

# PRAAYAS

## JEE 2026

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Mathematics

# Basic Maths

Lecture - 15

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# Topics *To be covered*



- A** Inequalities involving Modulus
- B** Problem Practice

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# Homework Discussion

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QUESTION

★★★★★KCLS★★★★★

KTK 04



If  $(x_1, y_1)$  and  $(x_2, y_2)$  are the solution of the system of equation.

$$\log_{225}(x) + \log_{64}(y) = 4$$

$$\log_x(225) - \log_y(64) = 1,$$

then show that the value of  $\log_{30}(x_1y_1x_2y_2) = 12$ .

let  $\log_{225} x = a, \log_{64} y = b$

$$a + b = 4 \Rightarrow b = 4 - a$$

$$\frac{1}{a} - \frac{1}{b} = 1$$

$$\frac{1}{a} - \frac{1}{4-a} = 1$$

$$4 - a - a = a(4 - a)$$

$$4 - 2a = 4a - a^2$$

$$a^2 - 6a + 4 = 0$$

$$\begin{cases} a_1 = \log_{225} x_1 \\ a_2 = \log_{225} x_2 \end{cases}$$

$$a_1 + a_2 = \log_{225} x_1 x_2$$

$$6 = \log_{225} x_1 x_2 \rightarrow x_1 x_2 = (225)^6$$

$$\begin{aligned} \log_{64} y_1 &= b_1 = 4 - a_1 \\ \log_{64} y_2 &= b_2 = 4 - a_2 \end{aligned}$$

$$\log_{64} y_1 y_2 = 8 - (a_1 + a_2)$$



$$\log(y_1 \cdot y_2) = 8 - 6 = 2$$

64

$$y_1, y_2 = 64^2$$

$$\log(x_1 x_2 y_1 y_2) = \log((225)^6 \cdot 64^2)$$

30

30

$$= \log_{30}(30^{12} \cdot 2^{12}) = \log_{30} 30^{12} = 12 \log_{30} 30 = 12 \text{ Ans}$$

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QUESTION

★★★★ KCLS ★★★★★

KTK 05



The sum of the roots of the equation  $x + 1 - 2 \log_2(3 + 2^x) + 2 \log_4(10 - 2^{-x}) = 0$ , is :

**A**  $\log_2 14$

~~**B**  $\log_2 11$~~

**C**  $\log_2 12$

**D**  $\log_2 13$

$$\log_2 2^{x+1} - 2 \log_2(3 + 2^x) + \frac{2}{2} \log(10 - \frac{1}{2^x}) = 0$$

$$\log_2 \left( \frac{2^{x+1} \cdot (10 - \frac{1}{2^x})}{(3 + 2^x)^2} \right) = 0$$

$$2^{x+1} \left( 10 - \frac{1}{2^x} \right) = 2^0 \cdot (3 + 2^x)^2$$

Let  $2^x = t$

$$2 \cdot t \left( 10 - \frac{1}{t} \right) = (3 + t)^2$$

$$20t - 2 = 9 + t^2 + 6t$$

$$t^2 - 14t + 11 = 0 \left\{ \begin{array}{l} t_1 = 2^{x_1} \\ t_2 = 2^{x_2} \end{array} \right.$$

$$\left. \begin{array}{l} t_1 \cdot t_2 = 2 \\ 11 = 2^{x_1 + x_2} \end{array} \right\}$$

Ans. B



$$2^{x_1+x_2} = 11$$

$$\log_2 2^{x_1+x_2} = \log_2 11$$

$$(x_1+x_2) = \log_2 11$$

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## Home Challenge-06



If  $x = \alpha$  is the solution of the equation  $|2 + \log_2 7x| - \log_2(x - 1) = 5$ , then find the value of  $(65)^{\frac{1}{3} \log_{\alpha^2+1} \alpha}$ . [Ans. 2]

Case ①  $2 + \log_2 7x \geq 0 \Rightarrow \log_2 7x \geq -2 \Rightarrow 7x \geq 2^{-2} = \frac{1}{4}$   
 $x \geq \frac{1}{28}$

$2 + \log_2 7x - \log_2(x-1) = 5$

$\log_2 \frac{7x}{x-1} = 3$

$\frac{7x}{x-1} = 8 \Rightarrow 7x = 8x - 8$

$x = 8$

Case ② If  $2 + \log_2 7x < 0$   
 $x < \frac{1}{28}$  (N.P)

Ans:  $x = 8 = \alpha$

$(65)^{\frac{1}{3} \log_{65} 8} = 8^{\frac{1}{3}} = 2$  Ans



# Aao Machaay Dhamaal Deh Swaal pe Deh Swaal

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$$\boxed{bpp \rightarrow 4}$$

$$\log_{(1/2)} \log_5 (\log_2 (x^2 - 6x + 40)) > 0$$

Solution:-

$$\log_5 (\log_2 (x^2 - 6x + 40)) > 1 \quad ??$$

$$\log_2 (x^2 - 6x + 40) > 5$$

$$x^2 - 6x + 40 > 32$$

$$x^2 - 6x + 8 > 0$$

$$(x - 4)(x - 2) > 0$$

$$x \in (-\infty, 2) \cup (4, \infty)$$

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(Sign change)

↓  
No Need

$$x^2 - 6x + 40 > 0$$

↓  
No Need

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The integral value of x satisfying the equation

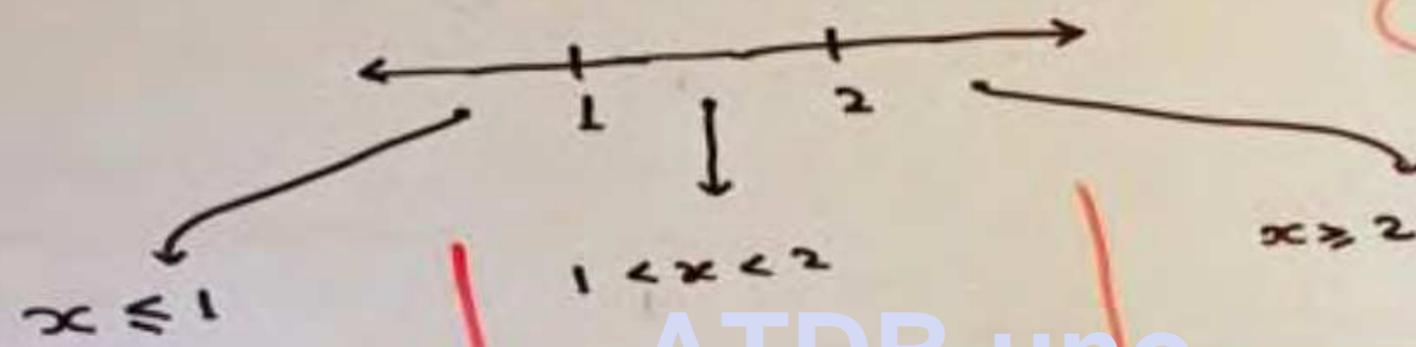
$$\Rightarrow |\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$$

$$|2 \log_3 x - 2| - |\log_3 x - 2| = 2 \quad \text{Let } \log_3 x = t$$

$$|2t - 2| - |t - 2| = 2$$

$$2|t - 1| - |t - 2| - 2 = 0$$

TAH-2



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$$-2t + 2 + t - 2 - 2 = 0$$

$$-t - 2 = 0$$

$$\boxed{t = -2} \text{ accepted}$$

$$\log_3 x = -2$$

$$x = 3^{-2}$$

$$\boxed{x = \frac{1}{9}}$$

x = 0.11...

$$2t - 2 + t - 2 - 2 = 0$$

$$3t - 6 = 0$$

$$\boxed{t = 2} \text{ rejected}$$

$$2t - 2 - t + 2 - 2 = 0$$

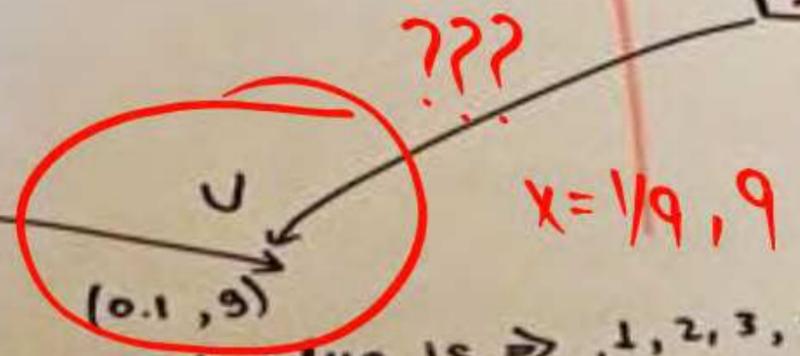
$$t - 2 = 0$$

$$\boxed{t = 2} \text{ accepted}$$

$$\log_3 x = 2$$

$$x = 3^2$$

$$\boxed{x = 9}$$



$$x = \frac{1}{9}, 9$$

integral value is  $\Rightarrow$  1, 2, 3, 4, 5, 6, 7, 8, 9



KTK2

$$x |x| - 5|x+2| + 6 = 0$$

$T_1$	-ve	<b>+ve</b> ??	+ve
$T_2$	-ve	-2	+ve

Case 2:  $-2 < x \leq 0$

**Galti Batao**

$$x^2 - 5x - 10 + 6 = 0$$

$$x^2 - 5x - 4 = 0$$

Case 1:  $x \leq -2$

$$-x^2 + 5x + 10 + 6 = 0$$

$$x^2 - 5x - 16 = 0$$

**$(x-8)(x+2) = 0$**

$$x = \cancel{8}, -2$$

Case 3:  $x \geq 0$

$$x = \frac{5 + \sqrt{41}}{2}, \frac{5 - \sqrt{41}}{2}$$

$$x^2 - 5x - 10 + 6 = 0$$

$$x^2 - 5x - 4 = 0$$

$$x = \frac{5 + \sqrt{41}}{2}, \frac{5 - \sqrt{41}}{2}$$

finally take union of  
3 cases  $\rightarrow$  only 3 roots  
are possible (B)  $\checkmark$

**QUESTION**



Values of x for which  $f(x) = \log_{|x-3|} \left( \frac{|x|-5}{2-|x|} \right)$  is defined.

