ELECTRIC CHARGES AND FIELDS

Basic Properties of Charges

Quantization of charge : Total charge on a body is always an integral multiple of a basic unit of charge denoted by e and is given by $q = \pm ne$

Conservation of charge : Total charge of an isolated system remains unchanged with time.

Additivity of charge : Total charge of a system is the algebraic sum (*i.e.* sum taking into account with proper signs) of all individual charges in the system.

Coulomb's Law

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|\vec{r}_{12}|^3}\vec{r}_{12} \text{ for like charges}$$

$$\vec{F}_{21} = \frac{q_1 q_2}{4\pi\varepsilon_0 |\vec{r}_{21}|^3}\vec{r}_{12} \text{ for unlike charges}$$

Using Superposition Principle

- The vector sum of forces would give us the total force.
- Force between two charges is unaffected by the presence of the other charges.

Electric Field

Electric Field : Electric field intensity at a point distant r from a point charge q in air is

$$\vec{E} = \frac{1}{4\pi\varepsilon_0 r^2}\hat{r}.$$

Basic characteristics

- Field lines start from positive charges and end at negative charges.
- Two field lines can never cross each other.
- Electrostatic field lines do not form any closed loops.

Electric Dipole

Electric Dipole : Every dipole is associated with a dipole moment $\vec{p}$ whose magnitude is equal to the product of the magnitude of either charge (q) and the distance $2\vec{a}$ between the charges, *i.e.*, $\vec{p} = q \times (2\vec{a})$.

Dipole in an external field experiences

Torque

$\vec{\tau} = \vec{p} \times \vec{E}$ or $\tau = pE\sin\theta$.

Electric field due to dipole

- On Equatorial line (Broad on position) $E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$
- On Axial line (End on position) $E = \frac{2p}{4\pi\varepsilon_0 r^3}$

Electric Flux

Electric Flux : Electric flux over an area in an electric field represents the total number of electric field lines crossing this area.

Gauss's Theorem : Total normal electric flux over a closed surface S in vacuum is $1/\varepsilon_0$ times the charge (Q) contained inside the surface.

$$\phi_E = \oint_S \vec{E}.d\vec{S} = \frac{Q}{\varepsilon_0}.$$

Applications

volume charge density, $\rho = \frac{q}{V}$

Electric field due to a uniformly charged thin spherical shell

At a point outside the shell *i.e.*, $r > R$, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}$

On the surface $r = R$, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$

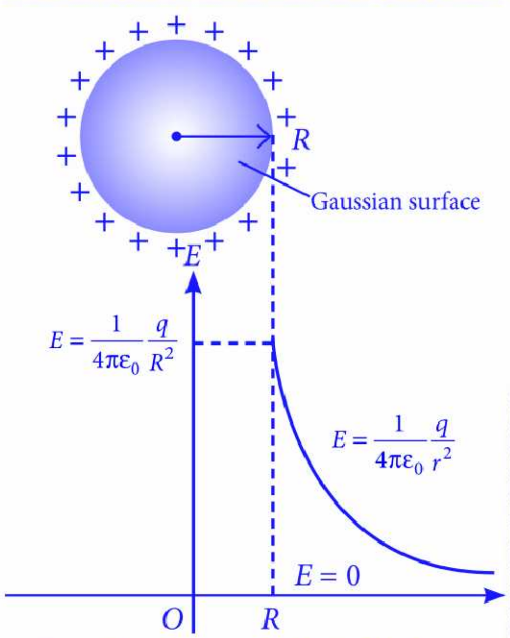
Inside the shell *i.e.*, $r < R$, $E = 0$

surface charge density, $\sigma = \frac{q}{A}$

Electric field due to a uniformly charged infinite thin plane sheet, $E = \frac{\sigma}{2\varepsilon_0}$.

linear charge density, $\lambda = \frac{q}{L}$

Electric field due to an infinitely long thin uniformly charged straight wire, $E = \frac{\lambda}{2\pi\varepsilon_0 r}$.



For uniformly charged spherical shell

ELECTROSTATIC POTENTIAL AND CAPACITANCE

Electrostatic Potential

Work done per unit positive test charge by an external force in bringing a unit positive charge from infinity to a point in the presence of another point charge.

$$V = -\frac{W}{q_0} = \frac{q}{4\pi\varepsilon_0 r}$$

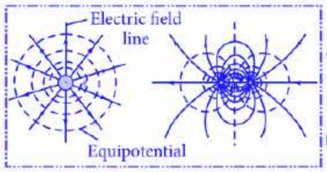
Electric Potential Energy

For a system of two charges

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}$$

Equipotential Surface

Surface having same electrostatic potential at every point.



Properties

- Do not intersect each other
- At every point $\vec{E} \perp$ surface
- Work done in moving a charge is zero $W_{net} = 0$
- Closely spaced in the region of strong field and vice-versa.

Electrostatic Potential Due to an Electric Dipole

At any arbitrary point ; $V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

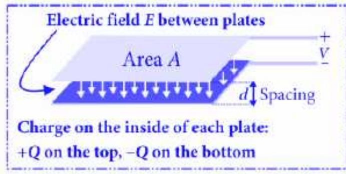
- At axial point $V = \frac{p}{4\pi\varepsilon_0 r^2}$
- At equatorial point $V = 0$

Potential energy of a dipole in external field

$U(\theta) = pE(\cos\theta_0 - \cos\theta)$

$\rightarrow$ When initially at $\theta_0 = 90°$

$U = -\vec{p}\cdot\vec{E}$



Relation between $\vec{E}$ and $\vec{V}$

$$\vec{E} = -\vec{\nabla} V$$

$$E = -\frac{dV}{dr}$$

Capacitor and Capacitance

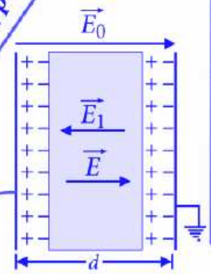
Capacitor is used to store electrical energy. Capacitance is defined as the ratio of the charge stored to the potential between the plates.

$$C = \frac{Q}{V}$$

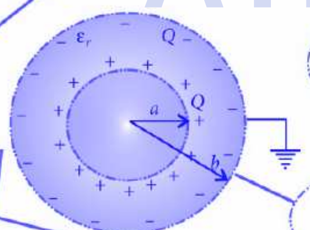
Combination of Capacitor

- Series combination $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$
- Parallel combination $C_p = C_1 + C_2$

Capacitances of different types of capacitors



- Parallel plate capacitor with dielectric slab of thickness t: $C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{K}\right)}$
- Parallel plate capacitor filled with dielectric: $C = \frac{K\varepsilon_0 A}{d}$
- Air filled parallel plate capacitor: $C = \frac{\varepsilon_0 A}{d}$
- Parallel plate capacitor with metallic conductor inserted in it: $C = \frac{\varepsilon_0 A}{(d - t)}$
- Spherical capacitor: $C = 4\pi\varepsilon_0 \frac{ab}{b - a}$



Electrostatic Shielding

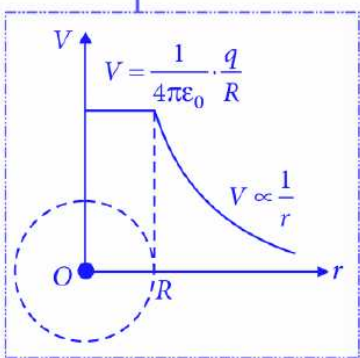
To shield an electronic circuit from external field by surrounding it with conducting walls.

