PRAYAS

JEE 2026

Mathematics

Basic Maths

Lecture - 17

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(IIT Kanpur)

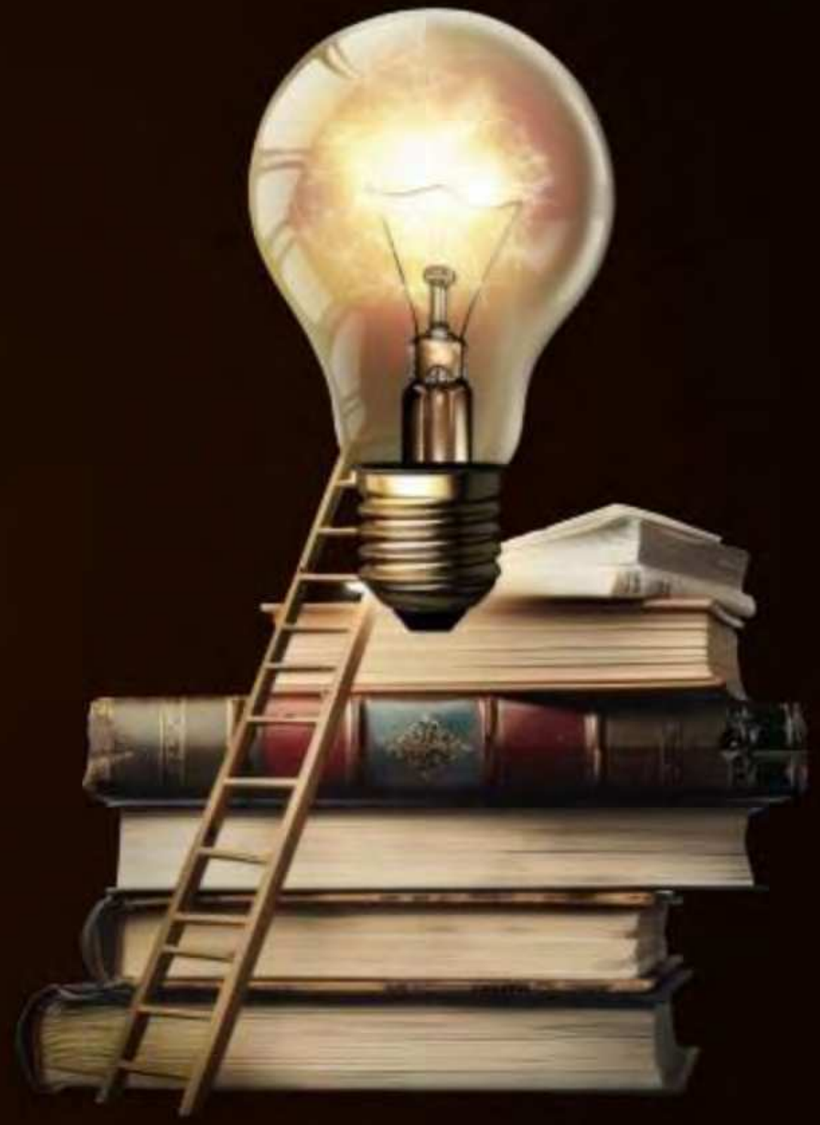
Topics *to be covered*

A Exponential Inequalities

B Characteristic & Mantissa

C Problem Practice

ATDB.uno



Homework Discussion

Bumper Practice Problems

1. $\sqrt[3]{\frac{x-2}{x-1}} < \sqrt[3]{\frac{1}{x-1}}$

2. $\sqrt{3x-2} < \sqrt{x+4}$

3. $\sqrt{4x-3} < \sqrt{2x+5}$

4. $\sqrt[3]{(3x-5)} < (x-1)$

5. $\sqrt[3]{3x-2} < x$

6. $\sqrt{(x+14)} < (x+2)$

7. $\sqrt{2x-2} < x-1$

8. $\sqrt{-x^2+4x-3} > (6-2x)$

9. $\sqrt{x-2} + \sqrt{x-1} > 2$

10. $7\sqrt{x} + 8\sqrt{-x} + \frac{15}{x^3} = 98$

$x \geq 0$; $-x \geq 0 \Rightarrow x \leq 0$; $x \neq 0$; $x \in \phi$

11. $\sqrt{2x-4} - \sqrt{x+5} = 1$

12. $\sqrt{x-1} + \sqrt{2x+6} = 6$

Answers

1. $x \in (1, 3)$

2. $x \in \left[\frac{2}{3}, 6\right)$

3. $x \in \left[\frac{3}{4}, 1\right)$

4. $x \in (-1, \infty) - \{2\}$

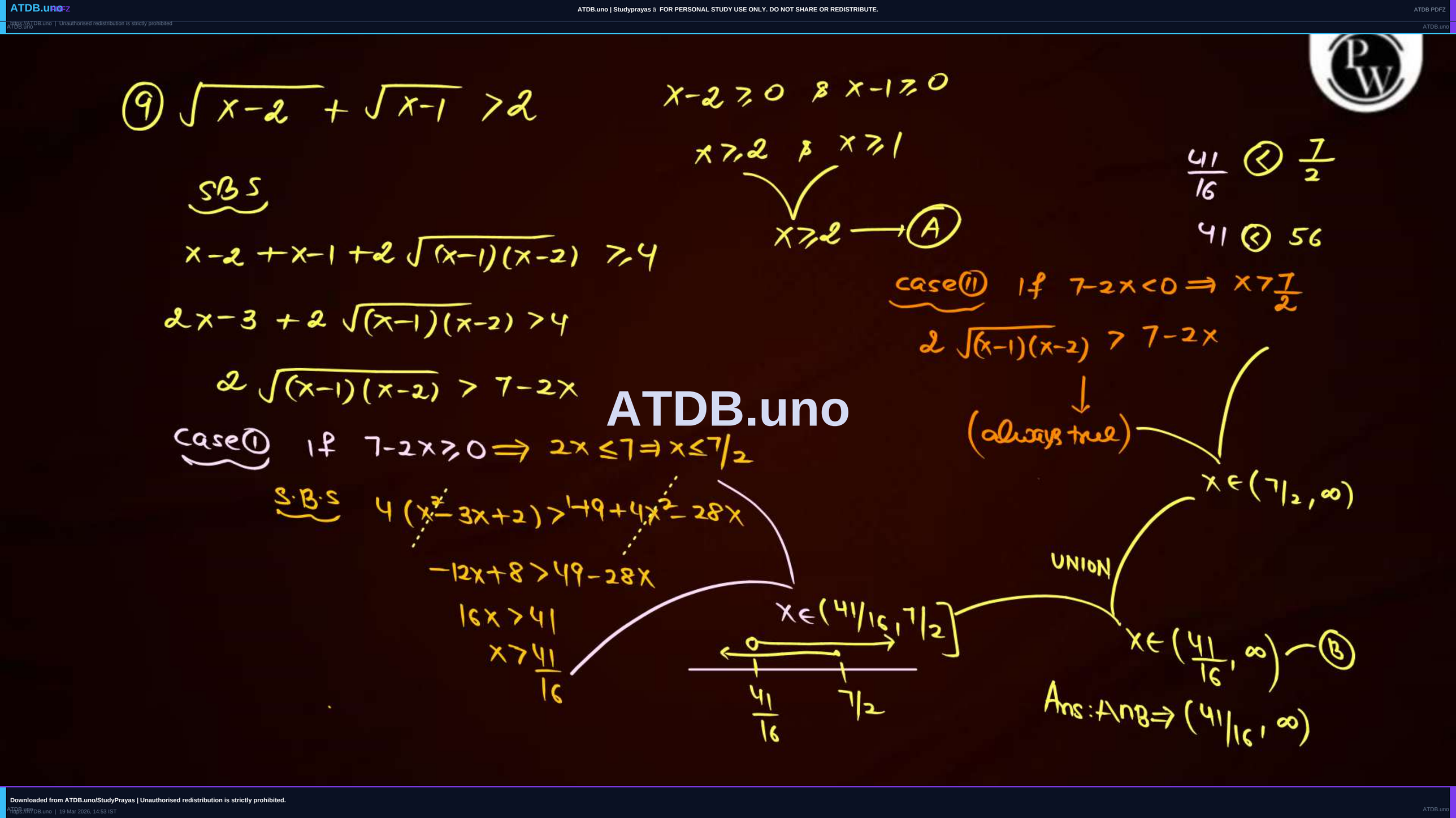
5. $x \in (-2, 1) \cup (1, \infty)$

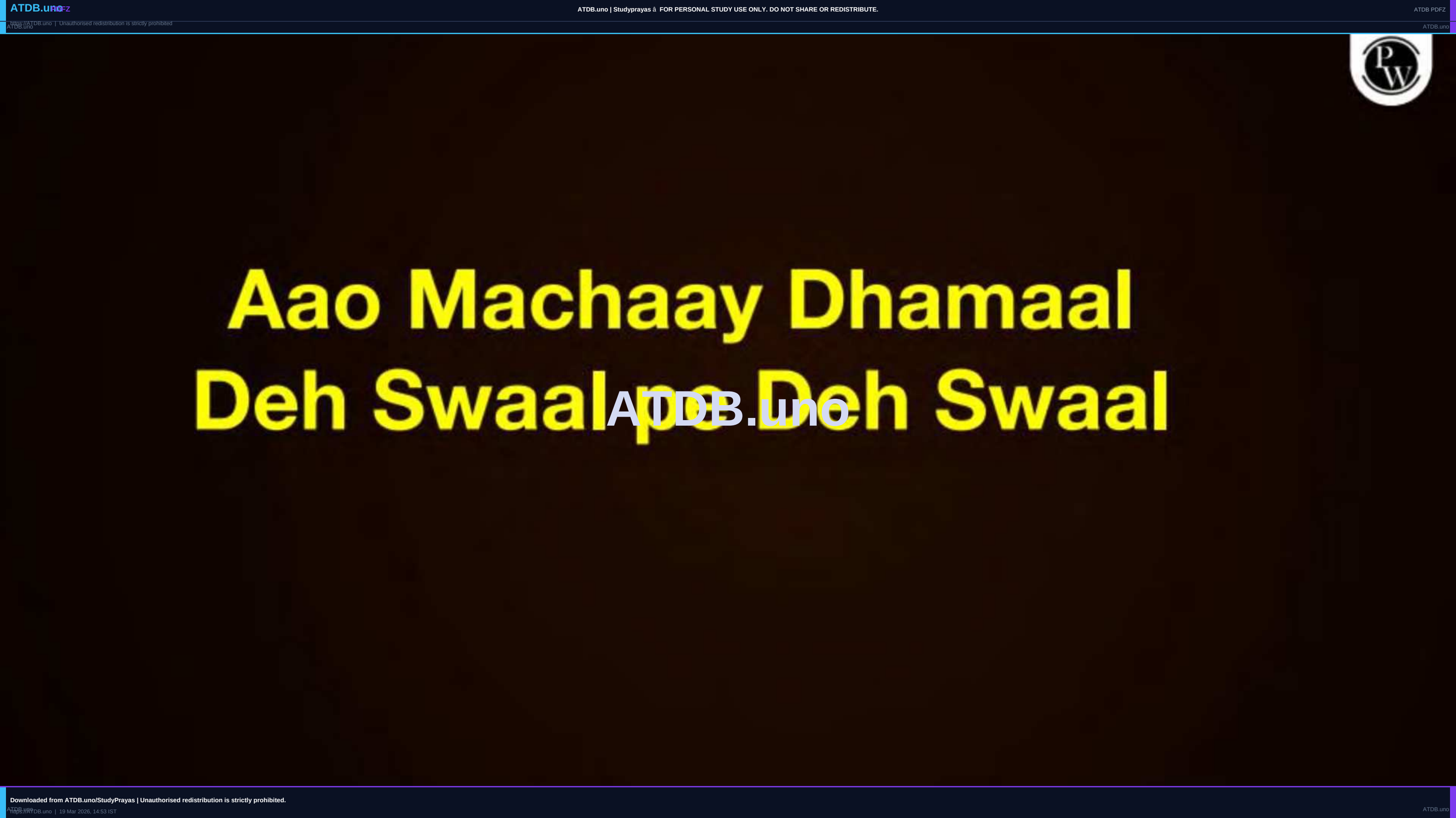
6. $x \in (2, \infty)$

7. $x \in (3, \infty)$

8. $x \in \left(\frac{13}{5}, 3\right)$

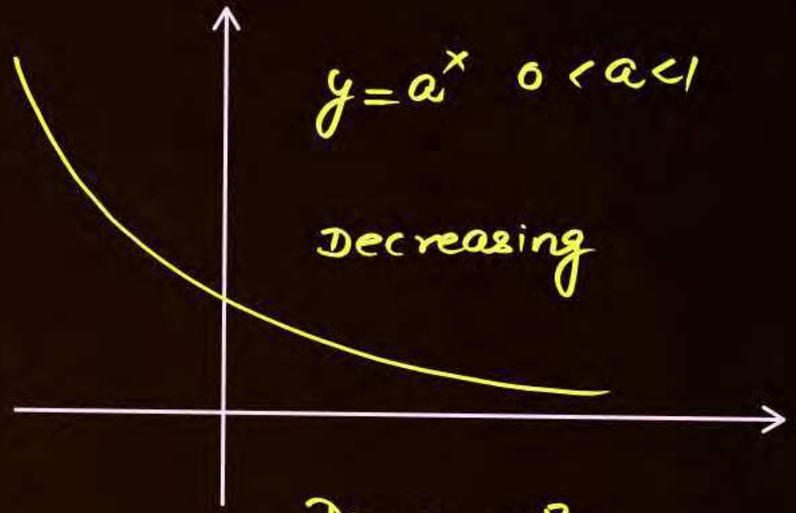
9. $x \in \left(\frac{41}{16}, \infty\right)$





Exponential Inequalities

Domain: R
Range: R^+



Domain : R
Range = R^+

Ex: $3^{x-2} > 3^{2x-3}$ find range of x

$x - 2 > 2x - 3$

$x < 1 \Rightarrow x \in (-\infty, 1)$

Ex: $\left(\frac{1}{2}\right)^{x^2-2x} > \left(\frac{1}{2}\right)^{4x-8}$ find range of x.

$x^2 - 2x < 4x - 8 \Rightarrow x^2 - 6x + 8 < 0$

$(x-2)(x-4) < 0$

$x \in (2, 4)$

QUESTION

(a) $(3)^{\log_2(x^2-3x+2)} > 3^1$

$\log_2(x^2-3x+2) > 1$ & $x^2-3x+2 > 0$ (No Need)

$x^2-3x+2 > 2^1$

$x(x-3) > 0$

$x \in (-\infty, 0) \cup (3, \infty)$

(b) $(1/3)^{(x^2+2x)} > (1/9)^{16-x}$

$\left(\frac{1}{3}\right)^{x^2+2x} > \left(\frac{1}{3}\right)^{32-2x}$

$x^2+2x < 32-2x$

$x^2+4x-32 < 0$

$x^2+8x-4x-32 < 0$

$(x+8)(x-4) < 0$

$x \in (-8, 4)$ Ans

M② Taking log to base 1/3 on both sides

$\log_{\frac{1}{3}}\left(\frac{1}{3}\right)^{x^2+2x} < \log_{\frac{1}{3}}\left(\frac{1}{3}\right)^{32-2x}$

$(x^2+2x) < 32-2x$

$x^2+4x-32 < 0$.

$\therefore x \in (-8, 4)$

QUESTION

(a) $2^x > 5$

Taking log to base 2 on both sides.

$\log_2 2^x > \log_2 5$

$x \log_2 2 > \log_2 5$

$x > \log_2 5$.

(b) $(1/2)^{\log_3(x^2-2x-3)} > 1$

$\left(\frac{1}{2}\right)^{\log_3(x^2-2x-3)} > \left(\frac{1}{2}\right)^0$

$\log_3(x^2-2x-3) < 0$ & $x^2-2x-3>0$

$x^2-2x-3 < 3^0$

$x^2-2x-3 < 1$

$x^2-2x-4 < 0$.

$x = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$

$(x-(1+\sqrt{5}))(x-(1-\sqrt{5})) < 0$

$x \in (1-\sqrt{5}, 1+\sqrt{5})$

$(x-3)(x+1) > 0$

$x \in (-\infty, -1) \cup (3, \infty)$

$x \in (1-\sqrt{5}, -1) \cup (3, \sqrt{5}+1)$

QUESTION

Find the number of integral values of x satisfying the inequality

$$\left(\frac{3}{4}\right)^{6x+10-x^2} < \frac{27}{64} = \left(\frac{3}{4}\right)^3$$

QUESTION [JEE Mains 2020 (8 Jan)]

Tahoz

Let S be the set of all real roots of the equation,
$3^x(3^x - 1) + 2 = |3^x - 1| + |3^x - 2|$. Then S :

(A) contains exactly two elements.

(B) is an empty set.

(C) is a singleton.

(D) contains at least four elements.

Ans. C

QUESTION [JEE Mains 2019 (10 April)]

Tan03

The number of real roots of the equation $5 + |2^x - 1| = 2^x(2^x - 2)$ **is**

A 2

B 1

C 3

D 4

Case (I) $2^x - 1 \geq 0$

$2^x \geq 1 = 2^0$

$x \geq 0$

Case (II) $2^x - 1 < 0 \rightarrow 2^x < 1 = 2^0 \Rightarrow x < 1$

