

PRAAYAS

JEE 2026

ATDB.uno

Mathematics

Basic Maths

Lecture - 08

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Topics *To be covered*



- A** Method of Intervals/ Wavy Curve Method
- B** Problem Practice

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Homework Discussion

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QUESTION [IIT-JEE 1980]

(Home Challenge-01)

$$\begin{aligned}\sqrt{x+y}^3 &= \left((x+y)^{\frac{1}{2}}\right)^3 \\ &= (x+y)^{3/2}\end{aligned}$$



If $x > y > 0$, then show that the expression $\left(\sqrt{2} \left(2x + \sqrt{x^2 - y^2}\right) \left(\sqrt{x - \sqrt{x^2 - y^2}}\right)\right)$ can be simplified to $\sqrt{(x+y)^3} - \sqrt{(x-y)^3}$.

$$E = (2x + \sqrt{x-y} \cdot \sqrt{x+y}) \left(\sqrt{2x - 2\sqrt{x^2 - y^2}}\right)$$

$$= (2x + \sqrt{x-y} \cdot \sqrt{x+y}) \sqrt{(x+y) - 2\sqrt{x-y}\sqrt{x+y}}$$

$$= (x+y + x-y + \sqrt{x-y}\sqrt{x+y}) \left(\sqrt{(\sqrt{x+y} - \sqrt{x-y})^2}\right)$$

$$= |\sqrt{x+y} - \sqrt{x-y}| (x+y + x-y + \sqrt{x+y} \cdot \sqrt{x-y})$$

$$= (\sqrt{x+y} - \sqrt{x-y}) (\sqrt{x+y}^2 + \sqrt{x-y}^2 + \sqrt{x+y} \cdot \sqrt{x-y}) = (\sqrt{x+y})^3 - (\sqrt{x-y})^3 = \sqrt{(x+y)^3} - \sqrt{(x-y)^3}$$

$$a^3 - b^3$$

QUESTION

(KTK 3)



(ADBST)

Given $3x^2 + x = 1$, then the value of $6x^3 - x^2 - 3x$ is equal to

- A -1
- B 0
- C 1
- D 2

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Ans. D

QUESTION

(KTK 7)



Show that the square of $\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$ is a rational number.

$$V = \frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{\sqrt{5^2 - 2 \cdot 5 \cdot 3\sqrt{3}}}{10 - \sqrt{76 + 2 \cdot 5 \cdot \sqrt{3}}} \cdot \frac{\sqrt{5^2 + (3\sqrt{3})^2 - 2 \cdot 5 \cdot 3\sqrt{3}}}{10 - \sqrt{(5\sqrt{3})^2 + 1 + 2 \cdot 5\sqrt{3} \cdot 1}}$$

$$= \frac{|5 - 3\sqrt{3}|}{10 - |1 + 5\sqrt{3}|}$$

$$= \frac{3\sqrt{3} - 5}{9 - 5\sqrt{3}} = \frac{3\sqrt{3} - 5}{\sqrt{3}(3\sqrt{3} - 5)} = \frac{1}{\sqrt{3}} \rightarrow V^2 = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3} \in \mathbb{Q}$$



Aao Machaay Dhamaal Deh Swaal pe Deh Swaal

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QUESTION



$$\text{Solve: } \frac{x^2 - 5x + 12}{x^2 - 4x + 5} > 3$$

$$D = (-4)^2 - 4 \cdot 1 \cdot 5 < 0$$

$a = 1, D < 0 \Rightarrow$ Denominator is always +ve

$$x^2 - 5x + 12 > 3x^2 - 12x + 15$$

$$2x^2 - 7x + 3 < 0$$

$$2x^2 - 6x - x + 3 < 0$$

$$2x(x-3) - 1(x-3) < 0$$

$$(2x-1)(x-3) < 0$$

$$\begin{array}{c} + \quad | \quad - \quad | \quad + \\ \hline \quad \quad \frac{1}{2} \quad \quad 3 \end{array}$$

$$x \in \left(\frac{1}{2}, 3\right)$$

Lal: $\frac{x^2 - 5x + 12}{x^2 - 4x + 5} - 3 > 0$

$$\frac{x^2 - 5x + 12 - 3x^2 + 12x - 15}{x^2 - 4x + 5} > 0$$

$$\frac{-2x^2 + 7x - 3}{x^2 - 4x + 5} > 0$$

$\rightarrow D < 0, a > 0$ always +ve

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$$-2x^2 + 7x - 3 > 0$$

$$2x^2 - 7x + 3 < 0$$

$$(2x-1)(x-3) < 0$$

$$x \in \left(\frac{1}{2}, 3\right)$$

QUESTION



Solve: $\frac{x+1}{x-1} \geq \frac{x+5}{x+1}$

$$\frac{x+1}{x-1} - \frac{x+5}{x+1} \geq 0$$

$$\frac{x^2 + 2x + 1 - (x^2 + 4x - 5)}{(x-1)(x+1)} \geq 0$$

$$\frac{-2x + 6}{(x-1)(x+1)} \geq 0$$

$$\frac{-2(x-3)}{(x-1)(x+1)} \geq 0$$

$$\frac{x-3}{(x-1)(x+1)} \leq 0$$

$$\frac{-1+1-1+1}{-1 \quad 1 \quad 3}$$

$$x \in (-\infty, -1) \cup (1, 3]$$

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QUESTION



Solve: $x^2 - 4x + 3 \geq 0$ and $x^2 - 4 \leq 0$

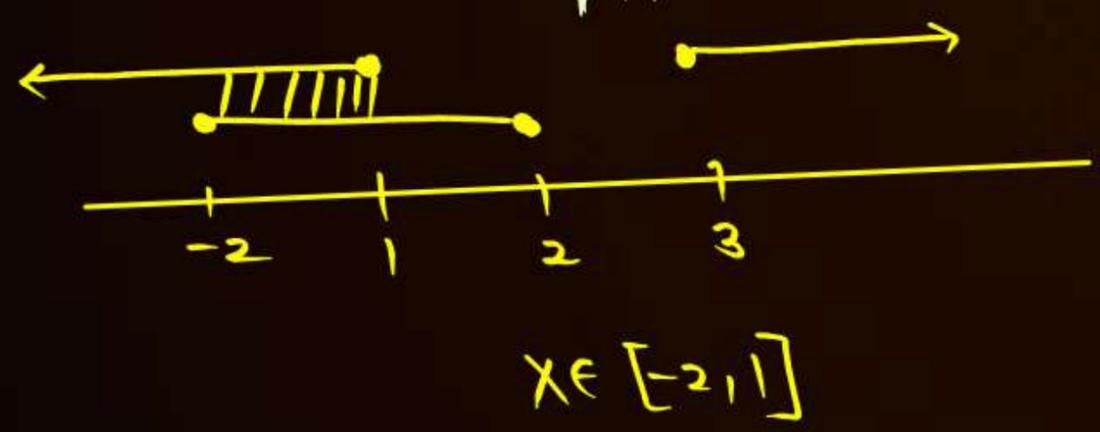
Chmatkaari Baba Na Bame

$(x-1)(x-3) \geq 0$ $(x-2)(x+2) \leq 0$

 $x \in (-\infty, 1] \cup [3, \infty)$ $x \in [-2, 2]$

$x^2 \leq 4$
 $x \leq \pm 2$

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Set A ke saaray element agar set B mai ho tab A, B kaa subset kehlaa hai
 $A \subseteq B$

QUESTION



$$\text{Solve: } (x - 1) (x^2 + 4x + 1) (x + 2) \leq 0$$

$$D = 16 - 4 = 12 > 0$$

$$\alpha, \beta = \frac{-4 \pm \sqrt{12}}{2} = -2 \pm \sqrt{3}$$

$$(x - 1) (x - (-2 + \sqrt{3})) (x - (-2 - \sqrt{3})) (x + 2) \leq 0$$

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Tah 01

QUESTION

Solve: $(x^2 - x - 6)(x^2 + 6x) \geq 0$

Tah02

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QUESTION

Solve: $x^2 - 5x + 6 \geq 0$ and $x^2 - 10x + 24 \leq 0$

Tah03

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QUESTION



$$\frac{x^2 - x - 6}{x^2 + 6x} \geq 0$$

Tahoy

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QUESTION



$$\text{Solve : } (x^2 + 3x + 1)(x^2 + 3x - 3) \geq 5$$

Tah 05

$$\text{let } x^2 + 3x = t$$

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QUESTION



Find Exhaustive set of values of x satisfying :