Lightning Conductor

Lightning conductors fitted above the highest part of a building to protect a tall building from being struck by lightning.

Electric Potential due to Uniformly Charged Spherical Shell

- Outside the shell $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}; r > R$
- On the shell $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}; r = R$
- Inside the shell $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$



Energy Stored in a Capacitor

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

Energy Density

$$u = \frac{U}{V} = \frac{1}{2}\varepsilon_0 E^2$$

Electric Potential Due to a Non-Conducting Solid Sphere

- outside the sphere $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r};\ r > R$
- on the sphere *i.e.*, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{R};\ r = R$
- inside the sphere $V = \frac{1}{4\pi\varepsilon_0}\frac{q(3R^2 - r^2)}{2R^3};\ r < R$

CURRENT ELECTRICITY

Current Electricity

Electric Current

- $I = \frac{q}{t} = \frac{ne}{t}$
- In case of an electron revolving in a circle of radius r with speed v, period of revolution is $T = \frac{2\pi r}{v}$
- Frequency of revolution $\upsilon = \frac{v}{2\pi r}$
- Current at any point of the orbit is $I = \frac{e}{T} = \frac{ev}{2\pi r}$

Electric Power

$$P = VI = I^2 R = \frac{V^2}{R}$$

Variation of Resistance with Temperature

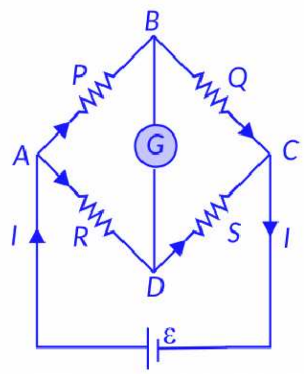
- Temperature coefficient of resistance, $\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$

If $T_1 = 0°C$ and $T_2 = T\,°C$ then $\alpha = \frac{R_T - R_0}{R_0 \times T}$ or $R_T = R_0(1 + \alpha T)$

Wheatstone Bridge

- It calculates the unknown resistence by balancing two legs of the bridge circuit

$\therefore \frac{P}{Q} = \frac{R}{S}$



Ohm's Law, Resistance and Resistivity

- $V = IR$
- Resistance of uniform conductor of length l and cross sectional area A, $R = \frac{\rho l}{A}$
- Resistivity or specific resistance, $\rho = \frac{RA}{l}$
- Effective specific resistance in series combination is

$$\frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2} \text{ (} A \text{ is same).}$$

- Effective specific resistance in parallel combination is

$$\frac{(A_1 + A_2)\rho_1\rho_2}{A_1\rho_2 + A_2\rho_1} \text{ (} l \text{ is same).}$$

Kirchhoff's Laws

- Law of conservation of charge applied at a junction, *i.e.*, $\Sigma I = 0$
- Law of conservation of energy applied in closed loop, *i.e.*, $\Sigma\varepsilon = \Sigma IR$

Current Density, Conductance and Conductivity

- Conductance, $G = \frac{1}{R}$
- Conductivity, $\sigma = \frac{1}{\rho} = \frac{l}{RA}$
- Current density, $J = \frac{I}{A} = \sigma E = nev_d$

Drift Velocity and Mobility of Charge

- Drift speed, $v_d = \frac{eE}{m}\tau$
- Mobility, $\mu_e = \frac{v_d}{E}$
- Current in terms of drift velocity,

$$I = neAv_d = \frac{ne^2 A\tau E}{m} = neA\mu_e E = neA\mu_e \frac{V}{l}$$

- In terms of relaxation time τ,

$$R = \frac{ml}{ne^2\tau A} \text{ and } \rho = \frac{m}{ne^2\tau}$$

Emf, Internal Resistance, Current in case of Grouping of Cells

- Emf of a cell, $\varepsilon = \frac{W}{q}$
- Terminal potential difference where current is being drawn from the cell, $V = \varepsilon - Ir$
- Terminal potential difference when the cell is being charged $V = \varepsilon + Ir$
- Internal resistance of a cell, $r = R\left[\frac{\varepsilon - V}{V}\right]$
- Grouping of identical cells:
  - Cells in series, $I = \frac{n\varepsilon}{R + nr}$ (n cells)
  - Cells in parallel, $I = \frac{m\varepsilon}{mR + r}$ (m cells)
  - Cells in mixed grouping, $I = \frac{mn\varepsilon}{mR + nr}$

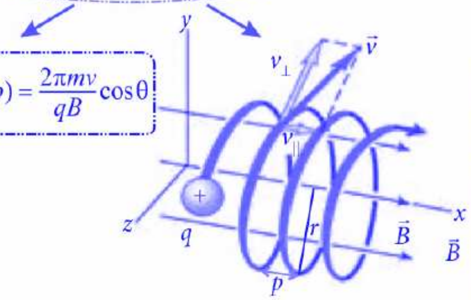
MOVING CHARGES AND MAGNETISM

For arbitrary angle θ (< 90°) $F = qvB \sin\theta$ and charge will attain helical path

For $\vec{v} \perp \vec{B}$, $\theta = 90°$, $F_{max} = qvB$ charge will attain circular path

For $\vec{v} \parallel \vec{B}$, $\theta = 0°$ ($F = 0$) no force is experienced

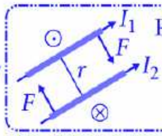
Pitch $(p) = \dfrac{2\pi mv}{qB}\cos\theta$



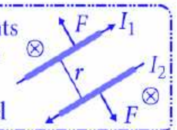
Magnetic Force

A charge q moving with velocity $\vec{v}$ in a uniform magnetic field $\vec{B}$ such that $\vec{v}$ makes an angle θ with $\vec{B}$, then magnetic force experienced by the charge is given by

$$\vec{F}_m = q(\vec{v} \times \vec{B})$$



Parallel currents attract while antiparallel currents repel



The force of attraction or repulsion acting on each conductor of length l due to currents in two parallel conductor is $F = \dfrac{\mu_0}{4\pi}\dfrac{2I_1I_2}{r}l$

Magnetic field at the centre of a circular coil

$$B = \frac{\mu_0 I}{2a}$$

Magnetic field at a point on the axis of the circular current carrying coil

$$B = \frac{\mu_0}{4\pi}\frac{2\pi NIa^2}{(a^2 + x^2)^{3/2}}$$

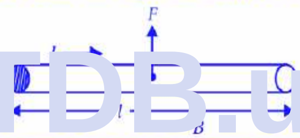
Magnetic Foce on a Current Carrying Conductor

$$\vec{F}_m = (nAl)e(\vec{v}_d \times \vec{B}) = [ne\vec{v}_d Al] \times \vec{B}$$
$$= (\vec{J}Al) \times \vec{B} = I(\vec{l} \times \vec{B})$$

For an arbitrary shape of wire,

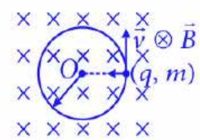
$$\vec{F} = \sum_i I(d\vec{l}_i \times \vec{B})$$

This force acts through the centre of the rod.



