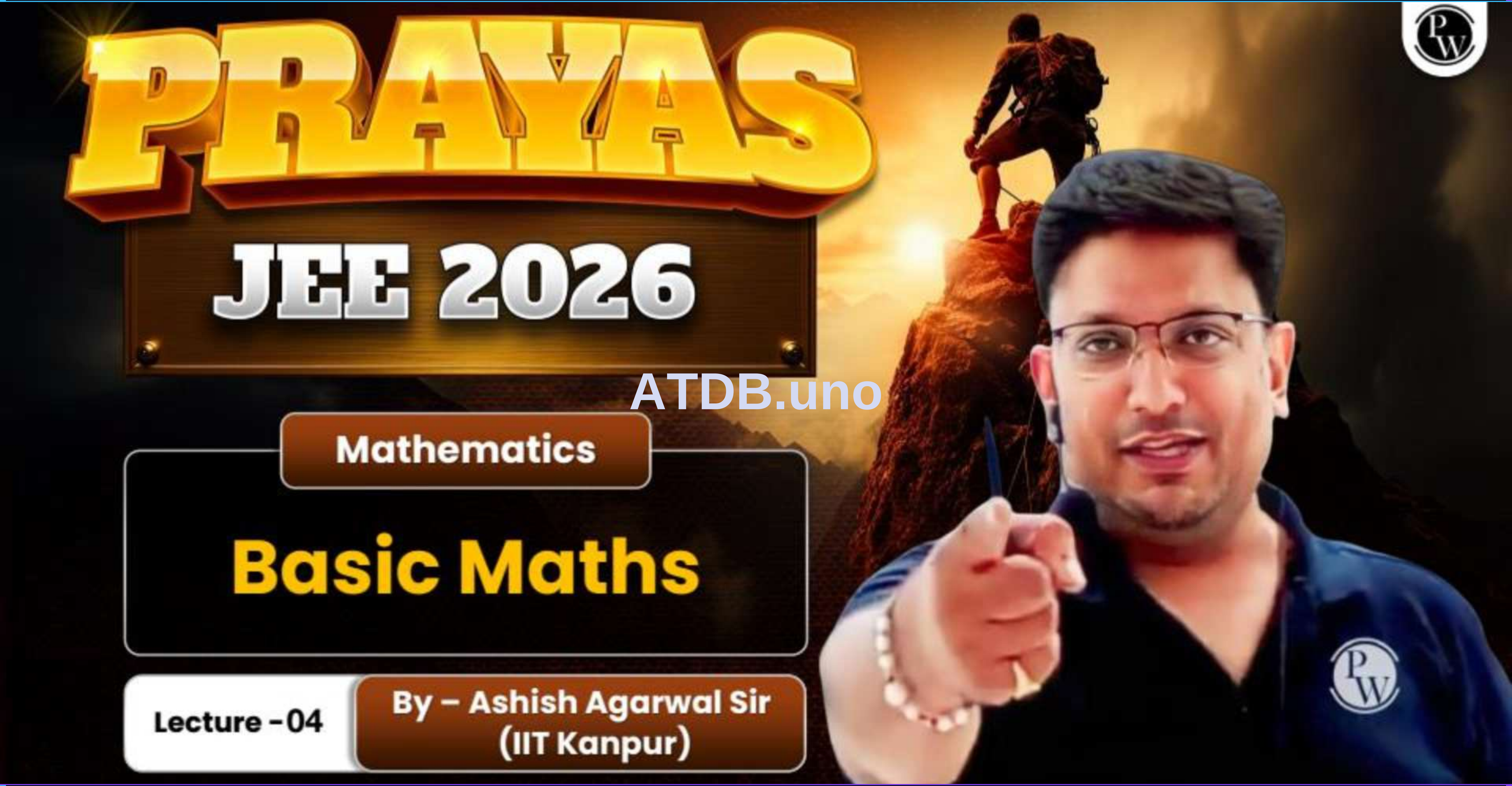

PRAYAS
JEE 2026
Mathematics
Basic Maths
Lecture -04
By – Ashish Agarwal Sir
(IIT Kanpur)

Topics *to be covered*

A Some Important Points

B Polynomials and their Factorization

Recap *of previous lecture*

$(x-1)(y-1)=14$

2	7	(3, 8)
7	2	(8, 3)
1	14	(2, 15)
14	1	(15, 2)
−2	−7	(−1, −6), (−6, −1)
−14	−1	(−13, 0)
−1	−14	(0, −13)

$y-1=-2$
$x-1=-7$

Fill in the Blanks:

1. If $(x-1)(y-1) = 14$ where $x, y \in I$ then (x, y) can be ______

2. If $n = \dfrac{27}{x} - x$ where $n, x \in N$ then (n, x) can be ______ $x=1$, $n=26$; $x=3$, $n=6$ → $(26, 1)$, $(6, 3)$

   $n \in N$ x should be divisor of 27

3. <u>12345</u>x is divisible by 9 then x is ______ 3, $\cancel{2}$ $x \in \{0, 1, 2, --, 9\}$

4. If 52541x is divisible by 4 then x can be ______ 2, 6

5. $24^2 + 29^2 =$ ______ $576 + 841 = 1417$

6. If 6794x is divisible by 6 then x can be ______ 4

   x = even

   26 + x should be divisible by 3

Recap of previous lecture

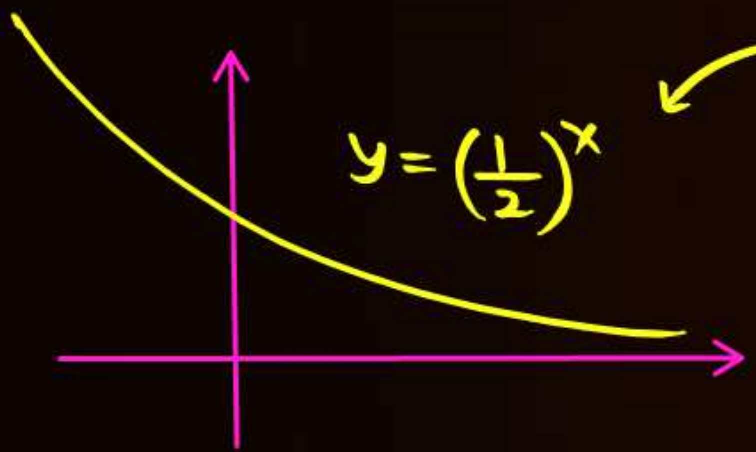
State True or False

$(-2)^{-4} = \frac{1}{(-2)^4} = \frac{1}{16} > 0$ $\quad (0)^4 = 0$ neither +ve nor -ve

Even (Any real No:) ≥ 0

1. Even power of every real number is always positive. (F)
2. Odd power of a real number can be positive or negative or zero depending on number. (T)
3. $x^{2n} \geq 0 \ \forall \ x \in R, n \in N$ (T)
4. x^{2n+1} is positive if x is positive, $n \in N$ (T)
5. x^{2n+1} is negative if x is negative, $n \in N$ (T)
6. If x is a positive real then $x^{\text{any power}}$ is always positive. (T)

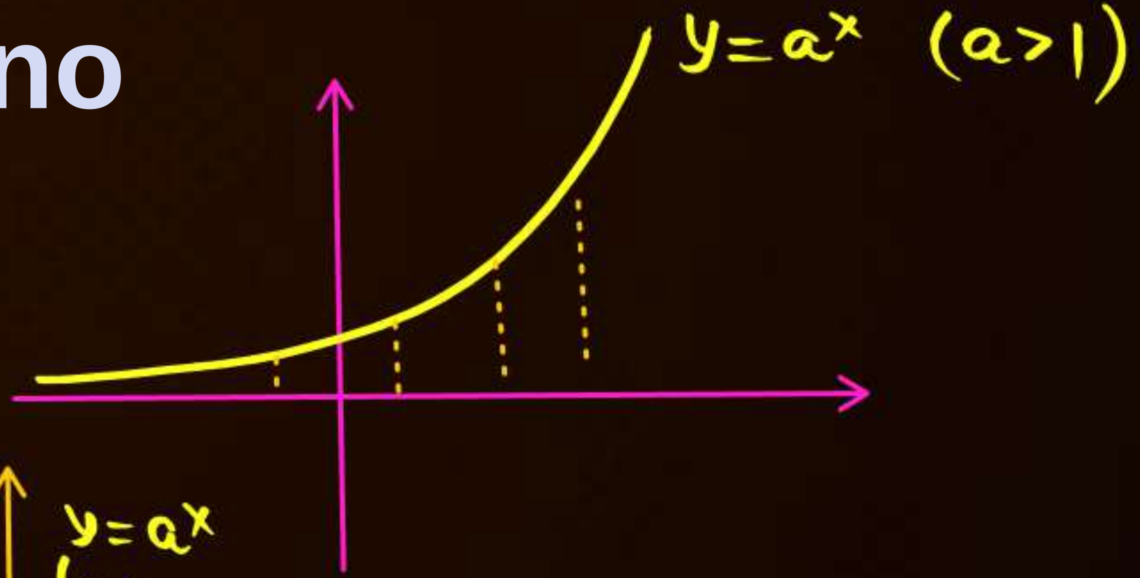
Any real NO: (Any +ve real) > 0



Ex: $\left(\frac{1}{2}\right)^x > 0$

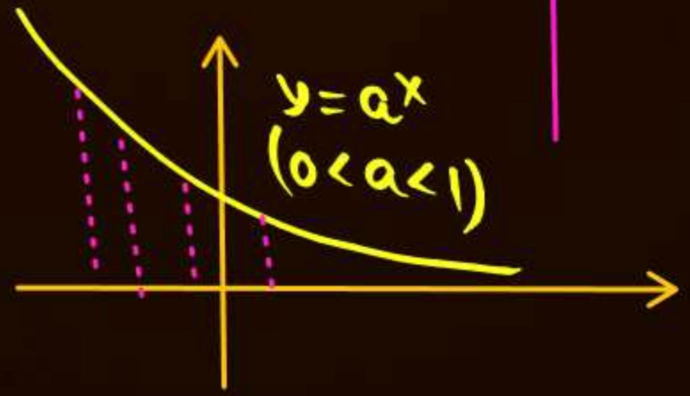
Ex: $2^{-3} = \frac{1}{2^3} = \frac{1}{8} > 0$

