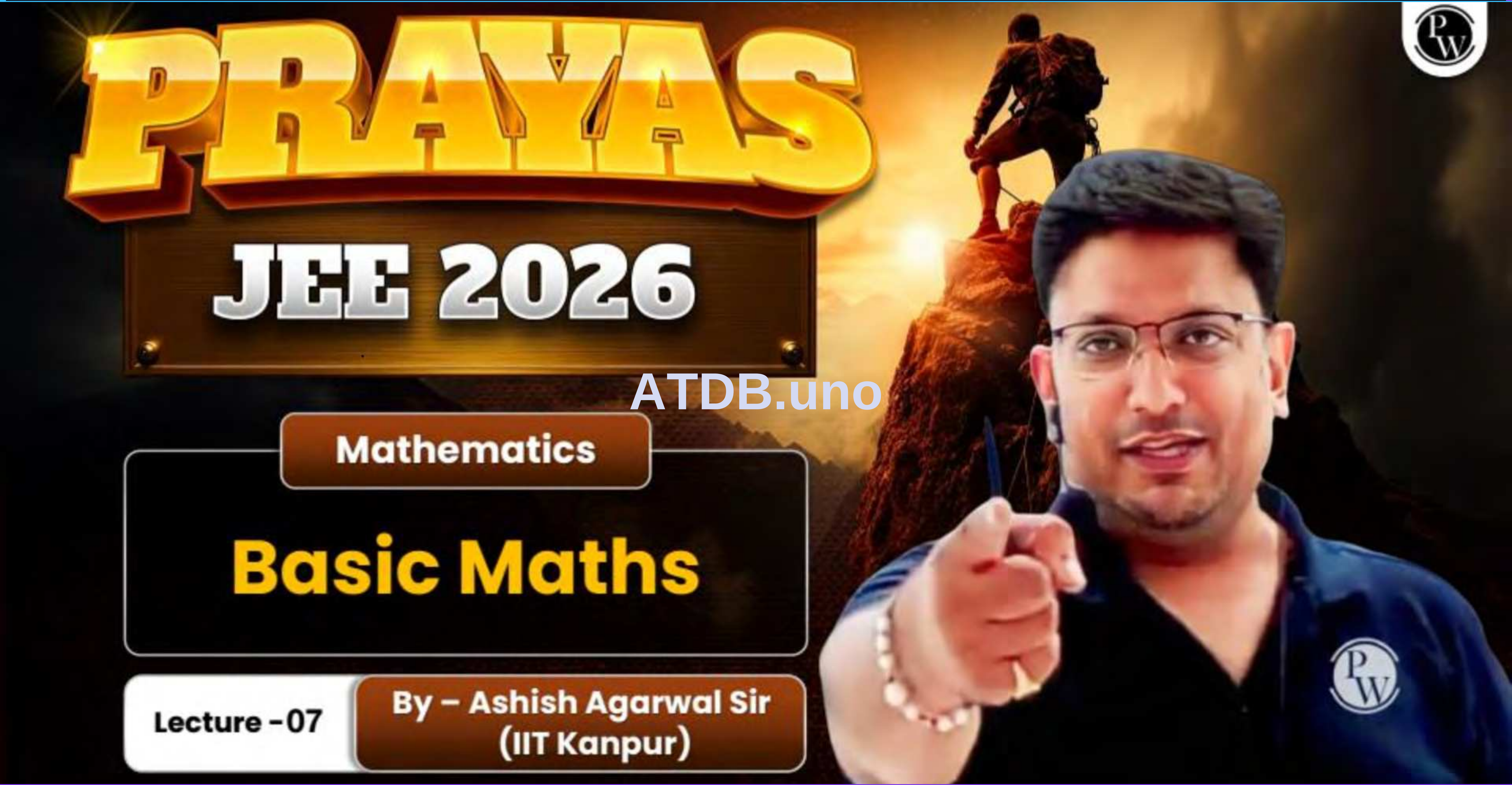

PRAYAS
JEE 2026
ATDB.uno
Mathematics
Basic Maths
Lecture -07
By – Ashish Agarwal Sir
(IIT Kanpur)

Topics *to be covered*

A Problem Practice

B Important Points in Inequalities

C Method of Intervals/ Wavy Curve Method

Homework Discussion

QUESTION

TAH 10

Let $a, b, c \in N (a > b)$ satisfy $c^2 - a^2 - b^2 = 101$ with $ab = 72$. Then which of the following can be correct?

(A) b and c are coprime

(B) c is an odd prime

(C) $(a + b + c)$ is even

(D) $a + b = c + 1$

$2ab = 144$

$c^2 - a^2 - b^2 - 2ab = -43$

$c^2 - (a+b)^2 = -43$

$(a+b)^2 - c^2 = 43$

$(a+b+c)(a+b-c) = 43$ — Prime NO:

$a+b+c = 43$

$a+b-c = 1$

$2c = 42$ — $c = 21$

$a+b = 22$, $ab = 72$

M① $a = 18, b = 4$ (By observation)

M② $a + \frac{72}{a} = 22$

$a^2 - 22a + 72 = 0$

$a = \cancel{4}, 18$

$b = \cancel{18}, 4$

Ans. A, D

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

QUESTION [JEE Mains 2016]

The sum of all real values of x satisfying $(x^2 - 5x + 5)^{x^2 + 4x - 60} = 1$ is

(A) 5

(B) 3

(C) –4

(D) 6

$a^x = 1 \begin{cases} x = 0, a \neq 0 \\ a = 1 \\ a = -1 \ \& \ (-1)^x = 1 \end{cases}$

$x^2 + 4x - 60 = 0 \ \& \ x^2 - 5x + 5 \neq 0$ put here

$(x+10)(x-6) = 0$

$x = 6, -10$

$36 - 30 + 5 \neq 0$

$100 + 50 + 5 \neq 0$

$\Downarrow$

$x = 6, -10$ ✓

OR $x^2 - 5x + 5 = 1$

$x^2 - 5x + 4 = 0$

$x = 1, 4$

OR $x^2 - 5x + 5 = -1$ & $(-1)^{x^2 + 4x - 60} = 1$

$x^2 - 5x + 6 = 0$ put here. $x = 2$ ✓

$x = 2, 3$ $x = 3$ ✗

Sum of soln $= 6 + (-10) + 1 + 4 + 2 = 3.$

QUESTION

The square root of $11 + \sqrt{112}$ is -

(A) $\sqrt{7} + 2$

(B) $\sqrt{7} + \sqrt{2}$

(C) $2 - \sqrt{7}$

(D) None

$$\sqrt{11 + 2\sqrt{28}}$$
$$= \sqrt{11 + 2 \cdot 2\sqrt{7}}$$
$$= \sqrt{2^2 + \sqrt{7}^2 + 2 \cdot 2 \cdot \sqrt{7}}$$
$$= \sqrt{(2+\sqrt{7})^2}$$
$$= |2+\sqrt{7}| = 2+\sqrt{7}$$

QUESTION

The square root $5 + 2\sqrt{6}$ is -

Jaho!

