

PRAAYAS

JEE 2026

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Mathematics

Basic Maths

Lecture - 10

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Topics *To be covered*



- A** Logarithm and its Properties
- B** Problem Practice

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Homework Discussion

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QUESTION

(Home Challenge-02)



Solve in real numbers the equation

$$\sqrt{x_1 - 1} + 2\sqrt{x_2 - 4} + \dots + n\sqrt{x_n - n^2} = \frac{1}{2}(x_1 + x_2 + \dots + x_n) \text{ for } x_1, x_2, x_3, \dots, x_{n-1}, x_n.$$

$$2\sqrt{x_1 - 1} + 4\sqrt{x_2 - 4} + \dots + 2n\sqrt{x_n - n^2} = x_1 + x_2 + \dots + x_n$$

$$\begin{aligned} & \sqrt{x_1 - 1}^2 - 2\sqrt{x_1 - 1} + 1 + \sqrt{x_2 - 4}^2 - 4\sqrt{x_2 - 4} + 4 + \sqrt{x_3 - 9}^2 - 6\sqrt{x_3 - 9} + 9 + \dots \\ & \dots + \sqrt{x_n - n^2}^2 - 2n\sqrt{x_n - n^2} + n^2 = 0 \end{aligned}$$

$$\Rightarrow (\sqrt{x_1 - 1} - 1)^2 + (\sqrt{x_2 - 4} - 2)^2 + (\sqrt{x_3 - 9} - 3)^2 + \dots + (\sqrt{x_n - n^2} - n)^2 = 0$$

$$\sqrt{x_1 - 1} = 1 \Rightarrow x_1 = 2$$

$$\sqrt{x_2 - 4} = 2 \Rightarrow x_2 = 8$$

$$\sqrt{x_3 - 9} = 3 \Rightarrow x_3 = 18$$

$$\sqrt{x_n - n^2} = n \Rightarrow x_n = 2n^2$$



Aao Machaay Dhamaal Deh Swaal pe Deh Swaal

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$$\log_b a = \frac{\log_c a}{\log_c b} \quad \left\{ \begin{array}{l} \log_b a = \frac{1}{\log_a b} \\ a^{\log_c b} = b^{\log_c a} \end{array} \right.$$

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$$D_3: \log_{a_1} a_2 \cdot \log_{a_2} a_3 \cdot \log_{a_3} a_4 \dots \log_{a_{n-1}} a_n = \log_{a_1} a_n$$

proof: $\log_{a_1} a_2 \cdot \log_{a_2} a_3 \cdot \log_{a_3} a_4 \dots \log_{a_{n-1}} a_n$

$$= \frac{\log_{10} a_2}{\log_{10} a_1} \cdot \frac{\log_{10} a_3}{\log_{10} a_2} \cdot \frac{\log_{10} a_4}{\log_{10} a_3} \dots \frac{\log_{10} a_n}{\log_{10} a_{n-1}}$$

$$= \frac{\log_{10} a_n}{\log_{10} a_1} = \log_{a_1} a_n = \text{RHS}$$



$$\text{Ex: } \log_2 3 \cdot \log_3 4 \cdot \log_4 5 \cdots \log_{31} 32 = ?$$
$$\log_2 32 = \log_2 2^5 = 5 \log_2 2 = 5.$$

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$$D_4: \log_{a^y} m^x = \frac{x}{y} \log_a m \text{ where } a, m > 0, a \neq 1, x, y \in \mathbb{R}, y \neq 0$$

$$\text{proof: } \log_{a^y} m^x = \frac{\log_a m^x}{\log_a a^y} = \frac{x \log_a m}{y \log_a a} = \frac{x \log_a m}{y} = \frac{x}{y} \log_a m$$

$$\text{Ex: } \log_{16} 32 = \log_{2^4} 2^5 = \frac{5}{4} \log_2 2 = \frac{5}{4}$$

$$\text{Ex: } \log_{\sqrt[3]{9}} \sqrt[4]{3} = \log_{3^{2/3}} 3^{1/4} = \frac{1/4}{2/3} \log_3 3 = \frac{3}{8} \text{ Ans}$$



$$D_5: a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$$

$$\underbrace{\text{LHS}}_{a^{\sqrt{\log_b a}}} = a^{\frac{\log_b a}{\sqrt{\log_b a}}} = \left(a^{\log_b a} \right)^{\frac{1}{\sqrt{\log_b a}}}$$

$$\left(\begin{array}{l} \because a^{\frac{m}{n}} = (a^m)^{\frac{1}{n}} \\ \because \sqrt{x} = \frac{x}{\sqrt{x}} \end{array} \right)_{x \in \mathbb{R}^+}$$

$$\underbrace{a^{\frac{\log_b a}{\sqrt{\log_b a}}}}_{b^{\sqrt{\log_a a}}} = \underline{\text{RHS}}$$

$$\text{Ex: } 2^{\sqrt{\log_2 3}} - 3^{\sqrt{\log_3 2}} = ?$$

$$\cancel{3^{\sqrt{\log_3 2}}} - \cancel{3^{\sqrt{\log_3 2}}} = 0$$



Kaam Ki Baat



Common base : $\log x = \log_{10} x$

Natural base : $\log_e x = \ln(x)$

(where $e \cong 2.718$ & $e^2 \cong 7.389$)

In Calculus : $\log x$ is considered as \log with natural base.

$$\ln 2 \cong 0.693,$$

$$\ln 3 \cong 1.098,$$

$$\ln 5 \cong 1.609,$$

$$\ln 7 \cong 1.945,$$

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$$\log_{10} e \cong 2.303$$

$$\log_{10} x = \frac{\ln x}{2.303}$$

How??

$$\star \ln x + \ln y = \ln(xy)$$

$$\star \ln x^n = n \ln x$$

$$\log_{10}^x = \frac{\log_e^x}{\log_{10} e} = \frac{\ln x}{2.303}$$

QUESTION



	Column 1	Column 2
(a)	$\frac{1}{9}^{\log_3 7}$ (R)	(P) $1/27$
(b)	$2^{-\log_{1/2} 7}$ (S)	(Q) $1/2$
(c)	$8^{\frac{1}{\log_3 2}}$ (P)	(R) $1/49$
(d)	$\left(\frac{1}{4}\right)^{\log_2 6}$	(S) 7
(e)	$3^{-\log_3 2}$	(T) $1/36$

$\left(\frac{1}{4}\right)^{\log_2 6} = 6^{\log_2 \frac{1}{4}} = 6^{-2} = \frac{1}{36}$
 $3^{-\log_3 2} = (2^{\log_3 3})^{-1} = 2^{-1} = \frac{1}{2}$ (Q)

