

PRAAYAS

JEE 2026

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Mathematics

Basic Maths

Lecture - 05

By - Ashish Agarwal Sir
(IIT Kanpur)



Topics *To be covered*



- A** Polynomials and their Factorization
- B** An Important Point

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Recap of previous lecture



Fill in the Blanks:

1. $x^2 + y^2 + 16z^2 - 4x - 4y + 8z + 9 = 0, x, y, z \in \mathbb{R}$ then $x + y + \frac{1}{z} = \underline{0}$

$$x^2 - 4x + 4 + y^2 - 4y + 4 + (4z)^2 + 2 \cdot 4z \cdot 1 + 1^2 = 0$$

$$(x-2)^2 + (y-2)^2 + (4z+1)^2 = 0 \Rightarrow x=2, y=2, z=-1/4$$

2. $(x^2 + 1) + \frac{1}{(x^2+1)}$ has minimum value 2 which is attained at

$$x = \underline{0}$$

$$x^2 + 1 = 1$$

$$\Downarrow \\ x^2 = 0$$

3. $(x^2 - 4)^2 + (y - 2)^4 + (z - \cot \pi/2)^6 = 0$ then $x + y + z$ can be 0, 4

$$x^2 = 4, y = 2, z = \cot \pi/2 = 0$$

$$x = -2, 2 \quad \swarrow \searrow \\ x + y + z = 4, 0$$

Recap *of previous lecture*



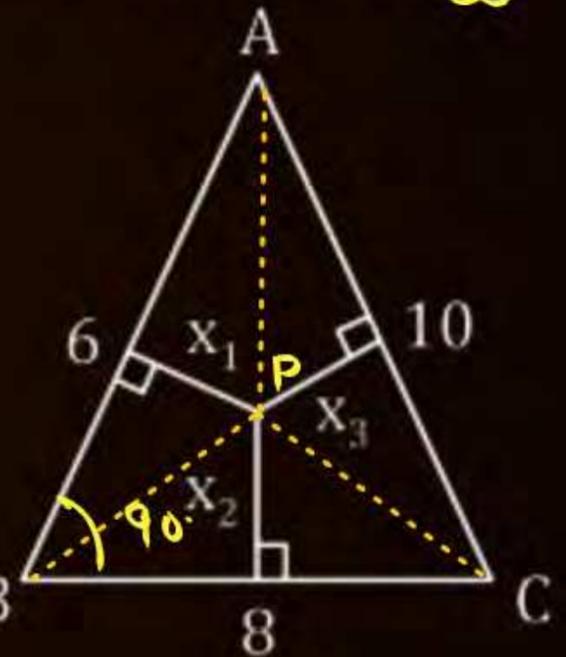
4. The sides of a $\triangle ABC$ are as shown in the figure. Let P be any internal point of this triangle and x_1, x_2 & x_3 denote the distance between P and sides of triangle. The value of $\left(\frac{3x_1+4x_2+5x_3}{6}\right)$ is

$6^2 + 8^2 = 10^2 \Rightarrow \angle ABC = \frac{\pi}{2}$

$ar \triangle BPC = \frac{1}{2} \cdot 8 \cdot x_2$
 $ar \triangle APC = \frac{1}{2} \cdot 10 \cdot x_3$
 $ar \triangle APB = \frac{1}{2} \cdot 6 \cdot x_1$

$\xrightarrow{+}$
 $3x_1 + 4x_2 + 5x_3$
 $= ar(\triangle ABC)$
 $= \frac{1}{2} \cdot 6 \cdot 8 = 24$

$\frac{3x_1 + 4x_2 + 5x_3}{6} = 4$



Recap *of previous lecture*



5. Perpendicular bisector of chord of a circle passes through it's Centre
6. Area of equilateral triangle of height $h \Rightarrow \underline{\Delta = \frac{h^2}{\sqrt{3}}}$
7. $P(x) = x^3 - 6x^2 + 11x - 6$ has $x-1$, $x-2$ & $x-3$ as factors.
 $P(1) = 1 - 6 + 11 - 6 = 0 \Rightarrow (x-1)$ is a factor. $P(3) = 27 - 54 + 33 - 6 = 0 \Rightarrow (x-3)$ is a factor
 $P(2) = 8 - 24 + 22 - 6 = 0 \Rightarrow (x-2)$ is a factor
8. Remainder when a polynomial of degree 5 is divided by a quadratic polynomial is of the form $ax+b$
9. $P(x) = x^4 - 6x^3 + 2x^2 + 2x + 1$ then $P(1) = \underline{1 - 6 + 2 + 2 + 1 = 0}$ means $x-1$ is a factor of $P(x)$.