(i) $x^3 - 3x^2 - x + 3 > 0$ — $x^2(x-3) - 1(x-3) > 0$

(ii) $x^4 - 3x^3 - x + 3 < 0$ — $(x^2-1)(x-3) > 0$

(iii) $x^4 + 6x^3 + 6x^2 + 6x + 5 \leq 0$ — $(x-1)(x+1)(x-3) > 0$



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$$x \in (-1, 1) \cup (3, \infty)$$

Tahos

$(3, \infty)$ ✓

$(-1, 1)$ ✓

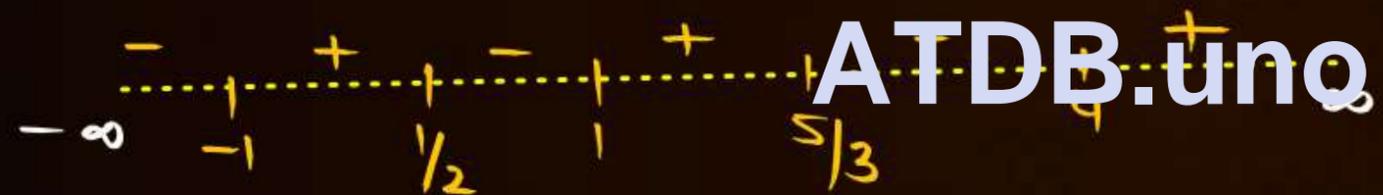
Exhaustive set of soln
||
complete set of soln.

QUESTION



$$\text{Solve: } \frac{(x+1)(1-2x)(x-1)}{(5-3x)(x-4)} \leq 0$$

$$\frac{(x+1) \cdot (-1) \cdot (2x-1)(x-1)}{(-1) \cdot (3x-5)(x-4)} \leq 0$$

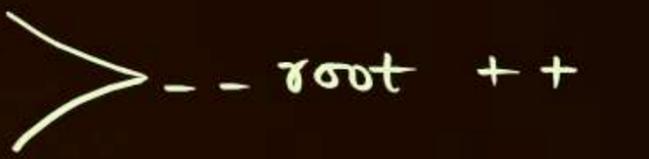


$$x \in (-\infty, -1] \cup \left[\frac{1}{2}, 1\right] \cup \left(\frac{5}{3}, 4\right)$$

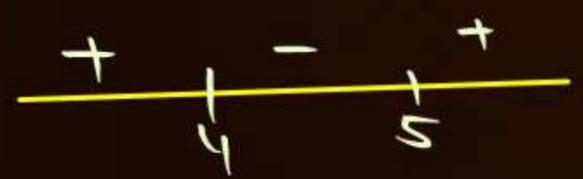


Why does method of Interval work?

$P(x) = x - 5 \rightarrow \text{root} = 5$



Ex: $(x-5)(x-4)$



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	F_1	F_2
5 ke aagay	+	+
4 se 5 ke Bich	-	+
4 se peechay	-	-



★ Every linear with odd power > 1 behaves like as if had power 1

$$\text{Ex: } (x-3)^3, \quad x-3, \quad (x-3)^{15}, \quad (x-3)^{101}$$

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3

★ (Linear in x)^{Even} ≥ 0

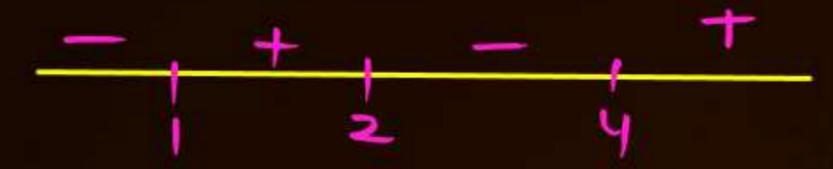


Case of Repeated Factors

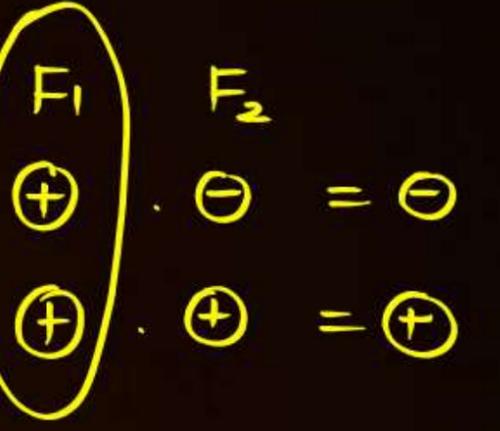


Treat as if power is 1

Ex: $(x-1)(x-2)^3(x-4)^5 \geq 0$

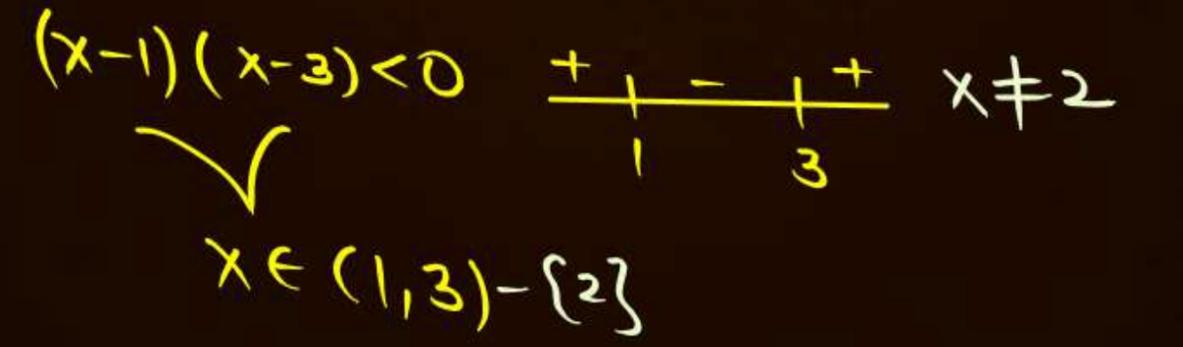


$x \in [1, 2] \cup [4, \infty)$



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Ex: $(x-1)(x-2)^2(x-3) < 0$





Kaam Ki Baat



B_1 : Every odd integral power of a linear factor is treated as 1.



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Kaam Ki Baat



B₂ : In case of even power of any factor, first we assume that it is always positive. So we delete it from the inequality but in the end we make a direct check at that value of x where the deleted factor is zero.



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QUESTION



Solve: $(x+1)(x-3)(x-2)^2 \geq 0$

Noble factor

$(x+1)(x-3) \geq 0$ — $x=2$ is also possible.