Motion of Charge in Uniform Transverse Magnetic Field

- Magnetic force on moving charge is given as
  $$F_m = qvB = \frac{mv^2}{r}$$
- Radius of circular path
  $$r = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2mE_K}}{qB} = \frac{\sqrt{2mqV}}{qB}$$
  $$r \propto v \propto p \propto \sqrt{E_K} \propto \sqrt{V}$$
- Time period of revolution
  $$T = \frac{2\pi R}{V} = \frac{2\pi m}{qB}$$



Biot Savart's Law

Magnetic field varies directly with current and length element and inversely with square of the distance.

$$dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}$$

MOVING CHARGES AND MAGNETISM

Torque on a current carrying coil placed in a uniform magnetic field

$$\tau = NIAB\sin\theta = MB\sin\theta$$

Ampere's Circuital Law

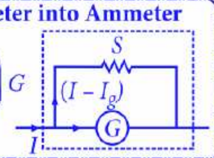
The line integral of magnetic field is equal to μ_0 times the current passing through area bounded by closed path.

$$\oint \vec{B}.d\vec{l} = \mu_0 I$$

Conversion

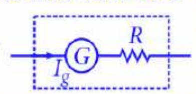
Galvanometer into Ammeter

$$S = \left(\frac{I_g}{I - I_g}\right)G$$



Galvanometer into Voltmeter

$$R = \frac{V}{I_g} - G$$



Magnetic field due to an infinitely long straight wire of radius a, carrying current I at a point

$$B = \begin{cases} \dfrac{\mu_0 Ir}{2\pi a^2} & ; \quad r < a \\ \dfrac{\mu_0 I}{2\pi a} & ; \quad r = a \\ \dfrac{\mu_0 I}{2\pi r} & ; \quad r > a \end{cases}$$

Moving Coil Galvanometer

Current I passing through the galvanometer is directly proportional to its deflection (θ). $I \propto \theta$ or $I = G\theta$.

where $G = \dfrac{k}{NAB}$ = galvanometer constant

Current sensitivity : $I_S = \dfrac{\theta}{I} = \dfrac{NAB}{k}$

Voltage sensitivity : $V_S = \dfrac{\theta}{IR} = \dfrac{NAB}{kR}$

Magnetic field due to a current carrying solenoid

$B_S = \mu_0 nI = (\mu_0 NI)/l$

MAGNETISM AND MATTER

THE BAR MAGNET

Bar Magnet as an Equivalent Solenoid

For solenoid of length $2l$ and radius a consisting n turns per unit length, the magnetic field is given by

$$B = \frac{\mu_0 2m}{4\pi r^3}$$ (where m = magnetic moment of solenoid $= n\,(2l)\,I\,(\pi a^2)$)

Magnetic Dipole in Magnetic Field

Torque on magnetic dipole,

$\tau = MB\sin\theta$

Torque on coil or loop,

$\vec{\tau} = \vec{M} \times \vec{B}$, here $\vec{M} = NI\vec{A}$,

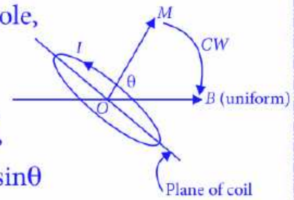
$\vec{\tau} = NI(\vec{A} \times \vec{B}),\ \tau = BINA\sin\theta$

$\theta = 90° \Rightarrow \tau_{max} = BINA$

$\theta = 0°$ or $180° \Rightarrow \tau_{min} = 0$

Potential energy of magnetic dipole,

$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$



Magnetisation and Magnetic Intensity

Relation between B, χ_m and H

$\vec{B} = \vec{B}_0 + \mu_0\vec{I}$

$\vec{B} = \mu_0(\vec{H} + \vec{I})$ $\quad (\because \vec{B}_0 = \mu_0\vec{H})$

$\vec{B} = \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$ $\quad (\because \vec{I} = \chi_m\vec{H})$

$\mu = \mu_0\mu_r = \mu_0(1 + \chi_m) \Rightarrow \mu_r = 1 + \chi_m$

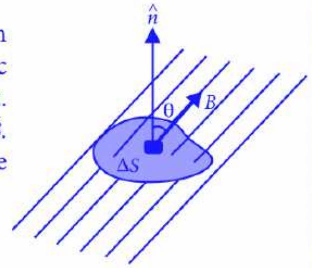
GAUSS'S LAW IN MAGNETISM

Magnetism and Gauss's Law

- The net magnetic flux through any closed surface is zero.

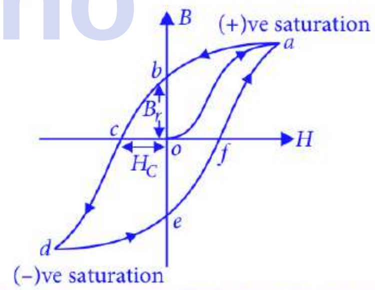
$$\phi_B = \sum_{\text{all}} \Delta\phi_B = \sum_{\text{all}} \vec{B}\cdot d\vec{S} = 0$$

- Incoming magnetic flux = Outgoing magnetic flux
- The Gauss's law of magnetism shows that isolated magnetic poles (monopoles) does not exist. There are no sources or sinks of $\vec{B}$. A dipole or current loop is the simplest magnetic element.



Magnetic Hysteresis

Hysteresis is the phenomenon of lagging of magnetic induction (B) or intensity of magnetisation (I) behind the magnetising field (H), when a specimen is taken through a cycle of magnetisation. From the hysteresis loop of material, we can study about retentivity, coercivity etc. of the material. The study of these characteristics enables us to select suitable materials for different purposes.

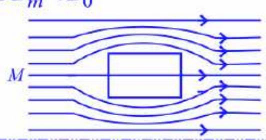


CLASSIFICATION OF MAGNETIC MATERIALS

Diamagnetic

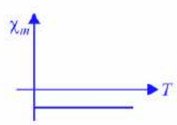
Poor magnetisation in opposite direction.

Here $B_m < B_0$



$\chi_m \rightarrow$ Small, negative and temperature independent

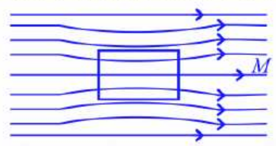
$\chi_m \propto T^0$



Paramagnetic

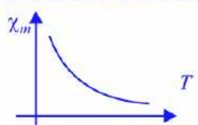
Poor magnetisation in same direction.