Recap *of previous lecture*



10. Remainder when a polynomial of degree 11 is divided by a cubic polynomial is of the form $ax^2 + bx + c$

11. We say $K(x)$ is a divisor $P(x)$ if remainder = 0

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12. Remainder when $P(x) = x^3 - x^2 + 1$ is divided by $2x - 3$ is $P(3/2) = (3/2)^3 - (3/2)^2 + 1$

$$P(x) \div ax + b \text{ Rem} = P\left(-\frac{b}{a}\right)$$

$$P(x) \div ax - b \text{ Rem} = P\left(\frac{b}{a}\right)$$

$$= \frac{27}{8} - \frac{9}{4} + 1$$

$$= \frac{27 - 18 + 8}{8} = \underline{\underline{17/8}}$$



Homework Discussion

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QUESTION



TAH 02



If $S_n = \frac{1}{\sqrt{1} + \sqrt{4}} + \frac{1}{\sqrt{4} + \sqrt{7}} + \frac{1}{\sqrt{7} + \sqrt{10}} + \dots$ n terms then -

4, 7, 10, ...
 $T_n = a + (n-1)d$
 $= 4 + (n-1)3 = 3n + 1$

A $S_8 = \frac{4}{3}$

B $S_{16} = 2$

C $S_{33} = 3$

D $S_{40} = \frac{10}{3}$

$S_n = \frac{1}{\sqrt{4} + \sqrt{1}} + \frac{1}{\sqrt{7} + \sqrt{4}} + \frac{1}{\sqrt{10} + \sqrt{7}} + \dots + \frac{1}{\sqrt{3n+1} + \sqrt{3n-2}}$

$S_n = \frac{1}{3} [(\sqrt{4} - 1) + (\sqrt{7} - \sqrt{4}) + (\sqrt{10} - \sqrt{7}) + \dots + (\sqrt{3n+1} - \sqrt{3n-2})]$

$S_n = \frac{\sqrt{3n+1} - 1}{3}$



Ans. A, B, C, D

QUESTION

TAH 05



If x & y are positive integers, such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$ & $x \geq y$, then the number of ordered pairs of (x, y) is

$$\frac{x+y}{xy} = \frac{1}{6}$$

$$6(x+y) = xy$$

$$6x + 6y - xy = 0$$

$$6x - 36 - y(x-6) = 0 - 36$$

$$6(x-6) - y(x-6) = -36$$

$$(x-6)(y-6) = 36$$

$x-6$	$y-6$	(x, y)
6	6	(12, 12)
9	4	(15, 10)
12	3	(18, 9)
36	1	(42, 7)
18	2	(24, 8)

Ans. 5



Aao Machaay Dhamaal Deh Swaal pe Deh Swaal

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QUESTION



If $(x^3 + ax^2 + bx + 6)$ has $(x - 2)$ as a factor and leaves a remainder 3 when divided by $(x - 3)$

Since $x-2$ is a factor of $P(x) = x^3 + ax^2 + bx + 6$

$$\Rightarrow P(2) = 0 \Rightarrow 8 + 4a + 2b + 6 = 0$$

$$2a + b = -7 \quad \text{--- (I)}$$

~~A~~ sum of absolute value of $(a + b)$ is 4

~~B~~ product of ab is 3

~~C~~ difference of a & b is 14

~~D~~ a^b is defined $(-3)^{-1} = \frac{1}{(-3)^1} = -\frac{1}{3}$

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Also $P(3) = 3$ (By Rem Thm)

$$27 + 9a + 3b + 6 = 3$$

$$9a + 3b = -30$$

$$3a + b = -10 \quad \text{--- (II)}$$

$$\begin{aligned} -a &= 3 \\ a &= -3 \\ b &= -1 \end{aligned}$$

$$\text{(A) } |a+b| = |-3-1| = 4$$

Ans. A, B, D

QUESTION

TAHOI



If $81x^5 + 27x^3 - 9x^2 + 50$ is divided by $(3x + 2)$ then remainder is λ then

- A** $\frac{3\lambda - 42}{10}$ is equal to 4
- B** if $\lambda = \frac{p}{q}$ then $(p + q)$ is divisible by 17 (where p & q are coprime)
- C** λ is a natural number
- D** $\left(\lambda - \frac{1}{3}\right)$ is divisible by 3

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Ans. A, B, D

QUESTION



Suppose $p(x)$ is a polynomial with integer coefficients. The remainder when $p(x)$ is divided by $x - 1$ is 1 and the remainder when $p(x)$ is divided by $x - 4$ is 10. If $r(x)$ is the remainder when $p(x)$ is divided by $(x - 1)(x - 4)$, find the value of $r(2006)$.

$$P(x) \div (x-1) \quad \text{Rem} = 1 \quad \text{---} \quad P(1) = 1$$

$$P(x) \div (x-4) \quad \text{Rem} = 10 \quad \text{---} \quad P(4) = 10$$

$$P(x) \div (x-1)(x-4) \quad \text{Rem} = r(x)$$

Rem theorem is applicable for linear Divisors

$$P(x) = Q(x) \cdot (x-1)(x-4) + (ax+b), \quad \text{i.e. } r(x) = ax+b.$$

degree 2

Now put $x=1$

$$P(1) = a + b = 1$$

put $x=4$

$$P(4) = 4a + b = 10$$

$$3a = 9$$

$$a = 3$$

$$b = -2$$

$$r(x) = 3x - 2$$

$$r(2006) = 6018 - 2 = 6016$$

Ans. 6016

QUESTION



Tah02

Let $f(x) = x^{135} + x^{125} - x^{115} + x^5 + 1$. If $f(x)$ is divided by $x^3 - x$ then the remainder is some function of x say $g(x)$. Find the value of $g(10)$.

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Ans. 21

QUESTION



Polynomial $P(x)$ contains only terms of odd degree. When $P(x)$ is divided by $(x - 3)$ the remainder is 6. If $P(x)$ is divided by $(x^2 - 9)$ then remainder is $g(x)$. Find value of $g(2)$.

