PRAYAS
JEE 2026

Mathematics

Basic Maths

Lecture - 16 | By – Ashish Agarwal Sir (IIT Kanpur)

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited

ATDB.uno

ATDB.uno

Topics *to be covered*

A Graphs involving Modulus

B Irrational Inequalities

C Problem Practice

ATDB.uno

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB.uno

QUESTION

(KTK 2)

$$\frac{1}{x-1} - \frac{4}{x-2} + \frac{4}{x-3} - \frac{1}{x-4} < \frac{1}{30}$$

$$\frac{1}{x-1} - \frac{1}{x-4} + \frac{4}{x-3} - \frac{4}{x-2} < \frac{1}{30}$$

$$\frac{x-4-x+1}{(x-1)(x-4)} + \frac{4x-8-4x+12}{(x-2)(x-3)} < \frac{1}{30}$$

$$\frac{-3}{x^2-5x+4} + \frac{4}{x^2-5x+6} < \frac{1}{30}$$

Let

$$\frac{4}{t+6} - \frac{3}{t+4} < \frac{1}{30}$$

$$\frac{4t+16-3t-18}{(t+4)(t+6)} < \frac{1}{30}$$

$$\frac{(t-2)}{t^2+10t+24} - \frac{1}{30} < 0$$

$$\frac{30t-60-t^2-10t-24}{30(t+4)(t+6)} < 0$$

$$\frac{-t^2+20t-84}{(t+4)(t+6)} < 0$$

$$\frac{t^2-20t+84}{(t+4)(t+6)} > 0$$

$$\frac{(t-6)(t-14)}{(t+4)(t+6)} > 0$$

$$\frac{(x^2-5x-6)(x^2-5x-14)}{(x^2-5x+4)(x^2-5x+6)} > 0$$

$$\frac{(x-6)(x+1)(x-7)(x+2)}{(x-4)(x-1)(x-2)(x-3)} > 0$$

Ans. $x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

QUESTION (KTK 4)

For the equation $\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$, which of the following do not hold good?

A no real solution

B one prime solution

C one integral solutions

D no irrational solution

$$\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$$

$$\frac{\log_3 x}{\log_3 3\sqrt{x}} + \frac{\log_3 \sqrt{x}}{\log_3 3x} = 0$$

$$\frac{\log_3 x}{1 + \frac{1}{2}\log_3 x} + \frac{\frac{1}{2}\log_3 x}{1 + \log_3 x} = 0 \qquad \text{let } \log_3 x = t$$
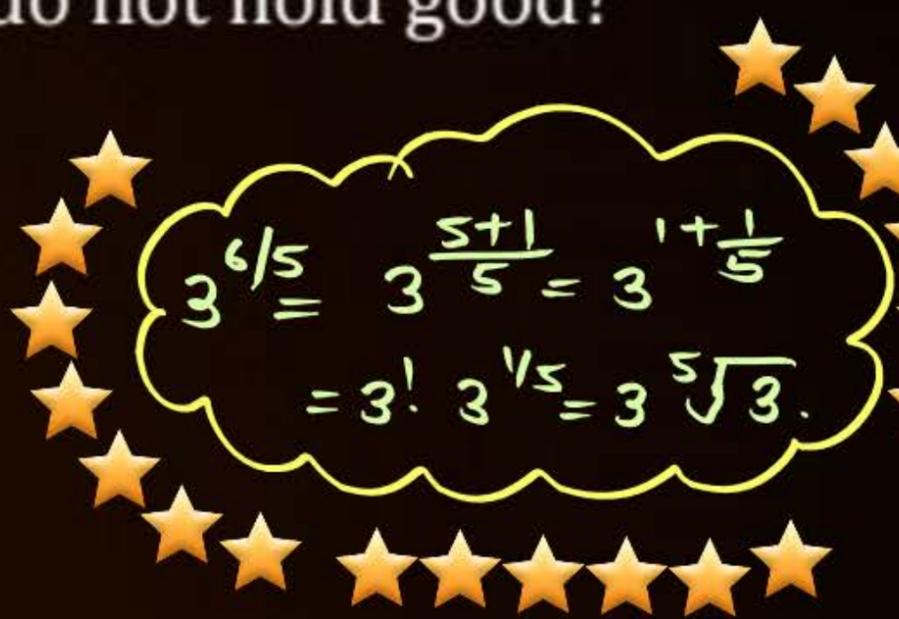
$$t\left(\frac{1}{1 + t/2} + \frac{1}{2(1+t)}\right) = 0$$

$$t = 0 \quad \text{or} \quad \frac{2}{2+t} + \frac{1}{2(1+t)} = 0$$

$$x = 1 \qquad 4 + 4t + 2 + t = 0$$

$$3^{6/5} = 3^{\frac{5+1}{5}} = 3^{1+\frac{1}{5}}$$
$$= 3! \; 3^{1/5} = 3\sqrt[5]{3}.$$

$$t = -6/5$$

$$x = 3^{-6/5} = \frac{1}{3^{6/5}} = \frac{1}{3\sqrt[5]{3}}$$
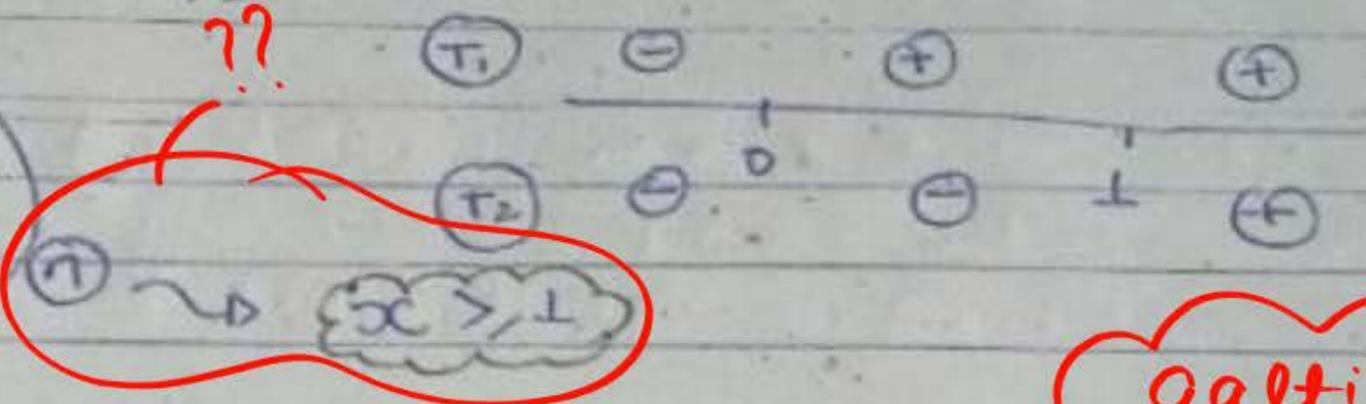
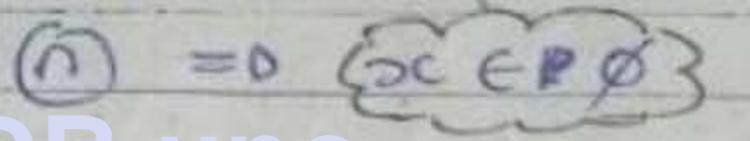Ans. A, B, D

Aao Machaay Dhamaal
Deh Swaalpe Deh Swaal

ATDB.unoFz

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

$$\text{Lecture} = 15$$

Ans.

Tah01. Solve $|x| + |x-1| \geq 7$.

$T_1 \qquad T_2$

$\Rightarrow$ Case 01 $\quad x \geq 1$.

$\Rightarrow \quad x + x - 1 \geq 7$.

$\Rightarrow \quad 2x - 8 \geq 0$.

$\Rightarrow \quad x - 4 \geq 0$.

$\cap \rightsquigarrow \boxed{x \geq 1}$

Case 02. $\quad 0 \leq x < 1$

$\Rightarrow \quad x - x + 1 \geq 7$

$\Rightarrow \quad 1 \geq 7 \quad (\text{Not possible})$

$\cap \Rightarrow \boxed{x \in \varnothing}$

Case 03. $\quad x < 0$:

$\Rightarrow \quad -x - x + 1 \geq 7$.

$\Rightarrow \quad -2x - 6 \geq 0$

$\Rightarrow \quad x + 3 \leq 0$.

$\cap$

$\rightsquigarrow \boxed{x < 0}$

-3

$\Rightarrow$ Union of all cases

$\rightsquigarrow \quad x \in (-\infty, 0) \cup [1, \infty)$

Ans.

galti Batao

??

Downloaded from ATDB.uno/StudyPrayas | Unauthorised redistribution is strictly prohibited.
ATDB.uno | 19 Mar 2026, 14:43 IST

ATDB.uno

ATDB.uno fz

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited
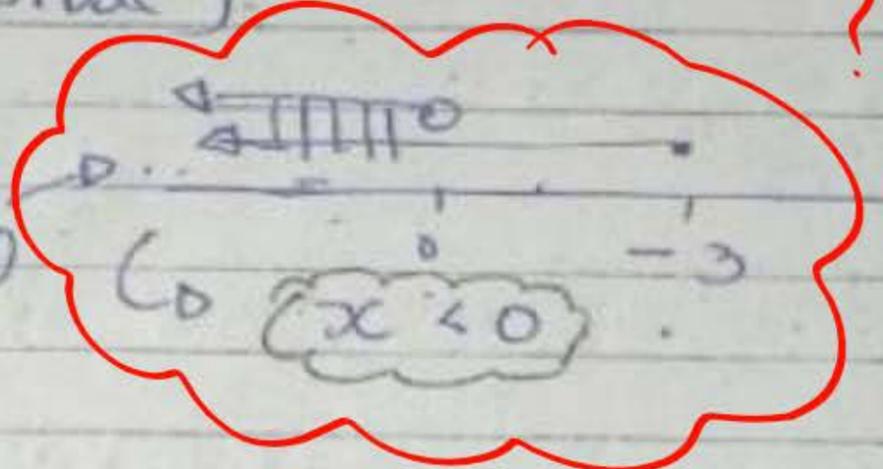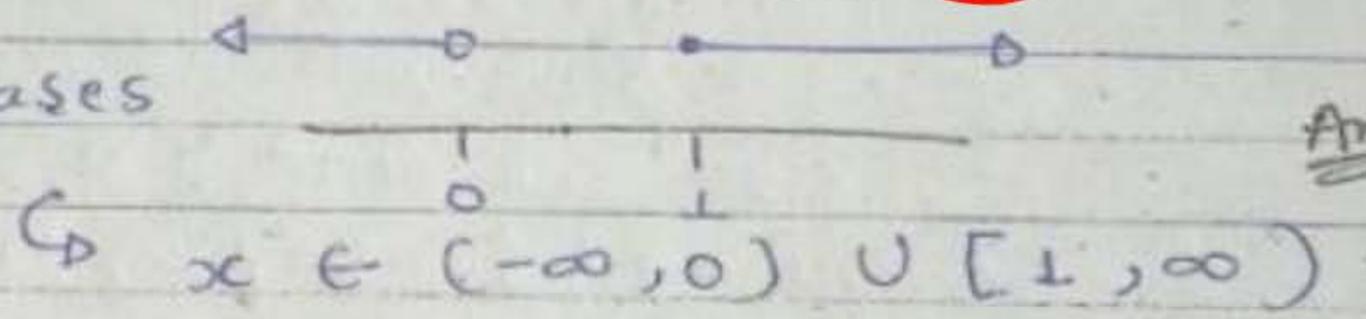
ATDB.uno

ATDB.uno

KTK-6

(A) $x^{\log_{10} x} = 100x$.

$\log_{10} x^{\log_{10} x} = \log_{10} 100x$

$\log_{10} x \cdot \log_{10} x = \log_{10} 100x$

$2\log_{10} x = \log_{10} 100x$??

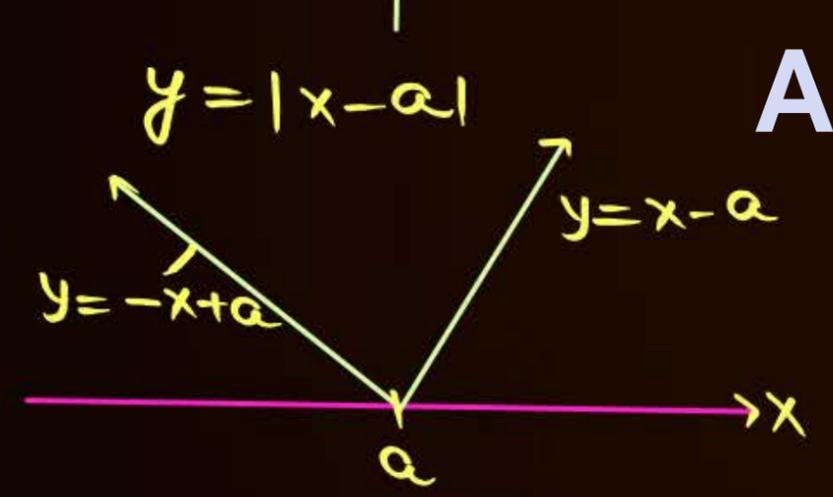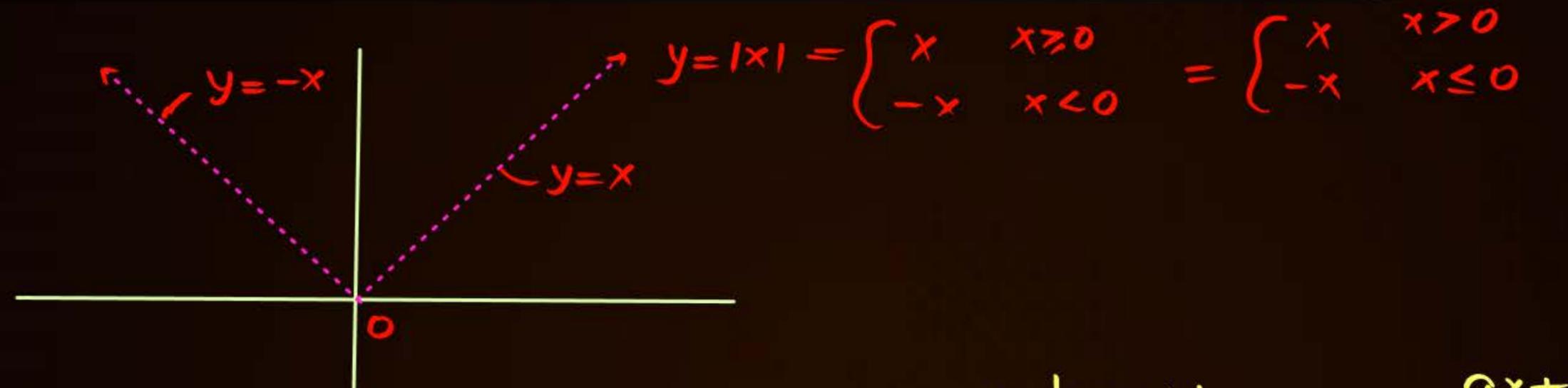$(\log_{10} x)^2 = \log_{10} 100x$