$|x-3| > 0$  &  $|x-3| \neq 1$  &  $\frac{|x|-5}{2-|x|} > 0$

$\downarrow$   
 $x \neq 3$

$x-3 \neq \pm 1$

$x \neq 3 \pm 1$

$x \neq 4, 2$

M(1)  $|x|=t$  (let) OR M(2)

$t-5 > 0$

$\frac{t-5}{t-2} < 0$

$t \in (2, 5)$

$2 < |x| < 5$

$x \in (-5, -2) \cup (2, 5)$

Case (1)  $x > 0$

$\frac{x-5}{2-x} > 0$

$\frac{x-5}{x-2} < 0$

$x \in (2, 5)$

Case (II) if  $x < 0$

$\frac{-x-5}{2+x} > 0$

$\frac{x+5}{x+2} < 0 \rightarrow x \in (-5, -2)$

$x \in (-5, -2) \cup (2, 5)$

Ans:  $(-5, -2) \cup (2, 5) - \{3, 4\}$

Ans.  $x \in (-5, -2) \cup (2, 5) - \{3, 4\}$

**QUESTION [JEE Mains 2022 (28 July)]**



★★★★ KCLS ★★★★★

Let  $S = \left\{ x \in [-6, 3] - \{-2, 2\} : \frac{|x+3|-1}{|x|-2} \geq 0 \right\}$  and  $T = \{x \in \mathbb{Z} : x^2 - 7|x| + 9 \leq 0\}$ .

Then the number of elements in  $S \cap T$  is :

**A** 7  
 -ve +ve +ve  
 -ve -3 -ve 0 +ve

$$\frac{|x+3|-1}{|x|-2} \geq 0$$

$T_2$

case(ii) if  $-3 < x < 0$

$$\frac{x+3-1}{-x-2} \geq 0$$

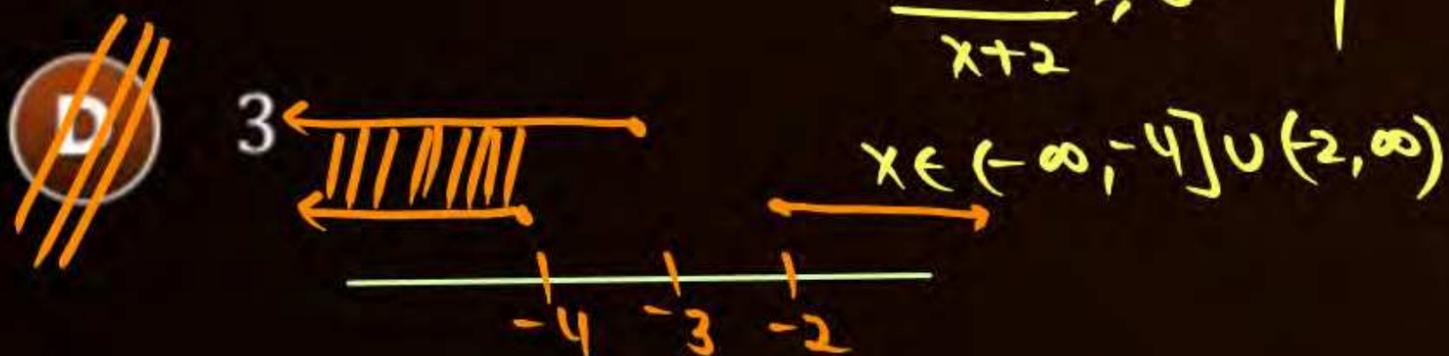
$$\frac{x+2}{x+2} \leq 0$$

**B** 5  
 case(i) if  $x \leq -3$

$$\frac{-x-3-1}{-x-2} \geq 0 \cap \frac{x+4}{x+2} \geq 0$$

**C** 4

$1 \leq 0, x \neq -2$   
 (N.P)  $\Rightarrow x \in \phi$  ✓



case(iii) if  $x \geq 0$

$$\frac{x+3-1}{x-2} \geq 0 \cap \frac{x+2}{x-2} \geq 0 \Rightarrow x \in (-\infty, -2] \cup (2, \infty)$$

Ans. D



$$x \in (-\infty, -4] \cup (2, \infty) \rightarrow [-6, -4] \cup (2, 3] = S$$

But for S  $x \in [-6, 3] - \{-2, 2\}$

for T

$$|x|^2 = x^2$$

$$x^2 - 7|x| + 9 \leq 0$$

$$|x|^2 - 7|x| + 9 \leq 0$$

$$\left(|x| - \frac{7 + \sqrt{13}}{2}\right) \left(|x| - \frac{7 - \sqrt{13}}{2}\right) \leq 0$$

$$|x| = \frac{7 \pm \sqrt{13}}{2}$$

$$|x| \in \left[\frac{7 - \sqrt{13}}{2}, \frac{7 + \sqrt{13}}{2}\right]$$

$$\frac{7 - \sqrt{13}}{2} < |x| < \frac{7 + \sqrt{13}}{2}$$

$$x \in \left[-\left(\frac{7 + \sqrt{13}}{2}\right), \left(\frac{7 - \sqrt{13}}{2}\right)\right] \cup \left[\frac{7 - \sqrt{13}}{2}, \frac{7 + \sqrt{13}}{2}\right]$$

$$x \in \mathbb{Z} \Rightarrow T = \{-5, -4, -3, -2, 2, 3, 4, 5\}$$

$$\begin{array}{c} \mathbb{R}^+ \\ \wedge \\ |x| \in [a, b] \\ x \in [-b, -a] \cup [a, b] \end{array}$$



$$SNT = \{-5, -4, 3\}$$



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**QUESTION**



Solve:  $|x^2 - x - 6| \leq x^2 + x - 10$

Case (i)

If  $x^2 - x - 6 \geq 0 \rightarrow (x-3)(x+2) \geq 0 \rightarrow x \in (-\infty, -2] \cup [3, \infty)$

$x^2 - x - 6 \leq x^2 + x - 10$

$2x \geq 4$

$x \geq 2$



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Case (ii)

If  $x^2 - x - 6 < 0 \rightarrow x \in (-2, 3)$

$-x^2 + x + 6 \leq x^2 + x - 10$

$2x^2 - 16 \geq 0$

$x^2 - (2\sqrt{2})^2 \geq 0$

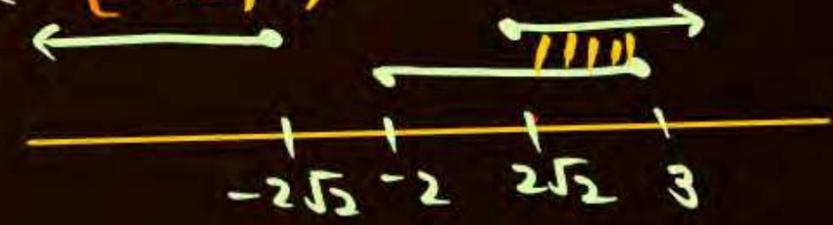
$(x-2\sqrt{2})(x+2\sqrt{2}) \geq 0$

$x \in (-\infty, -2\sqrt{2}] \cup [2\sqrt{2}, \infty)$

$x \in [2\sqrt{2}, 3)$

$x \in [2\sqrt{2}, 3) \cup [3, \infty)$

$x \in [2\sqrt{2}, \infty)$



## QUESTION



If  $||x - 2| - 3| \leq 2$  then complete set of values of  $x$  is

- A**  $[-3, 7]$
- ~~**B**  $[-3, 1] \cup [3, 7]$~~
- C**  $[1, 7]$
- D**  $[-3, 3] \cup [4, 7]$

$$-2 \leq |x-2|-3 \leq 2$$

$$1 \leq |x-2| \leq 5$$

$$|x-2| \geq 1$$

$$x-2 \leq -1 \text{ or } x-2 \geq 1$$

$$x \leq 1 \text{ or } x \geq 3$$

$$x \in (-\infty, 1] \cup [3, \infty)$$

$$|x-2| \leq 5$$

$$-5 \leq x-2 \leq 5$$

$$-3 \leq x \leq 7$$

$$x \in [-3, 7]$$

$$[-3, 1] \cup [3, 7]$$

$$|x| \leq a, a \in \mathbb{R}^+$$

$$-a \leq x \leq a$$

$$|x| > a, a \in \mathbb{R}^+$$

$$x < -a \text{ or } x > a$$

## QUESTION



$||x - 2| - 3| \leq 0$  then number of values of  $x$  satisfy the given inequality is