(a) $\left(\frac{1}{9}\right)^{\log_3 7} = 7^{\log_3 \frac{1}{9}} = 7^{-2} = \frac{1}{7^2} = \frac{1}{49}$
 (b) $2^{-\log_{1/2} 7} = (2^{\log_{1/2} 7})^{-1} = (7^{\log_{1/2} 2})^{-1} = (7^{-1})^{-1} = 7^1 = 7$
 (c) $8^{\frac{1}{\log_3 2}} = 8^{-\log_2 3} = (8^{\log_2 3})^{-1} = (3^{\log_2 8})^{-1} = (3^3)^{-1} = 3^{-3} = \frac{1}{27}$

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QUESTION



$$A = \log_{11}(11^{\log_{11} 1331});$$

$$C = \log_4(\log_2(\log_5 625));$$

Find the value of $A \times B \div C - D$

$$B = \log_{385} 5 + \log_{385} 11 + \log_{385} 7$$

$$D = 10^{\log_{100} 16}$$

$$A = \log_{11} 1331 = 3$$

$$B = \log_{385} (5 \times 11 \times 7) = \log_{385} 385 = 1$$

$$C = \log_4(\log_2 4) = \log_4 2 = \frac{1}{2}$$

$$D = 10^{\log_{100} 16} = 16^{\log_{100} 10} = 16^{\frac{\log_{10} 10}{\log_{10} 100}} = 16^{\frac{1}{2}} = \sqrt{16} = 4$$

$$A \times B \div C - D$$

$$3 \times 1 \div \frac{1}{2} - 4$$

$$3 \times 1 \times 2 - 4$$

$$= 6 - 4 = 2 \text{ Ans}$$

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QUESTION



Find the value of $\frac{1}{\log_3 2} + \frac{2}{\log_9 4} - \frac{3}{\log_{27} 8} = \log_2 3 + 2 \log_4 9 - 3 \log_8 27$

$$= \log_2 3 + 2 \cdot \frac{\log_3 3^2}{2^2} - 3 \frac{\log_3 3^3}{2^3}$$

$$= \log_2 3 + 2 \cdot \frac{2}{2} \log_2 3 - 3 \cdot \frac{3}{3} \log_2 3$$

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$$= \log_2 3 + 2 \log_2 3 - 3 \log_2 3$$

$$= 0$$

QUESTION



Find the value of $\log_2 10 - \log_8 125$.

$$\log_2 10 - \log_{2^3} 5^3$$

$$\log_2 10 - \frac{3}{3} \log_2 5$$

$$\log_2 10 - \log_2 5$$

$$= \log_2 \left(\frac{10}{5} \right) = \log_2 2 = 1$$

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QUESTION



If $\log_2 3 \cdot \log_3 4 \cdot \log_4 5 \dots \log_n (n+1) = 10$ find n .

$$\log_2(n+1) = 10$$

$$n+1 = 2^{10}$$

$$n = 1023 \text{ Ans}$$

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$$\log_{a_1} a_2 \cdot \log_{a_2} a_3 \cdot \log_{a_3} a_4 \dots \log_{a_n} a_{n+1}$$

$$\parallel$$

$$\log_{a_1} a_n$$

QUESTION

$$\text{Simplify: } \frac{81^{\frac{1}{\log_5 9}} + 3^{\frac{3}{\log_{\sqrt{6}} 3}}}{409} \cdot \left((\sqrt{7})^{\frac{2}{\log_{25} 7}} - (125)^{\log_{25} 6} \right)$$

$$\frac{81^{\log_9 5} + 3^{3 \cdot \log_{\sqrt{6}} 3}}{409} \left(\sqrt{7}^{2 \log_7 25} - 125^{\log_{25} 6} \right)$$

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$$= \frac{5^{\log 81} + \sqrt{6}^{3 \log 3}}{409} \left(25^{2 \log \sqrt{7}} - 6^{\log 125} \right)$$

$$= \frac{(5^2 + \sqrt{6}^3)}{409} \left(25^{2 \cdot 1/2} - 6^{\log_{5^2} 5^3} \right)$$

$$= \frac{(25 + 6\sqrt{6})}{409} \cdot (25 - 6^{3/2}) = \frac{(25 + 6\sqrt{6})(25 - 6\sqrt{6})}{409} = \frac{625 - 216}{409} = 1$$

$$a^{m \log_c b} = b^{m \log_c a}$$

$$(a^{\log_c b})^m$$

$$(b^{\log_c a})^m$$

$$= b^{m \log_c a}$$

$$6^{3/2} = 6^{1+1/2}$$

$$= 6^1 \cdot 6^{1/2}$$

$$= 6\sqrt{6}$$



QUESTION



Simplify : $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3}$

$$\cancel{5^{\log_3 7}} + \cancel{7^{\log_5 3}} - \cancel{5^{\log_3 7}} - \cancel{7^{\log_5 3}} = 0$$

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QUESTION



Prove that $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2} = 3$

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QUESTION



Simplify & compute :

$$\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$$



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QUESTION [JEE Advanced 2018]

The value of $((\log_2 9)^2)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is

Tah 03

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QUESTION [JEE Advanced 2013]

If $3^x = 4^{x-1}$, then $x =$

- ~~A~~ $\frac{2 \log_3 2}{2 \log_3 2 - 1}$
- ~~B~~ $\frac{2}{2 - \log_2 3}$
- ~~C~~ $\frac{1}{1 - \log_4 3}$
- ~~D~~ $\frac{2 \log_2 3}{2 \log_2 3 - 1}$

$a = b \iff \log_c a = \log_c b, a, b \in \mathbb{R}^+, c > 0, c \neq 1$

$3^x = 4^{x-1}$

$\log_2 3^x = \log_2 4^{x-1}$

$x \log_2 3 = (x-1) \cdot 2$

$x \log_2 3 = 2x - 2$

$2 = 2x - x \log_2 3$

$x(2 - \log_2 3) = 2$

$x = \frac{2}{2 - \log_2 3}$ (B)

$x = \frac{2}{2 - \log_2 3} = \frac{2}{2 - \frac{1}{\log_2 3}}$
 $x = \frac{2 \log_2^2 3}{2 \log_2^2 3 - 1}$ (A)

$x = \frac{2}{2 - \log_2 3} = \frac{2}{2 - \log_2 3}$
 $= \frac{2}{2 - 2 \log_2 3} = \frac{2}{2(1 - \log_2 3)} = \frac{1}{1 - \log_2 3}$ (C)



Logarithmic Equations



$$a^{m+n} = a^m \cdot a^n$$

$$a^{\log_a b} = b$$

$$5^2 = 5 \times 5 = 25$$

$$5^{2+\log_5 x}$$

$$\parallel$$

$$5^2 \cdot 5^{\log_5 x}$$

$$25 \cdot x$$

$$25x.$$

* Base of logarithm should be same throughout

* Try simplify each term if possible. Ex:

$$5^{2+\log_5 x}$$

$$\parallel$$

$$5^2 \cdot 5^{\log_5 x}$$

$$\parallel$$

$$25x$$

* If you take log on both sides of Eqn then take it to base already given in Question.