(A) $\sqrt{3} + 2$

(B) $\sqrt{3} - \sqrt{2}$

(C) $\sqrt{2} - \sqrt{3}$

(D) $\sqrt{3} + \sqrt{2}$

QUESTION

If $x = 3 + 3^{1/3} + 3^{2/3}$, then the value of $x^3 - 9x^2 + 18x - 12$ is

(A) ~~0~~

(B) –1

(C) 1

(D) 2

$x - 3 = 3^{1/3} + 3^{2/3}$

CBS

$x^3 - 27 - 3\cdot x\cdot 3(x-3) = 3 + 3^2 + 3\cdot \underbrace{3^{1/3}\cdot 3^{2/3}}_{3^1}(3^{1/3} + 3^{2/3})$

$x^3 - 9x^2 + 27x - 27 = 12 + 9(x-3)$

$x^3 - 9x^2 + 27x - \cancel{27} = 12 + 9x - \cancel{27}$

$x^3 - 9x^2 + 18x - 12 = 0$

QUESTION

The number $(7+5\sqrt{2})^{1/3}+(7-5\sqrt{2})^{1/3}$, is equal to

$$x=(7+5\sqrt{2})^{1/3}+(7-5\sqrt{2})^{1/3}$$

C.B.S

$$x^3=7+\cancel{5\sqrt{2}}+7-\cancel{5\sqrt{2}}+3\cdot(7+5\sqrt{2})^{1/3}(7-5\sqrt{2})^{1/3}(x)$$

$$x^3=14+3(49-50)^{1/3}x=14+3(-1)^{1/3}x$$

$$x^3=14-3x$$

$$x^3+3x-14=0 \qquad P(x)=x^3+3x-14$$

$$P(2)=2^3+3\cdot2-14=0$$

$$x^2(x-2)+2x(x-2)+7(x-2)=0$$

$$(x-2)(x^2+2x+7)=0$$

$$x=2 \text{ or } x^2+2x+7=0 \quad D=2^2-4\cdot7<0$$

$$\Downarrow$$

(No real roots)

$x=2$ Ans.

QUESTION

If $\dfrac{\ell}{\sqrt{10}+\sqrt{14}+\sqrt{15}+\sqrt{21}} = \dfrac{\sqrt{10}-\sqrt{14}-\sqrt{15}+\sqrt{21}}{k}$, then

(A) $k = \ell/2$

(B) $\ell = k/2$

(C) $\ell = 2/k$

(D) None of these

$$\frac{\ell}{\sqrt{10}+\sqrt{14}+\sqrt{15}+\sqrt{21}} = \frac{\sqrt{10}-\sqrt{14}-\sqrt{15}+\sqrt{21}}{k}$$

$$\ell k = ((\sqrt{10}+\sqrt{21}) + (\sqrt{14}+\sqrt{15}))\;((10+\sqrt{21})-(\sqrt{14}+\sqrt{15}))$$

$$\ell k = (\sqrt{10}+\sqrt{21})^2 - (\sqrt{14}+\sqrt{15})^2$$

$$\ell k = 31 + \cancel{2\sqrt{210}} - (29 + \cancel{2\sqrt{210}})$$

$$\ell k = 2$$

$$\ell = 2/k$$

QUESTION

If $\dfrac{4}{2+\sqrt{3}+\sqrt{7}} = \sqrt{a}+\sqrt{b}-\sqrt{c}$, then which of the following can be true-

(A) $a = 1, b = 4/3, c = 7/3$

(B) $a = 1, b = 2/3, c = 7/9$

(C) $a = 2/3, b = 1, c = 7/3$

(D) $a = 7/9, b = 4/3, c = 1$

$$\frac{4}{(2+\sqrt{3})+\sqrt{7}} \times \frac{(2+\sqrt{3})-\sqrt{7}}{(2+\sqrt{3})-\sqrt{7}}$$

$$\frac{4(2+\sqrt{3}-\sqrt{7})}{(2+\sqrt{3})^2-\sqrt{7}^2}$$

$$= \frac{4(2+\sqrt{3}-\sqrt{7})}{7+4\sqrt{3}-7}$$

$$= \frac{2+\sqrt{3}-\sqrt{7}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}+\sqrt{9}-\sqrt{21}}{3}$$

$$= \frac{2}{3}\sqrt{3}+1-\frac{\sqrt{21}}{3} = \sqrt{1}+\sqrt{\frac{4}{3}}-\sqrt{7/3}$$

QUESTION [IIT-JEE 1980]

Tah 02

The expression $\frac{12}{3+\sqrt{5}+2\sqrt{2}}$ is equal to

(A) $1-\sqrt{5}+\sqrt{2}+\sqrt{10}$

(B) $1+\sqrt{5}+\sqrt{2}-\sqrt{10}$

(C) $1+\sqrt{5}-\sqrt{2}+\sqrt{10}$

(D) $1-\sqrt{5}-\sqrt{2}+\sqrt{10}$

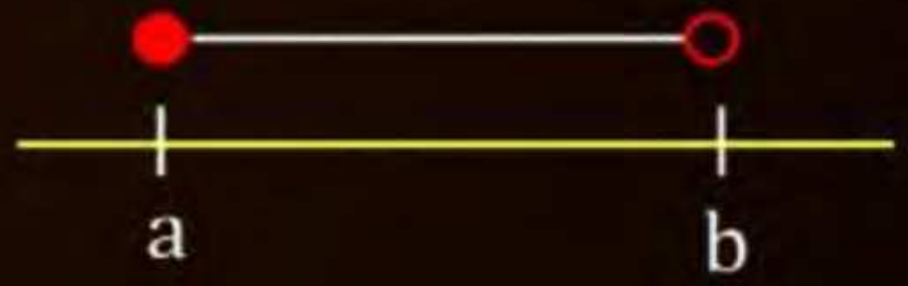
Interval Notation

(i) $(a, b) = \{x \in R : a < x < b\}$ open interval

(ii) $[a, b] = \{x \in R : a \leq x \leq b\}$ closed interval.

(iii) $(a, b] = \{x \in R : a < x \leq b\}$ open closed interval

(iv) $[a, b) = \{x \in R : a \leq x < b\}$ closed open interval.



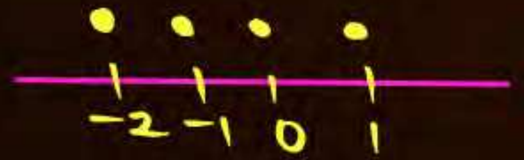
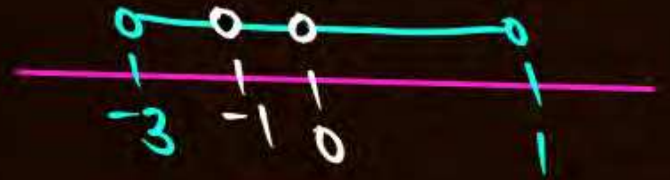
Intervals

An interval is a subset of real number R. If a, b $\in$ R & a < b then we can define four types of intervals :

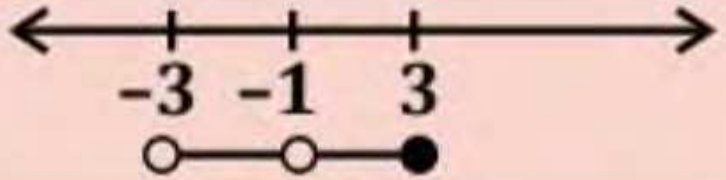
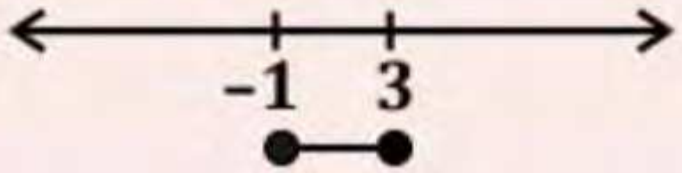
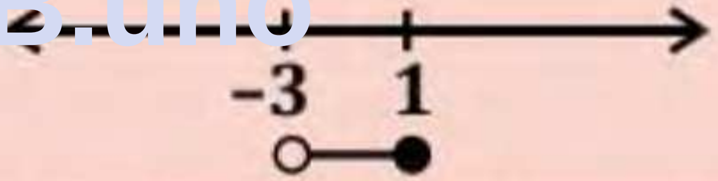
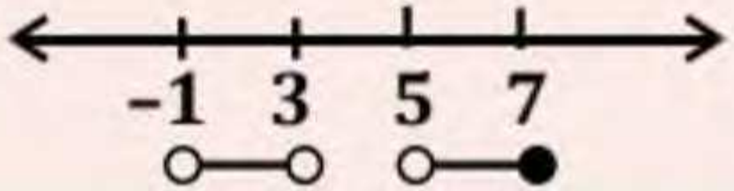
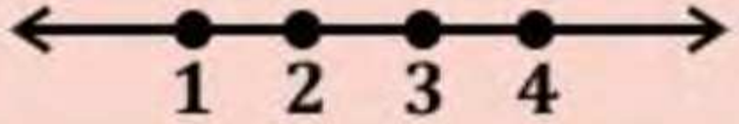
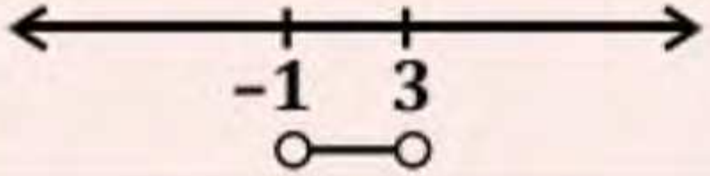
Name	Representation	Description
1. Open Interval		
2. Closed Interval		
3. Open Closed Interval		
4. Closed Open Interval		