Here $B_m > B_0$



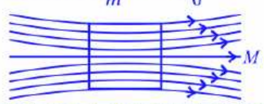
$\chi_m \rightarrow$ Small, positive and varies inversely with temperature

$\chi_m \propto \frac{1}{T}$ (Curie's law)



Ferromagnetic

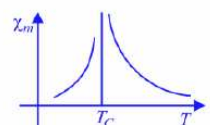
Strong magnetisation in same direction. Here $B_m >>> B_0$



$\chi_m \rightarrow$ Very large, positive and temperature dependent

$\chi_m \propto \frac{1}{T - T_C}$ (Curie-Weiss law)

(for $T > T_C$)



ELECTROMAGNETIC INDUCTION

Magnetic Flux and Faraday's Law

- Magnetic flux $\phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$
- Faraday's law : Whenever magnetic flux linked with a coil changes, an emf is induced in the coil.
  - Induced emf, $\varepsilon = -N\dfrac{d\phi_B}{dt}$
  - Induced current, $I = \dfrac{\varepsilon}{R} = N\dfrac{(-d\phi_B / dt)}{R}$
  - Induced charge flow, $\Delta Q = I\Delta t = -N\dfrac{\Delta\phi_B}{R}$

Inductance

- Emf induced in the coil/conductor, $\varepsilon = -L\dfrac{dI}{dt}$
- Coefficient of self induction, $L = \dfrac{N}{I}\phi_B = \dfrac{-\varepsilon}{dI/dt}$
- Self inductance of a long solenoid, $L = \mu_0\mu_r n^2 Al = \dfrac{\mu_0\mu_r N^2 A}{l}$
- Mutual inductance, $M = \dfrac{N_2\phi_2}{I_1} = \dfrac{-\varepsilon_2}{(dI_1/dt)} = \dfrac{-\varepsilon_1}{(dI_2/dt)}$
- Mutual inductance of two long coaxial solenoids,
  $$M = \mu_0\mu_r\pi r_1^2 n_2 l = \frac{\mu_0\mu_r N_1 N_2 A_1}{l}$$
- Coefficient of coupling, $k = \dfrac{M}{\sqrt{L_1L_2}}$

  For perfect coupling, $k = 1$ so, $M = \sqrt{L_1L_2}$

Combination of Inductors

- Inductors in series, $L_S = L_1 + L_2 \pm 2M$
- Inductors in parallel, $L_P = \dfrac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$
- If coils are far away, then $M = 0$.

  So, $L_S = L_1 + L_2$ and $L_P = \dfrac{L_1L_2}{L_1 + L_2}$

Magnetic Energy

- Energy stored in an inductor, $U_B = \dfrac{1}{2}LI^2$
- Energy stored in the solenoid, $U_B = \dfrac{1}{2\mu_0}B^2Al$
- Magnetic energy density, $u_B = \dfrac{U_B}{V} = \dfrac{B^2}{2\mu_0}$

Induced Electric Field

- It is produced by change in magnetic field in a region. This is non-conservative in nature.

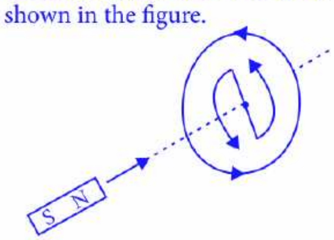
  $$\oint \vec{E}\cdot d\vec{l} = -\frac{d\phi_B}{dt} = -A\frac{dB}{dt} \neq 0$$
- This is also known as integral form of Faraday's law.

Lenz's Law

- The direction of the induced current is such that it opposes the change that has produced it.
- If a current is induced by an increasing(decreasing) flux, it will weaken (strengthen) the original flux.
- It is a consequence of the law of conservation of energy.

Application of Lenz's Law

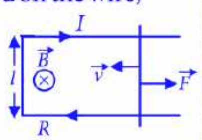
When a north pole of a bar magnet is moved towards a coil, the current induced in the coil will be in anti-clockwise direction (and vice versa) as shown in the figure.



Energy Consideration in Motional emf

- Emf in the wire, $\varepsilon = Bvl$
- Induced current, $I = \dfrac{\varepsilon}{R} = \dfrac{Bvl}{R}$
- Force exerted on the wire,

  $F = \dfrac{B^2l^2v}{R}$



- Power required to move the wire, $P = \dfrac{B^2l^2v^2}{R}$

  It is dissipated as Joule's heat.

Motional emf

- On a straight conducting wire, $\varepsilon = Bvl$
- On a rotating conducting wire about one end, $\varepsilon = \dfrac{B\omega l^2}{2}$

  Here, $\vec{B}$, $\vec{v}(=\omega r\hat{v})$ and $\vec{l}$ are perpendicular to each other.

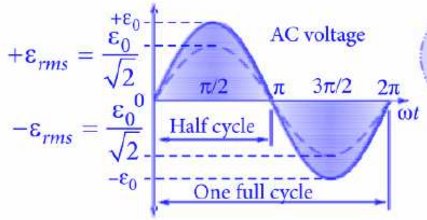
AC Generator

- Mechanical energy is converted into electrical energy by virtue of electromagnetic induction.
- Induced emf,

  $\varepsilon = NAB\omega\sin\omega t = \varepsilon_0\sin\omega t$
- Induced current,

  $I = \dfrac{NBA\omega}{R}\sin\omega t = I_0\sin\omega t$

ALTERNATING CURRENT



Alternating Current

Current which changes continuously in magnitude and periodically in direction.

Transformer

- Transformer ratio
  $$\frac{\varepsilon_S}{\varepsilon_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = k$$
- Efficiency of a transformer,
  $$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\varepsilon_S I_S}{\varepsilon_P I_P}$$

Step-up transformer, $\varepsilon_S > \varepsilon_P, I_S < I_P$ and $N_S > N_P$.

Step-down transformer, $\varepsilon_S < \varepsilon_P, I_S > I_P$ and $N_S < N_P$.

Alternating voltage

$\varepsilon = \varepsilon_0 \sin \omega t$

Applied across capacitor | **Applied across resistor** | **Applied across inductor**

Purely capacitive circuit

Current leads the voltage by a phase angle of $\pi/2$.

$I = I_0 \sin(\omega t + \pi/2)$; $I_0 = \frac{\varepsilon_0}{X_C} = \omega C \varepsilon_0$

where $X_C = 1/\omega C$

Purely resistive circuit

Alternating voltage is in phase with current.

$I = \varepsilon/R = I_0 \sin \omega t$

Purely inductive circuit

Current lags behind the voltage by a phase angle of $\pi/2$.

$I = I_0 \sin(\omega t - \pi/2)$; $I_0 = \varepsilon_0/X_L = \varepsilon_0/\omega L$

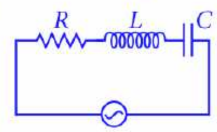
where $X_L = \omega L$

Combining LCR in series

Series LCR circuit

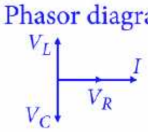
- $\varepsilon = \varepsilon_0 \sin \omega t$, $I = I_0 \sin(\omega t - \phi)$
  Impedance of the circuit: $Z = \sqrt{R^2 + (X_L - X_C)^2}$
  Where, $X_L = \omega L$, $X_C = \frac{1}{\omega C}$
- Phase difference between current and voltage is ϕ
  $\tan \phi = \frac{X_L - X_C}{R}$
- For $X_L > X_C$, ϕ is +ve. (Predominantly inductive)
- For $X_L < X_C$, ϕ is –ve. (Predominantly capacitive)

- Circuit diagram

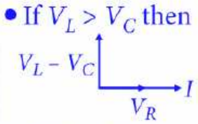


I is same for R, L and C.