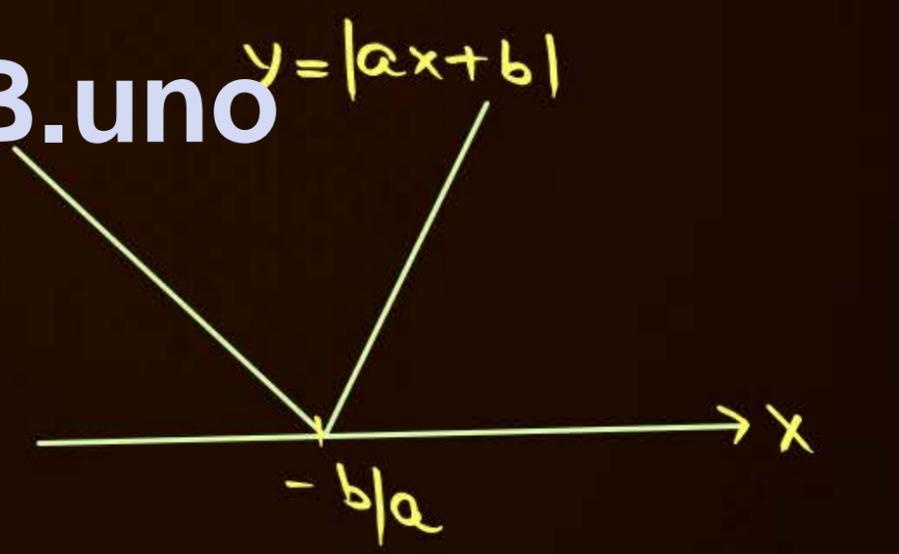$\log_{10} x^2 = \log_{10} 100x$

$x^2 = 100x$

$\boxed{x = 100}$

$\rightarrow$ rational
composite

ATDB.uno ᴘᴅꜰᴢ
https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno
ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.
ATDB PDFZ
ATDB.uno

Problems Involving Graph of Modulus Function

$$y = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases} = \begin{cases} x & x > 0 \\ -x & x \leq 0 \end{cases}$$

$y = -x$

$y = x$

O

$y = |x-a|$

$y = x - a$

$y = -x+a$

a

X

ATDB.uno

$y = |ax+b|$

$-b/a$

X

$ax+b = 0$

$x = -\dfrac{b}{a}$

Ex: $y = |3x - 2|$

slope +ve

slope -ve

$(0, 2)$

$y = -3x + 2$

$y = 3x - 2$

$m = slope = 3$

$\dfrac{2}{3}$

Ex: $y = |2x + 3|$

$L_1 : y = 2x + 3$

$L_2 : y = -2x - 3$

$-\dfrac{3}{2}$
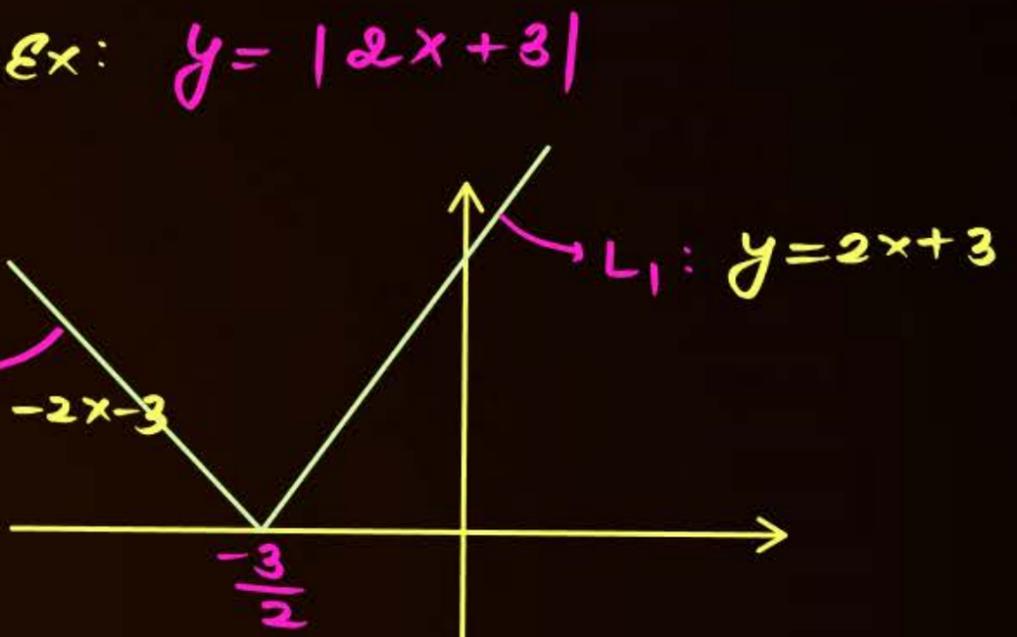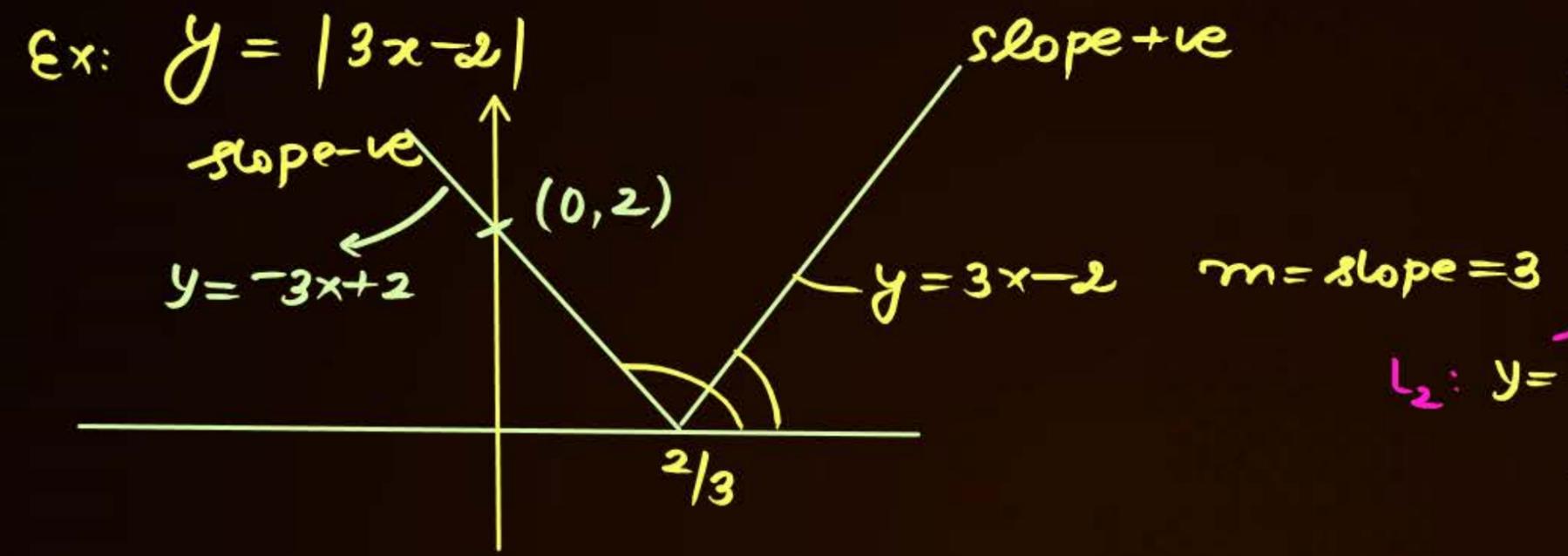
Ex: $y = |x - 1| + |x - 2|$

$$\dfrac{-ve \quad +ve \quad +ve}{-ve \quad -ve \quad +ve}$$

$$y = \begin{cases} -x+1-x+2 & x \le 1 \\ x-1+2-x & 1 < x < 2 \\ x-1+x-2 & x > 2 \end{cases} = \begin{cases} 3-2x & x \le 1 \\ 1 & 1 < x < 2 \\ 2x-3 & x > 2 \end{cases}$$

$y = 3 - 2x$

$y = 2x - 3$

$y = 1$
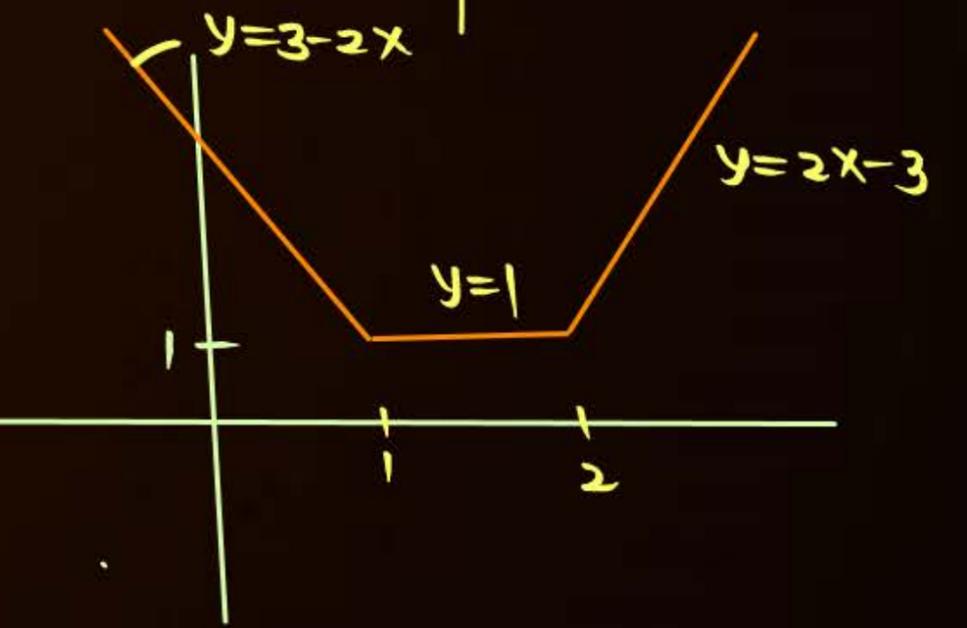
$$y = |x-1| - |x-3|$$

$$
y = \begin{cases} -x+1+x-3 & x \leq 1 \\ x-1+x-3 & 1 < x < 3 \\ x-1-(x-3) & x \geq 3 \end{cases}
= \begin{cases} -2 & x \leq 1 \\ 2x-4 & 1 < x < 3 \\ 2 & x \geq 3 \end{cases}
$$

$y = k$

$y = 2$

$y = |x-1| - |x-3|$

$y = 2x-4$

$y = -2$

$|x-1| - |x-3| = k$

find Range of k for

(a) no real soln $\rightsquigarrow k \in (-\infty, -2) \cup (2, \infty)$

(b) Exactly one real soln $\rightsquigarrow k \in (-2, 2)$

(c) ∞ many soln $\rightsquigarrow k = 2, -2$

$y = a$ is a line parallel to x axis

No: of real solns of $f(x) = g(x)$ is obtained by no: of points of Intersection of graphs $y = f(x)$ & $y = g(x)$

$f(r) = g(r)$

$f(\beta) = g(\beta)$

$f(\alpha) = g(\alpha)$

$y = g(x)$

$y = f(x)$

$\alpha \quad \beta \quad r$

$y = 3$

$y = 0$

$y = -2$

NO! of real soln = 3

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB.uno

QUESTION

Draw graph of $y = |x - 2|$.

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB.uno

QUESTION

$f|x - 1| + |x - 2| = k$ then find range of k for which the equation has

(a)   No solution $\leadsto$ $K \in (-\infty, 1)$

(b)   Two solution only $\leadsto$ $K \in (1, \infty)$

(c)   Infinitely many solution $\leadsto$ $K = 1$.