Ans. B

QUESTION [JEE Mains 2016]

Tanoy

If x is a solution of the equation, $\sqrt{2x+1}-\sqrt{2x-1}=1, \left(x \geq \frac{1}{2}\right)$, then $\sqrt{4x^2-1}$ is equal to:

A $\frac{3}{4}$

B $\frac{1}{2}$

C 2

D $2\sqrt{2}$

QUESTION [JEE Mains 2022 (30 June)]

Tah 05

Let $S_1 = \left\{x \in R - \{1,2\}: \frac{(x+2)(x^2+3x+5)}{-2+3x-x^2} \geq 0\right\}$ and $S_2 = \{x \in R: 3^{2x} - 3^{x+1} - 3^{x+2} + 27 \leq 0\}$.

Then, $S_1 \cup S_2$ is equal to :

$3^{2x} - 3^x \cdot 3 - 9 \cdot 3^x + 27 \leq 0$

let $3^x = t$

(A) $(-\infty, -2] \cup (1, 2)$

(B) $(-\infty, -2] \cup [1, 2]$

(C) $(-2, 1] \cup [2, \infty)$

(D) $(-\infty, 2]$

Ans. B

QUESTION [JEE Mains 2022 (25 June)]

Tah06

Let $A = \{x \in R: |x + 1| < 2\}$ and $B = \{x \in R: |x - 1| \geq 2\}$.
Then which one of the following statements is NOT true?

A $A - B = (-1, 1)$

B $B - A = R - (-3, 1)$

C $A \cap B = (-3, -1]$

D $A \cup B = R - [1, 3)$

Ans. B

QUESTION [JEE Mains 2021 (18 March)]

The value of $3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\cdots\infty}}}}$ is equal to

(A) $1.5+\sqrt{3}$

(B) $2+\sqrt{3}$

(C) $3+2\sqrt{3}$

(D) $4+\sqrt{3}$

$$x = 3+\cfrac{1}{4+\cfrac{1}{3+\cfrac{1}{4+\cfrac{1}{3+\cdots\infty}}}}$$

$$x = 3+\cfrac{1}{4+\cfrac{1}{x}}$$

$$x = 3+\frac{x}{4x+1} = \frac{12x+3+x}{4x+1}$$

$$4x^2+x = 13x+3$$

$$4x^2-12x-3=0$$

$$x = \frac{12\pm\sqrt{144+48}}{8}$$

$$x = \frac{12\pm 4\sqrt{9+3}}{8} = \frac{3+2\sqrt{3}}{2}$$

$$x = 1.5+\sqrt{3}.$$

Ans. A

QUESTION [JEE Mains 2021 (17 March)]

Tah 07

The value of $4+\cfrac{1}{5+\cfrac{1}{4+\cfrac{1}{5+\cfrac{1}{4+\cdots\infty}}}}$ is :

(A) $2+\frac{2}{5}\sqrt{30}$

(B) $2+\frac{4}{\sqrt{5}}\sqrt{30}$

(C) $5+\frac{2}{5}\sqrt{30}$

(D) $4+\frac{4}{\sqrt{5}}\sqrt{30}$

Ans. A

QUESTION [JEE Mains 2020 (5 Sept)]

Tan08

The product of the roots of the equation $9x^2 - 18|x| + 5 = 0$ is :

(A) $\frac{5}{9}$

(B) $\frac{5}{27}$

(C) $\frac{25}{81}$

(D) $\frac{25}{9}$

Ans. C

QUESTION [JEE Mains 2017 (9 April)]

Jan 09

The sum of all the real values of x satisfying the equation $2^{(x-1)(x^2+5x-50)} = 1$ is

A 16

B 14

C −4

D −5

Ans. C

Antilog

$\text{antilog}_{10}(\log_{10} 20) = 20$

$\text{antilog}_{2}(\log_{2} 6) = 6.$

$\text{antilog}_{a}(\log_{a} b) = b$

let $\log_a b = x$

$b = a^x$

$\text{antilog}_a x = a^x$

Ex: $\text{antilog}_2 3 = 2^3 = 8.$

Ex: $\text{antilog}_{10} 100 = 10^{100}$

Antilogarithm

(a) $\text{antilog}_{100}\left(-\frac{1}{2}\right) = 100^{-\frac{1}{2}} = \frac{1}{\sqrt{100}} = \frac{1}{10} = 0.1$

(b) $\text{antilog}_{8}\left(\frac{2}{3}\right) = 8^{2/3} = \left(8^{\frac{1}{3}}\right)^2 = 2^2 = 4$

Characteristic & Mantissa

logarithm of any no: to a given base always has two parts an integral part called characteristic and a fractional part called Mantissa

Characteristic ∈ Integer.

Mantissa ∈ [0,1)

Ex: $\log_2 16 = 4.0$

Integral part → characteristic = 4

fractional part → mantissa = 0

Ex: find characteristic of $\log_2 17$

clearly: $2^4 < 17 < 2^5 \rightarrow 4 < \log_2 17 < 5 \rightarrow \log_2 17 = 4.\text{something}$

characteristic = 4

QUESTION

Find the number of positive integers which have the characteristic 3 if the base of the logarithm is 5.

Let $\log_5 N = 3 \cdot \text{something}$.

$1 \le N < 10$

$N = 1, 2, 3, 4, 5, 6, 7, 8, 9$

No: of values of $N = 9$

$3 \le \log_5 N < 4$

$5^3 \le N < 5^4$

$125 \le N < 625$.

No: of possible Integral values of $N = 500$ Ans.