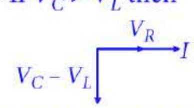
- Phasor diagram



- If $V_L > V_C$ then



- If $V_C > V_L$ then



Power in ac circuit

Average power (P_{av})

$P_{av} = \varepsilon_{rms} I_{rms} \cos \phi$

$= \frac{\varepsilon_0 I_0}{2} \cos \phi$

Power factor

- **Power factor:** $\cos \phi = \frac{R}{Z}$
- In pure resistive circuit, $\phi = 0°$; $\cos \phi = 1$
- In purely inductive or capacitive circuit
  $\phi = \pm \frac{\pi}{2}$; $\cos \phi = 0$
- In series LCR circuit,
  At resonance, $X_L = X_C$
  $\therefore$ $Z = R$ and $\phi = 0°, \cos \phi = 1$

Resonant series LCR circuit

When $X_L = X_C$, $Z = R$, current becomes maximum.

Resonant frequency $\omega_r = \frac{1}{\sqrt{LC}}$

Quality factor

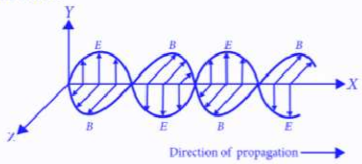
It is a measure of sharpness of resonance.

$\therefore \quad Q = \frac{1}{R}\sqrt{\frac{L}{C}}$

ELECTROMAGNETIC WAVES

ELECTROMAGNETIC WAVES

An electromagnetic wave is a wave radiated by an accelerated charge as coupled electric and magnetic field oscillating perpendicular to each other and to the direction of propagation of wave.



Magnitude of $\vec{E}$ and $\vec{B}$ are related as $\frac{E_0}{B_0} = c$

Speed of an electromagnetic wave in free space is given by

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

Ampere's circuital law

The line integral of magnetic field around any closed path in vacuum is equal to μ_0 times the total current passing through that closed path.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I$$

Properties of electromagnetic waves

- Do not carry any charge.
- It is not deflected by electric and magnetic field.
- Travel with speed of light in vacuum.
- Frequency does not change when it goes from one medium to another, but its wavelength changes.
- Transverse in nature.
- Do not require any material medium for propagation.

Electromagnetic spectrum

The orderly distribution of electromagnetic waves in accordance with their wavelength or frequency into distinct groups.

Displacement current

It is the current which is produced when electric field and hence electric flux changes with time.

$$I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

Modified Ampere's law

This law implies the fact that not only a conduction current but a displacement current, associated with a hanging electric field, also produces a magnetic field.

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 (I_C + I_D)$$

$$= \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

Maxwell's four equations

1. $\oint_s \vec{E} \cdot \vec{ds} = \frac{q}{\varepsilon_0}$
2. $\oint_s \vec{B} \cdot \vec{ds} = 0$
3. $\oint_c \vec{E} \cdot \vec{dl} = \frac{-d\phi_B}{dt}$
4. $\oint \vec{B} \cdot \vec{dl} = \mu_0 (I_C + I_D)$

$$= \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$

$$= \mu_0 \left(I_C + \varepsilon_0 \frac{d}{dt} \oint_s \vec{E} \cdot d\vec{s} \right)$$

Uses

Radiowaves
- Used in radio communication

Microwaves
- Radar communication
- Analysis of fine details of molecular and atomic structure.

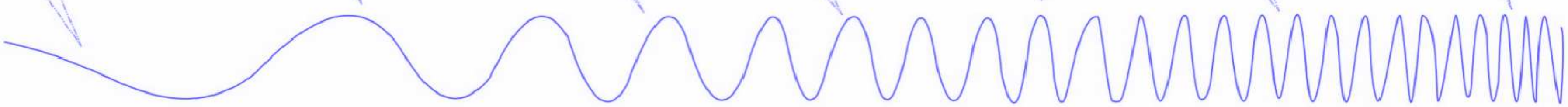
Infrared
- Useful for elucidating molecular structure.
- Useful for haze photography.

Visible light
- Detected by stimulating nerve endings of human retina.
- Can cause chemical reaction.

Ultraviolet
- Can cause many chemical reactions, *e.g.*, the tanning of the human skin.
- Ionize atoms in atmosphere, resulting in the ionosphere.

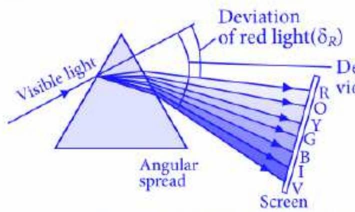
X-rays
- Penetrate matter (*e.g.*, radiography).
- Ionize gases.
- Cause fluorescence.
- Cause photoelectric emission from metals.

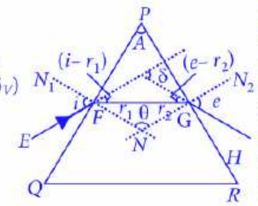
Gamma rays
- In the treatment of cancer and tumours.



Radiation Type	Radio	Microwave	Infrared	Visible	Ultraviolet	X-rays	Gamma rays
Wavelength (m)	$> 10^{-1}$	10^{-1} to 10^{-3}	10^{-3} to 10^{-6}	7×10^{-7} to 4×10^{-7}	4×10^{-7} to 1.5×10^{-9}	10^{-9} to 10^{-12}	$< 10^{-12}$

RAY OPTICS AND OPTICAL INSTRUMENTS





OPTICAL FIBRES

are used for transmitting optical signal through long distance. Optical fibres based on the phenomenon of TIR.

TOTAL INTERNAL REFLECTION

TIR conditions

- Light must travel from denser to rarer.
- Incident angle $i >$ critical angle i_c

Relation between μ and i_c: $\mu = \dfrac{1}{\sin i_c}$

REFRACTION OF LIGHT

Snell's law: When light travels from medium a to medium b, ${}^a\mu_b = \dfrac{\mu_b}{\mu_a} = \dfrac{\sin i}{\sin r}$

Refractive index,

$$\mu = \frac{\text{velocity of light in vaccum}}{\text{velocity of light in medium}} = \frac{c}{v}$$

Real and apparent depth

$$\mu = \frac{\text{real depth } (x)}{\text{apparent depth } (y)}$$

REFLECTION OF LIGHT

- According to the laws of reflection, $\angle i = \angle r$
- If a plane mirror is rotated by an angle θ, the reflected rays rotates by an angle 2θ.

