

PRAAYAS

JEE 2026

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Mathematics

Basic Maths

Lecture - 02

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Topics *To be covered*



- A** Important Notations
- B** Types of Integers & Natural Numbers
- C** Two Golden Inequalities
- D** Some Important Points

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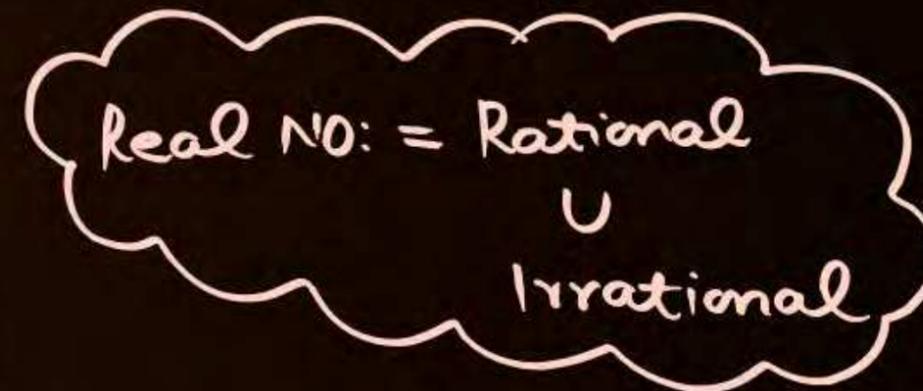
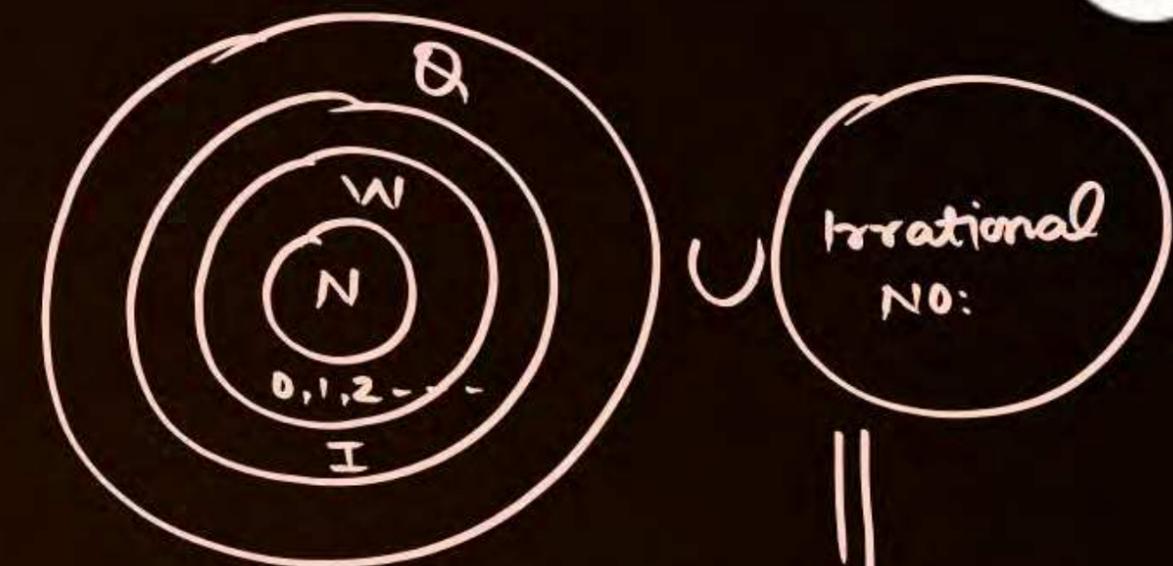


Recap *of previous lecture*



State True or False

1. Every Natural number is an integer. (T)
2. Every integer is a rational number. (T)
3. 0 is a rational number. (T)
4. Every rational number is a terminating decimal. (F)
5. $\sqrt{2} + \sqrt{3}$ is an irrational number but not a real number. (F)
6. Every real number is a complex number. (T)
7. 5 is a natural number but not an integer. (F)
8. $\frac{343}{240}$ is a terminating decimal. (F) $\frac{343}{240} = \frac{343}{3 \times 2^3 \times 2 \times 5}$



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Recap *of previous lecture*



State True or False

9. $\frac{547}{2500}$ is a terminating decimal (T)
 $\underbrace{5^2 \times 5^2 \times 2^2}_{2500} = 5^4 2^2$

$$x^3 \in \mathbb{Q}$$

$$x^3 \cdot x^3 \in \mathbb{Q}$$

10. If x^3 & x^7 are both non zero rational then x should also be rational (T) $x^6 \in \mathbb{Q}$

$$\text{Now } x^7 \in \mathbb{Q}$$

$$\frac{x^7}{x^6} \in \mathbb{Q}$$

$$x \in \mathbb{Q}$$

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Fill in the blank



Fill in the blanks with N, I, Q.