* verify All answers from original Eqn and check if any term becomes undefined or imaginary

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

$$n > 2, n \in \mathbb{N}$$

$$2 = \sqrt{4} = 4^{1/2}$$

QUESTION

★★★ASRQ★★★



$\frac{\log_2(9-2^x)}{x-3} = 1$. Then number of solution is

A 3

B 2

C 1

D 0

$$\log_2(9-2^x) = x-3$$

$$9-2^x = 2^{x-3}$$

$$9-2^x = 2^x \cdot 2^{-3}$$

$$9-2^x = \frac{2^x}{2^3} \quad \left. \vphantom{\frac{2^x}{2^3}} \right\} \text{let } 2^x = a$$

$$9-a = \frac{a}{8}$$

$$72-8a = a$$

$$9a = 72$$

$$a = 8$$

$$2^x = 8$$

$$2^x = 2^3 \Rightarrow x=3 \quad \text{(rejected)}$$

QUESTION



$\log_{x-1} 4 = 1 + \log_2(x-1)$ then x is

~~A~~ 3

$$\log_{x-1} 2^2 = 1 + \log_2(x-1)$$

B 2

$$2 \log_{x-1} 2 = 1 + \log_2(x-1)$$

~~C~~ 5/4

$$\frac{2}{\log_2(x-1)} = 1 + \log_2(x-1) \rightarrow \text{let } \log_2(x-1) = t$$

D 4/5

$$\frac{2}{t} = 1 + t$$

$$t^2 + t - 2 = 0$$

$$(t+2)(t-1) = 0$$

$$t = -2, 1$$

$$\log_2(x-1) = -2, 1$$

$$x-1 = 2^{-2} \text{ or } x-1 = 2^1$$

$$x = 1 + \frac{1}{2^2} = 5/4, x = 3$$

$$x = 5/4, 3$$

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QUESTION



$$a^x = N \Leftrightarrow \log_a N = x$$

$(x + 1)^{\log_{10}(x+1)} = 100(x + 1)$ then x is

- ~~A~~ 99
- ~~B~~ -0.9
- C 0.9
- D 10

Taking log on both sides to base 10

$$\log_{10} (x+1)^{\log_{10}(x+1)} = \log_{10} (100 \cdot (x+1))$$

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$$\log_{10}(x+1) \cdot \log_{10}(x+1) = \log_{10} 100 + \log_{10}(x+1)$$

let $\log_{10}(x+1) = t$

$$t^2 = 2 + t$$

$$t^2 - t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$$t = 2, -1$$

$$\log_{10}(x+1) = 2 \text{ or } \log_{10}(x+1) = -1$$

$$x+1 = 10^2 \text{ or } x+1 = 10^{-1}$$

$$x = 99, x = \frac{1}{10} - 1 = -\frac{9}{10} = -0.9$$



QUESTION

★★★KCLS★★★

$\log_5(\sqrt[x]{5} + 125) = \log_5 6 + 1 + \frac{1}{2x}$ then x is

$a^{m+n+p} = a^m \cdot a^n \cdot a^p$

- A** -1
- B** 1/2
- C** 1/16
- D** 1/4

$\log_5(5^{1/x} + 125) = \log_5 6 + 1 + \frac{1}{2x}$

$5^{1/x} + 125 = 5^{\log_5 6 + 1 + \frac{1}{2x}} = 5^{\log_5 6} \cdot 5^1 \cdot 5^{\frac{1}{2x}}$

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$5^{1/x} + 125 = 6 \cdot 5 \cdot 5^{1/2x}$

$5^{1/x} + 125 = 30 \cdot 5^{1/2x}$

$t^2 + 125 = 30t$

$t^2 - 30t + 125 = 0$

$t = 5, 25.$

~~$5^{1/x} = t$~~

~~$5^{1/2x} = t^2$~~

let $5^{1/2x} = t$

$(5^{1/2x})^2 = t^2$

$5^{1/x} = t^2$

Gadho/Gadhiyaa a isaa naa korol!



$$5^{\frac{1}{2x}} = 5^1, 5^2$$

$$\frac{1}{2x} = 1, 2$$

$$2x = 1, \frac{1}{2}$$

$$x = \frac{1}{2}, \frac{1}{4}$$

$x \notin \mathbb{N}$



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(wrong)

(No soln)

Reason

$\sqrt[x]{5}$, $x \in \mathbb{N}, x > 2$

QUESTION



$$\log_5(5^{1/x} + 125) = \log_5 6 + 1 + \frac{1}{2x} \text{ then } x \text{ is}$$

A -1

~~**B** 1/2~~

C 1/16

~~**D** 1/4~~

$${}^x\sqrt{5} = 5^{1/x}, x \in \mathbb{N}$$

$$5^{1/x} = {}^x\sqrt{5} \text{ only if } x \in \mathbb{N}, x \geq 2$$

$$5^{3/4} \neq \sqrt[3]{4}\sqrt{5}$$

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QUESTION

$5^{1+\log_4 x} + 5^{\log_{\frac{1}{4}} x - 1} = \frac{26}{5}$ then x is

- A 1
- B 1/16
- C 1/2
- D 16

$$5^1 \cdot 5^{\log_4 x} + 5^{\log_{\frac{1}{4}} x} \cdot 5^{-1} = \frac{26}{5}$$

$$5 \cdot 5^{\log_4 x} + \frac{5^{\log_4 x}}{5} = \frac{26}{5}$$

$$5 \cdot 5^{\log_4 x} + \frac{5^{-\log_4 x}}{5} = \frac{26}{5}$$

$$5 \cdot 5^{\log_4 x} + \frac{1}{5 \cdot 5^{\log_4 x}} = \frac{26}{5}$$

$$5t + \frac{1}{5t} = \frac{26}{5}$$

$$a^{m-n} = \frac{a^m}{a^n} = a^m \cdot a^{-n}$$

Handwritten notes in a cloud:

- * $a^{-m} = \frac{1}{a^m}$
- * $\log_2 4 - 1 = 2 - 1 = 1$
- * $\log_3(28-1) = \log_3 27 = 3$

Gadho/Gadhiyoo aisee naa karo $5 \cdot 5^{1/x} = 25^{1/x}$

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$$25t^2 + 1 = 26t$$

$$25t^2 - 26t + 1 = 0$$

$$25t^2 - 25t - t + 1 = 0$$

$$(25t - 1)(t - 1) = 0$$

$$t = \frac{1}{25}, 1$$

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$$5^{\log_x 4} = 5^{-2} \text{ or } 5^{\log_x 4} = 5^0$$

$$\log_x 4 = -2 \text{ or } \log_x 4 = 0$$

$$x = 4^{-2}, x = 4^0$$

$$x = \frac{1}{16}, 1$$

QUESTION



*** ASRQ ***

Find the square of the sum of the roots of the equation

$$\log_3 x \cdot \log_4 x \cdot \log_5 x = \log_3 x \cdot \log_4 x + \log_4 x \cdot \log_5 x + \log_5 x \cdot \log_3 x.$$

