



STRAIGHT LINE

01 Slope of a Straight Line

If the line makes an angle θ with positive direction of x-axis, then $\tan \theta$ is called slope of the line and is denoted by m .

02 Various forms of Line

01 Slope intercept form: The line with slope m and y intercept c is $y=mx+c$

02 Slope point form : The line with slope m and passing through the point (x_1, y_1) is $y-y_1=m(x-x_1)$.

03 Two point form : The line passing through the point (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} (x - x_1)$$

04 Intercept form :

$$\frac{x}{a} + \frac{y}{b} = 1$$

Here, a and b are x intercept and y intercept respectively which may be positive or negative

05 Normal form : The line whose normal makes an angle α with positive x axis and has length $=p$ is

$$x \cos \alpha + y \sin \alpha = p.$$

06 6. Distance or parametric form :

$$\frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r$$

07 General form of line : The equation $ax + by + c = 0$ where a and b are not simultaneously zero is called general form of line.

Note

(a) x - intercept made by $ax + by + c = 0$ is $-\frac{c}{a}$.

(b) y - intercept made by $ax + by + c = 0$ is $-\frac{c}{b}$.

(c) Slope of the line $ax + by + c = 0$ is $\left(-\frac{a}{b}\right)$.

(d) Area of triangle which the line $ax+by+c=0$ makes with coordinate axes $= \frac{c^2}{2ab}$.

03 Angle Between Two Lines

Let the slope of the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are respectively m_1 and m_2 and if the angle between these lines be θ , then

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{a_1 b_2 - a_2 b_1}{a_1 a_2 + b_1 b_2} \right|$$

(a) Condition for the lines to be parallel is

$$m_1 = m_2 \text{ or } \frac{a_1}{a_2} = \frac{b_1}{b_2}$$

(b) Condition for the lines to be coincidental is

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

(c) Condition for the lines to be perpendicular is

$$m_1 m_2 = -1$$

$$a_1 a_2 + b_1 b_2 = 0$$

04 Family of Lines

01 Family of lines which are parallel to the line $ax + by + c = 0$ is $ax + by + \lambda = 0$ where $\lambda \in \mathbb{R}$

02 Family of lines which is perpendicular to the line $ax + by + c = 0$ is $bx - ay + \lambda = 0$ where $\lambda \in \mathbb{R}$

03 Family of lines passing through the intersection point of $L_1 = a_1x + b_1y + c_1 = 0$ and $L_2 = a_2x + b_2y + c_2 = 0$ is $L_1 + \lambda L_2 = 0$ where, $\lambda \in \mathbb{R}$

05 Distance between a Point and a line

Let (x_1, y_1) be the given point and $ax + by + c = 0$ be the given line then distance between them, is

$$p = \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

06 Distance between Two Parallel Lines

Let the equation of two parallel lines be $ax + by + c = 0$ and $ax + by + c' = 0$,

$$\text{then distance between them is given by } P = \left| \frac{c - c'}{\sqrt{a^2 + b^2}} \right|$$

Note

01 If the foot of perpendicular drawn from point (x_1, y_1) to the line $ax + by + c = 0$ be (h, k) , then,

$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = -\left(\frac{ax_1 + by_1 + c}{a^2 + b^2} \right)$$

02 If the image of point (x_1, y_1) in the line mirror $ax + by + c = 0$ be (α, β) , then $\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = -2 \left(\frac{ax_1 + by_1 + c}{a^2 + b^2} \right)$

07 Concurrent Lines

Three or more lines are said to be concurrent if they have only one point in common. Let the three concurrent lines are $a_r x + b_r y + c_r = 0$

where $r = 1, 2, 3$, then

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

Note

If lines are concurrent Δ must be zero but $\Delta = 0$ not necessarily imply the lines are concurrent.

08 Comparison of Two Points with Respect to a Line

Let the given line be $L(x, y) = ax + by + c = 0$ and the points are $P(x_1, y_1)$ and $Q(x_2, y_2)$, then

1. If $L(x_1, y_1) \cdot L(x_2, y_2) > 0$ points P and Q lies on the same side of line $L=0$

2. If $L(x_1, y_1) \cdot L(x_2, y_2) < 0$ points P and Q lies on the opposite side of line $L=0$



09 Angle Bisectors of Angle Between Two Lines

The equations of angle bisectors of the angle between the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

is given by

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

Note

In the above equation if c_1 and c_2 are of same sign, then taking the sign same as the sign of $a_1a_2 + b_1b_2$ we always get angle bisector of the given lines. Also by taking + in above formula we get the bisector of that angle region which contains origin.

- Equation of straight line passing through given point (x_1, y_1) and making a given angle α with the given line

$$y = mx + c, \text{ are } y - y_1 = \frac{m - \tan\alpha}{1 + m \tan\alpha}(x - x_1) \text{ or } y - y_1 = \frac{m + \tan\alpha}{1 - m \tan\alpha}(x - x_1)$$

- The image of the line $ax + by + c = 0$ in the line $X = \lambda$ is $a(2\lambda - x) + by + c = 0$

- The image of the line $ax + by + c = 0$ in the line $y = \lambda$ is $ax + b(2\lambda - y) + c = 0$

10 Non-homogeneous equation of degree 2

The equation $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \dots$ (i) is called non-homogeneous equation of degree 2.

$$\text{Let, } \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

- Equation (i) represents a pair of straight lines if $\Delta = 0$, $h^2 \geq ab$, $g^2 \geq ac$ and $f^2 \geq bc$.

- If lines given by (i) have from $y = m_1x + c_1$ and $y = m_2x + c_2$ then $m_1 = \frac{-2h \pm \sqrt{4h^2 - 4ab}}{2a}$ or $m_2 = \frac{-2h \mp \sqrt{4h^2 - 4ab}}{2a}$

- If angle between the lines given by (i) be θ , then $\tan\theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$

Condition for line pair (i) to represents a pair of parallel lines is $h^2 = ab$

Condition for line pair (i) to represents a pair of perpendicular lines is $a + b = 0$

- If the line pair given by (i) be the pair of parallel lines, then distance between them is $= 2\sqrt{\frac{g^2 - ac}{a(a+b)}}$ or $2\sqrt{\frac{f^2 - bc}{b(a+c)}}$

- Condition for the line pair (i) to represent coincidental lines is $h^2 = ab$ and $g^2 = ac$ and $f^2 = bc$

- Point of intersection of the lines given by (i) is $\left(\frac{hf - bg}{ab - h^2}, \frac{hg - af}{ab - h^2} \right) = (\alpha, \beta)$

- Equation of angle bisector of the angle between line pair (i) is $\frac{(x - \alpha)^2 - (y - \beta)^2}{a - b} = \frac{(x - \alpha)(y - \beta)}{h}$

Note

In homogeneous case, $ax^2 + 2hxy + by^2 = 0$ replace g, f, c by 0.

11 Points to Remember

- Equation of lines perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$ is given by $bx^2 - 2hxy + ay^2 = 0$

- Two pair of straight lines viz. $a_1x^2 + 2h_1xy + b_1y^2 = 0$ and $a_2x^2 + 2h_2xy + b_2y^2 = 0$ have

$$(a) \text{ a line in common if } \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}^2 = 4 \begin{vmatrix} a_1 & h_1 \\ a_2 & h_2 \end{vmatrix} \begin{vmatrix} h_1 & b_1 \\ h_2 & b_2 \end{vmatrix}$$

$$(b) \text{ both lines in common if } \frac{a_1}{a_2} = \frac{h_1}{h_2} = \frac{b_1}{b_2}.$$

- Equation of line pair joining the point of intersection of curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and line $lx + my = 1$ with origin is given by $ax^2 + 2hxy + by^2 + (2gx + 2fy)(lx + my) + c(lx + my)^2 = 0$

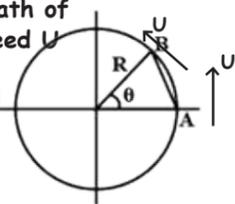


- Distance = Length of actual path
- Displacement = Length of shortest path
- Distance ≥ |displacement|



A particle moves from A to B in a circular path of radius R covering an angle θ with uniform speed U

- Distance = $\widehat{AB} = R\theta$
- Displacement = $AB = 2R\sin(\frac{\theta}{2})$
- Ratio of Displacement to Distance = $\frac{\sin(\frac{\theta}{2})}{\frac{\theta}{2}}$
- Time $t = \frac{R\theta}{U}$
- Average Velocity = $\frac{2U\sin(\frac{\theta}{2})}{\frac{\theta}{2}}$
- Average Acceleration = $\frac{U^2\sin(\frac{\theta}{2})}{R\frac{\theta}{2}}$



For uniform motion

Displacement = velocity x time
Average speed = |average velocity| = |instantaneous velocity|

Time average speed

$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{v_1 t_1 + v_2 t_2 + v_3 t_3 + \dots}{t_1 + t_2 + t_3 + \dots}$$

If $t_1 = t_2 = t_3 = \dots = t_n$
then
 $v_{av} = \frac{v_1 + v_2 + v_3 + \dots + v_n}{n}$

for v_1 & v_2 ,
 $v_{avg} = \frac{v_1 + v_2}{2}$ (Arithmetic mean of speeds)

Distance average speed

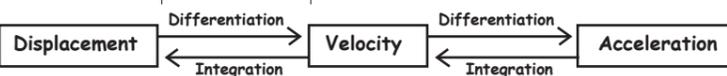
$$v_{av} = \frac{\text{Total distance covered}}{\text{Total time elapsed}} = \frac{s_1 + s_2 + s_3 + \dots + s_n}{t_1 + t_2 + t_3 + \dots + t_n} = \frac{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \dots + \frac{s_n}{v_n}}{\frac{s_1}{v_1} + \frac{s_2}{v_2} + \frac{s_3}{v_3} + \dots + \frac{s_n}{v_n}}$$

If $s_1 = s_2 = s_3 = \dots = s_n$
then
 $v_{av} = \frac{1}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} + \dots + \frac{1}{v_n}}$ for v_1 & v_2 ,
 $v_{avg} = \frac{2v_1 v_2}{v_1 + v_2}$ (Harmonic mean of speeds)

Instantaneous Velocity $v = \frac{dx}{dt}$ $\Delta x = \int v dt$

Instantaneous Acceleration $a = \frac{dv}{dt}$ $\Delta v = \int a dt$

Case 1: $v = f(t)$ or $x = f(t)$
Case 2: $V = f(x)$
Case 3: $t = f(x)$
then
 $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$
 $a = \frac{dv}{dx} \frac{dx}{dt}$
 $a = -(\text{double diff. of } t \text{ w.r.t. } x) \times V^3$



Motion with constant acceleration: Equations of motion

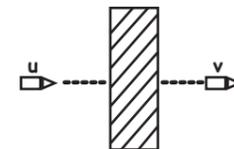
(i) $v = u + at$
(ii) $S = ut + \frac{1}{2} at^2$

- A Person travels from A to B covers unequal distances in equal interval of time with constant acceleration a then
initial velocity $U = \frac{3S_1 - S_2}{2t}$
Acceleration $a = \frac{S_2 - S_1}{t^2}$



(iii) $v^2 = u^2 + 2a.s$

- The number of planks required to stop the bullet
 $N = \frac{u^2}{u^2 - v^2}$

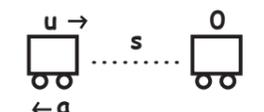


- The two ends of a train moving with constant acceleration pass a certain point with velocities u and v. The velocity with which the middle point of the train passes the same point is

$v_{Mid} = \sqrt{\frac{u^2 + v^2}{2}}$



- Calculation of stopping distance $s = \frac{u^2}{2a}$



(iv) $s_n = u + \frac{a}{2}(2n-1)$

- Ratio of distance travelled in equal interval of time in a uniformly accelerated motion from rest
 $S_1 : S_2 : S_3 = 1 : 3 : 5$



- for uniform accelerated motion $v_{avg} = \frac{u+v}{2}$

Different Cases	v-t graph	s-t graph
1. Uniform motion		
2. Uniformly accelerated motion with u = 0 at t = 0		
3. Uniformly accelerated with u ≠ 0 at t = 0 & s = 0 at t = 0		
4. Uniformly accelerated motion with u ≠ 0 and s = s0 at t = 0		
5. Uniformly retarded motion till velocity becomes zero		
6. Uniformly retarded then accelerated in opposite direction		

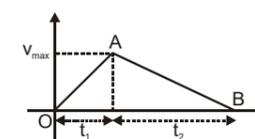
Important points about graphical analysis of motion

- Instantaneous velocity is the slope of position-time curve $[v = \frac{dx}{dt}]$
- Area of v-t curve gives displacement $[\Delta x = \int v dt]$
- Slope of velocity-time curve = instantaneous acceleration $[a = \frac{dv}{dt}]$
- Area of a-t curve gives change in velocity. $[\Delta v = \int a dt]$

MOTION ALONG A STRAIGHT LINE

A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β , to come to rest. If the total time elapsed is t, then

$v_{max} = \frac{\alpha\beta}{\alpha+\beta} t$ Total Distance = $\frac{1}{2} \left(\frac{\alpha\beta}{\alpha+\beta} \right) t^2$

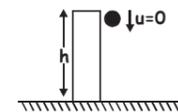


MOTION UNDER GRAVITY

Sign Convention

- (i) initial velocity
+ve = upward motion
-ve = downward motion
- (ii) Acceleration
Always -ve
- (iii) Displacement
+ve = final position is above initial position
-ve = final position is below initial position
Zero = final position & initial position are at same level

- Object is dropped from top of a tower
(i) Ratio of displacement in equal interval of time $S_1 : S_2 : S_3 : \dots = 1 : 3 : 5 : \dots$
(ii) Ratio of time of covering equal distance
 $t_1 : (t_2 - t_1) : (t_3 - t_2) : \dots : (t_n - t_{n-1}) = 1 : (\sqrt{2} - \sqrt{1}) : (\sqrt{3} - \sqrt{2}) : \dots : (\sqrt{n} - \sqrt{n-1})$

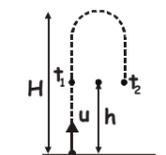


- If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance) then
(i) Maximum height attained $H = \frac{u^2}{2g}$
(ii) Time of ascent = time of descent $\frac{u}{g}$
(iii) Total time of flight = $\frac{2u}{g}$
(iv) Velocity of fall at the point of projection = u (downwards)



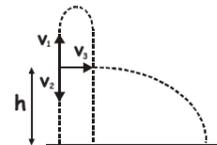
- At any point on its path the body will have same speed for upward journey and downward journey. If a body thrown upwards crosses a point in time t_1 & t_2 respectively then

height of point $h = \frac{1}{2} g t_1 t_2$ Maximum height $H = \frac{1}{8} g (t_1 + t_2)^2$



- Time of flight = $t_1 + t_2 = \frac{2u}{g}$
- A body is thrown upward, downward & horizontally with same speed takes time t_1, t_2 & t_3 respectively to reach the ground then

$t_3 = \sqrt{t_1 t_2}$ & height from where the particle was throw is $h = \frac{1}{2} g t_1 t_2$



01

Which of the following statements is/are incorrect?
 I. Aqueous solution of sugar conducts electricity.
 II. Conductance of electricity increases with an increase in concentration of common salt in aqueous glucose solution.
 III. Aqueous solution of acetic acid mainly contains unionized acetic acid molecules and only some CH_3COO^- and H_3O^+ ions.
 (a) III only (b) I only
 (c) II and III only (d) I, II and III

ELECTROLYTE

Substance which allow the flow of electricity through their aqueous solution or through their molten form.

- 01 STRONG ELECTROLYTE electrolyte which dissociates completely into ions. eg : HCl, NaOH
- 02 WEAK ELECTROLYTE electrolyte which dissociates partially into ions. eg : CH_3COOH , NH_4OH

IONIC EQUILIBRIUM

Equilibrium established in between ions and unionised Salt in a weak electrolyte.