Ex: find No: of possible +ve Integers for which log to base 7 has characteristic 2.

$$\log_7 N = 2 \cdot \text{something}$$

$$2 \le \log_7 N < 3$$

$$49 \le N < 343$$

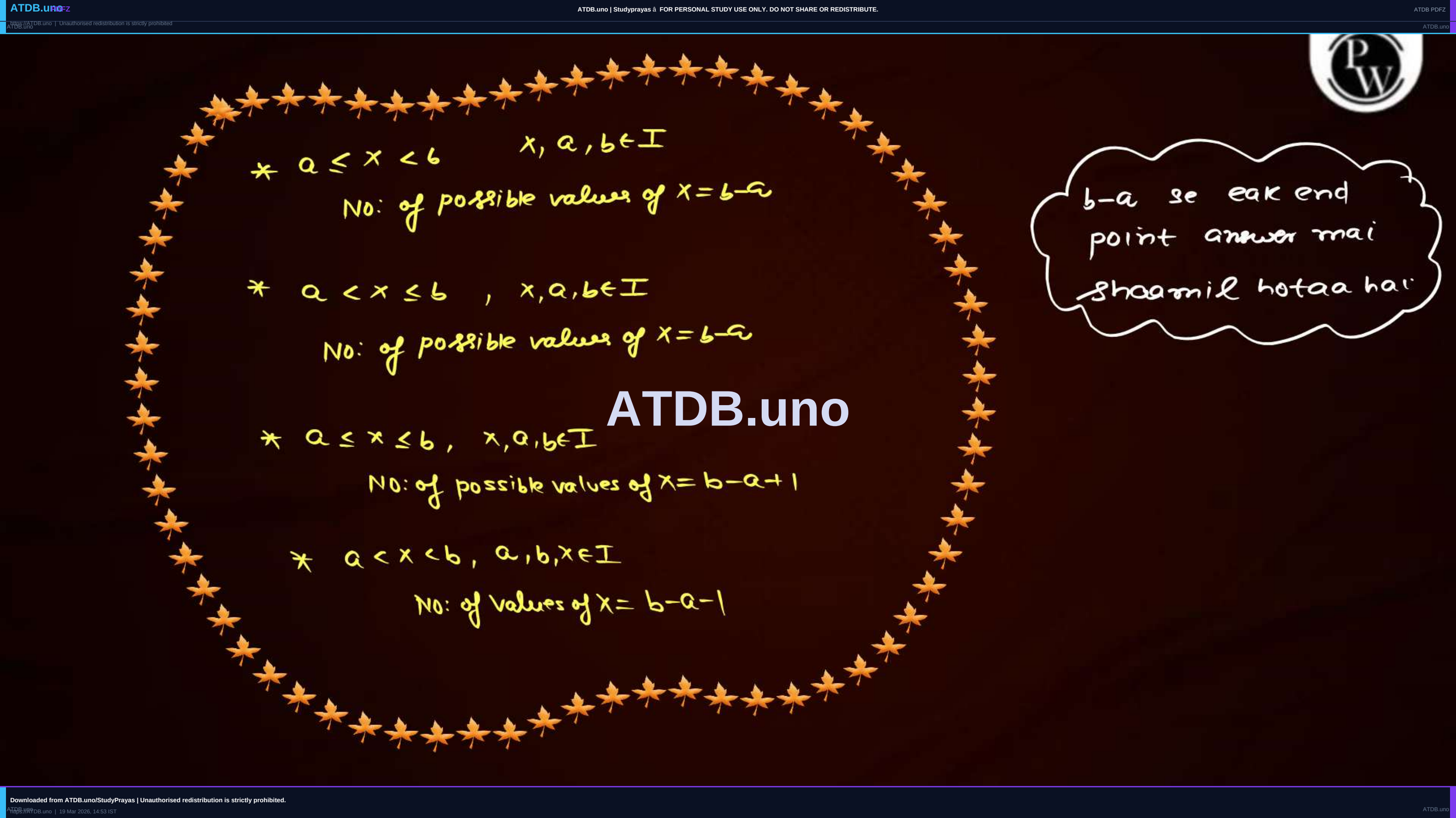
No: of Integral values of $N = 343 - 49$

$= 294$ Ans.

Ex: find characteristic of $\log_4 500$

$4^4 < 500 < 4^5$.

$4 < \log_4 500 < 5$ $\log_4 500 = 4 \cdot \text{something}$ ⌐ characteristic = 4



$\log_{10} N$ kI KAHAANI

$f \in [0,1)$

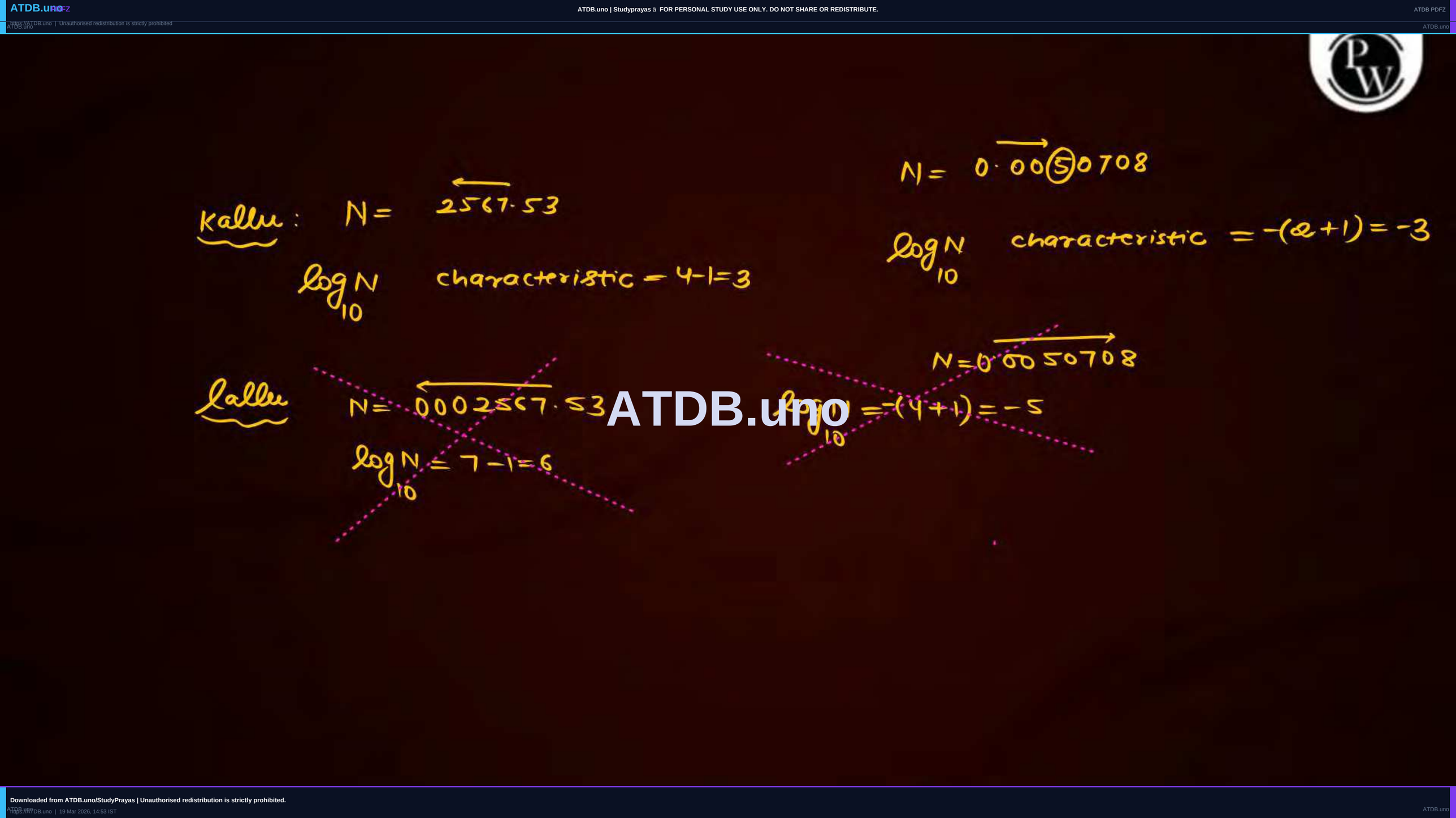
Ex: $\log_2 1 = 0$, $\log_{10} 1 = 0$

Ex: $\log_2 0$ is not defined in reals

If $N \geq 1$

		characteristic
$N = \overleftarrow{1.63}, \overleftarrow{9.85}$	$1 \leq N < 10 \Rightarrow 0 \leq \log_{10} N < 1 \Rightarrow \log_{10} N = 0 + f$	0
$N = \overleftarrow{95.62}, \overleftarrow{88.55}$	$10 \leq N < 100 \Rightarrow 1 \leq \log_{10} N < 2 \Rightarrow \log_{10} N = 1 + f$	1
$N = \overleftarrow{110.23}, \overleftarrow{999.25}$	$100 \leq N < 1000 \Rightarrow 2 \leq \log_{10} N < 3 \Rightarrow \log_{10} N = 2 + f$	2

If $N > 1$, $\log_{10} N$ has characteristic = (No: of Significant digits to left of decimal in N) $- 1$



$\log_{10} N$ kI KAHAANI

Ex: $\log_2 1 = 0$, $\log_{10} 1 = 0$

Ex: $\log_2 0$ is not defined in reals.

$f \in [0, 1)$

If $0 < N < 1$

		characteristic
$N = \overrightarrow{0.689}, \overrightarrow{0.976} * \frac{1}{10} \le N < 1 \Rightarrow -1 \le \log_{10} N < 0 \Rightarrow \log_{10} N = -1 + f$		-1
$N = \overrightarrow{0.078}, \overrightarrow{0.09705} * \frac{1}{100} \le N < \frac{1}{10} \Rightarrow -2 \le \log_{10} N < -1 \Rightarrow \log_{10} N = -2 + f$	(0.01 ↑, 0.1 ↑)	-2
$N = \overrightarrow{.0078}, \overrightarrow{0.00965} * \frac{1}{1000} \le N < \frac{1}{100} \Rightarrow -3 \le \log_{10} N < -2 \Rightarrow \log_{10} N = -3 + f$	(0.01, 0.01)	-3

If $0 < N < 1$, $\log_{10} N$ has characteristic $= -$ (No: of 0's immediately to right of decimal in N before a + 1 significant digit starts)

Common logarithm of N is logarithm of N to base 10 . i.e. $\log_{10} N$

Natural logarithm of N is logarithm of N to base e. i.e., $\log_e N = \ln N$

Note :

Logarithm of an positive real number N to base $a > 0, a \neq 1$ always has two parts

(1) An integral part called as characteristic.