SIMPLE MICROSCOPE

Magnifying power

- For final image to be formed at D (least distance) $M = 1 + \dfrac{D}{f}$
- For final image formed at infinity

$$M = \frac{D}{f}$$

REFLECTING TELESCOPE

Magnifying power

$$M = \frac{f_o}{f_e} = \frac{R/2}{f_e}$$

REFRACTION THROUGH PRISM

Relation between μ and δ_m

$$\mu = \frac{\sin\left(\dfrac{A+\delta_m}{2}\right)}{\sin\left(\dfrac{A}{2}\right)}$$

where, δ_m = angle of minimum deviation, A = angle of prism

or $\delta = (\mu - 1)A$ (Prism of small angle)

Angular dispersion

$$= \delta_V - \delta_R = (\mu_V - \mu_R)A$$

Dispersive power,

$$\omega = \frac{\delta_V - \delta_R}{\delta} = \frac{\mu_V - \mu_R}{\mu - 1}$$

Mean deviation, $\delta = \dfrac{\delta_V + \delta_R}{2}$

RAY OPTICS AND OPTICAL INSTRUMENTS

TELESCOPE

Astronomical telescope

- For final image formed at D (least distance) $M = -\dfrac{f_o}{f_e}\left(1 + \dfrac{f_e}{D}\right)$
- In normal adjustment, image formed at infinity $M = -f_o/f_e$

POWER OF LENSES

Power of lens : $P = \dfrac{1}{f\,(\text{in m})}$

Combination of lenses:

Power: $P = P_1 + P_2 - dP_1P_2$

(d = small separation between the lenses)

For $d = 0$ (lenses in contact)

Power: $P = P_1 + P_2 + P_3 + \ldots$

THIN SPHERICAL LENS

Thin lens formula : $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Magnification: $m = \dfrac{v}{u} = \dfrac{h_i}{h_o}$

REFRACTION BY SPHERICAL SURFACE

Relation between object distance (u), image distance (v) and refractive index (μ)

$$\frac{\mu_{\text{denser}}}{v} - \frac{\mu_{\text{rarer}}}{u} = \frac{\mu_{\text{denser}} - \mu_{\text{rarer}}}{R}$$

(Holds for any curved spherical surface.)

Lens maker's formula

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

REFLECTION BY SPHERICAL MIRRORS

Mirror formula, $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{1}{f} = \dfrac{2}{R}$

Magnification, $m = -\dfrac{v}{u} = \dfrac{h_i}{h_o}$

COMPOUND MICROSCOPE

Magnifying power, $M = m_o \times m_e$

- For final image formed at D (least distance) $M = -\dfrac{L}{f_o}\left(1 + \dfrac{D}{f_e}\right)$
- For final image formed at infinity

$$M = -\frac{L}{f_o} \cdot \frac{D}{f_e}$$

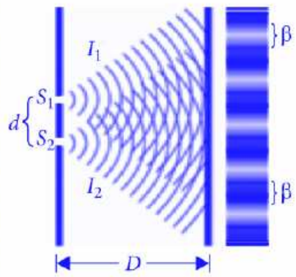
TERRESTRIAL TELESCOPE

For normal adjustment $M = \dfrac{f_o}{f_e}$

Distance between objective and eyepiece $d = f_o + 4f + f_e$

WAVE OPTICS

Huygens principle : According to Huygens principle,
- Every point on given wavefront (primary wavefront) acts as a fresh source of new disturbance, called secondary wavelets.
- The secondary wavelets spread out in all the directions with the speed of light in the medium.
- A surface touching these secondary wavelets tangentially in the forward direction at any instant gives the new (secondary) wavefront at that instant.
- Law of reflection $\angle i = \angle r$ & law of refraction $\frac{\sin i}{\sin r} = \mu$ can be explained by Huygens wave theory.



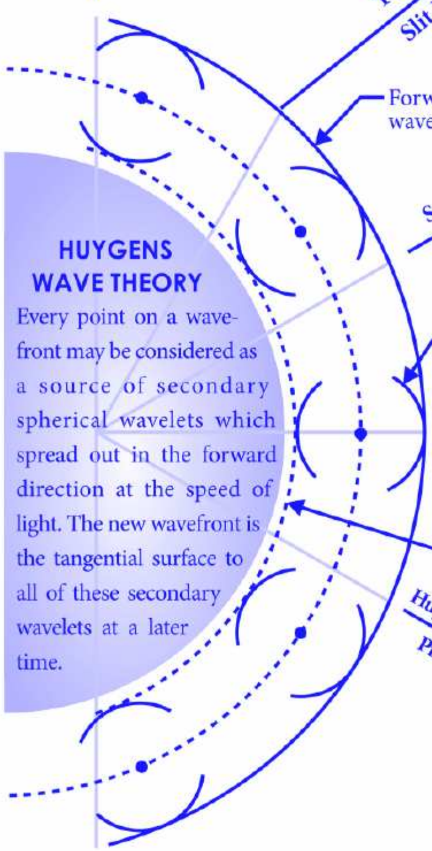
Young's Double Slit Experiment

INTERFERENCE OF LIGHT

- The superposition of two coherent waves resulting in a pattern of alternating dark and bright fringes of equal width.
- Position of bright fringes $x_n = \frac{n\lambda D}{d}$
- Position of dark fringes $x'_n = \frac{(2n-1)\lambda D}{2d}$
- Fringe width $\beta = \frac{\lambda D}{d}$
- Ratio of slit width with intensity : $\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$

HUYGENS WAVE THEORY

Every point on a wavefront may be considered as a source of secondary spherical wavelets which spread out in the forward direction at the speed of light. The new wavefront is the tangential surface to all of these secondary wavelets at a later time.



Superposition of Light Waves

COHERENT SOURCES

The source which emit continuous light waves of same wavelength, same frequency and in same phase are coherent sources.

Incoherent Sources

Two sources of light which do not emit light waves with a constant phase difference are called incoherent source.

ADDITION OF COHERENT WAVE

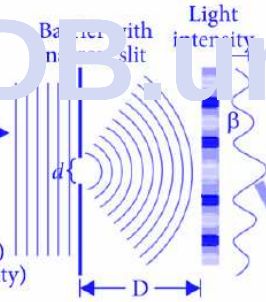
- Intensity distribution : If a, b are the amplitudes of interfering waves due to two coherent sources and ϕ is constant phase difference between the two waves at any point P, then the resultant amplitude at P will be

$$R = \sqrt{a^2 + b^2 + 2ab\cos\phi}$$

- Resultant intensity
$I_R = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$
- For bright fringes,
$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$ at $\phi = 0^o, 2\pi, 4\pi...$
- For dark fringes,
$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$ at $\phi = \pi, 3\pi, 5\pi...$
- For $I_1 = I_2 = I_0$; $I_R = 4I_0\cos^2\frac{\phi}{2}$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$$

Single Slit Diffraction



Huygen Fresnel Principle

FRESNEL'S DISTANCE

Ray optics as a limiting case of wave optics
- Diffraction at circular aperture
Linear spread, $x = Z\theta$
Areal spread, $x^2 = (Z\theta)^2$
$\left\{\theta = \frac{1.22\lambda}{a}\right\}$
- Fresnel's distance : Distance at which diffraction spread is equal to the size of aperture, $Z_F = \frac{a^2}{\lambda}$
- Size of Fresnel zone, $a_F = \sqrt{\lambda Z}$

DIFFRACTION

Single slit experiment
- Angular position of n^{th} minima, $\theta_n = \frac{n\lambda}{a}$
- Angular position of n^{th} maxima, $\theta'_n = \frac{(2n+1)\lambda}{2a}$
- Width of central maximum $\beta_o = 2\beta = \frac{2D\lambda}{a}$

WAVEFRONT

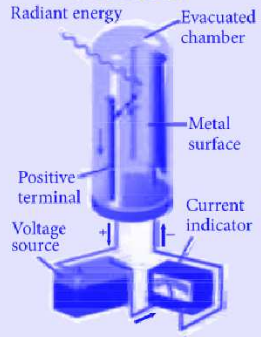
- The locus of all particles of the medium vibrating in the same phase at a given instant is known as wavefront.
- Depending on the shape of sources of light, wavefront can be of three types
  – Spherical wavefront : When the source of light is a point source, the wavefront is spherical.
  – Cylindrical wavefront : When the source of light is linear, the wavefront is cylindrical.
  – Plane wavefront : When the point source or linear source of light is at very large distance, a small portion of spherical or cylindrical wavefront appears to be plane. Such a wavefront is known as plane wavefront.