Recap of previous lecture



b. Rational numbers \cap irrational numbers = ϕ

c. Rational numbers \cup irrational numbers = Real NO:s

d. $N \cap W = \underline{N}$, $N \cup Q = \underline{Q}$

e. $I \cap Q = \underline{I}$, $I \cup Q = \underline{Q}$

f. $0.52\bar{3} = \frac{157}{300}$

g. $\frac{\text{Any number}}{0} = \text{Not defined}$

h. $x - \frac{2}{x+1} = 1 - \frac{2(x-1)}{x^2-1}$ then number of solutions = 0

~~$x - \frac{2}{x+1} = 1 - \frac{2}{x+1}$~~ $x \neq 1$
 $x = 1$

A ke saaray elements B mai hai

A is subset of B then
 * $A \cap B = A$
 * $A \cup B = B$

$x = 0.523333 \dots$
 $100x = 52.3333 \dots$ (i)
 $1000x = 523.3333 \dots$ (ii)
 (ii) - (i) $\Rightarrow 900x = 471 \Rightarrow x = \frac{471}{900} = \frac{157}{300}$

$$\begin{aligned}
 \text{Ex: } x &= 0.3\bar{4} \\
 x &= 0.3444\text{---} \\
 10x &= 3.4444\text{---} \\
 100x &= 34.444\text{---} \\
 \hline
 90x &= 31 \\
 x &= \frac{31}{90}
 \end{aligned}$$

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$$\begin{aligned}
 \text{Ex: } x &= 2.\bar{3}2 \\
 x &= 2.323232\text{---} \\
 100x &= 232.323232\text{---} \\
 \hline
 99x &= 230 \\
 x &= \frac{230}{99} \quad \underline{\text{Ans}}
 \end{aligned}$$



Important Notations



0 is neither +ve nor -ve

(i) \mathbb{R}^+ : Set of all positive real numbers i.e. all real numbers greater than 0 i.e.
 $\mathbb{R}^+ = (0, \infty)$



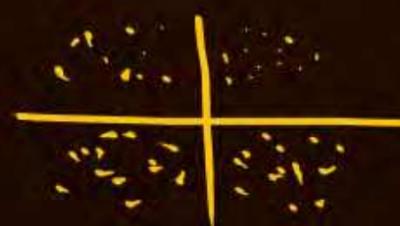
(ii) \mathbb{R}^- : Set of all negative real numbers i.e. all real numbers less than 0 i.e.
 $\mathbb{R}^- = (-\infty, 0)$



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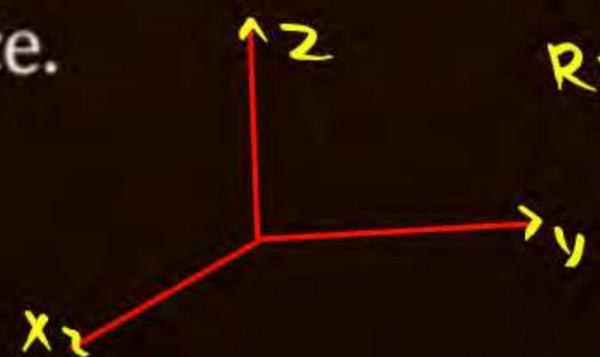
(iii) \mathbb{R}_0 : Set all real numbers except zero, $\mathbb{R}_0 = (-\infty, \infty) - \{0\}$.

(iv) $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$ = All points In 2-D plane



$$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2 = \{(x, y) \mid x, y \in \mathbb{R}\}$$

(v) $\mathbb{R}^3 = \mathbb{R} \times \mathbb{R} \times \mathbb{R}$ = All points in 3-D space.



$$\mathbb{R} \times \mathbb{R} \times \mathbb{R} = \mathbb{R}^3 = \{(x, y, z) \mid x, y, z \in \mathbb{R}\}$$



A close look at Set of Integers



Positive integers : $\{1, 2, 3, 4, \dots\}$

Negative integers : $\{-1, -2, -3, \dots\}$

Non Negative integers : $\{0, 1, 2, 3, 4, \dots\}$

None Positive integers : $\{0, -1, -2, -3, \dots\}$

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QUESTION



Indicate which numbers in the given sets are (a) Natural numbers (b) Whole numbers (c) Integers (d) Rational numbers (e) Irrational numbers.