KUCH SYMBOLS

- $\bullet$ Matlab Included
- $\circ$ Matlab not included
- ( ) Matlab End points Excluded
- [ ] Matlab End points included
- { } Matlab Discrete set of points → $x = 1, 2, 3, 4 \Rightarrow x \in \{1, 2, 3, 4\}$
- $\cup$ Matlab Milaa do | Mix kardo → $A = \{1, 2, 3\}$, $B = \{2, 4, 5\}$ → $A \cup B = \{1, 2, 3, 4, 5\}$
- $\cap$ Matlab Intersection | common → $A \cap B = \{2\}$
- $\in$ Matlab Belongs to

Inequality	Number line Representation	Interval or set representation
(i) $-3 \le x \le 5, x \in R$		$[-3, 5]$
(ii) $-2 < x \le 5, x \in R$		$(-2, 5]$
(iii) $-1 \le x < 3, x \in R$		$[-1, 3)$
(iv) $-2 \le x \le 1, x \in I$		$\{-2, -1, 0, 1\}$
(v) $-3 < x < 5, x \in N$		$\{1, 2, 3, 4\}$
(vi) $-3 < x < 1, x \in R$ but $x \ne -1, 0$		$(-3, 1) - \{-1, 0\}$ OR $(-3, -1) \cup (-1, 0) \cup (0, 1)$

Match the Column:

Column-I		Column-II	
1.	(–3, 1] R	P.	
2.	{1, 2, 3, 4} T	Q.	
3.	[–1, 3] Q	R.	
4.	(–1, 3) U	S.	
5.	(–1, 3) ∪ (5, 7] S	T.	
6.	(–3, –1) ∪ (–1, 3] P	U.	

Intersection & Union

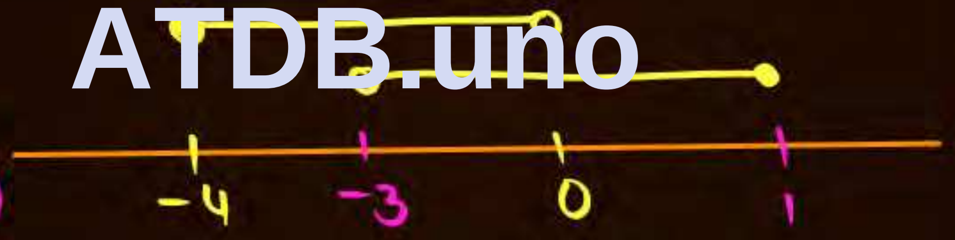
Find $A \cup B$ & $A \cap B$

1. $A = (-3, 1]$ & $B = [-4, 0)$
2. $A = (-3, -1)$ & $B = [-1, 2]$
3. $A = (-2, 3)$ & $B = (-1, 2]$

Tah03A

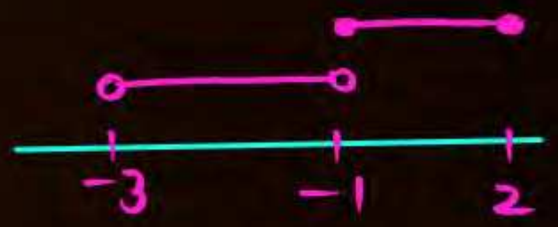
(1)

$A \cap B = (-3, 0)$



$A \cup B = [-4, 1]$

(2)



$A \cap B = \phi$

$A \cup B = (-3, 2]$

QUESTION

Evaluate the following — Jah02(B)

(i) $(-\infty, 3) \cap [-2, \infty)$

(ii) $(-4, 1] \cap (-3, 4)$

(iii) $(0, 5] \cap (1, \infty)$

(iv) $[0, 3) \cup [2, 6)$

(v) $[2, \infty) \cup (4, \infty)$

(vi) $[-1, 1) \cup [2, 5]$

Inequality Solve Karnay kaa Matlab?

Solving an inequality means finding the value of variable for which inequality holds.

Ex: $x + 3 > 4$

$x > 4 - 3$

$x > 1$ ~ sol. of inequality

Ex: $2x + 3 \le -4 + 3x$.

$7 \le x$

$x \ge 7$.

Kaam ki Baat

B1 : We can add (or subtract) any number 'k' on both sides of inequality. Doing this will not change the sign of inequality.

Ex: $-7 \le x+3 \le 5$

subtracting 3 from each side of ineq.

$-7-3 \le x \le 5-3$

$-10 \le x \le 2$

$\downarrow$

$x \in [-10, 2]$

Ex: $-2 \le x-4 < -1$

Add 4 to each side of ineq

$2 \le x < 3.$

$\Downarrow$

$x \in [2, 3)$

Kaam ki Baat

B2 : We can multiply (or divide) any non-zero number 'k' on both sides of inequality and sign of inequality will change according to sign of 'k' that is

- If $k > 0$ then sign of inequality will remains same,
- If $k < 0$ then sign of inequality will get reversed.

Reason

$-5 > -10$ Divide by -1

$5 < 10$

$-2 < 3$ Multiply by -1

$2 > -3$

Ex: $2x + 3 < 7$

$2x < 7 - 3$

$2x < 4$ Divide by 2

$x < \frac{4}{2}$

Ex: $5 - 3x > 8$

$-3x > 8 - 5$

$-3x > 3$

$x < \frac{3}{-3} = -1$

$x \in (-\infty, -1)$

Kaam ki Baat

B3 : Squaring (raising even power both side) can be done when both sides of inequality are non negative.

Ex: $3 > 2$ $\xrightarrow{SBS}$ $9 > 4$

Power 4: $3^4 > 2^4$

$81 > 16$.

$-3 < -2$ $\xrightarrow{SBS}$ $9 < 4$ ✗

$-2 < 5$ $\xrightarrow{S.B.S}$ $4 < 25$

$-9 < 3$ $\xrightarrow{S.B.S}$ $81 < 9$ ✗

B4 : Raising both sides to odd power is fine.

Ex: $-3 < -2$ $\xrightarrow{CBS}$ $-27 < -8$

Ex: $-2 < 5$ $\xrightarrow{CBS}$ $-8 < 125$

Algebra of Inequalities

1. Inequalities can be added provided they have same sign of inequality.
But inequalities can not be subtracted.

Ex: $x < 2$, $y < 3$ ⟹ $x + y < 5$

Ex: $x < 2$, $y \le -1$ ⟹ $x + y < 1$

Ex: $x \le 3$, $y \le -1$ ⟹ $x + y \le 2$

Ex: $x > 3$, $y < 2$ ⟹ $-y > -2$
$x - y > -2 + 3$
$x - y > 1$

Ex: $x > 5$, $y > 2$ ⟹ $x + y > 7$

$-5 > -y > -7$

Ex: If $-2 < x < 3$, $5 < y < 7$, find range of $x + y$ & $(x - y)$.