(2) A fractional part called as mantissa fraction part always lies in [0, 1)

QUESTION

Using log 2 = 0.3010 and log 3 = 0.4771

Find the number of digits

(i) $(2)^{200}$

$X = 2^{200}$

$\log_{10} X = \log_{10} 2^{200} = 200 \log 2$

$\log_{10} X = 200 \times 0.3010$

$\log_{10} X = 60.2 = 60 + 0.2$

(No: of digits in X to left of decimal -1) = Characteristic $= 60$

No: of digits in $X = 61$

(ii) $3^{12} \times 2^8 = X$

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$X = 3^{12} \times 2^8$

$\log_{10} X = \log_{10}(3^{12} \times 2^8) = 12 \log 3 + 8 \log 2$

$\log_{10} X = 12 \times 0.4771 + 8 \times 0.3010$

$\log_{10} X = 8.1332$

No: of digit in $X = 9$

QUESTION

$\log_{10} 7 = 0.8451$

Find the number of zeroes after decimal before a significant figures start in

(i) $\left(\frac{9}{8}\right)^{-100}$ (ii) $(0.35)^{12}$ (iii) $\frac{1}{2^{40}}$

$X = \left(\frac{9}{8}\right)^{-100} \rightarrow 0.000000766\underline{9}159\ldots$

$\log_{10} X = -100(\log 9 - \log 8)$

$= -100(2\log 3 - 3\log 2)$

$= -100(2 \times 0.4771 - 3 \times 0.3010)$

$\log_{10} X = -5.12 = -5 - 0.12 = -5 - 1 + 1 - 0.12 = -6 + 0.88$

Mantissa is always positive it lies in $[0,1)$

Characteristic $= -6$.

No: of zeros in X immediately after decimal $= 5$

$-(n+1) = -6$

$n+1 = 6$

$n = 5$

$X = (0.35)^{12}$

$\log_{10} X = 12 \log_{10}(35/100)$

$= 12(\log_{10} 7 + \log_{10} 5 - 2)$

$= 12(0.8451 + \log_{10}(10/2) - 2)$

$= 12(0.8451 + 1 - \log_{10} 2 - 2)$

$= 12(0.8451 + 1 - 0.3010 - 2)$

$= -5.4708 = -5 - 1 + 1 - 0.4708$

$= -6 + .5392$

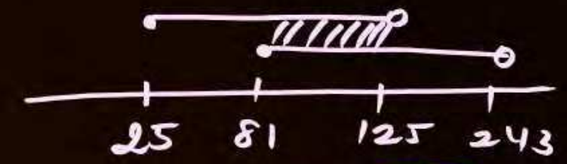
No: of zeros in X After decimal before S.D $= 5$ Ans

QUESTION

Let $\log_3 N = \alpha_1 + \beta_1$, $\log_5 N = \alpha_2 + \beta_2$ and $\log_7 N = \alpha_3 + \beta_3$ where $\alpha_1, \alpha_2, \alpha_3$ are integers and $\beta_1, \beta_2, \beta_3 \in [0, 1)$.

(i) Find the number of integral values of N if $\alpha_1 = 4$ and $\alpha_2 = 2$

(ii) Find the largest integral values of N if $\alpha_1 = 5$ and $\alpha_2 = 3$ and $\alpha_3 = 2$



Tah 10

$\log_3 N = \alpha_1 + \beta_1$

$\log_5 N = \alpha_2 + \beta_2$

$\log_7 N = \alpha_3 + \beta_3$

$[0,1)$

Mantissa

Characteristic

(i) $\alpha_1 = 4$, $\alpha_2 = 2$ (given)

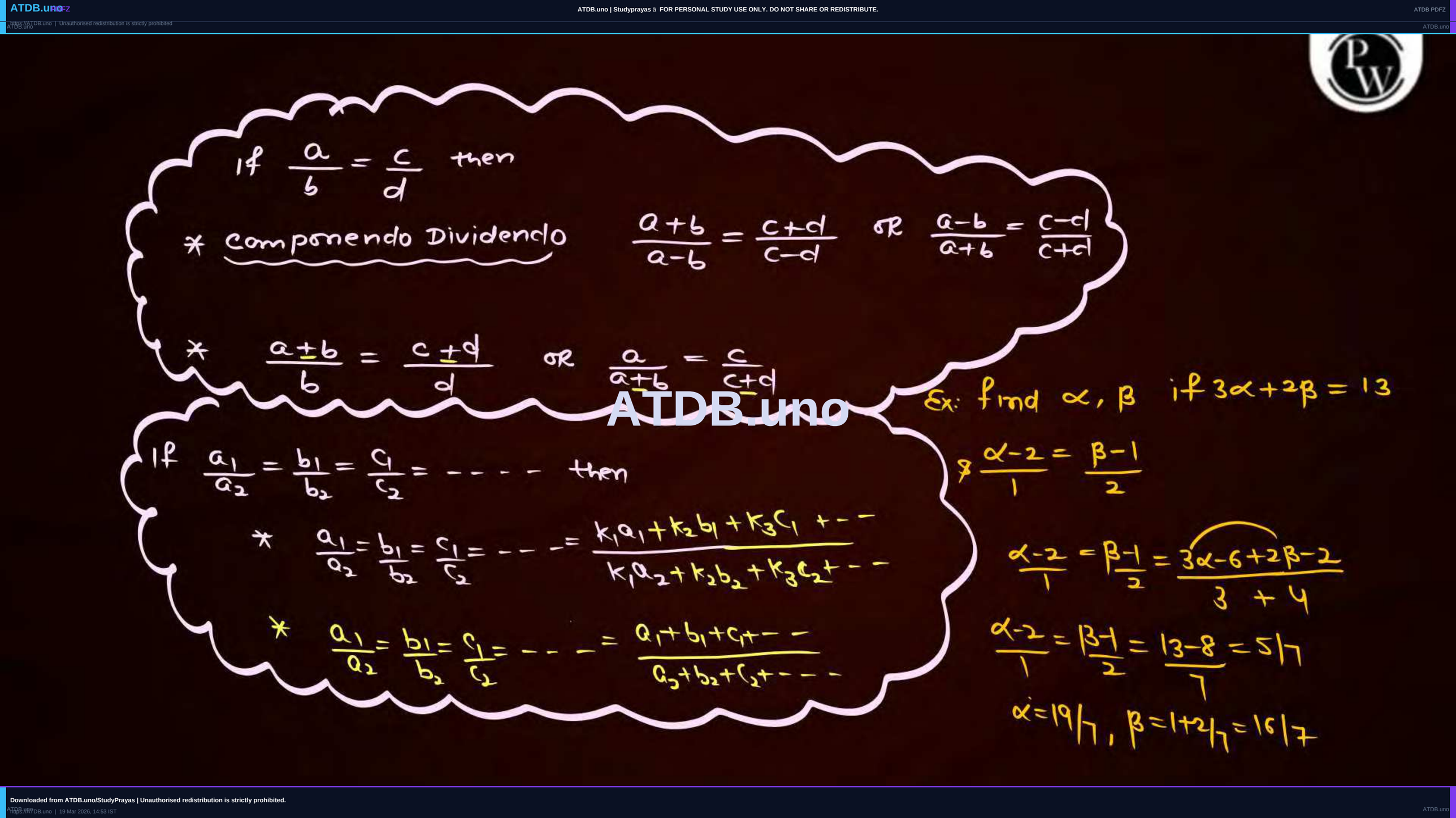
$4 \le \log_3 N = 4 + \beta_1 < 5 \longrightarrow 3^4 \le N < 3^5 \Rightarrow 81 \le N < 243$

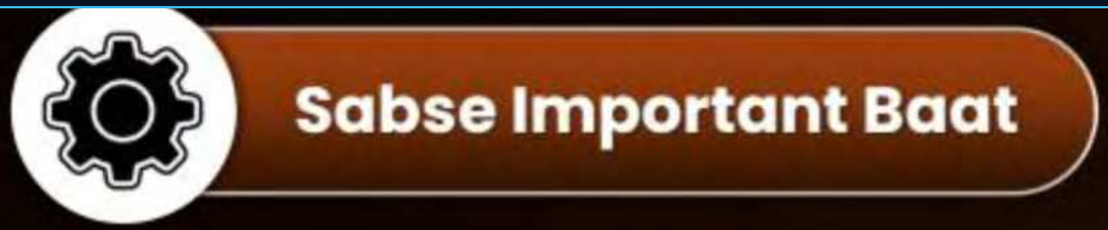
$2 \le \log_5 N = 2 + \beta_2 < 3 \longrightarrow 5^2 \le N < 5^3 \Rightarrow 25 \le N < 125$

$\Downarrow$

$81 \le N < 125.$

No. of Integral values of $N = 125 - 81 = 44$





Sabhi Class Illustrations Retry Karnay hai...

Home Challenge-09

If the least integral value satisfying the equation $\log_3 \sqrt{x^2 - 4x + 4} = 2^{\log_2(\log_3(|x|-2))}$ is α, then find the number of zeroes after decimal and before first significant digit in the number of $(\alpha)^{-4\alpha}$. [Ans. 9]

Today's KTK

No Selection —TRISHUL Apnao IIT Jao→ Selection with Good Rank

QUESTION

1. The common value of x satisfying $\frac{x-1}{x+2} \geq 0$ and $\frac{2x-5}{x-2} \leq 1$ is

(A) $(2, \infty]$ (B) $(2, 3]$ (C) $(-\infty, 3]$ (D) None of these

[Ans. B]

2. The set of values of x satisfying the inequality $2x - 7 < 4x - 2$ and $-5 \leq 2x + 6 < 4$ is given as

(A) $\left[\frac{-11}{2}, \frac{-5}{2}\right]$ (B) $\left[\frac{-11}{2}, \frac{-5}{2}\right)$ (C) $\left(-\frac{5}{2}, -1\right)$ (D) None of these

[Ans. C]

3. The smallest integer k satisfying the inequality $\frac{x-5}{x^2+5x-14} > 0$ is

(A) –7 (B) –6 (C) 6 (D) None of these

[Ans. B]

4. Number of integer values of x satisfying the inequality $\frac{x^2+6x-7}{|x+4|} < 0$ is

(A) 6 (B) 7 (C) 8 (D) None of these

[Ans. A]

QUESTION

5. Set of real values of x satisfying $|x + 4| = 3x - 2$ is given as

(A) $\left\{-\frac{1}{2}, 3\right\}$ (B) $\left\{-\frac{1}{2}\right\}$ (C) $\{3\}$ (D) None of these

[Ans. C]

6. The complete set of solution of $2|x + 1| + |x - 3| = 4$ is given by

(A) $\left\{\frac{5}{3}\right\}$ (B) $\{-1\}$ (C) $\left\{-1, \frac{5}{3}\right\}$ (D) None of these

[Ans. B]

7. The inequality $\frac{|x-3|}{x^2-5x+6} \geq 2$ is given as

(A) $\left[\frac{3}{2}, 2\right] \cup \left[2, \frac{5}{2}\right]$ (B) $\left[\frac{3}{2}, 2\right]$ (C) $\left[\frac{3}{2}, \frac{5}{2}\right]$ (D) $\left[\frac{3}{2}, 2\right)$ [Ans. D]

8. Values of x satisfying the equality $|x^2 + 8x + 7| = |x^2 + 4x + 4| + |4x + 3|$ for $x \in R$ are

(A) $(-2, \infty)$ (B) $\left(\frac{3}{4}, \infty\right)$ (C) $\{2\} \cup \left[-\frac{3}{4}, \infty\right)$ (D) $\left[-\frac{4}{3}, \infty\right)$

[Ans. C]

QUESTION

9. The equation $4^{\left(\frac{1}{x}-2\right)} = \frac{1}{2}\ln\sqrt{e}$ has the solution-

(A) –1 (B) 1 (C) 2 (D) None [Ans. B]

10. Solution of the equation $2^{x+2} \cdot 27^{x(x-1)} = 9$ are given by-

(A) $\log_2(2/3), 1$ (B) $2, 1 - \log_2 3$ (C) $-2, 1 - \log_2 3$ (D) None of these [Ans. C]

11. If $x^{[\log_3 x^2 + (\log_3 x)^2 - 10]} = \frac{1}{x^2}$ then x is equal to

(A) 9, 1/9 (B) 9, 1/81 (C) 1, 1/9 (D) 2, 2/9 [Ans. B]