Ex $7^{-2} > 0$



Ex: $\left(\frac{1}{3}\right)^{-3} = 27 > 0.$

Ex: $3^0 = 1 > 0$



Recap of previous lecture

7. $x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2 = 0$ where $x_i \in R, i = 1, 2, 3, \ldots n$ then $x_1 = x_2 = x_3 = \ldots = x_N = 0$ (T)

8. 6545 is divisible by 7. (T) $654 - 2 \times 5 = 644$

$64 - 8 = 56$

9. 32436 is divisible by 4, 6 & 9. (T)

10. $55^2 + 44^2 = \underline{3025 + 1936} = 4961$

11. $1 + 3 + 5 + 7 + \ldots$ upto 10 terms = $10^2 = 100$

Sum of first n odd numbers $= n^2$

Ex: $1+3+5+ \ldots$ upto n terms

$= \frac{n}{2}(2\cdot 1 + (n-1)2)$

$= n/2(2+2n-2) = n^2$

Homework Discussion

QUESTION

Let a, b, c are real numbers and satisfy a = 8 – b and c^2 = ab – 16, then $\frac{a}{b}$ is equal to

$$c^2 = (8-b)b - 16.$$

$$c^2 = 8b - b^2 - 16$$

$$c^2 + b^2 - 8b + 16 = 0$$

$$c^2 + b^2 - 2\cdot 4\cdot b + 4^2 = 0$$

$$c^2 + (b-4)^2 = 0$$

$$c = 0,\ b = 4$$

QUESTION

(KTK 1)

a, b, c are reals such that $a+b+c=3$ and $\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{10}{3}$. The value $E=\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$ is

(A) 9

(B) 7

(C) 5

(D) 3

$$\frac{3}{a+b}+\frac{3}{b+c}+\frac{3}{c+a}=10$$

$$\frac{a+b+c}{a+b}+\frac{a+b+c}{b+c}+\frac{a+b+c}{c+a}=10$$

$$1+\frac{c}{a+b}+\frac{a}{b+c}+1+1+\frac{b}{c+a}=10$$

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=7$$

$$\frac{x}{a+b} \neq \frac{x}{a} + \frac{x}{b}$$

Gadho/Gadhiyoo aisaa naa karo

QUESTION

(KTK 2)

Solve the equations : $\begin{cases} 2^x + 3^y = 41 \\ 2^{x+2} + 3^{y+2} = 209 \end{cases}$

Ans. x = 5 and y = 2

QUESTION

(KTK 3)

What is the area of an equilateral triangle inscribed in a circle of radius 4 cm?

(A) $12\ cm^2$

(B) $9\sqrt{3}\ cm^2$

(C) $8\sqrt{3}\ cm^2$

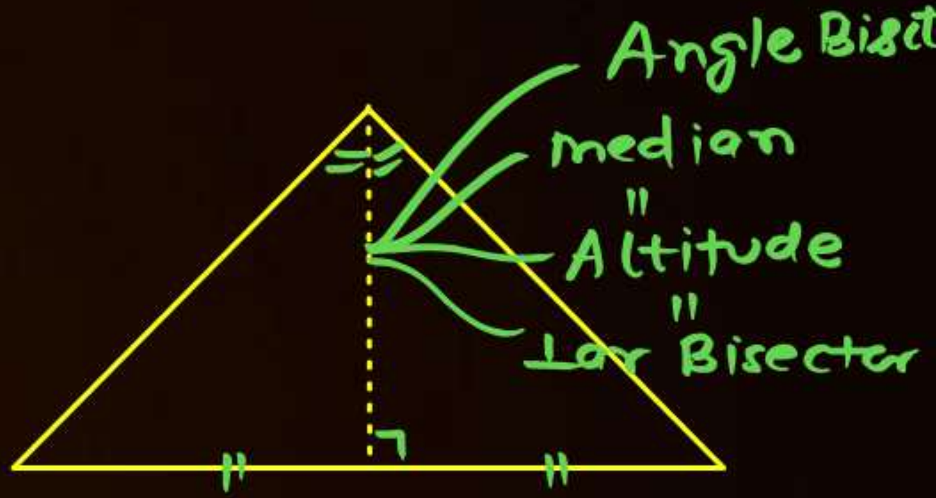
(D) $12\sqrt{3}\ cm^2$

$\sin 30^\circ = \frac{OM}{4}$

$OM = 4\sin 30^\circ = 2$

$h = 4 + 2 = 6$

Area of Equilateral Triangle $= \frac{h^2}{\sqrt{3}} = \frac{36}{\sqrt{3}} = 12\sqrt{3}\ cm^2$



⊥ar Bisector of any chord of a circle passes through its centre

Ans. D

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

Appke Doubts Humaray Solutions

Anshika Shahu · 11 hours ago

DC

sir agr koi homework question nhi ho tb kia google ya kisi aur platform ki hint ke liye help le skte try krne ke liye ya fr chod de ki aap kara doge

2 Same Doubts · 0 Reported — Mark Popular

Recap of previous lecture

State True or False

1. Every prime except 2 is odd. (T)
2. Every prime ≥ 5 is of type $6k \pm 1$, $k \in I^+$. (T)
3. Every number of type $6k \pm 1$, $k \in I^+$ is prime. (F)
4. Sum of two primes is also a prime. (F)
5. Every composite number has more than two positive factors. (T)
6. Every natural number is either prime or composite. (F)

2 is the only even prime

1 is neither prime nor composite

$6k \pm 1$

Doubt 11 hours ago — piyush mishra

sir 2 number me agar k ki jagah per 6 lele toh 35 ya 37 ayega lekin 35 prime nahi h

0 Same Doubts

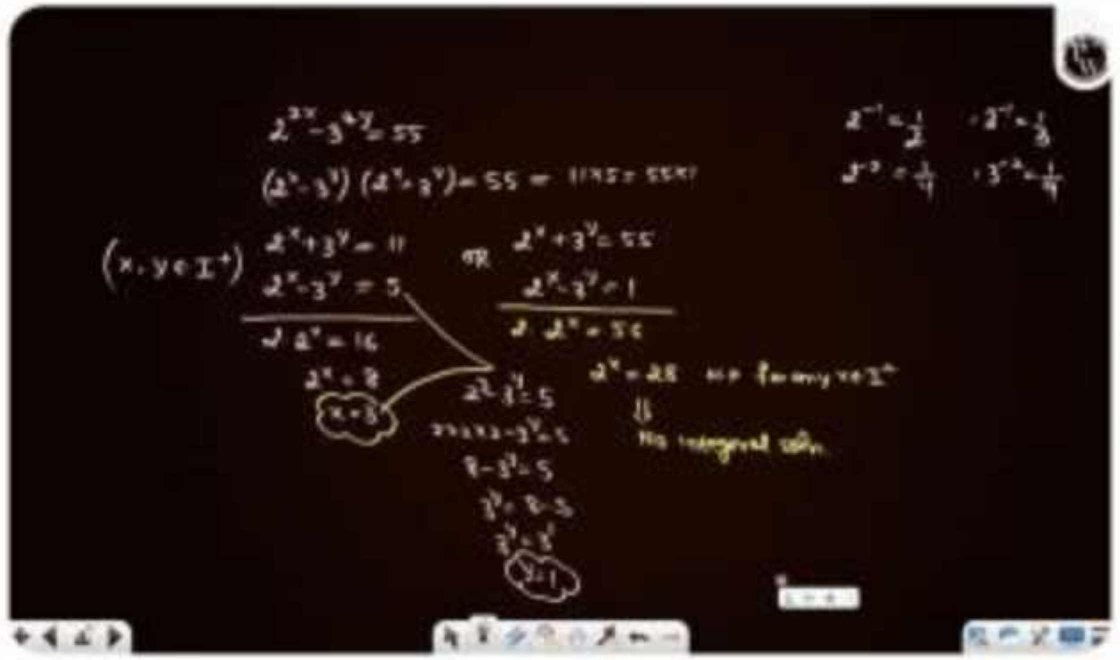
Doubt 21 hours ago — Prakhar

sir 1st me 9 odd hai pr prime nahi hai toh ye false hona chaiye tha?