$P(x)$ — contains terms of only odd degree

Ex: $P(x) = x^7 - 6x^3 + 5x$

$P(x) = x^9 - 5x^3 - 2x$

$P(-x) = -x^7 + 6x^3 - 5x$

$= -(x^7 - 6x^3 + 5x)$

$= -P(x)$

$g(x) = 2x$

$g(2) = 4$

observe!! $P(-x) = -P(x)$

Now

$P(x) \div x-3 \rightarrow \text{Rem} = 6 = P(3)$

$P(x) \div x^2-9 \rightarrow \text{Rem} = ?$

$P(x) = (x^2-9)Q(x) + ax+b$, $g(x) = ax+b$

$P(3) = 0 + 3a + b = 6$

$P(-3) = 0 - 3a + b = -6$

$2b = 0 \rightarrow b = 0$

$a = 2$

$P(-3) = -P(3)$
 $= -6$

Ans. 4



Factors of a Polynomial



$$\bullet \quad ax^2 + bx + c \begin{cases} \alpha \\ \beta \end{cases} \Rightarrow ax^2 + bx + c = a(x - \alpha)(x - \beta)$$

$$(x-1)^2(x-2)=0$$

roots 1, 1, 2

soln : 1, 2

$$\bullet \quad ax^3 + bx^2 + cx + d \begin{cases} \alpha \\ \beta \\ \gamma \end{cases} \Rightarrow ax^3 + bx^2 + cx + d = a(x - \alpha)(x - \beta)(x - \gamma)$$

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$$\bullet \quad ax^4 + bx^3 + cx^2 + dx + e \begin{cases} \alpha \\ \beta \\ \gamma \\ \delta \end{cases} \Rightarrow ax^4 + bx^3 + cx^2 + dx + e = a(x - \alpha)(x - \beta)(x - \gamma)(x - \delta)$$

QUESTION



Find three degree polynomial $P(x)$ whose zeroes are $-\frac{1}{3}$, $-\frac{2}{3}$, 4 and $P(1) = 1$

$$\begin{array}{l}
 P(x) \begin{cases} -\frac{1}{3} \\ -\frac{2}{3} \\ 4 \end{cases} \\
 \downarrow \\
 \text{degree}
 \end{array}$$

$$P(x) = a \cdot \left(x - \left(-\frac{1}{3}\right)\right) \left(x - \left(-\frac{2}{3}\right)\right) (x - 4)$$

$$P(x) = a \left(x + \frac{1}{3}\right) \left(x + \frac{2}{3}\right) (x - 4)$$

$$P(1) = a \left(\frac{4}{3}\right) \left(\frac{5}{3}\right) (-3) = 1$$

$$a \left(\frac{-20}{3}\right) = 1$$

$$a = -\frac{3}{20}$$

$$P(x) = -\frac{3}{20} \left(x + \frac{1}{3}\right) \left(x + \frac{2}{3}\right) (x - 4)$$

QUESTION



If 4 degree polynomial $P(x)$ is such that $P(1) = 1, P(2) = 2, P(3) = 3$ and $P(4) = 4$. Then find $P(5)$ (leading coefficient is 1).

$$g(x) = x^4 + (\)x^3 + (\)x^2 + (\)x + c - x$$

$P(x)$

consider $g(x) = P(x) - x$
 degree $g(x) = 4$
 leading coeff = 1

$$g(1) = P(1) - 1 = 0$$

$$g(2) = P(2) - 2 = 0$$

$$g(3) = P(3) - 3 = 0$$

$$g(4) = P(4) - 4 = 0$$

$$g(x) = 1 \cdot (x-1)(x-2)(x-3)(x-4)$$

$$P(x) - x = (x-1)(x-2)(x-3)(x-4)$$

$$P(x) = (x-1)(x-2)(x-3)(x-4) + x$$

$$P(5) = (5-1)(5-2)(5-3)(5-4) + 5 = 4! + 5 = 29$$

Concept : consider $g(x) = P(x) - x$ Jo apko lagay $P(x)$ hai

Mazdoori naa Kare
 $P(x) = ax^4 + bx^3 + cx^2 + dx + e$
 $P(1) = 1 \Rightarrow a + b + c + d + e = 1$
 $P(2) = 2 \Rightarrow 16a + 8b + 4c + 2d + e = 2$

QUESTION



Tah 03

If $p(x)$ is a polynomial of 3 degree for which $p(1) = 1$, $p(2) = 4$, $p(3) = 9$ then find the value of $p(5)$ and leading coefficient be 2.

$$g(x) = p(x) - x^2$$

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Ans. 73

QUESTION



There is a 4 degree polynomial $p(x)$ with
 $P(1) = \frac{1}{2}, P(2) = \frac{2}{3}, P(3) = \frac{3}{4}, P(4) = \frac{4}{5}$ and $P(5) = \frac{5}{6}$ then find $P(6)$.