DUAL NATURE OF RADIATION AND MATTER

Particle Nature of Radiation

Photoelectric Cell

Photocell



- An electrical device which converts light energy into electrical energy, is called as photocell or photoelectric cell.
- It works on the principle of photoelectric emission of electrons.

Application of Photoelectric Effect

Photoelectric Effect

- The phenomenon of emission of electrons from a metal surface when an electromagnetic wave of suitable frequency is incident on it is called photoelectric effect.

Einstein's Photoelectric Equation

- $E = K_{max} + \phi_0$, where ϕ_0 = work function, E = energy of incident light, K_{max} = maximum K.E. of e^-

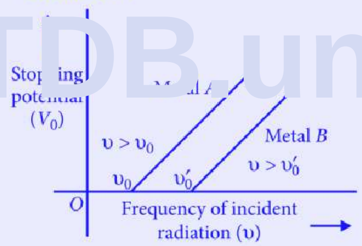
$$\Rightarrow \quad h\upsilon = \frac{1}{2}mv_{max}^2 + h\upsilon_0$$

$$\Rightarrow \quad \frac{1}{2}mv_{max}^2 = h(\upsilon - \upsilon_0)$$

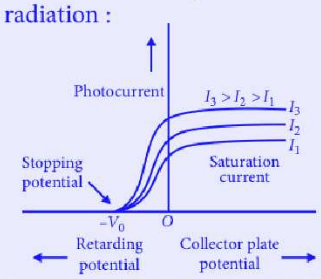
Experimental Study and Conclusion of Photoelectric Effect

- At constant frequency υ and potential V (Photo-current) $i_p \propto I$ (intensity)
- At constant frequency and intensity, the minimum negative potential at which the photocurrent becomes zero is called stopping potential (V_0).
- At stopping potential V_0, K_{max} of $e^- = eV_0$
- For a given frequency of the incident radiation, the V_0 is independent of I.
- The V_0 varies linearly with υ.

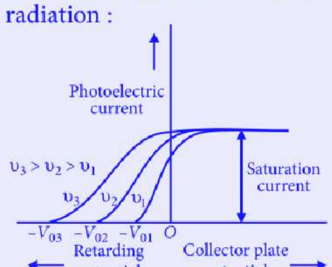
- Variation of stopping potential V_0 with frequency υ of incident radiation :



- Variation of photocurrent with collector plate potential for different intensity of incident radiation :



- Variation of photocurrent with collector plate potential for different frequencies of incident radiation :



Wave Nature of Matter

de-Broglie Hypothesis

- Due to symmetry in nature, the particle in motion also possess wave-like properties. And these waves are called matter waves.

Application of de Broglie Waves

de-Broglie Wavelength

$$\lambda = \frac{h}{p}$$

- For electron having K.E. (K) is

$$\lambda = \frac{h}{\sqrt{2mK}}, \text{ here } p = \sqrt{2mK}$$

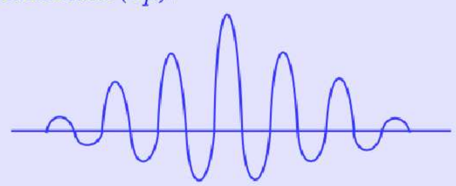
- For a charged particle accelerated by potential V is

$$\lambda = \frac{h}{\sqrt{2qmV}}, \text{ here } p = \sqrt{2qmV}$$

Electron Microscope

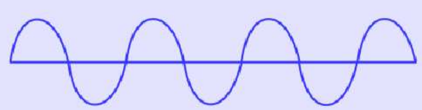
- Electron microscope is a device designed to study very minute objects.
- Based on principle of de Broglie wave and the fast moving electrons can be focussed by E or B field in a same way as beam of light is focussed by glass lenses.

- **The wave packet description of an electron :** The wave packet corresponds to a spread of wavelength around some central wavelength (and hence by de Broglie relation, a spread in momentum). Consequently, it is associated with an uncertainty in position (Δx) and an uncertainty in momentum (Δp).



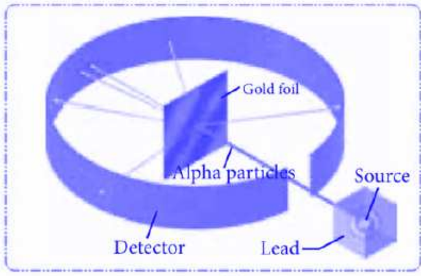
(a) A localised wave packet

- The matter wave corresponding to a definite momentum of an electron extends all over space. In this case, $\Delta p = 0$ and $\Delta x \to \infty$.



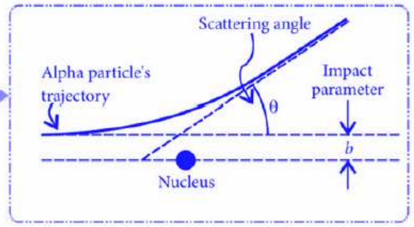
(b) An extended wave with fixed wavelength

ATOMS



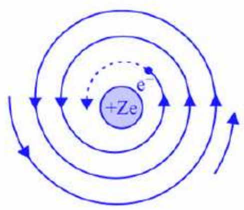
Rutherford's Model of Atom

- K.E. of α–particles, $K = \frac{1}{2}mv^2$
- Distance of closest approach,
$$r_0 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2Ze^2}{K} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{4Ze^2}{mv^2}$$
- Impact parameter,
$$b = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{K} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2\cot\frac{\theta}{2}}{\frac{1}{2}mv^2}$$
- **Conclusion :** An atom consists of a small and massive central core in which entire positive charge and whole mass of atom is concentrated.

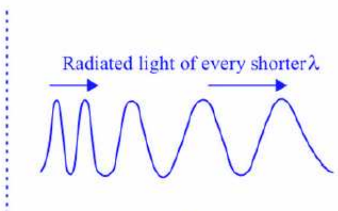


Drawbacks of Rutherford's atomic model

- According to electromagnetic theory, an accelerated charge particle always radiates energy. Electrons loses its energy and its radius of orbit decreases continuously and finally it would spiral into the nucleus. But in practical atoms do not collapse.
- An electron revolving in any orbit emits continuous radiation of all wavelengths. Actually the elements are found to emit spectral lines of definite frequencies and not all of them.