$x \in (-\infty, -1] \cup [3, \infty) \cup \{2\}$

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$0 \geq 0 \rightarrow \text{True}$

$a > b$ kaa matlab

$a > b$ or $a = b$

Dono mai se koi
kum se kum koi
Ek sahi ho jayay
toh $a > b$ ko sahi
kehte hai

Ex: $2 \geq 1$ (T)

Ex: $3 \geq 3$ (T)

QUESTION

Solve: $(x + 1)(x - 3)(x - 2)^2 \geq 0$

Noble factor



$$x \in (-\infty, -1] \cup [3, \infty) \cup \{2\}$$

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$\cup = \text{or}$
 $\cap = \text{and}$

$$0 \geq 0 \rightarrow \text{True}$$

$a > b$ kaa matlab

$$a > b \text{ or } a = b$$

Dono mai se koi
kum se kum koi
sahi ho jayay
toh $a > b$ ko sahi
kehte hai

Ex: $2 > 1$ (T)

Ex: $3 > 3$ (T)



QUESTION



Solve: $(x + 4)^5 (x + 3)^6 (x + 2)^7 (x - 1)^8 \leq 0$

$$x + 3 = 0$$

$$x = -3$$

$$x - 1 = 0$$

$$x = 1$$

M ①

$(x + 4)^5 (x + 2)^7 \leq 0$, $x = -3, 1$ are also possible



$$x \in [-4, -2] \cup \{1\}$$

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M ②



$$x \in [-4, -2] \cup \{1\}$$

QUESTION



Number of positive integral solution of $\frac{x^3(2x-3)^2(x-4)^6}{(x-3)^2(3x-8)} \leq 0$

$$\frac{x^3}{3x-8} \leq 0, \quad x = \frac{3}{2}, 4 \text{ are also possible}$$

Already present
x=3 is N.P
Not in Answer

$$\begin{array}{c} + \quad - \quad + \\ \hline 0 \quad 8/3 \end{array}$$

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$$x \in [0, 8/3) \cup \{4\}$$

Integer $x = 0, 1, 2, 4$
values of

No. of Integral values = 4 = Number of non negative Integral values.
No. of positive Integral values = 3

QUESTION

Solve: $(x - 1)^2 (x + 1)^3 (x - 4) < 0$

Tahot

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QUESTION



The complete solution set of inequality $\frac{(x-5)^{1005}(x+8)^{1008}(x-1)}{x^{1006}(x-2)^3(x-3)^5(x-6)(x+9)^{1010}} \leq 0$ is

A $(-\infty, -9) \cup (-8, 0) \cup (0, 1) \cup (2, 3) \cup [5, 6)$ $\frac{(x-5)^{1005}(x-1)}{(x-2)^3(x-3)^5(x-6)} \leq 0$, $x = -8$ is also possible
 $x = 0, -9$ are not possible.

B $(-\infty, -9) \cup (-9, 0) \cup (0, 1) \cup (2, 3) \cup (5, 6)$

C $(-\infty, -9) \cup (-9, 0) \cup (0, 1] \cup (2, 3) \cup [5, 6)$

D $(-\infty, 0) \cup (0, 1] \cup (2, 3) \cup [5, 6)$

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$x \in (-\infty, 1] \cup (2, 3) \cup [5, 6) - \{0, -9\}$

OR

$x \in (-\infty, -9) \cup (-9, 0) \cup (0, 1] \cup (2, 3) \cup [5, 6)$
 $\rightarrow -8$ is here

QUESTION



Solve: $\frac{(x-1)^2(x+1)^3}{x^4(x-2)} \leq 0$



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QUESTION



Find the exhaustive solutions set of

$$\frac{(2x - 5)^{100}(x + 3)(2x + 1)^{101}}{(x^2 - 4)^{151}(3x - 4)^{197}} < 0$$

Tan09

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$$\text{Ans. } (-\infty, -3) \cup \left(-2, -\frac{1}{2}\right) \cup \left(\frac{4}{3}, 2\right)$$



If a Factor is Eliminated

The factor which is eliminated from Numerator & Den. its root is never a part of answer

$$\frac{(x-1)(x-2)(x-3)}{(x^2-4)} < 0$$

$$\frac{(x-1)\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x+2)} < 0$$

$$\frac{(x-1)(x-3)}{x+2} < 0, x \neq 2$$



$$x \in (-\infty, -2) \cup (1, 3) - \{2\}$$

OR

$$x \in (-\infty, -2) \cup (1, 2) \cup (2, 3)$$



QUESTION



Solve for x :
$$\frac{(x^2 - 8x + 7)(x^2 - 5x + 4)(x - 3)^2}{(x^2 - 3x + 2)} \leq 0.$$

$$\frac{(x-1)(x-7)(x-1)(x-4)(x-3)^2}{(x-1)(x-2)} \leq 0$$

$$\frac{(x-7)(x-1)(x-4)}{x-2} \leq 0$$

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 $x=3$ is also possible
 $x \neq 1$



$$x \in (1, 2) \cup [4, 7] \cup \{3\}$$

QUESTION

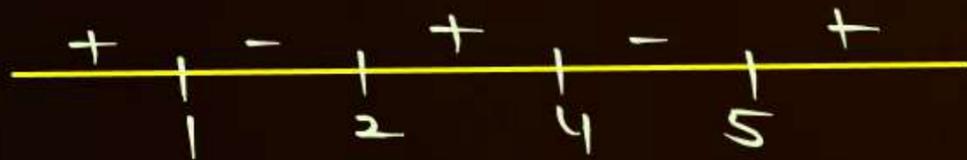


Solve for x: $\frac{(x-3)^3(x-1)(x-2)}{(x-4)(x-3)(x-5)} \geq 0$

$$\frac{(x-3)^2(x-1)(x-2)}{(x-4)(x-5)} \geq 0, x \neq 3$$

$$\frac{(x-1)(x-2)}{(x-4)(x-5)} \geq 0, x \neq 3$$

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$$x \in (-\infty, 1] \cup [2, 4) \cup (5, \infty) - \{3\}$$

OR

$$x \in (-\infty, 1] \cup [2, 3) \cup (3, 4) \cup (5, \infty)$$

QUESTION



Solve:
$$\frac{(x^2 - 6x + 8)^3 (x^2 - 7x + 6)^2}{(x^2 - 5x + 6)(x - 1)^3} \leq 0$$

$$x^2 - 7x + 6 = (x - 1)(x - 6)$$

$$\frac{(x - 2)^3 (x - 4)^3 (x - 1)^2 (x - 6)^2}{(x - 2)(x - 3)(x - 1)^3} \leq 0$$

$$\frac{(x - 2)^2 (x - 4)^3 (x - 6)^2}{(x - 3)(x - 1)} \leq 0 \quad x \neq 2, 1$$

$x = 6$ is possible.

$$\frac{(x - 4)^3}{(x - 3)(x - 1)} \leq 0$$

$$\begin{array}{ccccccc} & - & & + & & - & & + \\ & | & & | & & | & & | \\ \hline & 1 & & 3 & & 4 & & \end{array}$$

$$x \in (-\infty, 1) \cup (3, 4] \cup \{6\}$$

QUESTION



Solve: $\frac{(x^2 - 4x + 5)^2 (x - 3)^2 (x + 1)^3}{(x - 1)(x - 5)^3 (x^2 - 7x + 12)} > 0$

Tahio

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QUESTION



Find the exhaustive solutions set of $\frac{(x-4)(2x-5)^{27}(x^2-9)^{10}(x+4)^{93}}{(x^2-25)(x+3)^{91}(x^2+10)^5} > 0$. *Tah!!*

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$$\text{Ans. } (-\infty, -5) \cup (-4, -3) \cup \left(\frac{5}{2}, 3\right) \cup (3, 4) \cup (5, \infty)$$



Double Inequality



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QUESTION



Solve the system of in equations $-5 \leq \frac{2-3x}{4} \leq 9$

$$-20 \leq 2-3x \leq 36$$

$$-22 \leq -3x \leq 34$$

$$\frac{-22}{-3} \geq x \geq \frac{34}{-3}$$

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$$-\frac{34}{3} \leq x \leq \frac{22}{3}$$

QUESTION



Solve the system of in equations $2x - 1 > x + \frac{7-x}{3} > 2, x \in \mathbb{R}$

Tahiz

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QUESTION



Solve : $1 < \frac{3x^2 - 7x + 8}{x^2 + 1} \leq 2$

$$x^2 + 1 = x^2 + 0x + 1$$

$$D = 0^2 - 4 \cdot 1 \cdot 1 < 0$$

$$a = 1 > 0, D < 0$$

always +ve

$$x^2 + 1 < 3x^2 - 7x + 8 \leq 2x^2 + 2$$

$$2x^2 - 7x + 7 > 0$$

$$D = (-7)^2 - 4 \cdot 2 \cdot 7 < 0$$

$$a = 2 > 0$$

∴

$$x \in \mathbb{R}$$

$$x^2 - 7x + 6 \leq 0$$

$$(x-1)(x-6) \leq 0$$

$$x \in [1, 6]$$

$$x \in [1, 6]$$

$$g(x) \leq f(x) \leq h(x)$$



Ans.

