

PRAAYAS

JEE 2026

ATDB.uno

Mathematics

Basic Maths

Lecture - 14

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Topics *To be covered*



- A** Inequalities involving Modulus
- B** Problem Practice

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Homework Discussion

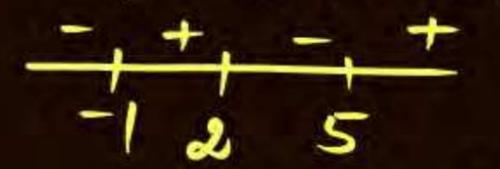
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QUESTION



Solve : $\log_{\frac{1}{x}} \left(\frac{2(x-2)}{(x+1)(x-5)} \right) \geq 1$

TAH 02 $\frac{2(x-2)}{(x+1)(x-5)} > 0$



$x \in (-1, 2) \cup (5, \infty)$

Case ① If $\frac{1}{x} > 1 \Rightarrow \frac{1}{x} - 1 > 0 \Rightarrow \frac{1-x}{x} > 0$
 $\Rightarrow \frac{x-1}{x} < 0 \Rightarrow x \in (0, 1)$

Case ② If $0 < \frac{1}{x} < 1$
 $\frac{1}{x} > 0 \Rightarrow x > 0$
 $\frac{1-x}{x} < 0 \Rightarrow \frac{x-1}{x} > 0$
 $x \in (-\infty, 0) \cup (1, \infty)$

(A)

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$\frac{2(x-2)}{(x+1)(x-5)} \geq \frac{1}{x}$

$\frac{2(x-2)}{(x+1)(x-5)} - \frac{1}{x} \geq 0$

$\frac{2x^2 - 4x - x^2 + 4x + 5}{x(x+1)(x-5)} \geq 0$



$\frac{x^2+5}{x(x+1)(x-5)} \geq 0$
 Always +ve

$\frac{1}{x(x+1)(x-5)} \geq 0 \Rightarrow x \in (-1, 0) \cup (5, \infty)$

$\frac{2(x-2)}{(x+1)(x-5)} \leq \frac{1}{x}$

$\frac{x^2+5}{x(x+1)(x-5)} \leq 0$

$x \in (-\infty, -1) \cup (0, 5)$

$x \in (1, 5)$

$x \in \phi$



$$\begin{aligned} & \text{A} \cap \text{B} \\ & (-1, 2) \cup (5, \infty) \\ & (1, 5) \cup \phi = \text{B} \\ & \text{B} = (1, 5) \\ & \cap \\ & x \in (1, 2) \text{ Ans} \end{aligned}$$

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QUESTION [JEE Mains 2025 (8 April)]

TAH 06



The sum of the squares of the roots of $|x - 2|^2 + |x - 2| - 2 = 0$ and the squares of the roots of $x^2 - 2|x - 3| - 5 = 0$, is

A 24

B 26

C 36

D 30

Case (i) if $x - 3 > 0 \Rightarrow x > 3$

$$x^2 - 2x + 6 - 5 = 0$$

$$x^2 - 2x + 1 = 0$$

$$x = 1$$

$x \in \phi$

Case (ii) if $x - 3 < 0 \Rightarrow x < 3$

$$x^2 + 2x - 6 - 5 = 0$$

$$x^2 + 2x - 11 = 0$$

$$x = \frac{-2 \pm \sqrt{48}}{2}$$

$$x = -1 \pm 2\sqrt{3} \Rightarrow x = 2\sqrt{3} - 1, -2\sqrt{3} - 1$$

$$|x - 2| = t$$

$$t^2 + t - 2 = 0$$

$$t = -2, 1$$

$$|x - 2| = -2, 1$$

$$x - 2 = -1, 1$$

$$x = 1, 3$$

Sum of squares of roots = $1 + 9 + (2\sqrt{3} - 1)^2 + (-2\sqrt{3} - 1)^2$

$$= 10 + 13 - 4\sqrt{3} + 13 + 4\sqrt{3}$$

$$= 36 \text{ Ans}$$

Ans. C

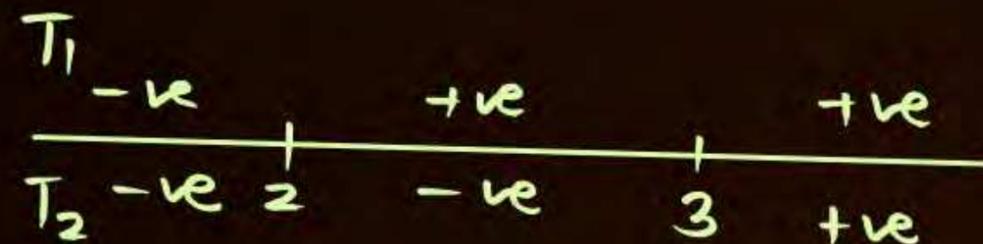
QUESTION [JEE Mains 2025 (7 April)]

TAH 07



The number of real roots of the equation $x|x - 2| + 3|x - 3| + 1 = 0$ is:

- A** 4
- B** 3
- C** 2
- D** 1



$$\frac{-1 + \sqrt{33}}{2} < 3$$

$$-1 + \sqrt{33} < 6$$

$$\sqrt{33} < 7$$

case (i) if $x \leq 2$

$$-x(x-2) - 3(x-3) + 1 = 0$$

$$-x^2 + 2x - 3x + 10 = 0$$

$$-x^2 - x + 10 = 0$$

$$x^2 + x - 10 = 0$$

$$x = \frac{-1 \pm \sqrt{41}}{2} = -\frac{1 + \sqrt{41}}{2}, -\frac{1 - \sqrt{41}}{2}$$

case (ii) if $2 < x < 3$

$$x(x-2) - 3(x-3) + 1 = 0$$

$$x^2 - 5x + 10 = 0$$

No real roots.

case (iii) if $x > 3$

$$x(x-2) + 3(x-3) + 1 = 0$$

$$x^2 - 2x + 3x - 9 + 1 = 0$$

$$x^2 + x - 8 = 0$$

$$x = \frac{-1 \pm \sqrt{33}}{2} = -\frac{1 - \sqrt{33}}{2}, \frac{-1 + \sqrt{33}}{2}$$

$x = \frac{-1 - \sqrt{41}}{2}$ Ans.