$2 \cdot P(1) = 1$ $3 \cdot P(2) = 2$ $4 \cdot P(3) = 3$ $5 \cdot P(4) = 4$ $6 \cdot P(5) = 5$

★★★★KCLS★★★★

Consider $g(x) = h(x) - x \implies$
 $g(x)$ degree = 5

$g(x) = (x+1)P(x) - x$
 $g(1) = 2 \cdot P(1) - 1 = 0$
 $g(2) = 0, g(3) = 0, g(4) = 0, g(5) = 0$

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lagta hai !!
 $P(x) = \frac{x}{x+1}$
 $(x+1)P(x) = x$
 $g(x) = P(x) - \frac{x}{x+1}$
 Not a polynomial
 let $(x+1)P(x) = h(x)$
 $h(x) = x$

$g(x) = a(x-1)(x-2)(x-3)(x-4)(x-5)$
 $(x+1) \cdot P(x) - x = a(x-1)(x-2)(x-3)(x-4)(x-5)$
 $P(x) = \frac{a(x-1)(x-2)(x-3)(x-4)(x-5) + x}{(x+1)}$

But $P(x)$ is a polynomial

Ans. 5/6



$$R(x) = a(x-1)(x-2) \dots (x-5) + x \text{ should have } x+1 \text{ as a factor.}$$

$$\Downarrow$$

$$R(-1) = 0 \rightarrow a(-2)(-3)(-4)(-5)(-6) - 1 = 0$$

$$a = \frac{-1}{720}$$

$$P(x) = \frac{a(x-1)(x-2)(x-3)(x-4)(x-5) + x}{x+1}$$

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$$P(6) = \frac{-\frac{1}{720} \cdot 120 + 6}{7} = \frac{-\frac{1}{6} + 6}{7} = \frac{\frac{35}{6}}{7 \times 6} = \frac{5}{6} \text{ Ans}$$



A Golden Point



$$E = a^2 + b^2 + c^2 - ab - bc - ca \text{ (where } a, b, c \in \mathbb{R}\text{)}$$

$$E = \frac{1}{2} (2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$$

$$= \frac{1}{2} (a^2 + c^2 - 2ac + a^2 + b^2 - 2ab + b^2 + c^2 - 2bc)$$

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$$= \frac{1}{2} \left(\underset{\geq 0}{(a-c)^2} + \underset{\geq 0}{(a-b)^2} + \underset{\geq 0}{(b-c)^2} \right) \geq 0$$

$$\textcircled{1} \quad E = a^2 + b^2 + c^2 - ab - bc - ca \geq 0$$

$$\textcircled{2} \quad a^2 + b^2 + c^2 - ab - bc - ca = 0 \Leftrightarrow a = b = c$$

QUESTION



If a, b, c denotes the length of sides of the triangle such that $9a^2 + 16b^2 + 25c^2 - 12ab - 20bc - 15ac = 0$ then ratio $a : b : c$ is-

~~A~~ 20 : 15 : 12

$$(3a)^2 + (4b)^2 + (5c)^2 - 3a \cdot 4b - 4b \cdot 5c - 5c \cdot 3a = 0$$

B 3 : 4 : 5

$$3a = 4b = 5c = 60\lambda$$

$$a = 20\lambda$$

$$b = 15\lambda$$

$$c = 12\lambda$$

$$a : b : c = 20\lambda : 15\lambda : 12\lambda$$

$$= 20 : 15 : 12$$

C 2 : 1 : 2

D 1 : 2 : 3

Ans. A

QUESTION



★★★★ KCLS ★★★★★

Find all real numbers x satisfying's $2^x + 3^x - 4^x + 6^x - 9^x = 1$.

$$4^x + 9^x + 1 - 2^x - 3^x - 6^x = 0$$

$$(2^x)^2 + (3^x)^2 + (1^x)^2 - 2^x \cdot 3^x - 3^x \cdot 1^x - 2^x \cdot 1^x = 0$$

$$2^x = 3^x = 1^x$$

$$2^x = 3^x = 1$$

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$$x = 0 \text{ Ans}$$

Saari Class Illustrations ATDB.uno Retry karni Hai



No HOMEWORK TODAY!!

ENJOY LIFE...
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Solution to Previous TAH

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QUESTION

TAH 01



If $x = \sqrt{33 - 20\sqrt{2}}$ & $y = \sqrt{54 - 20\sqrt{2}}$ then value of $x - y$ is equal to

A $3(1 + \sqrt{2})$

B $7(\sqrt{2} - 1)$

C $\frac{-7}{1 + \sqrt{2}}$

D $7(1 + \sqrt{2})$

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Ans. C

$$x = \sqrt{33 - 20\sqrt{2}}$$

$$y = \sqrt{54 - 20\sqrt{2}} \quad \text{then } x - y = ?$$

(Tah - 01)

$$x = \sqrt{(5)^2 + (2\sqrt{2})^2 - 2 \times 5 \times 2\sqrt{2}}$$

$$x = \sqrt{(5 - 2\sqrt{2})^2}$$

$$x = |5 - 2\sqrt{2}|$$

↳ +ve

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$$y = \sqrt{(5\sqrt{2})^2 + (2)^2 - 2 \cdot 5\sqrt{2} \cdot 2}$$

$$y = \sqrt{(5\sqrt{2} - 2)^2}$$

$$y = \sqrt{(2 - 5\sqrt{2})^2}$$

$$y = |2 - 5\sqrt{2}|$$

↳ -ve

$$x - y = (5 - 2\sqrt{2}) - (-(2 - 5\sqrt{2}))$$

$$x - y = 5 - 2\sqrt{2} + 2 - 5\sqrt{2} = 7 - 7\sqrt{2} = 7(1 - \sqrt{2})$$

$$x - y = \frac{-7(\sqrt{2} - 1) \times (\sqrt{2} + 1)}{(\sqrt{2} + 1)} = \frac{-7(2 - 1)}{(\sqrt{2} + 1)} = \frac{-7}{(\sqrt{2} + 1)}$$

$$\therefore x - y = \frac{-7}{\sqrt{2} + 1} \quad \text{Ans}$$

AKASH KUMAR



$$\boxed{TAH-1} \quad \boxed{507}$$

→ $x = \sqrt{33 - 20\sqrt{2}}$ & $y = \sqrt{54 - 20\sqrt{2}}$ then value of $x - y$ is equal to.