Case ① if $\log_3 x \cdot \log_4 x \cdot \log_5 x \neq 0$

$$1 = \frac{1}{\log_5 x} + \frac{1}{\log_3 x} + \frac{1}{\log_4 x}$$

$$1 = \log_x 5 + \log_x 3 + \log_x 4$$

$$1 = \log_x (5 \times 3 \times 4)$$

$$1 = \log_x 60$$

$$x = 60$$

Case ② if $\log_3 x \cdot \log_4 x \cdot \log_5 x = 0$

i.e. if $x = 1$

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LHS = RHS

↓

$x = 1$ is also a soln

Ans: $(60 + 1)^2 = 3721$

Gadho/Gadhiyo
aisaa naa karo

QUESTION

★★KCLS★★



$$\text{Solve for } x : \log^2(4 - x) + \log(4 - x) \cdot \log\left(x + \frac{1}{2}\right) - 2\log^2\left(x + \frac{1}{2}\right) = 0.$$

$$\text{let } \log(4-x) = a$$

$$\log\left(x + \frac{1}{2}\right) = b$$

$$a^2 + ab - 2b^2 = 0$$

$$a^2 + 2ab - ab - 2b^2 = 0$$

$$a(a+2b) - b(a+2b) = 0$$

$$(a+2b)(a-b) = 0$$

$$a = b, -2b.$$

$$\log(4-x) = \log\left(x + \frac{1}{2}\right) \text{ or } \log(4-x) = -2\log\left(x + \frac{1}{2}\right)$$

$$4-x = x + \frac{1}{2}$$

$$x = \frac{7}{4}$$

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$$A \cdot f^{2n}(x) + B \cdot f^n(x) \cdot g^m(x) + C \cdot g^{2m}(x) = 0$$

Try splitting middle term as in quad

$$\text{Ex: } x^4 + 5x^2y^2 + 6y^4$$

$$x^4 + 3x^2y^2 + 2x^2y^2 + 6y^4$$

$$(x^2 + 2y^2)(x^2 + 3y^2)$$



$$(4-x)(2x+1)^2 = 4$$

$$(4-x)(4x^2+1+4x) = 4$$

$$16x^2 + 4 + 16x - 4x^3 - x - 4x^2 = 4$$

$$-4x^3 + 12x^2 + 15x = 0$$

$$-x(4x^2 - 12x - 15) = 0$$

$$x=0, \quad 4x^2 - 12x - 15 = 0$$

$$x = \frac{12 \pm \sqrt{144 + 240}}{8}$$

$$x = \frac{12 \pm \sqrt{4 \times 16 \times 6}}{8}$$

$$x = \frac{3 \pm 2\sqrt{6}}{2}$$

$$x = 0, 7/4, \frac{3+2\sqrt{6}}{2}, \frac{3-2\sqrt{6}}{2}$$

$$x + \frac{1}{2} < 0$$

$$x + \frac{1}{2} = \frac{3-2\sqrt{6}}{2} + \frac{1}{2}$$

$$= \frac{4-2\sqrt{6}}{2} = -ve$$

384

96.

QUESTION



Indicate all correct alternatives, where base of the log is 2.

Tahoy

The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has :

- A** At least one real solution
- B** Exactly 3 real solutions
- C** Exactly one irrational solution
- D** Imaginary roots

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QUESTION



The equation $x^{[(\log_3 x)^2 - \frac{9}{2} \log_3 x + 5]} = 3\sqrt{3}$ has

Tah05

- A** Exactly 3 real solutions
- B** At least one real solution
- C** Exactly one irrational solution
- D** Complex roots

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Saari Class Illustrations ATDB.uno Retry karni Hai



Home Challenge-03



Positive integers a and b satisfy the condition $\log_2 [\log_2^a (\log_2^b (2^{1000}))] = 0$. Then the possible values of $a + b$ is/are:

- A** 501
- B** 252
- C** 128
- D** 66

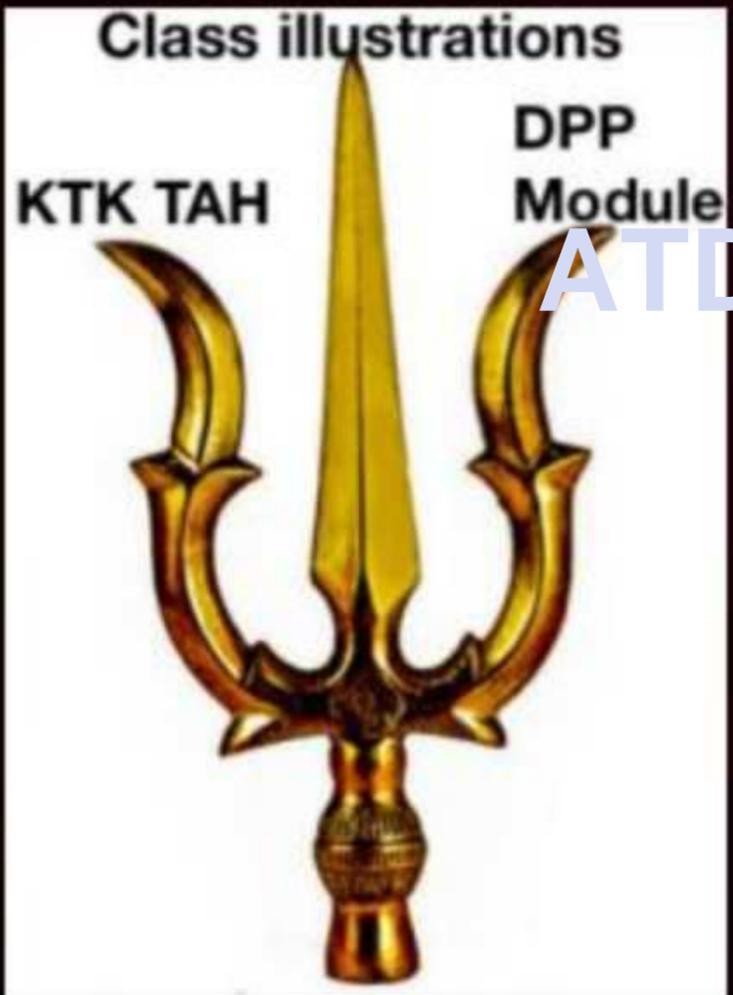
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Today's KTK



No Selection TRISHUL Selection with Good Rank
Apnao IIT Jao



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QUESTION

(KTK 1)



Solve the following inequalities:

$$(1) \frac{x}{x-5} > \frac{1}{2}$$

$$(2) \frac{x-5}{x-9} \leq 0$$

$$(3) x \leq 3 - \frac{1}{x-1}$$

$$(4) \frac{1}{x} \leq 1$$

$$(5) \frac{5x}{3x-1} \leq 0$$

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Ans. (1) $x \in (-\infty, -5) \cup (5, \infty)$, (2) $x \in (-\infty, 0) \cup [1, \infty)$,
(3) $x \in [5, 9)$, (4) $x \in \left[0, \frac{1}{3}\right)$, (5) $x \in (-\infty, 1) \cup \{2\}$

QUESTION

(KTK 2)



Complete solution set of inequality $\frac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1$ is

- A** $(-\infty, 0]$
- B** $(-\infty, 0] \cup (2, 3)$
- C** $[2, 3]$
- D** $(-\infty, 2) \cup (3, \infty)$

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Ans. B

QUESTION**(KTK 3)**

Find sum of all integral values of x satisfying $\frac{x^2 - 5x + 6}{x^2 - x - 12} \leq 0$.