$5 < y < 7$
$3 < x + y < 10$

multiply by -1

$-2 < x < 3$
$-7 < -y < -5$
$-9 < x - y < -2$

Ex: $x \ge 5$, $y < 3$ ⟹ $x - y \in (2, \infty)$

$\Downarrow$

$x \ge 5$
$-y > -3$
$x - y > 2$

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QUESTION

Solve: (a) $5x + 2 < 17$

$5x < 15$

$x < 3$

$x \in (-\infty, 3)$

(b) $-2x > 5$

$x < -\frac{5}{2}$

$x \in (-\infty, -5/2)$

ATDB.uno

QUESTION

Solve: $5x - 6 > 7x + 8$

$-6 - 8 > 7x - 5x$

$-14 > 2x$

$2x < -14$

$x < -7$

$x \in (-\infty, -7)$

QUESTION

Solve: $\dfrac{(5x-8)}{3} \geq \dfrac{(4x-7)}{2}$

$10x - 16 \geq 12x - 21$

$+21 - 16 \geq 12x - 10x$

$5 \geq 2x$

$x \leq 5/2$

$x \in (-\infty, 5/2]$

Ex: $\dfrac{5x-8}{-3} \geq \dfrac{4x-7}{-2}$

$-10x + 16 \geq -12x + 21$

$12x - 10x \geq 21 - 16$

$2x \geq 5$

$x \geq 5/2$

Ex: $\dfrac{5x-8}{-3} \geq \dfrac{4x-7}{-2}$ multiply by -1

$\dfrac{5x-8}{3} \leq \dfrac{4x-7}{2}$

$10x - 16 \leq 12x - 21$

$5 \leq 2x$

$2x \geq 5$

$x \geq 5/2$

$x \in [5/2, \infty)$

Ex: $\dfrac{5x-8}{-3} \geq \dfrac{4x-7}{2}$

$10x - 16 \leq -12x + 21$

$22x \leq 37$

$x \leq 37/22$

QUESTION

$$\frac{2x-3}{5} \geq 1$$

$$\frac{2x-3}{-5} \geq 1$$

TAHO4

$$\frac{x-3}{x-2} \geq 5$$

$$\frac{x-3}{x-2} - 5 \geq 0$$

$$\frac{x-3-5x+10}{x-2} \geq 0$$

$$\frac{-4x+7}{x-2} \geq 0$$

$$-1.\frac{(4x-7)}{x-2} \geq 0$$

$$\frac{4x-7}{x-2} \leq 0$$

$+ \quad - \quad +$

$\frac{7}{4} \quad 2$

$$x \in \left[7/4, 2\right)$$

QUESTION

Solve: $\dfrac{1}{3x-2} < 0$

$3x - 2 < 0$

$x < 2/3$ Ans

Wavy Curve method/Method of Intervals

Used for solving polynomial & $\underbrace{\text{Rational}}$ Inequalities In a variable

$\text{Ratio} = \dfrac{\text{polynomial}}{\text{polynomial}}$

Steps Involving Wavy Curve Method

Step-1 Create Zero in RHS and Simplify. — Eak side zero Banaao

Step-2 Convert LHS into Linear Factors. — Break polynomial into linear factors

Step-3 Make the coefficient of x positive in all the linear factors. — In each linear factor coeff of x should be +ve

Step-4 Find the critical points by equating each linear factor to zero and plot them on number line. — Equate each linear to 0 & find x

Step-5 Start writing plus minus sign alternate from the right most end of the number line.

Ex: $x^2 - 4x + 8 < 2x$

$x^2 - 6x + 8 < 0$

$(x-2)(x-4) < 0$

$x \in (2, 4)$

Steps Involving Wavy Curve Method

Step-6 (For $> 0, \geq 0$) select region with +ve

(For $< 0, \leq 0$) select region with –ve

Step 7 For $>$ or $<$ sign- all critical points are open bracket.

For $\geq$ or $\leq$ sign- numerator critical points are closed bracket whereas denominator critical points are open bracket.

Kuch Yaad Rakhne waali Baatein

1. Values of x corresponding to denominator are never included in answer.

2. Coefficient of x in every linear factor should be positive if not then make it positive.

Kaam Ki Baat

"Rational inequality mai cross multiplication nhi karna chahiye jab tak voh factor hamesha positive yaa negative naa ho"

QUESTION

$$(x^2 + x - 6)(x^2 - 2x - 8) \geq 0$$

$(x+3)(x-2)(x-4)(x+2) \geq 0$ → +ve Region

$x \in (-\infty, -3] \cup [-2, 2] \cup [4, \infty)$

Factorizing a Quadratic $P(x) = ax^2 + bx + c$

$P(x) = ax^2 + bx + c$

Discriminant
- $D \geq 0$ → can be splitted in two real linear factors.
- $D < 0$ → can not be splitted in two real linear factors

step 1

$$\boxed{D = b^2 - 4ac}$$

step 2 If $D \geq 0$ → find roots $ax^2 + bx + c = 0$

$$\alpha, \beta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

step 3 $ax^2 + bx + c = a(x - \alpha)(x - \beta)$

Some Golden Points

1. If $a > 0$ and $D < 0$ then $y = ax^2 + bx + c > 0$ for all $x \in R$

2. If $a < 0$ and $D < 0$ then $y = ax^2 + bx + c < 0$ for all $x \in R$

Ex: $f(x) = -x^2 + 2x - 3$

$D = (2)^2 - 4\cdot(-1)(-3)$

$= 4 - 12 = -8 < 0$

$a = -1 < 0,\ D < 0$

$f(x) = -x^2 + 2x - 3 < 0 \ \forall x \in R$

Ex: $f(-1) = -1 - 2 - 3 < 0$

Ex: $f(5) = -25 + 10 - 3 < 0$

Ex: $f(x) = x^2 + x + 2$

$D = 1 - 4\cdot 2 = -7 < 0$

$\Downarrow$

$a = 1 > 0,\ D < 0$

$\Downarrow$

$f(x) = x^2 + x + 2 > 0 \ \forall x \in R$

Ex: $f(-2) = 4 - 2 + 2 = 4 > 0$

QUESTION

Solve: $x^2 - 5x + 2 > 0$

$x^2 - 5x + 2$

step① $D = (-5)^2 - 4 \cdot 1 \cdot 2 = 25 - 8 = 17 > 0$

step② $x^2 - 5x + 2 = 0$

$$x = \frac{5 \pm \sqrt{25-8}}{2}$$

$$x = \frac{5 \pm \sqrt{17}}{2}$$

$$x^2 - 5x + 2 = 1 \cdot \left(x - \frac{5+\sqrt{17}}{2}\right)\left(x - \frac{5-\sqrt{17}}{2}\right)$$

$$\left(x - \frac{5+\sqrt{17}}{2}\right)\left(x - \frac{5-\sqrt{17}}{2}\right) > 0$$

$+ \quad \frac{5-\sqrt{17}}{2} \quad - \quad \frac{5+\sqrt{17}}{2} \quad +$

$$x \in \left(-\infty, \frac{5-\sqrt{17}}{2}\right) \cup \left(\frac{5+\sqrt{17}}{2}, \infty\right)$$

QUESTION

Solve: $3x^2 - 7x + 6 < 0$

Gadho / Gadhiyoo aisaa naa Karo

$3x^2 - 9x + 2x + 6 < 0$

$(3x+2)(x-2) < 0$

$D = (-7)^2 - 4 \cdot 3 \cdot 6 = 49 - 72 < 0$

$a = 3 > 0, \ D < 0$

$3x^2 - 7x + 6 > 0 \ \forall x \in R$

$3x^2 - 7x + 6 < 0$

$x \in \phi$

QUESTION

Solve: $x(4-x)(x-6)>0$

coeff of x in each linear should be +ve

$x \cdot -1 \cdot (x-4)(x-6)>0$

$x(x-4)(x-6)<0$

$x \in (-\infty, 0) \cup (4, 6)$

QUESTION

Solve: $x(7 - x)(2 - x)(x - 5) \leq 0$

Tah 05

QUESTION

Solve: $\dfrac{(2x+1)\,(x-3)\,(x+7)\,(x+4)}{(7x+4))\,(x+2)} \geq 0$

$x \in (-\infty, -7] \cup [-4, -2) \cup \left(-\frac{4}{7}, -\frac{1}{2}\right] \cup [3, \infty)$

Saari Class Illustrations

Retry karni Hai

QUESTION [IIT-JEE 1980]

(Home Challenge-01)

If $x > y > 0$, then show that the expression $\left(\sqrt{2}\left(2x+\sqrt{x^2-y^2}\right)\left(\sqrt{x-\sqrt{x^2-y^2}}\right)\right)$ can be simplified to $\sqrt{(x+y)^3}-\sqrt{(x-y)^3}$.