Ans. D

QUESTION [JEE Mains 2024 (5 April)]

TAH 08



The number real solutions of the equations $x|x + 5| + 2|x + 7| - 2 = 0$ is

T_1	-ve	-ve	+ve
T_2	-ve	+ve	+ve

-7 -5

Case ① if $x \leq -7$

$$-x^2 - 5x - 2x - 14 - 2 = 0$$

$$x^2 + 7x + 16 = 0$$

$$D < 0$$

↓
No real roots.

Case ② if $-7 < x < -5$

$$-x^2 - 5x + 2x + 14 - 2 = 0$$

$$-x^2 - 3x + 12 = 0$$

$$x^2 + 3x - 12 = 0$$

$$x = \frac{-3 \pm \sqrt{57}}{2}$$

$$x = \underline{\underline{\frac{-3 - \sqrt{57}}{2}}}, \frac{-3 + \sqrt{57}}{2}$$

Case ③ if $x \geq -5$

$$x^2 + 5x + 2x + 14 - 2 = 0$$

$$x^2 + 7x + 12 = 0$$

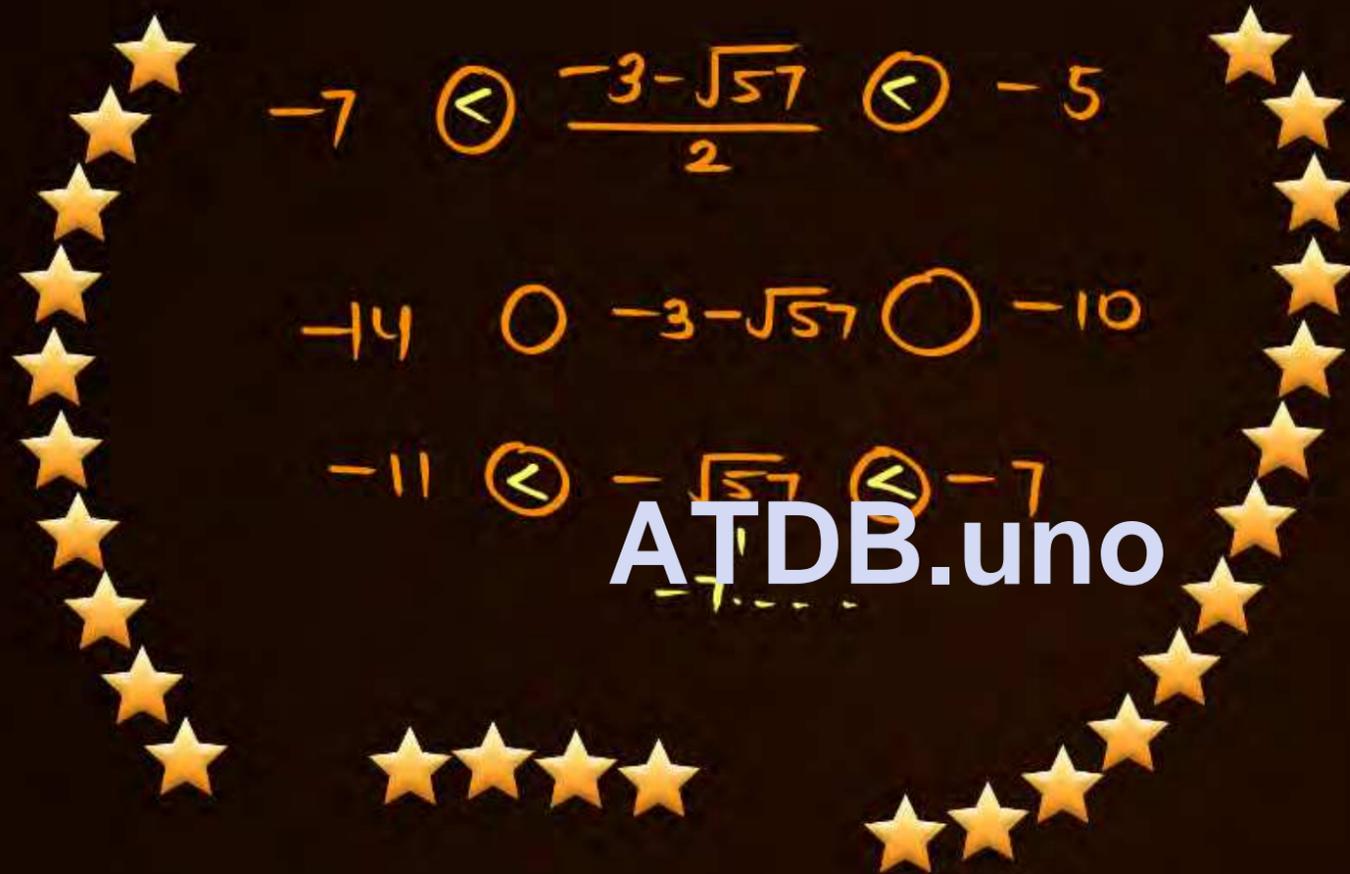
$$x = -4, -3$$

∪

$$x = -3, -4, \frac{-3 - \sqrt{57}}{2}$$

Ans. 3




$$\begin{aligned} -7 & \leq \frac{-3-\sqrt{57}}{2} \leq -5 \\ -4 & \leq -3-\sqrt{57} \leq -10 \\ -11 & \leq -\sqrt{57} \leq -7 \end{aligned}$$