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QUESTION

Solve following double inequalities :



(i) $-3 < \frac{2x-7}{5} \leq 8$

(ii) $x^2 + 2 \leq 3x < 2x^2 - 5$

(iii) $-2 < \frac{x-5}{2x+1} < 5$

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Solution to Previous TAH

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QUESTION



The square root $5 + 2\sqrt{6}$ is -

A $\sqrt{3} + 2$

B $\sqrt{3} - \sqrt{2}$

C $\sqrt{2} - \sqrt{3}$

D $\sqrt{3} + \sqrt{2}$

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The square root of $5 + 2\sqrt{6}$ is :- Tah-01

$$\sqrt{5 + 2\sqrt{6}} = \sqrt{(\sqrt{3})^2 + (\sqrt{2})^2 + 2\sqrt{3}\sqrt{2}}$$

$$= \sqrt{(\sqrt{3} + \sqrt{2})^2} = \sqrt{3} + \sqrt{2} \text{ Ans}$$



Tah-01.

$$\sqrt{5+2\sqrt{6}} = \sqrt{(1)^2 + (\sqrt{6})^2 + 2 \cdot 1 \cdot \sqrt{6}}$$

$$= \sqrt{(1+\sqrt{6})^2}$$

$$= |1+\sqrt{6}|$$

$$= 1+\sqrt{6} \text{ Ans.}$$

LECTURE - 07

TAH-01

Q) The square root $5 + 2\sqrt{6}$ is :-

(A) $\sqrt{3} + 2$

(B) $\sqrt{3} - \sqrt{2}$

(C) $\sqrt{2} - \sqrt{3}$

(D) $\sqrt{3} + \sqrt{2}$

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soln

$$\sqrt{5 + 2\sqrt{6}}$$

$$= \sqrt{3 + 2 + 2\sqrt{3}\sqrt{2}}$$

$$= \sqrt{(\sqrt{3} + \sqrt{2})^2} = |\sqrt{3} + \sqrt{2}|$$

$$= \sqrt{3} + \sqrt{2} \text{ ans}$$

OPTION-D



QUESTION [IIT-JEE 1980]

The expression $\frac{12}{3+\sqrt{5}+2\sqrt{2}}$ is equal to

A $1 - \sqrt{5} + \sqrt{2} + \sqrt{10}$

B $1 + \sqrt{5} + \sqrt{2} - \sqrt{10}$

C $1 + \sqrt{5} - \sqrt{2} + \sqrt{10}$

D $1 - \sqrt{5} - \sqrt{2} + \sqrt{10}$

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Tah-02.

$$\frac{12}{3 + \sqrt{5} + 2\sqrt{2}} = \frac{12}{(3 + \sqrt{5}) + 2\sqrt{2}} \times \frac{(3 + \sqrt{5}) - 2\sqrt{2}}{(3 + \sqrt{5}) - 2\sqrt{2}}$$

$$= \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{(3 + \sqrt{5})^2 - (2\sqrt{2})^2} = \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{14 + 6\sqrt{5} - 8}$$

$$= \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{6 + 6\sqrt{5}} = \frac{2}{1} \frac{12(3 + \sqrt{5} - 2\sqrt{2})}{6(1 + \sqrt{5})}$$

$$= \frac{6 + 2\sqrt{5} - 4\sqrt{2}}{(1 + \sqrt{5})} \times \frac{(1 - \sqrt{5})}{(1 - \sqrt{5})}$$

$$= \frac{6 - 6\sqrt{5} + 2\sqrt{5} - 10 - 4\sqrt{2} + 4\sqrt{10}}{-4}$$

$$= \frac{-4 - 4\sqrt{5} - 4\sqrt{2} + 4\sqrt{10}}{-4}$$

$$= \frac{-4(1 + \sqrt{5} + \sqrt{2} - \sqrt{10})}{-4}$$

$$= 1 + \sqrt{5} + \sqrt{2} - \sqrt{10} \quad \text{Ans.}$$

krish keshri
jharkhand

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Intersection & Union



Find $A \cup B$ & $A \cap B$

1. $A = (-3, 1]$ & $B = [-4, 0)$

2. $A = (-3, -1)$ & $B = [-1, 2]$

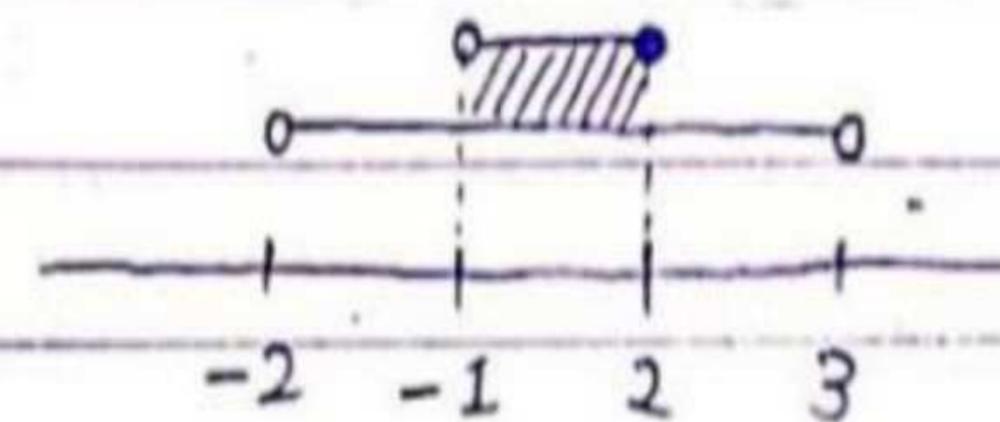
3. $A = (-2, 3)$ & $B = [-1, 2]$

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Tah-03.

(A)



$$\Rightarrow A \cap B = (-1, 2]$$

$$A = (-2, 3)$$

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$$B = (-1, 2]$$

$$\Rightarrow A \cup B = (-2, 3)$$

QUESTION



Evaluate the following

(i) $(-\infty, 3) \cap [-2, \infty)$

(ii) $(-4, 1] \cap (-3, 4)$

(iii) $(0, 5] \cap (1, \infty)$

(iv) $[0, 3) \cup [2, 6)$

(v) $[2, \infty) \cup (4, \infty)$

(vi) $[-1, 1) \cup [2, 5]$

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TAH-3B! Evaluate the following!

Solⁿ (i) $(-\infty, 3) \cap [-2, \infty)$

$\Rightarrow x \in [-2, 3)$.

(ii) $(-4, 1] \cap (-3, 4)$

$\Rightarrow x \in (-3, 1]$.