(a)



(b)

Important Expressions from Bohr's Atomic Model

Electron orbits and their energy

- Radius of permitted n^{th} orbits,
$$r_n = \frac{n^2h^2}{4\pi^2mkZe^2} \Rightarrow r_n \propto n^2$$
- Velocity of electron in n^{th} orbit,
$$v_n = \frac{2\pi kZe^2}{nh} \Rightarrow v_n \propto \frac{1}{n}$$
- Energy of electron in n^{th} orbit
$$E_n = \frac{-2\pi^2mk^2Z^2e^4}{n^2h^2} \Rightarrow E_n \propto \frac{1}{n^2}$$
where the symbols have their usual meanings.

Bohr's Model of Atom

- Bohr developed a theory of hydrogen and hydrogen like atoms which have only one orbital electron.
- Angular momentum of the electron in a stationary orbit is an integral multiple of $h/2\pi$.

i.e., $L = \frac{nh}{2\pi}$ or, $mvr = \frac{nh}{2\pi}$

Ionization Energy and Ionization Potential

Ionization energy and potential of an electron in hydrogen atom for n^{th} state.

- Ionization energy $= \frac{13.6Z^2}{n^2} eV$
- Ionization potential $= \frac{13.6Z^2}{n^2}$ V

Atomic Spectra

	Initial state	Final state	Wavelength formula	First member-second member	Series limit $n_i \to \infty$ to n_f	Maximum wavelength (n_f+1) to n_f	Lines found in
Lyman	$n_i = 2, 3, 4, 5, 6, ...$	$n_f = 1$	$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right)$	$n_i = 2$ to $n_f = 1$ $n_i = 3$ to $n_f = 1$	From ∞ to 1 $\lambda = \frac{1}{R}$ $\lambda = 911$ Å	From 2 to 1 $\lambda = \frac{4}{3R}$ $\lambda = 1216$ Å	UV region
Balmer	$n_i = 3, 4, 5, 6, 7, ...$	$n_f = 2$	$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$	$n_i = 3$ to $n_f = 2$ $n_i = 4$ to $n_f = 2$	From ∞ to 2 $\lambda = \frac{4}{R}$ $\lambda = 3646$ Å	From 3 to 2 $\lambda = \frac{36}{5R}$ $\lambda = 6563$ Å	Visible region
Paschen	$n_i = 4, 5, 6, 7, 8, ...$	$n_f = 3$	$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n_i^2}\right)$	$n_i = 4$ to $n_f = 3$ $n_i = 5$ to $n_f = 3$	From ∞ to 3 $\lambda = \frac{9}{R}$ $\lambda = 8204$ Å	From 4 to 3 $\lambda = \frac{144}{7R}$ $\lambda = 18753$ Å	IR region
Brackett	$n_i = 5, 6, 7, 8, 9, ...$	$n_f = 4$	$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$	$n_i = 5$ to $n_f = 4$ $n_i = 6$ to $n_f = 4$	From ∞ to 4 $\lambda = \frac{16}{R}$ $\lambda = 14585$ Å	From 5 to 4 $\lambda = \frac{400}{9R}$ $\lambda = 40515$ Å	IR region
Pfund	$n_i = 6, 7, 8, 9, 10, ...$	$n_f = 5$	$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n_i^2}\right)$	$n_i = 6$ to $n_f = 5$ $n_i = 7$ to $n_f = 5$	From ∞ to 5 $\lambda = \frac{25}{R}$ $\lambda = 22790$ Å	From 6 to 5 $\lambda = \frac{900}{11R}$ $\lambda = 74583$ Å	Far IR region

Line Spectra of Hydrogen Atom

- While transition between different atomic levels, light radiated in various discrete frequencies are called spectral series of hydrogen atom.
- Rydberg formula :

Wave number $\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$

R = Rydberg's constant $= 1.097 \times 10^7$ m^{-1}

TYPES OF SEMICONDUCTORS

Extrinsic Semiconductors

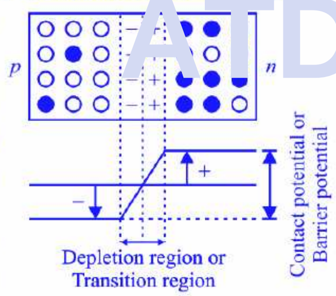
- The semiconductor whose conductivity is mainly due to impurity. On the basis of doping there are two type of extrinsic semiconductors.
  - *p*-type semiconductor ($n_h >> n_e$) : Obtained by doping a trivalent impurity atom.
  - *n*-type semiconductor ($n_e >> n_h$) : Obtained by doping a pentavalent impurity atom.
- Electrical conductivity, $\sigma = e\,[n_e\mu_e + n_h\mu_h]$

Intrinsic Semiconductors

The pure semiconductors having thermally generated current carriers.
$n_e = n_h = n_i$; where, n_e = Electron density, n_h = Hole density, n_i = Density of intrinsic carriers.

Formation of *p-n* Junction

- Due to high carrier concentration difference, holes diffuses from *p*-side to *n*-side and electron diffuses from *n*-side to *p*-side, produces diffusion current.
- Due to barrier potential, majority charge carriers are forced to move to other side of the junction and such movement produces drift current.

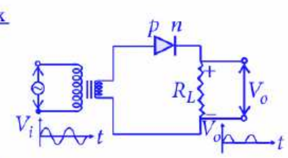


Rectifier

Used to convert ac signal into dc signal.

Half Wave Rectifier

- $I_{max} = \dfrac{\varepsilon_{max}}{(r_f + R_L)}$; $I_{dc} = \dfrac{I_{max}}{\pi}$
- Output frequency = Input frequency
- Efficiency = $\dfrac{P_{dc}}{P_{ac}} = 40.6\%$

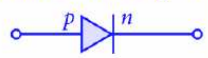


Semiconductor Diode

- ***P-N* Junction Diode :** In layman language, when a *p*-type semiconductor is brought into contact with an *n*-type semiconductor such that structure remains continuous at boundary, *p-n* junction diode is formed.
- Forward bias characteristic
  - Width of depletion layer decreases
  - Effective barrier potential decreases
  - Low resistance offered at junction and high current flow of the order of mA.
- Reverse Bias Characteristics
  - Width of depletion layer increases
  - Effective barrier potential increases
  - High resistance offered at the junction and low current flow of the order of μA.
  - Reverse break down occurs at a high reverse biased voltage where the ordinary diodes get damaged.

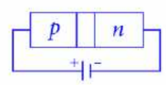
Characteristic and Symbol of *p-n* Junction Diode

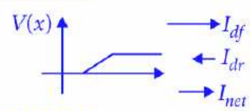
- The most important characteristic of a *p-n* junction is its ability to conduct current in one direction only.



- In the reverse direction, it offers very high resistance.

Forward Biased





- Effective barrier potential decreases.
- Depletion width decreases.
- Low resistance offered at junction.
- High current flows through the circuit.

Full Wave Rectifier

- $I_{max} = \dfrac{\varepsilon_{max}}{(r_f + R_L)}$; $I_{dc} = \dfrac{2I_{max}}{\pi}$
- Output frequency = 2 × Input frequency
- Efficiency = $\dfrac{P_{dc}}{P_{ac}} = 81.2\%$