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QUESTION [JEE Mains 2024 (5 April)]

TAH 09



The number of distinct real roots of the equation $|x| |x + 2| - 5|x + 1| - 1 = 0$ is

$|x| |x + 2| - 5|x + 1| - 1 = 0$

(T₁ T₂ T₃)

Hogayaa ↑
Nahi huaa ↓

T ₁	-ve	-ve	-ve	+ve
T ₂	-ve	-ve	+ve	+ve
T ₃	-ve	-ve	+ve	+ve

Case ①

If $x \leq -2$

$-x \cdot -(x+2) + 5(x+1) - 1 = 0$
 $x^2 + 2x + 5x + 4 = 0$
 $x^2 + 7x + 4 = 0$

$x = \frac{-7 + \sqrt{33}}{2}, \frac{-7 - \sqrt{33}}{2}$

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Case ② if $-2 < x < -1$

$-x(x+2) + 5(x+1) - 1 = 0$
 $-x^2 + 3x + 4 = 0$
 $x^2 - 3x - 4 = 0$
 $(x-4)(x+1) = 0$
 $x = 4, -1$
 (rejected)

Case ③ if $-1 \leq x \leq 0$

$-x^2 - 2x - 5x - 5 - 1 = 0$
 $x^2 + 7x + 6 = 0$
 $x = -6, -1$
 $x = -1$

Case ④

if $x > 0$

$x^2 + 2x - 5x - 5 - 1 = 0$
 $x^2 - 3x - 6 = 0$
 $x = \frac{3 \pm \sqrt{33}}{2}$
 $x = \frac{3 + \sqrt{33}}{2}$

Ans. 3

QUESTION [JEE Mains 2024 (8 April)]**TAH 11**

The number of distinct real roots of the equation $|x + 1| |x + 3| - 4|x + 2| + 5 = 0$ is

Redo

ATDB.uno**Ans. 2**



Lo Karo Duvaadaar Practice!!



$$1. \log_5(x^2 - 3x + 3) > 0$$

$$3. \log_{\left(\frac{1}{2}\right)}[\log_5(x^2 - 7x + 17)] > 0$$

$$5. \log_3[\log_5 \log_2(x^2 - 9x + 50)] > 0$$

$$7. \log_{0.5}(x^2 - 5x + 6) > -1$$

$$9. \log_{\left(\frac{1}{4}\right)}\left(\frac{35-x^2}{x}\right) \geq -\frac{1}{2} \quad \& \quad \frac{35-x^2}{x} > 0$$

$$\frac{35-x^2}{x} \leq \left(\frac{1}{4}\right)^{-\frac{1}{2}} = 2.$$

ReTry

$$2. \log_7[\log_5(x^2 - 7x + 15)] > 0$$

$$4. \log_{\left(\frac{1}{2}\right)}(\log_5(\log_2(x^2 - 6x + 40))) > 0$$

$$6. \log_6\left(\frac{x-2}{6-x}\right) > 0$$

$$8. \log_8(x^2 - 4x + 3) < 1$$

$$x^2 - 4x + 3 < 8 \quad \& \quad x^2 - 4x + 3 > 0$$

$$x^2 - 4x - 5 < 0 \quad (x-3)(x-1) > 0$$

$$(x-5)(x+1) < 0$$

$$x \in (-\infty, 1) \cup (3, \infty)$$

$$x \in (-1, 5)$$

$$x \in (-1, 1) \cup (3, 5)$$



$$\frac{35-x^2}{x} \leq 2$$

$$2 + \frac{x^2-35}{x} \geq 0$$

$$\frac{x^2+2x-35}{x} \geq 0$$

$$\frac{(x+7)(x-5)}{x} \geq 0$$

$$\begin{array}{c} - \quad + \quad - \quad + \\ | \quad | \quad | \quad | \\ -7 \quad 0 \quad 5 \end{array}$$

$$x \in [-7, 0) \cup [5, \infty)$$

$$\frac{35-x^2}{x} > 0$$

$$\frac{(x-\sqrt{35})(x+\sqrt{35})}{x} < 0$$

$$\begin{array}{c} - \quad + \quad - \quad + \\ | \quad | \quad | \quad | \\ -\sqrt{35} \quad 0 \quad \sqrt{35} \end{array}$$