QUESTION

Draw graph of $y = |x - 1| + |x - 2| + |x - 3|$.

$$y = \begin{cases} -x+1-x+2-x+3, & x \le 1 \\ x-1-x+2-x+3, & 1 < x < 2 \\ x-1+x-2-x+3 & 2 \le x \le 3 \\ x-1+x-2+x-3 & x > 3 \end{cases}$$

$$= \begin{cases} 6-3x & x \le 1 \\ 4-x & 1 < x < 2 \\ x & 2 \le x \le 3 \\ & x > 3 \end{cases}$$

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ
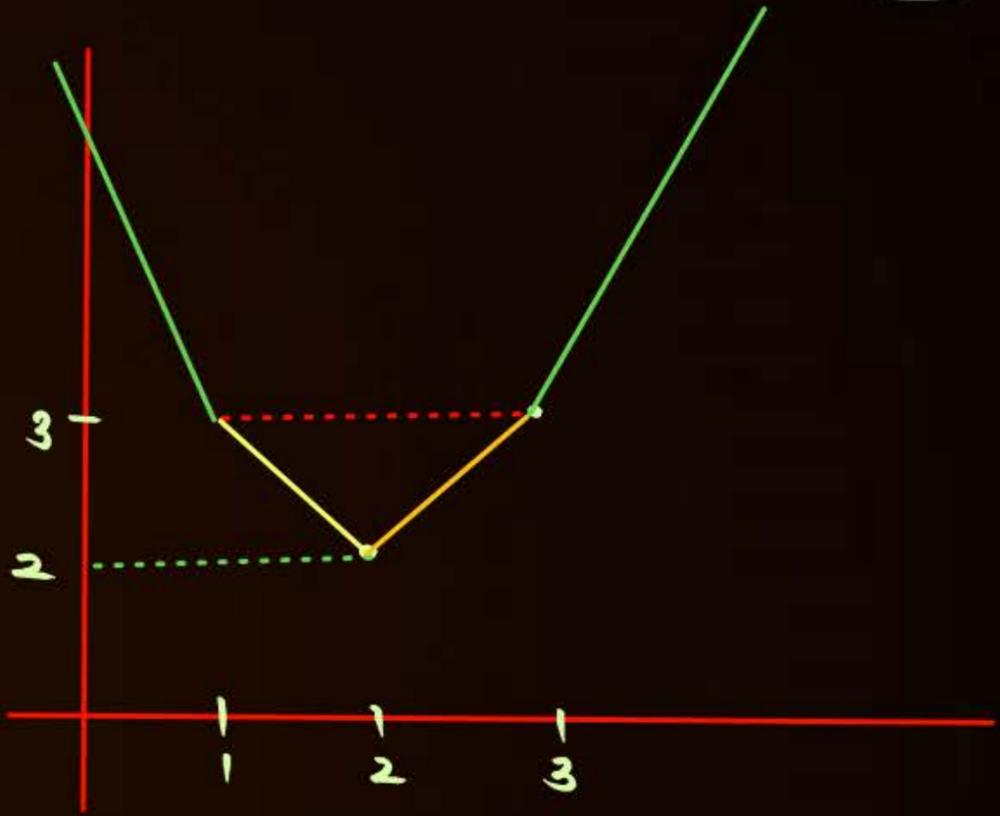
ATDB.uno

QUESTION

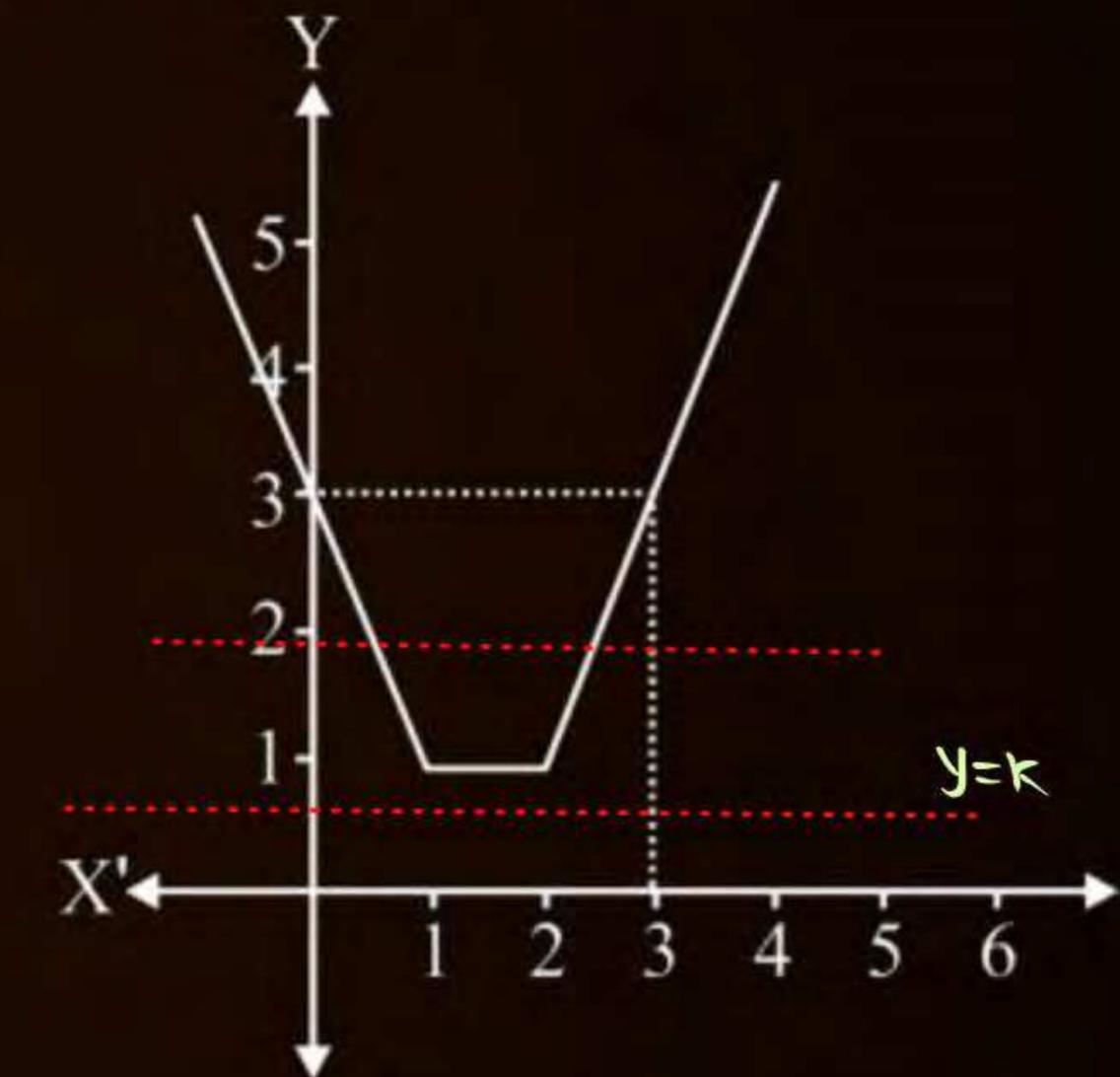If $|x - 1| + |x - 2| + |x - 3| = k$ then find range of k for which the equation has

(a)   No solution $\rightsquigarrow$ $k \in (-\infty, 2)$

(b)   Only one solution $\rightsquigarrow$ $K = 2$

(c)   Two solution $\rightarrow k \in (2, \infty)$

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno
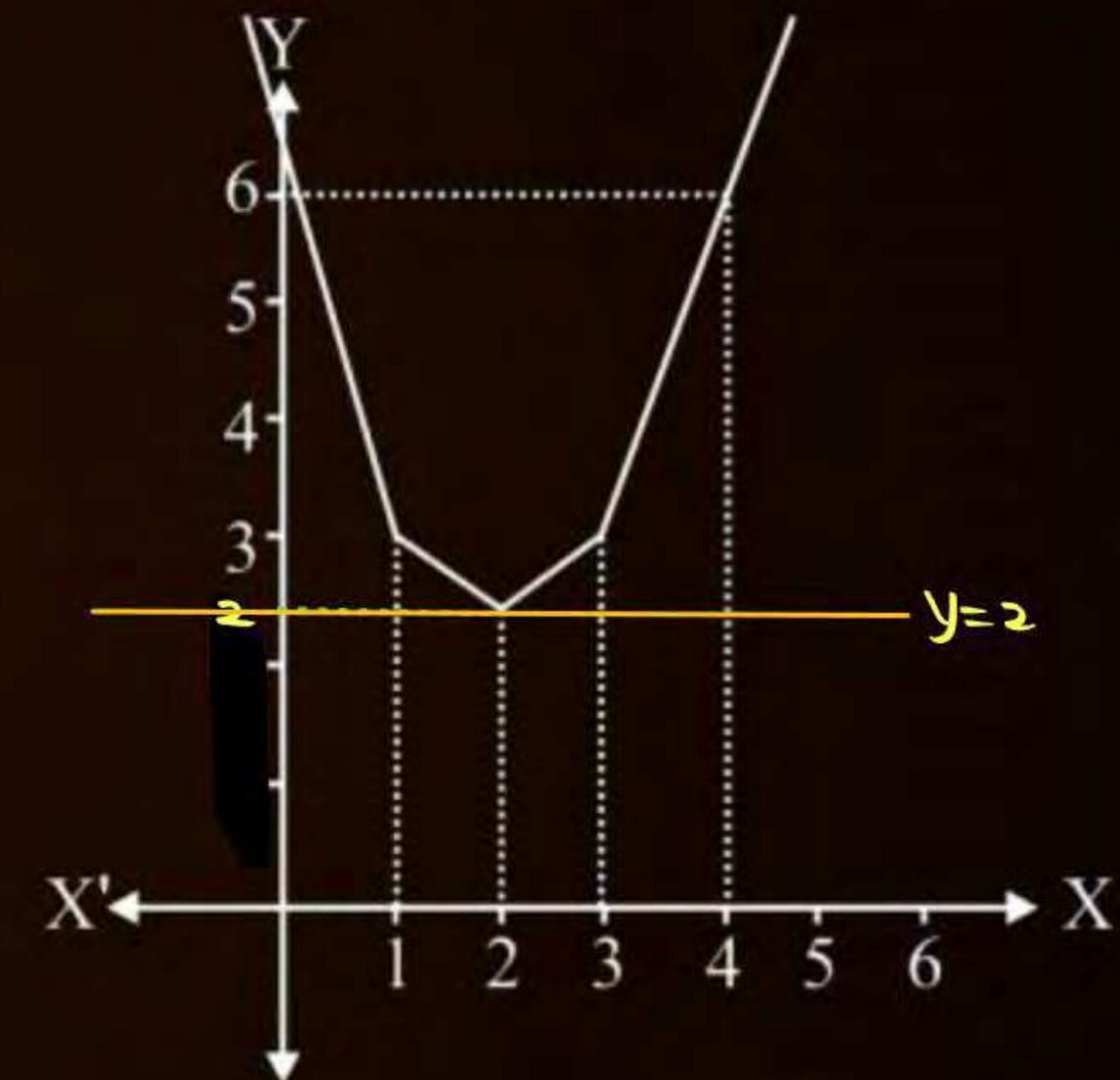
ATDB.uno

QUESTION

Tahol

If $|x - 5| - |x - 7| = k$ then for

A   K = 2 the equation has infinitely many solution

B   K = –2 the equation has infinitely many solution

C   K ∈ (2, ∞) the equation has no solution

D   K ∞ (–2, 2) the equation has exactly one solution

QUESTION

Tah02

Draw graph of $y = |x| + |x - 2|$ and answer the following.

Find the range of k for which the equation $|x| + |x - 2| = k$ has

A.   No Solution

B.   Infinitely Many Solutions

C.   Only two solutions

ATDB.uno

QUESTION

Tah03

Draw the graph of $y = |x - 4| - |x + 3|$ and answer the following.

Find the range of k for which the equation $|x - 4| - |x + 3| = k$ has

A. No Solution

B. Infinitely Many Solutions

C. Only two solutions

ATDB.uno

ATDB.uno | Studyprayas — FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited

ATDB.uno

ATDB.uno

QUESTION

Tanoy

Consider the function $f(x) = |x - 1| - 2|x + 2| + |x + 3|$

Column-I	Column-II
(A) If $f(x) = k$ has no solution, then $k \in$	(p) $(2, 4)$
(B) If $f(x) = k$ has one solution, then $k \in$	(q) $(-\infty, -2) \cup (4, \infty)$
(C) If $f(x) = k$ has two solution, then $k \in$	(r) $(-2, 2) \cup \{4\}$
(D) If $f(x) = k$ has more then two solution, then $k \in$	(s) $\{-2, 2\}$

$\text{Ex: } y = |x-1| + |x-3|$

$$y = \begin{cases} -x+1-x+3 & x<1 \\ x-1-x+3 & 1 \leq x \leq 3 \\ x-1+x-3 & x>3 \end{cases} = \begin{cases} 4-2x & x<1 \\ 2 & 1 \leq x \leq 3 \\ 2x-4 & x>3 \end{cases}$$
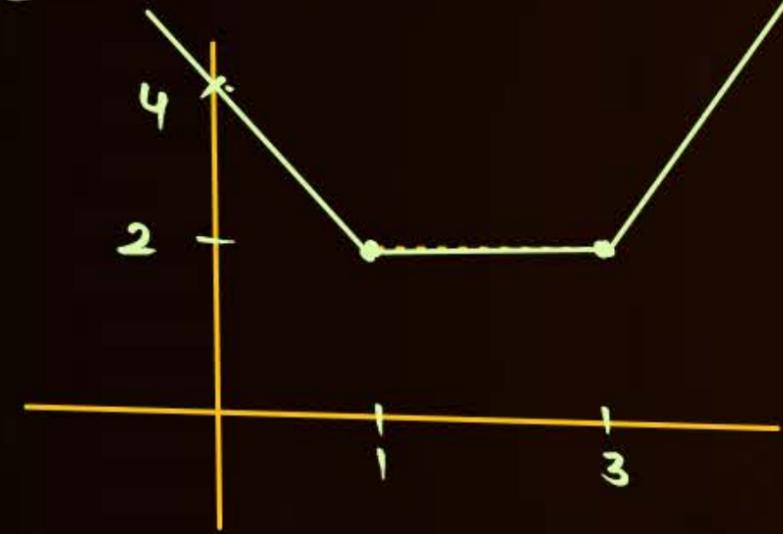
$f(x) = |x-1| + |x-3|$

Range of $f(x) = [2, \infty)$

Yaxis pe Range milti hai

$x=1 < x=2 < x=3$

$y = |x-1| + |x-2| + |x-3|$

$y_{min} \ @ \ x=2$

Range $[1+0+1, \infty) = [2, \infty)$

$x=1 < x=2 < x=3 < x=4$

$y = |x-1| + |x-2| + |x-3| + |x-4|$

$y_{min} \ @ \ x \in [2,3]$

Range: $[1+0+1+2, \infty) = [4, \infty)$

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ
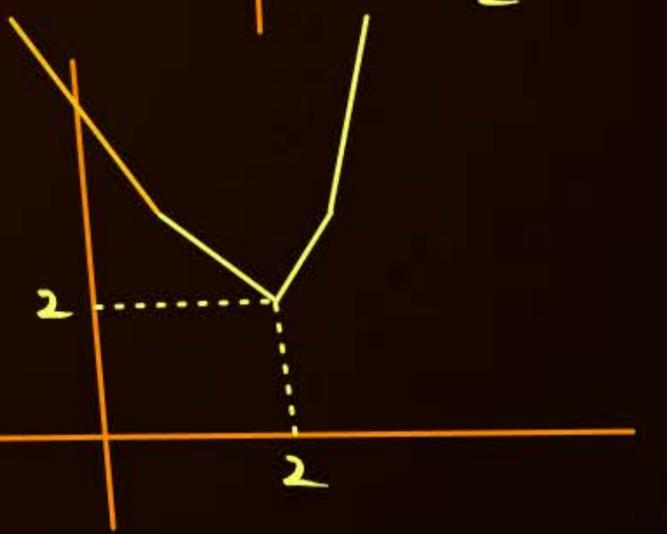
QUESTION

$y = |x - 1| + |x - 2| + |x - 3|$ and find y_{min}.

$$y_{min} @ \quad x = 2$$

$$y_{min} = 1 + 0 + 1 = 2$$

Ex: $y = |x+1| + |x-3| + |x+5| + |x-4| + |x-7|$

$x = -5 < x = -1 < x = 3 <$

$y = |x+5| + |x+1| + |x-3| + |x-4| + |x-7|$

y_{min} at $x = 3$.