Eg. In weak electrolytes, equilibrium is established between ions and the unionized molecules. This type of equilibrium I.involves ions in aqueous solution. II.is called physical equilibrium. III.is due to complete ionization IV.is called ionic equilibrium. The correct statement(s) is/are
 (A) I and IV only (B) II and III only (C) IV only (D) I, II, III and IV
ANS : (A)

02

Which of the following is conjugate acid of SO_4^{2-} ?
 (a) HSO_4^- (b) H^+
 (c) H_2SO_4 (d) SO_4^{2-}

ACID-BASE CONCEPTS

- 01 ARRHENIUS CONCEPT Acids are H^+ ions donors and bases are OH^- ion donors.
 $\text{HCl} \rightarrow \text{H}^+ + \text{Cl}^-$ (acid) $\text{NaOH} \rightarrow \text{Na}^+ + \text{OH}^-$ (base)
- 02 BRONSTED - LOWRY CONCEPT Acids are protons (H^+ ion) donors and bases are proton acceptors.
 Eg : $\text{CH}_3\text{COOH} + \text{H}_2\text{O} \rightleftharpoons \text{CH}_3\text{COO}^- + \text{H}_3\text{O}^+$
 acid base Base acid
 base + $\text{H}^+ \rightarrow$ Conjugate acid acid - $\text{H}^+ \rightarrow$ Conjugate base
 Conjugate acid of $\text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+$ Conjugate base of $\text{H}_2\text{O} \rightarrow \text{OH}^-$
- 03 LEWIS CONCEPT Bases are lone pair e^- donors & acids are lone pair e^- acceptors.
 Lewis acid - Substances having Vacant space in their valence shell to accommodate lone pair of e^-
 Eg : BF_3 , AlCl_3 , H^+ , Ag^+
 Lewis base - Substances having lone pair of e^- in their valence shell
 Eg : NH_3 , H_2O , Cl^- , OH^-

03

Which of the following species can act both as an acid as well as a base?
 (a) SO_4^{2-} (b) HSO_4^-
 (c) PO_4^{3-} (d) Cl^-

IONISATION AND IONISATION CONSTANT

In all ionisation reactions concentration of water $[\text{H}_2\text{O}]$ is taken as constant

- 01 IONISATION OF WATER
 $\text{H}_2\text{O} + \text{H}_2\text{O} \rightleftharpoons \text{H}_3\text{O}^+ + \text{OH}^-$
 Ionisation constant of water, $K_w = [\text{H}_3\text{O}^+][\text{OH}^-]$ or $K_w = [\text{H}^+][\text{OH}^-]$
- 02 IONISATION OF ACID
 $\text{CH}_3\text{COOH} + \text{H}_2\text{O} \rightleftharpoons \text{CH}_3\text{COO}^- + \text{H}_3\text{O}^+$
 Ionisation constant, $K_a = \frac{[\text{CH}_3\text{COO}^-][\text{H}_3\text{O}^+]}{[\text{CH}_3\text{COOH}]}$ or $K_a = \frac{[\text{CH}_3\text{COO}^-][\text{H}^+]}{[\text{CH}_3\text{COOH}]}$
- 03 IONISATION OF BASE
 Eg : NH_4OH
 Ionisation constant of base $K_b = \frac{[\text{NH}_4^+][\text{OH}^-]}{[\text{NH}_4\text{OH}]}$

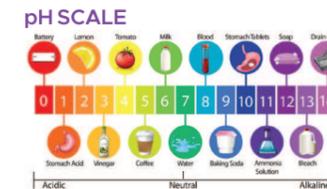
04

What will be the pH of a soft drink if hydrogen ion concentration in sample is $3.8 \times 10^{-3} \text{ M}$?
 (a) 3.8 (b) 5.04
 (c) 2.42 (d) 9.2

VALUE OF IONIC PRODUCT

At 25°C , $K_w = [\text{H}_3\text{O}^+][\text{OH}^-] = 1 \times 10^{-14}$
 01 If $[\text{H}^+] = [\text{OH}^-] = 1 \times 10^{-7} \text{ mol/L}$: solution will be neutral
 02 If $[\text{H}^+] > [\text{OH}^-]$ i.e $[\text{H}^+] > 1 \times 10^{-7} \text{ mol/L}$ solution will be acidic
 03 If $[\text{H}^+] < [\text{OH}^-]$ i.e $[\text{H}^+] < 1 \times 10^{-7} \text{ mol/L}$ solution will be basic

pH
 $\text{pH} = -\log [\text{H}^+]$
 pH of a neutral solution is 7
 pH of an acidic solution is < 7
 pH of a basic solution is > 7



05

What will be the ionisation constant of formic acid if its 0.01 M solution is 14.5% ionised?
 (a) 2.1×10^{-4} (b) 14.5
 (c) 0.145 (d) 1.45×10^{-4}

IMPORTANT EQUATIONS

- $\text{pH} = -\log [\text{H}^+]$ $\text{pKa} = -\log [K_a]$
- $\text{pOH} = -\log [\text{OH}^-]$ $\text{pKb} = -\log [K_b]$
- $\text{pKw} = \text{pH} + \text{pOH} = 14$
- $K_a \times K_b = K_w = 1 \times 10^{-14}$ at 25°C
- $\text{pKa} + \text{pKb} = 14$ at 25°C

DEGREE OF DISSOCIATION (α)

$$\alpha = \frac{\text{Number of moles ionised}}{\text{Total no. of moles}}$$
 Eg : 30% ionisation means $\alpha = \frac{30}{100} = 0.3$

Weak acid
 $K_a = C\alpha^2$
 $\alpha = \sqrt{K_a/c}$
 $[\text{H}_3\text{O}^+] = C\alpha$
 $[\text{H}^+] = \sqrt{K_a \cdot c}$

Weak base
 $K_b = C\alpha^2$
 $\alpha = \sqrt{K_b/c}$
 $[\text{OH}^-] = C\alpha$
 $[\text{OH}^-] = \sqrt{K_b \cdot c}$

06

Equimolar solutions of HF, HCOOH and HCN at 298 K have the values of K_a as 6.8×10^{-4} , 1.8×10^{-4} and 4.8×10^{-9} respectively. What will be the order of their acidic strength?
 (a) $\text{HF} > \text{HCN} > \text{HCOOH}$
 (b) $\text{HF} > \text{HCOOH} > \text{HCN}$
 (c) $\text{HCN} > \text{HF} > \text{HCOOH}$
 (d) $\text{HCOOH} > \text{HCN} > \text{HF}$

ACID STRENGTH

- acid strength $\propto K_a$
- acid strength $\propto \frac{1}{\text{pKa}}$
- For an acid HA
 As HA bond strength decreases, acid strength increases
- As we move from top to bottom, size of anion increases \therefore HA bond strength decreases & acid strength increases.
 eg :- Acid strength of $\text{HI} > \text{HBr} > \text{HCl} > \text{HF}$
- As we move from left to right, electronegativity increases \therefore HA bond becomes more polar & acid strength increases.
 eg :- Acid strength of $\text{HF} > \text{H}_2\text{O} > \text{NH}_3 > \text{CH}_4$

07

For a polybasic acid, the dissociation constants have a different values for each step, e.g.,
 $\text{H}_2\text{A} \rightleftharpoons \text{H}^+ + \text{HA}^-$; $K = K_{a1}$
 $\text{HA}^- \rightleftharpoons \text{H}^+ + \text{A}^{2-}$; $K = K_{a2}$
 $\text{HA}^{2-} \rightleftharpoons \text{H}^+ + \text{A}^{3-}$; $K = K_{a3}$
 What is the observed trend of dissociation constants in successive stages?
 (a) $K_{a1} > K_{a2} > K_{a3}$ (b) $K_{a1} = K_{a2} = K_{a3}$
 (c) $K_{a1} < K_{a2} < K_{a3}$ (d) $K_{a1} = K_{a2} + K_{a3}$

POLYPROTIC ACIDS & POLYHYDROXY BASES

- Acids having ionisable hydrogen are known as protic acids or basic acids. eg :- HCl, H_2SO_4 , $\text{H}_2\text{C}_2\text{O}_4$, H_3PO_4 , CH_3COOH etc
- Here H_2SO_4 , $\text{H}_2\text{C}_2\text{O}_4$ & H_3PO_4 are having more than one ionisable hydrogen, they are known as polyprotic acids
- Polyprotic acids ionises in stepwise.
- Here the removal of H^+ ion from neutral molecule is easy, but it is difficult to remove further H^+ ions from their corresponding anions $\therefore K_{a1} > K_{a2} > K_{a3}$
- In case of polyhydroxy bases or polyacidic bases like $\text{Fe}(\text{OH})_3$, $\text{Ba}(\text{OH})_2$ etc $K_{b1} > K_{b2} > K_{b3}$

08

Which of the following salts does not show its correct nature mentioned against it?
 (a) NaCl solution - Neutral
 (b) NaCN solution - Acidic
 (c) NH_4NO_3 solution - Acidic
 (d) KF solution - Basic

HYDROLYSIS OF SALTS

- $\text{Salt} + \text{H}_2\text{O} \xrightarrow{\text{hydrolysis}} \text{acid} + \text{base}$
- SALT OF STRONG ACID & STRONG BASE
 Strong acid (SA) & strong base (SB) \rightarrow Neutral, It's pH = 7
 eg :- NaCl $\text{NaOH} + \text{HCl} \rightarrow$ Neutral Na_2SO_4 $\text{NaOH} + \text{H}_2\text{SO}_4 \rightarrow$ Neutral
 SB SA SB SA
 Note : Neutral salts do not undergo hydrolysis
 - SALT OF STRONG ACID & WEAK BASE (SA + WB \rightarrow acidic), It's pH < 7
 eg :- NH_4Cl $\text{NH}_4\text{OH} + \text{HCl} \rightarrow$ acidic (cationic hydrolyses)
 WB SA
 - SALT OF WEAK ACID & STRONG BASE (WA + SB \rightarrow Basic), It's pH > 7
 eg :- CH_3COONa $\text{CH}_3\text{COOH} + \text{NaOH} \rightarrow$ Basic (anionic hydrolysis)
 - SALT OF WEAK ACID & WEAK BASE
 $\text{WA} + \text{WB} \rightarrow$ slightly acidic/basic
 $\text{pH} = 7 + \frac{1}{2} [\text{pKa} - \text{pKb}]$
 eg : $(\text{NH}_4)_2\text{CO}_3$

09

Which one of the following pairs of solution is not an acidic buffer?
 (a) CH_3COOH and CH_3COONa
 (b) H_2CO_3 and Na_2CO_3
 (c) H_3PO_4 and Na_3PO_4
 (d) HClO_4 and NaClO_4

COMMON ION EFFECT

- Ionisation of a weak electrolyte can be suppressed by the addition of salt which contain common ion.
- Ionisation of a weak acid, CH_3COOH can be suppressed by the addition of a salt like CH_3COONa . Common ion : CH_3COO^-
- Ionisation of a weak base, NH_4OH can be suppressed by the addition of a salt like NH_4Cl . Common ion : NH_4^+

BUFFER SOLUTIONS

Solution which can resist the change in pH on addition of small drops of acids or base.

- Acidic buffer : Mixture of equimolar concentrations of a weak acid and it's salt with a strong base. $\text{CH}_3\text{COOH} / \text{CH}_3\text{COONa}$.

$$\text{pH} = \text{pKa} + \log \frac{[\text{salt}]}{[\text{acid}]}$$
 Henderson - Hasselbalch equation
- Basic buffer : Mixture of equimolar concentrations of a weak base and it's salt with a strong acid. $\text{NH}_4\text{OH} / \text{NH}_4\text{Cl}$.

$$\text{pOH} = \text{pKb} + \log \frac{[\text{salt}]}{[\text{base}]}$$

10

The solubility of $\text{AgCl}_{(s)}$ with solubility product 1.6×10^{-10} in 0.1 M NaCl solution would be
 (a) $1.26 \times 10^{-5} \text{ M}$ (b) $1.6 \times 10^{-9} \text{ M}$
 (c) $1.6 \times 10^{-11} \text{ M}$ (d) zero.

SOLUBILITY PRODUCT

- Solubility is the concentration of an ion in a saturated solution(S)
- Solubility product is the equilibrium constant for solubility equilibrium
- Sparingly soluble salt \rightarrow salts which can dissolve only slightly into ions eg: AgCl , BaSO_4 , PbCl_2 , etc.
- K_{sp} can be calculated easily by using a sparingly soluble salt.
 Consider the ionisation of a sparingly soluble salt AgCl

$$\text{AgCl} \rightleftharpoons \underset{S}{\text{Ag}^+} + \underset{S}{\text{Cl}^-}$$

$$K_{sp} = [\text{Ag}^+][\text{Cl}^-] = S \times S = S^2$$
- If the value of ionic product = K_{sp} , solution is saturated.
- If the value of ionic product $< K_{sp}$, solution is unsaturated.
- If the value of ionic product $> K_{sp}$, solution is super saturated, here precipitation takes place

IONIC EQUILIBRIUM



01

RATE OF A CHEMICAL REACTION

decrease in conc of reactant / Time taken OR Increase in conc of reactant / Time taken

AVERAGE RATE

Consider a reaction: $A + B \rightarrow C + D$

$$-\frac{\Delta[A]}{\Delta t} = -\frac{\Delta[B]}{\Delta t} = \frac{\Delta[C]}{\Delta t} = \frac{\Delta[D]}{\Delta t}$$

INSTANTANEOUS RATE

Consider a reaction: $aA + bB \rightarrow cC + dD$

$$-\frac{1}{a} \frac{d[A]}{dt} = -\frac{1}{b} \frac{d[B]}{dt} = \frac{1}{c} \frac{d[C]}{dt} = \frac{1}{d} \frac{d[D]}{dt}$$

Unit of Rate = $\text{mol litre}^{-1} \text{s}^{-1}$

Q During the decomposition of H_2O_2 , 48 g O_2 is formed per minute at a certain point of time.

The rate of formation of water at this point is

- (a) $0.75 \text{ mol min}^{-1}$ (b) 1.5 mol min^{-1}
(c) $2.25 \text{ mol min}^{-1}$ (d) 3.0 mol min^{-1}

02

FACTORS INFLUENCING RATE OF REACTION

Factors	Effect on reaction rate
Increase in concentration	Increases
Increase in temperature	Increases
Presence of catalyst	Increases

Q Which of the following will lead to an increase in the rate of the reaction?

- a) Decrease in temperature
b) Decreasing concentration of reactants
c) Addition of catalyst
d) Addition of inhibitor

03

RATE LAW

Consider a general reaction,
 $aA + bB \rightarrow \text{product}$
Rate = $k[A]^x[B]^y$ (law of mass action)
Rate = $k[A]^x[B]^y$ (rate law expression)
 x & y are determined experimentally and may or may not be equal to a & b
 x & y represents the order of reaction with respect to A & B

RATE CONSTANT

- Larger the value of k , faster is the reaction.
- The value of k changes only with temperature for given reaction.

Unit of rate constant = $(\text{mol})^{1-a-b} \text{L}^{a+b} \text{s}^{-1}$

Q The rate constant of a zero-order reactions has the unit

- (a) s^{-1} (b) $\text{mol L}^{-1} \text{s}^{-1}$
(c) $\text{L}^2 \text{mol}^{-2} \text{s}^{-1}$ (d) $\text{L mol}^{-1} \text{s}^{-1}$

04

ORDER AND MOLECULARITY

Consider a general reaction,
 $aA + bB \rightarrow \text{product}$
Rate = $k[A]^x[B]^y$
molecularity = $a + b$
order = $x + y$

Molecularity	Order
Theoretical concept. It cannot be zero, fractional, infinite and imaginary.	An experimentally determined quantity. It can be equal to zero, positive, negative and fractional.

Q When the rate of the reaction is equal to the rate constant, the order of the reaction is

- (a) zero order
(b) first order
(c) second order
(d) third order

05

PSEUDO ORDER REACTIONS

Consider the reaction
 $\text{C}_{12}\text{H}_{22}\text{O}_{11} + \text{H}_2\text{O} \xrightarrow{\text{H}^+} \text{C}_6\text{H}_{12}\text{O}_6 + \text{C}_6\text{H}_{12}\text{O}_6$
In these reactions, concentration of water (one of the reactants) is in excess and its concentration remains constant throughout the reaction.
Thus, rate $\propto [\text{C}_{12}\text{H}_{22}\text{O}_{11}]$
Therefore, order = 1

Q For a pseudo first-order reaction, what is the unit of the rate of the reaction?