12. Complete set of values of x satisfying the inequality $x - 3 < \sqrt{x^2 + 4x - 5}$ is :

(A) $(-\infty, 5] \cup [1, \infty)$ (B) $(-5, 3]$ (C) $[3, 5)$ (D) $(-5, 3)$ [Ans. A]

Homework From Module

Prarambh (Topicwise) : Q1 to Q25

Prabal (JEE Main Level) : Q1 to Q33

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Solution to Previous TAH

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QUESTION

If $|x - 5| - |x - 7| = k$ then for

A K = 2 the equation has infinitely many solution

B K = –2 the equation has infinitely many solution

C $K \in (2, \infty)$ the equation has no solution

D K ∞ (–2, 2) the equation has exactly one solution

Tab 01

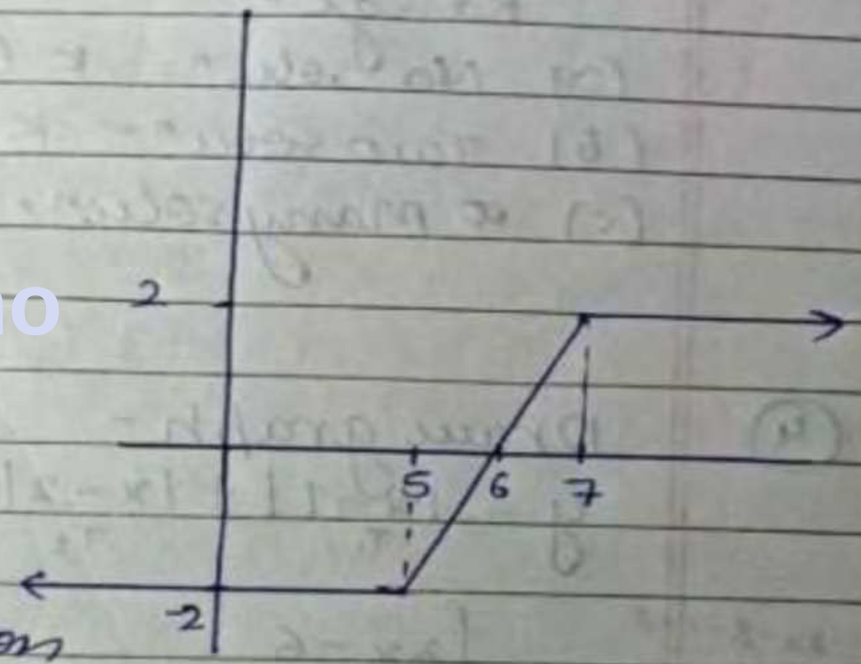
$$|x-5| - |x-7| = K$$

(T_1, T_2)

T_1	−ve		+ve	+ve
T_2	−ve	5	−ve	7 +ve

$$y = \begin{cases} -x+5+x-7 & x<5 \\ x-5+x-7 & 5 \le x < 7 \\ x-5-x+7 & x \ge 7 \end{cases}$$

$$y = \begin{cases} -2 & x \le 5 \\ 2x-12 & 5 \le x < 7 \\ 2 & x \ge 7 \end{cases}$$



(A) $K = 2$, ∞ many solutions
(B) $K = -2$, ∞ many solution
(C) $K \in (2, \infty)$ eqⁿ has no solution
(D) $K \in (-2, 2)$ the eqⁿ has exactly one solution

all are correct

Tah-01. $|x-5| - |x-7| = K$ then for

A) $K=2$ the eqⁿ has infinitely many soln.

B) $K=-2$ the eqⁿ has infinitely many soln.

C) $K \in (2,\infty)$ the eqⁿ has no soln.

D) $K \in (-2,2)$ the eqⁿ has exactly one soln.

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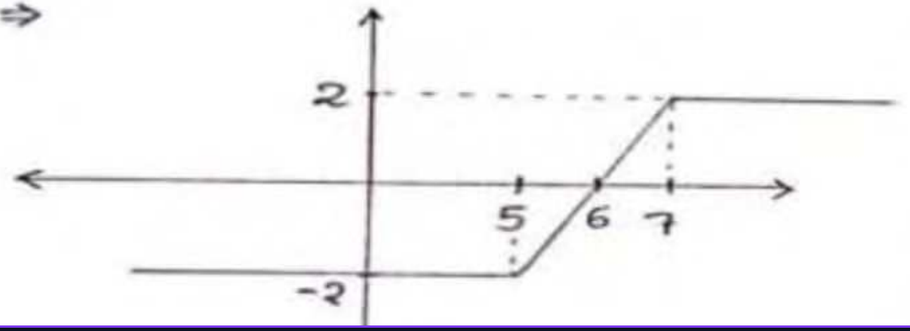
$\Rightarrow$

T_1	$-ve$	$+ve$	$+ve$
	5	7	
T_2	$-ve$	$-ve$	$+ve$

$\Rightarrow$

$$\left.\begin{array}{ll} -x+5+x-7, & x \le 5 \\ x-5+x-7, & 5 < x < 7 \\ x-5-x+7, & x \ge 7 \end{array}\right\} \Rightarrow \left.\begin{array}{ll} -2, & x \le 5 \\ 2x-12, & 5 < x < 7 \\ 2, & x \ge 7 \end{array}\right\}$$

$\Rightarrow$



(A)

(B)

(C)

(D)

All option's correct.

QUESTION

Draw graph of $y = |x| + |x - 2|$ and answer the following.

Find the range of k for which the equation $|x| + |x - 2| = k$ has

A. No Solution

B. Infinitely Many Solutions

C. Only two solutions

Tah-02. Draw graph of $y = |x| + |x-2|$ and answer the following. Find the range of K for which the eqⁿ:

$|x| + |x-2| = K$ has:

(A) No soln.

(B) Infinitely many soln.

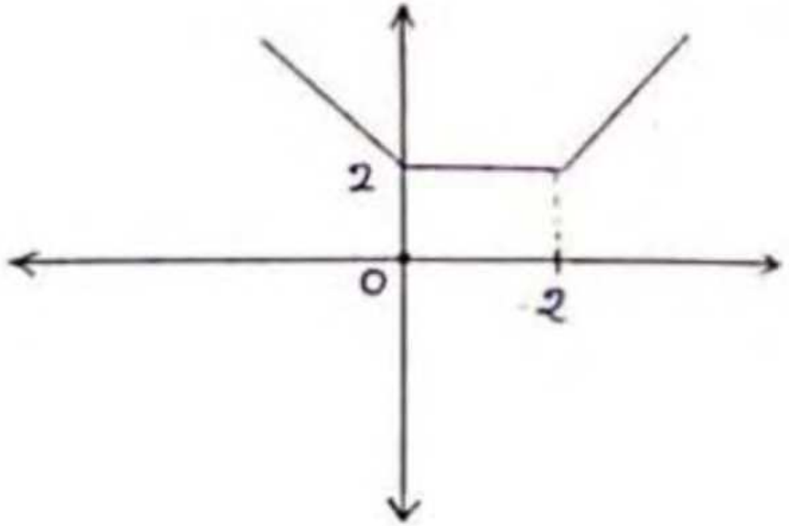
(C) Only two soln.

$\Rightarrow$

	-ve	+ve	+ve
T_1		0	2
T_2	-ve	-ve	+ve

$\Rightarrow$

$$\begin{cases} -x - x + 2 = -2x + 2, & x \leq 0 \\ x - x + 2 = 2, & 0 < x < 2 \\ x + x - 2 = 2x - 2, & x \geq 2 \end{cases}$$



(A) $K \in (-\infty, 2)$.

(B) $K = 2$.

(C) $K \in (2, \infty)$.

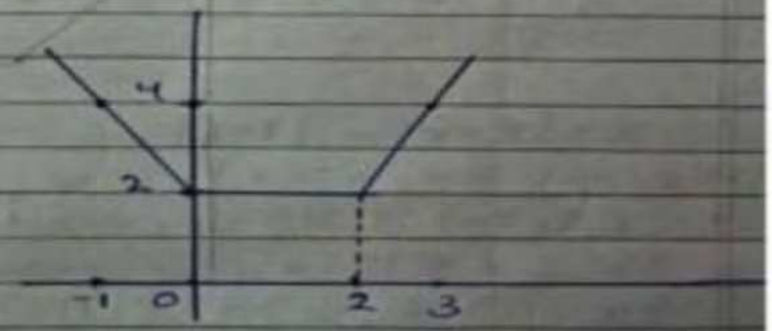
Tan02

$y = \underset{T_1}{|x|} + \underset{T_2}{|x-2|}$

T_1	-ve		tve		tve
T_2	-ve	0	-ve	2	tve

$$y = \begin{cases} -x-x+2 & x \le 0 \\ x-x+2 & 0 < x < 2 \\ x+x-2 & x \ge 2 \end{cases}$$

$$y = \begin{cases} 2-2x & x \le 0 \\ 2 & 0 < x < 2 \\ 2x-2 & x \ge 2 \end{cases}$$



Range of K :-

$|x| + |x-2| = K$

(A) No solution

$K \in (-\infty, 2)$

(B) Infinitely many solution.

i.e. $K = 2$

(C) only one solution.

$K \in \phi$

QUESTION

Draw the graph of $y = |x - 4| - |x + 3|$ and answer the following.

Find the range of k for which the equation $|x - 4| - |x + 3| = k$ has

A. No Solution

B. Infinitely Many Solutions

C. Only two solutions

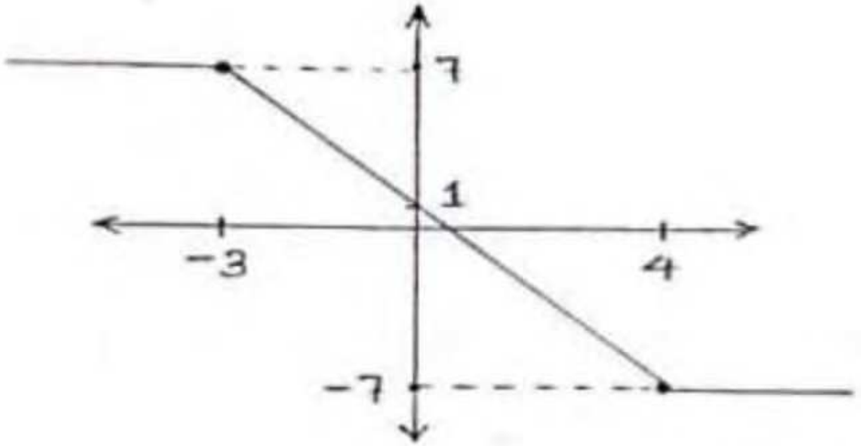
Tah-03.

Draw the graph of $y = |x-4| - |x+3|$ and answer the following. Find the range of K for which the eqⁿ $|x-4| - |x+3| = K$ has:

(A) No solution.

(B) Infinitely Many Soⁿ.

(C) Only two Soⁿ.



T_1: -ve, -ve, +ve (at -3, 4)

T_2: -ve, +ve, +ve

$$\Rightarrow \begin{cases} -x+4+x+3, & x \le -3 \\ -x+4-x-3, & -3 < x < 4 \\ +x-4-x-3, & x \ge 4 \end{cases}$$

$$\Rightarrow \begin{cases} 7, & x \le -3 \\ -2x+1, & -3 < x < 4 \\ -7, & x \ge 4 \end{cases}$$

(A) $K \in (-\infty, -7) \cup (7, \infty)$.