$$\text{Sol}^n \quad x - y = \sqrt{33 - 20\sqrt{2}} - \sqrt{54 - 20\sqrt{2}}$$

$$= \sqrt{33 - 2 \times 10 \times \sqrt{2}} - \sqrt{54 - 2 \times 10 \times \sqrt{2}}$$

$$= \sqrt{33 - 2 \times 5 \times 2\sqrt{2}} - \sqrt{54 - 2 \times 10 \times \sqrt{2}}$$

$$= \sqrt{(5)^2 + (\sqrt{8})^2 - 2 \times 5 \times \sqrt{8}} - \sqrt{54 - 2 \times 10 \times \sqrt{2}}$$

$$= \sqrt{(5 - \sqrt{8})^2} - \sqrt{54 - 2 \times 2 \times 5\sqrt{2}}$$

$$= |5 - \sqrt{8}| - \sqrt{(2)^2 + (\sqrt{50})^2 - 2 \times 2 \times \sqrt{50}}$$

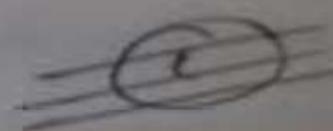
$$= 5 - \sqrt{8} - \sqrt{(\sqrt{50} - 2)^2}$$

$$= 5 - \sqrt{8} - (|\sqrt{50} - 2|)$$

$$= 5 - \sqrt{8} - \sqrt{50} + 2$$

$$= 7 - 2\sqrt{2} - 5\sqrt{2}$$

$$= 7(1 - \sqrt{2}) \times \frac{(1 + \sqrt{2})}{(1 + \sqrt{2})} = \frac{7(1 - 2)}{1 + \sqrt{2}} = \frac{-7}{1 + \sqrt{2}} \quad // \text{Ans.}$$



$2 \times 5 \times 2\sqrt{2}$	20
$= 2 \times 5 \times 2\sqrt{2}$	20
$(\sqrt{8})^2$	8
$2 \times 5 \times \sqrt{8}$	$10\sqrt{8}$
$5 - \sqrt{8}$	

$2 \times 2 \times 5\sqrt{2}$	20
$2 \times 2 \times 5\sqrt{2}$	20
$(\sqrt{50})^2$	50
$2 \times 2 \times \sqrt{50}$	$4\sqrt{50}$
$\sqrt{50} - 2$	

QUESTION

TAH 02



If $S_n = \frac{1}{\sqrt{1} + \sqrt{4}} + \frac{1}{\sqrt{4} + \sqrt{7}} + \frac{1}{\sqrt{7} + \sqrt{10}} + \dots$ n terms then -

A $S_8 = \frac{4}{3}$

B $S_{16} = 2$

C $S_{33} = 3$

D $S_{40} = \frac{10}{3}$

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Ans. A, B, C, D



Tah: ②

$$S_n = \frac{1}{\sqrt{1} + \sqrt{4}} + \frac{1}{\sqrt{4} + \sqrt{7}} + \frac{1}{\sqrt{7} + \sqrt{10}} + \dots + \frac{1}{\sqrt{3n-2} + \sqrt{3n+1}}$$

(A) $S_8 = 9/2$ (B) $S_{16} = 2$ (C) $S_{32} = 3$ (D) $S_{40} = 10/3$

Sol. $1, 4, 7, \dots, n \rightarrow T_n = 1 + (n-1)3 = 3n-2$

$$S_n = \frac{1}{\sqrt{1} + \sqrt{4}} + \frac{1}{\sqrt{4} + \sqrt{7}} + \frac{1}{\sqrt{7} + \sqrt{10}} + \dots + \frac{1}{\sqrt{3n-2} + \sqrt{3n+1}}$$

$$S_n = \frac{\sqrt{4} - \sqrt{1}}{4-1} + \frac{\sqrt{7} - \sqrt{4}}{7-4} + \frac{\sqrt{10} - \sqrt{7}}{10-7} + \dots + \frac{\sqrt{3n+1} - \sqrt{3n-2}}{(3n+1) - (3n-2)}$$

$$S_n = \frac{(\sqrt{4} - \sqrt{1}) + (\sqrt{7} - \sqrt{4}) + (\sqrt{10} - \sqrt{7}) + \dots + (\sqrt{3n+1} - \sqrt{3n-2})}{3}$$

$$= \frac{\sqrt{3n+1} - 1}{3}$$

24/04/2025

Himanshu Ramuka.

S.O.: Sushil Ramuka.

Rajasthan.