- (a) s^{-1}
(b) $\text{mol L}^{-1} \text{s}^{-1}$
(c) $\text{mol}^{-1} \text{L s}^{-1}$
(d) $\text{mol}^{-2} \text{L}^2 \text{s}^{-1}$

CHEMICAL KINETICS

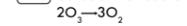
06

ELEMENTARY & COMPLEX REACTIONS

Reactions occurring only in one step are called elementary reactions while that involving a sequence of elementary reactions, are called complex reactions.

In case of complex reactions, the slowest step is called rate determining step.

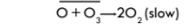
Note Consider the reaction



Step - 1



Step - 2



From slow step

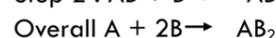
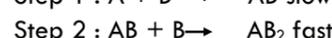
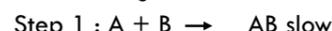
$$r = k[\text{O}][\text{O}_3]$$

Here

$$[\text{O}] \propto \frac{[\text{O}_3]}{[\text{O}_2]}$$

$$r = k[\text{O}_3]^2[\text{O}_2]^{-1}$$

Q Suppose the reaction: $A + 2B \rightarrow AB_2$ occurs by the following mechanism:



- (a) $k[A]$ (b) $k[B]$ (c) $k[A][B]$ (d) $k[B]^2$

07

INTEGRATED RATE EQUATIONS

Zero order

$$k = \frac{[A]_0 - [A]_t}{t}$$

First order

$$k = \frac{2.303 \log \frac{[A]_0}{[A]_t}}{t}$$

Second order

$$k = \frac{1}{t} \left[\frac{1}{[A]_t} - \frac{1}{[A]_0} \right]$$

Q A first order reaction has a specific reaction rate of 10^{-2} sec^{-1} . How much time will it take for 20 g of the reactant to reduce to 5 g?

- (a) 138.6 sec (b) 346.5 sec
(c) 693.0 sec (d) 238.6 sec

08

HALF LIFE PERIOD

Zero order

$$t_{1/2} = \frac{[A]_0}{2k}$$

First order

$$t_{1/2} = \frac{0.693}{k}$$

Second order

$$t_{1/2} = \frac{1}{k[A]_0}$$

FIRST ORDER TRICKS

$$t_{75\%} = 2t_{1/2}$$

$$t_{90\%} = \frac{2.303}{k} \log \frac{100}{10}$$

$$t_{99.9\%} = 10t_{1/2}$$

Q The half-life period of zero order reaction is directly proportional to the _____

- a) Rate constant
b) Initial concentration of reactants
c) Final concentration of reactants
d) Concentration of products

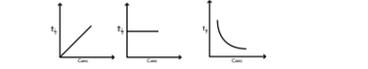
09

GRAPHICAL REPRESENTATION

1. Concentration - time graph



2. Rate - concentration graph



3. Half life - concentration graph



Q The graph of $t_{1/2}$ versus initial concentration 'a' is for

- (a) First order
(b) Second order
(c) Zero order
(d) Can't predict

10

ARRHENIUS EQUATION

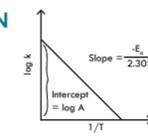
$$k = A e^{-E_a/RT}$$

$$\log k = \log A - \frac{E_a}{2.303R} \left(\frac{1}{T} \right)$$

NOTE

For every 10° rise in temperature, rate becomes double and hence, rate constant becomes double.

A reaction with higher value of E_a will have smaller value of rate constant.



Q The slope of Arrhenius plot ($\ln k$ vs $1/T$) of first order reaction is $-5 \times 10^3 \text{ K}$. The value of E_a of the reaction is [Given: $R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$]

- (a) -83 kJ mol^{-1} (b) 41.5 kJ mol^{-1}
(c) 83 kJ mol^{-1} (d) 166 kJ mol^{-1}

INERTIA

A body cannot change its state of rest or uniform motion along a straight line. This property is called inertia. Inertia has no unit and no dimension.

INERTIA

- Inertia of Rest
- Inertia of Motion
- Inertia of Direction

- Inertia of Rest
Inability to change state of rest by itself.

- Inertia of Motion
Inability of a body to change its state of uniform motion by itself.

- Inertia of Direction
Inability of a body to change direction of motion by itself.

Newton's Second Law
F_{net} = Rate of change of linear momentum.

Instantaneous
$$\vec{F} = \frac{d\vec{p}}{dt}$$

Average
$$\vec{F}_{av} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

MOMENTUM
$$\vec{p} = m\vec{v}$$

-It is a vector quantity having direction same as that of velocity
-Unit is kg m/s.

NEWTON'S THIRD LAW

-To every action, there is always an equal (in magnitude) and opposite (in direction) reaction.

- Forces in nature always occur in pairs.
- A single isolated force is not possible.
- Counter force experienced by a body- reaction
- Action and reaction never act on the same body

* Force exerted on body A by body B (action)
* force exerted on body B by body A (reaction)

$$\vec{F}_{Action} = -\vec{F}_{Reaction}$$

Some common forces

1. Normal Reaction

- Occurs when two surfaces are in contact with each other
Always perpendicular to the surface.

$$N_{Bj} = N_{jB}$$

$$N_{Bj} = N_{Bj}$$
 (normal on block by ground)
$$N_{jB} = N_{jB}$$
 (normal on ground by block)

Tension
Restoring force developed when a longitudinal force is applied on a body

Ideal Rope

- *Massless
- *Tension is same everywhere
- *Opposes only elongation
- *On compression it becomes slack.
- *Tension always acts away from the object.

Inclined Forces

- If, $F\sin\theta < Mg$ block remains in contact with ground & accelerates horizontally
- If, $F\sin\theta = Mg$ block just leaves contact with ground
- If, $F\sin\theta > Mg$ the block leaves contact with ground and it begins to accelerate obliquely.



MOTION OF CONNECTED BODIES

$F_1 > F_2$
 $F_1 - F_2 = Ma$
 $a = \frac{F_1 - F_2}{M}$

Condition	Free body diagram	Equation	Force and acceleration
		$F - f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$f = m_2 a$	$f = \frac{m_2 F}{m_1 + m_2}$
		$f = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - f = m_2 a$	$f = \frac{m_1 F}{m_1 + m_2}$
		$F - f_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$f_1 - f_2 = m_2 a$	$f_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$
		$f_2 = m_3 a$	$f_2 = \frac{m_3 F}{m_1 + m_2 + m_3}$

here, f, f_1 and f_2 are normal reactions between blocks

LIQUID JETS

When jet is stopped at wall

Area = A , $\frac{m}{\Delta x}$, $v=0$

$$F_{jet} = \frac{P_f - P_i}{\Delta t} = \frac{m \times 0 - mu}{\Delta t} = -\rho A u^2$$

$$F_{wall} = \rho A u^2$$

When liquid jet bounce back

$$F_{jet} = \frac{P_f - P_i}{\Delta t} = \frac{-2mu}{\Delta t} = -2\rho A u^2$$

$$F_{wall} = 2\rho A u^2$$

When liquid jet strikes obliquely

$$F_{jet} = -2\rho A u^2 \cos\theta$$

$$F_{wall} = 2\rho A u^2 \cos\theta$$

Change in momentum = $-2mu \cos\theta$

can support both elongation and compression

Rod

Elongation Compressive

SINGLE BLOCK

Horizontal Force
Acceleration is along x-axis only
Along y-axis $a_y = 0$

$$N - Mg = Ma_y$$

$$\therefore a_y = 0$$

$$N = Mg$$

Along x-axis
$$F - 0 = Ma$$

$$a = \frac{F}{M}$$

MOTION OF BLOCKS CONNECTED BY MASSLESS STRING

Condition	Free body diagram	Equation	Tension and acceleration
		$T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$F - T = m_2 a$	$T = \frac{m_1 F}{m_1 + m_2}$
		$F - T = m_1 a$	$a = \frac{F}{m_1 + m_2}$
		$T = m_2 a$	$T = \frac{m_2 F}{m_1 + m_2}$
		$T_1 = m_1 a$	$a = \frac{F}{m_1 + m_2 + m_3}$
		$T_2 - T_1 = m_2 a$	$T_1 = \frac{m_1 F}{m_1 + m_2 + m_3}$
		$F - T_2 = m_3 a$	$T_2 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$

LAWS OF MOTION

PULLEY-BLOCK SYSTEM

Ideal pulley
 $M_1 > M_2$
$$a = \frac{F}{M} = \frac{M_1 - M_2}{M_1 + M_2} g$$

$$T = \frac{2M_1 M_2}{M_1 + M_2} g$$

$$a = \frac{M_1 g}{M_1 + M_2}$$

$$T = \frac{M_1 M_2}{M_1 + M_2} g$$

INCLINED PLANE + PULLEY

	$m g \sin\theta$	$m a = T - m g \sin\theta$	$a = \left[\frac{m_2 - m_1 \sin\theta}{m_1 + m_2} \right] g$
	$m g \cos\theta$	$m a = m g \cos\theta - T$	$T = \frac{m_1 (m_2 + m_1 \sin\theta)}{m_1 + m_2} g$
	$m g \sin\theta$	$T - m g \sin\theta = m a$	$a = \frac{(m_2 \sin\theta - m_1 \sin\theta)}{m_1 + m_2} g$
	$m g \cos\theta$	$m a = m g \sin\theta - T$	$T = \frac{m_2 m_1 (\sin\theta + \sin\theta)}{m_1 + m_2} g$
	$m g \sin\theta$	$m g \sin\theta - T = m a$	$a = \frac{m_2 \sin\theta}{m_1 + m_2 + m_3} g$
	$m g \cos\theta$	$T = m a$	$T = \frac{m_3 m_2 \sin\theta}{m_1 + m_2 + m_3} g$

THICK ROPE

Tension will be different at different points.

Mass per unit length = $\frac{M}{L}$
Mass of x length of rope = $\frac{M}{L} x$

Note :
Mass of given length = $\frac{\text{total mass}}{\text{total length}} \times \text{given length}$

$\frac{\text{mass}}{\text{length}} = \text{constant}$

$$a_{rope} = \frac{F}{M}$$

For (L-x) rope length, $\frac{M}{L} (L-x) = m_2$
For x rope length, $\frac{M}{L} x = m_1$

$$\vec{F} = m\vec{a} \Rightarrow T = m_2 \frac{F}{M}$$

LIFT PROBLEMS

Apparent weight of body in a lift
Reading of weighing machine = reaction force exerted by weighing machine

Apparent weight, $(W_{apparent}) = \text{Reaction force (R)}$

Case 1: Lift is at rest
 $R = mg$, $W_{apparent} = W_{actual} = mg$

Case 2: Lift moving up or down with constant velocity
 $R = mg$, $W_{apparent} = W_{actual} = mg$

Case 3: Accelerated upward at a rate of 'a'
 $R - mg = ma \Rightarrow R = m(g+a) = W_{app}$
 $W_{apparent} > W_{actual} \rightarrow \text{Feels over weight}$
Accelerated upward at a rate of 'g'
 $R - mg = mg$, $R = 2mg$, $W_{app} = 2 \times W_{act}$

Case 4: Accelerated downward at a rate of 'a'
 $mg - R = ma$, $R = m(g-a) = W_{app}$, $W_{app} < W_{act}$
Accelerated downward at a rate of 'g' [Freefall]
 $mg - R = mg$, $R = mg - mg = 0$, $W_{app} = 0$

If $a > g$:
body leaves contact from ground and begins free-fall

Lift moves from ground floor to first floor

C: v decrease, $v=0, a=0$, $N=mg$ (true weight), first floor

B: v increase, $a_1 \uparrow$, $N=m(g+a_1)$, Ground floor

A: $v=0, a=0$, $N=mg$ (true weight)

Graph

Graph 1: $N = m(g+a_1)$ (increasing), $N = m(g-a_2)$ (decreasing), $a=0$ (constant)

Graph 2: $N = m(g+a_1)$ (increasing), $N = m(g-a_2)$ (decreasing), $a=0$ (constant)

Acceleration, $a_1 = \tan\theta_1$
Retardation, $a_2 = \tan\theta_2$

FRAME OF REFERENCE & PSEUDO FORCE

Frame of Reference
A frame in which observer is situated and makes his observation

Inertial frame of reference	Non-Inertial frame of reference
At rest or moving with uniform velocity along straight line. i.e unaccelerated	Accelerated frame of reference.
Newton's law of motion hold's $\vec{F}_{net} = m\vec{a}$	Newton's law of motion not applicable. $\vec{F}_{net} + \vec{F}_{pseudo} = m\vec{a}$ $\vec{F}_{pseudo} = -m\vec{a}_c$

RELATIVE SLIPPING

Minimum force required to push the inclined plane such that "m" does not slip with respect to "M"

$$F = (m+M) g \tan \theta, \quad a = g \tan \theta$$

Minimum acceleration of "M" such that there is no relative slipping.

Minimum mass M_0 such that there is no relative slipping

$$M_0 = \frac{M+m}{\cot \theta - 1}$$

EQUILIBRIUM & LAMI'S THEOREM

$$\frac{F_1}{\sin \alpha} = \frac{F_2}{\sin \beta} = \frac{F_3}{\sin \gamma}$$

PARALLELOGRAM LAW

$$\vec{F} = \vec{F}_1 + \vec{F}_2, \quad F = \sqrt{F_1^2 + F_2^2}, \quad \tan \theta = \frac{F_2}{F_1}$$

MAN-CAGE PROBLEM

Man holds the cage stationary

$$T = \frac{(m+M)g}{2}$$

$$N = \frac{(m-M)g}{2}$$

- $m < M$
- if $m > M$ man cannot hold the cage stationary

IMPULSE

If a large force acting for short period of time, there will be a sudden change in momentum

$$\vec{F}_{imp} = \frac{d\vec{p}}{dt}$$

$$\vec{P}_f - \vec{P}_i = \int \vec{F}_{imp} dt = \text{area under } F-t \text{ graph}$$

$$\vec{I} = \text{Impulse} = \vec{P}_f - \vec{P}_i = \int \vec{F}_{imp} dt = \text{area of } F-t \text{ graph}$$

Case-1

Impulse

$$I = P_f - P_i = mV_2 - (-mV_1)$$

$$I = m(V_1 + V_2)$$

Average impulsive force

$$F_{avg} = \frac{\text{impulse}}{t_o} = \frac{\Delta P}{t_o} = \frac{m(v_1 + v_2)}{t_o}$$

Case-2

Impulse

$$m(v+v) = J = m(2v)$$

$$J = 2mv$$

Average impulsive force

$$F_{avg} = \frac{\Delta P}{\Delta t} = \frac{2mv}{t_o}$$

ROD SLIDING ON A WALL

velocity of B towards A = $v \sin \theta$

velocity of A away from B = $u \cos \theta$

these velocities should be equal,

$$v \sin \theta = u \cos \theta \Rightarrow v = u \cot \theta$$

FRICTION

Static friction

- It is a self adjusting force.
- The opposing force that comes into play, when object tends to slip over the surface of other object, but slipping has not yet started.
- As applied force increases static friction also increases.
- The body doesn't move until a maximum value of static friction is attained
- This value is called limiting friction or $(f_s)_{max}$

$$(f_s)_{max} = \mu_s N$$

Kinetic friction

If the applied force is increased further and slipping between surfaces start, the friction opposing the slipping is called kinetic friction.