- A 0
- B 1
- C 2
- D Infinite

$$||x-2|-3| \leq 0 \quad \text{But } ||x-2|-3| \geq 0$$

$$\Rightarrow ||x-2|-3| = 0$$

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$$|x-2|-3=0$$

$$|x-2|=3$$

$$x-2 = -3, 3$$

$$x = -1, 5.$$

## QUESTION



$$\text{Solve: } |x| + |x - 1| \geq 7$$

$$\begin{array}{c} T_1 \\ T_2 \end{array} \begin{array}{c} - \quad + \quad + \\ \hline - \quad 0 \quad -ve \quad + \end{array}$$

Tahoi

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**QUESTION**

Solve:  $|x + 1| + |x - 1| > 2$



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## QUESTION



If  $|x^2 - x - 10| > |x^2 - 11x - 22|$  then find the possible set of all values of  $x$ .

$$|x^2 - x - 10|^2 > |x^2 - 11x - 22|^2$$

$$(x^2 - x - 10)^2 > (x^2 - 11x - 22)^2$$

$$(x^2 - x - 10 + x^2 - 11x - 22)(x^2 - x - 10 - x^2 + 11x + 22) > 0$$

$$(2x^2 - 12x - 32)(10x + 12) > 0$$

$$(x^2 - 6x - 16)(5x + 6) > 0$$

$$(x - 8)(x + 2)(5x + 6) > 0$$

$$\begin{array}{ccccccc} & - & & + & & - & & + \\ & | & + & | & - & | & + \\ \hline & - & 2 & - & 6 & / & 5 & 8 \end{array}$$

$$x \in (-2, -6/5) \cup (8, \infty)$$

$$\begin{array}{l} |x| > |y| \\ \Rightarrow x^2 > y^2 \end{array}$$

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**QUESTION**

Solve:  $|x - 6| > |x^2 - 5x + 9|$



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## QUESTION



$$\text{Solve: } (|x - 1| - 3)(|x + 2| - 5) < 0$$

$$|x - 1| - 3 < 0 \text{ \& } |x + 2| - 5 > 0$$

$$|x - 1| < 3 \text{ \& } |x + 2| > 5$$

$$-3 < x - 1 < 3 \text{ \& } x + 2 < -5 \text{ or } x + 2 > 5$$

$$-2 < x < 4 \text{ \& } x < -7 \text{ or } x > 3$$

$$x \in (-2, 4) \text{ \& } x \in (-\infty, -7) \cup (3, \infty)$$

$$x \in (3, 4)$$

UNION

$$x \in (-7, -2) \cup (3, 4)$$

$$\text{OR } |x - 1| - 3 > 0 \text{ \& } |x + 2| - 5 < 0$$

$$|x - 1| > 3 \text{ \& } |x + 2| < 5$$

$$x - 1 < -3 \text{ or } x - 1 > 3 \text{ \& } -5 < x + 2 < 5$$

$$x < -2 \text{ or } x > 4 \text{ \& } -7 < x < 3$$

$$x \in (-\infty, -2) \cup (4, \infty) \text{ \& } x \in (-7, 3)$$

$$x \in (-7, -2)$$

## QUESTION



$$\text{Solve: } (|x - 1| - 3)(|x + 2| - 5) < 0$$

Таһоу

M(2)

$$\begin{array}{c} T_1 \\ T_2 \end{array} \begin{array}{cccc} - & - & + & + \\ \hline - & - & + & + \end{array}$$

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# Using Triangle Inequality



$P_9: ||a| - |b|| \leq |a + b| \leq |a| + |b|$

$|a+b| = |a| + |b| \iff ab > 0$

Ex:  $|2 + (-3)| \leq |2| + |-3|$   
 $1 \leq 5$

$|a+b| = ||a| - |b|| \iff ab \leq 0$

Ex:  $|-5 + (-6)| \leq |-5| + |-6|$   
 $11 \leq 11$

$||a| - |-b|| \leq |a - b| \leq |a| + |-b|$

$P_{10}: ||a| - |b|| \leq |a - b| \leq |a| + |b|$

$|a-b| = |a| + |b| \iff a \cdot b \leq 0$

$||a| - |b|| = |a - b| \iff ab > 0$

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# QUESTION

$\overset{a}{|x-2|} + \overset{b}{|x-5|} = 3$

Solve:  $|x-2| + |x-5| = 3$

$$|a| + |b| \geq |a-b|$$

M(1)  $3 = |x-2| + |x-5| \geq |x-2-x+5| = 3$

$$(x-2)(x-5) \leq 0$$

M(3)

$$|x-2| + |-(5-x)| = 3$$

$$|x-2| + |5-x| = 3$$

$$3 = |x-2| + |5-x| \geq |x-2+5-x| = 3$$

M(2)

$$\begin{array}{cccc} - & + & + & \\ | & | & | & \\ -2 & -x & 5 & + \end{array}$$

$x \in [2, 5]$

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$$|x-2| + |x-5| = 3$$

$$(x-2)(5-x) \geq 0$$

$$(x-2)(x-5) \leq 0$$

$$x \in [2, 5]$$

Case (I)  $x \leq 2$

$$-(x-2) - (x-5) = 3$$

$$-x+2-x+5=3$$

$$2x=4$$

$$x=2 \checkmark$$

Case (II)  $2 < x < 5$

$$x-2+5-x=3 \quad \cap \quad x \in (2, 5)$$

$$3=3 \quad \sim \quad x \in \mathbb{R}$$

Case (III)  $x > 5$

$$x-2+x-5=3$$

$$2x=10$$

$$x=5 \checkmark$$

UNION

$$x \in [2, 5]$$

## QUESTION



Solve:  $\left| \frac{x}{x-1} \right| + |x| = \frac{x^2}{|x-1|}$

$\underbrace{\quad}_a + \underbrace{\quad}_b = \quad$

$$|a| + |b| = |a+b|$$

$$\Downarrow$$

$$a \cdot b \geq 0$$

$$\frac{x}{x-1} \geq 0$$

$$\frac{x^2}{x-1} \geq 0$$

$$\frac{1}{x-1} \geq 0, \quad x=0 \text{ is also possible}$$

$$x-1 > 0$$

$$x > 1$$

$$\Rightarrow \text{Ans: } x \in \{0\} \cup (1, \infty)$$

observe!!  $a+b = \frac{x}{x-1} + x = \frac{x+x^2-x}{x-1} = \frac{x^2}{x-1}$

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## QUESTION



Solve:  $\sqrt{x + 2\sqrt{x-1}} + \sqrt{x - 2\sqrt{x-1}} = 2$

$$\sqrt{x-1+1+2\sqrt{x-1}} + \sqrt{x-1+1-2\sqrt{x-1}} = 2$$

$$\sqrt{\sqrt{x-1}^2+1^2+2\sqrt{x-1}\cdot 1} + \sqrt{\sqrt{x-1}^2+1^2-2\sqrt{x-1}\cdot 1} = 2$$

$$|\sqrt{x-1} + 1| + |\sqrt{x-1} - 1| = 2$$

$$2 = |\sqrt{x-1} + 1| + |\sqrt{x-1} - 1| \geq |\sqrt{x-1} + 1 - (\sqrt{x-1} - 1)| = 2$$

By Triangle Ineq.

$$(\sqrt{x-1} + 1)(\sqrt{x-1} - 1) \leq 0$$

$$(x-1-1) \leq 0$$

$$x \leq 2 \rightarrow x \in (-\infty, 2] \text{ Ans } (-\infty, 2] \cap [1, \infty)$$

$$x \in [1, 2]$$

clearly  $x-1 \geq 0 \rightarrow x \geq 1$

$$x + 2\sqrt{x-1} \geq 0 \quad x - 2\sqrt{x-1} \geq 0$$

True

$$x \geq 2\sqrt{x-1}$$

$$x^2 \geq 4x - 4$$

$$x^2 - 4x + 4 \geq 0$$

$$(x-2)^2 \geq 0$$

True.

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## QUESTION



Solve for values of x :

$$(i) \quad |3x - 5| + |8 - x| = |3 + 2x|$$

$$(ii) \quad |x^2 - 5x + 6| + |x^2 - 4| = 5|x - 2|$$

$$|3x - 5| + |8 - x| \geq |3x - 5 + 8 - x| = |3 + 2x|$$

By Triangle Ineq

$$(3x - 5)(8 - x) \geq 0$$

$$(3x - 5)(x - 8) \leq 0$$

$$x \in [5/3, 8]$$

$$|x^2 - 5x + 6| + |x^2 - 4| \geq |x^2 - 5x + 6 - x^2 + 4|$$

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$$= 5|x - 2|$$

$$\Downarrow$$

$$(x^2 - 5x + 6)(x^2 - 4) \leq 0$$

$$(x - 2)(x - 3)(x - 2)(x + 2) \leq 0$$

$$(x - 3)(x + 2) \leq 0, \quad x = 2 \text{ is also possible}$$

$$x \in [-2, 3] \text{ Ans}$$

## QUESTION



Tah05

$$(a) \quad |x^2 - 9| + |16 - x^2| = 7$$

$$\text{Ans: } x \in [-4, -3] \cup [3, 4]$$

$$(b) \quad \text{Solve: } |(x^2 + 2x + 2) + (3x + 7)| < |x^2 + 2x + 2| + |3x + 7|$$

$$\text{Ans: } x < -7/3$$

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$$||a|-|b|| = |a+b| \Leftrightarrow ab < 0$$

$$|a+b| = |a|+|b| \Leftrightarrow ab > 0$$

$$||a|-|b|| \leq |a+b| \leq |a|+|b|$$

$$|a+b| < |a|+|b| \Leftrightarrow ab < 0$$

$$||a|-|b|| < |a+b| \Leftrightarrow ab > 0$$

$$||a|-|b|| = |a-b| \Leftrightarrow ab > 0$$

$$|a-b| = |a|+|b| \Leftrightarrow ab \leq 0$$

$$\star ||a|-|b|| \leq |a-b| \leq |a|+|b|$$

$$||a|-|b|| < |a-b| \Leftrightarrow ab < 0$$

$$|a-b| < |a|+|b| \Leftrightarrow ab > 0$$

## QUESTION



$$\text{Solve: } |x^2 - 2x| + |x - 4| > |x^2 - 3x + 4|$$

$$|a| + |b| > |a - b|$$

$$(x^2 - 2x) + (x - 4) > 0$$

$$x(x - 2) + (x - 4) > 0$$

$$\begin{array}{c} - & + & - & + \\ \hline & 0 & 2 & 4 \end{array}$$

$$x \in (0, 2) \cup (4, \infty)$$

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**Sabse Important Baat**



**Sabhi Class Illustrations **ATDB.uno** Retry Karnay hai...**



## Home Challenge-07



Find the sum of all the integral solution(s) of the equation  $3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}}\right)^2$ . [Ans. 3]

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# Today's BPP

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# Bumper Practice Problems



Solve the following inequality for x.