(i) $\{-10, -\sqrt{2}, -\frac{3}{4}, 0, \frac{4}{5}, \sqrt{4}, \pi, 7, \frac{18}{2}, 100\}$

Handwritten annotations for set (i):

- -10 : I, Q
- $-\sqrt{2}$: R-Q
- $-\frac{3}{4}$: Q
- 0 : I, W, Q
- $\frac{4}{5}$: I, W, Q
- $\sqrt{4}$: I, W, Q
- π : R-Q
- 7 : I, N, W, Q
- $\frac{18}{2}$: I, N, W, Q
- 100 : I, N, W, Q

(ii) $\{-\sqrt[3]{8}, \frac{0}{3}, \sqrt[3]{7}, \sqrt{\frac{4}{9}}, 1.\overline{126}\}$

Handwritten annotation: Tah 01(A)

* $\sqrt{4} = 2$
principal root of 4

- * $\sqrt{16} = 4$
- * $\sqrt{9} = 3$
- * $\sqrt{0} = 0$

$\sqrt{4} \notin R-Q$ Gadho/Gadhiyoo
Dhyaan se Dekho

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Ex: $x^2 = 4$
 $x = \pm 2$

But $\sqrt{4} = 2$

\sqrt{a} , $a > 0$ aisa non -ve NO: jiska square 'a' ho

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QUESTION



The greatest integer lying between -10 and -15 is

- A** -10
- B** -11
- C** -15
- D** -14

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Ans. B

QUESTION



Number of whole numbers lying between -5 and 5

A 10

B 3

C 4

~~**D** 5~~



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Any Statement in Mathematics is
True when it is universally true
under given conditions

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Any statement is false if we find an
example against the statement



QUESTION

Which of the following conditions imply that the real number x is rational ?

I. $x^{1/2}$ is rational (given)

$$x^{1/2} \in \mathbb{Q} \Rightarrow x^{1/2} \cdot x^{1/2} \in \mathbb{Q} \Rightarrow x \in \mathbb{Q}$$

(given)

II. x^2 and x^5 are rational

$$x^2, x^5 \in \mathbb{Q} \Rightarrow x^2 \cdot x^2 \in \mathbb{Q} \text{ i.e. } x^4 \in \mathbb{Q}$$

(given)

III. x^2 and x^4 are rational

$$\frac{x^5}{x^4} = x \in \mathbb{Q}, x \neq 0$$

A I and II only

$$\sqrt{2}^2, \sqrt{2}^4 = 2, 4 \in \mathbb{Q}$$

But $\sqrt{2} \notin \mathbb{Q}$

$$\text{If } x=0 \Leftrightarrow x^2, x^5 \in \mathbb{Q}$$

B I and III only

C II and III only

D I, II, and III

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Ans. A

QUESTION



Let a, b, c be positive integer S.t. $\frac{a\sqrt{2}+b}{b\sqrt{2}+c}$ is a rational number, then which of the following values of a, b, c is/are possible?

~~A~~ $a = 1, b = 2, c = 4$

~~B~~ $a = 4, b = 6, c = 9$

~~C~~ $a = 2, b = 5, c = 8$

~~D~~ $a = 3, b = 9, c = 27$

$$\frac{a\sqrt{2}+b}{b\sqrt{2}+c} \in \mathbb{Q} \text{ (given)}$$

$$a, b, c \in \mathbb{I}^+$$

$$b^2 - ac \in \mathbb{I}$$

$$b^2 - ac \neq \sqrt{2}$$

$$\frac{a\sqrt{2}+b}{b\sqrt{2}+c} \cdot \frac{b\sqrt{2}-c}{b\sqrt{2}-c} \in \mathbb{Q}$$