PULLING FORCE & PUSHING FORCE

Pulling Force:

$$F \cos \theta \geq \mu_s (mg - F \sin \theta) \Rightarrow F_{pulling} = \frac{\mu_s mg}{\cos \theta + \mu_s \sin \theta}$$

Pushing Force:

$$F \cos \theta \geq \mu_s (mg + F \sin \theta) \Rightarrow F_{pushing} = \frac{\mu_s mg}{\cos \theta - \mu_s \sin \theta}$$

$F_{pushing} > F_{pulling}$

ANGLE OF FRICTION

Angle(θ) made by resultant of normal (N) & frictional force(f_s) with normal

$$R = \sqrt{N^2 + f_s^2}$$

R is resultant

$$\tan \theta = \frac{f_s}{N}$$

When $f_s = (f_s)_{max}$ $R = N \sqrt{1 + \mu_s^2}$

$\tan \theta = \mu_s$, μ_s is Coefficient of friction

i) $f = 0$

$$\tan \theta = 0 \Rightarrow \theta = 0$$

ii) $0 < f_s < (f_s)_{max}$

iii) $f_s = (f_s)_{max}$

$$0 \leq \tan \theta \leq \mu_s$$

$$N < R < N \sqrt{1 + \mu_s^2}$$

$$R_{max} = N \sqrt{1 + \mu_s^2}$$

$$\tan \theta_{max} = \mu_s$$

iv) When slipping has started,

$$f = f_k$$

$$R_{max} = N \sqrt{1 + \mu_k^2}$$

θ = Angle of kinetic friction

ANGLE OF REPOSE

Angle made by inclined plane such that a block kept on it just begins to slide

Depends only on μ_s and is independent of mass.

$$\tan \theta_0 = \mu_s$$

Variation of R

As angle of inclined plane increases, R remains constant (=mg) and when sliding starts R starts decreasing.

Variation of angle of friction.

As angle of inclined plane increases, angle of friction will also increase and as sliding starts its value becomes constant and $\tan \theta = \mu_k$

Minimum & Maximum force (applied parallel to inclined plane)

$F_{min} = mg (\sin \theta - \mu_s \cos \theta)$

$F_{max} = mg (\sin \theta + \mu_s \cos \theta)$

$mg (\sin \theta - \mu_s \cos \theta) \leq F \leq mg (\sin \theta + \mu_s \cos \theta)$

HORIZONTAL TRUCK BOX

Case-1
Box does not slip.

$$f_s \leq \mu_s N$$

$$\Rightarrow ma_T \leq \mu_s mg$$

$$\Rightarrow a_T \leq \mu_s g$$

Case-2
Box slips

$$a_T > \mu_s g$$

VERTICAL TRUCK BOX

Case-1
Box does not slip.

$$f_s \leq \mu_s N$$

$$\Rightarrow mg \leq \mu_s ma_T$$

$$\Rightarrow \mu_s \geq \frac{g}{a_T}$$

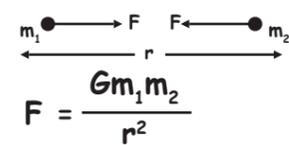
Case-2
Box slips

$$\mu_k \leq \frac{g}{a_T}$$

LAWS OF MOTION

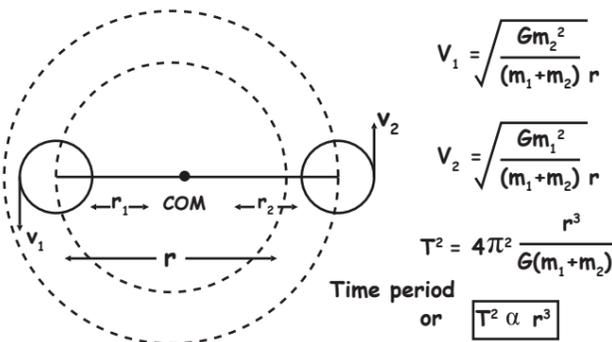


NEWTON'S LAW OF GRAVITATION



$F = \frac{Gm_1m_2}{r^2}$
 G - Universal gravitational constant
Value of G
 $6.67 \times 10^{-11} \text{ Nm}^2\text{Kg}^{-2}$ (SI or MKS)
 $6.67 \times 10^{-8} \text{ dyne cm}^2\text{g}^{-2}$ (CGS)
Dimensional formula [G]
 $M^{-1}L^3T^{-2}$

ROTATION OF 2 MASSES UNDER MUTUAL GRAVITATIONAL FORCE OF ATTRACTION



$$v_1 = \sqrt{\frac{Gm_2^2}{(m_1+m_2)r}}$$

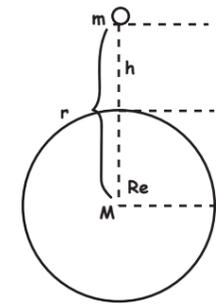
$$v_2 = \sqrt{\frac{Gm_1^2}{(m_1+m_2)r}}$$

$$T^2 = 4\pi^2 \frac{r^3}{G(m_1+m_2)}$$

Time period or $T^2 \propto r^3$

VARIATION IN THE VALUE OF ACCELERATION DUE TO GRAVITY

• Variation due to height 'h'



$$g' = \frac{GM}{(R+h)^2}$$

$$g' = \frac{gR^2}{(R+h)^2}$$

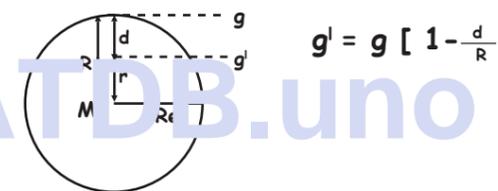
General Equation

Approximate equation
 $h \ll R$ ($h < 100 \text{ km}$)
 use, $g' = g \left(1 - \frac{2h}{R}\right)$

Note the point
 If $h \ll R$, then decrease in the value of g with height

Absolute decrease = $\Delta g = g - g' = \frac{2hg}{R}$
 Fractional decrease = $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{2h}{R}$
 Percentage decrease = $\frac{\Delta g}{g} \times 100 = \frac{2h}{R} \times 100$

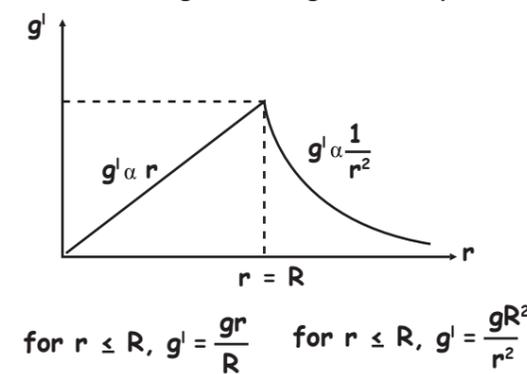
• Variation due to depth 'd'



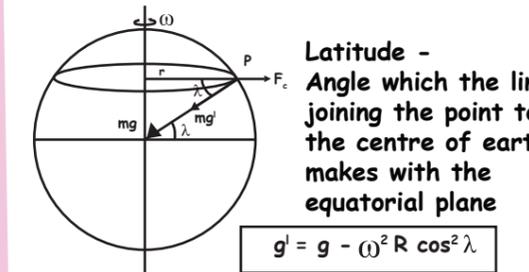
$$g' = g \left[1 - \frac{d}{R}\right]$$

Absolute decrease = $\frac{\Delta g}{g} = g - g' = \frac{dg}{R}$
 Fractional decrease = $\frac{\Delta g}{g} = \frac{g-g'}{g} = \frac{d}{R}$
 Percentage decrease = $\frac{\Delta g}{g} \times 100 = \frac{d}{R} \times 100$

Very imp graph
 The graphical representation of change in the value of g' with height and depth



• Variation of g due to rotation of earth



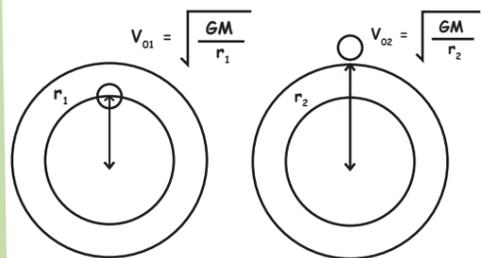
Latitude -
 Angle which the line joining the point to the centre of earth makes with the equatorial plane

$$g' = g - \omega^2 R \cos^2 \lambda$$

Note \Rightarrow value of $\omega^2 R = 0.034$
 For poles $\lambda = 90^\circ$ $g' = g$
 There is no effect of rotational motion of the earth on the value of g at poles.

For equator $\lambda = 0^\circ$ $g' = g - \omega^2 R$
 The effect of rotational motion of the earth on the value of g at the equator is maximum.
 When a body of mass m is moved from equator to the poles, weight increases by an amount $m(g_p - g_e) = m\omega^2 R$

WORK DONE IN MOVING OBJECT FROM ONE ORBIT TO ANOTHER



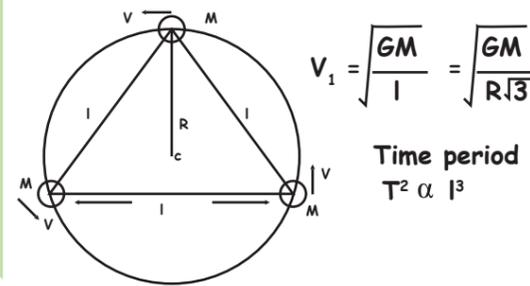
CONCEPT - WORK DONE BY EXTERNAL AGENT = CHANGE IN MECHANICAL ENERGY

$$W = E_2 - E_1 = \frac{GMm}{2} \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

IMPORTANT POINTS ABOUT GRAVITATIONAL FORCE

1. Gravitational force
 * Always attractive in nature
 * Independent of the nature of medium between masses
 * Independent of presence or absence of other bodies
2. Are central forces, acts along the centre of gravity of two bodies.
3. Conservative force
4. Force between any two masses - Gravitational force
 Force between earth and any other body - Force of gravity

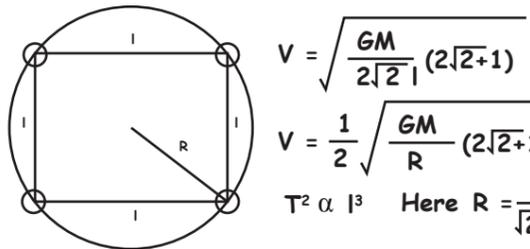
THREE MASSES(EQUAL) REVOLVING UNDER MUTUAL GRAVITATIONAL FORCE



$$v_1 = \sqrt{\frac{GM}{l}} = \sqrt{\frac{GM}{R\sqrt{3}}}$$

Time period $T^2 \propto l^3$

FOUR EQUAL MASSES UNDER MUTUAL GRAVITATIONAL FORCE

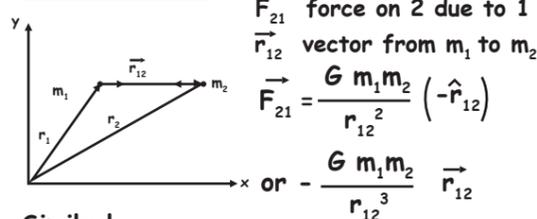


$$v = \sqrt{\frac{GM}{2\sqrt{2}l}} (2\sqrt{2}+1)$$

$$v = \frac{1}{2} \sqrt{\frac{GM}{R}} (2\sqrt{2}+1)$$

$T^2 \propto l^3$ Here $R = \frac{l}{\sqrt{2}}$

VECTOR FORM

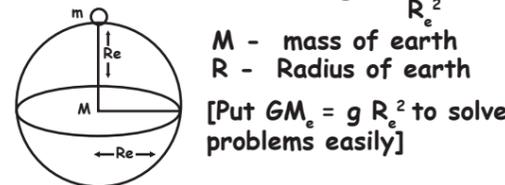


Similarly \vec{F}_{12} force on 1 due to 2
 $\vec{F}_{21} = \frac{Gm_1m_2}{r_{12}^2} (\hat{r}_{12})$ or $\frac{Gm_1m_2}{r_{12}^3} \vec{r}_{12}$
 Clearly Newton's third law $\vec{F}_{21} = -\vec{F}_{12}$

Gravitational force is a two body interaction. Force between two particles does not depend on the presence or absence of other particles. The principle of superposition is valid here. "Force on a particle due to a no. of particles is the resultant of forces due to individual particles."

GRAVITY

Acceleration due to gravity
 On the surface of earth $g = \frac{GM_e}{R_e^2}$

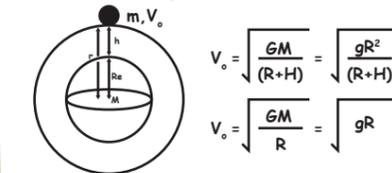


g IN TERMS OF DENSITY OF EARTH

$g = 4\pi G\rho R_e$ $g \propto \rho R_e$
 "If density is mentioned use the"

ORBITAL VELOCITY

Orbit at a height 'h' from the surface



$$v_0 = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}}$$

$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

If orbit is closer to earth's surface (neglect 'h') $v_0 = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

(called minimum orbit, velocity-first cosmic velocity)

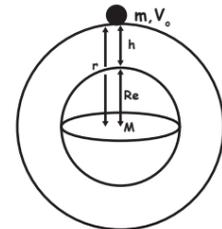
Note - for easy calculations
 $\sqrt{gR} = 8 \text{ km/s}$ or $\sqrt{\frac{GM}{R}} = 8 \text{ km/s} = 8 \times 10^3 \text{ m/s}$
 or $\frac{GM}{R} = 64 \times 10^6$

KE, PE OR TE FOR AN ORBITING SATELLITE

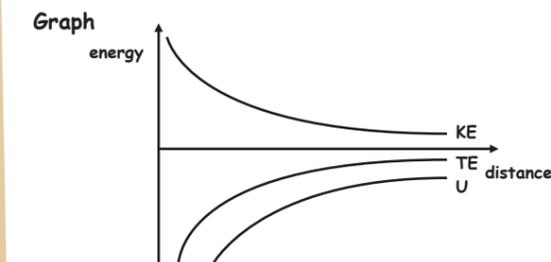
$$KE = \frac{GMm}{2r}$$

$$U = -\frac{GMm}{r}$$

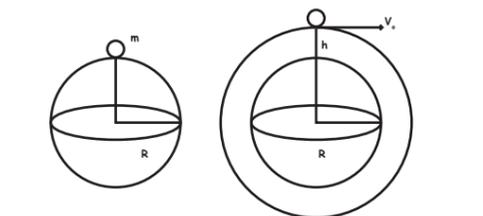
$$TE = -\frac{GMm}{2r}$$



Relation KE, U & TE
 $U = 2 \times T.E$
 $K.E = -T.E$



WORK DONE IN MOVING OBJECT FROM SURFACE TO CIRCULAR ORBIT



$$W = E_f - E_i$$

$$W = E_{\text{total}} - U_i = \frac{-GMm}{2(R+h)} + \frac{GMm}{R}$$

GRAVITATION



Centre of Mass

- Avg. position of all the parts of the system, weighted according to their mass
- For homogeneous objects, centre of mass lies at their geometric centre
- Centre of mass may or may not lie inside the object

Centre of mass for various shapes

Uniformly distributed mass centre of mass

- Rod**
 $(\frac{L}{2}, 0)$
- Square Lamina**
 $(\frac{L}{2}, \frac{L}{2})$
- Semicircular Ring**
 $(0, \frac{2R}{\pi})$
- Hemispherical shell**
 $(0, \frac{R}{2})$
- Solid circular cone**
 $(0, \frac{h}{4})$
- Solid hemisphere**
 $(0, \frac{3R}{8})$

Motion of centre of mass

velocity of centre of mass

$$\vec{V}_{cm} = \frac{M_1\vec{V}_1 + M_2\vec{V}_2 + M_3\vec{V}_3 + \dots}{M_1 + M_2 + M_3 + \dots}$$

Acceleration of centre of mass

$$\vec{a}_{cm} = \frac{M_1\vec{a}_1 + M_2\vec{a}_2 + M_3\vec{a}_3 + \dots}{M_1 + M_2 + M_3 + \dots}$$

Moment of Inertia

i) for discrete system of particles

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \dots + m_nr_n^2$$

ii) for continuous body

$$I = \int dI = \int r^2 dm$$

Perpendicular axis theorem

$$I_z = I_x + I_y$$

(Only valid for laminar bodies)

Note:-
 X and Y axis must lie in the plane of body
 Z-axis must be ⊥ to the plane of the body
 Axes need not pass through center of mass

Centre of Mass For System of n Particles

$$\vec{r}_{cm} = \frac{\sum m\vec{r}}{\sum m}$$

General Equation

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \dots + m_n\vec{r}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

In terms of Cartesian co-ordinates

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots + m_nx_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \dots + m_ny_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$z_{cm} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \dots + m_nz_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

Isolated System

- No net external force acting on the system
- bodies within the system can have mutual force between them