Range $[8+4+0+1+4, \infty) = [17, \infty)$

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB.uno

QUESTION

Given, $f(x) = |x| + 2|x - 1| + |x - 2| + |x - 4| + |x - 6| + 2|x - 10|$ where $x \in R$. Find the minimum value of f.

$$x = 0 < x = 1 \leq x = 1 < x = 2 < x = 4 < x = 6 < x = 10 \leq x = 10$$

$$y = |x| + |x-1| + |x-1| + |x-2| + |x-4| + |x-6| + |x-10| + |x-10|$$

$$y_{min} \ @ \ x \in [2, 4]$$

$$y_{min} = 2 + 0 + 2 + 4 + 8 + 8 = 26$$

$$\text{Range } f(x) = [26, \infty)$$

Ex find range of $f(x) = |3x+4| + |2x-3|$

$\left|-\frac{4}{3} - \frac{3}{2}\right| = \left|\frac{-8-9}{6}\right| = \frac{17}{6}$

$$f(x) = 3\left|x + \frac{4}{3}\right| + 2\left|x - \frac{3}{2}\right|$$

$$f(x) = \left|x + \frac{4}{3}\right| + \left|x + \frac{4}{3}\right| + \left|x + \frac{4}{3}\right| + \left|x - \frac{3}{2}\right| + \left|x - \frac{3}{2}\right|$$

$\uparrow$

$y_{min} @ x = \frac{-4}{3}$

$$\text{Range} = \left[0 + 0 + 0 + \frac{17}{6} + \frac{17}{6}, \infty\right)$$

$$= \left[17/3, \infty\right)$$

$$y = |3x+4| + |2x-3|$$
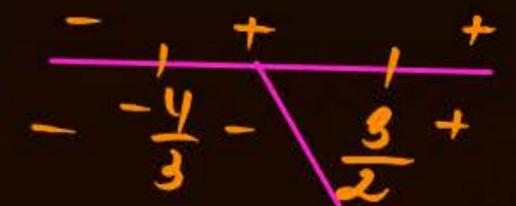
$$y = \begin{cases} -3x-4-2x+3, & x \le -\dfrac{4}{3} \\[2mm] 3x+4-2x+3, & -\dfrac{4}{3} < x < \dfrac{3}{2} \\[2mm] 3x+4+2x-3 & x \geqslant 3/2 \end{cases}$$
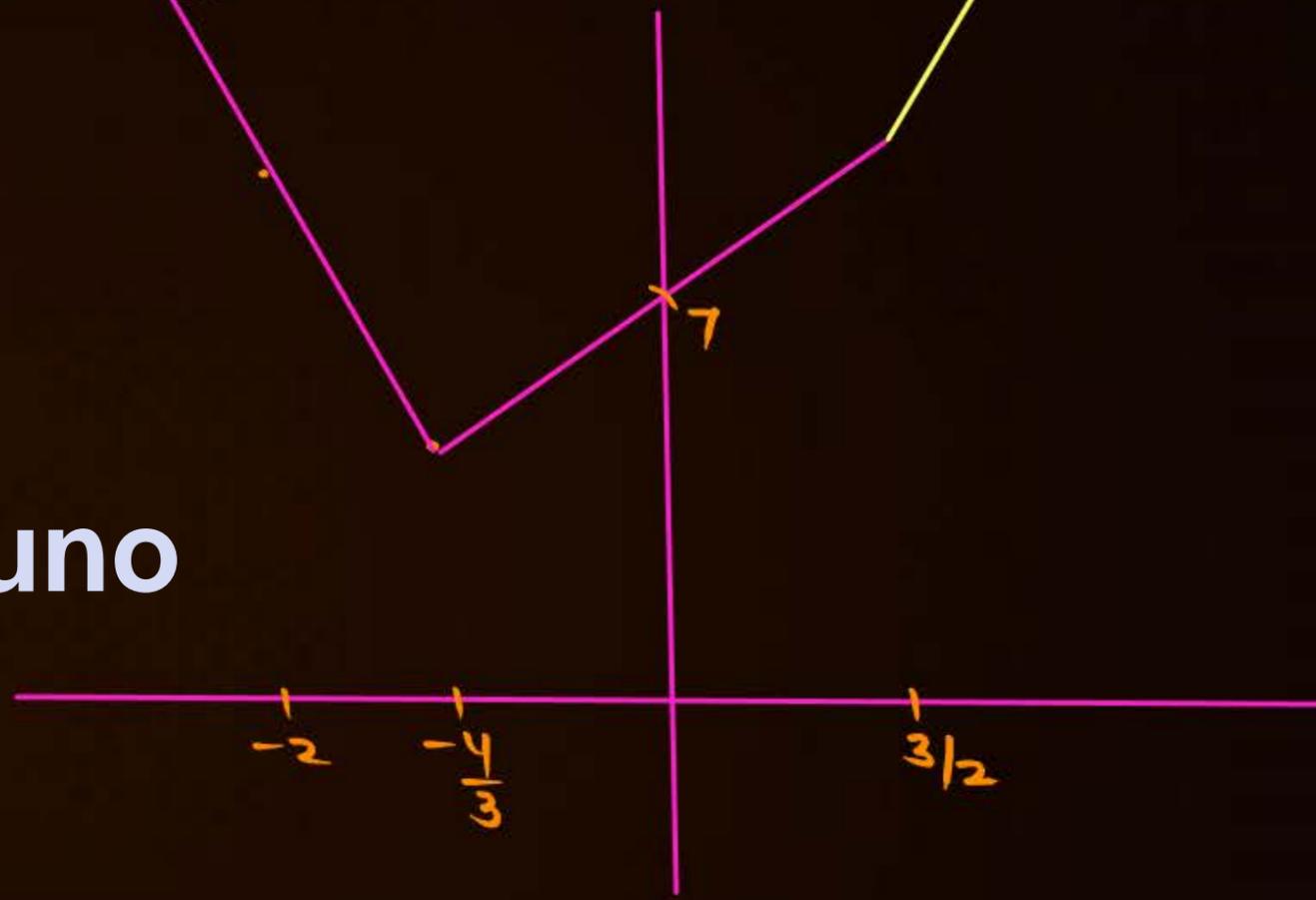
$$y = \begin{cases} -5x-1 & x \le -\dfrac{4}{3} \\[2mm] x+7 & -\dfrac{4}{3} < x < 3/2 \\[2mm] 5x+1 & x \geqslant 3/2 \end{cases}$$

x	$-4/3$	-2
y	$17/3$	9

x	$-4/3$	0
y	$17/3$	7

Irrational Inequalities

$$f(x) \geqslant 0 \quad \sim \quad \text{☁} \quad \text{(A)}$$

$$\sqrt{f(x)} \quad < \quad g(x)$$
$$\leq$$

Case① if $g(x) \geqslant 0$ ⟶ ☁

$$f(x) \quad < \quad g^2(x)$$
$$\leq$$

$\cap$

B

Case ⑪   if $g(x) < 0$ ⟶ ☁

$$\sqrt{f(x)} < g(x)$$
$$\downarrow$$
$$x \in \phi$$

$$x \in \phi$$

final Ans: $A \cap B$

Irrational Inequalities

$$\sqrt{f(x)} \geq g(x)$$

$$f(x) \geq 0 \quad \text{---} \quad \text{(A)}$$

Case ① if $g(x) \geq 0$

$$f(x) \geq g^2(x)$$

$$\cap$$

$$\cup$$

final Ans: $A \cap B$

Case ⑪ if $g(x) < 0$

$$\sqrt{f(x)} \geq g(x)$$

$$\downarrow$$

always true

B

Irrational Inequalities

$$\sqrt{f(x)} \gtrless \sqrt{g(x)}$$

$$f(x) \geq 0 \quad \& \quad g(x) \geq 0$$

S.B.S

$$f(x) \gtrless g(x)$$

n

n
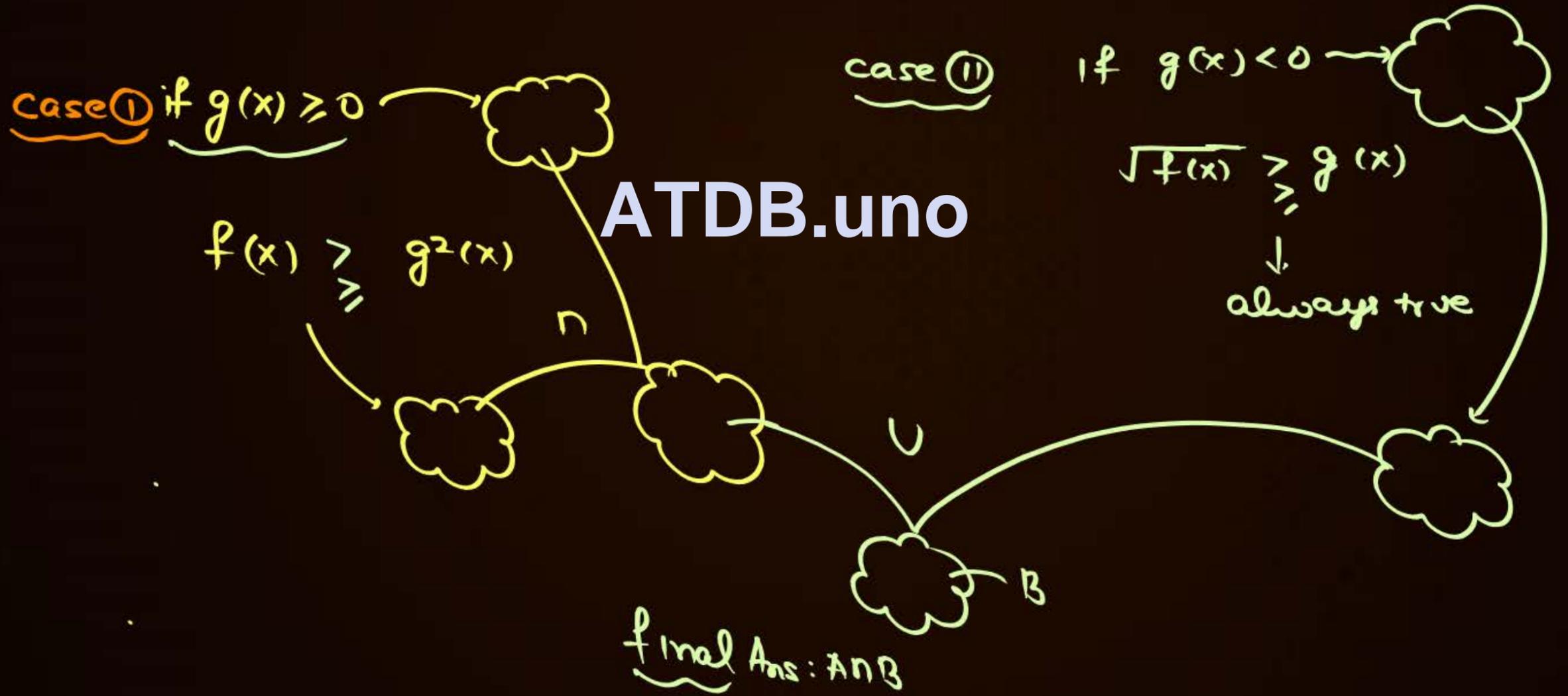
Final Ans.

QUESTION

Solve for x if, $4 - x < \sqrt{x^2 - 2x}$

$x^2 - 2x \geq 0$

$x(x-2) \geq 0$

non-ve

$x \in (-\infty, 0] \cup [2, \infty)$

(A)

Ineq mai dono side hamesha odd power toh laga sakte hai but Even power ke liyay dono Sides negative nahi honi chahiyay

case ① if $4 - x \geq 0 \rightarrow x \leq 4$

$4 - x < \sqrt{x^2 - 2x}$

non-ve   non-ve

S.B.S

$(4-x)^2 < x^2 - 2x$

$16 + x^2 - 8x < x^2 - 2x$

$6x > 16$

$x > 8/3$

8/3   4
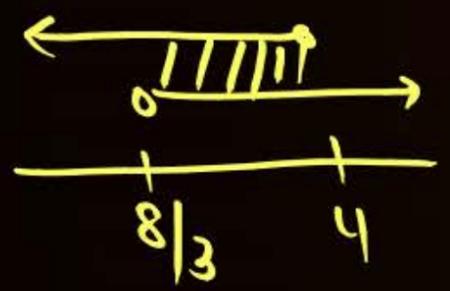
case ② if $4 - x < 0 \implies x > 4$

$4 - x < \sqrt{x^2 - 2x}$

-ve   non-ve

always true.