$$x \in (-\infty, -\sqrt{35}) \cup (0, \sqrt{35})$$

$$x \in [-7, -\sqrt{35}) \cup [5, \sqrt{35})$$



$$\textcircled{3} \log_{\frac{1}{2}} (\log_5 (x^2 - 7x + 17)) > 0$$

$$\log_5 (x^2 - 7x + 17) < 1$$

$$x^2 - 7x + 17 < 5$$

$$x^2 - 7x + 12 < 0$$

$$(x-4)(x-3) < 0$$

$$x \in (3, 4)$$

$$\& \log_5 (x^2 - 7x + 17) > 0$$

$$x^2 - 7x + 17 > 1$$

$$x^2 - 7x + 16 > 0$$

$$D < 0, a > 0$$

$$x \in \mathbb{R}$$

$$\& x^2 - 7x + 17 > 0$$

$$D < 0, a = 1$$

$$x \in \mathbb{R}$$

$$x \in (3, 4)$$



$$\log_3 (\log_5 (\log_2 (x^2 - 9x + 50))) > 0$$

$$\log_5 \log_2 (x^2 - 9x + 50) > 3^0 = 1$$

$$\log_2 (x^2 - 9x + 50) > 5$$

$$x^2 - 9x + 50 > 2^5 = 32$$

$$x^2 - 9x + 18 > 0$$

$$(x - 6)(x - 3) > 0$$

$$x \in (-\infty, 3) \cup (6, \infty)$$

$$\log_5 (\log_2 (x^2 - 9x + 50)) > 0 \quad \text{(No Need)}$$

$$\log_2 (x^2 - 9x + 50) > 0 \quad \text{(No Need)}$$

$$x^2 - 9x + 50 > 0$$

$$\text{(No Need)}$$

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Answers



1. $(-\infty, 1) \cup (2, \infty)$

2. $(-\infty, 2) \cup (5, \infty)$

3. $(3, 4)$

4. $(2, 4)$

5. $(-\infty, 3) \cup (6, \infty)$

6. $(4, 6)$

7. ~~$(1, 4)$~~ $(1, 2) \cup (3, 4)$

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8. $(-1, 1) \cup (3, 5)$

9. $(-1, 0) \cup (5, \infty)$



$$\textcircled{7} \log_{0.5} (x^2 - 5x + 6) > -1$$

$$x^2 - 5x + 6 < (0.5)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2$$

$$x^2 - 5x + 4 < 0$$

$$(x-1)(x-4) < 0$$

$$x \in (1, 4)$$

$$x^2 - 5x + 6 > 0$$

$$(x-2)(x-3) > 0$$

$$x \in (-\infty, 2) \cup (3, \infty)$$

$$x \in (1, 2) \cup (3, 4)$$

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Home Challenge-05



If the value of x which satisfies the equation $2 \log_3 \sqrt{3^{1-x} + 2} = 1 + \log_3(4 \cdot 3^x - 1)$ is given by, $1 - \log_3 k$, then find the value of k . [Ans. 4]

$$\log_3 \sqrt{3^{1-x} + 2}^2 = 2 \log_3 3 + \log_3(4 \cdot 3^x - 1)$$

$$\sqrt{x^2} = |x|$$

$$\sqrt{x^2}^2 = x$$

$$\log_3(3^{1-x} + 2) = \log_3(3 \cdot (4 \cdot 3^x - 1))$$

$$3^{1-x} + 2 = 12 \cdot 3^x - 3$$

$$\frac{3}{3^x} + 2 = 12 \cdot 3^x - 3$$

$$\text{Let } 3^x = t \quad \frac{3}{t} + 2 = 12t - 3$$

$$12t^2 - 5t - 3 = 0$$

$$12t^2 - 9t + 4t - 3 = 0$$

$$(3t+1)(4t-3) = 0$$

$$t = -1/3, 3/4$$

$$3^x = -1/3, 3/4$$



$$3^x = 3/4$$

$$\log_3 3^x = \log_3 (3/4)$$

$$x = 1 - \log_3 4$$

$k=4$

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Aao Machaay Dhamaal Deh Swaal pe Deh Swaal

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QUESTIONTahoi

The number of solutions of the equation

$$\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x + 1)^2 - 4 = 0, x > 0, \text{ is :}$$

ATDB.uno**Ans. 1**

Question



Tah02

Find the integral value of x satisfying the equation $|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$.

[Ans. 9]

$$|2\log_3 x - 2| - |\log_3 x - 2| = 2$$

$$\text{Let } \log_3 x = t$$

$$2|t - 1| - |t - 2| = 2$$

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QUESTION



$$(x-1) |x^2 - 4x + 3| + 2x^2 + 3x - 5 = 0$$

case ① if $x^2 - 4x + 3 > 0 \Rightarrow (x-1)(x-3) > 0$
 $x \in (-\infty, 1) \cup [3, \infty)$

$$(x-1)(x^2 - 4x + 3) + 2x^2 + 5x - 2x - 5 = 0$$

$$(x-1)^2(x-3) + (2x+5)(x-1) = 0$$

$$(x-1) [(x-1)(x-3) + 2x+5] = 0$$

$x=1$ or $x^2 - 4x + 3 + 2x + 5 = 0$
 $x^2 - 2x + 8 = 0$

No real roots.
 $D < 0$

case ② if $x^2 - 4x + 3 < 0$
 $x \in (1, 3)$

$$-(x-1)(x^2 - 4x + 3) + (2x+5)(x-1) = 0$$

$$(x-1)(-x^2 + 4x - 3 + 2x + 5) = 0$$

$$-x^2 + 6x + 2 = 0$$

$$x=1 \quad \text{or} \quad x^2 - 6x - 2 = 0$$

$$x = \frac{6 \pm \sqrt{44}}{2}$$

$$x = 3 + \sqrt{11}, 3 - \sqrt{11}$$

$x \notin \phi$

$x=1$

QUESTION

$$|x^2 + 4x + 3| + 2x + 5 = 0$$

Tah03

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QUESTION



$$|x - |4 - x|| = 2x + 4$$

$A \cup \phi = A$
 $A \cap \phi = \phi$

$$|4 - x| = |-(x - 4)| = |-1||x - 4| = |x - 4|$$

$|x - y| = |y - x|$

$$|x - |x - 4|| = 2x + 4$$

Case ① if $x - 4 > 0 \Rightarrow x > 4$

$$|x - x + 4| = 2x + 4$$

$4 = 2x + 4$
 $x = 0$
 \Downarrow
 No soln
 $x \in \phi$

Case ② if $x - 4 < 0 \Rightarrow x < 4$

$$|x + x - 4| = 2x + 4$$

$$|2x - 4| = 2x + 4$$

Case a if $2x - 4 > 0$
 $x > 2$

$$2x - 4 = 2x + 4$$

$$-4 = 4$$

(N.P)