(iii) $(0, 5] \cap (1, \infty)$

$\Rightarrow x \in (1, 5]$

(iv) $[0, 3) \cup [2, 6)$

$\therefore x \in [0, 6)$

(v) $[2, \infty) \cup (4, \infty)$

$\Rightarrow x \in [2, \infty)$

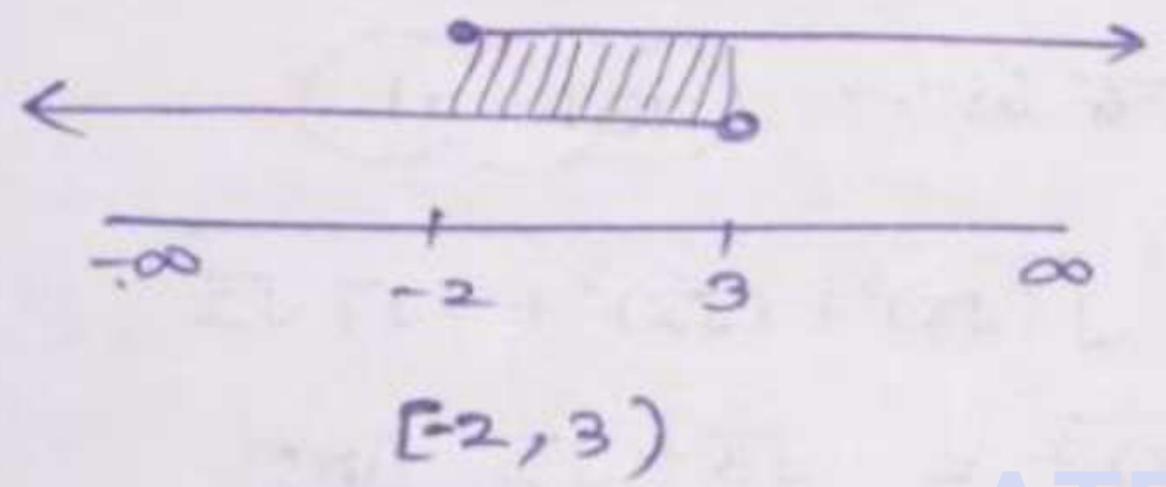
(vi) $[-1, 1) \cup [2, 5]$

$\therefore x \in [-1, 1) \cup [2, 5]$

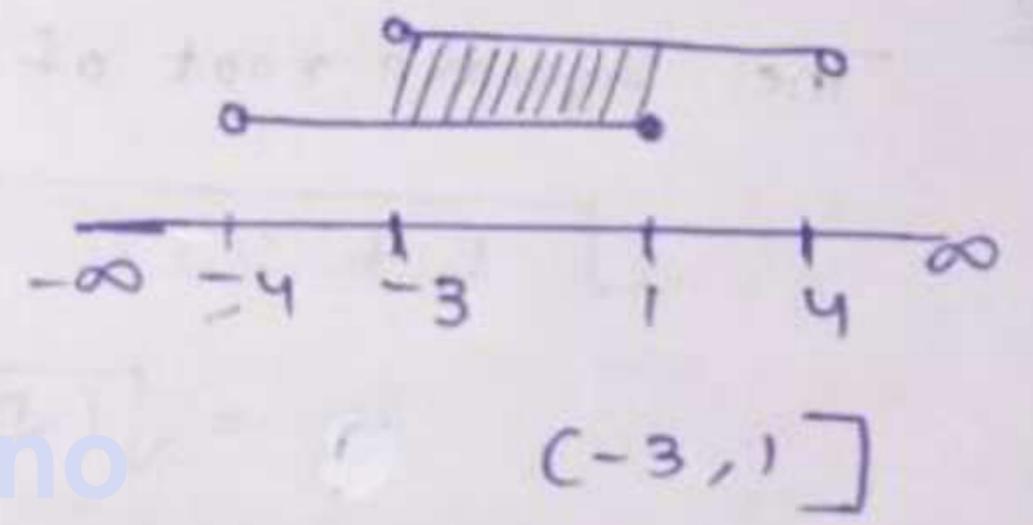


Q Evaluate the following :-

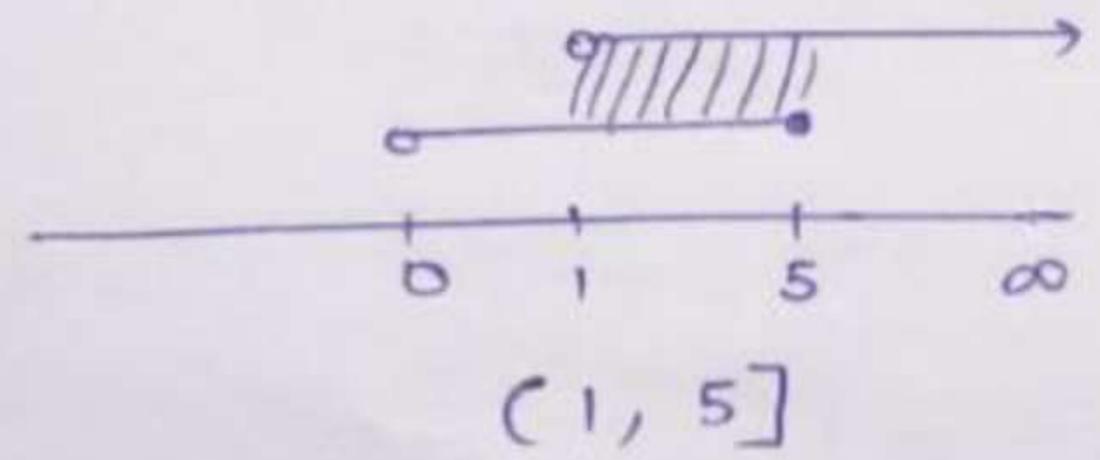
(i) $(-\infty, 3) \cap [-2, \infty)$



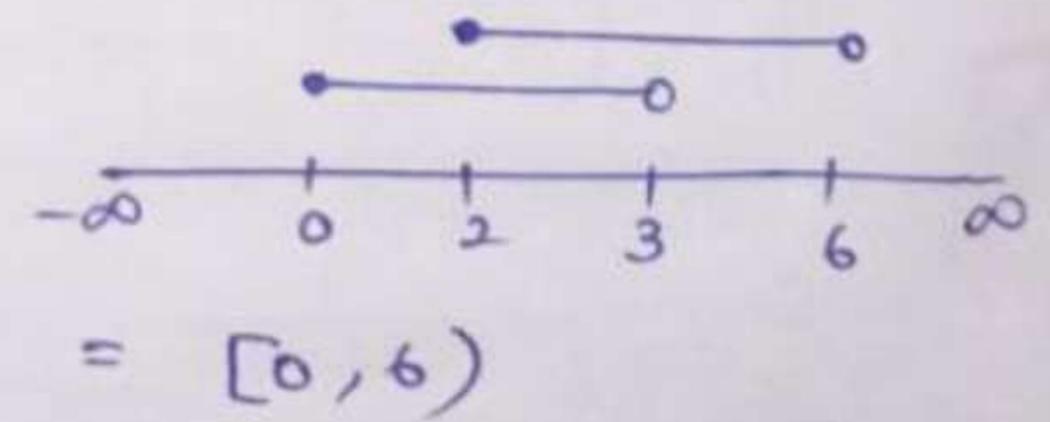
Tah-03 (B)
(ii) $(-4, 1] \cap (-3, 4)$



(iii) $(0, 5] \cap (1, \infty)$



(iv) $[0, 3) \cup [2, 6)$



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QUESTION



$$\frac{2x - 3}{5} \geq 1$$

$$\frac{2x - 3}{-5} \geq 1$$

$$\frac{x - 3}{x - 2} \geq 5$$

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$$\textcircled{1} \quad \frac{2x-3}{5} \geq 1$$

$$\frac{2x-3}{5} - \frac{1}{1} \geq 0$$

$$\frac{2x-3-5}{5} \geq 0$$

$$\frac{2x-8}{5} \geq 0$$

$$2x-8 \geq 0$$

$$x-4 \geq 0$$

$$x \geq 4$$

$$x \in [4, \infty)$$

$$\textcircled{2} \quad \frac{2x-3}{-5} \geq 1$$

$$2x-3 \leq -5$$

$$2x \leq -2$$

$$x \leq -1$$

$$x \in (-\infty, -1]$$

Tah-04

ATDB.uno

QUESTION



$$\text{Solve: } x(7 - x)(2 - x)(x - 5) \leq 0$$

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Solve: $x(7-x)(2-x)(x-5) \leq 0$