ATDB.uno**Ans. 3**

QUESTION

(KTK 4)



Which of the following does not hold true for the expression

$$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1} ?$$

- A** $E = 2$ if $x \leq -1$
- B** $E = -2x$ if $-1 < x < 1$
- C** $E = -2$ if $x \geq 1$
- D** $E = -2$ for all x

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Ans. D

QUESTION



KTK 07

If $x \in [-5, 7]$, then number of integral values of x satisfying $\frac{2x+3}{x^2+x-12} < \frac{1}{2}$ is

A 5

B 6

C 7

D 8

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Ans. C

QUESTION

KTK 08



Solution set of the inequality $x + 1 \leq \frac{6}{x}$ is

- A** $(0, 2]$
- B** $[-3, 2)$
- C** $(-\infty, -3] \cup (0, 2]$
- D** $[-3, 0) \cup (2, \infty)$

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Ans. C

QUESTION

KTK 09



The set of all values of x for which $\frac{(x+1)(x-3)^2(x-5)(x-4)^3(x-2)}{x} < 0$ is

- A** $(-\infty, -1) \cup (0, 2) \cup (4, 5)$
- B** $(-1, 0) \cup (2, 4) \cup (5, \infty)$
- C** $(-1, 0) \cup (2, 3) \cup (4, 5)$
- D** $(-\infty, -1) \cup (0, 2) \cup [3, 5)$

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Ans. A

QUESTION

KTK 10



Which of the following sets does not satisfy the inequality $\frac{1}{x-2} + \frac{1}{x-1} \geq \frac{1}{x}$?

A $(-\sqrt{2}, 0)$

B $(1, \sqrt{2})$

C $(2, \infty)$

D $(0, 1)$

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Ans. D



Solution to Previous TAH

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QUESTION

$$\text{Solve: } (x^2 - x - 1)(x^2 - x - 7) < -5$$

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$$\text{Ans. } x \in (-2, -1) \cup (2, 3)$$

Sol

$$\text{Let, } x^2 - x = t$$

$$(t-1)(t-7) < -5$$

$$t^2 - 7t - t + 7 < -5$$

$$t^2 - 8t + 7 < -5$$

$$t^2 - 8t + 12 < 0$$

$$t^2 - 6t - 2t + 12 < 0$$

$$t(t-6) - 2(t-6) < 0$$

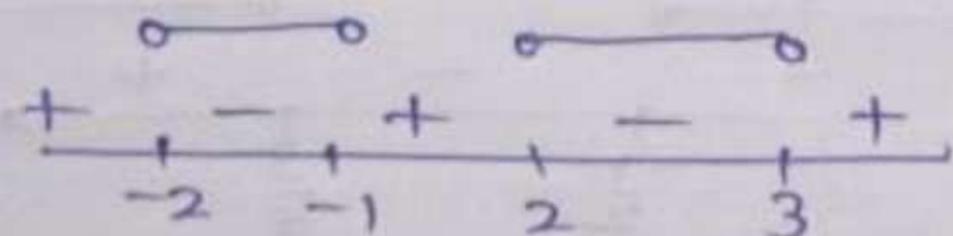
$$(t-2)(t-6) < 0$$

$$(x^2 - x - 2)(x^2 - x - 6) < 0$$

$$(x^2 - 2x + x - 2)(x^2 - 3x + 2x - 6) > 0$$

$$x(x-2) + 1(x-2) \quad x(x-3) + 2(x-3)$$

$$(x+1)(x-2)(x+2)(x-3) < 0$$



$$x \in (-2, -1) \cup (2, 3)$$

Tah-01



• Q (TAH)-1! Solve: $(x^2 - x - 1)(x^2 - x - 7) < -5$.

Soln

let $x^2 - x = t$.