$x \in (4, \infty)$

$x \in (8/3, 4]$

$\cap$

UNION

$x \in (8/3, 4] \cup (4, \infty) = (8/3, \infty)$ (B)

$$A \cap B$$

$$x \in (-\infty, 0] \cup [2, \infty)$$

$$x \in (8/3, \infty)$$

Ans: $\left(8/3, \infty\right)$

ATDB.uno

ATDB.uno | Studyprayas å FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno
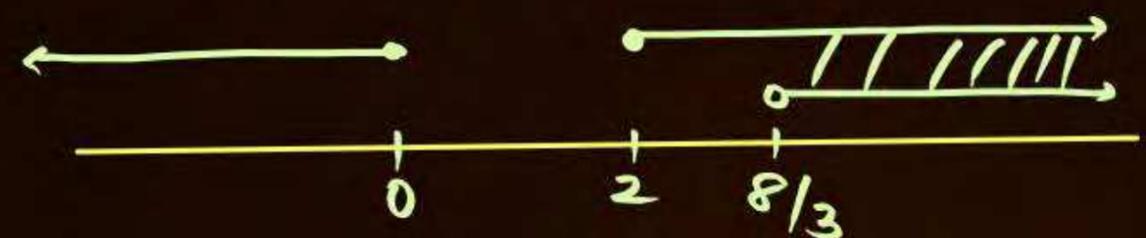
ATDB.uno

QUESTION

Solve for x, if $\sqrt{x^2 - 3x + 2} > x - 2$

$$x^2 - 3x + 2 \geq 0$$
$$(x-1)(x-2) \geq 0$$
$$x \in (-\infty, 1] \cup [2, \infty) \quad \text{—} \quad \boxed{A}$$

case ① If $x - 2 \geq 0 \Rightarrow x \geq 2$

non-ve $\leftarrow$ $\sqrt{x^2 - 3x + 2} > x - 2$ $\rightarrow$ non-ve

S.B.S $\quad x^2 - 3x + 2 > x^2 - 4x + 4$

$x > 2$

$x \in (2, \infty)$

$\cap$

case ⑪ If $x - 2 < 0 \Rightarrow x < 2$

$\sqrt{x^2 - 3x + 2} > x - 2$ $\quad$ —ve

non-ve $\qquad$ always true

$x < 2$

UNION

$x \in (-\infty, \infty) - \{2\}$ —$\boxed{B}$

Ans: $A \cap B = (-\infty, 1] \cup (2, \infty)$ $\underline{Ans}$

QUESTION

Solution of the inequality, $x - 3 < \sqrt{x^2 + 4x - 5}$

Tah 5

A $(-\infty, -5] \cup [1, \infty)$

B $(-\infty, -5] \cup (7/5, \infty)$

C $(-5, 3]$

D $(7/5, \infty)$

ATDB.uno

https://ATDB.uno | Unauthorised redistribution is strictly prohibited

ATDB.uno | Studyprayas à FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB.uno

QUESTION

Solve following inequalities

(i) $\dfrac{\sqrt{2x-1}}{x-2} < 1$ Ans. $\left[\dfrac{1}{2}, 2\right) \cup (5, \infty)$

(ii) $x - \sqrt{1 - |x|} < 0$ Ans. $[-1, (\sqrt{5}-1)/2]$

(iii) $\sqrt{x^2 - x - 6} < 2x - 3$ Ans. $x \in [3, \infty)$

(iv) $\sqrt{x^2 - 6x + 8} \le \sqrt{x+1}$ Ans. $x \in \left[\dfrac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \dfrac{7+\sqrt{21}}{2}\right]$

(iii)

$x^2 - x - 6 \ge 0 \quad (x-3)(x+2) \ge 0$

$x \in (-\infty, -2] \cup [3, \infty)$ ──(A)

Tah06

SBS

Case① if $2x-3 \ge 0 \Rightarrow x \ge \dfrac{3}{2}$

$\sqrt{x^2 - x - 6} < 2x - 3$ ⌐ non-ve

non-ve

$x^2 - x - 6 < 4x^2 + 9 - 12x$

$3x^2 - 11x + 15 > 0$

always +ve i.e $x \in R$

$x \in [3/2, \infty)$

case⑪ If $2x-3 < 0 \Rightarrow x < 3/2$

$\sqrt{x^2 - x - 6} < 2x - 3$ ⌐ -ve

non-ve → Not possible

$x \in \phi$

$x \in [3/2, \infty)$

$\cup$

Ⓑ

Ans: $A \cap B = [3, \infty)$

-2   3/2   3

ATDB.unofz

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited
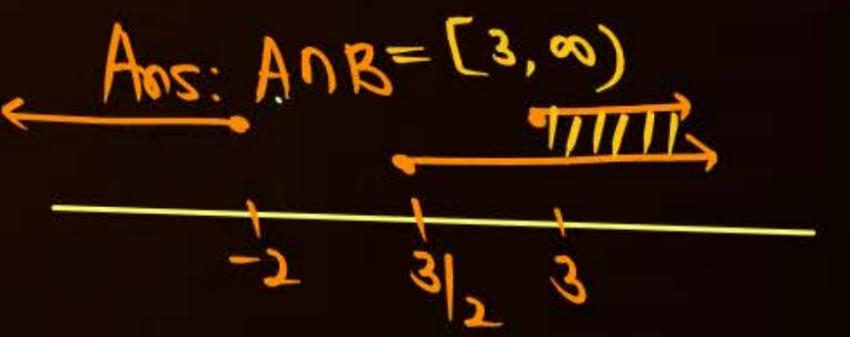
ATDB.uno

ATDB.uno

(iv) $\sqrt{x^2-6x+8} \leq \sqrt{x+1}$

SBS

$x^2-6x+8 \leq x+1$

$x^2-7x+7 \leq 0$

$\left(x - \dfrac{7+\sqrt{21}}{2}\right)\left(x - \dfrac{7-\sqrt{21}}{2}\right) \leq 0$

$x = \dfrac{7 \pm \sqrt{21}}{2}$

$x \in \left[\dfrac{7-\sqrt{21}}{2}, \dfrac{7+\sqrt{21}}{2}\right]$

$x^2-6x+8 \geqslant 0$ & $x+1 \geqslant 0$

$(x-2)(x-4) \geqslant 0$ & $x \geqslant -1$

$x \in (-\infty, 2] \cup [4, \infty)$ $\cap$

$x \in [-1, 2] \cup [4, \infty)$

$\cap$

$x \in \left[\dfrac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \dfrac{7+\sqrt{21}}{2}\right]$
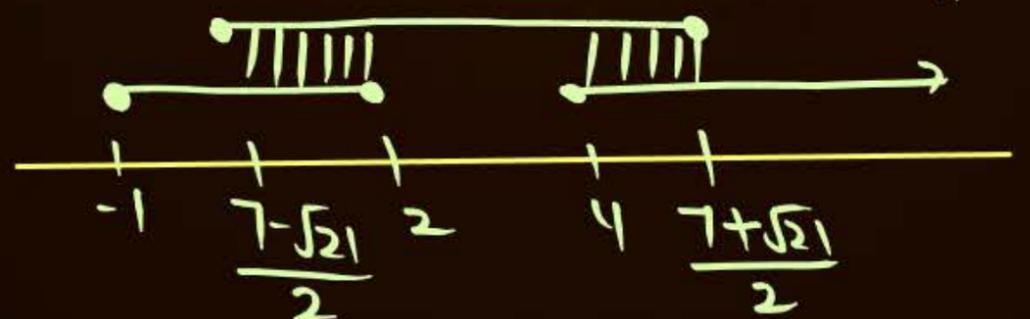
Number line with points: -1, $\dfrac{7-\sqrt{21}}{2}$, 2, 4, $\dfrac{7+\sqrt{21}}{2}$

ATDB.uno
ATDB.uno | Studyprayas à FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.
ATDB PDFZ
https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno
ATDB.uno

QUESTION [JEE Mains 2023 (1 Feb)]

Let $S = \{x : x \in \mathbb{R}$ and $(\sqrt{3} + \sqrt{2})^{x^2 - 4} + (\sqrt{3} - \sqrt{2})^{x^2 - 4} = 10\}$. Then $n(S)$ is equal to

A 6

B 4

C 0

D 2

$$\log_{10}\left((\sqrt{3}+\sqrt{2})^{x^2-4} + (\sqrt{3}-\sqrt{2})^{x^2-4}\right) = \log_{10} 10$$

$$\log_{10}(\sqrt{3}+\sqrt{2})^{x^2-4} + \log_{10}(\sqrt{3}-\sqrt{2})^{x^2-4} = \log_{10} 10$$

$$\log_{10}(\sqrt{3}+\sqrt{2})^{x^2-4} + \log_{10}(\sqrt{3}-\sqrt{2})^{x^2-4} = \log_{10} 10$$

Gadho/Gadhiyoo aisaa naa karo

$$\log_a(b+c) \neq \log_a b + \log_a c$$

$$Ex: \log_2(8+4) \neq \log_2 8 + \log_2 4 = 3+2=5$$

$$\| \\ \log_2 12$$

Ans. B

ATDB.uno
ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.
ATDB PDFZ
https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno
ATDB.uno

QUESTION [JEE Mains 2023 (1 Feb)]

Let $S = \{x : x \in \mathbb{R}$ and $(\sqrt{3} + \sqrt{2})^{x^2-4} + (\sqrt{3} - \sqrt{2})^{x^2-4} = 10\}$. Then $n(S)$ is equal to

A 6

B 4

C 0

D 2

$$\log_{10}(\sqrt{3}+\sqrt{2})^{x^2-4} + \log_{10}(\sqrt{3}-\sqrt{2})^{x^2-y} = \log_{10}\frac{100}{10}$$

$(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2}) = 3-2 = 1$

$$\sqrt{3} - \sqrt{2} = \frac{1}{\sqrt{3}+\sqrt{2}}$$

$$(\sqrt{3}+\sqrt{2})^{x^2-4} + \left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)^{x^2-y} = 10$$

Gadho/Gadhiyoo aisaa naa koro

Let $(\sqrt{3}+\sqrt{2})^{x^2-4} = t$

$(\sqrt{3}+\sqrt{2})^{x^2-4} + \dfrac{1^{x^2-4}}{(\sqrt{3}+\sqrt{2})^{x^2-4}} = 10$

$t + \dfrac{1}{t} = 10$

$(\sqrt{3}+\sqrt{2})^{x^2-4} + \dfrac{1}{(\sqrt{3}+\sqrt{2})^{x^2-4}} = 10$

$t^2 - 10t + 1 = 0$

$t = \dfrac{10 \pm \sqrt{96}}{2} = \dfrac{10 \pm 4\sqrt{6}}{2}$

$t = 5 \pm 2\sqrt{6} = (\sqrt{3}+\sqrt{2})^2$

Ans. B

ATDB.uno

ATDB.uno | Studyprayas à FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

$$(\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}+\sqrt{2})^2 \text{ or } (\sqrt{3}-\sqrt{2})^2$$

$$x^2-4=2 \qquad \text{or} \qquad (\sqrt{3}+\sqrt{2})^{x^2-4} = (\sqrt{3}-\sqrt{2})^2 = \frac{1}{(\sqrt{3}+\sqrt{2})^2} = (\sqrt{3}+\sqrt{2})^{-2}$$

$$x = \pm\sqrt{6} \qquad\qquad\qquad\qquad x^2-4=-2$$

$$x^2=2$$

$$x=\pm\sqrt{2}$$

$$\text{Ans: } S = \{\sqrt{6}, -\sqrt{6}, \sqrt{2}, -\sqrt{2}\}$$

$$n(S)=4$$

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB.uno

https://ATDB.uno | Unauthorised redistribution is strictly prohibited

ATDB.uno

QUESTION [JEE Mains 2024 (1 Feb)]

Tah 07

Let $S = \{x \in R: (\sqrt{3} + \sqrt{2})^x + (\sqrt{3} - \sqrt{2})^x = 10\}$. Then the number of elements in S is :

A 4

B 0

C 2

D 1

Ans. C

QUESTION [JEE Mains 2023 (31 Jan)]

Tah08

The number of real roots of the equation $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$, is

A 0

B 1

C 3

D 2

Ans. B

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

QUESTION

Tah 09

Solve: $\sqrt{x^2 - 6x + 9} + \sqrt{x^2 + 8x + 16} = 7$

ATDB.unofz

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno

ATDB.uno

QUESTION [JEE Mains 2019 (8 April)]

Tan08

The sum of the solutions of the equation $|\sqrt{x} - 2| + \sqrt{x}(\sqrt{x} - 4) + 2 = 0$ $(x > 0)$ is equal to:

(A) 9

(B) 12

(C) 4

(D) 10

Ans. D

ATDB.uno

ATDB.uno | Studyprayas à FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ
ATDB PDFZ
ATDB.uno
https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno
ATDB.uno

Home Challenge-08

Let $f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)}$. If $\prod_{r=-10}^{10} f(\ln 2^r) = e^\lambda$, then find the value of $(2\lambda - 15)$. [Ans. 6]

Today's BPP

ATDB.uno ATDBFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited

ATDB.uno

ATDB.uno

Bumper Practice Problems

1. $\sqrt[3]{\dfrac{x-2}{x-1}} < \sqrt[3]{\dfrac{1}{x-1}}$

2. $\sqrt{3x-2} < \sqrt{x+4}$

3. $\sqrt{4x-3} < \sqrt{2x+5}$

4. $\sqrt[3]{(3x-5)} < (x-1)$

5. $\sqrt[3]{3x-2} < x$

6. $\sqrt{(x+14)} < (x+2)$

7. $\sqrt{2x-2} < x-1$

8. $\sqrt{-x^2+4x-3} > (6-2x)$

9. $\sqrt{x-2} + \sqrt{x-1} > 2$

10. $7\sqrt{x} + 8\sqrt{-x} + \dfrac{15}{x^3} = 98$

11. $\sqrt{2x-4} - \sqrt{x+5} = 1$

12. $\sqrt{x-1} + \sqrt{2x+6} = 6$

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB.uno

Answers

1. $x \in (1, 3)$

2. $x \in \left[\frac{2}{3}, 6\right)$

3. $x \in \left[\frac{3}{4}, 1\right)$

4. $x \in (-1, \infty) - \{2\}$

5. $x \in (-2, 1) \cup (1, \infty)$

6. $x \in (2, \infty)$

7. $x \in (3, \infty)$

8. $x \in \left(\frac{13}{5}, 3\right)$

9. $x \in \left(\frac{41}{16}, \infty\right)$

Homework From Module

Prarambh (Topicwise) : Q1 to Q25

Prabal (JEE Main Level) : Q1 to Q33

Solution to Previous TAH

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

QUESTION

Solve: $|x| + |x - 1| \geq 7$

ATDB.uno

ATDB.uno | Studyprayas à FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

Lecture 15 : 08/05/2025

 # IAH-01 :

$$\xleftarrow{\quad\bullet\quad\quad\bullet\quad}_{\;0\qquad\;1}$$
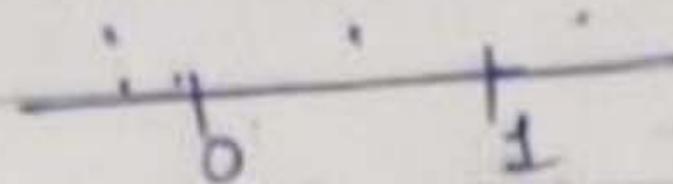
(i) If $x \leq 0$,

$-x - x + 1 \geq 7$

$-2x \geq 6$

$x \leq -3$

$\Rightarrow \boxed{x \in (-\infty, -3)}$

(ii) If $0 < x < 1$,

$+x + x \geq 7$

$1 \geq 7$

(Not possible)

$\Rightarrow x \in \phi$

(iii) If $x \geq 1$

$x + x - 1 \geq 7$

$2x \geq 8$

$x \geq 4$

$\Rightarrow \boxed{x \in (4, \infty)}$

$$\boxed{x \in (-\infty, -3) \cup (4, \infty)} \quad \underline{Ans.}$$

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited

ATDB.uno

ATDB.uno

Q1 $|x| + |x-1| \geq 7$

$\boxed{x < 0}$

$-x - x + 1 \geq 7$

$-2x \geq 6$

$\boxed{x \leq -3}$

$\boxed{x \in (-\infty, -3]}$

$\boxed{0 \leq x < 1}$

$+x - x + 1 \geq 7$

$1 \geq 7 \quad X \text{ (NP)}$

$\boxed{x \geq 1}$

$x + x - 1 \geq 7$

$2x \geq 8$

$\boxed{x \geq 4}$

$\boxed{x \in [4, \infty)}$

$\boxed{x \in (-\infty, -3] \cup [4, \infty)}$

ATDB.uno PDFZ

ATDB.uno | Studyprayas å FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

QUESTION

Solve: $|x + 1| + |x - 1| > 2$

Downloaded from ATDB.uno/StudyPrayas | Unauthorised redistribution is strictly prohibited.
ATDB.uno | 19 Mar 2026, 14:43 IST

ATDB.uno

ATDB.uno fz

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited

ATDB.uno

ATDB.uno

IAH-02

$$\overline{\underset{-1}{|} \quad \underset{1}{|}}$$
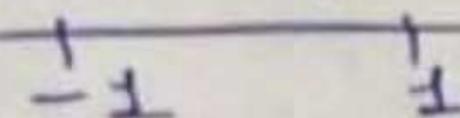
(i) If $x \leq -1$,

$\to x - 1 - x + 1 > 2$

$-2x > 2$

$x < -1$

$\Rightarrow x \in (-\infty, -1)$

(ii) If $-1 \leq x < 1$

$x + 1 - x + 1 > 2$

$2 > 2$

(Not possible

$\Rightarrow x \in \phi$

(iii) If $x \geq 1$

$x + 1 + x - 1 > 2$

$2x > 2$

$x > 1$

$\Rightarrow x \in (1, \infty)$

$$\boxed{x \in (-\infty, -1) \cup (1, \infty)} \quad \text{Ans}$$

Method-2 : Using triangle inequality.