(a)  $|x| > 2$

(b)  $|x - 1| > 3$

(c)  $|x - 2| < 1$

(d)  $|x + 1| \geq 2$

(e)  $|x - 1| \leq 5$

(f)  $|2x - 3| > 7$

(g)  $|3x + 5| < 2$

(h)  $|4x + 6| > 5$

(i)  $|2x - 3| > -2$

(j)  $|4x - 9| \leq 7$

(k)  $|3x + 5| \geq 2$

(l)  $|2x + 3| \geq 0$

(m)  $|4 - 3x| < -2$

(n)  $|5 - 3x| \leq 0$

(o)  $\left| \frac{x-3}{x-5} \right| > 1$

(p)  $\left| \frac{x+4}{x+2} \right| \leq 1$

(q)  $|x^2 - 4x| < 5$

(r)  $|x - 3| > -1$

(s)  $|3x - 2.5| \geq 2$

(t)  $|x - 2| \leq |x + 4|$

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# Answers



(a)  $(-\infty, -2) \cup (2, \infty)$

(b)  $(-\infty, -2) \cup (4, \infty)$

(c)  $x \in (1, 3)$

(d)  $x \in (-\infty, -3] \cup [1, \infty)$

(e)  $x \in (-4, 6)$

(f)  $(-\infty, -2) \cup (5, \infty)$

(g)  $\left(-\frac{7}{3}, -1\right)$

(h)  $x \in \left(-\infty, -\frac{11}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$

(i)  $x \in (-\infty, \infty)$

(j)  $x \in \left[\frac{1}{2}, 4\right]$

(k)  $x \in \left(-\infty, -\frac{1}{3}\right] \cup [-1, \infty)$

(l)  $x \in \mathbb{R}$

(m)  $x \in \phi$

(n)  $x \in \left\{\frac{5}{3}\right\}$

(o)  $(4, \infty) - \{5\}$

(p)  $(-\infty, -3]$

(q)  $(-1, 5)$

(r)  $x \in (-\infty, \infty)$

(s)  $x \in \left(-\infty, \frac{1}{6}\right] \cup \left[\frac{3}{2}, \infty\right)$

(t)  $x \in [-1, \infty)$

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**QUESTION**

Solve the following equations

(a)  $|x - 1| + |x - 3| = 2$

(b)  $|x| + |x + 5| = 5$

(c)  $|x - 1| + |x - 4| = 2$

(d)  $|x^2 - 2x| + |x - 4| = |x^2 - 3x + 4|$

**ATDB.uno**

**Answers:**

(a)  $[1, 3]$

(b)  $[-5, 0]$

(c)  $\{\}$

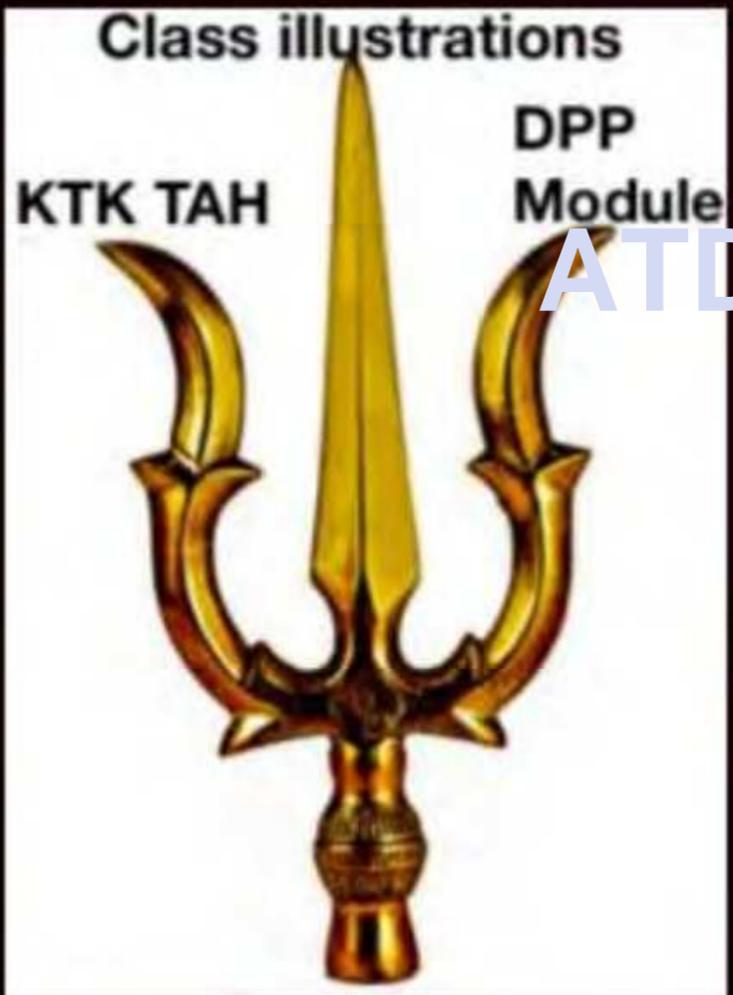
(d)  $x \in (-\infty, 0] \cup [2, 4]$



# Today's KTK



No Selection TRISHUL Selection with Good Rank  
Apnao IIT Jao



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## QUESTION

(KTK 1)



## Column-I

$$(A) \frac{\log_2 32}{\log_3 \sqrt{243}}$$

$$(B) \frac{2\log 6}{\log 12 + \log 3}$$

$$(C) \log_{1/4} \left(\frac{1}{16}\right)^{-2}$$

$$(D) \frac{\log_5 16 - \log_5 4}{\log_5 128}$$

## Column-II

(P) positive integer

(Q) negative integer

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(R) rational but not integer

(S) Prime

Ans. A  $\rightarrow$  P, S; B  $\rightarrow$  P; C  $\rightarrow$  Q; D  $\rightarrow$  R

## QUESTION

(KTK 2)



$$\frac{1}{x-1} - \frac{4}{x-2} + \frac{4}{x-3} - \frac{1}{x-4} < \frac{1}{30}$$

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$$\text{Ans. } x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$$

## QUESTION

(KTK 3)



The equation  $\frac{\log_8\left(\frac{8}{x^2}\right)}{(\log_8 x)^2} = 3$  has

- A** no integral solution
- B** one natural solution
- C** two real solutions
- D** one irrational solution

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Ans. B, C

## QUESTION

(KTK 4)



For the equation  $\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$ , which of the following do not hold good?

- A** no real solution
- B** one prime solution
- C** one integral solutions
- D** no irrational solution

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Ans. A, B, D

**QUESTION****(KTK 5)**

If  $\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4 (x^2) + \log_8 (x^3) + \log_{16} (x^4) = 40$  then  $x$  is equal to

**ATDB.uno****Ans. 256**

## QUESTION

(KTK 6)



Column-I		Column-II	
(A)	If $x_1$ and $x_2$ satisfy the equation $x^{\log_{10} x} = 100x$ then the value of $x_1 x_2$ equals	(P)	irrational
(B)	Sum of the squares of the roots of the equation $\log_2(9 - 2^x) = 3 - x$ is	(Q)	rational
(C)	If $\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \frac{1}{3}$ then $x$ is	(R)	prime
(D)	Let $\log_b a = 3$ , $\log_b c = -4$ . If the value of $x$ satisfying the equation $a^{3x} = c^{x-1}$ is expressed in the form $p/q$ , where $p$ and $q$ are relatively prime then $p + q$ is	(S)	composite

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Ans. A  $\rightarrow$  Q, S; B  $\rightarrow$  Q, S; C  $\rightarrow$  P; D  $\rightarrow$  QR



## Homework From Module



**Prarambh (Topicwise) : Q1 to Q25**

**Prabal (JEE Main Level) : Q1 to Q33**

# ATDB.uno



# Solution to Previous KTKs

## ATDB.uno

**QUESTION****(KTK 1)**

$$\text{Solve for } x : 3|x^2 - 4x + 2| = 5x - 4$$

**ATDB.uno****Ans.  $x = 2, 5$**



\* KTR

DATE: 11/11/2020  
PAGE: 1/1

1.