Case b if $2x - 4 < 0$
 $x < 2$

$$-2x + 4 = 2x + 4$$

$x = 0$

$x = 0$

$x = 0$ Ans

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QUESTION



$$|x^2 - 3|x| + 2| = x^2 - 2x$$

Case ① if $x > 0$

$$|x^2 - 3x + 2| = x^2 - 2x$$

Case (a) if $x^2 - 3x + 2 > 0$
 $(x-1)(x-2) > 0$
 $x \in (-\infty, 1] \cup [2, \infty)$

$$x^2 - 3x + 2 = x^2 - 2x$$

$x = 2$

Case (b) if $x^2 - 3x + 2 < 0$
 $x \in (1, 2)$

$$-(x^2 - 3x + 2) = x^2 - 2x$$

$$-x^2 + 3x - 2 = x^2 - 2x$$

$$2x^2 - 5x + 2 = 0$$

$$2x^2 - 4x - x + 2 = 0$$

$$(2x-1)(x-2) = 0$$

$x = 2, 1/2$

$x = 2$

Case ② if $x < 0$

$$|x^2 + 3x + 2| = x^2 - 2x$$

Case (a) $x^2 + 3x + 2 > 0$
 $(x+1)(x+2) > 0$
 $x \in (-\infty, -2] \cup [-1, \infty)$

$$x^2 + 3x + 2 = x^2 - 2x$$

$$5x = -2$$

$$x = -2/5$$

$x = -2/5$

Case (b) $x^2 + 3x + 2 < 0$
 $x \in (-2, -1)$

$$-x^2 - 3x - 2 = x^2 - 2x$$

$$2x^2 + x + 2 = 0$$

$D < 0$

No real roots.



Final Ans: $2, -\frac{2}{5}$

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Inequalities Involving Modulus



P₆: $|x| \geq |y| \Leftrightarrow x^2 \geq y^2$

$|x| \geq |y|$ — Both sides non-ve
S.B.S $x^2 \geq y^2$
 $(x-y)(x+y) \geq 0$

P₇: $|x| \geq a, a \in \mathbb{R}^+$
 $x \leq -a$ or $x \geq a$



$|x| > a, a \in \mathbb{R}^+$
 $x > a$ or $x < -a$

P₈: $|x| \leq a, a \in \mathbb{R}^+$
 $-a \leq x \leq a$



$|x| < a, a \in \mathbb{R}^+$
 $-a < x < a$

QUESTION



Solve following inequalities:

(i) $|x| \geq 2 \rightarrow x \geq 2 \text{ or } x \leq -2 \Rightarrow x \in (-\infty, -2] \cup [2, \infty)$

(ii) $|x| < 5 \rightarrow -5 < x < 5 \Rightarrow x \in (-5, 5)$

(iii) $3 < |x| \leq 7 \rightarrow x \in [-7, -3) \cup (3, 7]$

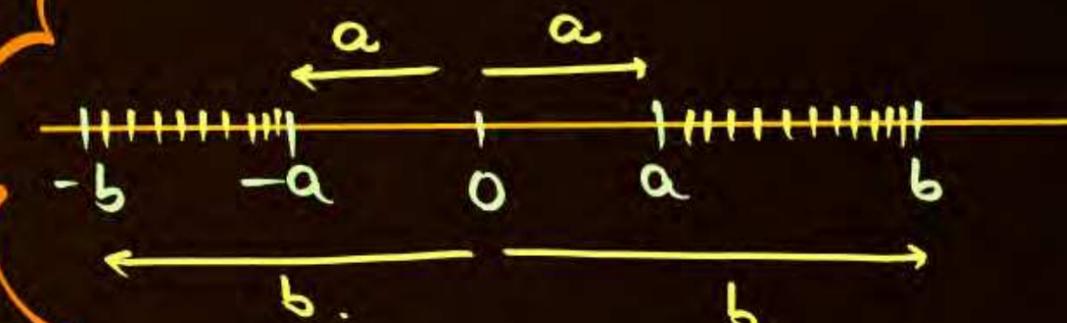
(iv) $|x| + 5 \geq 0 \rightarrow |x| \geq -5 \rightarrow x \in \mathbb{R}$

(v) $3|x| + 2 < 0 \rightarrow |x| < -2/3 \rightarrow x \in \phi$

(vi) $3|x| + 2 \leq 0 \rightarrow |x| \leq -2/3 \rightarrow x \in \phi$

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$$a \leq |x| \leq b, \quad a, b \in \mathbb{R}^+$$



$$x \in [-b, -a] \cup [a, b]$$

QUESTION



If $|x - 3| \geq 2$ then

- ~~A~~ $x \geq 5$ or $x \leq 1$
- B $x \geq 5$ or $x \geq 1$
- C $x \leq -5$ or $x \geq 1$
- D $x \leq -1$ or $x \geq 5$

$$|x| \geq a \begin{cases} x > a \\ \text{or} \\ x \leq -a \end{cases}$$

$$x - 3 > 2 \text{ or } x - 3 \leq -2$$

$$x > 5 \text{ or } x \leq 1$$

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QUESTION



If $|3 - 2x| \leq 7$ then

~~A~~ $-2 \leq x \leq 5$

B $-5 \leq x \leq 2$

C $-2 \leq x \leq 2$

D $-5 \leq x \leq -2$

$$|x| \leq a$$
$$-a \leq x \leq a$$

$$|3 - 2x| \leq 7$$

$$-7 \leq 3 - 2x \leq 7$$

$$-10 \leq -2x \leq 4$$

$$\frac{-10}{-2} \geq x \geq \frac{4}{-2}$$

$$5 \geq x \geq -2$$

$$-2 \leq x \leq 5.$$

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$$|-2| = |2|$$

$$|-x| = |x|$$

$$|4-x| = |-(x-4)|$$

$$= |x-4|$$

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QUESTION



If $|x - 2| \leq x^2 - x - 1$ then find the possible set of all values of x .