$M_1r_1 = M_2r_2$ $a_{cm} = 0, v_{cm} = \text{constant}$

Moment of Inertia (for a point object)

$$I = mr^2$$

m = Mass of body
 r = Perpendicular distance of the body from the axis of rotation

Moment of Inertia

- Tensor Quantity
- Rotational analogue of mass

Parallel Axis Theorem

$$I_{parallel} = I_{COM} + Ma^2$$

Conditions:-

- the two axes must be parallel to each other
- One of the axis must pass through centre of mass

Moment Of Inertia For Various Objects

Disc <ul style="list-style-type: none"> $I = MR^2$ (about diameter) $I = \frac{5}{4}MR^2$ (about edge) $I = \frac{MR^2}{2}$ (about central axis) $I = \frac{3}{2}MR^2$ (about edge) 	Ring <ul style="list-style-type: none"> $I = MR^2$ (about diameter) $I = \frac{3}{2}MR^2$ (about edge) $I = 2MR^2$ (about edge) 	Thin Rod <ul style="list-style-type: none"> $I = \frac{ml^2}{12}$ (about center) $I_{end} = \frac{ml^2}{3}$ (about end)
Hollow cylinder <ul style="list-style-type: none"> $I = MR^2$ (about central axis) $I = 2MR^2$ (about edge) 	Solid cylinder <ul style="list-style-type: none"> $I = \frac{3}{2}MR^2$ (about central axis) $I = \frac{MR^2}{2}$ (about central axis) 	Hollow sphere <ul style="list-style-type: none"> $I = \frac{2}{3}MR^2$ (about diameter) $I = \frac{5}{3}MR^2$ (about edge)
Solid sphere <ul style="list-style-type: none"> $I = \frac{7}{5}MR^2$ (about diameter) $I = \frac{2}{5}MR^2$ (about edge) 		

Centre Of Mass For Continuous Body

$$\vec{r}_{cm} = \frac{\int dm \cdot \vec{r}}{\int dm}$$

λ = mass per unit length
 σ = mass per unit area
 ρ = mass per unit volume

- Mass distributed over length ⇒ $dm = \lambda \cdot dl$
- Mass distributed over area ⇒ $dm = \sigma \cdot dA$
- Mass distributed over volume ⇒ $dm = \rho \cdot dV$

Cavity in object

If some mass is removed from a body, COM will shift towards the side with more mass

COM of remaining body

Assuming COM of original body is at the origin

$$\vec{r}_{rem} = \frac{-M_{cav} \times \vec{r}_{cav}}{M_{rem}}$$

Two Point Masses

$$I_{COM} = m_1r_1^2 + m_2r_2^2$$

$$r_1 = \frac{m_2r}{m_1 + m_2}, \quad r_2 = \frac{m_1r}{m_1 + m_2}$$

$$I_{com} = m_{red} r^2, \quad m_{red} = \frac{m_1m_2}{m_1 + m_2}$$

Factors Affecting Moment of Inertia

- Mass of the body
- Axis of rotation
- Mass distribution

Moment of Inertia along the centre of mass and perpendicular to the plane surface

- Ring**
 - MR^2 (⊥ to plane)
 - $\frac{MR^2}{2}$ (Parallel to plane)
 - $\frac{MR^2}{4}$ (Parallel to plane)
- Disc**
 - $\frac{MR^2}{2}$ (⊥ to plane)
 - $\frac{MR^2}{4}$ (Parallel to plane)
 - $\frac{MR^2}{2}$ (Parallel to plane)
- Square sheet**
 - $\frac{Ml^2}{6}$ (⊥ to plane)
 - $\frac{Ml^2}{12}$ (Parallel to plane)
 - $\frac{Ml^2}{12}$ (Parallel to plane)
- Rectangular sheet**
 - $\frac{M}{12}(l^2 + b^2)$ (⊥ to plane)
 - $\frac{Ml^2}{12}$ (Parallel to plane)
 - $\frac{Mb^2}{12}$ (Parallel to plane)

ROTATIONAL MOTION 01

Radius of Gyration

Definition: The distance of a point mass from the axis whose mass is equal to the mass of whole body and whose moment of inertia is equal to moment of inertia of the body about that axis

$I = \frac{mR^2}{2}$

Disc

$I = mk^2$

$\frac{mR^2}{2} = mk^2$
 $k = \frac{R}{\sqrt{2}}$

k is the radius of gyration

Torque

Torque $\tau_o = r F \sin \theta$

$= F \sin \theta \times r = F_{\perp} r$ (1)

$= F \times r \sin \theta = F_{\perp} r$ (2)

$\vec{\tau}_o = \vec{r} \times \vec{F}$ (Vector form)

If force is radial i.e. $\theta = 0^\circ$ or 180°
Torque $\tau = 0$

If force is tangential and \perp to radius vector i.e. $\theta = 90^\circ$
Torque, $\tau = \tau_{\max} = rF$

Equilibrium

For translational equilibrium

$F_{\text{net}} = 0$
 τ_{net} may or maynot be zero

Rotational Equilibrium

$\tau_{\text{net}} = 0$
 F_{net} may or maynot be zero

Static Equilibrium

Combination of both translational and rotational equilibrium

$F_{\text{net}} = 0 \Rightarrow$ Forces are balanced
 $\tau_{\text{net}} = 0 \Rightarrow \tau_{\text{clockwise}} = \tau_{\text{anticlockwise}}$

Principle of moments

When a body is in rotational equilibrium sum of clock wise moments about any point is equal to sum of anticlockwise moments about that point

$F_1 \times d_1 = F_2 \times d_2$

Load x load arm = Effort x effort arm

$m_1 g x_1 = m_2 g x_2 + m_3 g x_3$

$T = \frac{2m_1 m_2 g}{m_1 + m_2}$
 $2T x_1 = m_3 g x_2$

Angular acceleration

$\tau = I\alpha$
 τ - torque
 I moment of inertia
 α angular acceleration

Initial angular acceleration when a rod is released

Initial angular acceleration when a body is released from an angle θ

$\tau = I\alpha$
 $(mg \sin \theta) r_{\text{cm}} = I\alpha$
 $\alpha = \frac{(mg \sin \theta) r_{\text{cm}}}{I}$
For rod $r_{\text{cm}} = \frac{l}{2}$

Translation - rotation combination

$\alpha = \frac{\tau_{\text{applied}} - \tau_{\text{opposition}}}{\text{Total } I}$
 $\alpha = \frac{m_2 R g - m_1 R g}{m_1 R^2 + m_2 R^2 + I}$

Angular momentum & its conservation

Angular momentum of a point mass:-

Angular momentum about origin
 $\vec{L} = \vec{r} \times \vec{p}$
 $= m(\vec{r} \times \vec{v})$
 $\vec{p} = m\vec{v}$

1) When $\theta = 0^\circ$ or 180°
 $L_o = mvr \sin 180^\circ = 0$
OR $mvr \sin 0^\circ = 0$
Angular momentum is minimum

1) When $\theta = 90^\circ$
 $|\vec{L}| = r p \sin \theta$
 $= r p \sin 90^\circ$
 $= r p$
 $= L_{\text{max}}$

Spin angular momentum

$L_{\text{axis}} = I\omega$

Conservation of Angular momentum

If there is no external torque, angular momentum is conserved

$\tau = \frac{dL}{dt}$
If $\tau = 0 \Rightarrow \frac{dL}{dt} = 0$
 $L = \text{constant}$
 $I\omega = \text{constant}$

$I_1 \omega_1 = I_2 \omega_2$

$\tau \propto$ moment of inertia increases angular velocity decreases and if moment of inertia decreases angular velocity increases

Moment of inertia when two discs are joined

Discs initially rotating in same direction:-

$\omega_f = \frac{I_1 \omega_1 + I_2 \omega_2}{I_1 + I_2}$

Discs initially rotating in opposite direction:-

$\omega_f = \frac{I_1 \omega_1 - I_2 \omega_2}{I_1 + I_2}$

Discs initially rotating in opposite direction:-

$\omega_f = \frac{I_1 \omega_1 - I_2 \omega_2}{I_1 + I_2}$

Work, Energy & Power in rotation

1) Work done by a torque,
 $W = \tau \theta$ (if torque is uniform)
 $= \int \tau d\theta$ (if torque is non uniform)

2) K.E for are rotating body $= \frac{1}{2} I \omega^2$
 $= \frac{1}{2} I v^2$
 $= \frac{1}{2} L \omega$

3) Work-Energy theorem
 $\Sigma W = \Delta K = \frac{1}{2} I (\omega_2^2 - \omega_1^2)$

Energy loss when 2 discs are joined:-

1) Same direction:-
 $E_{\text{lost}} = \Delta K.E = \frac{I_1 I_2}{2(I_1 + I_2)} (\omega_1 - \omega_2)^2$

2) Opposite direction:-
 $E_{\text{lost}} = \Delta K.E = \frac{I_1 I_2}{2(I_1 + I_2)} (\omega_1 + \omega_2)^2$

Mechanical energy conservation

Angular velocity with which the rod hits the ground without slipping, released from rest

$\omega = \sqrt{\frac{3g}{l}}$

Rolling Motion

Translatory + Rotatory = Rolling

Velocity in rolling

- Condition for rolling without slipping:- $V = R\omega$
- Velocity of any point on rolling object, $V_p = \omega d = \frac{v d}{R}$

d is the distance from point of contact

Energy in rolling motion

1) Translatory Motion
 $T_{k.E} = \frac{1}{2} m v^2$

2) Spinning motion/rotational motion
 $R_{k.E} = \frac{1}{2} I \omega^2 = \frac{1}{2} m k^2 \frac{v^2}{R^2} = \frac{1}{2} m v^2 \times \left(\frac{k^2}{R^2}\right)$

3) Rolling motion
 $T_{k.E} + R_{k.E} = \frac{1}{2} m v^2 + \frac{1}{2} m v^2 \times \frac{k^2}{R^2} = \frac{1}{2} m v^2 \left(1 + \frac{k^2}{R^2}\right)$
 $\frac{k_{\text{total}}}{k_{\text{trans}}} = \left(1 + \frac{k^2}{R^2}\right)$

Motion on an inclined plane

Velocity at bottom $= \sqrt{\frac{2gh}{1 + \frac{k^2}{R^2}}}$
 $v \propto \frac{1}{\sqrt{1 + \frac{k^2}{R^2}}}$

$\frac{k^2}{R^2} \uparrow \Rightarrow v \downarrow \Rightarrow \text{Time } \uparrow$

Velocity: solid sphere > Disc > Hollow > Sphere > Ring
Time to reach bottom: Ring > Hollow sphere > Disc > solid sphere

Value of velocity:-

- Ring/Hollow cylinder $= \sqrt{gh}$
- Disc/Solid cylinder $= \sqrt{\frac{4}{3} gh}$
- Hollow sphere $= \sqrt{\frac{6}{5} gh}$
- Solid sphere $= \sqrt{\frac{10}{7} gh}$

Acceleration
 $a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}}$ $a \propto \frac{1}{1 + \frac{k^2}{R^2}}$

Time of descend:-
 $t = \frac{1}{\sin \theta} \sqrt{\frac{2h}{g} \left(1 + \frac{k^2}{R^2}\right)}$
 $t \propto \sqrt{1 + \frac{k^2}{R^2}}$
Ring > Hollow sphere > Disc > solid sphere

ROTATIONAL MOTION 02

WORK IS SAID TO BE DONE when a force applied on the body displaces the body through a certain distance

WORK DONE BY CONSTANT FORCE

$W = F \cos \theta \times S = \vec{F} \cdot \vec{S}$

NATURE OF WORK DONE

1) Positive work ($0^\circ \leq \theta < 90^\circ$)

2) Negative work ($90^\circ < \theta \leq 180^\circ$)

2) Zero work
Work done becomes 0 for three conditions

- Force is perpendicular to displacement
- if there is no displacement
- if there is no force acting on the body

WORK DONE BY VARIABLE FORCE

$dW = \vec{F} \cdot d\vec{s}$
 $W = \int \vec{F} \cdot d\vec{s} = \int F ds \cos \theta$

in terms of rectangular components
 $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$
 $d\vec{s} = dx \hat{i} + dy \hat{j} + dz \hat{k}$
 $W = \int F_x dx + \int F_y dy + \int F_z dz$

Graphical representation of work done

$dW = F \cdot dx$
 $W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} F \cdot dx$

CONSERVATIVE & NON CONSERVATIVE FORCE

Conservative: work done does not depend on path followed
Non-conservative: work depends on the path followed

- $W_{A \rightarrow B}(\text{Path 1}) = W_{A \rightarrow B}(\text{Path 2}) = W_{A \rightarrow B}(\text{Path 3})$ (for conservative force)
- $W_{A \rightarrow B}(\text{Path 1}) \neq W_{A \rightarrow B}(\text{Path 2}) \neq W_{A \rightarrow B}(\text{Path 3})$ (for non conservative force)

Note :
Work done for a complete cycle for a conservative force is zero

WORK DONE BY DIFFERENT FORCES

$W_1 = mgh = mgh$
 $W_2 = mg \times l \sin \theta = mg \times l \times \frac{h}{l} = mgh$
 $W_3 = mgh_1 + 0 + mgh_2 + 0 + mgh_3 + 0 + mgh_4$

Work done by spring force
magnitude of spring force, $F_s = -kx$

$W_s = \int_{x_i}^{x_f} \vec{F}_s \cdot d\vec{x} = - \int_{x_i}^{x_f} kx dx = -\frac{1}{2} k(x_f^2 - x_i^2)$

THE CHAIN

$L \rightarrow$ Total length
($1/n$)th part of length hanging
 $M \rightarrow$ Mass of chain
Work done in pulling the hanging portion back on the table
 $W = \frac{MgL}{2n^2}$

WORK ENERGY & POWER

ENERGY

- Capacity of doing work
- Scalar quantity
- Dimension ML^2T^{-2}

Relation between different units
1eV = 1.6×10^{-19} Joules
1kWh = 3.6×10^6 Joules
1calorie = 4.18 Joules
1 Joule = 10^7 erg

Kinetic Energy

- Energy possessed by virtue of motion
- Always positive
- Depends on frame of reference

Work Energy Theorem
Change in kinetic energy of a body is equal to network done on the body
 $K_2 - K_1 = \int \vec{F} \cdot d\vec{r}$

$K.E = \frac{1}{2} mv^2$

ENERGY WITH OTHER QUANTITIES

Linear momentum: $P = \sqrt{2mK}$
Variation of graph of kinetic Energy

1) m constant, $E \propto v^2$
2) m constant, $E \propto P^2$
3) P constant, $E \propto \frac{1}{m}$
4) P constant, $E \propto P$

POTENTIAL ENERGY

- Defined only for conservative force
- Energy possessed by a body by virtue of its position/configuration
- Can either be positive, negative or zero according to point of reference
- Force always acts from higher potential to lower potential

Identifying forces with the help of potential energy

- Force opposing the motion:-
On increasing x , if U increases
 $\frac{dU}{dx} = \text{positive}$ (BC portion of graph)
- Force supporting the motion:-
On increasing x , if U decreases
 $\frac{dU}{dx} = \text{negative}$ (AB portion of graph)
- Zero force:-
On increasing x , if U does not change
 $\frac{dU}{dx} = 0$

B, C points on the graph

Types of Potential Energy

- Elastic Potential Energy
- Electric Potential Energy
- Gravitational Potential Energy

• Types of equilibrium

If net force acting on a particle is zero it is said to be in equilibrium

STABLE

- If particle displaced from equilibrium position, force acting will try to bring the particle back to the equilibrium position
- Potential energy is minimum at stable equilibrium
- $F = -\frac{dU}{dx} = 0$
- $\frac{d^2U}{dx^2} = \text{positive}$

UNSTABLE

- If particle displaced from equilibrium position, force acting on it tries to displace it further away from equilibrium position
- Potential energy is maximum at unstable equilibrium
- $F = -\frac{dU}{dx} = 0$
- $\frac{d^2U}{dx^2} = \text{negative}$

NOETRAL

- If particle is slightly displaced from equilibrium, then it does not experience a force or continues to be in equilibrium
- Potential energy is constant
- $F = -\frac{dU}{dx} = 0$
- $\frac{d^2U}{dx^2} = 0$