$|x + 1| + |x - 1| > 2$

$|a| + |b| > |a - b| \iff ab > 0$

$\Rightarrow (x + 1) \cdot (x - 1) > 0$

$\Rightarrow \boxed{x \in (-\infty, -1) \cup (1, \infty)} \quad \text{Ans}$

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB.uno

Tah. 02 $\quad |x+1| + |x-1| > 2$

	T_1		
T_1	$-ve$	$+ve$	$+ve$
T_2	$-ve$ -1 $-ve$		$+ve$

Case I → $x \leq +1$

$$-(x+1) - (x-1) > 2$$
$$-x-1 - x+1 > 2$$
$$-2x > 2$$
$$-2x - 2 > 0$$
$$-2(x+1) > 0$$
$$x+1 < 0$$
$$x \in (-\infty, -1)$$

Case II $-1 < x < 1$

$$x+1 - (x-1) > 2$$
$$x+1 - x+1 > 2$$
$$2 > 2$$
$$\text{No } \rho_o$$

Case III → $x \geq 1$

$$(x+1) + (x-1) > 2$$
$$x+1 + x-1 > 2$$
$$2x > 2$$
$$x > 1$$
$$x \in (1, \infty)$$

$$x \in (-\infty, -1) \cup (1, \infty)$$

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

QUESTION

Solve: $|x - 6| > |x^2 - 5x + 9|$

ATDB.uno
ATDB PDFZ
ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.
ATDB.uno
ATDB PDFZ
ATDB.uno

$\dfrac{Tan-05}{8}$

$$|x-6| > |x^2 - 5x + 9|$$

$$(x-6)^2 > (x^2 - 5x + 9)^2$$

$$(x - 6 + x^2 - 5x + 9)(x - 6 - x^2 + 5x - 9) > 0$$

$$(x^2 - 4x + 3)(-x^2 + 6x - 15) > 0$$

$$(x^2 - 4x + 3)(x^2 - 6x + 15) < 0$$
$$\hookrightarrow D < 0$$

$$(x^2 - 3x - x + 3)(\cancel{x^2}) < 0$$

$$x(x-3) - 1(x-3) < 0$$

$$x \quad (x-1)(x-3) < 0$$

$$\boxed{x \in (+1, +3)}$$
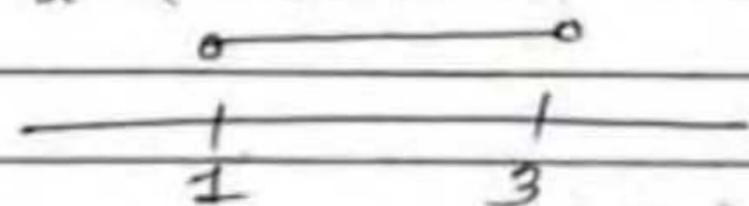
TAH-03 :

$$(x-6)^2 \cdot > (x^2-5x+9)^2$$

$$(x-6+x^2-5x+9)(x-6-x^2+5x-9) > 0$$

$$(x^2-4x+3) \cdot (-x^2+6x-15) > 0$$

$$(x-3)(x-1)(x^2-6x+15) < 0 \implies (x-3) \cdot (x-1) < 0$$

$$\downarrow D<0, a>0$$

$$(\text{always +ve})$$

$$\boxed{x \in (1,3)} \; \text{Ans}$$

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

QUESTION

Solve: $(|x - 1| - 3)(|x + 2| - 5) < 0$

Downloaded from ATDB.uno/StudyPrayas | Unauthorised redistribution is strictly prohibited.
ATDB.uno | 19 Mar 2026, 14:43 IST

ATDB.uno

Page No:

Date:

IAH-04

$$\overset{\bullet}{\underset{-2}{\longleftarrow}} \quad \overset{\bullet}{\underset{1}{|}}$$
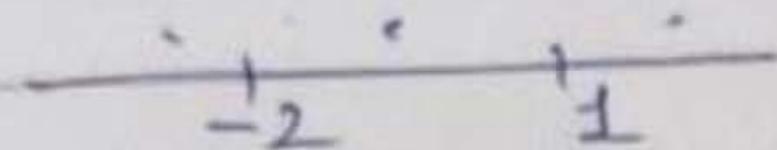
(i) If $x \leq -2$,

$(-x+1-3)\cdot(-x-2-5) < 0$

$(x+2)\cdot(x+7) < 0$

$x \in (-7,-2)$

$\Rightarrow \boxed{x \in (-7,-2)}$

(ii) If $-2 < x < 1$

$(-x+1-3)(x+2-5) < 0$

$(x+2)\cdot(x-3) > 0$

$x \in (-\infty,-2) \cup (3,\infty)$

$\Rightarrow x \in \phi$

(iii) If $x \geq 1$

$(x-1-3)\cdot(x+2-5) < 0$

$(x-4)(x-3) < 0$

$x \in (3,4)$

$\Rightarrow \boxed{x \in (3,4)}$

$$x \in (-7,-2) \cup (3,4) \quad \underline{Ans}$$

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

$$(|x-1| - 3)(|x+2| - 5) < 0$$

Either $|x-1| - 3 > 0$ & $|x+2| - 5$ X This method was already told by sir...

(H-?)   ___ Case ① when $x \leq -2$.

$$(-x+1-3)(-x-2-5) < 0$$
$$(x+2)(x+7) < 0$$
$$(-7, -2)$$

When $-2 < x \leq 1$
$$(-x+1-3)(x \cdot$$
$$(x+2)(x-3) > 0$$
$$(-\infty, -2) \cup (3, \infty).$$ Case Rejected.

when $x \geq 1$.
$$(x-4)(x-3) < 0$$
$$x \in (3, 4)$$

$$(-7, -2) \cup (3, 4)$$ Ans

QUESTION

(a) $|x^2 - 9| + |16 - x^2| = 7$

Ans: $x \in [-4, -3] \cup [3, 4]$

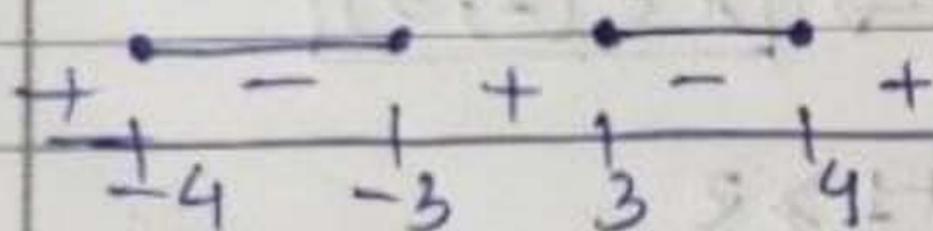(b) Solve: $|(x^2 + 2x + 2) + (3x + 7)| < |x^2 + 2x + 2| + |3x + 7|$

Ans: $x < -7/3$

(a) $|x^2-9| + |16-x^2| = 7$

$\quad |a| + |b| = |a+b| \iff ab \geqslant 0$

$(x^2-9) \cdot (16-x^2) \geqslant 0$

$(x^2-9)(x^2-16) \leqslant 0$

$(x-3)(x+3)(x-4)(x+4) \leqslant 0$

$\Rightarrow \{x \in [-4,-3] \cup [3,4]\}$ _Ans_

(b) $|(x^2+2x+2)+(3x+7)| < |x^2+2x+2| + |3x+7|$

$\quad |a+b| \quad < \quad |a| + |b| \quad \iff \quad ab < 0$

$(x^2+2x+2) \cdot (3x+7) < 0 \Rightarrow 3x+7 < 0$

$(D < 0, \, a > 0)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad \{x < \dfrac{-7}{3}\}$ _Ans_

Ques 05 (a) $|x^2 - 9| + |16 - x^2| = 7$

$\underset{a}{\underbrace{\quad}} \qquad \underset{b}{\underbrace{\quad}}$

$|a| + |b| \geq |a+b|$

$|x^2 - 9| + |16 - x^2| \geq |x^2 - 9 + 16 - x^2| = 7$

holds equality when —

$ab \geq 0$

$(x^2 - 9)(16 - x^2) \geq 0$

$(x-3)(x+3)(x^2 - 16) \leq 0$

$(x-3)(x+3)(x-4)(x+4) \leq 0$

$$\overset{+}{\underset{-4}{\quad}} \; \overset{-}{\underset{-3}{\quad}} \; \overset{+}{\underset{3}{\quad}} \; \overset{-}{\underset{4}{\quad}} \; \overset{+}{\quad}$$

$x \in [-4, -3] \cup [3, 4]$

(b) $|(x^2 + 2x + 2) + (3x + 7)| < |x^2 + 2x + 2| + |3x + 7|$

$\underset{a}{\underbrace{\quad}} \qquad \underset{b}{\underbrace{\quad}}$

$|a + b| < |a| + |b|$

strictly increase when $ab < 0$

Therefore —

$(x^2 + 2x + 2)(3x + 7) < 0$

$\hookrightarrow \quad D < 0$
$\qquad a > 1 \quad (\text{always } +ve)$

$(3x + 7) < 0$

$x < \dfrac{-7}{3}$

$x \in \left(-\infty, \dfrac{-7}{3}\right)$

Solution to Previous KTKs

QUESTION

(KTK 1)

Column-I

(A) $\dfrac{\log_2 32}{\log_3 \sqrt{243}}$

(B) $\dfrac{2\log 6}{\log 12 + \log 3}$

(C) $\log_{1/4}\left(\dfrac{1}{16}\right)^{-2}$

(D) $\dfrac{\log_5 16 - \log_5 4}{\log_5 128}$

Column-II

(P) positive integer

(Q) negative integer

(R) rational but not integer

(S) Prime

Ans. $A \to P, S; B \to P; C \to Q; D \to R$

ATDB.uno
https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno
ATDB.uno | Studyprayas å FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.
ATDB PDFZ
ATDB PDFZ
ATDB.uno

A.) $\quad \log_{10} x \cdot \log_{10} x = \log_{10} 100x$

$$\log_{10}^2 x = 2 + \log_{10} x$$

$$t^2 - t - 2 = 0$$

$$t^2 - 2t + t - 2 = 0$$

$$(t-2)(t+1) = 0$$

$$\log_{10} x = 2 \quad \text{or} \quad \log_{10} x = -1$$

$$\boxed{x = 100 \text{ or } \frac{1}{10}}$$

$$\boxed{x_1 x_2 = 10}$$

$\bigcirc Q_{15}$

B.) $\quad 9 - 2^x = 2^{3-x}$

$$9 - 2^x = \frac{2^3}{2^x}$$

$$9 - t = \frac{8}{t}$$

$$9t - t^2 = 8$$

$$0 = t^2 - 9t + 8$$

$$0 = t^2 - 8t - t + 8$$

$$0 = (t-8)(t-1)$$

$$2^x = 2^3 \quad \text{or} \quad 2^x = 2^0 \qquad \bigcirc Q$$

$$x = 3 \quad \text{or} \quad 0 \qquad \boxed{x_1 \, x_2 = 0}$$

C.) $\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \frac{1}{3}$

$\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right) = \frac{1}{2}$

$\log_{\frac{1}{2}} x = \frac{1}{2}$

$$\boxed{x = \frac{1}{\sqrt{2}}}$$ (P)

D.) $\log_b a = 3$, $\quad \log_b c = -4$, $\quad a^{3x} = c^{x-1}$

$a = b^3$, $\quad c = b^{-4}$

$b^{9x} = b^{4-4x}$

$b^{9x} = \dfrac{b^4}{b^{-4x}}$

$b^{13x} = b^4$

$13x = 4$

$x = \dfrac{4}{13}$,

$$\boxed{p + q = 17}$$ Q, R

ATDB.uno

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno

ATDB.uno

(A) $\dfrac{\log_2 2^5}{\log_3 3^{5/2}} \Rightarrow \dfrac{5}{5} \times 2 \Rightarrow ② \Rightarrow \boxed{P, S}$

(B) $\dfrac{2\log 6}{\log 12 + \log 3} \Rightarrow \dfrac{\log 6^2}{\log 6^2} \Rightarrow ① \Rightarrow \boxed{P}$

(C) $\log_{\frac{1}{4}} (16)^2 \Rightarrow \log_{4^{-1}} (4)^4 \Rightarrow \boxed{\!-4\!} \Rightarrow \boxed{Q}$

(D) $\dfrac{\log_5 4}{\log_5 2^7} \Rightarrow \dfrac{2\log_5 2}{7\log_5 2} \Rightarrow \left(\dfrac{2}{7}\right) \Rightarrow \boxed{R}$