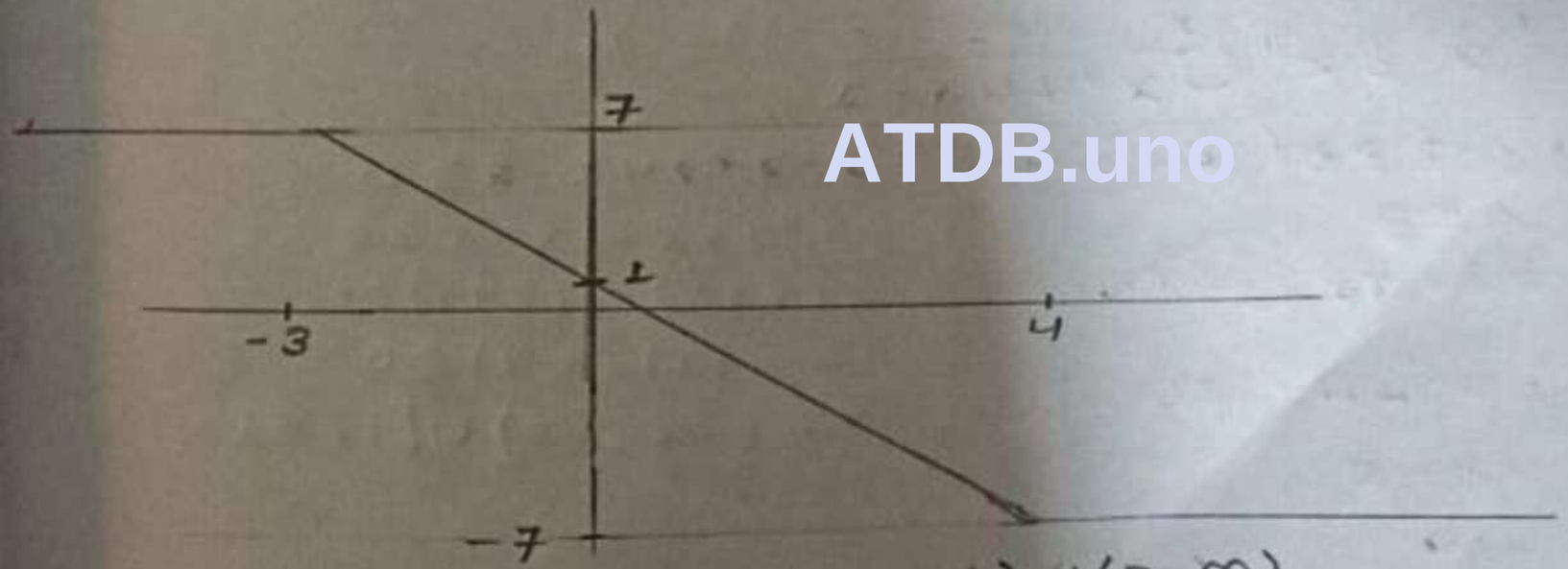
(B) $K \in \{-7, 7\}$.

(C) $K \in \phi$.

TAH03: $Y = |x-4| - |x+3|$

T_1	−ve		−ve		+ve
T_2	−ve	−3	+ve	4	+ve

$$Y = \begin{cases} -x+4+x+3 & x \le -3 \\ -x+4-x+3 & -3 < x < 4 \\ x-4-x-3 & x \ge 4 \end{cases} = \begin{cases} 7 & x \le -3 \\ -2x+1 & -3 < x < 4 \\ -7 & x \ge 4 \end{cases}$$



A. NO SOLUTION → $K \in (-\infty, -7) \cup (7, \infty)$

B. ∞ many soln → $K \in (-7, 7)$

C. only two soln → $K \in (-7, 7)$

QUESTION

Consider the function $f(x) = |x - 1| - 2|x + 2| + |x + 3|$

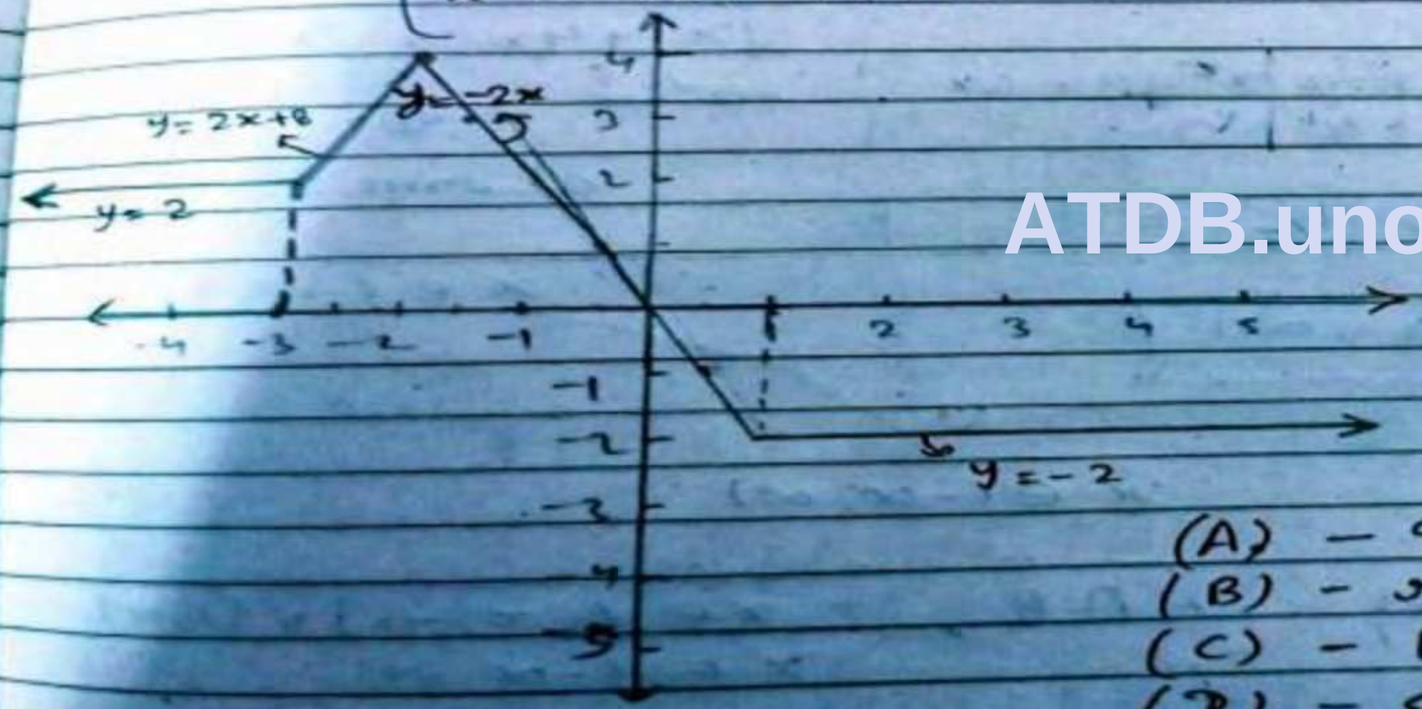
Column-I		Column-II	
(A)	If $f(x) = k$ has no solution, then $k \in$	(p)	$(2, 4)$
(B)	If $f(x) = k$ has one solution, then $k \in$	(q)	$(-\infty, -2) \cup (4, \infty)$
(C)	If $f(x) = k$ has two solution, then $k \in$	(r)	$(-2, 2) \cup \{4\}$
(D)	If $f(x) = k$ has more then two solution, then $k \in$	(s)	$\{-2, 2\}$

[IAH-04] $f(x) = |x-1| - 2|x+2| + |x+3|$

	$x<-3$	$-3 \le x \le -2$	$-2 \le x \le 1$	$x>1$
$x-1$	$-ve$	$-ve$	$-ve$	$+ve$
$-2(x+2)$	$-ve$	$+ve$	$+ve$	$+ve$
$x+3$	$-ve$	$-ve$	$+ve$	$+ve$

$$y = \begin{cases} -x+1+2x+4-x-3 & x<-3 \\ -x+1+2x+4+x+3 & -3 \le x \le -2 \\ -x+1-2x-4+x+3 & -2 \le x \le 1 \\ x-1-2x+4+x+3 & x>1 \end{cases} = \begin{cases} 2 & x<-3 \\ 2x+8 & -3 \le x \le -2 \\ -2x & -2 \le x \le 1 \\ -2 & x>1 \end{cases}$$

$-2x$	$-2 \le x \le 1$
x	-1.5
y	3



(A) – q = $(-\infty,-2) \cup (4,\infty)$

(B) – r = $(-2,2) \cup \{4\}$

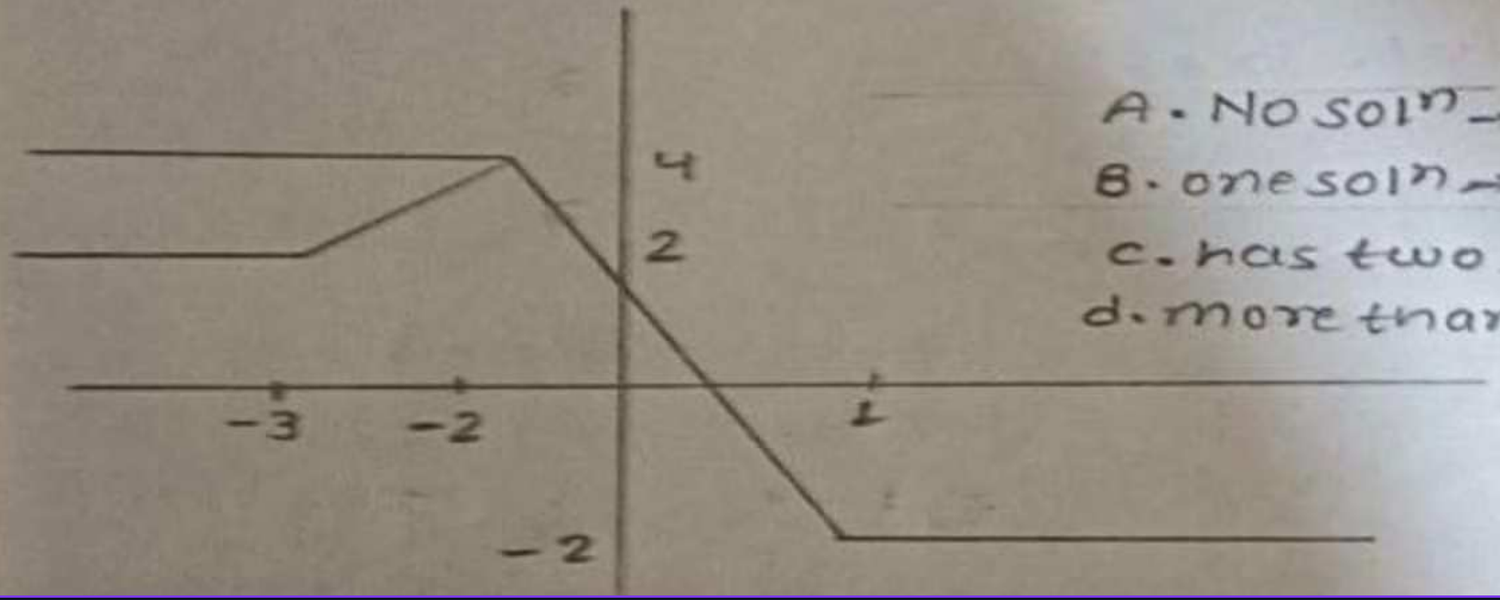
(C) – P = $(2,4)$

(D) – s = $\{-2,2\}$

TAH 04: $f(x) = |x^{T_1}-2| - 2|x^{T_2}+2| + |x+3|$

T_1	-ve	-ve	-ve	+ve	
T_2	-ve	-3 -ve	-2 +ve	⊥ +ve	
T_3	-ve	+ve	+ve	+ve	

$$f = \begin{cases} -x+1+2x+4-x-3 & x \le -3 \\ -x+1+2x+4+x+3 & -3 < x < -2 \\ -x+1-2x-4+x+3 & -2 \le x < 1 \\ x-1-2x-4+x+3 & x \ge 1 \end{cases} = \begin{cases} 2 & x \le -3 \\ 2x+8 & -3 < x < -2 \\ -2x & -2 \le x < 1 \\ -2 & x \ge 1 \end{cases}$$



A. No soln → $k \in (-\infty, -2) \cup (4, \infty)$

B. one soln → $k \in (-2, 2) \cup \{4\}$

c. has two soln → $k \in (2, 4)$

d. more than two soln → $k \in \{-2, 2\}$

Tah-01

Consider the func^n $f(x) = |x-1| - 2|x+2| + |x+3|$.