CONSERVATION OF ENERGY

For an isolated system or body in the presence of only conservative forces, the sum of kinetic and potential energies at any point remains constant throughout the motion
 $K.E + P.E = \text{constant}$

POWER

- Rate at which body does work
- Average power (P_{av}) = $\frac{W}{t}$
- Instantaneous power
 $(P_{inst}) = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$

Relation between units:

- 1 watt = 1 joule/sec = 10^7 erg/sec
- 1 HP = 746 watt, 1 MW = 10^6 watt
- 1 KW = 10^3 watt

- If work done by two bodies is same then power $\propto \frac{1}{\text{time}}$
- Unit of power multiplied by time always gives work
1 kWh = 3.6×10^6 Joules
- Slope of work-time curve gives instantaneous power

• Area under power-time graph gives work done

$P = \frac{dW}{dt} \Rightarrow dW = P dt$
 $W = \int_{t_1}^{t_2} P dt$

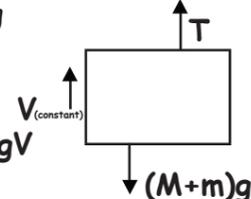
$W = \text{Area under P-t graph}$

Position and velocity in terms of power (P=constant)

- 1) Velocity, $V = \left[\frac{2Pt}{m} \right]^{1/2}$
- 2) Position, $S = \left[\frac{8Pt^3}{9m} \right]^{1/2} + 3/2$

Power delivered by an elevator

$a=0, T=(M+m)g$
 $\vec{P} = \vec{T} \cdot \vec{V}$
 $= TV$
 Power, $P=(M+m)gV$



Power of a water drawing pump

- Power, $P = \frac{dW}{dt} = \frac{dm}{dt} \left[gh + \frac{V^2}{2} \right]$
- h = height of water level
- $\frac{dm}{dt} \Rightarrow$ mass flow rate of pump

$V \rightarrow$ velocity of the water outlet

- Power required to just lift water, $V=0$

$P = gh \left(\frac{dm}{dt} \right)$

Efficiency of pump

$\mu = \frac{\text{Output Power}}{\text{Input Power}}$

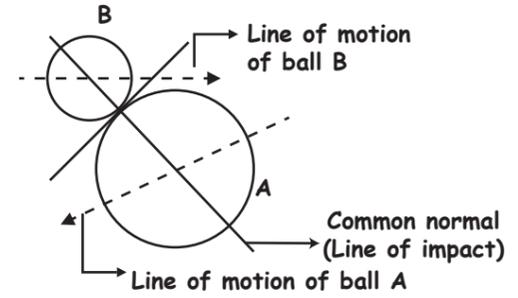


WORK ENERGY & POWER

Collision is the event in which impulsive force acts between two or more bodies which results in change of their velocities.

Line of impact

Line passing through common normal to surfaces in contact during impact



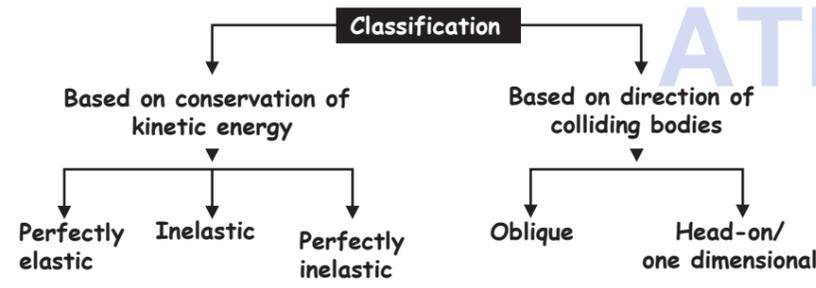
Coefficient of restitution (e)

$$e = \frac{\text{Velocity of separation along the line of impact}}{\text{Velocity of approach along the line of impact}}$$

$$= \frac{\text{Relative velocity along the line of impact after collision}}{\text{Relative velocity along the line of impact before collision}}$$

Conditions

1. For elastic collision: $e=1$
2. For inelastic collision: $e < 1$
3. For perfectly inelastic collision: $e=0$



Perfectly elastic collision

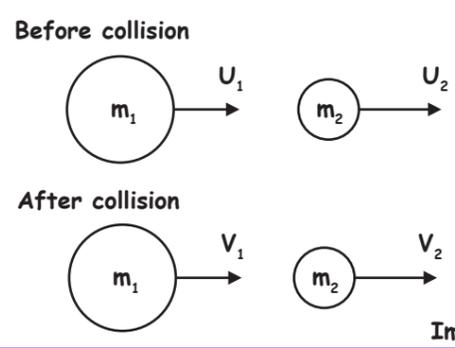
K.E before and after collision is same

Inelastic collision

K.E before and after collision is not same

Head-on collision / One dimensional collision

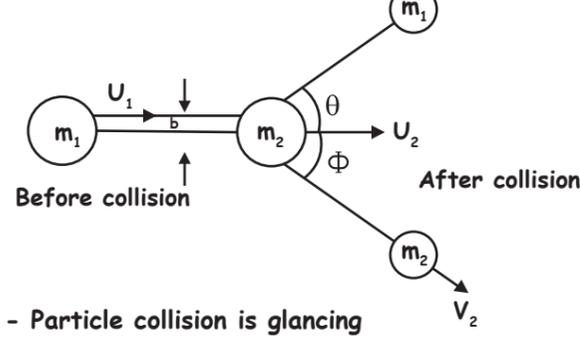
Initial velocities of the bodies are along the line of impact.



COLLISION

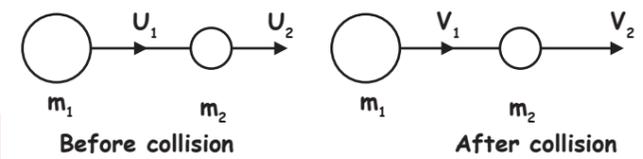
If initial velocities of the bodies are not along the line of impact.

Oblique collision



- Particle collision is glancing
- Directions of motion after collision are not along initial line of motion
- Impact parameter $0 < b < (r_1 + r_2)$ where r_1, r_2 are radii of colliding bodies

Perfectly elastic Head-on collision



Special cases:

- 1) Projectile and target having same mass $m_1 = m_2$, then $v_1 = u_2, v_2 = u_1$, the velocities get interchanged.
- 2) If massive projectile collides with a light target i.e. $m_1 \gg m_2$, then $v_1 = u_1, v_2 = -u_2 + 2u_1$
- 3) If a light projectile collides with a very heavy target, $m_1 \ll m_2$, then $v_1 = -u_1 + 2u_2, v_2 = u_2$

Energy transfer from projectile to target

- 1) Fractional decrease in kinetic energy of projectile (If target is at rest)

$$\frac{\Delta K}{K} = \frac{4m_1 m_2}{(m_1 - m_2)^2 + 4m_1 m_2}$$

Greater the difference in masses, less will be transfer of K.E and vice versa

Transfer of K.E will be maximum when difference in masses is maximum

If $m_2 = nm_1, \frac{\Delta K}{K} = \frac{4n}{(1+n)^2}$

Inelastic collision

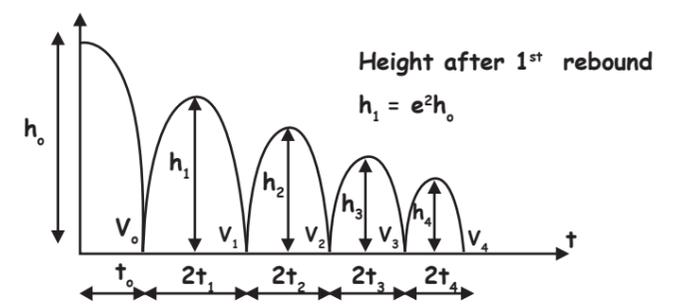
$$e = \frac{V_2 - V_1}{U_1 - U_2} = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

Velocity after collision $V_1 = \frac{(1+e)m_2 U_2}{m_1 + m_2} + \frac{(m_1 - em_2)U_1}{m_1 + m_2}$

Ratio of velocities $V_2 = \frac{(1+e)m_1 U_1}{m_1 + m_2} + \frac{(m_2 - em_1)U_2}{m_1 + m_2}$

Loss in kinetic energy $\Delta K = \frac{1}{2} \left[\frac{m_1 m_2}{m_1 + m_2} \right] (1 - e^2) (U_1 - U_2)^2$

Rebounding of ball



Total height covered by the ball before it stops bouncing

$$H = h_0 \left[\frac{1+e^2}{1-e^2} \right]$$

Total time taken by the ball until it stops bouncing $T = \left(\frac{1+e}{1-e} \right) \frac{2h_0}{g}$

Perfectly inelastic collision

Colliding bodies stick together After collision are moving in the same



Loss in kinetic energy $\Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (U_1 - U_2)^2$

Colliding bodies are moving in the opposite direction

$$V = \frac{m_1 U_1 - m_2 U_2}{m_1 + m_2}, \text{ Change in kinetic energy } \Delta K = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (U_1 - U_2)^2$$

1 Electrochemical Cell

Left side	Right side
Oxidation	Reduction
Anode	Cathode
Negative	Positive

4 EMF of a cell

$$E_{cell}^0 = E_{Cathode}^0 - E_{Anode}^0$$

$$E_{cell} = RP_{Cathode} - RP_{Anode}$$

$$E_{cell} = RP_{Cathode} + OP_{Anode}$$

$$E_{cell} = OP_{Anode} - OP_{Cathode}$$

In cell, Cathode with high RP, Anode with low RP makes spontaneous reactions

2 Representation of cell

Zn | Zn²⁺ || Cu²⁺ | Cu
R_a | P_a || R_c | P_c

Product at anode, Reactant at cathode

- Electrode potential (E_{Mⁿ⁺/M})
- E.P = Reduction Potential (R.P) = -Oxidation potential (O.P)
- If R.P = x, then O.P = -x
- Representation of Reduction half reaction: Mⁿ⁺ + ne⁻ → M
- Standard Reduction Potential (SRP) (E^o_{Mⁿ⁺/M})
- R.P at 1M and 298K. SRP is calculated by using SHE
- Representation of SHE: H⁺(1M) | H₂(g, 1 bar) | Pt(s), E^o_{SHE} = 0

5 Nernst equation

$$E_{cell} = E_{cell}^0 - \frac{0.0591}{n} \log \left[\frac{\text{Product}}{\text{Reactant}} \right]$$

For Zn | Zn²⁺ || Cu²⁺ | Cu

$$E_{cell} = E_{cell}^0 - \frac{0.0591}{2} \log \left[\frac{Zn^{2+}}{Cu^{2+}} \right]$$

For Ni | Ni²⁺ || Ag⁺ | Ag

$$E_{cell} = E_{cell}^0 - \frac{0.0591}{2} \log \left[\frac{Ni^{2+}}{(Ag^+)^2} \right]$$

R₁ | P₁ || R₂ | P₂
If R₂ ↑, P₁ ↓ then E_{cell} ↑

3 Electrochemical series

[Oxidation] → [Reduction]	E ^o (volts)
Lithium: Li ⁺ (aq) + e ⁻ ↔ Li(s)	-3.03
Potassium: K ⁺ (aq) + e ⁻ ↔ K(s)	-2.92
Calcium: Ca ²⁺ (aq) + 2e ⁻ ↔ Ca(s)	-2.87
Sodium: Na ⁺ (aq) + e ⁻ ↔ Na(s)	-2.71
Magnesium: Mg ²⁺ (aq) + 2e ⁻ ↔ Mg(s)	-2.37
Aluminum: Al ³⁺ (aq) + 3e ⁻ ↔ Al(s)	-1.66
Zinc: Zn ²⁺ (aq) + 2e ⁻ ↔ Zn(s)	-0.76
Iron: Fe ²⁺ (aq) + 2e ⁻ ↔ Fe(s)	-0.44
Lead: Pb ²⁺ (aq) + 2e ⁻ ↔ Pb(s)	-0.13
Hydrogen: 2H ⁺ (aq) + 2e ⁻ ↔ H ₂ (g)	0.00
Copper: Cu ²⁺ (aq) + 2e ⁻ ↔ Cu(s)	+0.34
Silver: Ag ⁺ (aq) + e ⁻ ↔ Ag(s)	+0.80
Gold: Au ³⁺ (aq) + 3e ⁻ ↔ Au(s)	+1.50
Fluorine: F ₂ (g) + 2e ⁻ ↔ 2F ⁻ (aq)	+2.87

SRP ↑ = O.A., SRP ↓ = R.A.
Metals with high SRP = less reactive
Metals with low SRP = highly reactive

6 Application of Nernst Equation

- Electrode Potential: E_{Mⁿ⁺/M} = E^o_{Mⁿ⁺/M} - (0.0591/n) log (1/[Mⁿ⁺])
- Nernst equation in SHE:
 - E_{H⁺/H₂} = -0.0591 log (P_{H₂}/[H⁺]²)
 - If P_{H₂} = 1 atm (R.P.) = E_{H⁺/H₂} = -0.0591 pH (O.P.) = E_{H₂/H⁺} = +0.0591 pH
- Concentration Cells: Zn | Zn_{(C₁)²⁺ || Zn_{(C₂)²⁺ | Zn}}
- E_{cell} = (0.0591/n) log (C₂/C₁)
- C₂/C₁ > 1 ⇒ log (C₂/C₁) > 0 ∴ E_{cell} > 0

7 EMF; K_c & ΔG

$$E_{cell}^0 = \frac{0.0591}{n} \log K_c, \log K_c = \frac{nE_{cell}^0}{0.0591}$$

$$\Delta G = -nFE_{cell}$$

Spontaneous	Non-spontaneous
ΔG < 0	ΔG > 0
E ^o _{cell} > 0	E ^o _{cell} < 0
log K _c > 0	log K _c < 0
K _c > 1	K _c < 1

Galvanisation is applying coating of Zn

1 Electrolytic cell

ANODE	CATHODE
• Anion goes to anode	• Cation goes to cathode
• +ve electrode	• -ve electrode
• Oxidation	• Reduction
• A → A ⁺ + e ⁻	• B + 1e ⁻ → B ⁻
• A → A ⁿ⁺ + ne ⁻	• B ⁿ⁺ + ne ⁻ → B ⁻

2 Product of electrolysis

Deposition order of cation: (order of R.P)
Li⁺ < K⁺ < Ca²⁺ < Na⁺ < Mg²⁺ < Al³⁺ < Zn²⁺ < Fe²⁺ < Ni²⁺ < H⁺ < Cu²⁺ < Hg²⁺ < Ag⁺ < Au³⁺

Deposition order of anion
SO₄²⁻ < NO₃⁻ < OH⁻ < Cl⁻ < Br⁻ < I⁻

Note: 1) For conc. H₂SO₄
Anode: H⁺ + 1e⁻ → 1/2 H₂
Cathode: 2SO₄²⁻ → S₂O₈²⁻ + 2e⁻ (peroxo disulphate ion)

2) Very dil. NaCl(H₂O >> NaCl)
Anode: H⁺ + 1e⁻ → 1/2 H₂
Cathode: 2OH⁻ → 1/2 O₂ + H₂O + 2e⁻

3) For CuSO₄ with Cu electrode
Anode: Cu → Cu²⁺ + 2e⁻
Cathode: Cu²⁺ + 2e⁻ → Cu

Electroplating

ELECTROCHEMISTRY

3 Faraday's law

Product formed

$$m = \frac{EM}{96500} \times It$$

$$EM = \frac{AM}{\text{valency}}$$

1F = charge of 1 mole of e⁻ = 96500 C

Na⁺ + e⁻ → Na ⇒ 1F
Mg²⁺ + 2e⁻ → Mg ⇒ 2F
Al³⁺ + 3e⁻ → Al ⇒ 3F

1F displaces & gives 1 equivalent of product

1F = 96500 C

- 1mol ⇔ 23g ⇔ Na⁺
- O₂ ⇒ 8g ⇒ 1/4 mole = 5.6L
- H₂ ⇒ 1g ⇒ 1/2 mole = 11.2L
- Cl₂ ⇒ 35.5g ⇒ 1/2 mole = 11.2L
- 1mol ⇔ 9g ⇔ Al³⁺
- Ag⁺ ⇒ 108g ⇒ 1mol
- Cu²⁺ ⇒ 31.75g ⇒ 1/2 mole