$-(x^2 - x - 1) \leq x - 2 \leq x^2 - x - 1$ nahi karna

yaaha kar sakete hai $D < 0, a > 0$
 $|x^2 - 5x + 6| \leq x^2 + x + 1$ always +ve

$|x - 2| \leq x^2 - x - 1$

(Any real) Even power ≥ 0
NO

case (i) if $x - 2 > 0 \Rightarrow x > 2$

$x - 2 \leq x^2 - x - 1$
 $x^2 - 2x + 1 \geq 0$
 $(x - 1)^2 \geq 0$
 \downarrow
 $x \in \mathbb{R}$



case (ii) if $x - 2 < 0 \Rightarrow x < 2$

$-(x - 2) \leq x^2 - x - 1$
 $-x + 2 \leq x^2 - x - 1$
 $x^2 - 3 \geq 0$
 $x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$
 \cap
 $x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, 2)$

$-(x^2 + x + 1) \leq x^2 - 5x + 6 \leq x^2 + x + 1$

$2x^2 - 4x + 7 \geq 0$ $6x \geq 5$
 $D < 0$ $x \geq 5/6$

always +ve
 \downarrow
 $x \in \mathbb{R}$ \cap $x \in [5/6, \infty)$

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final ans: $x > 2 \cup x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, 2)$

$x \in (-\infty, -\sqrt{3}] \cup [\sqrt{3}, \infty)$



ATDB.uno



Sabse Important Baat



Sabhi Class Illustrations [ATDB.uno](https://atdb.uno) Retry Karnay hai...



Home Challenge-06



If $x = \alpha$ is the solution of the equation $|2 + \log_2 7x| - \log_2(x - 1) = 5$, then find the value of $(65)^{\frac{1}{3} \log_{\alpha^2+1} \alpha}$. [Ans. 2]

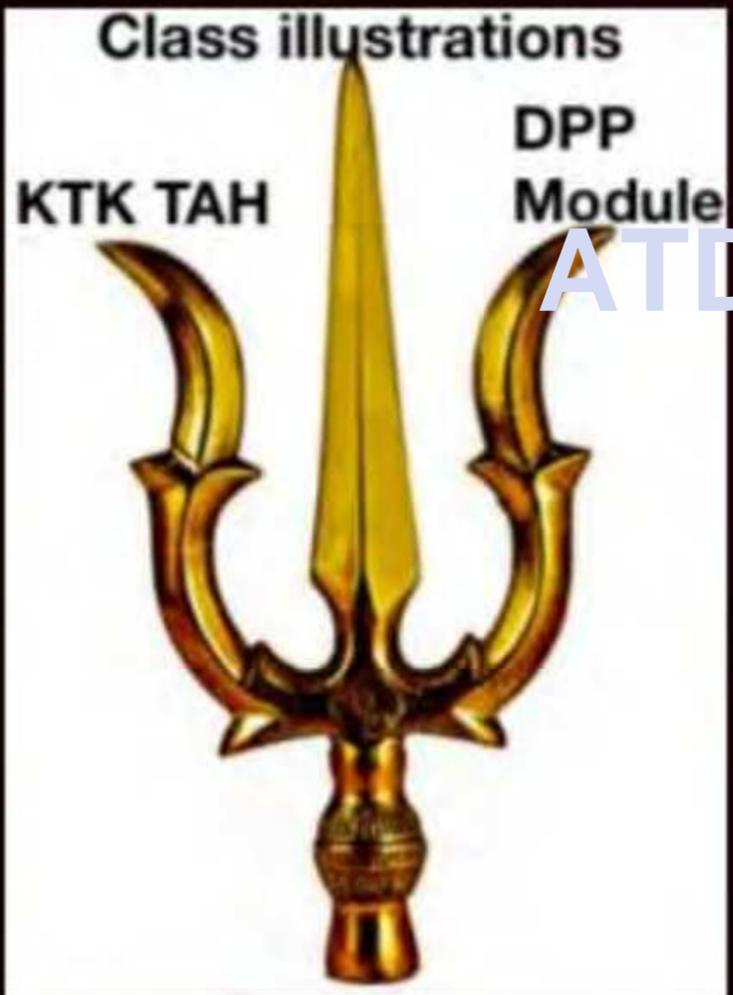
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Today's KTK



No Selection TRISHUL Selection with Good Rank
Apnao IIT Jao



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QUESTION**(KTK 1)**

$$\text{Solve for } x : 3|x^2 - 4x + 2| = 5x - 4$$

ATDB.unoAns. $x = 2, 5$

QUESTION

(KTK 2)



The number of real roots of the equation $x|x| - 5|x + 2| + 6 = 0$, is :

A 4

B 3

C 5

D 6

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Ans. B

QUESTION



KTK 03

Solve for x :

$$\log 4 + \left(1 + \frac{1}{2x}\right) \log 3 = \log(\sqrt[x]{3} + 27)$$

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Ans. $x \in \phi$

QUESTION

KTK 04



If (x_1, y_1) and (x_2, y_2) are the solution of the system of equation.

$$\log_{225}(x) + \log_{64}(y) = 4$$

$$\log_x(225) - \log_y(64) = 1,$$

then show that the value of $\log_{30}(x_1y_1x_2y_2) = 12$.

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QUESTION

KTK 05



The sum of the roots of the equation $x + 1 - 2 \log_2(3 + 2^x) + 2 \log_4(10 - 2^{-x}) = 0$, is :

A $\log_2 14$

B $\log_2 11$

C $\log_2 12$

D $\log_2 13$

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Ans. B

QUESTION

KTK 06



Evaluate
$$\frac{\left((64)^{\frac{1}{\log_5 8}} + 2^{\frac{2}{\log_{\sqrt{5}} 2}} \right) \left((\sqrt{11})^{\frac{2}{\log_{25} 11}} - (64)^{\log_8 \sqrt{5}} \right)}{300}$$

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Ans. 2

QUESTION

KTK 09



$$\text{Simplify : } 5^{\log_{\frac{1}{5}}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2} \frac{1}{10+2\sqrt{21}}$$

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Ans. 6



Homework From Module



Prarambh (Topicwise) : Q1 to Q17

Prabal (JEE Main Level) : Q1 to Q7

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Solution to Previous TAH

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QUESTION [JEE Mains 2021]