$$\frac{2ab - ac\sqrt{2} + b\sqrt{2} - bc}{2b^2 - c^2} \in \mathbb{Q}$$

$$\frac{2ab - bc + \sqrt{2}(b^2 - ac)}{2b^2 - c^2} \in \mathbb{Q}$$

$$b^2 - ac = 0$$

$$b^2 = ac$$



Types of Integers and Natural Numbers



1. **Even Integers:** $2n, n \in I = \{\dots, -6, -4, -2, 0, 2, 4, 6 \dots\}$
2. **Odd Integers:** $2n + 1$ or $2n - 1, n \in I = \{\dots, -5, -3, -1, 1, 3, 5 \dots\}$
3. **Prime Numbers:** 2, 3, 5, 7, 11, 13 All natural numbers with only two divisors 1 & itself.
4. **Composite Numbers:** All natural numbers with more than two distinct positive divisors.
 - Ex 4 \rightarrow Divisor: 2, 4, 1
 - 6 \rightarrow Divisors: 2, 3, 6, 1
5. **Relatively Prime / Coprime:** Two natural numbers whose HCF is one.
 - Ex: $\begin{matrix} 2 \times 3 \\ 8 \times 21 \\ 7 \times 16 \end{matrix}$
6. **Twin Prime Numbers:** Two prime numbers with a difference of two are called twin primes.
 - Ex: 3 & 5, 5 & 7, 17 & 19

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1 is neither prime nor Composite Number

$N - \text{prime} - \{1\} = \text{Composite NO.}$
NO: 8

Every prime NO: > 5 is of type $6k \pm 1$, $k \in \mathbb{N}$

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$$5 = 6 \cdot 1 - 1$$

$$7 = 6 \cdot 1 + 1$$

$$11 = 6 \cdot 2 - 1$$

$$13 = 6 \cdot 2 + 1$$

$$17 = 6 \cdot 3 - 1$$

$$23 = 6 \cdot 4 - 1$$

$$19 = 6 \cdot 3 + 1$$

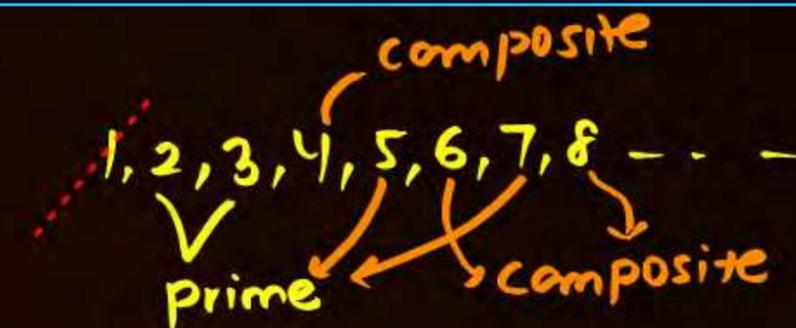
But Every NO: of form $6k \pm 1$
is not prime

$$6 \cdot 4 + 1 = 25 \notin \text{prime}$$

$$6 \cdot 6 - 1 = 35 \notin \text{prime.}$$



Kaam ki Baatien



1. 2 is smallest prime.

2. 4 is the smallest composite number.

3. 1 is neither prime nor composite.

4. Every prime ≥ 5 is of type ATDB.uno where ATDB.uno but every number of form ATDB.uno is not ATDB.uno.

QUESTION



If 'a' and 'b' are two distinct prime numbers lying between 1 and 10, which of the following can be the sum of 'a' and 'b' -



~~A~~ 5

B 6

~~C~~ 7

~~D~~ 8

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Ans. A, C, D



Two Golden Inequalities

$$G_1: x + \frac{1}{x} \geq 2, \quad x \in \mathbb{R}^+$$

$$G_2: x + \frac{1}{x} \leq -2, \quad x \in \mathbb{R}^-$$

★ any +ve Real + its reciprocal ≥ 2

★ any -ve Real + its reciprocal ≤ -2

$$x + \frac{1}{x} = 2, x \in \mathbb{R}^+ \iff x = 1$$

$$x + \frac{1}{x} = -2, x \in \mathbb{R}^- \iff x = -1$$

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Proof: Since $x \in \mathbb{R}^+ \Rightarrow \sqrt{x} \in \mathbb{R}^+$
 (i) $(\sqrt{x} - \frac{1}{\sqrt{x}})^2 \geq 0$ $\frac{1}{\sqrt{x}} \in \mathbb{R}^+$

$$x + \frac{1}{x} - 2\sqrt{x} \cdot \frac{1}{\sqrt{x}} \geq 0$$

$$x + \frac{1}{x} \geq 2 \quad (\text{Hence proved})$$

(ii) Since $x \in \mathbb{R}^-$ let $x = -t$, $t > 0 \Rightarrow t + \frac{1}{t} \geq 2$ from (i)
 \Downarrow
 $t = -x$ $-x - \frac{1}{x} \geq 2 \Rightarrow x + \frac{1}{x} \leq -2$