$$(t - 1)(t - 7) < -5$$

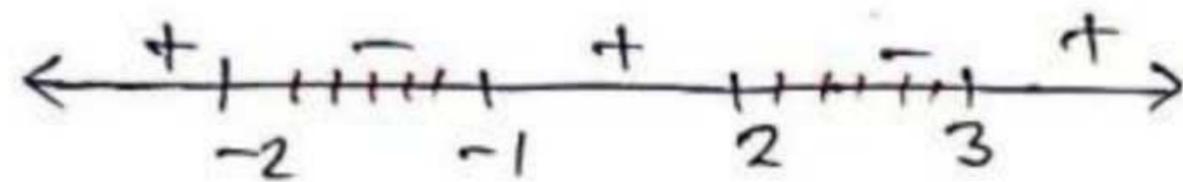
$$\text{or, } t^2 - 7t - t + 7 < -5$$

$$\text{or, } t^2 - 8t + 12 < 0$$

$$\text{or, } (t - 6)(t - 2) < 0$$

$$\text{or, } (x^2 - x - 6)(x^2 - x - 2) < 0$$

$$\text{or, } (x - 3)(x + 2)(x - 2)(x + 1) < 0$$



$$\therefore x \in (-2, -1) \cup (2, 3)$$

TAH 1



QUESTION



If $\frac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0$, then $x \in$

A $(-\infty, -4]$

B $(-1, 0]$

C $(3, \infty)$

D $\{1\}$

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Ans. A, B, C, D



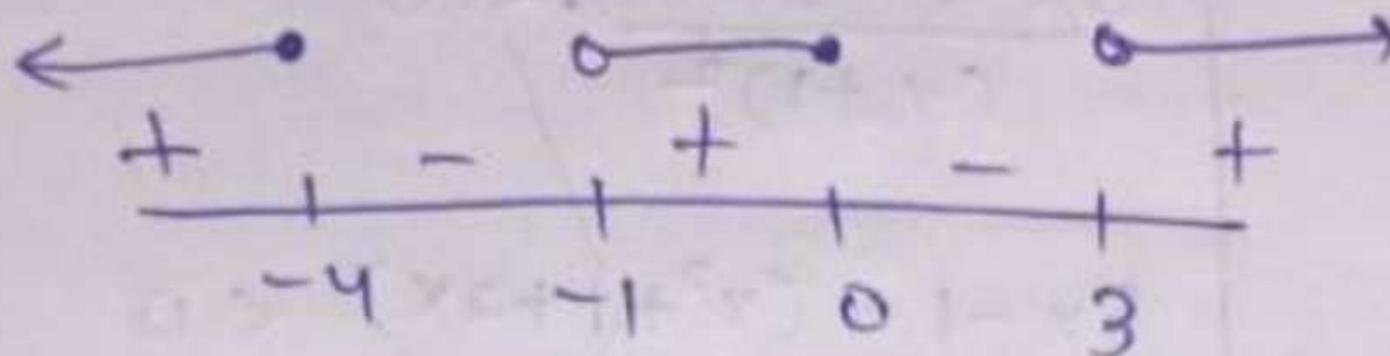
$$\text{Q.} \quad \text{If } \frac{x^3(x-1)^2(x+4)}{(x+1)(x-3)} \geq 0 \text{ then } x \in$$

Tah-02

$$\frac{x^3(x+4)}{(x+1)(x-3)} \geq 0$$

$x=1$ is also possible

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$$x \in (-\infty, -4] \cup (-1, 0] \cup (3, \infty) \cup \{1\}$$



TAH-2:

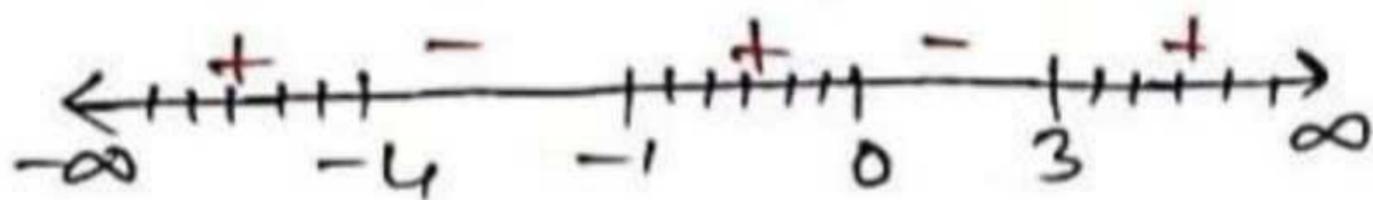
$$\frac{x^3 (x-1)^2 (x+4)}{(x+1)(x-3)} \geq 0 ; x \in ?$$

TAH 2

Soln

$$\frac{x^3 (x-1)^2 (x+4)}{(x+1)(x-3)} \geq 0.$$

$$\Rightarrow \frac{x(x+4)}{(x+1)(x-3)} \geq 0 ; x=1 \text{ is also possible,}$$



$$\therefore x \in (-\infty, -4] \cup (-1, 0] \cup (3, \infty) \cup \{1\}$$

QUESTION

TAH 03



$$1. \text{ Solve } \frac{x(3-4x)(x+1)}{(2x-5)} < 0$$

$$[\text{Ans. } x \in (-\infty, -1) \cup (0, 3/4) \cup (5/2, \infty)]$$

$$2. \text{ Solve } \frac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2x^5} \leq 0$$

$$[\text{Ans. } x \in (-\infty, -3/2) \cup (0, 4/3] \cup [4, \infty)]$$

$$3. \text{ Solve } \frac{(x-3)(x+5)(x-7)}{|x-4|(x+6)} \leq 0$$

$$[\text{Ans. } x \in (-6, -5] \cup [3, 4) \cup (4, 7]]$$

$$4. \text{ Solve } \frac{5x+1}{(x+1)^2} < 1$$

$$[\text{Ans. } x < 0 \text{ or } x > 3, x \neq -1]$$

$$5. \text{ Solve } \frac{x^4}{(x-2)^2} > 0$$

$$[\text{Ans. } x \in \mathbb{R} - \{0, 2\}]$$

$$6. \text{ Solve } \frac{6x^2-5x-3}{x^2-2x+6} \leq 4$$

$$[\text{Ans. } -\frac{9}{2} \leq x \leq 3]$$

$$7. \text{ Solve } \frac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$$

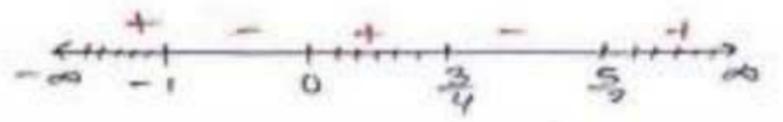
$$[\text{Ans. } x \in (-\infty, -2] \cup \{1\}]$$

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TAH-3: Solve the inequalities

Soln:- (i) $\frac{x(3-4x)(x+1)}{(2x-5)} < 0$
 $\Rightarrow \frac{x(4x-3)(x+1)}{(2x-5)} > 0$

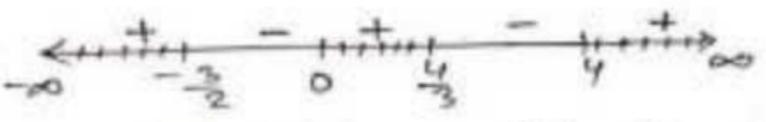
TAH.3
 by Reed
 from WB



$\therefore x \in (-\infty, -1) \cup (0, \frac{3}{4}) \cup (\frac{5}{2}, \infty)$ Ans.

(ii) $\frac{(2x+3)(4-3x)^2(x-4)}{(x-2)^2 \cdot x^5} \leq 0$

$\Rightarrow \frac{(2x+3)(3x-4)(x-4)}{x} \geq 0 ; x \neq 2$



$x \in (-\infty, -\frac{3}{2}] \cup (0, \frac{4}{3}] \cup [4, \infty)$ Ans.

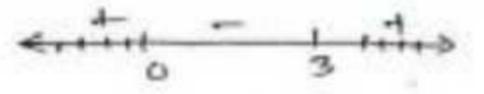
(iii) $\frac{5x+1}{(x+1)^2} < 1$

$\Rightarrow \frac{5x+1-x^2-2x-1}{(x+1)^2} < 0$

$\Rightarrow \frac{-x^2+3x}{(x+1)^2} < 0$

$\Rightarrow \frac{x(x-3)}{(x+1)^2} > 0$

$\Rightarrow x(x-3) > 0 ; x \neq -1$



$\therefore x \in (-\infty, 0) \cup (3, \infty) - \{-1\}$



(iv) $\frac{x^4}{(x-2)^2} > 0$

$\Rightarrow \dots \frac{x^4}{(x-2)^2}$ always +ve for $\forall x \in \mathbb{R} - \{0\}$
 always +ve for $\forall x \in \mathbb{R} - \{2\}$
 [x ≠ 0]
 [x ≠ 2]
 "Ans" $x \in \mathbb{R} - \{0, 2\}$