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

(KTK 1)

If a and b are rational numbers and $a + b\sqrt{2} = 5(\sqrt{2} - 3) + \sqrt{8}$ then find value of $a^2 + b^2 =$

Ans. 274

QUESTION

(KTK 2)

If value of $\left(x+\frac{1}{x}=5\right)$ then find value of :

(i) $x^2+\frac{1}{x^2}$ (ii) $x-\frac{1}{x}$ (iii) $x^4+\frac{1}{x^4}$

(iv) $x^4-\frac{1}{x^4}$ (v) $x^3+\frac{1}{x^3}$

Ans. (i) 23, (ii) $\pm\sqrt{21}$, (iii) 527, (iv) $\pm 115\sqrt{21}$, (v) 110

QUESTION

(KTK 3)

Given $3x^2 + x = 1$, then the value of $6x^3 - x^2 - 3x$ is equal to

(A) -1

(B) 0

(C) 1

(D) 2

Ans. D

QUESTION

(KTK 4)

If $\sqrt{9^x} = \sqrt[3]{9^2}$, then x =

(A) $\frac{2}{3}$

(B) $\frac{4}{3}$

(C) $\frac{1}{3}$

(D) $\frac{5}{3}$

Ans. B

QUESTION

(KTK 5)

$$\frac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{77}} =$$

(A) $x^{76/15}$

(B) $x^{78/15}$

(C) $x^{79/15}$

(D) $x^{77/15}$

Ans. D

QUESTION

(KTK 6)

If $\dfrac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^m} = \dfrac{1}{27}$, where m and n are natural numbers, then find the value of (m – n) is ________

Ans. 1

QUESTION

(KTK 7)

Show that the square of $\frac{\sqrt{25-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$ is a rational number.

QUESTION

(KTK 8)

If $5^{10x} = 4900, 2^{\sqrt{y}} = 25$ then the value of $\frac{\left(5^{(x-1)}\right)^5}{4^{-\sqrt{y}}}$ is

(A) $\frac{14}{5}$

(B) 5

(C) $\frac{28}{5}$

(D) 14

Ans. D

Solution to Previous TAH

QUESTION

TAH 01

The number of real number pairs (x, y) which will satisfy the equation $x^2 - xy + y^2 = 4(x + y - 4)$ is

Ans. 1

26 | 4 | 25

TAH

Ques The no of real no pairs (x, y) which will satisfy the equation $x^2 - xy + y^2 = 4(x+y-4)$ is

$$x^2 - xy + y^2 = 4(x+y-4)$$

$$x^2 - xy + y^2 = 4x + 4y - 16$$

$$2x^2 - 2xy + 2y^2 = 8x + 8y - 32$$

$$x^2 + x^2 - 2xy + y^2 + y^2 = 8x + 8y - 32$$

$$(x-y)^2 + x^2 + y^2 - 8x - 8y + 32 = 0$$

$$(x-y)^2 + x^2 - 8x + 16 + y^2 - 8y + 16 = 0$$

$$(x-y)^2 + (x-4)^2 + (y-4)^2 = 0$$

QUESTION

TAH 02

Factorize the following

(i) $x^3 - 13x - 12$ [Ans. $(x+1)(x-4)(x+3)$]

(ii) $x^3 - 7x - 6$ [Ans. $(x+2)(x-3)(x+1)$]

(iii) $x^3 - 6x^2 + 11x - 6$ [Ans. $(x-1)(x-2)(x-3)$]

(iv) $2x^3 + 9x^2 + 10x + 3$ [Ans. $(x+1)(x+3)(2x+1)$]

(v) $x^3 - 9x^2 + 23x - 15$ [Ans. $(x-1)(x-3)(x-5)$]

(vi) $2x^3 - 9x^2 + 13x - 6$ [Ans. $(x-1)(x-2)(2x-3)$]

(vii) $x^3 - 4x^2 + 5x - 2$ [Ans. $(x-2)(x-1)^2$]

Ta 02

(i) $x^3-13x-12$

$\rightarrow x^2(x+1)-x(x+1)-12(x+1)$

$\Rightarrow (x+1)(x^2-x-12)$

$\therefore (x+1)(x-4)(x+3)$

(iii) $x^3-6x^2+11x-6$

$-ab-bc-ca$

$x^2(x-1)-5x(x-1)+6(x-1)$

$(x-1)(x-2)(x-3)$

(v) $x^3-9x^2+23x-15$

$\Rightarrow x^2(x-1)-8x(x-1)+15(x-1)$

$\Rightarrow (x-1)(x-3)(x-5)$

(vi) x^3-4x^2+5x-2

$\Rightarrow x^2(x-1)-3x(x-1)+2(x-1)$

$\Rightarrow (x-1)(x-2)(x-1)$

(ii) x^3-7x-6

$x^2(x+1)-x(x+1)-6(x+1)$

$(x+1)(x-3)(x+1)$

(iv) $2x^3+9x^2+10x+3$

$=2x^2(x+1)+7x(x+1)+3(x+1)$

$=(x+1)(x+3)(2x+1)$

(vi) $2x^3-9x^2+13x-6$

$\Rightarrow 2x^2(x-1)-7x(x-1)+6(x-1)$

$\Rightarrow (x-1)(x-2)(x-3)$

QUESTION

TAH 03

Given that $a + b + c = 3$, $a^2 + b^2 + c^2 = 5$ and $a^3 + b^3 + c^3 = 7$, then the value of $a^4 + b^4 + c^4$ is equal to

Ieh 03 → Solution: → $a+b+c=3,\ a^2+b^2+c^2=5$

$$a^3+b^3+c^3=7$$

$$a^3+b^3+c^3+3abc=(a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$7+3abc=3(5-ab-bc-ca)$$

$$7+3abc=15-3(ab+bc+ca)$$

$$3abc+3(ab+bc+ca)=15-7$$

$$3abc+3(ab+bc+ca)=8$$

$$3abc=8-6=2$$

$$a+b+c=3$$

SBS

$$(a+b+c)^2=3^2$$

$$a^2+b^2+c^2+2(ab+bc+ca)=9$$

$$5+2(ab+bc+ca)=9$$

$$ab+bc+ca=\cancel{4}/\cancel{2}$$

$$ab+bc+ca=2$$

$a^2b^2+b^2c^2+c^2a^2=3$

$$(a^2+b^2+c^2)^2=(5)^2$$

$$a^4+b^4+c^4+2(a^2b^2+b^2c^2+c^2a^2)=25$$

$$a^4+b^4+c^4+2\times3=25$$

$$a^4+b^4+c^4=25-6=19$$ Ans

QUESTION

TAH 04

Find the square root of $10 + \sqrt{24} + \sqrt{60} + \sqrt{40}$.