TAH 01



The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is

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Lec-13

Tah-01 $\log_4(x-1) = \log_2(x-3)$

$$\log_2(x-1) = \log_2(x-3)$$

$$\Rightarrow \frac{1}{2} \log_2(x-1) = \log_2(x-3)$$

$$\Rightarrow \log_2(x-1) = 2 \log_2(x-3)$$

$$\Rightarrow (x-1) = (x-3)^2$$

$$\Rightarrow x-1 = x^2 + 9 - 6x$$

$$\Rightarrow x^2 - 7x + 10 = 0$$

$$\Rightarrow (x-5)(x-2) = 0$$

$$\Rightarrow n = 5, 2x$$

$$\boxed{x=2}$$

$$\underline{\underline{\text{no. of sol}^n = 1 \text{ An}}}$$

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QUESTION**TAH 03**

$$\left| \left(\frac{x^2 - 6x + 8}{x^2 - 4x + 3} \right) \right| = - \left(\frac{x^2 - 6x + 8}{x^2 - 4x + 3} \right)$$

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TAH 03-0

$$\left| \frac{(x^2 - 6x + 8)}{(x^2 - 4x + 3)} \right| = - \frac{(x^2 - 6x + 8)}{(x^2 - 4x + 3)}$$

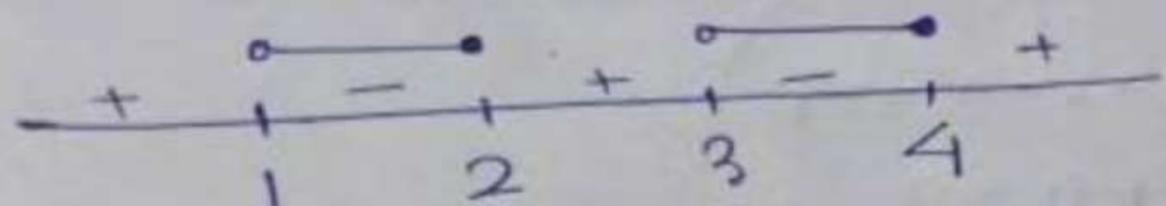
$\begin{matrix} \text{''} \\ |f(x)| \end{matrix}$ $\begin{matrix} \text{''} \\ -f(x) \end{matrix}$

$$f(x) \leq 0$$

⇓

$$\frac{x^2 - 6x + 8}{x^2 - 4x + 3} \leq 0$$

$$\frac{(x-2)(x-4)}{(x-1)(x-3)} \leq 0$$



$$\therefore x \in (1, 2] \cup (3, 4]$$

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QUESTION



TAH 04

If $|3x - 2| + x = 11$ then x is

- A** $13/4$
- B** $9/2$
- C** $-9/2$
- D** $-13/4$

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4 $|3x-2|+x = 11$

Case 1

if $3x-2 > 0 \Rightarrow x > \frac{2}{3}$

$$3x-2+x=11$$

$$4x-2=11$$

$$2x = \frac{11}{2} + 1$$

$$x = \frac{13}{4}$$

$$x = \frac{13}{4}$$

Case 2: if $3x-2 < 0 \Rightarrow x < \frac{2}{3}$

$$-3x+2+x=11$$

$$-2x+2=11$$

$$(x-1) = -\frac{11}{2}$$

$$x = -\frac{9}{2}$$

$$x = -\frac{9}{2}$$

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from mp

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$$x = \frac{13}{4}, -\frac{9}{2}$$

(A), (B)



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TAH 04:

$$|3x - 2| + x = 11$$

Case I: $3x - 2 \geq 0 \Rightarrow x \geq \frac{2}{3}$

$$3x - 2 + x = 11$$

$$4x = 13$$

$$x = \frac{13}{4}$$

Case II: $3x - 2 < 0 \Rightarrow x < \frac{2}{3}$

$$-3x + 2 + x = 11$$

$$-2x = 9$$

$$x = -\frac{9}{2}$$

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$$x = \frac{13}{4}$$

$$x = -\frac{9}{2}$$

$$\boxed{x = \frac{13}{4}} \text{ or } \boxed{x = -\frac{9}{2}}$$

QUESTION

TAH 05



$|x - 1| + |x - 2| + |x - 3| = 9$ then x can be

A -5

B 9

C -1

D 5

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Ques 5

$$|x-1| + |x-2| + |x-3| = 9$$

T_1	-	+	+	+
T_2	-	-	+	+
T_3	-	-	-	+

Case 1 : if $x \leq 1$

$$-(x-1) - (x-2) - (x-3) = 9$$

$$-x+1 - x+2 - x+3 - 9 = 0$$

$$-3x - 3 = 0$$

$$x = -1$$

$$x = -1$$

Case 4 if $x > 3$

$$x-1 + x-2 + x-3 - 9 = 0$$

$$3x - 15 = 0$$

$$x = 5$$

$$x = 5$$

Case 2: if $1 < x \leq 2$

$$(x-1) - (x-2) - (x-3) = 9$$

$$x-1 - x+2 - x+3 - 9 = 0$$

$$-x - 5 = 0$$

$$x = -5$$

\cap

\emptyset

Case 3 if $2 < x \leq 3$

$$(x-1) + x-2 - x+3 - 9 = 0$$

$$x = 9$$

\cap

\emptyset

take union all 4 cases

$$x = -1, 5 \quad (c), (d) \text{ etc}$$



TAH 05 :

$$|x-1| + |x-2| + |x-3| = 9$$

	1	2	3
$T_1 \rightarrow$	-ve	+ve	+ve
$T_2 \rightarrow$	-ve	-ve	+ve
$T_3 \rightarrow$	-ve	-ve	+ve