4 Electrolytic conduction

Resistance (R) = ρ l / A, Unit of R = Ω, ρ = Ωm, C = Ω⁻¹S = mho, K = Ω⁻¹m⁻¹ or Sm⁻¹, 1Scm⁻¹ = 100 Sm⁻¹

Conductance (C) = 1/R

Conductivity (K) = 1/ρ

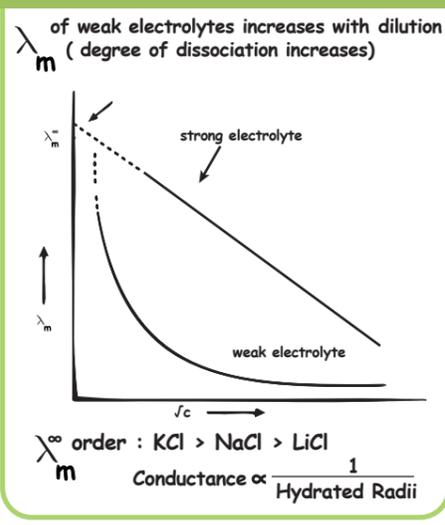
Molar Conductivity (λ _m)	Equivalent Conductivity (λ _{eq})
λ _m = 1000 K / M	λ _{eq} = 1000 K / N
K → Scm ⁻¹	K → Scm ⁻¹
M → mol L ⁻¹	N → eq L ⁻¹
λ _m → Scm ² mol ⁻¹	λ _{eq} → Scm ² eq ⁻¹

1Scm² mol⁻¹ = 10⁻⁴ Sm² mol⁻¹, λ_m = λ_{eq} × Z, N > M ∴ λ_m > λ_{eq}

For H₂SO₄, Z = 2 (2H⁺)
NaCl, Z = 1 (1Na⁺)
Al₂(SO₄)₃, Z = 6 (2Al³⁺)

λ_m for SE increases with dilution (interionic attraction decreases)

At √c = 0, λ_m = λ_m[∞] (limiting molar conductivity)

$$\lambda_m = \lambda_m^\infty - b\sqrt{c}$$
 (Debye-Huckel Onsagar equation)


5 Kohlrausch's law

$$\lambda_m^\infty(AB_2) = \lambda_m^\infty(A^{2+}) + 2\lambda_m^\infty(B^-)$$

$$\lambda_{eq}^\infty(AB_2) = \lambda_m^\infty(A^{2+}) + \lambda_{eq}^\infty(B^-)$$

For Al₂(SO₄)₃

$$\lambda_m^\infty(Al_2(SO_4)_3) = 2\lambda_m^\infty(Al^{3+}) + 3\lambda_m^\infty(SO_4^{2-})$$

$$\lambda_{eq}^\infty(Al_2(SO_4)_3) = \lambda_{eq}^\infty(Al^{3+}) + \lambda_{eq}^\infty(SO_4^{2-})$$

Application

$$\lambda_m^\infty NH_4OH = \lambda_m^\infty NH_4Cl + \lambda_m^\infty NaOH - \lambda_m^\infty NaCl$$

$$\lambda_m^\infty CH_3COOH = \lambda_m^\infty CH_3COONa + \lambda_m^\infty HCl - \lambda_m^\infty NaCl$$

$$\lambda_m^\infty BaSO_4 = \lambda_m^\infty BaCl_2 + \lambda_m^\infty Na_2SO_4 - 2\lambda_m^\infty NaCl$$

Internal Energy(U)

(1) $\Delta U = \Delta Q_v = nC_v \Delta T = n \frac{f}{2} R \Delta T$
 $= \frac{nR \Delta T}{\gamma - 1} = \frac{\Delta(PV)}{\gamma - 1}$
 $= \frac{P_f V_f - P_i V_i}{\gamma - 1}$

(Internal Energy is only the function of temperature of the gas)

• First law of T.D $\Rightarrow Q = \Delta U + W$
 • Q, W \Rightarrow path functions
 • $\Delta U \Rightarrow$ state function

• $W_1 \neq W_2$
 • $Q_1 \neq Q_2$
 • $\Delta U_1 = \Delta U_2 = U_B - U_A$

WORK

Work done: path function
 $W = \int F \cdot dx = \int P dv$
 • Unit: Joule(J)

Isobaric process $\Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0$
 Cyclic process
 $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A$
 $\Delta U = U_A - U_A = 0$

Sign Convention

• $+ve Q$ Heat added to system
 • $-ve Q$ Heat removed from system
 • $+ve W$ Work done by system
 • $-ve W$ Work done on system

HEAT

Heat: path function
 \Rightarrow unit: calorie/Joule

For any process,
 $Q = nC \Delta T$
 where, C = Specific heat capacity for the process

Adiabatic process $\Rightarrow \Delta Q = 0$ [No heat transfer]
 At constant volume $\Rightarrow Q_v = \Delta U = nC_v \Delta T$
 At constant pressure $\Rightarrow Q_p = \Delta U + W = nC_p \Delta T$

Work done from P-V Graph

Area under P-V diagram gives work done by the gas

Compression

$|W_{adiabatic}| > |W_{isothermal}| > |W_{isobaric}| > |W_{isochoric}|$

Isobaric process

$W = P(V_2 - V_1)$

Isochoric process

$W = P \Delta V = 0$

Cyclic process

- W = area inside the graph
- For clockwise process, $W = -ve$
- For anti-clockwise process, $W = +ve$

$W = \frac{\pi}{4} (P_2 - P_1)(V_2 - V_1)$
 $W = \frac{1}{2} (P_2 - P_1)(V_2 - V_1)$

Expansion

$W_{isobaric} > W_{isothermal} > W_{adiabatic} > W_{isochoric}$

$W_1 \neq W_2$
 $W_1 > W_2$

Thermodynamic processes

01 Adiabatic process

- $Q = 0$ [no exchange of heat]
- Rapid or spontaneous process/insulated vessel

Compression
 $Q = \Delta U + W$
 $Q = 0 \Rightarrow \Delta U = -W$
 $W = -ve \Rightarrow \Delta U = +ve$
 $\Delta U \uparrow \Rightarrow$ Temperature \uparrow
 \Rightarrow Pressure \uparrow

Expansion
 $W = -ve \Rightarrow \Delta U = +ve$
 $\Delta U \downarrow \Rightarrow$ Temperature \downarrow
 \Rightarrow Pressure \downarrow

Equation of state

$PV^\gamma = \text{constant}$
 $TV^{\gamma-1} = \text{constant}$
 $PT^{\frac{\gamma}{\gamma-1}} = \text{constant}$

Work done by the gas
 $W = -\Delta U = nC_v(T_1 - T_2)$
 $= n \frac{f}{2} R(T_1 - T_2)$
 $W = \frac{nR}{\gamma-1}(T_1 - T_2)$
 $W = \frac{P_i V_i - P_f V_f}{\gamma-1}$

Slope of adiabatic process
 $= \gamma \times$ slope of isothermal process
 specific heat of gas $\Rightarrow C = 0$
 $C = \frac{Q}{\Delta T} \rightarrow Q = 0$
 $C = 0$

02 Isothermal process

$\Rightarrow \Delta T = 0 \Rightarrow \Delta U = 0$
 eg:- perfectly conducting slow process

$Q = \Delta U + W$
 $Q = W$

equation of state $\Rightarrow PV = \text{Constant}$
 $P_1 V_1 = P_2 V_2$

Workdone by the gas
 $W = 2.303 nRT \log \left(\frac{V_2}{V_1} \right)$
 $W = 2.303 nRT \log \left(\frac{P_1}{P_2} \right)$

Slope:
 Slope of adiabatic process
 $= \gamma \times$ slope of isothermal process
 specific heat $C = \infty$

03 Isochoric process

$\Delta P = 0$

$Q = \Delta U + W$
 equation of state $\Rightarrow V \propto T$
 $\frac{V_1}{V_2} = \frac{T_1}{T_2}$

Work done by the gas
 $W = P \Delta V = P(V_2 - V_1) = nR(T_2 - T_1)$
 specific heat
 $\Rightarrow C_p = \left(1 + \frac{f}{2}\right) R$
 $= \frac{\gamma R}{\gamma - 1}$

04 Isochoric process

$\Delta V = 0$ or $V = \text{constant}$
 equation of state $\Rightarrow P \propto T \Rightarrow \frac{P_1}{P_2} = \frac{T_1}{T_2}$

Work done by the gas
 $\Delta V = 0 \Rightarrow W = 0$
 specific heat
 $\Rightarrow C_v = \frac{f}{2} R$
 $= \frac{R}{\gamma - 1}$

Heat Engine

'Device that converts heat into work'

High temp. T_1 Source
 $Q_1 \rightarrow$ heat absorbed by engine
 $W = Q_1 - Q_2$
 $Q_2 \rightarrow$ heat released by engine
 Low temp. T_2 Sink (surrounding)

efficiency(η)
 $\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$
 $\eta_{\text{Carnot}} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$
 $\eta_{\text{max}} \Rightarrow$ When $Q_2 = 0$ or $T_2 = 0K$ (not possible)

Carnot Engine

\rightarrow Ideal engine
 $\rightarrow \eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$

Refrigerator and heat pump

Source T_1 (High Temp.)
 Q_1
 W (work done)
 Q_2
 Sink T_2 (Low Temp.)

Coefficient of performance of refrigeration(β)
 $\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} \quad \beta = \frac{T_2}{T_1 - T_2}$

Coefficient of performance of heat pump
 $\beta = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{T_1}{T_1 - T_2}$

Relationship between

$\beta = \frac{1 - \eta}{\eta}$
 $(COP)_{\text{heat pump}} = 1 + (COP)_{\text{refrigerator}}$

Cascaded engine

Same work output
 $W_1 = W_2$
 $T_1 - T = T - T_3$
 $2T = T_1 + T_3$
 $T = \frac{T_1 + T_3}{2}$

Same efficiency
 $1 - \frac{T}{T_1} = 1 - \frac{T_3}{T}$
 $T^2 = T_1 T_3$
 $T = \sqrt{T_1 T_3}$

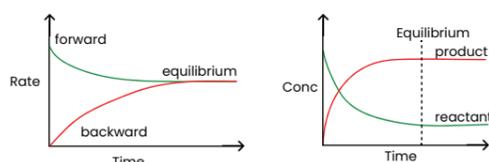


THERMODYNAMICS

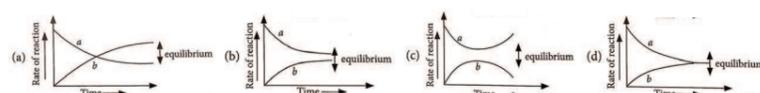
EQUILIBRIUM

- The chemical reactions which takes place in both directions are called reversible reactions
- Equilibrium is the end state of a reversible reaction.
- Gaseous Equilibrium is established only in a closed container.
- At equilibrium, the rate of forward and backward reactions are equal.
- At equilibrium, the concentration of reactants & products becomes constant.

GRAPHICAL REPRESENTATIONS



Q. For the equilibrium $A \rightleftharpoons B$, the variation of the rate of the forward (a) & reverse (b) reaction with time is given by



PHYSICAL EQUILIBRIUM

- Such equilibrium is established in physical reactions.
- It is dynamic in nature.

LIQUID \rightleftharpoons VAPOUR EQUILIBRIUM

- Here vapour pressure is constant at a constant temp for given.

SOLID \rightleftharpoons LIQUID EQUILIBRIUM

- Established only at a constant temperature
- Ice-water equilibrium established at 0°C (at 1 atm)

SOLID IN LIQUID EQUILIBRIUM

- Established only in a Saturated solution
- eg: Saturated sugar solution.
Sugar (dissolved) \rightleftharpoons Sugar (undissolved)

GAS IN LIQUID EQUILIBRIUM

- Here solubility depends upon pressure (Henry's law)
- eg: Soda water
 CO_2 (dissolved) \rightleftharpoons CO_2 (undissolved)

Q. Which of the given statements does not elucidate the equilibrium state precisely?

- The equilibrium can be approached from either direction.
- The equilibrium can be attained only if the system is an isolated system.
- The free energy change at constant pressure and temperature is zero.
- It is dynamic in nature.

CHEMICAL EQUILIBRIUM

- Chemical equilibrium approaches from both forward & backward direction
- Chemical equilibrium is dynamic in nature.

EQUILIBRIUM CONSTANT $[K_c]$

- For a general reversible reaction
 $aA + bB \rightleftharpoons cC + dD$

$$K_c = \frac{[C]^c [D]^d}{[A]^a [B]^b}$$

REACTION QUOTIENT $[Q]$

- At any time during the reaction $aA + bB \rightleftharpoons cC + dD$ the ratio $\frac{[C]^c [D]^d}{[A]^a [B]^b}$ is known as concentration quotient, Q_c .

- At equilibrium $Q_c = K_c$

CHARACTERISTICS OF K [eqb. const]

- Value of K does not depends upon initial concentration of reactants and products.
- Value of K does not depends upon the direction from which equilibrium is attained.

Q In the given reaction: $A + 2B \rightleftharpoons 2C$, 2 moles each of A & B present in 10 L of solution combine to form 1 mole of C. Calculate K_c for the reaction.

- (A) 1.5 (B) 6.67 (C) 0.15 (D) 2.3

CHEMICAL EQUILIBRIUM

Applications of K & Q

- Value of K depends only on temperature.
- If K for the reaction $aA + bB \rightleftharpoons cC + dD$ is K , then K for the reaction $cC + dD \rightleftharpoons aA + bB$ will be $\frac{1}{K}$
- If K for the reaction $aA + bB \rightleftharpoons cC + dD$ is K , then K for the reaction $naA + nbB \rightleftharpoons ncC + ndD$ will be $(K)^n$
- During the addition of two reactions having equilibrium constants K_1 & K_2 , then the net Constant $K = K_1 \times K_2$
- During the subtraction of a reaction having constant K_2 from a reaction having constant K_1 , then the net constant $K = K_1 / K_2$
- If $Q < K$, the reaction will proceed in forward direction
- If $Q > K$, the reaction will proceed in backward direction
- If $Q = K$, the system is in equilibrium.
- If $K > 10^3$, the reaction is almost complete in forward direction.
- If $K < 10^{-3}$, the reaction is in backward direction.
- If K is in b/w 10^3 & 10^{-3} almost same reaction takes place in both forward and backward direction

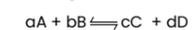
Q At a given temperature, the equilibrium constants for the reactions,
 $NO(g) + \frac{1}{2}O_2(g) \rightleftharpoons NO_2(g)$ & $2NO_2(g) \rightleftharpoons 2NO(g) + O_2(g)$

are K_1 and K_2 respectively. If K_1 is 4×10^{-3} . then K_2 will be

- (A) 8×10^{-3} (B) 16×10^{-3} (C) 6.25×10^4 (D) 6.25×10^6

HOMOGENEOUS EQUILIBRIUM

- If they are in solid or liquid phase (or aqueous), K can be represented as K_c .
- If they are in gaseous phase, K can be represented as K_p
- For a general reaction



$$K_c = \frac{[C]^c [D]^d}{[A]^a [B]^b} \quad \& \quad K_p = \frac{P_c^c P_d^d}{P_a^a P_b^b}$$

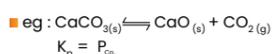
$$K_p = K_c (RT)^{\Delta n_g} \quad \Delta n_g = n_p - n_r$$

UNIT OF EQUILIBRIUM CONSTANT

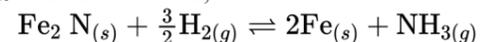
- Unit of $K_c = (\text{mol/L})^{\Delta n_g}$
- Unit of $K_p = (\text{atm})^{\Delta n_g}$
- If $\Delta n_g = 0$, equilibrium constant has no unit

HETEROGENEOUS EQUILIBRIUM

- In heterogeneous equilibrium, concentration of pure solid & pure liquid is taken as one



Q For the reaction,



- (a) $K_c = K_p(RT)$ (b) $K_c = K_p(RT)^{-1/2}$
(c) $K_c = K_p(RT)^{1/2}$ (d) $K_c = K_p(RT)^{3/2}$

LE CHATELIER'S PRINCIPLE

According to Le-chatelier's principle, if a system at equilibrium is subjected to a change in concentration, temperature or pressure, the equilibrium will shift automatically in one direction which will nullify the effect of the change.