(A) $S_8 = \frac{\sqrt{3(8)+1} - 1}{3} = 9/2$

(B) $S_{16} = \frac{\sqrt{3(16)+1} - 1}{3} = 6/2 = 3$

(C) $S_{32} = \frac{\sqrt{3(32)+1} - 1}{3} = 99/3 = 3$

(D) $S_{40} = \frac{\sqrt{3(40)+1} - 1}{3} = \frac{\sqrt{121} - 1}{3} = \frac{11-1}{3} = 10/3$

QUESTION



TAH 03

If $x = \sqrt{2 + \sqrt{3}} + \sqrt{4 - \sqrt{15}}$ then value of $\sqrt{2}x$ is equal to

A $\sqrt{5} - \sqrt{3}$

B $\sqrt{5} - 1$

C $\sqrt{3} + \sqrt{5}$

D $\sqrt{5} + 1$

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Ans. D



(9)

TRAB 3 $x = \sqrt{2+\sqrt{3}} + \sqrt{4-\sqrt{15}}$ then $\sqrt{2}x = ?$

Solⁿ

$$\begin{aligned}\sqrt{2}x &= \sqrt{2}\sqrt{2+\sqrt{3}} + \sqrt{2}\sqrt{4-\sqrt{15}} \\ &= \sqrt{4+2\sqrt{3}} + \sqrt{8-2\sqrt{15}} \\ &= \sqrt{(\sqrt{3}+1)^2} + \sqrt{(\sqrt{5}-\sqrt{3})^2} \\ &= |\sqrt{3}+1| + |\sqrt{5}-\sqrt{3}| \\ &= \sqrt{3}+1 + \sqrt{5}-\sqrt{3} \\ &= 1+\sqrt{5} \quad \text{Ans}\end{aligned}$$

QUESTION

TAH 04



If $a = \sqrt{6 + 2\sqrt{5}} - \sqrt{6 - 2\sqrt{5}}$; $b = \sqrt[3]{6\sqrt{3} + 10} + \sqrt[3]{10 - 6\sqrt{3}}$, then the value of (ab) is equal to

A 8

B 12

C 4

D 6

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Ans. C



Q If $a = \sqrt{6 + 2\sqrt{5}} - \sqrt{6 - 2\sqrt{5}}$; $b = \sqrt[3]{6\sqrt{3} + 10} + \sqrt[3]{10 - 6\sqrt{3}}$,

Sol

then the value of (ab) is equal to $-$ Tah -04

$$a = \sqrt{(\sqrt{5})^2 + (1)^2 + 2\sqrt{5}} - \sqrt{(\sqrt{5})^2 + (1)^2 - 2\sqrt{5}}$$

$$a = \sqrt{(\sqrt{5} + 1)^2} - \sqrt{(\sqrt{5} - 1)^2}$$

$$a = |\sqrt{5} + 1| - |\sqrt{5} - 1| = \sqrt{5} + 1 - (\sqrt{5} - 1)$$

$$a = \sqrt{5} + 1 - \sqrt{5} + 1 = 2$$

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$$b = \sqrt[3]{6\sqrt{3} + 10} + \sqrt[3]{10 - 6\sqrt{3}}$$

$$b^3 = \underbrace{(6\sqrt{3} + 10)}_{C \cdot AS} + (10 - 6\sqrt{3}) + \underbrace{3 \sqrt[3]{6\sqrt{3} + 10} \sqrt[3]{10 - 6\sqrt{3}}}_{(\sqrt[3]{6\sqrt{3} + 10} + \sqrt[3]{10 - 6\sqrt{3}})}$$

$$b^3 = 6\sqrt{3} + 20 - 6\sqrt{3} + 3 \sqrt[3]{100 - 108} \cdot b$$

$$b^3 = \cancel{6\sqrt{3}} + 20 - \cancel{6\sqrt{3}} + 3 \sqrt[3]{(-2)^3} \cdot b$$

$$b^3 = 20 - 6b$$



$$b^3 + 6b - 20 = 0 \rightarrow (b-2)(b^2 + 2b + 10) = 0$$

$$P(b) = b^3 + 6b - 20$$

$$P(2) = 8 + 12 - 20 = 0$$

$(b-2)$ is a factor

$$b = 2 \text{ or } b^2 + 2b + 10 = 0$$

$$D = 4 - 10 \cdot 4 < 0$$

↓
No real roots

$$b(b^2 + 6) = 20$$

$$b = 20 \text{ or } b^2 + 6 = 20$$

Gadho | Gadhiyo aisa naa
karo

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QUESTION

TAH 05



If x & y are positive integers, such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$ & $x \geq y$, then the number of ordered pairs of (x, y) is

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Ans. 5



Ques
 If x & y are positive integers, such that
 $\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$ & $x \neq y$, then the
 number of ordered pairs of (x, y) is .