$$3 | x^2 - 4x + 2 | = 5x - 4$$

$$x^2 - 4x + 2 \geq 0$$

$$\frac{4 \pm \sqrt{16 - 8}}{2} \rightarrow 2 \pm \sqrt{2}$$

$$(-\infty, 2 - \sqrt{2}] \cup [2 + \sqrt{2}, \infty)$$

$$x^2 - 4x + 2 \leq 0$$

$$(2 - \sqrt{2}, 2 + \sqrt{2})$$

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$$3x^2 - 12x + 6 = 5x - 4$$

$$3x^2 - 17x + 10 = 0$$

$$3x^2 - 15x - 2x + 10 = 0$$

$$3x(x - 5) - 2(x - 5) = 0$$

$$(3x - 2)(x - 5) = 0$$

$$x = \frac{2}{3}, 5$$

$$-3x^2 + 12x - 6 = 5x - 4$$

$$3x^2 - 7x + 2 = 0$$

$$3x^2 - 6x - x + 2 = 0$$

$$3x(x - 2) - 1(x - 2) = 0$$

$$(3x - 1)(x - 2) = 0$$

$$x = \frac{1}{3}, 2$$

$$x = 2, 5$$



KTK-10) Solve for  $x$ :  $3|x^2-4x+2| = 5x-4$

Case-1:

$$x^2-4x+2 \geq 0$$

$$x = \frac{4 \pm \sqrt{16-4(2)}}{2}$$

$$x = \frac{4 \pm 2\sqrt{2}}{2} \Rightarrow x = 2+\sqrt{2}, 2-\sqrt{2}$$

$$(x-(2+\sqrt{2}))(x-(2-\sqrt{2})) \geq 0$$

$$x \in (-\infty, 2-\sqrt{2}] \cup [2+\sqrt{2}, \infty)$$

Case-2:

$$x^2-4x+2 < 0$$

$$(x-(2+\sqrt{2}))(x-(2-\sqrt{2})) < 0$$

$$x \in (2-\sqrt{2}, 2+\sqrt{2})$$

$$3(x^2-4x+2) - 5x + 4 = 0$$

$$3x^2 - 12x + 6 - 5x + 4 = 0$$

$$3x^2 - 17x + 10 = 0$$

$$3x^2 - 15x - 2x + 10 = 0$$

$$3x(x-5) - 2(x-5) = 0$$

$$(x-5)(3x-2) = 0$$

$$x = 5, x = \frac{2}{3}$$

$$[x = 5]$$

$$3(-x^2+4x-2) - 5x + 4 = 0$$

$$-3x^2 + 12x - 6 - 5x + 4 = 0$$

$$-3x^2 + 7x - 2 = 0$$

$$3x^2 - 7x + 2 = 0$$

$$3x^2 - 6x - x + 2 = 0$$

$$3x(x-2) - 1(x-2) = 0$$

$$(3x-1)(x-2) = 0$$

$$[x = 2], x = \frac{1}{3}$$

$\cup$

$$[x = 2, 5] \underline{\underline{\text{Ans.}}}$$

**Kriti Mathur**  
**Raj.**



KTK1

$$3|x^2-4x+2| = 5x-4$$

Case ①  $x^2-4x+2 \geq 0$

Case ②:  $x^2-4x+2 < 0$

$$(x-(2+\sqrt{2}))(x-(2-\sqrt{2})) \geq 0$$

$$x \in (2-\sqrt{2}, 2+\sqrt{2})$$

$$x \in (-\infty, 2-\sqrt{2}) \cup (2+\sqrt{2}, \infty)$$

$$3x^2-12x+6 = 5x-4 = 0$$

$$3x^2-17x+10 = 0$$

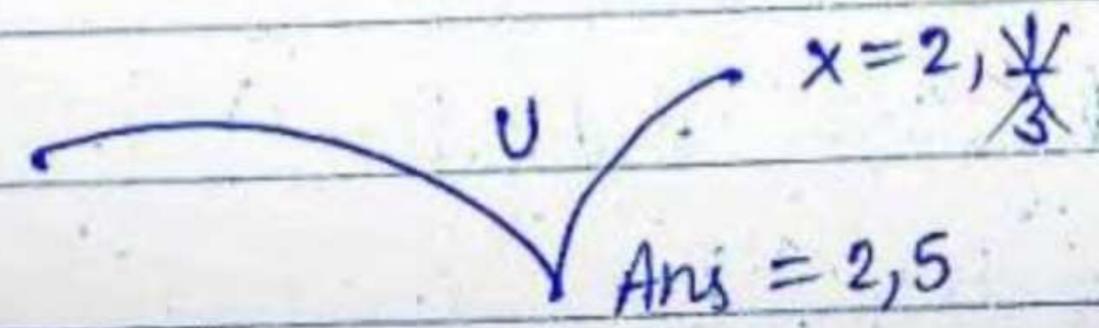
$$x = \frac{17 \pm 13}{6}$$

$$x = 5, \frac{2}{3}$$

$$3x^2 + 0x - 6 - 5x + 4 = 0$$

$$3x^2 - 7x + 2 = 0$$

$$x = \frac{7 \pm 5}{6} \quad (x-6)(x-1) = 0$$
  
$$x = 6, x = 1.$$



sakshisahu

## QUESTION

(KTK 2)



The number of real roots of the equation  $x|x| - 5|x + 2| + 6 = 0$ , is :

**A** 4

**B** 3

**C** 5

**D** 6

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Ans. B



KTK-2) The number of real roots of the equation

$$x^{\tau_1} |x| - 5 |x+2|^{\tau_2} + 6 = 0$$

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Raj.**

$\tau_1$	-ve		-ve		+ve
$\tau_2$	-ve	-2	+ve	0	+ve

Case-1

If  $x \geq 0$

$$x(x) - 5(x+2) + 6 = 0$$

$$x^2 - 5x - 10 + 6 = 0$$

$$x^2 - 5x - 4 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 4(-4)}}{2}$$

$$x = \frac{5 \pm \sqrt{41}}{2}$$

$$\left[ x = \frac{5 + \sqrt{41}}{2}, \frac{5 - \sqrt{41}}{2} \right]$$

Case-2

If  $-2 < x < 0$

$$x(-x) - 5(x+2) + 6 = 0$$

$$-x^2 - 5x - 10 + 6 = 0$$

$$-x^2 - 5x - 4 = 0$$

$$x^2 + 5x + 4 = 0$$

$$x^2 + 4x + x + 4 = 0$$

$$x(x+4) + 1(x+4) = 0$$

$$(x+1)(x+4) = 0$$

$$[x = -1], x = -4$$

Case-3

If  $x \leq -2$

$$x(-x) - 5(-x-2) + 6 = 0$$

$$-x^2 + 5x + 16 = 0$$

$$x^2 - 5x - 16 = 0$$

$$x = \frac{5 \pm \sqrt{25 - 4(-16)}}{2}$$

$$x = \frac{5 \pm \sqrt{89}}{2}$$

$$\left[ x = \frac{5 - \sqrt{89}}{2}, \frac{5 + \sqrt{89}}{2} \right]$$

No. of real roots = 3



the number of non roots of the equation  $x|x| - 5|x+2| + 6 = 0$ , is:  
 let assume to be  $x|x| - 5|x+2| + 6 = 0$

$t_1$	-ve	+	+	+
$t_2$	-ve	-2	1ve	0

Case I  $x \geq 0$

$$x^2 - 5x - 10 + 6 = 0$$

$$\Rightarrow x^2 - 5x - 4 = 0$$

$$\Rightarrow x = \frac{5 \pm \sqrt{25+16}}{2}$$

$b^2 - 4ac$   
 $= 25 - 4 \cdot 1 \cdot (-4)$

$$\Rightarrow x = \frac{5 \pm \sqrt{41}}{2} \Rightarrow x = \left( \frac{5 + \sqrt{41}}{2} \right), \left( \frac{5 - \sqrt{41}}{2} \right)$$

Case II  $-2 < x < 0$

$$-x^2 - 5x - 10 + 6 = 0$$

$$\Rightarrow x^2 + 5x + 4 = 0$$

$$\Rightarrow x = \frac{-5 \pm \sqrt{25-16}}{2}$$

$$\Rightarrow x = \frac{-5+3}{2}, \frac{-5-3}{2}$$

$$x = -1, -4$$

Case III  $x \leq -2$

$$-x^2 + 5x + 10 + 6 = 0$$

$$\Rightarrow x^2 - 5x - 16 = 0$$

$$\Rightarrow x = \frac{5 \pm \sqrt{89}}{2}$$

$$x = \frac{5 + \sqrt{89}}{2}, \frac{5 - \sqrt{89}}{2}$$

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## QUESTION



KTK 03

Solve for x :

$$\log 4 + \left(1 + \frac{1}{2x}\right) \log 3 = \log(\sqrt[x]{3} + 27)$$

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Ans.  $x \in \phi$



$$3. \log 4 + \left(1 + \frac{1}{2x}\right) \log 3 = \log \left(\sqrt[x]{3} + 27\right)$$

$$\text{Sol}^n - \log 4 + \log 3 + \log 3^{1/2x} = \log \left(\sqrt[x]{3} + 27\right)$$

$$\log (12 \cdot 3^{1/2x}) \stackrel{\log}{=} \log \left(\sqrt[x]{3} + 27\right)$$