If $x \in \mathbb{R}$ then $x^2 \geq 0$

$$\text{Ex: } (-\sqrt{2})^2 = 2$$

$$(-5)^2 = 25$$

$$(3)^2 = 9$$

$$0^2 = 0$$

$$f(x) = x + \frac{1}{x} \quad \text{Range: } (-\infty, -2] \cup [2, \infty)$$

$f(x) = k$ has a real soln
if $k \in \text{Range of } f(x)$

$$\text{Ex: } \sin x = 2 \rightarrow \text{NO soln}$$

$$\text{Ex: } \cos x = -3 \rightarrow \text{NO soln.}$$

QUESTION [JEE Mains 2023 (24 Jan)]



The number of real solutions of the equation $3\left(x^2 + \frac{1}{x^2}\right) - 2\left(x + \frac{1}{x}\right) + 5 = 0$, is

- A** 3
- B** 4
- C** 0
- D** 2

$$x + \frac{1}{x} = t$$

$$\text{S.B.S } x^2 + \frac{1}{x^2} + 2x \cdot \frac{1}{x} = t^2$$

$$x^2 + \frac{1}{x^2} = t^2 - 2$$

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$$3(t^2 - 2) - 2t + 5 = 0$$

$$3t^2 - 2t - 1 = 0$$

$$3t^2 - 3t + t - 1 = 0$$

$$(3t+1)(t-1) = 0$$

$$t = -\frac{1}{3}, 1$$

No real soln if Range of $x + \frac{1}{x}$

$x + \frac{1}{x} = -\frac{1}{3}$ or $x + \frac{1}{x} = 1$

↳ No real soln

↳ 'coz $-\frac{1}{3}$ does not lie in Range of $x + \frac{1}{x}$

Ans. C

QUESTION



Consider the expressions

$$E_1 = x + \frac{1}{x}, x \in \mathbb{R}^-; \quad E_2 = y^2 + \frac{1}{y^2}, y \in \mathbb{R}_0; \quad E_3 = z^2 + \frac{1}{z^2 + 1} + 2, z \in \mathbb{R}$$

If $\alpha = \max E_1, \beta = \min E_2, \gamma = \min E_3$ then $\alpha + \beta + \gamma$ is



3

$$E_1 = x + \frac{1}{x} \quad x \in \mathbb{R}^- \Rightarrow E_1 = x + \frac{1}{x} \leq -2, \quad E_2 = y^2 + \frac{1}{y^2}, y \in \mathbb{R}_0 \Rightarrow \text{clearly } y^2 > 0 \text{ i.e. } y^2 \in \mathbb{R}^+$$

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$$\alpha = \max E_1 = -2 \text{ @ } x = -1$$

$$\Rightarrow E_2 = y^2 + \frac{1}{y^2} \geq 2$$

$$\beta = \min E_2 = 2 \text{ @ } y^2 = 1 \Rightarrow y = \pm 1$$



2

$$E_3 = z^2 + 1 + \frac{1}{z^2 + 1} + 1, z \in \mathbb{R}$$

$$z^2 \geq 0 \Rightarrow 1 + z^2 \geq 1$$

$$(z^2 + 1) + \frac{1}{z^2 + 1} \geq 2$$

$$1 + z^2 \in \mathbb{R}^+$$

$$(z^2 + 1) + \frac{1}{z^2 + 1} + 1 \geq 3$$

$$\gamma = \min E_3 = 3 \text{ @ } z^2 + 1 = 1 \Rightarrow z = 0$$

$$\alpha + \beta + \gamma = -2 + 2 + 3 = 3$$



7

Ans. A

QUESTION

Tah 02



If a, b, c are non-zero real numbers, then the minimum value of expression

$$\left(\frac{(a^4 + 3a^2 + 1)(b^4 + 5b^2 + 1)(c^4 + 7c^2 + 1)}{a^2b^2c^2} \right) \text{ is}$$