→ * $\boxed{\text{TAH-04}}$

$$\sqrt{10+\sqrt{24}+\sqrt{60}+\sqrt{40}}$$

$$\sqrt{10+2\sqrt{2}\times\sqrt{3}+2\times\sqrt{5}\times\sqrt{3}+2\times\sqrt{2}\times\sqrt{5}}$$

$$=\sqrt{(\sqrt{3})^2+(\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{2}\times\sqrt{3}+\sqrt{5}\sqrt{3}+\sqrt{2}\sqrt{5})}$$

$$=\sqrt{(\sqrt{3}+\sqrt{5}+\sqrt{2})^2}$$

$$=\sqrt{3}+\sqrt{5}+\sqrt{2}$$ // Ans.

$$\begin{array}{r|l} 2 & 24 \\ 2 & 12 \\ 2 & 6 \\ 3 & 3 \\ & 1 \end{array} \qquad \begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 5 & 15 \\ 3 & 3 \\ & 1 \end{array} \qquad \begin{array}{r|l} 2 & 40 \\ 2 & 20 \\ 2 & 10 \\ 5 & 5 \\ & 1 \end{array}$$

QUESTION

TAH 05

If $a_1 + a_2 + a_3 + a_4 = -3$ and $a_1^2 + a_2^2 + a_3^2 + a_4^2 = 63$ then find value of :

$a_1a_2 + a_2a_3 + a_1a_3 + a_3a_4 + a_1a_4 + a_2a_4 = ?$

(where $a_1, a_2, a_3, a_4 \in R$)

If $a_1 + a_2 + a_3 + a_4 = -3$ and $a_1^2 + a_2^2 + a_3^2 + a_4^2 = 63$ then find value of :—

$$a_1a_2 + a_2a_3 + a_1a_3 + a_3a_4 + a_1a_4 + a_2a_4 = ?$$

(where $a_1, a_2, a_3, a_4 \in R$)

Tah – 05

Sol

$$a_1 + a_2 + a_3 + a_4 = -3$$

S.B.S

$$a_1^2 + a_2^2 + a_3^2 + a_4^2 + 2(a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4)$$

$$\underbrace{a_1^2 + a_2^2 + a_3^2 + a_4^2} + 2(a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4) = 9$$

$$63 + 2(a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4) = 9$$

$$(a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4) = \frac{9-63}{2} = -\frac{54}{2}$$

$$= -27 \text{ Ans}$$

* IT AH-05

$a_1 + a_2 + a_3 + a_4 = -3$, $a_1^2 + a_2^2 + a_3^2 + a_4^2 = 63$

$\downarrow$ SBS

$\rightarrow (a_1 + a_2 + a_3 + a_4)^2 = 9$

$\Rightarrow \underbrace{a_1^2 + a_2^2 + a_3^2 + a_4^2}_{63} + 2(a_1a_2 + a_1a_3 + a_1a_4 + a_2a_3 + a_2a_4 + a_3a_4) =$

$\rightarrow \cancel{2}(a_1a_2 + a_1a_3 + a_1a_4 + a_2a_4 + a_3a_4) = 9 - 63 = -\cancel{54}\ -27$

$$\boxed{a_1a_2 + a_1a_3 + a_1a_4 + a_2a_4 + a_3a_4 = -27}$$

// Ans.

QUESTION

TAH 06

If $\sqrt[4]{\sqrt[3]{x^2}} = x^k$, then k =

(A) $\frac{2}{6}$

(B) 6

(C) $\frac{1}{6}$

(D) 7

Task 06 → Solution: → $\sqrt[4]{\sqrt[3]{x^2}} = x^k$

$$\left(\left(x^2\right)^{\frac{1}{3}}\right)^{\frac{1}{4}} = x^k$$

$$\left(\left(x^2\right)^{\frac{1}{4}}\right)^{\frac{1}{3}} = x^k$$

$$x^{\frac{1}{6}} = x^k$$

$$\boxed{k = \frac{1}{6}}$$

QUESTION

TAH 07

The numerical value of $(x^{1/a-b})^{1/a-c} \times (x^{1/b-c})^{1/b-a} \times (x^{1/c-a})^{1/c-b}$ is (a, b, c are distinct real numbers)

(A) 1

(B) 8

(C) 0

(D) None

Q The numerical value of (Tah - 07)

$$\left(x^{1/a-b}\right)^{1/a-c} \times \left(x^{1/b-c}\right)^{1/b-a} \times \left(x^{1/c-a}\right)^{1/c-b} \text{ is}$$

Sol

$$x^{\frac{(-1)}{(a-b)(c-a)}} \times x^{\frac{(-1)}{(a-b)(b-c)}} \times x^{\frac{(-1)}{(b-c)(c-a)}}$$

$$x^{\frac{(-1)}{(a-b)(c-a)} + \frac{(-1)}{(a-b)(b-c)} + \frac{(-1)}{(b-c)(c-a)}}$$

$$x^{\frac{(-1)(b-c) + (-1)(c-a) + (-1)(a-b)}{(a-b)(b-c)(c-a)}}$$

$$x^{\frac{(c-b) + (a-c) + (b-a)}{(a-b)(b-c)(c-a)}} = x^{\frac{\cancel{c}-\cancel{b}+\cancel{a}-\cancel{c}+\cancel{b}-\cancel{a}}{(a-b)(b-c)(c-a)}}$$

$$= x^0 = 1 \quad \underline{\text{Ans}}$$

QUESTION

TAH 08

$\sqrt{5+\sqrt{5+\sqrt{5}}+\cdots\infty}$ is equal to

(A) 5

(B) $5+\sqrt{5}$

(C) $\dfrac{1+\sqrt{21}}{2}$

(D) $\dfrac{\sqrt{5}-1}{2}$

TAH-8: $\sqrt{5+\sqrt{5+\sqrt{5+\ldots\infty}}}$ is equal to:

(a) 5 (b) $5+\sqrt{5}$ (c) $\frac{1+\sqrt{21}}{2}$ (d) $\frac{\sqrt{5}-1}{2}$.

Solⁿ: let $\sqrt{5+\sqrt{5+\sqrt{5+\ldots\infty}}} = x$

TAH 8 by Reed west bengal

$\Rightarrow \sqrt{5+x} = x$

S.B.S.

$\Rightarrow 5+x = x^2$

$\Rightarrow x^2 - x - 5 = 0.$

$D > 0.$

$\therefore x = \frac{1 \pm \sqrt{1+20}}{2\cdot 1}$

or, $x = \frac{1 \pm \sqrt{21}}{2}$

$x = \frac{1+\sqrt{21}}{2}$ accepted

$x = \frac{1-\sqrt{21}}{2}$ Rejected

$\therefore$ Ans: (c) $\frac{1+\sqrt{21}}{2}$

Qn 08 → Solution: → $\sqrt{5+\sqrt{5+\sqrt{5+\ldots\infty}}}$

$$\sqrt{5+\sqrt{5+\sqrt{5+\ldots\infty}}} = x$$

$$\sqrt{5+x} = x$$

$$5+x = x^2$$

$$x^2-x-5=0$$

$$x = \frac{1 \pm \sqrt{1+20}}{2\times 1}$$

$$x = \frac{1 \pm \sqrt{21}}{2}$$

$$x = \frac{1+\sqrt{21}}{2}$$

Ans

QUESTION

TAH 09

If $a, b, c \in R$ and $a, b, c \neq 0$ such that $\frac{a}{b}+\frac{b}{c}+\frac{c}{a}=6$ and $\frac{b}{a}+\frac{c}{b}+\frac{a}{c}=8$, then $\frac{a^3}{b^3}+\frac{b^3}{c^3}+\frac{c^3}{a^3}-3$ is equal to

(A) 81

(B) 48

(C) 72

(D) 84

TAH-9: If $a, b, c \in R$ and $a, b, c \neq 0$ such that $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 6$ & $\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 8$ then $\frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3 = ??$

Soln:

$$\frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3\left(\frac{a}{b}\right)\left(\frac{b}{c}\right)\cdot\left(\frac{c}{a}\right)$$

$$= \left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a}\right)\left(\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} - \frac{ab}{\cancel{bc}} - \frac{bc}{\cancel{ca}} - \frac{ac}{\cancel{ba}}\right)$$

$$= \underbrace{\left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a}\right)}_{6}\left(\underbrace{\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2}}_{?} - \underbrace{\left(\frac{a}{c} + \frac{b}{a} + \frac{c}{b}\right)}_{8}\right) \quad \text{— (i)}$$

now, $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 6.$

S.B.S.