$$12z = z^2 + 27$$

$$z^2 - 12z + 27 = 0$$

$$(z-3)(z-9) = 0$$

$$3^{1/2x} = 3$$

$$2x \geq 2, \quad 2x \in \mathbb{N}$$

$$3^{1/2x} = 3^2$$

$$\frac{1}{2x} = 1$$

so,

$$\frac{1}{2x} = 2$$

$$2x = 1$$

$$2x = \frac{1}{2}$$

$$x \in \emptyset$$

$$x = \frac{1}{4}$$



KTK-3) Solve for  $x$ :

$$\log 4 + \left(1 + \frac{1}{2x}\right) \log 3 = \log (\sqrt[2]{3} + 27)$$

$$\log 4 + \log 3 + \frac{1}{2x} \log 3 = \log (3^{1/2} + 3^3)$$

$$\log 12 + \log 3^{1/2x} = \log (3^{1/2} + 3^3)$$

$$\log (12 \times 3^{1/2x}) = \log (3^{1/2} + 3^3)$$

$$12 \times 3^{1/2x} = 3^{1/2} + 27$$

$$\text{Let } 3^{1/2x} = t, \quad t^2 = 3^{1/x}$$

$$\therefore 12t = t^2 + 27$$

$$t^2 - 12t + 27 = 0$$

$$t^2 - 9t - 3t + 27 = 0$$

$$t(t-9) - 3(t-9) = 0$$

$$t = 3, \quad t = 9$$

$$3^{1/2x} = 3, \quad 3^{1/2x} = 3^2$$

$$\frac{1}{2x} = 1, \quad \frac{1}{2x} = 2$$

$$x = \frac{1}{2}, \quad x = \frac{1}{4}$$

Rejected as  $\sqrt[2]{3}, x \in \mathbb{N}$

$\Rightarrow x \in \emptyset$  Ans.

**Kriti Mathur  
Raj.**

## QUESTION

## KTK 04



If  $(x_1, y_1)$  and  $(x_2, y_2)$  are the solution of the system of equation.

$$\log_{225}(x) + \log_{64}(y) = 4$$

$$\log_x(225) - \log_y(64) = 1,$$

then show that the value of  $\log_{30}(x_1y_1x_2y_2) = 12$ .

# ATDB.uno

## QUESTION

KTK 05



The sum of the roots of the equation  $x + 1 - 2 \log_2(3 + 2^x) + 2 \log_4(10 - 2^{-x}) = 0$ , is :

**A**  $\log_2 14$

**B**  $\log_2 11$

**C**  $\log_2 12$

**D**  $\log_2 13$

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Ans. B

## QUESTION

KTK 06



Evaluate 
$$\frac{\left( (64)^{\frac{1}{\log_5 8}} + 2^{\frac{2}{\log_{\sqrt{5}} 2}} \right) \left( (\sqrt{11})^{\frac{2}{\log_{25} 11}} - (64)^{\log_8 \sqrt{5}} \right)}{300}$$

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Ans. 2



$$6. \left( (64)^{\frac{1}{\log_5 8}} + 2^{\frac{2}{\log_5 2}} \right) \left( (\sqrt{11})^{\frac{2}{\log_2 11}} - (64)^{\log_8 \sqrt{5}} \right)$$

$$\text{Sol}^n - \left( (8)^{2 \log_5 5} + 2^{2 \log_2 \sqrt{5}} \right) \left( 11^{\frac{1}{2} \times 2 \log_{11} 25} - 8^{2 \log_8 \sqrt{5}} \right)$$

$$= \frac{(25 + \sqrt{5}^2)}{300} \left( 25 - 8 \right) = \boxed{0}$$

$$= \frac{625 - 25}{300} = \frac{600}{300} = \frac{31}{15}$$

$$= \frac{600}{300} = \boxed{2}$$



$$\text{KTK-6)} \text{ Evaluate } \frac{(64)^{\frac{1}{\log_5 8}} + 2^{\frac{2}{\log_{\sqrt{5}} 2}}}{300} \left( (\sqrt{11})^{\frac{2}{\log_{25} 11}} - (64)^{\log_8 \sqrt{5}} \right)$$

$$\Rightarrow \frac{(64)^{\log_8 5} + 2^{\frac{2 \log_2 \sqrt{5}}{300}}}{300} \left( (\sqrt{11})^{2 \log_{25} 11} - (64)^{\log_8 \sqrt{5}} \right)$$

$$\Rightarrow \frac{(5^{\log_8 64} + \sqrt{5}^{\frac{2 \log_2 2}{300}})}{300} \left( 25^{2 \log_{(11)^{1/2}} 11} - \sqrt{5}^{\log_8 64} \right)$$

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$$\Rightarrow \frac{(5^2 + 5)}{300} (25 - 5) = \frac{(25 + 5)(25 - 5)}{300}$$

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**Raj.**

$$= \frac{625 - 25}{300} = \frac{600}{300}$$

$$= \underline{\underline{2 \text{ Ans.}}}$$

## QUESTION

KTK 07



$$\text{Simplify : } 5^{\log_{\frac{1}{5}}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2} \frac{1}{10+2\sqrt{21}}$$

ATDB.uno

Ans. 6



$$4. \quad 5^{\log_{\frac{1}{5}}\left(\frac{1}{2}\right)} + \log_{\sqrt{2}} \frac{4}{\sqrt{7} + \sqrt{3}} + \log_{\frac{1}{2}} \frac{1}{10 + 2\sqrt{21}}$$

$$= 2 + \log_2 \left(\frac{4}{\sqrt{7} + \sqrt{3}}\right)^2 + \log_2 (10 + 2\sqrt{21})$$

$$= 2 + \log_2 \left(\frac{16}{10 + 2\sqrt{21}}\right) + \log_2 (10 + 2\sqrt{21})$$

$$= 2 + \log_2 \left(\frac{16}{10 + 2\sqrt{21}} \times (10 + 2\sqrt{21})\right) = 2 + \log_2 16$$

16

$$\text{KTK-9)} \text{ Simplify : } 5^{-\frac{1}{5}} + \log_{\sqrt{2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{1/2} \frac{1}{10+2\sqrt{2}}$$

$$= 5^{\log_5 2^{-1}} + \log_{2^{1/2}} \frac{4}{\sqrt{7}+\sqrt{3}} + \log_{2^{-1}} \frac{1}{10+2\sqrt{2}}$$

$$= 5 \log_5 2 + 2 \log_2 \frac{4}{\sqrt{7}+\sqrt{3}} - \log_2 \frac{1}{10+2\sqrt{2}}$$

$$= 2 + \log_2 \left( \frac{4}{\sqrt{7}+\sqrt{3}} \right)^2 - \log_2 \left( \frac{1}{10+2\sqrt{2}} \right)$$

$$= 2 + \log_2 \left( \frac{\left( \frac{4}{\sqrt{7}+\sqrt{3}} \right)^2}{\frac{1}{10+2\sqrt{2}}} \right)$$

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$$= 2 + \log_2 \left( \frac{16}{(\sqrt{7}+\sqrt{3})^2} \times 10+2\sqrt{2} \right)$$

$$= 2 + \log_2 \left( \frac{16}{7+3+2\sqrt{21}} \times 10+2\sqrt{21} \right)$$

$$= 2 + \log_2 2^4 = 2 + 4 = \underline{\underline{6}} \text{ Ans.}$$





# Solution to Previous TAH

## ATDB.uno

**QUESTION [JEE Mains 2024 (8 April)]****TAH 11**

The number of distinct real roots of the equation  $|x + 1| |x + 3| - 4|x + 2| + 5 = 0$  is

**ATDB.uno****Ans. 2**



The number of distinct real roots of the equation  $|x + 1| |x + 3| - 4|x + 2| + 5 = 0$  is

$$\Rightarrow |x+1| |x+3| - 4|x+2| + 5 = 0$$

$t_1 \quad t_2 \quad t_3$

Case-1  $x \leq -3$

$$(x+1)(x+3) + 4(x+2) + 5 = 0$$

$$x^2 + 4x + 3 + 4x + 8 + 5 = 0$$

$$x^2 + 8x + 16 = 0$$

$$(x+4)^2 = 0$$

$$x = -4$$

Case-2

$$-3 < x \leq -2$$

$$-(x+1)(x+3) + 4(x+2) + 5 = 0$$

$$- [x^2 + 4x + 3] + 4x + 8 + 5 = 0$$

$$-x^2 - 4x - 3 + 4x + 13 = 0$$

$$0 = x^2 - 10$$

$$x = +\sqrt{10} \text{ or } -\sqrt{10} \Rightarrow \text{both rejected}$$

Ans. 2

Redo





Case-3  $-2 < x \leq -1$

$$-(x+1)(x+3) - 4(x+2) + 5 = 0$$

$$-x^2 - 4x - 3 - 4x - 8 + 5 = 0$$

$$x^2 + 8x - 6 = 0$$

$$x = \frac{-8 \pm \sqrt{64 + (4)(1)(6)}}{2}$$

$$= \frac{-8 \pm \sqrt{88}}{2}$$

$$= -4 \pm 2\sqrt{22}$$

↓  
rejected

Case-4  $x > -1$

$$(x+1)(x+3) - 4(x+2) + 5 = 0$$

$$x^2 + 4x + 3 - 4x - 8 + 5 = 0$$

$$x^2 = 0$$

$$x = 0$$

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Union of all the cases

give  $x = 0 \cup -4$

②

**QUESTION**

The number of solutions of the equation

$$\log_{(x+1)}(2x^2 + 7x + 5) + \log_{(2x+5)}(x + 1)^2 - 4 = 0, x > 0, \text{ is :}$$