Solⁿ

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}$$

$$\frac{x+y}{xy} = \frac{1}{6}$$

$$6x + 6y = xy$$

$$6x + 6y - xy = 0$$

$$6x - y(-6 + x) + 36 - 36 = 0$$

$$6x - 36 - y(x - 6) + 36 = 0$$

$$6(x - 6) - y(x - 6) + 36 = 0$$

$$(x - 6)(6 - y) + 36 = 0$$

$$(x - 6)(y - 6) = 36$$

36								
↓	↓	↓	↓	↓	↓	↓	↓	↓
1×36	36×1	2×18	18×2	4×9	9×4	6×6	3×12	12×3
$(7, 42)$	$(42, 7)$	$(8, 24)$	$(24, 8)$	$(10, 15)$	$(15, 9)$	$(12, 12)$	$(9, 18)$	$(18, 9)$

Atq $x, y \in \mathbb{I}^+$ & $x \neq y$

\therefore final pairs : $(42, 7), (24, 8), (15, 9), (18, 9)$
 $(12, 12)$

\therefore n(ordered pairs) = ~~4~~ // 5 // and



Solution to Previous KTKs

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QUESTION

(KTK 1)



The expression $\sqrt{12 + 6\sqrt{3}} + \sqrt{12 - 6\sqrt{3}}$ simplifies to

- A** 4
- B** $2\sqrt{3}$
- C** $3\sqrt{3}$
- D** 6

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Ans. D



Lecture 4 24 April 2025

KTK 1 The expression $\sqrt{12+6\sqrt{3}} + \sqrt{12-6\sqrt{3}}$ simplifies to

Solⁿ Let $X = \sqrt{12+6\sqrt{3}} + \sqrt{12-6\sqrt{3}}$

$$X = \sqrt{(3+\sqrt{3})^2} + \sqrt{(3-\sqrt{3})^2}$$

$$X = |3+\sqrt{3}| + |3-\sqrt{3}|$$

$$X = 3+\sqrt{3} + 3-\sqrt{3}$$

$$X = 6$$

= Ans

QUESTION

(KTK 2)



Let p, q be real numbers satisfying $p^2 - q^2 = 4$ and $2pq = 3$ then $(p^2 + q^2)$ is equal to

- A** 1
- B** 9
- C** 16
- D** 5

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Ans. D



Let p, q be real nos satisfying $p^2 - q^2 = 4$ and $2pq = 3$
then $(p^2 + q^2)$ is equal to.

Sol

KTK-02

$$p^2 - q^2 = 4 \rightarrow \text{SAS}$$

$$\{(p+q)(p-q)\}^2 = (4)^2$$

$$(p+q)^2 (p-q)^2 = 16$$

$$(p^2 + q^2 + 2pq)(p^2 + q^2 - 2pq) = 16$$

$$(p^2 + q^2)^2 - (2pq)^2 = 16$$

$$(p^2 + q^2)^2 - (3)^2 = 16$$

$$(p^2 + q^2)^2 = 16 + 9$$

$$(p^2 + q^2) = \sqrt{25} = 5$$

$$\boxed{p^2 + q^2 = 5} \quad \underline{\text{Ans}}$$

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KTK-03

QUESTION**(KTK 3)**

Value of x satisfying the equation $\sqrt{x^2 + 2x - 63} + |x^2 - 9x + 14| = 0$ is

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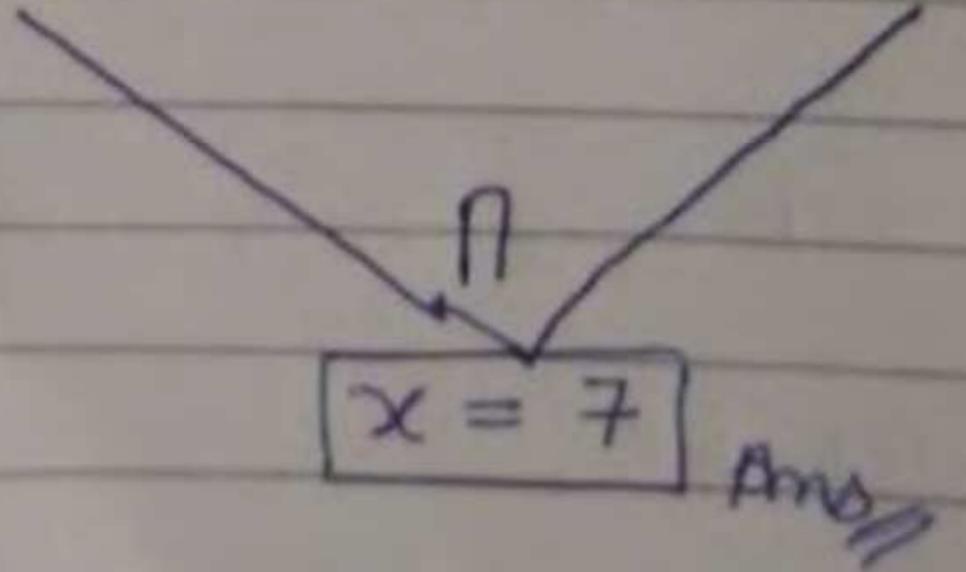
Q.3 value of x satisfying the equation $\sqrt{x^2+2x-63} + |x^2-9x+14| = 0$ is

Solⁿ

$$\sqrt{x^2+2x-63} + |x^2-9x+14| = 0$$

$(\geq 0) \quad (\geq 0)$

$$\Rightarrow \begin{aligned} x^2+2x-63 &= 0 & x^2-9x+14 &= 0 \\ (x-7)(x+9) &= 0 & (x-7)(x-2) &= 0 \\ x &= 7, -9 & x &= 2, 7 \end{aligned}$$



Ans //

QUESTION

(KTK 4)