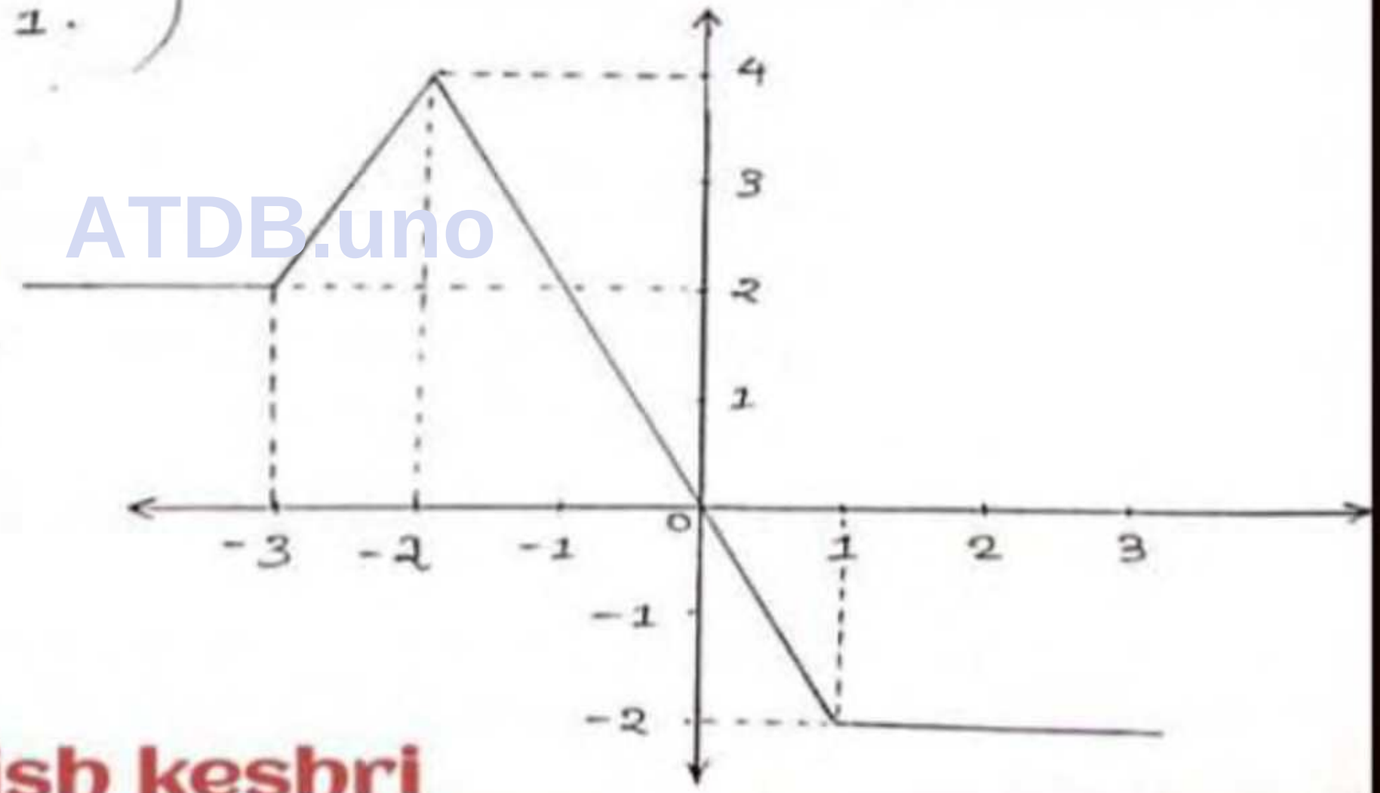
$$\begin{cases} -x+1+2x+4-x-3, & x \le -3 \\ -x+1+2x+4+x+3, & -3 < x \le -2 \\ -x+1-2x-4+x+3, & -2 < x < 1 \\ x-1-2x-4+x+3, & x \ge 1 \end{cases}$$

T_1 : -ve -ve -ve +ve (sign change at -3, -2, 1)

T_2 : -ve -ve +ve +ve

T_3 : -ve +ve +ve +ve

$$\Rightarrow \begin{cases} 2, & x \le -3 \\ 2x+8, & -3 < x \le -2 \\ -2x, & -2 < x < 1 \\ -2, & x \ge 1 \end{cases}$$



(A) $\rightarrow k \in (-\infty, -2) \cup (4, \infty)$.

(B) $\rightarrow k \in (-2, 2) \cup \{4\}$

(C) $\rightarrow k \in (2, 4)$

(D) $\rightarrow k \in \{-2, 2\}$.

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QUESTION

Solution of the inequality, $x - 3 < \sqrt{x^2 + 4x - 5}$

A $(-\infty, -5] \cup [1, \infty)$

B $(-\infty, -5] \cup (7/5, \infty)$

C $(-5,3]$

D $(7/5, \infty)$

Sakshi

Tak-5 $x-3 < \sqrt{x^2+4x-5}$ also $x^2+4x-5 \geq 0$

$(x+5)(x-1) \geq 0$

$x \in (-\infty, -5] \cup [1, \infty)$ — (A)

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(C1) if $x-3 \geq 0$

$x \geq 3$

non-ve ← $x-3 < \sqrt{x^2+4x-5}$ — non-ve

$x^2+9-6x-x^2-4x+5 < 0$

$14-10x < 0$

$x > 7/5$

$\cap$

$x \geq 3$ — union

(C2) $x-3 < 0 \Rightarrow x < 3$

$x-3 < \sqrt{x^2+4x-5}$ → non-ve

-ve ✓ always true

$x < 3$

$x \in (-\infty, \infty)$ — (B)

Final ans : A ∩ B $\Rightarrow (-\infty, -5] \cup [1, \infty)$ Ans (A)

TDh-02

Simplify: $x-3 < \sqrt{x^2+4x-5}$.

Case ① if $x-3 \geq 0 \Rightarrow x \geq 3$.

$\Rightarrow x-3 < \sqrt{x^2+4x-5}$

SBS.

$\Rightarrow x^2-6x+9 < x^2+4x-5$

$\Rightarrow 10x > 14 \Rightarrow x > \frac{14}{10} \Rightarrow x > \frac{7}{5}$

$x \in [3, \infty)$.

$\Rightarrow x^2+4x-5 \geq 0$

$\Rightarrow (x-1)(x+5) \geq 0$

$\Rightarrow x \in (-\infty, -5] \cup [1, \infty)$.

(A)

Case ② if $x-3 < 0 \Rightarrow x < 3$.

$\Rightarrow x-3 < \sqrt{x^2+4x-5}$

Always true $\Rightarrow x \in R$.

$x \in (-\infty, 3)$

$\cup$

$\Rightarrow x \in (-\infty, \infty)$

(B)

$\Rightarrow A \cap B \Rightarrow x \in (-\infty, -5] \cup [1, \infty)$ Ans.

JEE MAIN - 05

$$x - 3 < \sqrt{x^2 + 4x - 5}$$

$$\sqrt{x^2 + 4x - 5} > x - 3$$

$x^2 + 4x - 5 \geq 0$

$(x+5)(x-1) \geq 0$

$x \in (-\infty, -5] \cup [1, \infty)$

Case (I) $x - 3 > 0$

$x > 3$

$\sqrt{x^2 + 4x - 5} > x - 3$

non-ve ✓ | non-ve

non-ve

S.B.S

$x^2 + 4x - 5 > x^2 + 9 - 6x$

$10x - 14 > 0$

$x > 7/5$

$x \in (7/5, \infty)$

Case (II) $x - 3 < 0$

$x < 3$

$\sqrt{x^2 + 4x - 5} > x - 3$

non-ve ✓ | -ve

⇓

always true

$x \in (-\infty, 3)$

∪

$x \in (-\infty, \infty)$

Final Ans : $A \cap B$,

$x \in (-\infty, -5] \cup [1, \infty)$

$x \in (-\infty, \infty)$

$$x \in (-\infty, -5] \cup [1, \infty)$$

A

QUESTION

Solve following inequalities

(i) $\frac{\sqrt{2x-1}}{x-2} < 1$

Ans. $\left[\frac{1}{2}, 2\right) \cup (5, \infty)$

(ii) $x - \sqrt{1-|x|} < 0$

Ans. $[-1, (\sqrt{5}-1)/2)$

(iii) $\sqrt{x^2 - x - 6} < 2x - 3$

Ans. $x \in [3, \infty)$

(iv) $\sqrt{x^2 - 6x + 8} \leq \sqrt{x+1}$

Ans. $x \in \left[\frac{7-\sqrt{21}}{2}, 2\right] \cup \left[4, \frac{7+\sqrt{21}}{2}\right]$

Tah-06

(i) $\frac{\sqrt{2x-1}}{x-2} < 1$

$\Rightarrow 2x-1 \geq 0, \quad x-2 \neq 0$

$\Rightarrow x \geq \frac{1}{2}, \quad x \neq 2$ (A)

Case (1) if $x-2 > 0$

$\Rightarrow \sqrt{2x-1} < x-2$ (SBS)

$\Rightarrow 2x-1 < x^2-4x+4$

$\Rightarrow x^2-6x+5 > 0$

$\Rightarrow (x-5)(x-1) > 0$

$\Rightarrow x \in (-\infty,1) \cup (5,\infty)$

$x \in (5,\infty)$

Case-(2) if $x-2 < 0$

$\Rightarrow \frac{\sqrt{2x-1}}{(x-2)} < 1$

$(x-2)$ → -ve

$\Rightarrow$ any -ve < 1

always true.