(v) $\frac{6x^2-5x-3}{x^2-2x+6} \leq 4$

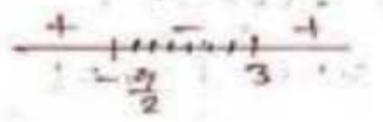
$\Rightarrow \frac{6x^2-5x-3-4(x^2-2x+6)}{x^2-2x+6} \leq 0$
 \downarrow
 $D < 0, a > 0 \Rightarrow$ always +ve

$\Rightarrow 2x^2+3x-27 \leq 0$

$\Rightarrow 2x^2+9x-6x-27 \leq 0$

$\Rightarrow (2x+9)(x-3) \leq 0$

\downarrow
 $x \in [-\frac{9}{2}, 3]$



TAH 3(part2)
 By Reed
 from WB

(vi) $\frac{(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$

$\Rightarrow \frac{6(x+2)(x-1)^2}{(x^2-3x+4)} \leq 0$

$\Rightarrow \frac{(x+2)}{1} \leq 0$ always +ve. } & x = 1 is also possible.

$\Rightarrow x \leq -2$ } $x \in (-\infty, -2] \cup \{1\}$



(i) $\frac{x(3-4x)(x+1)}{(2x-5)} < 0$

$\Rightarrow \frac{x(4x-3)(x+1)}{(2x-5)} > 0 \rightarrow +ve$

Number line: $-\infty \quad -1 \quad 0 \quad \frac{3}{4} \quad \frac{5}{2} \quad +\infty$

Signs: $+$ $-$ $+$ $-$ $+$

Roots: 0 0 0

$x \in (-\infty, -1) \cup (0, \frac{3}{4}) \cup (\frac{5}{2}, \infty)$ Ans.

Tab-03

(ii) $\frac{(2x+3)(4-3x)^3(x-4)}{(x-2)^2 x^5} \leq 0$

$\Rightarrow \frac{(2x+3)(3x-4)^3(x-4)}{x^5} \geq 0 \quad x \neq 2$

Number line: $-\infty \quad -\frac{3}{2} \quad 0 \quad \frac{4}{3} \quad 4 \quad \infty$

Signs: $+$ $-$ $+$ $-$ $+$

Roots: \bullet \circ \bullet \bullet

$x \in (-\infty, -\frac{3}{2}] \cup (0, \frac{4}{3}] \cup [4, \infty)$ Ans.

(iii) $\frac{(x-3)(x+5)(x-7)}{|x-4|(x+6)} \leq 0$

$\Rightarrow \frac{(x-3)(x+5)(x-7)}{(x+6)} \leq 0 \quad x \neq 4$

Number line: $-\infty \quad -6 \quad -5 \quad 3 \quad 7 \quad \infty$

Signs: $+$ $-$ $+$ $-$ $+$

Roots: \circ \bullet \bullet \bullet

(4) $\frac{5x+1}{(x+1)^2} < 1$

$\Rightarrow \frac{5x+1}{(x+1)^2} - 1 < 0$

$\Rightarrow \frac{5x+1 - (x+1)^2}{(x+1)^2} < 0$

$\Rightarrow \frac{5x+1 - x^2 - 2x - 1}{(x+1)^2} < 0$

$\Rightarrow \frac{-x^2 + 3x}{(x+1)^2} < 0$

$\Rightarrow \frac{-x(x-3)}{(x+1)^2} < 0$

$\Rightarrow \frac{x(x-3)}{(x+1)^2} \geq 0 \Rightarrow x(x-3) > 0 \quad x \neq -1$

+ve.

TAH-03
(04)

Number line: $-\infty \quad 0 \quad 3 \quad \infty$

Signs: $+$ $-$ $+$

Roots: \circ \bullet

$x \in (-\infty, 0) \cup (3, \infty)$ Ans.

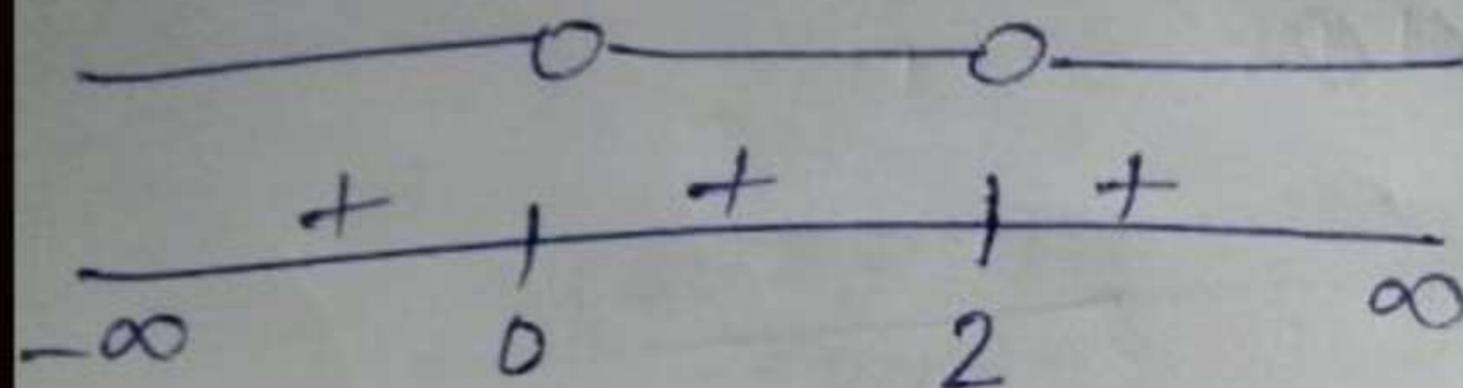
(5)



$$\#(5) \frac{x^4}{(x-2)^2} > 0 \rightarrow +ve$$

TAH-03
(05)