- Conc. of reactant increases \rightarrow shift towards forward reaction.
- Conc. of product decreases \rightarrow shift towards forward reaction.
- Conc. of reactant decreases \rightarrow shift towards backward reaction.
- Conc. of product increases \rightarrow shift towards backward reaction.
- Pressure increases \rightarrow shift towards lesser number of gaseous moles
- Pressure decreases \rightarrow shift towards higher number of gaseous moles
- No. of gaseous moles of reactants & products are equal, pressure has no effect.
- If temperature increases \rightarrow shift towards endothermic
- If temperature decreases \rightarrow shift towards exothermic
- Catalyst helps to attain eqm state easily. After the establishment of eqm, catalyst has no effect.
- Addition of inert gas at constant volume, no effect.
- Addition of inert gas at constant pressure \rightarrow shift towards higher number of gaseous moles

Q Which one of the following conditions will favour maximum formation of the product in the reaction $A_2(g) + B_2(g) \rightleftharpoons X_2(g)$. $\Delta_r H = -X \text{ kJ/mol}$?

- (A) Low temperature and high pressure (B) High temperature and high pressure
(C) Low temperature and low pressure (D) High temperature and low pressure

01 UNCERTAINTY PRINCIPLE

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Delta x \cdot m\Delta v \geq \frac{h}{4\pi}$$

Q. According to Heisenberg's uncertainty principle, $\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$ which of the following is correct ?

- If $\Delta x = 0$ then $\Delta p = \infty$
- If $\Delta v = 0$ then $\Delta p = 0$
- If $\Delta p = 0$ then $\Delta x = \infty$
- All are correct

Q. Find uncertainty in velocity, if uncertainty position is equal to uncertainty in momentum.

- $\frac{h}{2\sqrt{\pi m}}$
- $\frac{1}{2m} \sqrt{\frac{h}{\pi}}$
- $\frac{1}{m} \sqrt{\frac{h}{\pi}}$
- $\frac{1}{2} \sqrt{\frac{h}{\pi m}}$

Q. The uncertainty involved in the measurement of velocity within a distance of 0.1 \AA is:

- $5.79 \times 10^5 \text{ m/s}$
- $5.79 \times 10^7 \text{ m/s}$
- $5.79 \times 10^8 \text{ m/s}$
- $5.79 \times 10^9 \text{ m/s}$

PRINCIPLE QUANTUM NUMBER

In n^{th} Shell ,
 Number of subshells = n
 Number of orbitals = n^2
 Max. number of electrons = $2n^2$

Q. Find angular momentum of
 (i) 2s orbital (ii) 3d orbital
 (iii) 4p orbital (iv) e^- in 4th orbit

AZIMUTHAL QUANTUM NUMBER

- It describes shell or orbit
 $n = 1, 2, 3, 4, \dots$
 K, L, M, N, \dots
- It describes size & energy of shell.
 $r \propto n^2$ $E \propto \frac{1}{n^2}$
- It defines the angular momentum
 $mvr = \frac{nh}{2\pi}$

Q. Find maximum no. of e^- having
 (i) $n=4, s = -1/2$ (ii) $n=3, l=1, m=0$
 (iii) $n=2, l=0$ (iv) $n=3, l=1$

MAGNETIC QUANTUM NUMBER

- It describes subshell value from 0 to $n-1$
 $l=0 \rightarrow s$ $l=2 \rightarrow d$
 $l=1 \rightarrow p$ $l=3 \rightarrow f$
- Orbital angular momentum
 $= \sqrt{l(l+1)} \hbar, \hbar = \frac{h}{2\pi}$
- Maximum no. of orbital in a subshell = $2l + 1$
 Maximum no. of electrons in a subshell = $4l + 2$

If $l=2$
 1) Orbital = d
 2) No. of orbitals = $2(2+1)=5$
 ($d_{xy}, d_{xz}, d_{yz}, d_{x^2-y^2}, d_{z^2}$)
 3) Total $e^-s = 2(2l+1) = 10 e^-s$
 4) Orbital angular momentum = $= \sqrt{2(2+1)} \hbar = \sqrt{6} \hbar$

SPIN QUANTUM NUMBER

Value of $m = -l \leq m \leq l$
 Total values of $m = 2l + 1$

$n = 4$
 $l = 0$ $m = 0$
 $l = 1$ $m = -1, 0, +1$
 $l = 2$ $m = -2, -1, 0, +1, +2$
 $l = 3$ $m = -3, -2, -1, 0, +1, +2, +3$

SPIN
 — CLOCKWISE (+ 1/2)
 — ANTICLOCKWISE (- 1/2)

ATDB.uno STRUCTURE OF ATOM

ENERGY OF ORBITALS

- Mono electronic species
 Energy defined upon n
 $1s < 2s = 2p < 3s = 3p = 3d$
- Multi electronic species
 $3s < 3p < 4s < 3d$

(n+l) rule
 $\rightarrow As (n + l) \uparrow, E \uparrow$
 \rightarrow If $(n + l)$ is same, then $n \uparrow E \uparrow$

Orbital	2s	3d
(n+l) value	$n = 2$ $l = 0$ $n+l = 2$	$n = 3$ $l = 2$ $n+l = 5$

SHAPE OF ORBITALS

- s orbital - Spherical shape
- p orbital - dumb bell shape
- d orbital - double dumb bell shape

NODES

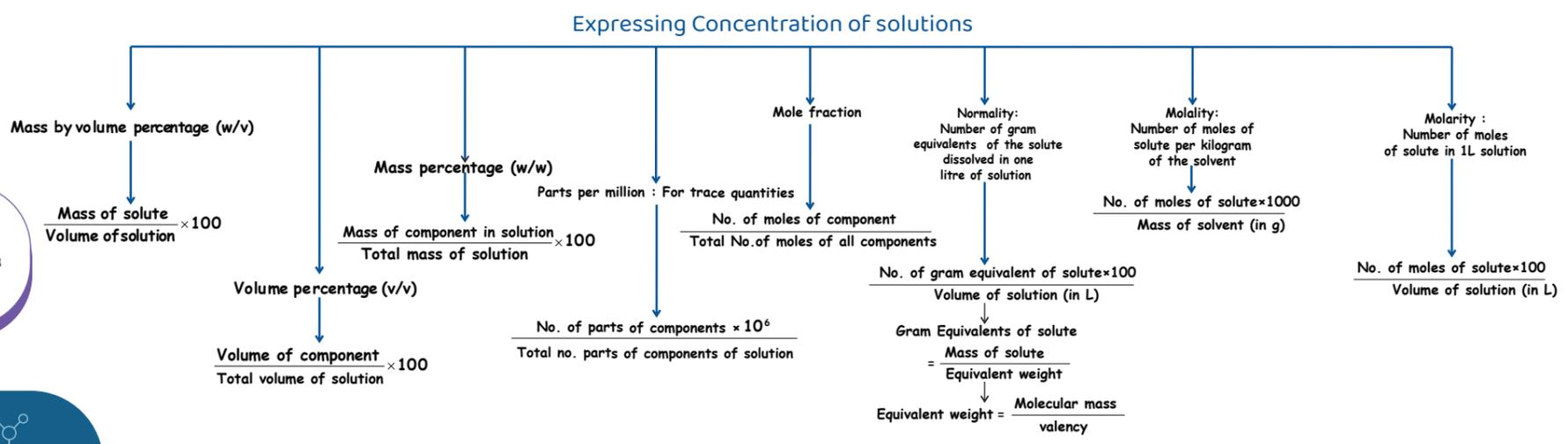
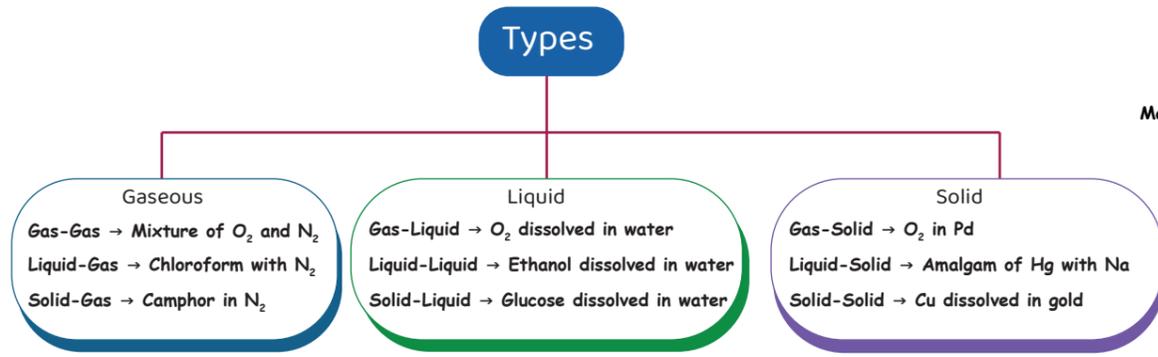
$\Psi \rightarrow e^-$ wave function
 $\Psi^2 \rightarrow$ probability of finding the electrons

- * Node \rightarrow Probability of finding the electron is zero.
- * Node plane \rightarrow Plane; where $\Psi^2 = 0$
- * Radial nodes $\rightarrow n - l - 1$
- * Angular nodes = l
- * Total nodes = $n - 1$

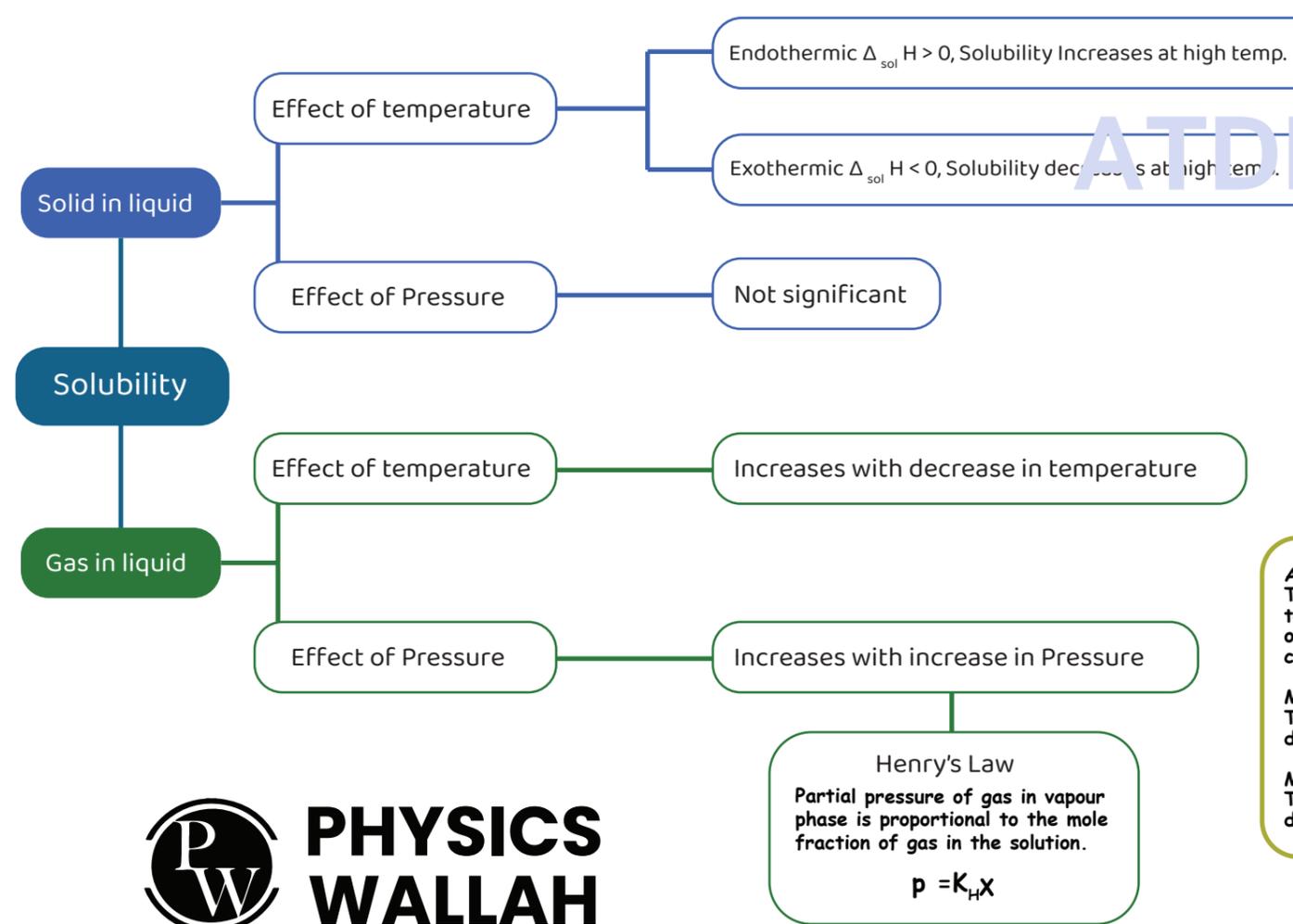
FILLING OF ATOMIC ORBITAL

- Aufbau principle**
 Electrons are filled in the increasing order of energy
 $1s < 2s < 2p < 3s < 3p < 4s < 3d \dots$
- Pauli's exclusion principle**
 No two electrons can have same four quantum numbers
 $1s^3$ - against Pauli's exclusion principle
- Hund's rule**
 Pairing is only takes place after each orbital is singly occupied.
 $\uparrow \downarrow \uparrow \uparrow$ - Against Hund's rule





SOLUTIONS



Colligative properties

- Osmotic pressure $\rightarrow \pi = CRT$
- Depression in freezing point $\rightarrow \Delta T_f = \frac{K_f \times W_2 \times 1000}{M_2 \times W_1}$
- Elevation of boiling point $\rightarrow \Delta T_b = \frac{K_b \times 1000 \times W_2}{M_2 \times W_1}$
- Relative lowering of vapour pressure (for ideal solutions) $\rightarrow \frac{W_2 \times M_1}{M_2 \times W_1} = \frac{P_1^0 - P_1}{P_1^0}$

Abnormal molecular mass

Molecular mass different from expected value

Van't Hoff factor (i) = $\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$

Raoult's Law

For any solution, the partial vapour pressure of each volatile component is directly proportional to its mole fraction in solution phase

- Obey Raoult's law
 Ideal solution $\rightarrow \Delta H_{\text{mix}} = \Delta V_{\text{mix}} = 0$
 eg: n-hexane and n-heptane, Ethyl bromide + Ethyl chloride, Chlorobenzene + Bromobenzene, etc.
- Do not Obey Raoult's law
 Non-Ideal solution $\rightarrow \Delta H_{\text{mix}} \text{ or } \Delta V_{\text{mix}} \neq 0$

Azeotropes:

The mixtures of liquids which boil at constant temperature like a pure liquid and possess same composition of components in liquid as well as vapour phase are called constant boiling mixtures or azeotropic mixtures.

- Minimum boiling azeotropes:** They are formed by those liquid pairs which show positive deviations from ideal behaviour. eg: ethanol-water mixture.
- Maximum boiling azeotropes:** They are formed by those liquid pairs which show negative deviations from ideal behaviour e.g nitric acid-water mixture.

Positive Deviation

$\Delta H_{\text{mix}} > 0$
 $\Delta V_{\text{mix}} > 0$

eg: Acetone + Ethyl alcohol, Water + Ethyl alcohol, Carbon tetrachloride + Chloroform, Chloroform + Ethanol, Acetone + Carbon disulphide, Acetone + Benzene, etc.

Negative Deviation

$\Delta H_{\text{mix}} < 0$
 $\Delta V_{\text{mix}} < 0$

eg: Acetone + Aniline, HCl + H₂O, HNO₃ + H₂O, H₂SO₄ + H₂O, Acetone + Chloroform, HNO₃ + Chloroform, Benzene + Chloroform, Pyridine + Glacial acetic acid, etc.