0 Same Doubts

Doubt 11 hours ago — Prashant

sir 2+3=5, 2+5=7 etc.. prime aa to raha



Doubt 11 hours ago

sir what about -11 & -5

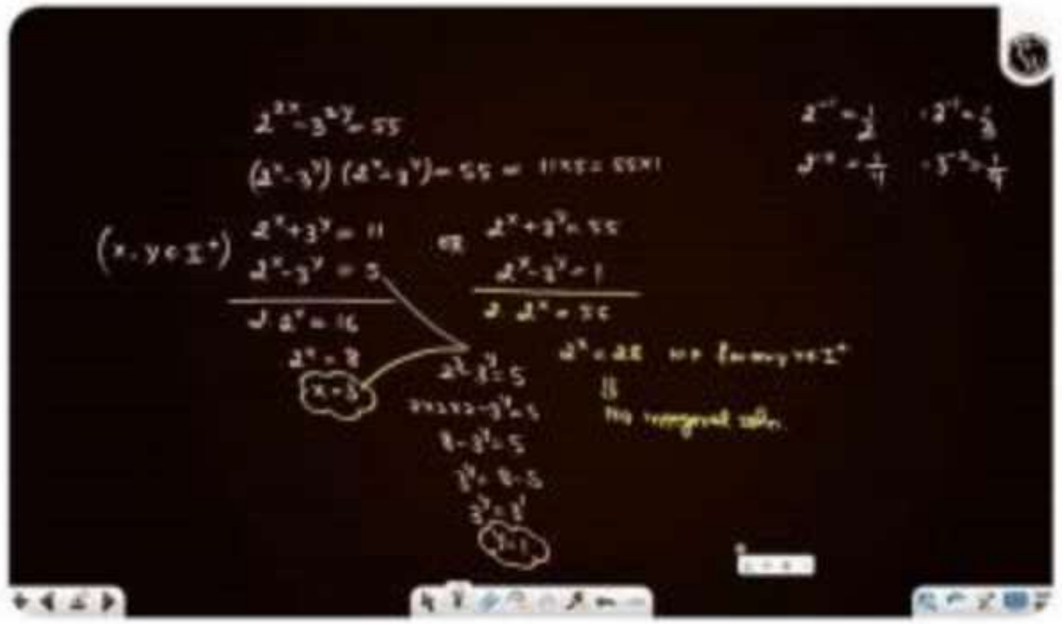
0 Same Doubts

11 hours ago

sir 55 ko 2ke power -3 ke power me express krke compare krke x and y ka value nhi nikal skte ?

0 Same Doubts · 0 Reported — Mark Popular

Appke Doubts Humaray Solutions



Doubt 11 hours ago

sir agar intezar ke jagah R deta to no of solutions

QUESTION KTK 3

	Column-I		Column-II
(A)	A rectangular box has volume 48, and the sum of the length of the twelve edges of the box is 48. The largest integer that could be the length of an edge of the box, is	(P)	1
(B)	The number of zeroes at the end in the product of first 20 prime numbers, is $2\cdot 3\cdot 5\, p_4 p_5 \cdots p_{20}$ is only one zero at end	(Q)	2
(C)	The number of solutions of $2^{2x} - 3^{2y} = 55$, in which x and y are integers, is	(R)	3
		(S)	4
		(T)	6

Ans. (A) T; (B) P; (C)

Doubt 11 hours ago

sir esma 4ki power x - 9 ki power x = 55 aur fir hit and trial kar lenaga aur sirf ek hi solution milega

QUESTION ★★★★ KCLS ★★★★

For each positive number x, let $f(x) = \dfrac{\left(x+\frac{1}{x}\right)^6 - \left(x^6+\frac{1}{x^6}\right) - 2}{\left(x+\frac{1}{x}\right)^3 + \left(x^3+\frac{1}{x^3}\right)}$. The minimum value of f(x) is

(A) 1
(B) 2
(C) 3
(D) 4
(E) 6

$X + \frac{1}{X} = 2$

Doubt 11 hours ago

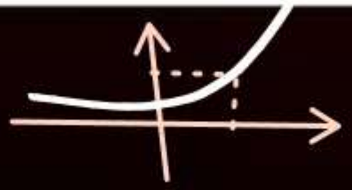
sir direct min 2 put karke solve kare to ans 6 hi ara hai pls check