$$\Rightarrow \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} + 2\left(\frac{ab}{bc} + \frac{bc}{ca} + \frac{ac}{ba}\right) = 36.$$

$$\Rightarrow \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} + 2\underbrace{\left(\frac{a}{c} + \frac{b}{a} + \frac{c}{b}\right)}_{8} = 36$$

$$\Rightarrow \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} + 2 \times 8 = 36$$

TAH 09 by Reed west bengal

$$\Rightarrow \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} = 36 - 16 = 20 \quad \text{— (ii)}$$

put (ii) in (i);

$$\frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3\cdot\frac{a}{b}\cdot\frac{b}{c}\cdot\frac{c}{a}$$

$$= \left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a}\right)\left(\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} - \left(\frac{a}{c} + \frac{b}{a} + \frac{c}{b}\right)\right)$$

$$= 6 \times (20 - 8)$$

$$= 6 \times 12 = 72. \text{ (Ans.)}$$

If $a, b, c \in R$ and $a, b, c \neq 0$ such that $\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 6$ and $\frac{b}{a} + \frac{c}{b} + \frac{a}{c} = 8$, then $\frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3 = ?$

TAH 09

जय श्री राम

$$\frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3 = \left(\frac{a}{b} + \frac{b}{c} + \frac{c}{a}\right)\left(\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} - \left(\frac{a}{c} + \frac{b}{a} + \frac{c}{b}\right)\right)$$

$$\frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3 = 6 \cdot \left(\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} - (8)\right) \quad \text{—— (1)}$$

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} = 6$$

S.B.S

$$\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} + 2 \cdot \frac{a}{c} + 2\frac{b}{a} + 2\frac{c}{b} = 36$$

$$\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} + 2\left(\frac{a}{c} + \frac{b}{a} + \frac{c}{b}\right) = 36$$

$$\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} + 2(8) = 36$$

$$\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} = 36 - 16 = 20 \quad \text{—— (2)}$$

$$\therefore \frac{a^3}{b^3} + \frac{b^3}{c^3} + \frac{c^3}{a^3} - 3 = 6(20 - 8) = 6 \times 12 = 72$$

Ans

QUESTION

TAH 10

Let $a, b, c \in N (a > b)$ satisfy $c^2 - a^2 - b^2 = 101$ with $ab = 72$. Then which of the following can be correct?

(A) b and c are coprime

(B) c is an odd prime

(C) $(a + b + c)$ is even

(D) $a + b = c + 1$

Ans. A, D

Let $a, b, c \in N$ $(a>b)$ satisfy $c^2 - a^2 - b^2 = 101$ with $ab = 72$. Then which of the following can be correct?

(a) b and c are prime.
(b) c is an odd prime.
(c) $(a+b+c)$ is even.
(d) $a+b = c+1$.

Soln:-

$c^2 - a^2 - b^2 = 101$ $\quad ab = 72$ —— (ii)

or, $c^2 = a^2 + b^2 + 101$ —— (i)

(i) + (ii) × 2

$c^2 = a^2 + b^2 + 101$
$144 = 2ab$

(add) $c^2 + 144 = (a+b)^2 + 101$

or, $(a+b)^2 - c^2 = 144 - 101$

or, $(a+b+c)(a+b-c) = 43$ $\quad$ 43×1

i.e. $a+b+c = 43$
$a+b-c = 1$

(subtract) $2c = 42$
or, $c = 21$

but $(a+b-c) \not> 43$, since it has to be the smaller one.

∴ $a+b+21 = 43$
or, $a+b = 43 - 21$
or $a+b = 22$

also $ab = 72$
or, $a = \frac{72}{b}$.

now,
$a+b = 22$
or, $\frac{72}{b} + b = 22$
or, $72 + b^2 = 22b$
or, $b^2 - 22b + 72 = 0$
or, $b^2 - 18b - 4b + 72 = 0$
or, $b(b-18) - 4(b-18) = 0$
or, $(b-18)(b-4) = 0$

$2 \mid 72$, $2 \mid 36$, $3 \mid 6$, 2
∵ $6 \times 3 = 18$
∵ $2 \times 2 = 4$
& $18 + 4 = 22$

∴ $b - 18 = 0$
or, $b = 18$
then, $a = \frac{72}{18}$
or, $a = 4$
not accepted ∵ $a > b$

or, $b - 4 = 0$
or, $b = 4$
then, $a = \frac{72}{4}$
or, $a = 18$
accepted

∴ $a = 18$ & $b = 4$.

∴ $a = 18$, $b = 4$, $c = 21$.

(a) → b, c are co prime. ✓ (HCF = 1)
(b) → c is an odd prime ✗ $(c = 21 = 3 \times 7)$
(c) → $(a+b+c)$ is even. ✗ [∵ $a+b+c = 43 \Rightarrow$ odd]
(d) → $a+b = c+1$ ✓ [∵ $a+b = 22 = c+1$]

∴ Ans ⇒ (a) & (d).

TAH 10
by Reed
West Bengal

Solution to Previous KTKs

QUESTION

(KTK 1)

If a, b, & c are three non zero real numbers such that $5a^2 + 4b^2 + 4c^2 - 8ab - 4ac = 0$ then the value of $a/b + b/c$ is ______

Ans. 3

1×-2

Lecture 6

Write on White
Date
Page

(KTK1) If a, b & c are three non zero real numbers such that $5a^2 + 4b^2 + 4c^2 - 8ab - 4ac = 0$ then the value of $a/b + b/c$ is ___

$5a^2 + 4b^2 + 4c^2 - 8ab - 4ac = 0$

$4a^2 + a^2 + 4b^2 + 4c^2 - 8ab - 4ac = 0$

$\{(2a)^2 + (2b)^2 - 2(2a)(2b)\} + \{a^2 + (2c)^2 - 2 \cdot a \cdot (2c)\} = 0$

$\underbrace{(2a - 2b)^2}_{\geq 0} + \underbrace{(a - 2c)^2}_{\geq 0} = 0$

$2a = 2b$ $\qquad$ $a - 2c = 0$

$\frac{a}{b} = 1$ $\qquad$ $a = 2c$

$\frac{2c}{b} = 1$

$\frac{b}{c} = 2$

$\frac{a}{b} + \frac{b}{c} \Rightarrow 1 + 2 = 3$ ✓

1. $a, b, c \in R - \{0\}$

$5a^2 + 4b^2 + 4c^2 - 8ab - 4ac = 0$ $\quad \frac{a}{b} + \frac{b}{c} = ?$

$4a^2 + 4b^2 - 8ab + a^2 + 4c^2 - 4ac = 0$

$(2a)^2 + (2b)^2 - 2(2a)(2b) + a^2 + (2c)^2 - 2 \cdot a \cdot 2c = 0$

$\underbrace{(2a - 2b)^2}_{\geq 0} + \underbrace{(a - 2c)^2}_{\geq 0} = 0$

$\therefore \cancel{2}a = \cancel{2}b \qquad a = 2c$

$a = b \qquad \frac{a}{2} = c \quad \text{or} \quad \frac{a}{c} = 2$

$\frac{a}{b} + \frac{b}{c} = \frac{1}{1} + \frac{a}{c} = 1 + 2 = 3$

QUESTION (KTK 2)

If a, b, & c are three non zero real numbers such that $2a^2 + b^2 + c^2 - 2ab - 2ac = 0$ then the value of $\frac{a+b}{c}$ is equal to ______