$(-\infty, 0) \cup (0, 2] \cup (5, 7) \cup (7, \infty)$

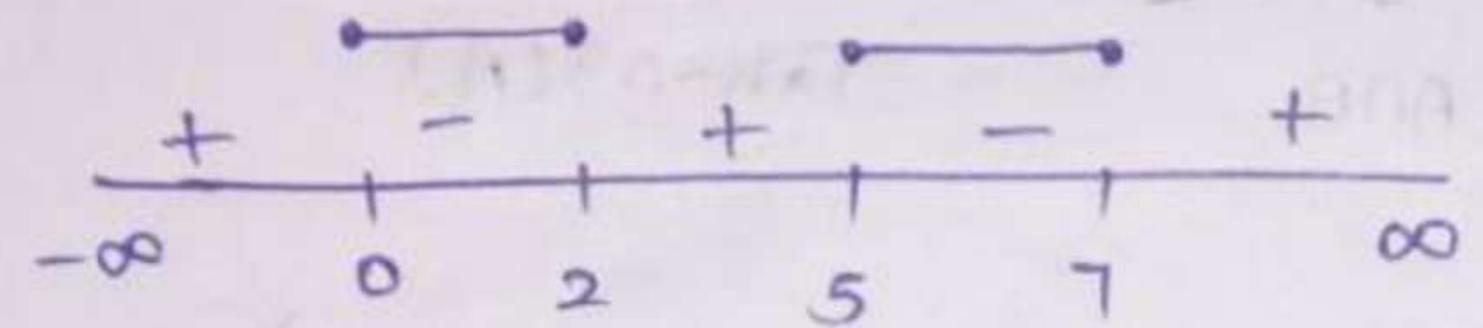
Method

Tah -05

$$x(7-x)(2-x)(x-5) \leq 0$$

$$x(-1)(x-7)(-1)(x-2)(x-5) \leq 0$$

$$x(x-7)(x-2)(x-5) \leq 0$$



$x \in [0, 2] \cup [5, 7]$ Ans



TAH-5!

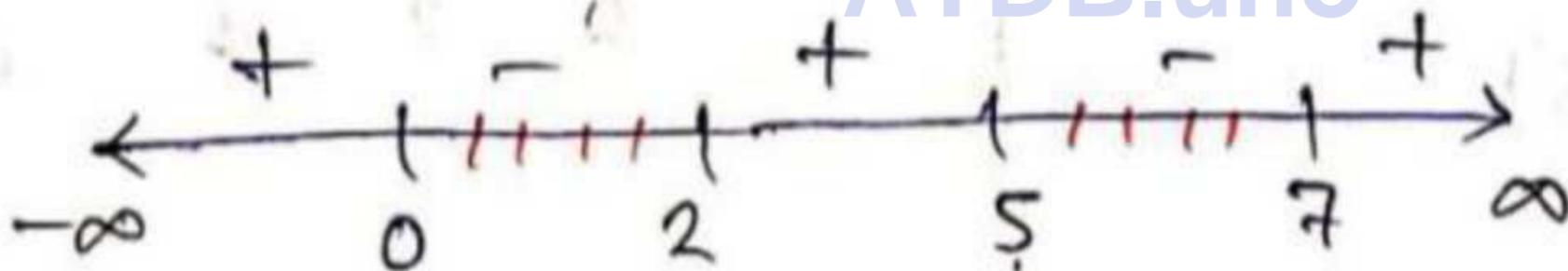
$$x(7-x)(2-x)(x-5) \leq 0.$$

Soln

$$x(7-x)(2-x)(x-5) \leq 0.$$

$$\text{or } x(x-7)(x-2)(x-5) \leq 0.$$

(Sign reverses
two times.)



$$\therefore x \in [0, 2] \cup [5, 7].$$

Ans.

QUESTION [IIT-JEE 1980]

(Home Challenge-01)



If $x > y > 0$, then show that the expression $\left(\sqrt{2} \left(2x + \sqrt{x^2 - y^2} \right) \left(\sqrt{x - \sqrt{x^2 - y^2}} \right) \right)$ can be simplified to $\sqrt{(x + y)^3} - \sqrt{(x - y)^3}$.

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Q-13: If $x > y > 0$, then show that the expression $(\sqrt{2}(2x + \sqrt{x^2 - y^2})) (\sqrt{x - \sqrt{x^2 - y^2}})$ can be simplified to $\sqrt{(x+y)^3} - \sqrt{(x-y)^3}$.

Soln. We can write $2x = (x+y) + (x-y)$

now, the expression,

$$E = \left\{ \sqrt{2} (2x + \sqrt{x^2 - y^2}) (\sqrt{x - \sqrt{x^2 - y^2}}) \right\}$$

$$\text{or, } E = \left\{ (x+y) + (x-y) + \sqrt{x^2 - y^2} \right\} \left\{ \sqrt{2x - 2\sqrt{x^2 - y^2}} \right\}$$

$$\text{or, } E = \left\{ (x+y) + (x-y) + \sqrt{x^2 - y^2} \right\} \left\{ \sqrt{(x+y) + (x-y) - 2\sqrt{(x+y)(x-y)}} \right\}$$

$$\text{or, } E = \left\{ (x+y) + (x-y) + \sqrt{(x+y)(x-y)} \right\} \left\{ \sqrt{(x+y)^2 + (x-y)^2 - 2\sqrt{(x+y)(x-y)}} \right\}$$

$$\text{or, } E = \left\{ (x+y) + (x-y) + \sqrt{(x+y)(x-y)} \right\} \left\{ \sqrt{(\sqrt{x+y} - \sqrt{x-y})^2} \right\}$$

$$\text{or, } E = \left\{ (x+y) + (x-y) + \sqrt{x+y} \sqrt{x-y} \right\} \left\{ |\sqrt{x+y} - \sqrt{x-y}| \right\}$$

$$\text{or, } E = \left\{ (\sqrt{x+y})^2 + (\sqrt{x-y})^2 + \sqrt{x+y} \sqrt{x-y} \right\} \left\{ \sqrt{x+y} - \sqrt{x-y} \right\}$$

$$\text{or, } E = \left\{ (\sqrt{x+y})^2 + (\sqrt{x-y})^2 + \sqrt{x+y} \sqrt{x-y} \right\} \left\{ \sqrt{x+y} - \sqrt{x-y} \right\}$$

$$\text{Let } \sqrt{x+y} = a \text{ and } \sqrt{x-y} = b,$$

$$\text{then, } E = (a^2 + b^2 + ab)(a-b)$$

$$\text{or, } E = a^3 - b^3$$

$$\text{i.e. } E = (\sqrt{x+y})^3 - (\sqrt{x-y})^3$$

$$\text{or, } E = \sqrt{(x+y)^3} - \sqrt{(x-y)^3}$$

HC 1
by Reed
from West Bengal



Solution to Previous KTKs

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QUESTION**(KTK 1)**

If a and b are rational numbers and $a + b\sqrt{2} = 5(\sqrt{2} - 3) + \sqrt{8}$ then find value of $a^2 + b^2 =$

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Ans. 274



Q11

If a and b are rational numbers and

$a + b\sqrt{2} = 5(\sqrt{2} - 3) + \sqrt{8}$, then find value of

$$a^2 + b^2 =$$

Sol

ATDB.uno

KTK-01

$$a + b\sqrt{2} = 5\sqrt{2} - 15 + 2\sqrt{2}$$

$$a + b\sqrt{2} = -15 + 7\sqrt{2}$$

$$a = -15$$

$$b = 7$$

$$\therefore a^2 + b^2 = (-15)^2 + (7)^2 = 225 + 49 = 274$$



- Q-5: If a and b are rational numbers, and
and $a + b\sqrt{2} = 5(\sqrt{2} - 3) + \sqrt{8}$ then find the
value of $a^2 + b^2$.