0 Same Doubts

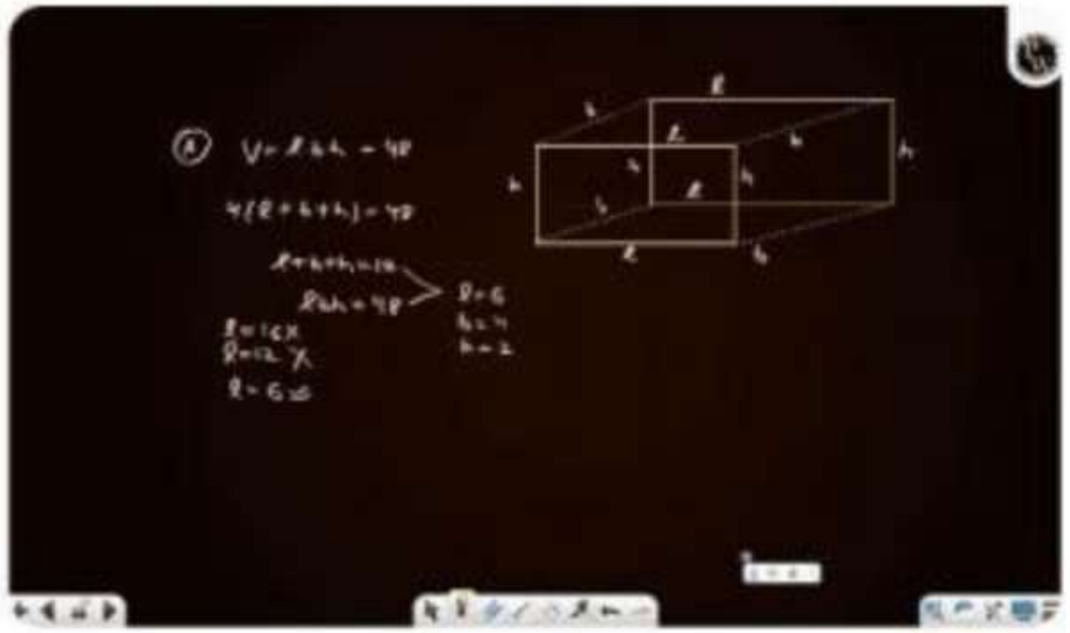
$2^{2x} - 3^{2y} = 55$

If $x, y \in R$ No: of soln $= \infty$

$4^x - 9^y = 55 \Rightarrow 4^x = 55 + 9^y$

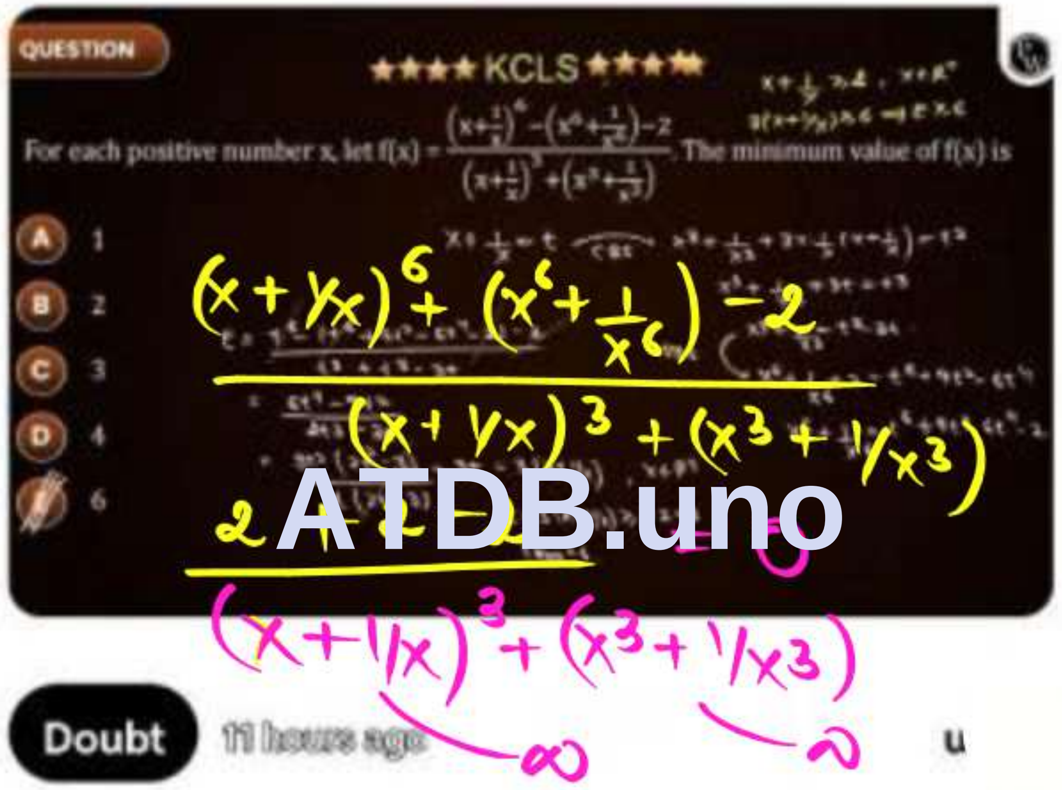


Appke Doubts Humaray Solutions



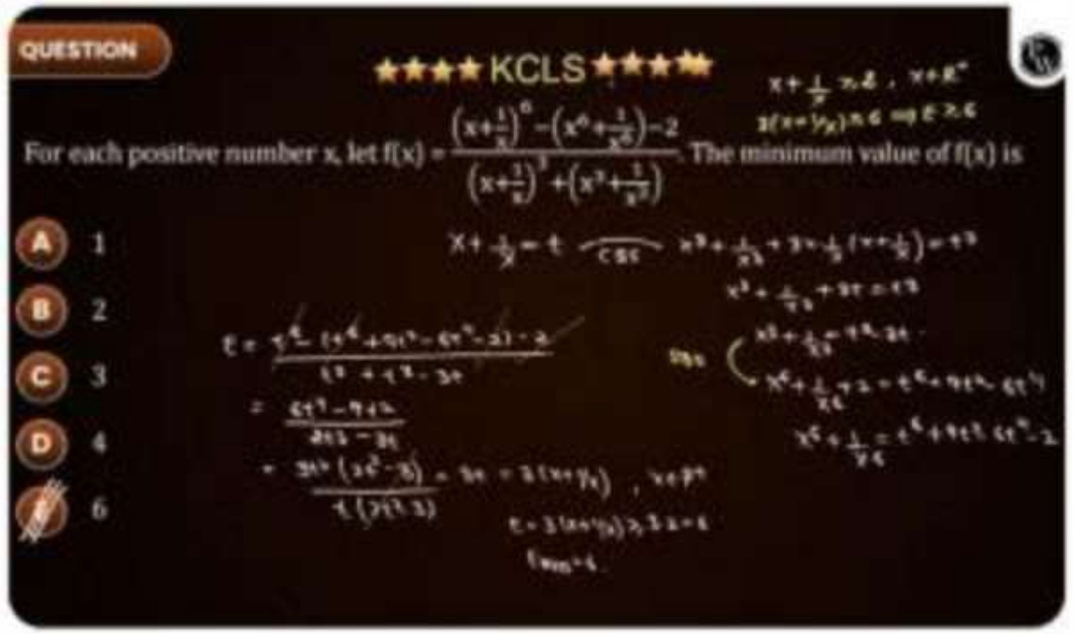
Doubt 11 hours ago

Sir A nahi samjh aa raha hai aap add aur multiply dono kar rahe hai kyu batayega n



Doubt 11 hours ago

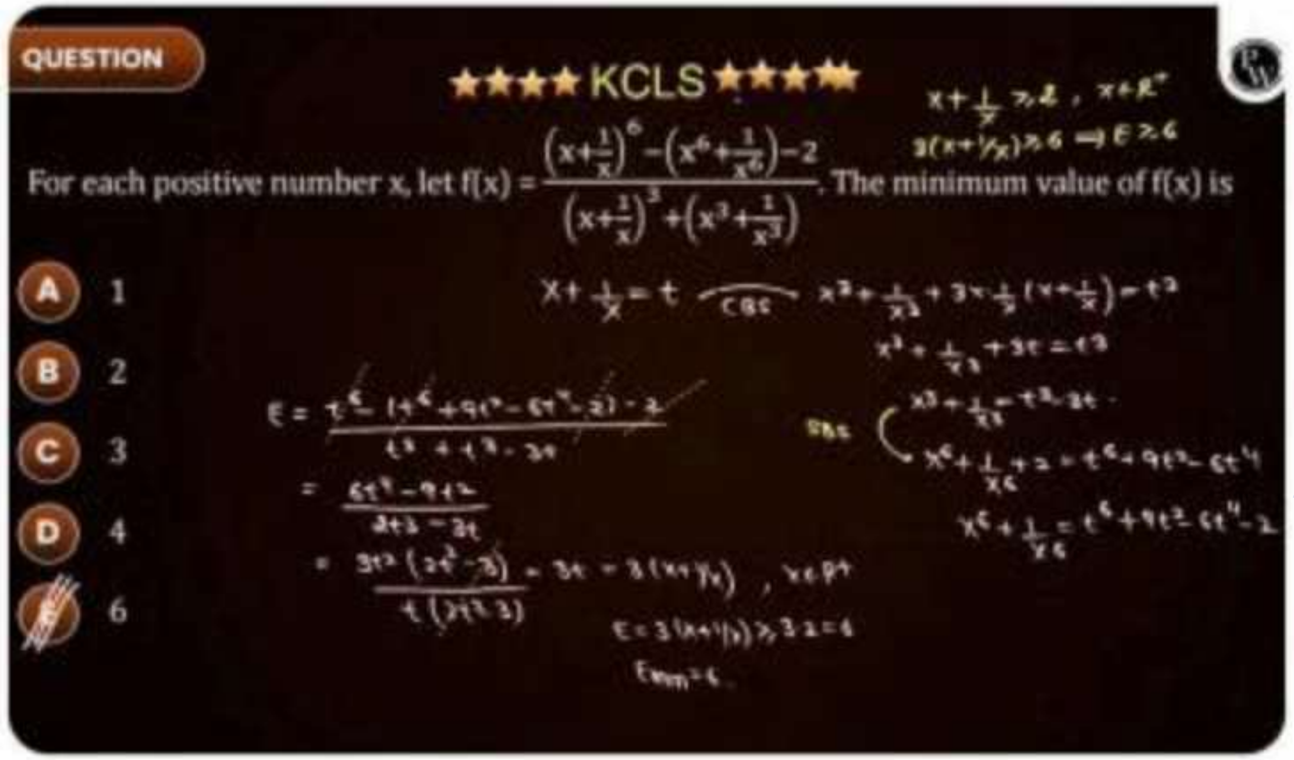
sir mein yah bhi to kar sakte the ki Main X Plus 1/ x ki minimum value direct likh Li as similarly others mein bhi kar liya and you answer a raha tha vah to sis a raha hai kyunki x positive diya hai toh min 2 max infinity hoti and in denominator same X + 1 by x wali form thi toh fx min krne ke denominator max kr dete fx=0 a jata hope you understand!!



Doubt 11 hours ago

Sir itna bada karne ki kya jarurat hai agar x6 + 1/x6 woh bhi 2 ke equal ya badi hogi toh harr jagah 2 substitute kardo ans 6 hi aa rahi hai

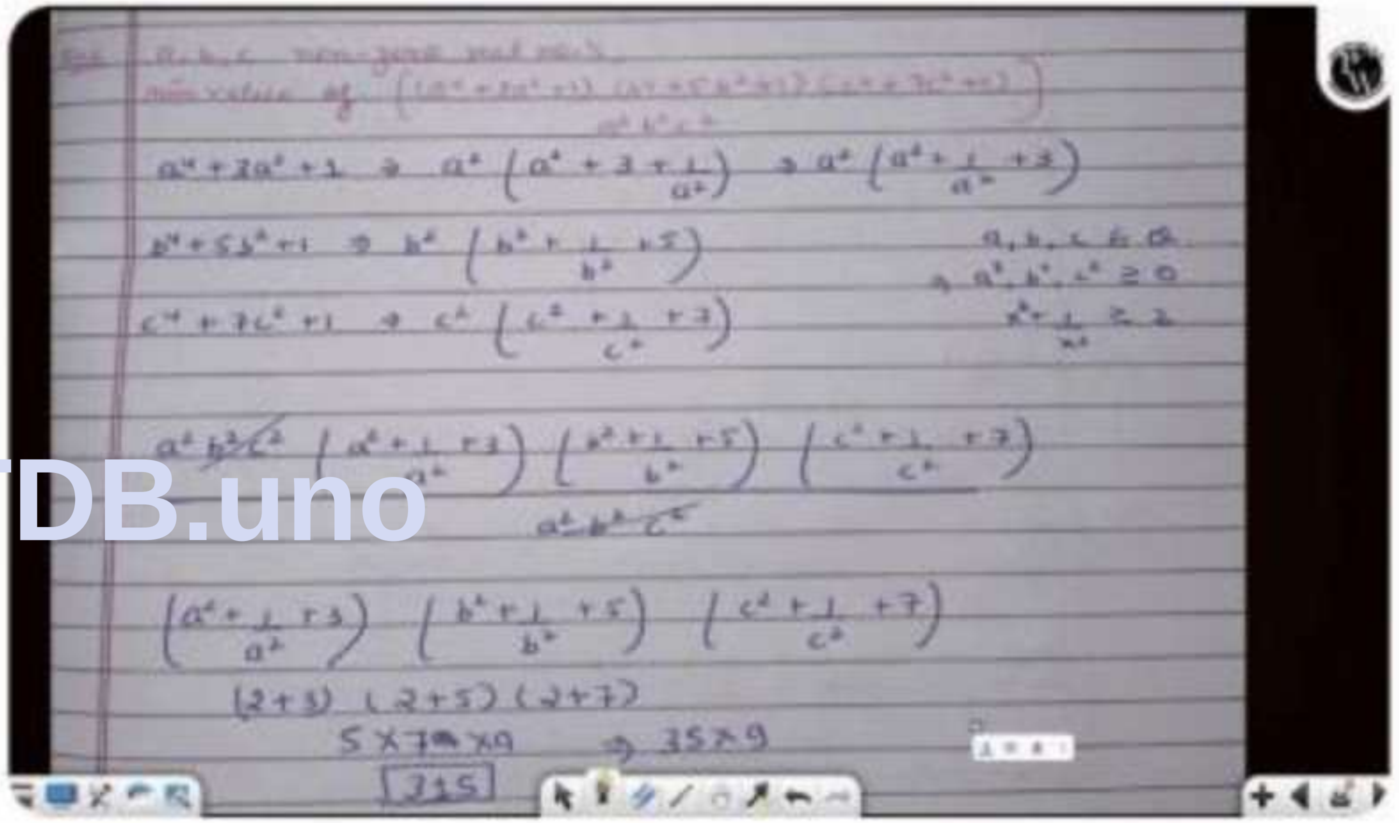
Appke Doubts Humaray Solutions



Doubt 21 hours ago

Sir numerator minimum aur denominator maximum karne par hoga?