Ans. 2

2) . $2a^2 + b^2 + c^2 - 2ab - 2ac = 0$

$a^2 + b^2 - 2ab + a^2 + c^2 - 2ac = 0$

$(a-b)^2 + (a-c)^2 = 0$

$\therefore \ a = b \quad a = c$

$\frac{a+b}{c} = \frac{a+a}{a} = \frac{2a}{a} = 2$ &

KTK-2

$a, b, c \in R_0$

$2a^2 + b^2 + c^2 - 2ab - 2ac = 0$, $\dfrac{a+b}{c} = ?$

$a^2 - 2ab + b^2 + a^2 - 2ac + c^2 = 0$

$(a-b)^2 + (a-c)^2 = 0$

$\geq \quad \geq 0$

$(a-b)^2 = 0 \quad (a-c)^2 = 0$

$a - b = 0 \quad a - c = 0$

$a = b \quad a = c$

$\boxed{a = b = c}$

fnf :- $\dfrac{a+a}{a} \Rightarrow \dfrac{2a}{a} \Rightarrow 2.$

QUESTION

(KTK 3)

If $x^2 + 16y^2 + 9z^2 = 4xy + 12yz + 3zx$ then find the value of $\frac{x+4y}{3z}$. (Given $x, y, z \in R_0$)

Ans. 2

KTK 8

$x^2 + 16y^2 + 9z^2 = 4xy + 12yz + 3zx$, $x, y, z \in R_0$, $\frac{x+4y}{3z} = ?$

$\Rightarrow$ $\cancel{x^2 - 2 \cdot 2x} y$

$x^2 + (4y)^2 + (3z)^2 - x \cdot 4y - 4y \cdot 3z - 3z \cdot x = 0$

It is posissble $\Longleftrightarrow$ $x = 4y = 3z = 12\lambda$

$\boxed{x = 12\lambda}$

$4y = 12\lambda \Rightarrow \boxed{y = 3\lambda}$

$3z = 12\lambda$

$\boxed{z = 4\lambda}$

$\frac{x+4y}{3z}$

$\Rightarrow \frac{12\lambda + 12\lambda}{12\lambda}$

$\Rightarrow \frac{24\lambda}{12\lambda}$

$\Rightarrow 2.$ fnf Q.E.D

KTK-3: If $x^2+16y^2+9z^2 = 4xy+12yz+3zx$ then find $\frac{x+4y}{3z} = ?$ $(x, y, z \in R_0)$

Soln:

$$x^2+16y^2+9z^2 = 4xy+12yz+3zx$$

$$\Rightarrow x^2+(4y)^2+(3z)^2 - x\cdot 4y - 4y\cdot 3z - x\cdot 3z = 0.$$

$$a^2 + b^2 + c^2 - ab - bc - ca = 0.$$

$$\Downarrow$$

$$\boxed{x = 4y = 3z}$$

$$\therefore \frac{x+4y}{3z} = \frac{x+x}{x} = \frac{2x}{x} = 2. \text{ (Ans)}$$

KTK 3,4
by Reed
west bengal

QUESTION

(KTK 4)

If the real numbers x, y, z are such that
$x^2 + 4y^2 + 16z^2 = 48$ and $xy + 4yz + 2zx = 24$,
what is the value of $x^2 + y^2 + z^2$?

Ans. 21

If the real no.s x, y, z are such that

$x^2 + 4y^2 + 16z^2 = 48$ & $xy + 4yz + 2zx = 24$.

then $x^2+y^2+z^2 = ?$

Solⁿ: $x^2 + 4y^2 + 16z^2 = 48$ — (i) $\quad xy + 4yz + 2zx = 24$ — (ii)

(i) − (2 × (ii)):

$x^2 + 4y^2 + 16z^2 - 2(xy + 4yz + 2zx) = 48 - (2 \times 24)$

$\Rightarrow x^2 + (2y)^2 + (4z)^2 - x \cdot 2y - 2y \cdot 2z - 4z \cdot x = 0.$

$a^2 + b^2 + c^2 - ab - bc - ca = 0.$

$\Downarrow$

$\boxed{x = 2y = 4z}$

put in eqnⁿ (i):

$x^2 + 4y^2 + 16z^2 = 48$

$\Rightarrow x^2 + (2y)^2 + (4z)^2 = 48$

$\Rightarrow x^2 + x^2 + x^2 = 48$

$\Rightarrow 3x^2 = 48$

$\Rightarrow x^2 = \frac{48}{3} = 16$

now

$x^2+y^2+z^2$

$= x^2 + (\frac{x}{2})^2 + (\frac{x}{4})^2$

$= x^2 + \frac{x^2}{4} + \frac{x^2}{16}$

$= \frac{16x^2 + 4x^2 + x^2}{16}$

$= \frac{21x^2}{16}$

$= \frac{21 \times 16}{16} = 21$ (Ans)

Q If the real numbers x, y, z are such that
$x^2 + 4y^2 + 16z^2 = 48$ and $xy + 4yz + 2zx = 24$
what is the value of $x^2 + y^2 + z^2$?

KTK-04

Sol

$$x^2 + 4y^2 + 16z^2 = 2(xy + 4yz + 2zx)$$

$$x^2 + 4y^2 + 16z^2 - 2xy - 8yz - 4zx = 0.$$

$$x^2 + (2y)^2 + (4z)^2 - x.2y - 2y.4z - 4z.x = 0$$

$\therefore$ $\boxed{x = 2y = 4z}$

$$(2y)^2 + 4y^2 + 16.\left(\frac{y}{2}\right)^2 = 48$$

$$4y^2 + 4y^2 + 8y^2 = 48$$

$$12y^2 = 48$$

$$y^2 = \frac{48}{12} = 4$$

$$\boxed{y^2 = 4}$$

$$x^2 + 4.4 + 16.\frac{4}{4} = 48$$

$$x^2 + 16 + 16 = 48$$

$$x^2 + 32 = 48$$

$$x^2 = 48 - 32 = 16$$

$$\boxed{x^2 = 16}$$

$$16 + 4(4) + 16z^2 = 48$$

$$16 + 16 + 16z^2 = 48$$

$$16z^2 = 48 - 32$$

$$16z^2 = 16$$

$\therefore$ $x^2 + y^2 + z^2$
$= 16 + 4 + 1 = 21$

QUESTION

(KTK 5)

If x, y, z are real numbers then find the minimum value of
$4x^2 + y^2 + 9z^2 - 4x - 2y - 6z + 17$.

Q. If x, y, z are real numbers then find the minimum value of

KTK-05

$$4x^2 + y^2 + 9z^2 - 4x - 2y - 6z + 17.$$

$$(2x)^2 + y^2 + (3z)^2 - 4x - 2y - 6z + 17.$$

$$(2x-1)^2 + (y-1)^2 + (3z-1)^2 + 17 - 3.$$

जय श्री राम

$$\begin{matrix} (2x-1)^2 & (y-1)^2 & (3z-1)^2 \\ \Downarrow & \Downarrow & \Downarrow \\ 0 & 0 & 0 \end{matrix}$$

for minimum value

$\therefore$ minimum value $= 14$ Ans

KTK-51 If x, y, z are real numbers then find the min. value of $4x^2 + y^2 + 9z^2 - 4x - 2y - 6z + 17$.

Sol^n $4x^2 + y^2 + 9z^2 - 4x - 2y - 6z + 17 = E$

$$E = (2x)^2 + y^2 + (3z)^2 - 2\cdot 2x\cdot 1 - 2\cdot y\cdot 1 - 2\cdot 3z\cdot 1 + 17$$

$$E = [(2x)^2 - 2\cdot 2x\cdot 1 + 1] + [(3z)^2 - 2\cdot 3z\cdot 1 + 1] + [y^2 - 2y + 1] + 17 - 3$$

$$E = \underbrace{(2x-1)^2}_{\geq 0} + \underbrace{(3z-1)^2}_{\geq 0} + \underbrace{(y-1)^2}_{\geq 0} + 14$$

KTK 5
by Reed
west bengal

$\therefore E|_{min} = 0 + 0 + 0 + 14 = 14$. (Ans.)


THANK
YOU