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$$x \neq 0, 2$$

$$x \in \mathbb{R} - \{2, 0\}$$

025, 07:24



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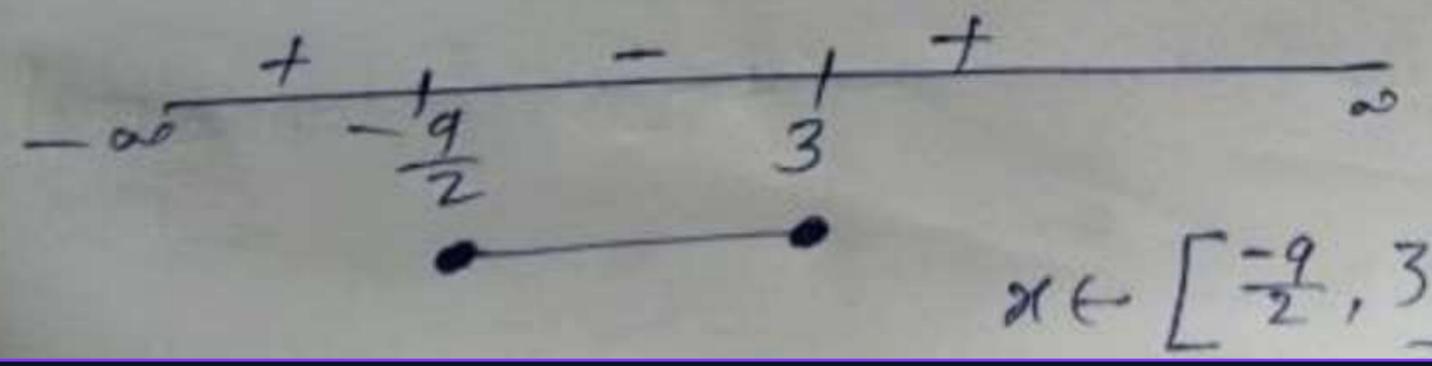
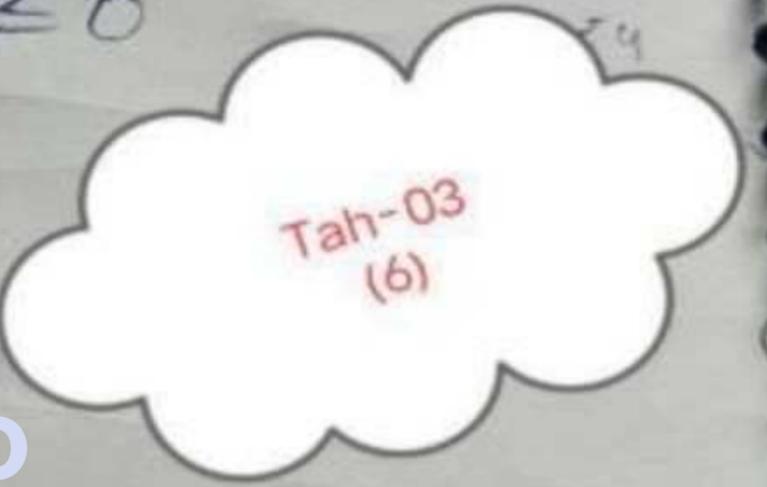
$$\frac{6x^2 - 5x - 3}{x^2 - 2x + 6} - 4 \leq 0$$

$$\Rightarrow \frac{6x^2 - 5x - 3 - 4x^2 + 8x - 24}{(x^2 - 2x + 6)} \leq 0$$

$$\Rightarrow \frac{2x^2 + 3x - 27}{x^2 - 2x + 6} \leq 0$$

$$\Rightarrow \frac{2x^2 + 9x - 6x - 27}{(x^2 - 2x + 6)} \leq 0$$

$$\Rightarrow \frac{(2x + 9)(x - 3)}{(x^2 - 2x + 6)} \leq 0$$



$D < 0$
 $a > 0$

$x^2 - 2x + 6$
always +ve

vivo Y29
Mirza Mishra, Mahauliya Bihar, May 2, 2025, 07:24



(7)

$$\frac{-(x+2)(x^2-2x+1)}{-4+3x-x^2} \geq 0$$

$$\Rightarrow \frac{(x+2)(x^2-2x+1)}{x^2-3x+4} \leq 0$$

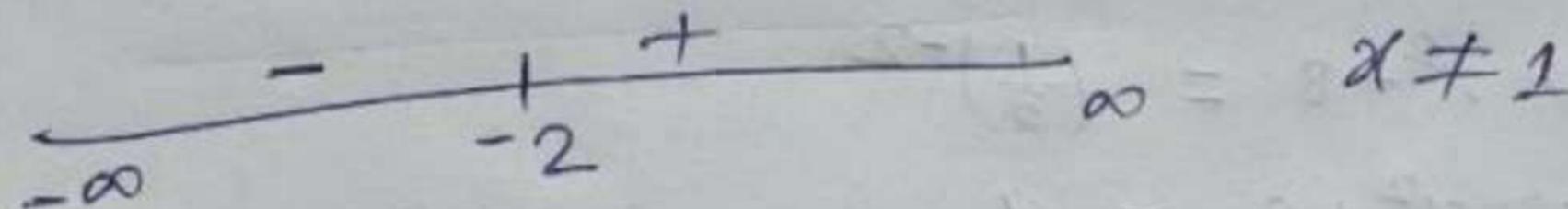
TAH-03
(07)

$$\Rightarrow \frac{(x+2)(x-1)(x-1)}{\cancel{x-4}}$$

$\Rightarrow D < 0$

$a > 0$

ATDB.uno



$$x \in (-\infty, -2]$$

QUESTION



Find all values of x for which the following equalities hold true.

(i) $\log_2 x^2 = 1$

(ii) $\log_3 x = \log_3 (2 - x)$

(iii) $\log_4 x^2 = \log_4 x$

(iv) $\log_{1/2} (2x + 1) = \log_{1/2} (x + 1)$

(v) $\log_{1/3} (x^2 + 8) = -2$

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TAH-4: Find all values of x for which the following equalities hold true:

(i) $\log_2 x^2 = 1$

(ii) $\log_4 x^2 = \log_4 x$

(iii) $\log_{1/3} (x^2 + 8) = -2$

(iv) $\log_3 x = \log_3 (2-x)$

(v) $\log_{1/2} (2x+1) = \log_{1/2} (x+1)$

Soln (i) $\log_2 x^2 = 1$

$$\Rightarrow x^2 = 2^1$$

$$\Rightarrow \boxed{x = \pm\sqrt{2}}$$

\therefore Equality holds at $x = \sqrt{2}, -\sqrt{2}$

(ii) $\log_4 x^2 = \log_4 x$

$$\Rightarrow x^2 = x$$

$$\Rightarrow x(x-1) = 0$$

$$\begin{array}{c|c} x=0 & x=1 \\ \text{N.P.} & \text{(Fine)} \end{array}$$

\therefore Equality holds at $x = 1$.

(iii) $\log_{1/3} (x^2 + 8) = -2$

$$\Rightarrow x^2 + 8 = \left(\frac{1}{3}\right)^{-2} = 9$$

$$\Rightarrow x^2 = 1$$

$$\Rightarrow \boxed{x = \pm 1}$$

\therefore Equality holds at $x = 1, -1$.

(iv) $\log_3 x = \log_3 (2-x)$

$$\Rightarrow x = 2-x$$

$$\Rightarrow 2x = 2$$

$$\Rightarrow \boxed{x = 1}$$

\therefore Equality holds at $x = 1$.

(v) $\log_{1/2} (2x+1) = \log_{1/2} (x+1)$

$$\Rightarrow 2x+1 = x+1$$

$$\Rightarrow \boxed{x = 0} \quad \checkmark \text{ (valid)}$$

\therefore Equality holds at $x = 0$.

TAH 4

BY REED

FROM WB



THANK YOU

ATDB.uno