The expression $\sqrt{(28 + 10\sqrt{3})} + \sqrt{(28 - 10\sqrt{3})}$ simplifies to

- A** 10
- B** 12
- C** $2\sqrt{3}$
- D** 5

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Ans. A



$$KTK = 4$$

The expression $\sqrt{28+10\sqrt{3}} + \sqrt{28-10\sqrt{3}}$ simplifies to
(A) 10 (B) 12 (C) $2\sqrt{3}$ (D) 5

$$\rightarrow \sqrt{28+10\sqrt{3}} + \sqrt{28-10\sqrt{3}}$$

$$= \sqrt{25+3+2 \times 5 \times \sqrt{3}} + \sqrt{25+3-2 \times 5 \times \sqrt{3}}$$

$$= \sqrt{(5+\sqrt{3})^2} + \sqrt{(5-\sqrt{3})^2}$$

$$= \underset{\substack{\downarrow \\ (+ve)}}{|5+\sqrt{3}|} + \underset{\substack{\downarrow \\ (+ve)}}{|5-\sqrt{3}|}$$

$$= 5 + \sqrt{3} + 5 - \sqrt{3} = 10 \text{ --- (A) (Ans)}$$



(9)

KTRK The expression $\sqrt{28 + 10\sqrt{3}} + \sqrt{28 - 10\sqrt{3}}$ simplifies to

solⁿ

$$\begin{aligned} \text{let } X &= \sqrt{28 + 10\sqrt{3}} + \sqrt{28 - 10\sqrt{3}} \\ &= \sqrt{(5 + \sqrt{3})^2} + \sqrt{(5 - \sqrt{3})^2} \\ &= |5 + \sqrt{3}| + |5 - \sqrt{3}| \\ &= 5 + \sqrt{3} + 5 - \sqrt{3} \\ &= 10 \\ &= \text{Ans} \end{aligned}$$

QUESTION**(KTK 5)**

Find all the integral solutions of the equation $xy = 2x - y$.

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Ans. $(0, 0), (-2, 4), (1, 1), (-3, 3)$



KTR'S

Find all Integral Solutions of eqⁿ $xy = 2x - y$

Solⁿ

$$xy = 2x - y$$

$$2x - y - xy = 0$$

$$2x - y(1+x) + 2 - 2 = 0$$

[Simon Factorisation]

$$2x + 2 - y(1+x) - 2 = 0$$

$$2(x+1) - y(x+1) - 2 = 0$$

$$(x+1)(2-y) - 2 = 0$$

$$(x+1)(y-2) = -2$$

$$1 \times -2$$

$$(0, 0)$$

$$-2 \times 1$$

$$(-3, 3)$$

$$-1 \times 2$$

$$(-2, 4)$$

$$2 \times -1$$

$$(1, 1)$$

All solⁿ are

$$(0, 0), (1, 1), (-2, 4), (-3, 3) //$$

KTK -5

Find all the integral solutions of eqn. $xy = 2x - y$.

↳

$$2x - y = xy$$

$$\Rightarrow 2x - y - xy = 0$$

$$\Rightarrow 2x - y(1+x) = 0$$

$$\Rightarrow 2 - 2 + 2x - y(1+x) = 0$$

$$\Rightarrow -2 + 2(1+x) - y(1+x) = 0$$

$$\Rightarrow -2 + (2-y)(1+x) = 0$$

$$\Rightarrow (2-y)(1+x) = 2$$

$$1 \times 2 \longrightarrow (x, y) = (1, 1)$$

$$2 \times 1 \longrightarrow (x, y) = (0, 0)$$

$$-1 \times -2 \longrightarrow (x, y) = (-3, 3)$$

$$-2 \times -1 \longrightarrow (x, y) = (-2, 4)$$

} (Ans)



Find the integral solutions of the equation

Sol

$$xy = 2x - y \quad (\text{KTK-5})$$

$$x \cancel{y} - 2x = -y$$

$$x(y-2) = -y$$

$$x = \frac{-y + 2 - 2}{y-2}$$

$$x = \frac{-(y-2) - 2}{(y-2)}$$

$$x = -1 - \frac{2}{y-2}$$

$y-2$ should be a divisor of 2

$$y-2 = -2, -1, 1, 2$$

$$y = 0, 1, 3, 4$$

$$x = 0, 1, -3, -2$$

$$\therefore (x, y) = (0, 0), (1, 1), (-3, 3)$$

$$(-2, 4)$$

4 pairs

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K (5): Find all the integral solutions of eq.ⁿ

$$xy = 2x - y$$

Sol.

$$\Rightarrow 2x - xy - y = 0$$

$$\Rightarrow x(2-y) - y + 2 - 2 = 0$$

$$\Rightarrow x(2-y) + 2 - y - 2 = 0$$

$$\Rightarrow (2-y)(x+1) = 2$$

$$1 \quad 2 \quad \longrightarrow (x, y) \equiv (1, 1)$$

$$2 \quad 1 \quad \longrightarrow (x, y) \equiv (0, 0)$$

$$-1 \quad -2 \quad \longrightarrow (x, y) \equiv (-3, 3)$$

$$-2 \quad -1 \quad \longrightarrow (x, y) \equiv (-2, 2)$$

24/04/2025

Himanshu Ramuka.

S.O.: Sushil Ramuka.

Rajasthan.

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THANK YOU

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