cos^2\theta$

$\dfrac{1}{r_1^2} + \dfrac{1}{r_2^2} = 9 - 5 = 4$

5. (3) Equation of normal at $(2\sec\theta, \tan\theta)$ is

$$\frac{2x}{\sec\theta} + \frac{y}{\tan\theta} = 5$$

Slope $= -1 \Rightarrow \quad -\dfrac{2\tan\theta}{\sec\theta} = -1$

$\Rightarrow \quad \theta = \dfrac{\pi}{6}$

$\therefore$ normal becomes $y = -x + \dfrac{5}{\sqrt{3}}$

$\because$ It is tangent to ellipse $\Rightarrow \quad a^2 + b^2 = \dfrac{25}{3}$

$\Rightarrow \quad \dfrac{9}{25}(a^2 + b^2) = 3$