**ATDB.uno**

**Ans. 1**

$$\log_{(x+1)}(2x^2+7x+5) + \log_{(2x+5)}(x+1)^2 - 4 = 0, x > 0, \text{ is:}$$

$$\log_{(x+1)}((2x+5)(x+1)) + \log_{(2x+5)}(x+1)^2 - 4 = 0$$

$$\log_{(x+1)}(2x+5) + \log_{(x+1)}(x+1) + 2 \log_{(2x+5)}(x+1) - 4 = 0$$

$$\text{Let } \log_{(x+1)}(2x+5) = t$$

$$t + 1 + \frac{2}{t} - 4 = 0$$

TAH-01

1 solution

$$t^2 - 3t + 2 = 0$$

$$t^2 - t - 2t + 2 = 0$$

$$t(t-1) - 2(t-1) = 0$$

$$(t-1)(t-2) = 0$$

$$t = 1$$

$$t = 2$$

$$\log_{(x+1)}(2x+5) = 1$$

$$2x+5 = x+1$$

$$x = -4$$

Put and check ✗

$$\log_{(x+1)}(2x+5) = 2$$

$$2x+5 = (x+1)^2$$

$$0 = x^2 - 4$$

$$0 = x^2 - 2^2$$

$$(x+2)(x-2) = 0$$

$$x = 2, -2$$

Put and check

-2 Not Possible



Date: / /

L-14

TAH-01

The number of solutions of the equation  $\log_{(x+1)}(2x^2+7x+5) + \log_{(2x+5)}(x+1)^2 - 4 = 0, x > 0,$

$$\log_{(x+1)}(2x^2+7x+5) + 2 \log_{(2x+5)}(x+1) - 4 = 0$$

$$\log_{(x+1)}((x+1)(2x+5)) + 2 \log_{(2x+5)}(x+1) - 4 = 0$$

$$\log_{(x+1)}(x+1) + \log_{(x+1)}(2x+5) + 2 \log_{(2x+5)}(x+1) - 4 = 0$$

$$1 + \log_{(x+1)}(2x+5) + \frac{2}{\log_{(x+1)}(2x+5)} - 4 = 0$$

$$\text{let } \log_{(x+1)}(2x+5) = t$$

$$t + \frac{2}{t} - 3 = 0$$

$$t^2 - 3t + 2 = 0$$

$$(t-1)(t-2) = 0$$

$$t = 1, 2$$

$$\log_{(x+1)}(2x+5) = 1, 2$$

$$\textcircled{1} 2x+5 = (x+1)^1 \text{ or } 2x+5 = (x+1)^2$$

$$x = -4$$

Not possible

$$2x+5 = x^2+2x+1$$

$$x^2 = 4$$

$$x = \pm 2$$

$$x = 2, -2$$

Not possible

Only  $x = 2$  ans.

Lecture - 14

Q. Find the number of solutions of the equation

$$\log_{(x+1)} (2x^2 + 7x + 5) + \log_{(2x+5)} (x+1)^2 - 4 = 0, \quad x > 0$$

Ans.

$$\log_{(x+1)} (2x^2 + 7x + 5) + \log_{(2x+5)} (x+1)^2 = 4$$

$$\log_{(x+1)} (2x^2 + 7x + 5) + 2 \log_{(2x+5)} (x+1) = 4$$

$$\log_{(x+1)} (2x+5)(x+1) + 2 \log_{(2x+5)} (x+1) = 4$$

$$= \log_{(x+1)} (x+1) + \log_{(x+1)} (2x+5) + \frac{2}{\log_{(x+1)} (2x+5)} = 4$$

$$\log_{(x+1)} (2x+5) + \frac{2}{\log_{(x+1)} (2x+5)} = 3$$

$$\text{Let } \log_{(x+1)} (2x+5) = t$$

$$t + \frac{2}{t} = 3 \Rightarrow t^2 + 2 = 3t$$

$$\Rightarrow t^2 - 3t + 2 = 0$$

$$\Rightarrow t^2 - 2t - t + 2 = 0$$

$$\Rightarrow t(t-2) - 1(t-2) = 0$$

$$\Rightarrow (t-1)(t-2) = 0$$

$$t = 1$$

$$\log_{(x+1)} (2x+5) = 1$$

$$2x+5 = x+1$$

$$x+5-1=0$$

$$x = -4$$

$$\text{As } x > 0$$

$$t = 2$$

$$\log_{(x+1)} (2x+5) = 2$$

$$(2x+5) = (x+1)^2$$

$$x^2 + 1 + 2x - 2x - 5 = 0$$

$$x^2 - 4 = 0$$

$$(x+2)(x-2) = 0$$



**Kriti Mathur  
Raj.**



Tah 1

$$\log(x+1)^{(2x^2+7x+5)} + \log(2x+5)^{(x+1)^2-4} = 0 \quad x > 0$$

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$$\log(x+1)^{(2x^2+5x+2x+5)} + \log(2x+5)^{(x+1)^2-4} = 0$$

$$\log(x+1)^{(x+1)(2x+5)} + \log(2x+5)^{(x+1)^2-4} = 0$$

$$\log(x+1)^{(x+1)} + \log(x+1)^{(2x+5)} + 2 \log(2x+5)^{(x+1)-4} = 0$$

Let  $\log(x+1)^{(2x+5)} = t$

$$1 + t + \frac{2}{t} - 4 = 0$$

$$t^2 + 2 - 3t = 0$$

$$t^2 - 3t + 2 = 0$$

$$(t-2)(t-1) = 0$$

$$t = 1, 2$$

only one sol<sup>n</sup>  $x=2$

**sakshisahu**

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$$\log(x+1)^{(2x+5)} = 1, 2$$

$$(2x+5) = (x+1)$$

$$x = -4$$

X rejected

$$2x+5 = (x+1)^2$$

$$\cancel{2x} + 5 = x^2 + 1 + \cancel{2x}$$

$$4 = x^2$$

$$x = \pm 2$$

$$x = 2, -\cancel{2}$$

## Question



Find the integral value of  $x$  satisfying the equation  $|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$ .

[Ans. 9]

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**SOURAV KALITA  
ASSAM**

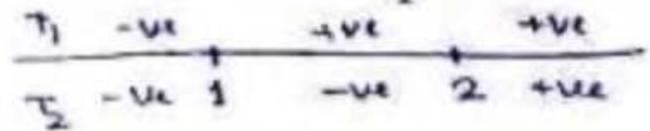
$$|\log_{13} x - 2| - |\log_3 x - 2| = 2$$

$$\log_{13} x = 2 \log_3 x$$

$$|2 \log_3 x - 2| - |\log_3 x - 2| = 2$$

Let  $\log_3 x = t$

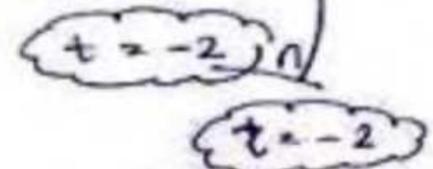
$$\frac{|2t-2|}{T_1} - \frac{|t-2|}{T_2} = 2$$



Case I:  $t \leq 1$

$$-2t + 2 + t - 2 = 2$$

$$-t = 2$$

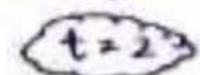


Case II:  $1 < t \leq 2$

$$2t - 2 + t - 2 = 2$$

$$3t = 6$$

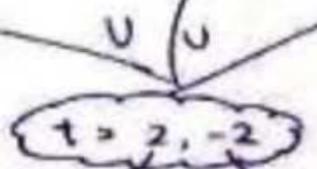
$$t = 2$$



Case III:  $t > 2$

$$2t - 2 - t + 2 = 2$$

$$t = 2$$



$$\log_3 x = 2$$

$$x = 3^2$$

$$x = 9$$

$$\log_3 x = -2$$

$$x = 3^{-2}$$

$$x = \frac{1}{9} \text{ (Rejected)}$$

∴ Ans: 9 ✓



TAM-02

Find the integral value of  $x$  satisfying the equation  $|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$ .