2 Same Doubts



Doubt 21 hours ago

sir a b c ko 1 rakhne pe ans aa ja raha h , is I m wrong anywhere?

0 Same Doubts

QUESTION

Given that $x^2 + y^2 = 8x + 6y + 11$, where x and y are integers. What is the smallest possible value of $|4x - 2y|$.

Diamond Points to Note

P$_4$: $\sqrt{x^2} = |x|$ Square root of a positive real number is always positive

❖ $\sqrt{\text{Zero}} = \sqrt{0} = 0$

$\sqrt[3]{-8} = -2$, $\sqrt[2]{-4}$ — Not defined in real

$\sqrt[5]{-32} = -2$

★ $\sqrt{x}$ or $\sqrt[2n]{x}$ is defined in real No: only if $x \geq 0$

★ $\sqrt{x}$ or $\sqrt[2n]{x} \geq 0 \ \forall\ x \geq 0$

★ $\sqrt{x^2} = |x|$, $\sqrt[4]{x^4} = |x|$ - - , $\sqrt[2n]{x^{2n}} = |x|$

★ $\sqrt[3]{x}$, $\sqrt[5]{x}$, - - . $\sqrt[2n+1]{x}$ is defined $\forall\ x \in R$

★ $\sqrt[2n+1]{x} = \begin{cases} +ve & x > 0 \\ -ve & x < 0 \\ 0 & x = 0 \end{cases}$

★ $\sqrt[2n+1]{x^{2n+1}} = x$

$\sqrt{4} = ?$

(-2)

Ex: $\sqrt{(-4)^2} \neq -4$ → sahi $\sqrt{(-4)^2} = |-4| = 4$

b'coz $\sqrt{(-4)^2} = \sqrt{16} = 4$

Ex: $\sqrt[6]{(-3)^6} = |-3| = 3$.

Ex: $\sqrt[5]{(-4)^5} = -4$

Ex: $\sqrt[7]{(-6)^7} = -6$

Ex $\sqrt[3]{2^3} = 2$

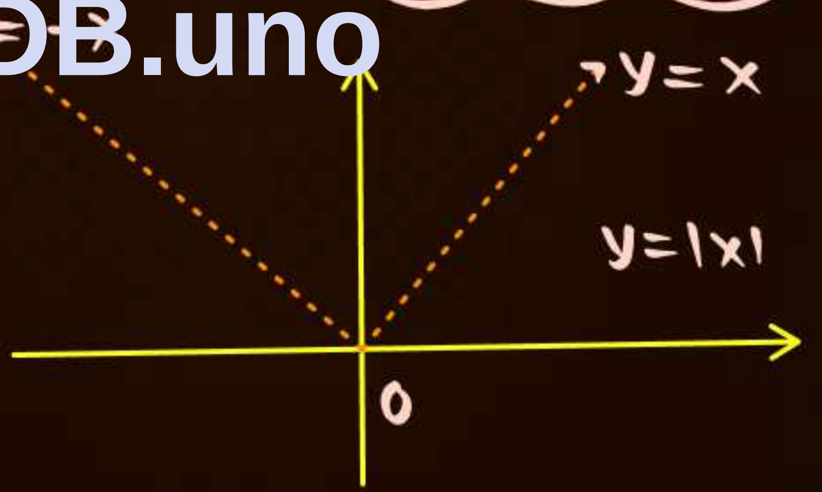
$\sqrt[2n]{x}$ stands for non-ve no. whose $(2n)^{th}$ power is x

$\sqrt[2n+1]{x}$ stands for a real no. whose $(2n+1)^{th}$ power is x

NICHOD!!

- $\sqrt[2n]{x} = y \geq 0$ i.e. even root of any non negative real is non negative.
- $\sqrt{x^2} = |x|$
- $\sqrt[4]{x^4} = |x|, \sqrt[6]{x^6} = |x|$ ....
- $\sqrt[2n]{x^{2n}} = |x|, n \in N$
- $\sqrt[3]{x^3} = x, \sqrt[5]{x^5} = x$ ....
- $\sqrt[2n+1]{x^{2n+1}} = x, n \in N$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$



* $|x| \geq 0$
* $|x| = x \Leftrightarrow x \geq 0$
* $|x| = -x \Leftrightarrow x \leq 0$

IQ Test

- ⭐ $|x| > x$ if $x \in$ $\underline{(-\infty, 0)}$
- ⭐ $|x| < x$ if $x \in$ $\underline{\phi}$
- ⭐ $|x| = 2$ if $x =$ $\underline{\pm 2}$
- ⭐ $|x| = \sqrt{2}$ if $x =$ $\underline{\pm\sqrt{2}}$
- ⭐ $|x| = -1$ if $x =$ Not possible.

Ex: $|-2| > -2$

$|-5| > -5$

$|-2| = 2$ "School mai"

$\downarrow$

$|-2| = -(-2) = 2$ "IIT mai"

↳ −ve

Ex: $-\sqrt{2}$ $\qquad$ $\sqrt{-2}$

-1.414 $\qquad$ $(1.414\ldots)i$

Yaad Rakhnaa

When quantity inside modulus is non-negative, it comes out as it is, and when the quantity inside modulus is negative, it comes out with minus sign

Teacher

"Find the value of $\sqrt{7-4\sqrt{3}}+\sqrt{7+4\sqrt{3}}$"

Kallu: $\sqrt{2^2+\sqrt{3}^2-2\cdot2\cdot\sqrt{3}}+\sqrt{2^2+\sqrt{3}^2+2\cdot2\cdot\sqrt{3}}$

$\sqrt{(2-\sqrt{3})^2}+\sqrt{(2+\sqrt{3})^2}$

$= 2-\sqrt{3}+2+\sqrt{3}=4$

Dono galat

Tillu: $\sqrt{\sqrt{3}^2+2^2-2\cdot2\sqrt{3}}+\sqrt{\sqrt{3}^2+2^2+2\cdot\sqrt{3}\cdot2}$

$\sqrt{(\sqrt{3}-2)^2}+\sqrt{(\sqrt{3}+2)^2}$

$=\underbrace{|\sqrt{3}-2|}_{-ve}+\underbrace{|\sqrt{3}+2|}_{+ve}$

$=-(\sqrt{3}-2)+\sqrt{3}+2$

$=4$

Lallu: $\sqrt{\sqrt{3}^2+2^2-2\cdot2\cdot\sqrt{3}}+\sqrt{\sqrt{3}^2+2^2+2\cdot2\cdot\sqrt{3}}$

$\sqrt{(\sqrt{3}-2)^2}+\sqrt{(\sqrt{3}+2)^2}$

$=\sqrt{3}-2+\sqrt{3}+2$

$=2\sqrt{3}.$

QUESTION

$$\frac{2}{\sqrt{2}} = \frac{\sqrt{2}\times\sqrt{2}}{\sqrt{2}} = \sqrt{2}$$

Let $n = \sqrt{6+\sqrt{11}} + \sqrt{6-\sqrt{11}} - \sqrt{22}$, then

(A) $n \geq 1$

(B) $0 < n < 1$

(C) $n = 0$

(D) $-1 < n < 0$

$$n = \sqrt{6+\sqrt{11}} + \sqrt{6-\sqrt{11}} - \sqrt{22}$$

$$n = \frac{\sqrt{2}\cdot\sqrt{6+\sqrt{11}}}{\sqrt{2}} + \frac{\sqrt{2}\cdot\sqrt{6-\sqrt{11}}}{\sqrt{2}} - \sqrt{22}$$

$$= \frac{\sqrt{12+2\sqrt{11}} + \sqrt{12-2\sqrt{11}}}{\sqrt{2}} - \sqrt{22}.$$

$$= \frac{\sqrt{\sqrt{11}^2+1^2+2\cdot\sqrt{11}\cdot 1} + \sqrt{\sqrt{11}^2+1^2-2\sqrt{11}}}{\sqrt{2}} - \sqrt{22}$$

$$= \frac{|\overset{+ve}{\sqrt{11}+1}| + |\overset{+ve}{\sqrt{11}-1}|}{\sqrt{2}} - \sqrt{22}. = \frac{\sqrt{11}+\cancel{1}+\sqrt{11}-\cancel{1}}{\sqrt{2}} - \sqrt{22}$$

$$= \frac{2\sqrt{11}}{\sqrt{2}} - \sqrt{22} = \sqrt{2}\times\sqrt{11} - \sqrt{22} = \sqrt{22}-\sqrt{22}$$