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ASSAM**

Case I: If $x \leq 1$

$$-x+1-x+2-x+3=9$$

$$-3x=3$$

$$x=-1$$

$$x=-1$$

Case II: If $1 < x \leq 2$

$$x-1-x+2-x+3=9$$

$$-x+4=9$$

$$x=-5$$

$$x = \phi$$

Case III: $2 < x \leq 3$

$$x-1+x-2-x+3=9$$

$$x=9$$

$$x = \phi$$

Case IV: $x > 3$

$$x-1+x-2+x-3=9$$

$$3x-6=9$$

$$x=5$$

$$x=5$$

$$\therefore \text{Ans: } \boxed{x = -1, 5} \checkmark$$

QUESTION [JEE Mains 2025 (7 April)]**TAH 07**

The number of real roots of the equation $x|x - 2| + 3|x - 3| + 1 = 0$ is:

- A** 4
- B** 3
- C** 2
- D** 1

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Ans. D



Q. 7)

$$x|x(-2) + 3|x(-3) + 1 = 0$$

Tah-7

$$\begin{array}{l} T_1 \quad \xrightarrow{T_1} \quad -ve \quad | \quad +ve \quad | \quad +ve \\ T_2 \quad -ve \quad | \quad -ve \quad | \quad +ve \end{array}$$

Case ①: If $x \leq 2$

$$-x(x-2) - 3(x-3) + 1 = 0$$

$$\Rightarrow -x^2 + 2x - 3x + 9 + 1 = 0$$

$$\Rightarrow x^2 + x - 10 = 0$$

$$\therefore x = \frac{-1 \pm \sqrt{41}}{2}$$

$$x = \frac{-1 + \sqrt{41}}{2}$$

$$x = \frac{-1 - \sqrt{41}}{2}$$

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 $x \in \phi$ Case ③: $x \geq 3$

$$x(x-2) + 3(x-3) + 1 = 0$$

$$x^2 - 2x + 3x - 9 + 1 = 0$$

$$x^2 + x - 8 = 0$$

$$\therefore x = \frac{-1 + \sqrt{33}}{2}, \frac{-1 - \sqrt{33}}{2}$$

 $x \in \phi$ Case-2: $2 < x < 3$

$$x(x-2) - 3(x-3) + 1 = 0$$

$$x^2 - 2x - 3x + 9 + 1 = 0$$

$$x^2 - 5x + 10 = 0$$

$$\Delta = 25 - 40 < 0$$

& $a > 0$ always +ve

Take Union of all cases

$$x = \frac{-1 - \sqrt{41}}{2}$$

Ans

QUESTION [JEE Mains 2024 (5 April)]**TAH 08**

The number real solutions of the equations $x|x + 5| + 2|x + 7| - 2 = 0$ is

ATDB.uno**Ans. 3**



$$x|x+5| + 2|x+7| - 2 = 0$$



$T_1 \rightarrow$ -ve -ve +ve
 $T_2 \rightarrow$ -ve +ve +ve

Case I: $x \leq -7$

$$\begin{aligned}
 x(-x-5) + 2(-x-7) - 2 &= 0 \\
 -x^2 - 5x - 2x - 14 - 2 &= 0 \\
 -x^2 - 7x - 16 &= 0 \\
 x^2 + 7x + 16 &= 0 \\
 D = 7^2 - 4(16) &= -ve \\
 &\text{Not possible.}
 \end{aligned}$$

Case II: $-7 < x \leq -5$

$$\begin{aligned}
 x(-x-5) + 2(x+7) - 2 &= 0 \\
 -x^2 - 5x + 2x + 14 - 2 &= 0 \\
 -x^2 - 3x + 12 &= 0 \\
 x^2 + 3x - 12 &= 0 \\
 D = 54 \\
 x = \frac{-3 \pm \sqrt{54}}{2}
 \end{aligned}$$

Case III: $x > -5$

$$\begin{aligned}
 x(x+5) + 2(x+7) - 2 &= 0 \\
 x^2 + 5x + 2x + 14 - 2 &= 0 \\
 x^2 + 7x + 12 &= 0 \\
 x^2 + 3x + 4x + 12 &= 0 \\
 (x+3)(x+4) &= 0 \\
 x &= -3, -4
 \end{aligned}$$

$$x = \frac{-3 - \sqrt{54}}{2}$$

$$x = -3, -4, \frac{-3 - \sqrt{54}}{2}$$

$$x = -3, -4$$

∴ Answer: (3) ✓

QUESTION [JEE Mains 2021]**TAH 10**

The number of the real roots of the equation $(x + 1)^2 + |x - 5| = \frac{27}{4}$ is

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Q.10) $(x+1)^2 + |x-5| = \frac{27}{4}$
Tah-10

Case ①: $x > 5$

$$(x+1)^2 + (x-5) = \frac{27}{4}$$

$$\Rightarrow x^2 + 2x + 1 + x - 5 = \frac{27}{4}$$

$$\Rightarrow 4x^2 + 12x - 16 = 27$$

$$\Rightarrow 4x^2 + 12x - 43 = 0$$

$$\Delta = 144 + 688 = 832 > 0$$

$$x = \frac{-12 \pm \sqrt{832}}{8}$$

$$x = \frac{-12 \pm 8\sqrt{13}}{8}$$

$x \in \emptyset$

Pranav Anand
Bihar

Case ②: $x < 5$

$$x^2 + 2x + 1 - x + 5 = \frac{27}{4}$$

$$4x^2 + 4x + 24 = 27$$

~~$$2x(2x+3) = 1(2)$$~~

$$4x^2 + 6x - 2x - 3 = 0$$

$$2x(2x+3) - 1(2x+3) = 0$$

$$(2x+3)(2x-1) = 0$$

$$x = -\frac{3}{2}, \frac{1}{2}$$

$$x = -\frac{3}{2}, \frac{1}{2}$$

$$x = -\frac{3}{2}, \frac{1}{2}$$

\therefore No of roots = ② Ans



THANK
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YOU