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Ans. 315



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$\sqrt{1} = 1$	$\sqrt{11} = 3.3166$
$\sqrt{2} = 1.4142$	$\sqrt{12} = 3.4641$
$\sqrt{3} = 1.732$	$\sqrt{13} = 3.6055$
$\sqrt{4} = 2$	$\sqrt{14} = 3.7416$
$\sqrt{5} = 2.236$	$\sqrt{15} = 3.8729$
$\sqrt{6} = 2.4494$	$\sqrt{16} = 4$
$\sqrt{7} = 2.6457$	$\sqrt{17} = 4.1231$
$\sqrt{8} = 2.8284$	$\sqrt{18} = 4.2426$
$\sqrt{9} = 3$	$\sqrt{19} = 4.3588$
$\sqrt{10} = 3.1622$	$\sqrt{20} = 4.4721$

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Yaad Karnay hai



$1^2 = 1$	$2^2 = 4$	$3^2 = 9$	$4^2 = 16$	$5^2 = 25$
$6^2 = 36$	$7^2 = 49$	$8^2 = 64$	$9^2 = 81$	$10^2 = 100$
$11^2 = 121$	$12^2 = 144$	$13^2 = 169$	$14^2 = 196$	$15^2 = 225$
$16^2 = 256$	$17^2 = 289$	$18^2 = 324$	$19^2 = 361$	$20^2 = 400$
$21^2 = 441$	$22^2 = 484$	$23^2 = 529$	$24^2 = 576$	$25^2 = 625$
$26^2 = 676$	$27^2 = 729$	$28^2 = 784$	$29^2 = 841$	$30^2 = 900$



Yaad Karnay hai



$31^2 = 961$	$32^2 = 1024$	$33^2 = 1089$	$34^2 = 1156$	$35^2 = 1225$
$36^2 = 1296$	$37^2 = 1369$	$38^2 = 1444$	$39^2 = 1521$	$40^2 = 1600$
$41^2 = 1681$	$42^2 = 1764$	$43^2 = 1849$	$44^2 = 1936$	$45^2 = 2025$
$46^2 = 2116$	$47^2 = 2209$	$48^2 = 2304$	$49^2 = 2401$	$50^2 = 2500$
$51^2 = 2601$	$52^2 = 2704$	$53^2 = 2809$	$54^2 = 2916$	$55^2 = 3025$
$56^2 = 3136$	$57^2 = 3249$	$58^2 = 3364$	$59^2 = 3481$	$60^2 = 3600$

**Don't Forget to
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Retry all the class illustrations**



Today's KTK



No Selection TRISHUL Selection with Good Rank
Apnao IIT Jao



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QUESTION

KTK 1



The equation $\frac{2x^2}{x-1} - \frac{2x+7}{3} + \frac{4-6x}{x-1} + 1 = 0$ has the roots-

- A** 4 and 1
- B** only 1
- C** only 4
- D** neither 4 nor 1

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Ans. C

QUESTION

KTK 2



Which one of the following does not reduce to $\sin x$ for every x , wherever defined, is

- A** $\frac{\tan x}{\sec x}$
- B** $\frac{\sin x}{\sec^2 x - \tan^2 x}$
- C** $\frac{\sin^2 x \sec x}{\tan x}$
- D** All reduce to $\sin x$

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Ans. D

QUESTION

KTK 3



Column-I		Column-II	
(A)	A rectangular box has volume 48, and the sum of the length of the twelve edges of the box is 48. The largest integer that could be the length of an edge of the box, is	(P)	1
(B)	The number of zeroes at the end in the product of first 20 prime numbers, is	(Q)	2
(C)	The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is	(R)	3
		(S)	4
		(T)	6

Ans. (A) T; (B) P; (C) P

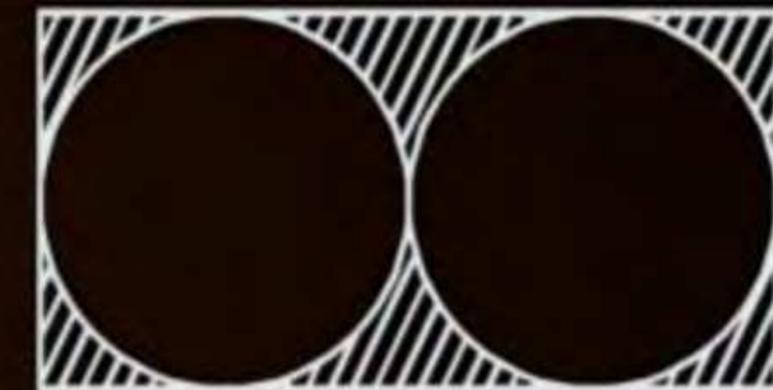
QUESTION

KTK 4



The ratio of total area of the rectangle to the total shaded area

- A** $\frac{2}{\pi}$
- B** $\frac{4}{4 - \pi}$
- C** $\frac{4 - \pi}{\pi}$
- D** $\frac{\pi}{4}$