ATDB.uno
ATDB PDFZ
https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno
ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.
ATDB.uno

Sol^n

$(A)\ \dfrac{\log_2 32}{\log_3 \sqrt{243}} = \dfrac{\log_2 (2)^5}{\log_3 \sqrt{3^5}} = \dfrac{5}{\frac{5}{2}} = \boxed{2}$

$\longrightarrow\ (P)\ \circ\ (S)$

$(B)\ \dfrac{2\log 6}{\log 12 + \log 3} = \dfrac{2\log 6}{\log 36} = \dfrac{2\log 6}{\log (6)^2} = \dfrac{2\log 6}{2\log 6}$

$= \boxed{1}$

$\longrightarrow\ (P)$

$(C)\ \log_{1/q}(1/16)^{-2} = \log_{1/q}\left(\dfrac{1}{q}\right)^{-9} = \log_{1/q}(1/q)^{-9} = \boxed{-9}$

$\longrightarrow\ (Q)$

$(D)\ \dfrac{\log_5 16 - \log_5 4}{\log_5 128} = \dfrac{\log_5 4}{\log_5 (2)^7} = \dfrac{\log_5 (2)^2}{\log_5 (2)^7}$

$= \dfrac{2\log_5 2}{7\log_5 2}$

$= \boxed{\dfrac{2}{7}}$

$\longrightarrow\ (R)$

QUESTION

(KTK 2)

$$\frac{1}{x-1} - \frac{4}{x-2} + \frac{4}{x-3} - \frac{1}{x-4} < \frac{1}{30}$$

Ans. $x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$

ATDB.unoFz

ATDB.uno | Studyprayas å FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno

ATDB.uno

$$\frac{1}{x-1} - \frac{4}{x-2} + \frac{4}{x-3} - \frac{1}{x-4} < \frac{1}{30}$$

$$\Rightarrow \quad \frac{1}{x-1} - \frac{1}{x-4} + \frac{4}{x-3} - \frac{4}{x-2} < \frac{1}{30}$$

$$\frac{(x-4) - (x-1)}{(x-1)(x-4)} + 4\left[\frac{(x-2) - (x-3)}{(x-3)(x-2)}\right] < \frac{1}{30}$$

$$\frac{-3}{(x-1)(x-4)} + \frac{4}{(x-3)(x-2)} < \frac{1}{30}$$

$$\frac{-3}{x^2 - 5x + 4} + \frac{4}{x^2 - 5x + 6} < \frac{1}{30}$$

$$x^2 - 5x = t$$

$$\frac{-3}{t+4} + \frac{4}{t+6} - \frac{1}{30} < 0$$

$$\frac{-3(30)(t+6) + 4(30)(t+4) - (t+4)(t+6)}{(t+4)(t+6)(30)} < 0$$

$$\frac{-90t - 540 + 120t + 480 - t^2 - 10t - 24}{(t+4)(t+6)} < 0$$

Ans. $x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$

ATDB.uno

ATDB.uno | Studyprayas å FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited
ATDB.uno

ATDB.uno

$$\frac{-t^2 + 20t - 84}{(t+4)(t+6)} < 0$$

$$\Rightarrow \frac{t^2 - 20t + 84}{(t+4)(t+6)} > 0$$
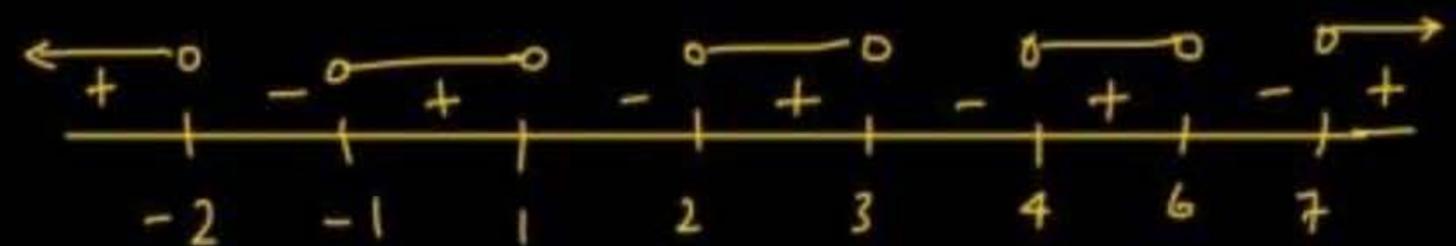
$$\Rightarrow \frac{t^2 - 14t - 6t + 84}{(t+4)(t+6)} > 0$$

$$\Rightarrow \frac{(t-14)(t-6)}{(t+4)(t+6)} > 0$$

$$\frac{(x^2 - 5x - 14)(x^2 - 5x - 6)}{(x^2 - 5x + 4)(x^2 - 5x + 6)} > 0$$

$$\frac{(x^2 - 7x + 2x - 14)(x^2 - 6x + x - 6)}{(x^2 - 5x + 4)(x^2 - 5x + 6)} > 0$$

$$\frac{(x-7)(x+2)(x-6)(x+1)}{(x-1)(x-4)(x-2)(x-2)} > 0$$

ATDB.uno

$$x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$$

Made with Goodnotes

QUESTION

The equation $\dfrac{\log_8\left(\dfrac{8}{x^2}\right)}{(\log_8 x)^2} = 3$ has

A no integral solution

B one natural solution

C two real solutions

D one irrational solution

ATDB.uno

Ans. B, C

KTK-03 | The eq^n $\dfrac{\log_8(8/x^2)}{(\log_8 x)^2} = 3$

$$\Rightarrow \dfrac{1 - \log_8 x^2}{(\log_8 x)^2} = 3$$

$$\Rightarrow 1 - 2\log_8 x = 3(\log_8 x)^2$$

let $\log_8 x = t$

$$1 - 2t = 3t^2$$

$$3t^2 + 2t - 1 = 0$$

$$3t^2 + 3t - t - 1 = 0$$

$$3t(t+1) - 1(t+1) = 0$$

$$(3t - 1)(t + 1) = 0$$

$$t = \dfrac{1}{3}, \; -1$$

$$\log_8 x = \dfrac{1}{3} \qquad\qquad \log_8 x = -1$$

$$x = 2 \qquad\qquad\qquad x = \dfrac{1}{8}$$

B) one natural sol^n

C) 2 real sol^n

KTK-3

$$\frac{\log_8\left(\frac{8}{x^2}\right)}{(\log_8 x)^2} = 3 \text{ has}$$

$$\log_8\left(\frac{8}{x^2}\right) = 3(\log_8 x)^2$$

$$\left[\log_8 8 - \log_8 x^2\right] = 3(\log_8 x)^2$$

Let $\log_8 x = t$

$$(1 - 2t) = 3t^2$$

$$3t^2 + 2t - 1 = 0$$

$$3t^2 + 3t - t - 1 = 0$$

$$3t[t+1] - 1(t+1) = 0$$

$$(t+1)(3t-1) = 0$$

$$\boxed{t = -1} \qquad \boxed{t = \frac{1}{3}}$$

$$\log_8 x = -1 \qquad \log_8 x = \frac{1}{3}$$

$$\boxed{x = \frac{1}{8}} \qquad \boxed{x = 2}$$

QUESTION (KTK 4)

For the equation $\log_{3\sqrt{x}} x + \log_{3x} \sqrt{x} = 0$, which of the following do not hold good?

A no real solution

B one prime solution

C one integral solutions

D no irrational solution

ATDB.uno

Ans. A, B, D

ATDB.uno

ATDB.uno | Studyprayas à FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

ATDB.uno

$$\log \quad \frac{\log_3 x}{\log_3 3\sqrt{x}} + \frac{\log_3 \sqrt{x}}{\log_3 3x} = 0$$

$$\frac{\log_3 x}{1 + \log_3 \sqrt{x}} + \frac{\log_3 \sqrt{x}}{1 + \log_3 x} = 0$$

$$\frac{\log_3 x}{1 + \frac{1}{2}\log_3 x} + \frac{1}{2}\frac{\log_3 x}{1 + \log_3 x} = 0$$

$$\frac{2t}{2 + t} + \frac{t}{2(1+t)} = 0$$

$$\frac{2t \cdot (2)(1+t) + t(2+t)}{2(1+t)(2+t)} = 0$$

$$\frac{4t + 4t^2 + 2t + t^2}{2(1+t)(2+t)} = 0$$

$$5t^2 + 6t = 0$$

$$t(5t + 6) = 0$$

$$t = 0, \frac{-6}{5}$$

$$\log_3 x = 0, \frac{-6}{5} \Rightarrow x = 1, (3)^{-6/5}$$

$$\boxed{x = 1, \frac{1}{3^{6/5}}}$$

Ans : $\boxed{(a), (b), (d)}$

QUESTION

(KTK 5)

If $\log_{\sqrt{2}} \sqrt{x} + \log_2 x + \log_4(x^2) + \log_8(x^3) + \log_{16}(x^4) = 40$ then x is equal to

Ans. 256

KTK-05 If $\log_{\sqrt{2}}\sqrt{x} + \log_2 x + \log_4(x^2) + \log_8(x^3) + \log_{16}(x^4) = 40$, then x is equal to

$$\Rightarrow \log_2 x + \log_2 x + \log_2 x + \log_2 x + \log_2 x = 40$$

$$5\log_2 x = 40$$

$$x = 2^8$$

$$= 256$$

KTK-05

$$\log_{\sqrt 2}\sqrt x$$

$$\log_{\sqrt 2}\sqrt x + \log_2 x + \log_4 x^2 + \log_8 x^3 + \log_{16} x^4 = 40$$

$$2\log_2 \sqrt x + \log_2 x + \frac{1}{2}\log_2 x^2 + \frac{1}{3}\log_2 x^3 + \frac{1}{4}\log_2 x^4 = 40$$

$$\log_2 x + \log_2 x + \log_2 x + \log_2 x + \log_2 x = 40$$

$$5t = 40$$

$$t = \frac{40}{5} = 8$$

$$\log_2 x = 8$$

$$\boxed{x = 2^8 = 256}$$

QUESTION (KTK 6)

Column-I		Column-II	
(A)	If x_1 and x_2 satisfy the equation $x^{\log_{10} x} = 100x$ then the value of $x_1 x_2$ equals	(P)	irrational
(B)	Sum of the squares of the roots of the equation $\log_2(9 - 2^x) = 3 - x$ is	(Q)	rational
(C)	If $\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \frac{1}{3}$ then x is	(R)	prime
(D)	Let $\log_b a = 3, \log_b c = -4$. If the value of x satisfying the equation $a^{3x} = c^{x-1}$ is expressed in the form p/q, where p and q are relatively prime then $p + q$ is	(S)	composite

Ans. $A \rightarrow Q, S; B \rightarrow Q, S; C \rightarrow P; D \rightarrow QR$

[FTK - 06]

(a) $x^{\log_{10}x} = 100x$

Taking log both sides

$\log_{10}x \cdot \log_{10}x = \log_{10}100x$

$(\log_{10}x)^2 = \log_{10}100 + \log_{10}x$

$(\log_{10}x)^2 = 2 + \log_{10}x$

Let $(\log_{10}x) = t$

$\log_{10}x = 2$

$\boxed{x_1 = 100}$

$\log_{10}x = -1$

$\boxed{x_2 = \dfrac{1}{10}}$

$t^2 = 2 + t$

$t^2 - t - 2 = 0$

$t^2 - 2t + t - 2 = 0$

$(t-2)(t+1) = 0$

$t = +2, -1$

$x_1 \cdot x_2 = 100 \times \dfrac{1}{10} = \boxed{10}$ // $A \to 5, 8$

(b) $\log_2(9 - 2^x) = 3 - x$

$2^{(3-x)} = 9 - 2^x$

$\dfrac{2^3}{2^x} = 9 - 2^x \implies 8 = 9 \cdot 2^x - 2^{2x}$

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited

ATDB.uno

ATDB.uno

$$2^x = t$$

$$t^2 - 9t + 8 = 0 \qquad 2^{2x} - 8t + t^2$$

$$t^2 - 8t - t + 8 = 0 \qquad t^2 - 8t + 9 = 0$$

$$(t-8)(t-1) = 0 \qquad t^2 = 9t$$

$$\boxed{t = 8, 1}$$

$$2^x = 8 \qquad\qquad 2^{2x} = 1$$

$$\boxed{x = 3} \qquad\qquad \boxed{x = 0} \qquad \boxed{B \rightarrow (3, 5)}$$

$$3^2 + 0^2 = 9$$

(c) $$\log_{\frac{1}{8}}\left(\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right)\right) = \frac{1}{3}$$

$$\log_{\frac{1}{4}}\left(\log_{\frac{1}{2}} x\right) = \left(\frac{1}{8}\right)^{\frac{1}{3}}$$

$$\log_{\frac{1}{2}} x = \frac{1}{4}\left(\frac{1}{2}\right)$$

$$\log_{\frac{1}{2}} x = \frac{1}{2}$$

$$x = \left(\frac{1}{2}\right)^{\frac{1}{2}}$$

$$\boxed{x = \frac{1}{\sqrt{2}}} \qquad \boxed{C \rightarrow P}$$

$$\log_b a = 3 \qquad \log_b c = -4 \quad , \quad a^{3x} = c^{x-1}$$

$$a^{3x} = c^{x-1}$$

Taking log both sides

$$3x \log_b a = (x-1)\log_b c$$

$$3x (3) = (x-1)(-4)$$

$$9x = -4x + 4$$

$$13x = 4$$

$$4 + 13$$

$$\boxed{17}$$

$$\boxed{D \rightarrow R}$$

$$\boxed{x = \frac{4}{13}}$$

RTK06

Q

(A) $x^{\log_{10}x} = 100x$

$$(\log_{10}x)^2 = \log_{10}10^2 + \log_{10}x$$

$$t^2 = 2 + t$$

$$t^2 - t - 2 = 0$$

$$\boxed{t = -1} \qquad \boxed{t = 2}$$

$$x = \frac{1}{10} \qquad x_2 = 100$$

$$\boxed{x_1 \, x_2 = 10}$$

(B) $\log_2(9 - 2^x) = 3 - x$

$$2^x \quad 9 - 2^x = \frac{8}{2^x}$$

$$9 - t = \frac{8}{t}$$

$$9t - t^2 = 8$$

$$t^2 - 9t + 8 = 0$$

$$t^2 - 8t - t + 8 = 0$$

$$t(t-8) - 1(t-8) = 0$$

$$\boxed{t = 1} \qquad \boxed{t = 8}$$

$$\boxed{x = 0} \qquad \boxed{x = 3}$$

$$\log_{\frac{1}{4}}(\log_5 x) = 2^{-3}$$

$$\log_{\frac{1}{2}} x = 2^{-2 \times \frac{1}{2}}$$

$$x = 2^{-1 \times \frac{1}{2}}$$

$$\log_{\frac{1}{2}} x = \frac{1}{2}$$

$$x = \left(\frac{1}{2}\right)^{\frac{1}{2}}$$

$$\boxed{x = \frac{1}{\sqrt{2}}}$$

(D)

$$\log_b a = 3 \quad ; \quad \log_b c = -4$$

$$\boxed{a = b^3} \qquad \boxed{c = b^{-4}}$$

$$a^{3x} = c^{x-1}$$

$$(a^3)^x \cdot$$

$$(a)^{3x} = \frac{(c)^x}{c}$$

$$(b^3)^{3x} = \frac{(b^{-4})^x}{b^{-4}}$$

$$b^{9x} = \frac{b^4}{b^{4x}}$$

$$b^{9x} = b^{4-4x}$$

$$9x = 4 - 4x$$

$$13x = 4 \qquad \boxed{x = \frac{4}{13}} \qquad \boxed{p = 4} \quad \boxed{q = 13}$$