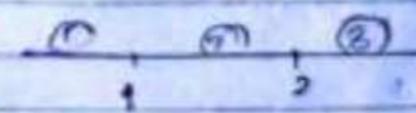
Soln

$$|2 \log_3 x - 2| - |\log_3 x - 2| = 2$$

Let  $\log_3 x = t$

$$\Rightarrow |2(t-1)| - |t-2| - 2 = 0$$

$$2|t-1| - |t-2| - 2 = 0$$



Case 1:  $t \leq 1$

$$-2(t-1) + (t-2) - 2 = 0$$

$$-2t + 2 + t - 2 - 2 = 0$$

$$-t - 2 = 0$$

$$t = -2$$

$$\log_3 x = -2$$

$x = 1/9$  ⇒ rejected Not Integer

Case 2:  $1 < t \leq 2$

$$2(t-1) + (t-2) - 2 = 0$$

$$2t - 2 + t - 2 - 2 = 0$$

$$3t - 6 = 0$$

$$t = 2$$

$$\log_3 x = 2 \Rightarrow x = 9$$



TAH2) Find the integral value of  $x$  satisfying the equation

$$|\log_{\sqrt{3}} x - 2| - |\log_3 x - 2| = 2$$

$$|\log_{3^{1/2}} x - 2| - |\log_3 x - 2| = 2$$

$$|\frac{1}{2} \log_3 x - 2| - |\log_3 x - 2| = 2$$

$$|2 \log_3 x - 2| - |\log_3 x - 2| = 2$$

$$\text{Let } \log_3 x = t \Rightarrow 2|t-1| - |t-2| = 2$$

$T_1$	-ve		+	+
$T_2$	-ve		-	+

Case-1

$$\text{If } x \geq 2$$

$$2(t-1) - (t-2) = 2$$

$$2t-2-t+2-2=0$$

$$t-2=0$$

$$[t=2]$$

$$\text{If } 1 \leq x < 2$$

$$2(t-1) - (-t+2) = 2$$

$$2t-2+t-2-2=0$$

$$3t-8=0$$

$$t = \frac{8}{3} \times$$

$$t = 2, -2$$

$$\text{at } t=2$$

$$\log_3 x = 2$$

$$[x=9]$$

$$\text{at } t=-2$$

$$\log_3 x = -2$$

$$[x = \frac{1}{9}]$$

Integral value of  $x = 9$

**Kriti Mathur  
Raj.**

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Case-3

$$\text{If } x < 1$$

$$2(-t+1) - (-t+2) - 2 = 0$$

$$-2t+2+t-2-2=0$$

$$-t-2=0$$

$$[t=-2]$$

## QUESTION



$$|x^2 + 4x + 3| + 2x + 5 = 0$$

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TAH 03:

$$|x^2 + 4x + 3| + 2x + 5 = 0$$

Case I:  $x^2 + 4x + 3 > 0 \rightarrow (x+3)(x+1) > 0$   
 $x \in (-\infty, -3] \cup [-1, \infty)$

$$x^2 + 4x + 3 + 2x + 5 = 0$$

$$x^2 + 6x + 8 = 0$$

$$(x+4)(x+2) = 0$$

$$x = \cancel{-2}, -4$$

$$x = -4$$

$$x = -4 \text{ or } -1 - \sqrt{3}$$

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**ASSAM**



Case II:  $x^2 + 4x + 3 < 0$   
 $\Rightarrow (x+3)(x+1) < 0$   
 $x \in (-3, -1)$

$$-x^2 - 4x - 3 + 2x + 5 = 0$$

$$-x^2 - 2x + 2 = 0$$

$$x^2 + 2x - 2 = 0$$

$$D = 4 - 4(-2)$$

$$= 12 > 0$$

$$x = \frac{-2 \pm \sqrt{12}}{2}$$

$$x = -1 \pm \sqrt{3}$$

$$x = -1 - \sqrt{3}$$

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TAH 3)

$$|x^2 + 4x + 3| + 2x + 5 = 0$$

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Case 1:

$$\text{If } x^2 + 4x + 3 \geq 0$$

$$x^2 + 3x + x + 3 \geq 0$$

$$x(x+3) + 1(x+3) \geq 0$$

$$(x+1)(x+3) \geq 0$$

$$x \in (-\infty, -3] \cup [-1, \infty)$$

$$x^2 + 4x + 3 + 2x + 5 = 0$$

$$x^2 + 6x + 8 = 0$$

$$x^2 + 4x + 2x + 8 = 0$$

$$x(x+4) + 2(x+4) = 0$$

$$(x+2)(x+4) = 0$$

$$x = -2, -4$$

$$[x = -4]$$

Case -2:

$$\text{If } x^2 + 4x + 3 < 0$$

$$(x+1)(x+3) < 0$$

$$x \in (-1, -3)$$

$$-x^2 - 4x - 3 + 2x + 5 = 0$$

$$-x^2 - 2x + 2 = 0$$

$$x^2 + 2x - 2 = 0$$

$$D = \sqrt{4 - 4(-2)}$$

$$D = \sqrt{12} = 2\sqrt{3}$$

$$x = \frac{-2 \pm 2\sqrt{3}}{2}$$

$$x = -1 + \sqrt{3}, -1 - \sqrt{3}$$

$$[x = -1 - \sqrt{3}]$$

$$\cup$$

$$[x = -4, -1 - \sqrt{3}] \text{ Ans.}$$

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# Solution to Previous BPPs

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## Lo Karo Duvaadaar Practice!!



$$1. \log_5(x^2 - 3x + 3) > 0$$

$$2. \log_7[\log_5(x^2 - 7x + 15)] > 0$$

$$3. \log_{\left(\frac{1}{2}\right)}[\log_5(x^2 - 7x + 17)] > 0$$

$$4. \log_{\left(\frac{1}{2}\right)}(\log_5(\log_2(x^2 - 6x + 40))) > 0$$

$$5. \log_3[\log_5 \log_2(x^2 - 9x + 50)] > 0$$

$$6. \log_6\left(\frac{x-2}{6-x}\right) > 0$$

$$7. \log_{0.5}(x^2 - 5x + 6) > -1$$

$$8. \log_8(x^2 - 4x + 3) < 1$$

$$9. \log_{\left(\frac{1}{4}\right)}\left(\frac{35-x^2}{x}\right) \geq -\frac{1}{2}$$



## Answers



1.  $(-\infty, 1) \cup (2, \infty)$

2.  $(-\infty, 2) \cup (5, \infty)$

3.  $(3, 4)$

4.  $(2, 4)$

5.  $(-\infty, 3) \cup (6, \infty)$

6.  $(4, 6)$

7.  $(1, 4)$

8.  $(1, 5)$

9.  $(-1, 0) \cup (5, \infty)$

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$$1) \quad x^2 - 3x + 3 > 1$$

$$x^2 - 2x - x + 2 > 0$$

$$(x-2)(x-1) > 0$$

$$x \in (-\infty, 1) \cup (2, \infty)$$

$$2.) \quad \log_5 (x^2 - 7x + 15) > 1$$

$$x^2 - 7x + 15 > 5$$

$$x^2 - 2x - 5x + 10 > 0$$

$$(x-2)(x-5) > 0$$

$$x \in (-\infty, 2) \cup (5, \infty)$$

$$\cap \quad x^2 - 7x + 15 > 0$$

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↓

$D < 0$

$$x^2 - 7x + 15 > 0 \quad \forall x \in \mathbb{R}$$



$$4) \log_{1/2} \left( \log_5 \left( \log_2 (x^2 - 6x + 40) \right) \right) > 0$$

$$\log_5 \left( \log_2 (x^2 - 6x + 40) \right) < 1$$

$$\log_2 (x^2 - 6x + 40) < 5$$

$$x^2 - 6x + 40 < 32$$

$$x^2 - 6x + 8 < 0$$

$$x^2 - 4x - 2x + 8 < 0$$

$$(x - 4)(x - 2) < 0$$

$$x \in (2, 4) \rightarrow i$$

$$\log_5 \left( \log_2 (x^2 - 6x + 40) \right) > 0$$

$$\log_2 (x^2 - 6x + 40) > 1$$

$$x^2 - 6x + 40 > 2$$

$$x^2 - 6x + 38 > 0$$

↓

$$D < 0, a > 0$$

↓

$$x^2 - 6x + 38 > 0 \quad \forall x \in \mathbb{R} \rightarrow ii$$

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$$\log_2 (x^2 - 6x + 40) > 0$$

$$x^2 - 6x + 40 > 1$$

$$x^2 - 6x + 39 > 0$$

↓

$$D < 0, a > 0$$

$$x^2 - 6x + 39 > 0 \quad \forall x \in \mathbb{R}$$

$$x \in \mathbb{R} \rightarrow \text{iii}$$

$$x^2 - 6x + 40 > 0$$

↓

$$D < 0, a > 0$$

$$x^2 - 6x + 40 > 0 \quad \forall x \in \mathbb{R} \rightarrow \text{iv}$$

i ∩ ii ∩ iii ∩ iv

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$$x \in (2, 4)$$



$$6.) \log_6 \left( \frac{x-2}{6-x} \right) > 0$$

$$\frac{x-2}{6-x} > 1$$

$$\frac{x-2}{6-x} - 1 > 0$$

$$\frac{x-2 - (6-x)}{6-x} > 0$$

$$\frac{x-2-6+x}{6-x} > 0$$

$$\frac{2x-8}{x-6} < 0$$

$$\Delta \quad \frac{x-2}{6-x} > 0$$

↳ No Need

$$\frac{x-4}{x-6} < 0$$

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$$x \in (4, 6)$$



Opp - 6

$$\log_6 \left( \frac{x-2}{6-x} \right) > 0$$

Solution:-

$$\frac{x-2}{6-x} > 1$$

$$\frac{x-2}{6-x} > 0$$

$$\frac{x-2}{6-x} - 1 > 0$$

↓  
No Need

$$\frac{x-2-6+x}{6-x} > 0$$

$$\frac{2x-8}{6-x} > 0$$

$$\frac{2(x-4)}{6-x} > 0$$

$$\frac{x-4}{x-6} < 0$$

$$x \in (4, 6)$$



$$8.) \log_8 (x^2 - 4x + 3) < 1$$

$$x^2 - 4x + 3 < 8$$

$$x^2 - 4x - 5 < 0$$

$$(x-5)(x+1) < 0$$

$$x \in (-1, 5)$$

$$\& \quad x^2 - 4x + 3 > 0$$

$$x^2 - 3x - x + 3 > 0$$

$$(x-3)(x-1) > 0$$

$$x \in (-\infty, 1) \cup (3, \infty)$$

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$$x \in (-1, 1) \cup (3, 5)$$



**THANK**  
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**YOU**