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Ans. B

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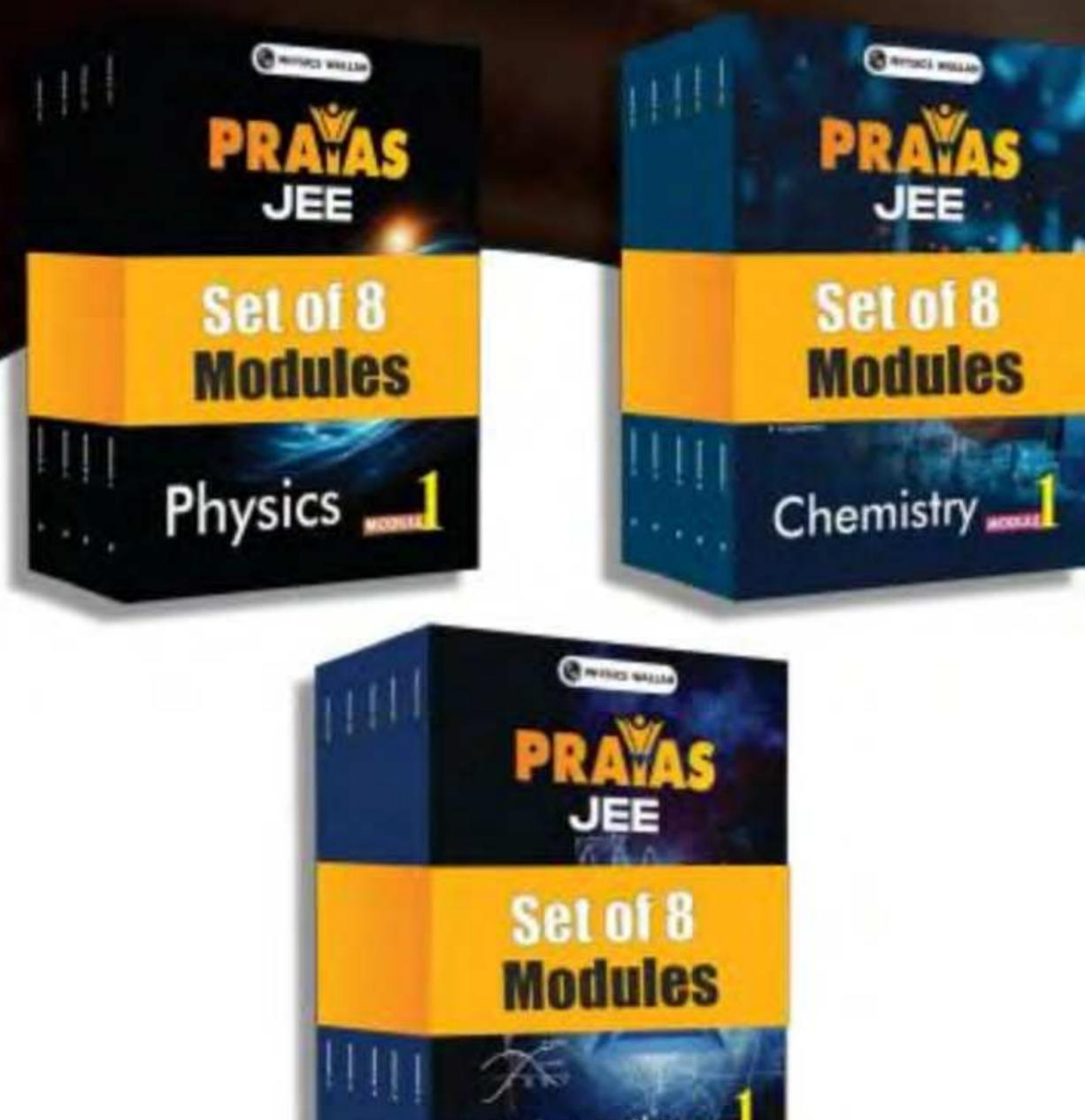
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