Home Challenge-07

Find the sum of all the integral solution(s) of the equation $3^{|x|} = \left(\dfrac{3}{(\sqrt{3})^{|x-2|}}\right)^2$. [Ans. 3]

ATDB.uno

ATDB.uno | Studyprayas   FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

Home Challenge-07

Find the sum of all integral $sol^n(s)$ of the eq^n

$$3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}}\right)^2$$

$\Rightarrow \quad 3^{|x|} = 3^{2-|x+2|}$

bases same, powers can be compared

$|x| = 2 + |x-2| \Rightarrow |x| + |x-2| - 2 = 0$

① $x \leq 0$
$-x - x + 2 - 2 = 0$
$-2x = 0$
$x = 0$

② $0 < x < 2$
$x - x + x - 2 = 0$
$\forall x \in R$
$x \in (0, 2)$

③ $x \geq 2$
$x + x - 2 - 2 = 0$
$2x = 4$
$x = 2$

$x \in [0, 2]$

$\therefore$ integral values of $x = 0, 1, 2$

Sum $= 0 + 1 + 2 = 3$

Home challenge -01

Q

$$3^{|x|} = \left(\frac{3}{(\sqrt{3})^{|x-2|}} \right)^2$$

$$3^{|x|} = 3^{2-|x-2|}$$

$$|x| = 2 - |x-2|$$

$\boxed{x < 0}$	$\boxed{0 < x < 2}$	$\xcancel{x>2}$ $\boxed{x > 2}$
$-x = 2 + x - 2$	$x = 2 + x - 2$	$x = 2 - x - 2$
	$x = x$	$2x = 0$
$2x = 0$	$x \in (0, 2]$ ✓	$\boxed{x = 0}$ ✗
$\boxed{x = 0}$ ✓		

integral sol^n $x = \{1, 0, 2\}$

sum = ③ A

Solution to Previous BPPs

ATDB.uno PDFZ

ATDB.uno | Studyprayas â FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

https://ATDB.uno | Unauthorised redistribution is strictly prohibited

ATDB.uno

ATDB.uno

Bumper Practice Problems

Solve the following inequality for x.

(a) $|x| > 2$

(b) $|x - 1| > 3$

(c) $|x - 2| < 1$

(d) $|x + 1| \geq 2$

(e) $|x - 1| \leq 5$

(f) $|2x - 3| > 7$

(g) $|3x + 5| < 2$

(h) $|4x + 6| > 5$

(i) $|2x - 3| > -2$

(j) $|4x - 9| \leq 7$

(k) $|3x + 5| \geq 2$

(l) $|2x + 3| \geq 0$

(m) $|4 - 3x| < -2$

(n) $|5 - 3x| \leq 0$

(o) $\left|\dfrac{x-3}{x-5}\right| > 1$

(p) $\left|\dfrac{x+4}{x+2}\right| \leq 1$

(q) $|x^2 - 4x| < 5$

(r) $|x - 3| > -1$

(s) $|3x - 2.5| \geq 2$

(t) $|x - 2| \leq |x + 4|$

Answers

(a) $(-\infty, -2) \cup (2, \infty)$

(b) $(-\infty, -2) \cup (4, \infty)$

(c) $x \in (1, 3)$

(d) $x \in (-\infty, -3] \cup [1, \infty)$

(e) $x \in (-4, 6)$

(f) $(-\infty, -2) \cup (5, \infty)$

(g) $\left(-\frac{7}{3}, -1\right)$

(h) $x \in \left(-\infty, \frac{-11}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)$

(i) $x \in (-\infty, \infty)$

(j) $x \in \left[\frac{1}{2}, 4\right]$

(k) $x \in \left(-\infty, \frac{-5}{3}\right] \cup [-1, \infty)$

(l) $x \in \mathbb{R}$

(m) $x \in \phi$

(n) $x \in \left\{\frac{5}{3}\right\}$

(o) $(4, \infty) - \{5\}$

(p) $(-\infty, -3]$

(q) $(-1, 5)$

(r) $x \in (-\infty, \infty)$

(s) $x \in \left(-\infty, \frac{1}{6}\right] \cup \left[\frac{3}{2}, \infty\right)$

(t) $x \in [-1, \infty)$

ATDB.uno | Studyprayas FOR PERSONAL STUDY USE ONLY. DO NOT SHARE OR REDISTRIBUTE.

ATDB PDFZ

QUESTION

Solve the following equations

(a) $|x - 1| + |x - 3| = 2$

(b) $|x| + |x + 5| = 5$

(c) $|x - 1| + |x - 4| = 2$

(d) $|x^2 - 2x| + |x - 4| = |x^2 - 3x + 4|$

Answers:

(a) $[1, 3]$

(b) $[-5, 0]$

(c) $\{\}$

(d) $x \in (-\infty, 0] \cup [2, 4]$

(B.P.P.)

(a) $|x| > 2 \Rightarrow \boxed{x \in (-\infty, -2) \cup (2, \infty)}$

(b) $|x-1| > 3 \Rightarrow x-1 < -3 \text{ or } x-1 > 3$

$x < -2 \text{ or } x > 4$

$\Rightarrow \boxed{x \in (-\infty, -2) \cup (4, \infty)}$

(c) $|x-2| < 1 \Rightarrow -1 < x-2 < 1$

$1 < x < 3 \Rightarrow \boxed{x \in (1, 3)}$

(d) $|x+1| \geq 2 \Rightarrow x+1 < -2 \text{ or } x+1 \geq 2$

$x \leq -3 \text{ or } x \geq 1$

$\Rightarrow \boxed{x \in (-\infty, -3] \cup [1, \infty)}$

(e) $|x-1| \leq 5 \Rightarrow -5 \leq x-1 \leq 5$

$-4 \leq x \leq 6 \Rightarrow \boxed{x \in [-4, 6]}$

(f) $|2x-3| > 7 \Rightarrow 2x-3 < -7 \text{ or } 2x-3 > 7$

$2x < -4 \text{ or } 2x > 10$

$x < -2 \text{ or } x > 5$

$\Rightarrow \boxed{x \in (-\infty, -2) \cup (5, \infty)}$

(g) $|3x+5| < 2 \Rightarrow -2 < 3x+5 < 2$

$-7 < 3x < -3$

$-\frac{7}{3} < x < -1 \Rightarrow \boxed{x \in \left(-\frac{7}{3}, -1\right)}$

(h) $|4x+6| > 5 \Rightarrow 4x+6 < -5 \text{ or } 4x+6 > 5$

$4x < -11 \qquad 4x > -1$

$x < \frac{-11}{4} \qquad x > \frac{-1}{4}$

$\Rightarrow \boxed{x \in \left(-\infty, \frac{-11}{4}\right) \cup \left(-\frac{1}{4}, \infty\right)}$

(i) $|2x-3| > -2 \Rightarrow \boxed{x \in R}$

(j) $|4x-9| \leq 7 \Rightarrow -7 \leq 4x-9 \leq 7$

$2 \leq 4x \leq 16$

$\frac{1}{2} \leq x \leq 4 \Rightarrow \boxed{x \in \left[\frac{1}{2}, 4\right]}$

(k) $|3x+5| \geq 2 \Rightarrow 3x+5 \leq -2 \text{ or } 3x+5 \geq 2$

$3x \leq -7 \qquad 3x \geq -3$

$x \leq -\frac{7}{3} \qquad x \geq -1$

$\Rightarrow \boxed{x \in \left(-\infty, -\frac{7}{3}\right] \cup [-1, \infty)}$

(l) $|2x+3| \geq 0 \Rightarrow \boxed{x \in R}$

(m) $|4-3x| < -2 \Rightarrow \boxed{x \in \phi}$

(n) $|5-3x| \leq 0 \Rightarrow |5-3x| = 0$

$5-3x = 0$

$3x = 5 \Rightarrow \boxed{x = \frac{5}{3}}$

(o) $\left|\frac{x-3}{x-5}\right| > 1 \Rightarrow \frac{x-3}{x-5} < -1 \text{ or } \frac{x-3}{x-5} > 1$

$\frac{x-3}{x-5} + 1 < 0 \text{ or } \frac{x-3}{x-5} - 1 > 0$

Left column:

$$\frac{x-3+x-5}{x-5} < 0 \quad \text{or} \quad \frac{x-3-x+5}{x-5} > 0$$

$$\frac{2x-8}{x-5} < 0 \quad \text{or} \quad \frac{2}{x-5} > 0$$

$$\frac{x-4}{x-5} < 0 \quad \text{or} \quad \frac{1}{x-5} > 0$$

$$x \in (4,5) \quad \text{or} \quad x-5>0 \Rightarrow x>5$$

$$\Rightarrow \boxed{x \in (4,5) \cup (5,\infty)}$$

(p) $\left|\frac{x+4}{x+2}\right| \le 1 \Rightarrow -1 \le \frac{x+4}{x+2} \le 1$

$$\frac{x+4}{x+2} \ge -1 \quad \& \quad \frac{x+4}{x+2} \le 1$$

$$\frac{x+4}{x+2}+1 \ge 0 \quad \& \quad \frac{x+4}{x+2}-1 \le 0$$

$$\frac{2x+6}{x+2} \ge 0 \quad \& \quad \frac{x+4-x-2}{x+2} \le 0$$

$$\frac{x+3}{x+2} \ge 0 \quad \& \quad \frac{2}{x+2} \le 0$$

$$x \in (-\infty,-3] \cup (2,\infty) \quad \& \quad x+2<0 \Rightarrow x<-2$$

$$\Downarrow$$

$$\boxed{x \in (-\infty,-3]} \quad \underline{Ans}$$

(q) $|x^2-4x| < 5 \Rightarrow -5 < x^2-4x < 5$

$$x^2-4x+5>0 \quad \& \quad x^2-4x-5<0$$

$$(D<0) \quad \& \quad (x-5)(x+1)<0$$

$$x \in R \quad \& \quad x \in (-1,5)$$

$$\Downarrow$$

$$\boxed{x \in (-1,5)} \quad \underline{Ans}$$

(r) $|x-3| \ge -1 \Rightarrow \boxed{x \in R}$

Right column:

(d) $|3x-2.5| \ge 2 \Rightarrow 3x-2.5 \le -2 \quad \text{or} \quad 3x-2.5 \ge 2$

$$3x \le 0.5 \qquad 3x \ge 4.5$$

$$x \le \frac{0.5}{3} \qquad x \ge \frac{4.5}{3} \Rightarrow x \ge 1.5$$

$$\Rightarrow \boxed{x \in \left(-\infty, \frac{0.5}{3}\right] \cup [1.5,\infty)}$$

(e) $|x-2| \le |x+4| \Rightarrow (x-2)^2-(x+4)^2 \le 0$

$$(x-2+x+4)(x-2-x-4) \le 0$$

$$(2x+2)(-6) \le 0$$

$$x+1 \ge 0$$

$$x \ge -1 \Rightarrow \boxed{x \in [-1,\infty)}$$

Q) Solve the following questions:

(a) $|x-1|+|x-3| = 2$

$|a|+|b| = |a-b|$

$\Rightarrow (x-1)(x-3) \le 0$

$ab \le 0 \Rightarrow \boxed{x \in [1,3]} \quad Ans$

(b) $|x|+|x+5| = 5$

$|a|+|b| = |a-b| \Longleftrightarrow ab \le 0$

$\Rightarrow x(x+5) \le 0 \Rightarrow \boxed{x \in [-5,0]} \quad Ans$

(c) $|x-1|+|x-4| = 2$ $\quad$ Ans: $x \in \phi$

(i) If $x \le 1$, $\quad$ (ii) If $1<x<4$ $\quad$ (iii) If $x>4$

$-x+1-x+4=2$ $\quad$ $x-1-x+4=2$ $\quad$ $x-1+x-4=2$

$3=2x$ $\quad$ $3=2$ $\quad$ $2x=7$

$x=3/2$ $\quad$ (Not possible) $\quad$ $x=7/2$

(reject) $\quad$ $x \in \phi$ $\quad$ (reject)
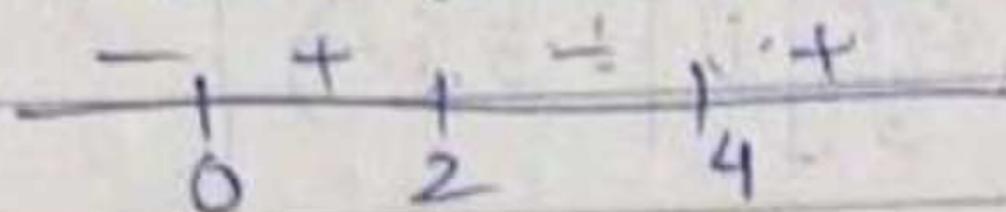
(d) $|x^2-2x| + |x-4| = |x^2-3x+4|$

$|a| + |b| = |a-b| \iff ab \leq 0$

$(x^2-2x)\cdot(x-4) \leq 0$

$x(x-2)(x-4) \leq 0 \implies x \in [0,2] \cup [4,$

$$x \in (-\infty, 0] \cup [2, 4]$$

$Ans.$