Solⁿ

$$a + b\sqrt{2} = 5(\sqrt{2} - 3) + \sqrt{8}$$

$$\text{or, } a + b\sqrt{2} = 5\sqrt{2} - 15 + 2\sqrt{2}$$

$$\text{or, } a + b\sqrt{2} = -15 + 7\sqrt{2}$$

On comparing,

$$a = -15 \quad \& \quad b = 7$$

$$\underline{\underline{\text{So,}}}$$

$$a^2 + b^2 = (-15)^2 + (7)^2$$

$$= 225 + 49$$

$$= 274 \quad (\underline{\underline{\text{Ans.}}})$$

KTK 1
by Reed
from WB

$$\underline{KTK - 1}$$

$$a + b\sqrt{2} = 5(\sqrt{2} - 3) + \sqrt{8} \quad \text{then find } a^2 + b^2 = ?$$

$$a + b\sqrt{2} = 5\sqrt{2} - 15 + 2\sqrt{2}$$

$$a + b\sqrt{2} = -15 + 7\sqrt{2}$$

Both side compare

$$a = -15, \quad b = 7$$

$$\begin{aligned} \therefore a^2 + b^2 &= (15)^2 + (7)^2 \\ &= 225 + 49 \\ &= 274 \end{aligned}$$

Prince Gupta
Gopalganj (Bihar)



QUESTION

(KTK 2)



If value of $\left(x + \frac{1}{x} = 5\right)$ then find value of :

(i) $x^2 + \frac{1}{x^2}$

(ii) $x - \frac{1}{x}$

(iii) $x^4 + \frac{1}{x^4}$

(iv) $x^4 - \frac{1}{x^4}$

(v) $x^3 + \frac{1}{x^3}$

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Ans. (i) 23, (ii) $\pm\sqrt{21}$, (iii) 527, (iv) $\pm 115\sqrt{21}$, (v) 110



Q If value of $\left(x + \frac{1}{x} = 5\right)$ then find value of KTK-02

(i) $x^2 + \frac{1}{x^2}$

(ii) $x - \frac{1}{x}$

Sol

$\frac{1}{x^2}$

$$x^2 + \frac{1}{x^2} + 2 = 25$$

$$x^2 + \frac{1}{x^2} = 23$$

Sol

$$\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2 = 23 - 2$$

$$\left(x - \frac{1}{x}\right)^2 = 21$$

$$\left(x - \frac{1}{x}\right) = \pm\sqrt{21}$$



(iii) $x^4 + \frac{1}{x^4}$

Sol

$$x^2 + \frac{1}{x^2} = 23$$

S.B.S

$$x^4 + \frac{1}{x^4} + 2 = (23)^2$$

$$x^4 + \frac{1}{x^4} = 529 - 2 = 527$$

(v) $x^3 + \frac{1}{x^3}$

Sol

$$x + \frac{1}{x} = 5$$

C.B.S

$$x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x} \left(x + \frac{1}{x} \right) = 125$$

$$x^3 + \frac{1}{x^3} + 3(5) = 125$$

$$x^3 + \frac{1}{x^3} = 125 - 15$$

$$x^3 + \frac{1}{x^3} = 110$$

(iv) $x^4 - \frac{1}{x^4}$

Sol

$$(x^2)^2 - \left(\frac{1}{x^2}\right)^2$$

$$\left(x^2 + \frac{1}{x^2}\right) \left(x^2 - \frac{1}{x^2}\right)$$

$$= \left(x^2 + \frac{1}{x^2}\right) \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x}\right)$$

$$= (23) (5) \pm \sqrt{21}$$

$$= \pm \sqrt{21} \ 115 = \pm 115\sqrt{21} \text{ Ans}$$

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QUESTION

(KTK 3)



Given $3x^2 + x = 1$, then the value of $6x^3 - x^2 - 3x$ is equal to

- A** -1
- B** 0
- C** 1
- D** 2

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Ans. D



Question

KT - 03

Given $3x^2 + x = 1$, the value of $6x^3 - x^2 - 3x$ is equal to

- (A)
(B)
(C)
(D)

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$$3x^2 + x = 1$$

$$3x^2 = 1 - x$$

$$6x^3 = 2x - 2x^2$$

$$\begin{aligned} 6x^3 - x^2 - 3x &= 2x - 2x^2 - 3x - x^2 \\ &= -x - 3x^2 \\ &= -(x + 3x^2) \\ &= -1 \end{aligned}$$

sol^m



KTK-3!
Q-7! Given $3x^2 + x = 1$, then the value of $6x^3 - x^2 - 3x$ is equal to:

- Ⓐ -1 Ⓑ 0 Ⓒ 1 Ⓓ 2.

Solⁿ method-1!

$$3x^2 + x = 1 \text{ --- (i)}$$

$$\Rightarrow 3x^2 = 1 - x \text{ --- (ii)}$$

$$6x^3 - x^2 - 3x$$

$$= x [2 \cdot 3x^2 - x - 3]$$

$$= x [2(1-x) - x - 3] \text{ from (ii)}$$

$$= x [2 - 2x - x - 3]$$

$$= x [-3x - 1]$$

$$= -3x^2 - x$$

$$= -(3x^2 + x) \text{ from (i)}$$

$$= -1. \text{ (Ans.)}$$

$\therefore \text{Ans.} \Rightarrow \text{Ⓐ } -1.$

method-2!

$$3x^2 + x = 1$$

$$\Rightarrow 3x^2 = 1 - x \text{ --- (i)}$$

$$6x^3 - x^2 - 3x$$

$$= 2x \cdot \frac{3x^2}{\downarrow} - x^2 - 3x$$

$$= 2x(1-x) - x^2 - 3x$$

$$= 2x - 2x^2 - x^2 - 3x$$

$$= -3x^2 - x$$

$$= -(3x^2 + x)$$

$$= -1. \text{ (Ans)}$$

KTK-3
By Reed
from w.B.

QUESTION

(KTK 4)



If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x =$

A $\frac{2}{3}$

B $\frac{4}{3}$

C $\frac{1}{3}$

D $\frac{5}{3}$

ATDB.uno

Ans. B



KTK-04

Ques) If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x =$

(A) $\frac{2}{3}$

$\frac{2}{3}$

(B) $\frac{4}{3}$

$\frac{4}{3}$

(C) $\frac{1}{3}$

$\frac{1}{3}$

(D) $\frac{5}{3}$

$\frac{5}{3}$

Solⁿ

$$\sqrt{9^x} = \sqrt[3]{9^2}$$

$$9^{x/2} = 9^{2/3}$$

$$\frac{x}{2} = \frac{2}{3}$$

$$x = \frac{4}{3}, \text{ ans}$$

OPTION-B



Q-8! If $\sqrt{9^x} = \sqrt[3]{9^2}$, then $x = ?$

- (a) $\frac{2}{3}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) $\frac{5}{3}$

Solⁿ

$$\sqrt{9^x} = \sqrt[3]{9^2}$$

$$\text{or } 9^{\frac{x}{2}} = 9^{\frac{2}{3}}$$

$$\text{or } \frac{x}{2} = \frac{2}{3}$$

$$\text{or } x = \frac{4}{3} \quad (\underline{\underline{\text{Ans.}}})$$

QUESTION

(KTK 5)



$$\frac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt[5]{x^3}} \times \sqrt[30]{x^{77}} =$$

A $x^{76/15}$

B $x^{78/15}$

C $x^{79/15}$

D $x^{77/15}$

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Ans. D



$$\text{Q} = \frac{\sqrt{x^3} \times 3\sqrt{x^5} \times 30\sqrt{x^{77}}}{5\sqrt{x^3}}$$

KTK-05

$$\text{Sol} = \frac{(x^3)^{1/2} \times (x^5)^{1/3} \times (x^{77})^{1/30}}{(x^3)^{1/5}}$$

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$$= \frac{x^{3/2} \times x^{5/3} \times x^{77/30}}{x^{3/5}} = \frac{x^{\frac{3}{2} + \frac{5}{3} + \frac{77}{30}}}{x^{3/5}}$$

$$= \frac{x^{\frac{40+50+77}{30}}}{x^{3/5}} = \frac{x^{86/15}}{x^{3/5}} = x^{\left(\frac{86}{15} - \frac{3}{5}\right)} = x^{\frac{77}{15}} \text{ Ans}$$



Q-9: $\frac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt{x^3}} \propto \sqrt[30]{x^{77}} = ?$