$n = 0$

QUESTION

Ta h01

If $x = \sqrt{33 - 20\sqrt{2}}$ & $y = \sqrt{54 - 20\sqrt{2}}$ then value of $x - y$ is equal to

(A) $3(1 + \sqrt{2})$

(B) $7(\sqrt{2} - 1)$

(C) $\dfrac{-7}{1 + \sqrt{2}}$

(D) $7(1 + \sqrt{2})$

Ans. C

QUESTION

Tah 02

If $S_n = \frac{1}{\sqrt{1}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{10}} +$ ..... n terms then -

(A) $S_8 = \frac{4}{3}$

(B) $S_{16} = 2$

(C) $S_{33} = 3$

(D) $S_{40} = \frac{10}{3}$

QUESTION

Let $0 < x < 1$ then $\sqrt{(x-1)^2} + \sqrt[4]{(2x+1)^4} - \sqrt[3]{\left(x-\frac{1}{2}\right)^3}$ is equal to

(A) $\frac{5}{2}$

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) dependent of x

let $V = \underbrace{|x-1|}_{-ve} + |2x+1| - \left(x-\frac{1}{2}\right)$

Now $0 < x < 1 \Rightarrow x-1 = -ve$

$0 < x < 1 \Rightarrow 0 < 2x < 2 \Rightarrow 2x+1 = +ve$

$V = -(\cancel{x}-1) + 2\cancel{x}+1 - \cancel{x}+\frac{1}{2}$

$V = 5/2$

Ans. A

QUESTION

Tah03

If $x = \sqrt{2+\sqrt{3}} + \sqrt{4-\sqrt{15}}$ then value of $\sqrt{2}x$ is equal to

(A) $\sqrt{5}-\sqrt{3}$

(B) $\sqrt{5}-1$

(C) $\sqrt{3}+\sqrt{5}$

(D) $\sqrt{5}+1$

Ans. D

QUESTION

Let $x = \sqrt[3]{2+\sqrt{5}} + \sqrt[3]{2-\sqrt{5}}$, then $x^3 + 3x$ is equal to

CBS

$$x^3 = 2+\cancel{\sqrt{5}}+2-\cancel{\sqrt{5}} + 3\cdot\sqrt[3]{2+\sqrt{5}}\cdot\sqrt[3]{2-\sqrt{5}}\underbrace{\left(\sqrt[3]{2+\sqrt{5}}+\sqrt[3]{2-\sqrt{5}}\right)}_{x}$$

$$x^3 = 4 + 3\cdot\sqrt[3]{4-5}\cdot x$$

$$x^3 = 4 + 3\sqrt[3]{-1}\cdot x = 4-3x$$

$$x^3+3x=4$$

(A) 1

(B) 2

(C) 3

(D) 4

Ans. D

QUESTION

Tah 04

If $a = \sqrt{6+2\sqrt{5}} - \sqrt{6-2\sqrt{5}}$; $b = \sqrt[3]{6\sqrt{3}+10} + \sqrt[3]{10-6\sqrt{3}}$, then the value of (ab) is equal to

(A) 8

(B) 12

(C) 4

(D) 6

QUESTION

If $\underbrace{|x^2 - 1|}_{\geq 0} + \underbrace{(x - 1)^2}_{\geq 0} + \underbrace{\sqrt{x^2 - 3x + 2}}_{\geq 0} = 0$, then value of x is :

(A) 1

(B) 4

(C) –2

(D) None of these

$|x^2 - 1| = 0 \rightarrow x^2 - 1 = 0 \Rightarrow x = \pm 1$

& $(x-1) = 0 \rightarrow x = 1$

& $x^2 - 3x + 2 = 0 \rightarrow x = 1, 2$

$x = 1$

QUESTION

The number of real solutions of the equation $(x-1)^4 + (x-2)^4 + (x-3)^4 = 0$, is

$\geq 0 \quad \geq 0 \quad \geq 0$

(A) 4

(B) 2

(C) 1

(D) 0

$(x-1)^4 = 0 \;—\; x=1$

$\&\ (x-2)^4 = 0 \;—\; x=2$

$\&\ (x-3)^4 = 0 \;—\; x=3$

$\cap \Rightarrow x \in \phi$

QUESTION

Find the number of solutions for the equation $|x-3|^2+|x-4|+x^2+7=0$.

Ashish Sir's Novel Concepts (ASNC)

Simon's Factoring Technique

① $pq - p - q = p(q-1) - q + 1 - 1$

$= p(q-1) - 1(q-1) - 1$

$= (p-1)(q-1) - 1$

② $pq + p + q = p(q+1) + q + 1 - 1$

$= p(q+1) + 1\cdot(q+1) - 1$

$= (p+1)(q+1) - 1$

QUESTION

If $m, n \in \mathbb{N}$ then find the number of ordered pairs (m, n) such that $\frac{2}{m} + \frac{2}{n} = 1$.

M(1)

Ans: 3 ordered pairs

$$\frac{2}{m} + \frac{2}{n} = 1$$

$$\frac{2n+2m}{mn} = 1$$

$$2m+2n = mn$$

$$2m+2n-mn = 0$$

$$2m - n(m-2) = 0$$

$$2m-4+4-n(m-2) = 0$$

$$2(m-2) - n(m-2) + 4 = 0$$

$$(2-n)(m-2) = -4$$

$$-(n-2)(m-2) = -4$$

$$(m-2)(n-2) = 4$$

$m-2$	$n-2$	(m,n)
1	4	(3, 6) ✓
4	1	(6, 3) ✓
2	2	(4, 4) ✓
−4	−1	~~(−2, 1)~~
−1	−4	~~(1, −2)~~
−2	−2	~~(0, 0)~~

QUESTION

If m, n ∈ N then find the number of ordered pairs (m, n) such that $\frac{2}{m} + \frac{2}{n} = 1$.

M2

$$\frac{2}{m} + \frac{2}{n} = 1$$

$$\frac{2n+2m}{mn} = 1$$

$$2m + 2n = mn$$

$$2n = mn - 2m$$

$$2n = m(n-2)$$

$$m = \frac{2n}{n-2} = \frac{2n-4+4}{n-2} = \frac{2(n-2)}{n-2} + \frac{4}{n-2}$$

$$m = 2 + \frac{4}{n-2}$$

$$m = 2 + \frac{4}{n-2}$$

$n-2$ should be a divisor of 4

$$n-2 = -1, -2, -4, 1, 2, 4$$

$$n = 1, 0, -2, 3, 4, 6.$$

$$m = -2, -, -, 6, 4, 3$$

$$(m,n) = (6,3), (3,6) (4,4)$$

QUESTION

Tan05

If x & y are positive integers, such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{6}$ & $x \geq y$, then the number of ordered pairs of (x, y) is

Polynomials

Ex: $p(x) = x^3 - 6x^2 + \frac{7}{x} - 3$ ✗ $\quad 7 \cdot x^{-1}$

Ex: $p(x) = x^6 - 6x^5 - 4x^3 + 7x + \sqrt{x}$ ✗ $\quad x^{\frac{1}{2}}$

An algebraic expression of the for

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x^1 + a_0 x^0, \text{ where}$$

(i) $a_n \neq 0$

(ii) power of x is whole number, is called a polynomial in one variable.

a_n = leading coeff
a_0 = constant term / absolute term.

Hence, $a_n, a_{n-1}, a_{n-2}, \ldots, a_0$ are coefficients of $x^n, x^{n-1}, \ldots, x^0$ respectively and $a_n x^n, a_{n-1} x^{n-1}, a_{n-2} x^{n-2}, \ldots$ are terms of the polynomial. Here the term $a_n x^n$ is called the Leading term and its coefficient a_n, the leading coefficient.

If leading coefficient is '1' then the polynomial is called as monic polynomial.

Ex: $p(x) = x^7 - 4x^3 + 3x + \pi$

yes a polynomial

7 = degree

leading coeff = 1

absolute term = π

Yes also a monic poly.

Degree of Polynomial

Degree of the polynomial in one variable is the largest exponent of the variable.

For example, the degree of the polynomial $3x^7 - 4x^6 + x + 9$ is 7 and the degree of the polynomial $5x^6 - 4x^2 - 6$ is 6.

Degree of Polynomial

$0 \cdot x^7, 0 \cdot x^3, 0x^{10}$

Polynomials classified by degree

Degree	Name	General form	Example
(undefined)	Zero polynomial	0	0
0	(Non-zero) constant polynomial	$a; (a \neq 0)$	1
1	Linear polynomial	$ax + b; (a \neq 0)$	$x + 1$
2	Quadratic polynomial	$ax^2 + bx + c; \ (a \neq 0)$	$x^2 + 1$
3	Cubic polynomial	$ax^3 + bx^2 + cx + d; (a \neq 0)$	$x^3 + 1$

Usually, a polynomial of degree n, for n greater than 3, is called a polynomial of degree n, although the phrases quadratic polynomial and quintic polynomial are sometimes used.

Remainder & Factor Theorem

Divisor	Rem
$x+a$	$P(-a)$
$x-a$	$P(a)$
$ax+b$	$P(-b/a)$

Remainder Theorem

Let P(x) be a polynomial of degree ≥ 1 and 'a' is any real number. If P(x) is divided by (x − a), then the remainder is P(a).

Ex: find remainder when $P(x) = x^4 - x^3 + 3x^2 - 2x + 1$ is divided by

a) $x-1 \longrightarrow$ Rem $= P(1) = 1-1+3-2+1 = 2$

b) $x+1 \longrightarrow$ Rem $= P(-1) = 1+1+3+2+1 = 8$

Factor Theorem

Let P(x) be a polynomial of degree ≥ 1 and 'a' be any real constant such that P(a) = 0, then (x − a) is a factor of P(x). Conversely, if (x − a) is a factor of P(x), then P(a) = 0.

Quotient

$$x-a\ \overline{)\ P(x)\ }\ \text{— dividend}$$

$Q(x)$ — Quotient

divisor — $x-a$

$r(x)$ — Rem.

proof: $P(x) = (x-a)\,Q(x) + \text{Rem}$

put $x=a$

$P(a) = 0\cdot Q(a) + \text{Rem}$

$P(a) = \text{Remiander}$

Remainder Theorem

Dividend = Quotient × Divisor + Rem

degree Rem < degree of divisor

Factor Thm

If Rem = 0 i.e $P(a) = 0$

$\Downarrow$

$(x-a)$ is a factor of $P(x)$

Note:

Let $P(x)$ be any polynomial of degree greater than or equal to one. If leading coefficient of $P(x)$ is 1 then $P(x)$ is called monic. (Leading coefficients means coefficients of highest power.)

Don’t Forget to

Retry all the class illustrations

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

(KTK 1)

The expression $\sqrt{12+6\sqrt{3}}+\sqrt{12-6\sqrt{3}}$ simplifies to

A. 4

B. $2\sqrt{3}$

C. $3\sqrt{3}$

D. 6

Ans. D

QUESTION

(KTK 2)

Let p, q be real numbers satisfying $p^2 - q^2 = 4$ and $2pq = 3$ then $(p^2 + q^2)$ is equal to

(A) 1

(B) 9

(C) 16

(D) 5

Ans. D

QUESTION

(KTK 3)

Value of x satisfying the equation $\sqrt{x^2 + 2x - 63} + |x^2 - 9x + 14| = 0$ is

QUESTION

(KTK 4)

The expression $\sqrt{(28+10\sqrt{3})}+\sqrt{(28-10\sqrt{3})}$ simplifies to

(A) 10

(B) 12

(C) $2\sqrt{3}$

(D) 5

Ans. A

QUESTION

(KTK 5)

Find all the integral solutions of the equation $xy = 2x - y$.

Ans. (0, 0), (−2, 4), (1, 1), (−3, 3)

Solution to Previous TAH

QUESTION

If a, b, c are distinct real numbers such that $a^2 - b = b^2 - c = c^2 - a$, then $(a + b)(b + c)(c + a) =$ ______________

PAGE NO.

TAH 01

Given: $\underbrace{a^2-b = b^2-c}_{\Downarrow} = c^2-a$

we have,

$$a^2-b = b^2-c$$

$$a^2-b^2 = b-c$$

$$(a+b)(a-b) = b-c \Rightarrow a+b = \frac{b-c}{a-b}$$

Ily, $b+c = \dfrac{c-a}{b-c}$ and $a+c = \dfrac{b-a}{a-c}$

Now, $(a+b)(b+c)(c+a) = \dfrac{(b-c)(c-a)(b-a)}{(b-c)(a-b)(a-c)} = 1$ Ans

Q-1! If a, b, c are distinct real numbers such
TAH-1 that $a^2-b=b^2-c=c^2-a$, then
$(a+b)(b+c)(c+a)=?$

Soln: $a^2-b=b^2-c=c^2-a$.

$a^2-b=b^2-c \Rightarrow a^2-b^2=b-c$ ——(i)
or, $(a-b)(a+b)=(b-c)$ —— (i)

$b^2-c=c^2-a \Rightarrow b^2-c^2=c-a$
or, $(b+c)(b-c)=(c-a)$ —— (ii)

$a^2-b=c^2-a \Rightarrow a^2-c^2=b-a$
or, $(a+c)(a-c)=(b-a)$ —— (iii)

(i) $\times$ (ii) $\times$ (iii):

$(a+b)(b+c)(a+c)(a-b)(b-c)(a-c)$
$=(b-c)(c-a)(b-a)$

or, $(a+b)(b+c)(c+a)\cancel{(a-b)}\cancel{(b-c)}\cancel{(c-a)}\times\cancel{(-1)}$
$=\cancel{(b-c)}\cancel{(c-a)}\cancel{(a-b)}\times\cancel{(-1)}$

or, $(a+b)(b+c)(c+a)=1$

[$\because a, b, c$ are distinct
$\Downarrow$
$a-b\neq b-c\neq c-a \neq 0$]

$\therefore$ Ans. $=1$.

TAH 01
SAYANTAN MANNA

QUESTION

If x, y & z are three real numbers such that $x^2 + 4y^2 + 9z^2 - 2x - 4y - 6z + 3 = 0$ then find the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$.

*. TRH-02 :-

Sol^n

$$x^2 + 4y^2 + 9z^2 - 2x - 4y - 6z + 3 = 0$$

$3 \to 1 + 1 + 1$ (split)

$$\Downarrow$$

$$\underbrace{x^2 - 2x + 1} + \underbrace{4y^2 - 4y + 1} + \underbrace{9z^2 - 6z + 1} = 0$$

$$(x-1)^2 + (2y-1)^2 + (3z-1)^2 = 0$$

$x - 1 = 0$	$2y - 1 = 0$	$3z - 1 = 0$
$x = 1$	$y = 1/2$	$z = 1/3$

Find

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{1} + 2 + 3 = 6$$

2) Tah 02

If x, y & z are three real numbers such that $x^2+4y^2+9z^2-2x-4y-6z+3=0$ then find the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$.

$$x^2-2x+1+4y^2-4y+1+9z^2-6z+1=0$$

$$(x-1)^2+(2y-1)^2+(3z-1)^2=0$$

$x-1=0$ $\quad$ $2y-1=0$ $\quad$ $3z-1=0$

$\underline{\underline{x=1}}$ $\quad$ $y=1/2$ $\quad$ $z=1/3$.

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{1}+\frac{1}{1/2}+\frac{1}{1/3}$$

$$=1+2+3$$

$$=6$$

Q-2: If x, y, z are three real numbers such that $x^2+4y^2+9z^2-2x-4y-6z+3=0$; then find the value of $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$.

Soln: $x^2+4y^2+9z^2-2x-4y-6z+3=0$.

or, $x^2-2x+4y^2-4y+9z^2-6z+3=0$.

or, $x^2-2\cdot x\cdot 1+1^2-1^2+(2y)^2-2\cdot 2y\cdot 1+1^2-1^2$
$+(3z)^2-2\cdot 3z\cdot 1+1^2-1^2+3=0$.

or, $(x-1)^2-1+(2y-1)^2-1+(3z-1)^2-1+3=0$.

or, $(x-1)^2+(2y-1)^2+(3z-1)^2-\cancel{3}+\cancel{3}=0$.

or, $\underbrace{(x-1)^2}_{\geq 0}+\underbrace{(2y-1)^2}_{\geq 0}+\underbrace{(3z-1)^2}_{\geq 0}=0$.

$\therefore x=1$ | $2y=1$, or, $y=\frac{1}{2}$ | $3z=1$, or, $z=\frac{1}{3}$

$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z}$

$=\frac{1}{1}+\frac{1}{1/2}+\frac{1}{1/3}$

$=1+2+3$

$=6$ (Ans.)