- (A) $x^{76/15}$ (B) $x^{78/15}$ (C) $x^{79/15}$ (D) $x^{77/15}$

Soln:-

$$\frac{\sqrt{x^3} \times \sqrt[3]{x^5}}{\sqrt{x^3}} \propto \sqrt[30]{x^{77}}$$

$$= \frac{x^{\frac{3}{2}} \times x^{\frac{5}{3}}}{x^{\frac{3}{2}}} \times x^{\frac{77}{30}}$$

$$= x^{\frac{3}{2} + \frac{5}{3} - \frac{3}{2}} \propto x^{\frac{77}{30}}$$

$$= x^{\frac{45 + 50 - 30}{30}} \propto x^{\frac{77}{30}}$$

$$= x^{\frac{77}{30}} \times x^{\frac{77}{30}}$$

$$= \left(x^{\frac{77}{30}}\right)^2 = x^{2 \times \frac{77}{30}} = x^{\frac{77}{15}}$$

\therefore Ans \Rightarrow (D) $x^{\frac{77}{15}}$

KTK 4,5
by Reed
from WB

QUESTION

(KTK 6)



If $\frac{9^n \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^m} = \frac{1}{27}$, where m and n are natural numbers, then find the value of (m - n) is _____

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Ans. 1



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Date / /

KTK-06

$$\text{Q. If } \frac{9^n \times 3^2 \times 3^{-\frac{n}{2} \times 2}}{3^{3m} \times 2^m} - 27^n = \frac{1}{27} \text{ where } m \text{ and } n \text{ are}$$

natural no's, find $(m-n)$.

$$\text{sol}^n: \frac{3^{2n+2+n} - 3^{3n}}{3^{3m} \times 2^m} = \frac{1}{27} \Rightarrow \frac{3^{3n+2} - 3^{3n}}{3^{3m} \times 2^m} = \frac{3^{3n}(3^2-1)}{3^{3m} \times 2^m} = \frac{1}{27}$$

$$\frac{3^{3n} \cdot 2^3 \cdot 3^3}{3^{3m} \cdot 2^m} = 1 \Rightarrow \frac{3^{3n+3} \cdot 2^3}{3^{3m} \cdot 2^m} = 1$$

$$\Rightarrow 3n+3 = 3m \quad ,, \quad m=3$$

$$3n+3 = 9 \Rightarrow 3n = 6 \Rightarrow n = 2$$

$$m-n = 3-2 = 1$$

$$= \frac{9^m \times 3^2 \times (3^{-n/2})^{-2} - (27)^n}{3^{3m} \times 2^m} = \frac{1}{27}, \text{ where } m \text{ and } n \text{ are}$$

natural numbers, then find the value of $(m-n)$ is:

KTK 6

Sol

$$\frac{3^{2m} \times 3^2 \times 3^n - (3^3)^n}{3^{3m} \times 2^m} = \frac{3^{2m+2+n} - 3^{3n}}{3^{3m} \times 2^m} = \frac{3^{3n}(3^2-1)}{3^{2m} \cdot 6^m}$$

$$= \frac{3^{3n} \cdot 8}{3^{2m} \cdot 6^m} = \frac{1}{27}$$

$$= \frac{3^{3n} \cdot 2^3}{3^{2m} \cdot 2^m \cdot 3^m} = \frac{1}{3^3}$$

$$3^{3n+3} \cdot 2^3 = 3^{3m} \cdot 2^m$$

$$\boxed{m=3}$$

$$3m = 3n + 3$$

$$9 \times 3 = 3n + 3$$

$$3n = 9 - 3$$

$$3n = 6$$

$$\boxed{n=2}$$

$$\therefore m-n = 3-2 = \textcircled{1} \text{ Ans}$$

QUESTION**(KTK 7)**

Show that the square of $\frac{\sqrt{25-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$ is a rational number.

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- Q-104 show that the square of

$$\frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$$

is a rational number.

Soln.

Method 1:

$$E = \frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}}$$

$$\text{or, } E = \frac{\sqrt{26-15\sqrt{3}}}{5\sqrt{2}-\sqrt{38+5\sqrt{3}}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$\text{or, } E = \frac{\sqrt{52-30\sqrt{3}}}{10-\sqrt{76+10\sqrt{3}}}$$

$$\text{or, } E = \frac{\sqrt{(3\sqrt{3})^2+5^2-2\cdot 3\sqrt{3}\cdot 5}}{10-\sqrt{(5\sqrt{3})^2+12+2\cdot 5\sqrt{3}\cdot 1}}$$

$$\text{or, } E = \frac{\sqrt{(3\sqrt{3}-5)^2}}{10-\sqrt{(5\sqrt{3}+1)^2}}$$

$$\text{or, } E = \frac{|3\sqrt{3}-5|}{10-|5\sqrt{3}+1|}$$

$$\text{or, } E = \frac{3\sqrt{3}-5}{10-5\sqrt{3}-1}$$

$$\text{or, } E = \frac{3\sqrt{3}-5}{9-5\sqrt{3}}$$

$$\text{or, } E = \frac{(3\sqrt{3}-5)}{\sqrt{3}(3\sqrt{3}-5)} = \frac{1}{\sqrt{3}}$$

$$\therefore E^2 = \left(\frac{1}{\sqrt{3}}\right)^2 = \frac{1}{3} \in \mathbb{Q}. \quad [\text{PROVED}]$$

KTK 7

by Reed

from WB

$$\sqrt{52} = (3\sqrt{3})^2 + 5^2$$

$$2\sqrt{30\sqrt{3}} = 2 \cdot 15 \cdot \sqrt{3} \quad \times$$

$$= 2 \cdot 5\sqrt{3} \cdot 3 \quad \times$$

$$= 2 \cdot 3\sqrt{3} \cdot 5 \quad \checkmark$$

$$(1^2 + 2^2) = 26 \neq 52$$

$$\sqrt{76} = (5\sqrt{3})^2 + 12$$

$$\therefore \sqrt{10\sqrt{3}} = 2 \cdot 5\sqrt{3} \cdot 1$$

$$(1^2 + 2^2) = 76$$

$$= 75 + 1 = 76$$

QUESTION

(KTK 8)



If $5^{10x} = 4900$, $2^{\sqrt{y}} = 25$ then the value of $\frac{(5^{(x-1)})^5}{4^{-\sqrt{y}}}$ is

A $\frac{14}{5}$

B 5

C $\frac{28}{5}$

D 14

ATDB.uno

Ans. D



Q If $5^{10x} = 4900$, $2^{\sqrt{y}} = 25$ then the value

of $\frac{(5^{x-1})^5}{4^{-\sqrt{y}}}$ is

KTK-08

Sol

$$\frac{5^{(5x-5)}}{4^{-\sqrt{y}}} = 5^{5x-5} \cdot 4^{\sqrt{y}} \Rightarrow \frac{5^{5x} (2^{\sqrt{y}})^2}{5^5}$$

$$\Rightarrow \frac{70 \times (25)^2}{5^5} \Rightarrow \frac{70 \times 5^4}{5^5} = \underline{\underline{14}} \text{ Ans}$$

Q-11: If $5^x = 4900$, $2^y = 25$, then the value of $\frac{(5^{2x-1})^5}{4^{10}}$ is:

- (A) $\frac{14}{5}$ (B) 5 (C) $\frac{28}{5}$ (D) 14.

Soln

$$\begin{aligned} & \frac{(5^{2x-1})^5}{4^{10}} \\ &= \frac{(5^x \times 5^{-1})^5}{(2^2)^{10}} \\ &= \frac{5^{5x} \times 5^{-5}}{(2^{10})^2} \\ &= \frac{5^{5x} \times (2^2)^2}{5^5} \\ &= \frac{70 \times (25)^2}{5^5} \\ &= \frac{14 \times 5 \times (5^2)^2}{5^5} \\ &= \frac{14 \times 5 \times 5^4}{5^5} \\ &= \frac{14 \times 5^3}{5^3} \\ &= 14 \text{ (Ans)} \end{aligned}$$

\therefore Ans \Rightarrow 14.

now,

$$\begin{aligned} 5^{10x} &= 4900 \\ \text{or, } 5^{\frac{10x}{2}} &= 4900^{\frac{1}{2}} \\ \text{or, } 5^{5x} &= 70 \end{aligned}$$

KTK 8
by Reed
from WB





THANK YOU

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