TAH 02
SAYANTAN MANNA
WEST BENGAL

QUESTION

Let a, b, c are real numbers and satisfy $a = 8 - b$ and $c^2 = ab - 16$, then $\frac{a}{b}$ is equal to

TAH 3

$$a = 8 - b \xrightarrow{Sq} (a+b)^2 = 8^2$$

$$a^2 + b^2 + 2ab = 64 \quad \text{— (I)}$$

$$c^2 = ab - 16 \Rightarrow ab = c^2 + 16 \quad \text{— (II)}$$

put ab in (I) eq

$$a^2 + b^2 + 2(c^2 + 16) = 64$$

$$\Rightarrow (a-b)^2 + 2ab + 2(c^2 + 16) = 64$$

$$\Rightarrow (a-b)^2 + 4c^2 + 64 = 64$$

$$\Rightarrow (a-b)^2 + 4c^2 = 0$$

$$\therefore a = b \quad \text{and} \quad c = 0$$

$$\boxed{\frac{a}{b} = 1} \text{ Ans.}$$

Ash Man Maity

3) Tan 08

Let a, b, c are real numbers and each of $a = 8 - b$ and $c^2 = ab - 16$, then $\frac{a}{b}$ is equal to

$$c^2 = (8-b)b - 16$$
$$c^2 = 8b - b^2 - 16$$
$$c^2 = -b^2 + 8b - 16$$
$$c^2 = -(b^2 - 8b + 16)$$

$c^2 \in R$, $c^2 \geq 0$

$$-(b^2 - 8b + 16) \geq 0$$
$$b^2 - 8b + 16 \geq 0$$

S = -8
P = 16
-4 -4

$$(b-4)(b-4) \geq 0$$
$$(b-4)^2 \geq 0$$
$$b - 4 = 0$$
$$b = 4$$

$a = 8 - b$
$a = 8 - 4$
$= 4$ //

$\frac{a}{b} = \frac{4}{4} = 1$ //

Solution to Previous KTKs

QUESTION

(KTK 1)

a, b, c are reals such that $a + b + c = 3$ and $\frac{1}{a+b} + \frac{1}{b+c} + \frac{1}{c+a} = \frac{10}{3}$.
The value $E = \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$ is

(A) 9

(B) 7

(C) 5

(D) 3

(KT-K-1)

a, b, c are reals such that $a+b+c=3$ and $\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{10}{3}$ Then value

$$E=\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b} \text{ is}$$

Abhishek Kumar

Sol: → $a+b+c=3$ — (i)

$$\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{10}{3} \text{ — (ii)}$$

eq (i)×(ii)

$$1+\frac{c}{(a+b)}+1+\frac{a}{(b+c)}+1+\frac{b}{(a+c)}=10$$

$$\frac{a}{(b+c)}+\frac{b}{(a+c)}+\frac{c}{(a+b)}=10-3$$

$$=7$$

(B)

[redacted]

a, b, c are reals such that $a+b+c=3$ and ~~a+b~~ Separate

$\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{10}{3}$, Find $E=\frac{a}{a+c}+\frac{b}{c+a}+\frac{c}{a+b}$

Sol: $$E=\frac{a}{a+c}+\frac{b}{c+a}+\frac{c}{a+b}$$

$$\rightarrow \frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}=\frac{10}{3} \text{ given}$$

Soumay Pandey

$$\rightarrow \frac{a+b+c}{a+b}+\frac{a+b+c}{b+c}+\frac{a+b+c}{c+a}=10$$

$\{\because a+b+c=3\}$

$$\rightarrow \left(\frac{a+b}{a+b}+\frac{c}{a+b}\right)+\left(\frac{a}{b+c}+\frac{b+c}{c+c}\right)+\left(\frac{a+c}{c+a}+\frac{b}{c+a}\right)=10$$

$$\rightarrow \left(1+\frac{c}{a+b}\right)+\left(1+\frac{a}{b+c}\right)+\left(1+\frac{b}{c+a}\right)=10$$

Finally

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}=10-3=7 \text{ Ans}$$

QUESTION

(KTK 2)

Solve the equations : $\begin{cases} 2^x + 3^y = 41 \\ 2^{x+2} + 3^{y+2} = 209 \end{cases}$

Ans. x = 5 and y = 2

Q-3!- Solve the equations!

$$\begin{cases} 2^x + 3^y = 41 \\ 2^{x+2} + 3^{y+2} = 209 \end{cases}$$

Soln:-

$2^x + 3^y = 41$ ——— (i)

$2^{x+2} + 3^{y+2} = 209$

or, $2^x \times 4 + 3^y \times 9 = 209$

or, $4 \cdot 2^x + 9 \cdot 3^y = 209$ ——— (ii)

Let $2^x = t$, $3^y = s$.

$t + s = 41$ ——— (i) $\times 4$

$4t + 9s = 209$ ——— (ii)

$4t + 4s = 164$

$4t + 9s = 209$

(Subtract) $5s = 45$

or, $\boxed{s = 9}$

or $3^y = 9$

or, $3^y = 3^2$

or, $y = 2$.

$\therefore t = 41 - 9$

or, $\boxed{t = 32}$

or, $2^x = 32$

or, $2^x = 2^5$

or, $x = 5$

$\therefore$ Ans. $\Rightarrow x = 5,\ y = 2$.

KTK 2
SAYANTAN MANNA
WEST BENGAL

QUESTION

(KTK 3)

What is the area of an equilateral triangle inscribed in a circle of radius 4 cm?

(A) $12\ cm^2$

(B) $9\sqrt{3}\ cm^2$

(C) $8\sqrt{3}\ cm^2$

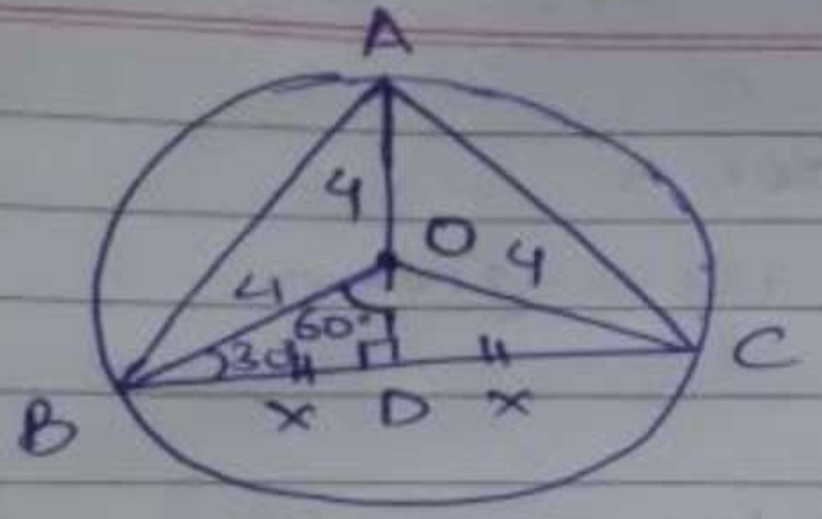
(D) $12\sqrt{3}\ cm^2$

Ans. D

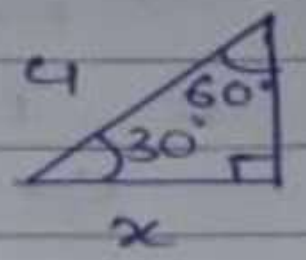
KTK 03 :- lect 3

Date ______
Page ______

Q What is the area of an equilateral triangle inscribed in circle of radius of 4cm?



In $\triangle OBD$,



$$\sin 60° = \frac{P}{H} = \frac{x}{4}$$

$$\frac{\sqrt{3}}{2} = \frac{x}{4}$$

$$\boxed{x = 2\sqrt{3}}$$

Now

$$BC = 2x = 2(2\sqrt{3}) = 4\sqrt{3}$$

$$\text{Area of } \triangle ABC = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} \cdot 4\sqrt{3} \cdot 4\sqrt{3}$$

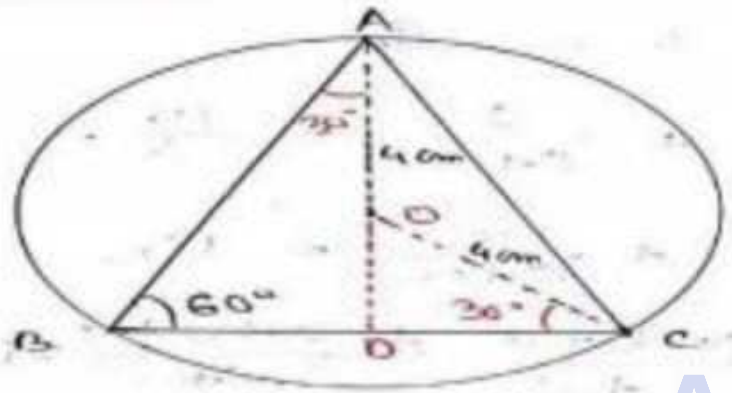
$$= 12\sqrt{3} \text{ ans}$$

Q-2: What is the area of an equilateral triangle inscribed in a circle of radius 4 cm?

(a) $12\ cm^2$ (b) $9\sqrt{3}\ cm^2$ (c) $8\sqrt{3}\ cm^2$ (d) $12\sqrt{3}\ cm^2$

Soln:

→ method-1:



in ΔCOD,

$\sin 30° = \frac{OD}{OC}$

or, $\frac{1}{2} = \frac{OD}{4}$

or, $OD = 2$ cm.

Height of ΔABC
$= AD$
$= AO + OD$
$= 4 + 2 = 6$ cm.

in ΔCOD,

$\cos 30° = \frac{CD}{OC}$

or, $\frac{\sqrt{3}}{2} = \frac{CD}{4}$

or, $CD = 2\sqrt{3}$

side of triangle ABC $= BC$
$= BD + CD$
$= 2 \times CD$
$= 2 \times 2\sqrt{3}$
$= 4\sqrt{3}$ cm.

$\therefore$ Area $= \frac{1}{2} \times$ Base $\times$ Height
$= \frac{1}{2} \times 6 \times 4\sqrt{3}$.
$= 12\sqrt{3}\ cm^2$.

$\therefore$ Ans ⇒ (d) $12\sqrt{3}\ cm^2$

Mann ki Baat

- Masum Gadhaa — A — 5.91 %
- Gadhaa Jo khud ko Godha samajhay — ~~B~~ — 4.1 %
- Voh Gadhaa jisay maloom hai — C — 76 %
- Voh gadhaa. — selection.
- Ghodaa — selection — D — 14 %


THANK
YOU