$\Rightarrow x \in R$

$x \in (-\infty,2)$

(B)

$\cup \quad x \in (-\infty,2) \cup (5,\infty)$

$\Rightarrow A \cap B$

$\Rightarrow x \in \left[\frac{1}{2},2\right) \cup (5,\infty)$ Ans

TAH (6): solve following inequalities:

(i). $\frac{\sqrt{2x-1}}{x-2} < 1$

$\sqrt{2x-1} > x-2$; $2x-1 \geq 0 \Rightarrow x \geq 1/2$ (A)

case (1) If $x-2 \geq 0$

$x \geq 2$

$2x-1 < (x-2)^2$

$2x-1 < x^2+4-4x$

$x^2-6x+5 > 0$

$x^2-5x-1x+5 > 0$

$x(x-5)-1(x-5) > 0$

$(x-1)(x-5) > 0$

$x \in (-\infty, 1) \cup (5, \infty)$

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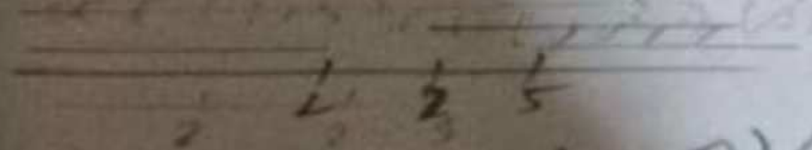


$x \in (5, \infty)$

case (2) If $x-2 < 0$

$x < 2$

$\sqrt{2x-1} > x-2$

↓

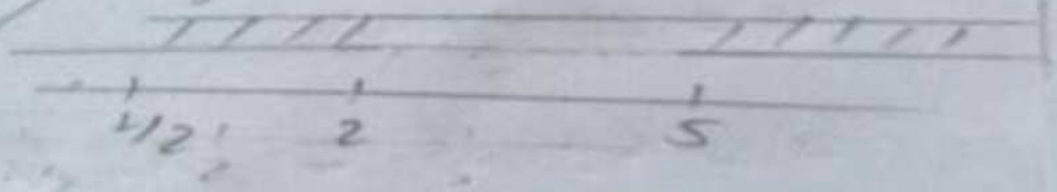
Always true

$x \in (-\infty, 2)$

∪

(B)

$x \in (-\infty, 2) \cup (5, \infty)$



$x \in [1/2, 2) \cup (5, \infty)$

Part (2) (ii) $x - \sqrt{1-|x|} < 0$.

$\Rightarrow x < \sqrt{1-|x|}$

$\Rightarrow 1-|x| \geq 0$

$\Rightarrow |x| \leq 1$

$\Rightarrow x \in [-1, 1]$. (A)

Case (1) if $x \geq 0$.

$\Rightarrow x < \sqrt{1-|x|}$ (SQUARE BOTH SIDES)

$\Rightarrow x^2 < 1-x$

$\Rightarrow x^2 + x - 1 < 0$

$\Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$

$x \in \left[0, \frac{-1+\sqrt{5}}{2}\right)$

Case (2) if $x < 0$.

$\Rightarrow x < \sqrt{1-|x|}$

$\Rightarrow x < \sqrt{1+x}$

↓ −ve ↓ Non −ve

$\Rightarrow$ (Always true)

$x \in R$

$x \in (-\infty, 0)$

$\cup \; x \in \left(-\infty, \frac{\sqrt{5}-1}{2}\right)$ (B)

$\Rightarrow A \cap B$

$\Rightarrow x \in \left[-1, \frac{\sqrt{5}-1}{2}\right)$ Ans.

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QUESTION [JEE Mains 2024 (1 Feb)]

Let $S = \{x \in R: (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$. Then the number of elements in S is :

A 4

B 0

C 2

D 1

Ans. C

JEE-07

$$S = \{x \in R : (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$$

$$(\sqrt{3}+\sqrt{2})(\sqrt{3}-\sqrt{2}) = 3-2 = 1$$

$$\sqrt{3}-\sqrt{2} = \frac{1}{\sqrt{3}+\sqrt{2}}$$

Now,

$$(\sqrt{3}+\sqrt{2})^x + \frac{1}{(\sqrt{3}+\sqrt{2})^x} = 10$$

$$[(\sqrt{3}+\sqrt{2})^x = t]$$

$$t + \frac{1}{t} = 10$$

$$t^2 - 10t + 1 = 0$$

$$t = \frac{10 \pm \sqrt{100-4}}{2} = \frac{10 \pm 4\sqrt{6}}{2} = 5 \pm 2\sqrt{6}$$

$$t = 5 \pm 2\sqrt{6} = (\sqrt{3} \pm \sqrt{2})^2$$

$$(\sqrt{3}+\sqrt{2})^x = (\sqrt{3}+\sqrt{2})^2 \quad \text{or} \quad (\sqrt{3}-\sqrt{2})^2$$

$$[x = 2]$$

$$(\sqrt{3}+\sqrt{2})^x = \left(\frac{1}{\sqrt{3}+\sqrt{2}}\right)^2$$

$$(\sqrt{3}+\sqrt{2})^x = (\sqrt{3}+\sqrt{2})^{-2}$$

$$[x = -2]$$

no. of element in S is 2.

Toh-07. (Thala for a reason).

Let $S = \{x \in R : (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10\}$. Then the no. of element in S is :

$\Rightarrow (\sqrt{3}+\sqrt{2})^x + (\sqrt{3}-\sqrt{2})^x = 10.$

$\Rightarrow t + \frac{1}{t} = 10$

$\Rightarrow t^2 + 1 - 10t = 0$

$\Rightarrow t^2 - 10t + 1 = 0$

$\Rightarrow t = \frac{10 \pm \sqrt{96}}{2} \Rightarrow t = \frac{2(5 \pm 2\sqrt{6})}{2}.$

let : $(\sqrt{3}+\sqrt{2})^x = t.$

And :

$\Rightarrow \frac{1}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

$\Rightarrow \frac{\sqrt{3}-\sqrt{2}}{1}$

So;

$\Rightarrow (\sqrt{3}-\sqrt{2})^x = \frac{1}{t}.$

$(\sqrt{3}+\sqrt{2})^x = 5+2\sqrt{6}$

$(\sqrt{3}+\sqrt{2})^x = (\sqrt{3})^2 + 2\cdot\sqrt{3}\cdot\sqrt{2} + (\sqrt{2})^2$

$(\sqrt{3}+\sqrt{2})^x = (\sqrt{3}+\sqrt{2})^2$

$x = 2$

$(\sqrt{3}+\sqrt{2})^x = 5-2\sqrt{6}.$

$\Rightarrow (\sqrt{3}+\sqrt{2})^x = (\sqrt{3}-\sqrt{2})^x$

$\Rightarrow (\sqrt{3}+\sqrt{2})^x = (\sqrt{3}+\sqrt{2})^{-2}$

$x = -2$

$n(S) = 2$ Ans.

QUESTION [JEE Mains 2023 (31 Jan)]

The number of real roots of the equation $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$, is

A 0

B 1

C 3

D 2

Ans. B

Tah-08

$\sqrt{x^2-4x+3} + \sqrt{x^2-9} = \sqrt{4x^2-14x+6}$ is :

$\Rightarrow \sqrt{(x-3)(x-1)} + \sqrt{(x-3)(x+3)} = \sqrt{(x-3)(4x-2)}$.

$\Rightarrow \sqrt{(x-3)(x-1)} + \sqrt{(x-3)(x+3)} - \sqrt{(x-3)(4x-2)} = 0$.

$\Rightarrow \sqrt{(x-3)} \left\{ \sqrt{(x-1)} + \sqrt{(x+3)} - \sqrt{(4x-2)} \right\} = 0$.

$\Rightarrow$ $=0$ $\quad$ $=0$

$\star$ $\sqrt{x-3} = 0$

$\Rightarrow x-3 = 0$

$\Rightarrow \boxed{x=3}$

$\Rightarrow \sqrt{x-1} + \sqrt{x+3} = \sqrt{4x-2}$. (SB·S)

$\Rightarrow x-1+x+3+2\cdot\sqrt{x-1}\cdot\sqrt{x+3} = 4x-2$.

$\Rightarrow 2x+2+2\sqrt{x^2+2x-3} = 4x-2$.

$\Rightarrow 2\sqrt{x^2+2x-3} = 2x-4$ 2

$\Rightarrow \sqrt{x^2+2x-3} = x-2$. (SBS)

$\Rightarrow x^2+2x-3 = x^2-4x+4$

$\Rightarrow 6x = 7 \Rightarrow \boxed{x=\frac{7}{6}}$ X

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$\Rightarrow$ no of roots = 1 Ans.

because does'l satisfy $\sqrt{x^2-9}$.

Q-3: The number of real roots of the equation:

$\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$ is:

TAH 08
BY REED
FROM WB

(a) 0. (b) 1 (c) 3 (d) 2.

Soln: $\sqrt{x^2-4x+3}+\sqrt{x^2-9}=\sqrt{4x^2-14x+6}$

or, $\sqrt{(x-1)(x-3)}+\sqrt{(x+3)(x-3)}=\sqrt{4x^2-12x-2x+6}$

or $\sqrt{(x-1)(x-3)}+\sqrt{(x+3)(x-3)}=\sqrt{(4x-2)(x-3)}$ —— (A)

or, $\sqrt{x-3}\left[\sqrt{x-1}+\sqrt{x+3}\right]=\sqrt{4x-2}\cdot\sqrt{x-3}$.

or, $\sqrt{x-3}\left[\sqrt{x-1}+\sqrt{x+3}\right]-\sqrt{x-3}\sqrt{4x-2}=0$.

or, $\sqrt{x-3}\left[\sqrt{x-1}+\sqrt{x+3}-\sqrt{4x-2}\right]=0$.

$\therefore \sqrt{x-3}=0$ or, $\sqrt{x-1}+\sqrt{x+3}-\sqrt{4x-2}=0$

or, $x-3=0$

or, $\boxed{x=3}$

(valid) ∵ no problem in (A).

$\therefore \boxed{x=3}$ → only soln.

∴ Ans ⇒ No. of real roots of the eqn is = (b) 1.

or, $\sqrt{x-1}+\sqrt{x+3}=\sqrt{4x-2}$

S.B.S.

or, $x-1+x+3+2\sqrt{x-1}\cdot\sqrt{x+3}=4x-2$

or, $2x+2+2\sqrt{x^2+2x-3}=4x-2$

or, $2\sqrt{x^2+2x-3}=2x-4=2(x-2)$

or, $\sqrt{x^2+2x-3}=x-2$

S.B.S.

or, $x^2+2x-3=x^2-4x+4$

or, $6x=7$

or, $\boxed{x=\frac{7}{6}}$

Discarded

{ $\frac{7}{6}$ → <3 }

bcoz $x^2-9 \Rightarrow$ -ve @ $x=\frac{7}{6}$

QUESTION

Solve: $\sqrt{x^2 - 6x + 9} + \sqrt{x^2 + 8x + 16} = 7$

[JAH-09]

$$\sqrt{x^2-6x+9} + \sqrt{x^2+8x+16} = 7$$

$$\sqrt{(x-3)^2} + \sqrt{(x+4)^2} = 7$$

$$|x-3| + |x+4| = 7$$

$|x-3| \to a$, $|x+4| \to b$

$$|a| + |b| = |a-b| \iff ab \leq 0$$

Now

$$(x-3)(x+4) \leq 0$$

$$x \in [-4, 3]$$

Tah-09

Solve: $\sqrt{x^2-6x+9} + \sqrt{x^2+8x+16} = 7$.

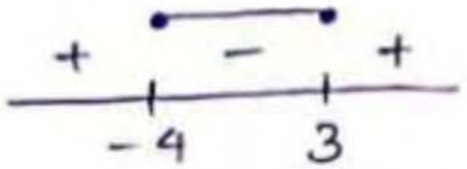
$\Rightarrow \sqrt{(x-3)(x-3)} + \sqrt{(x+4)(x+4)} = 7$

$\Rightarrow \sqrt{(x-3)^2} + \sqrt{(x+4)^2} = 7$

$\Rightarrow |x-3| + |x+4| = 7 \longrightarrow |x-3-x+4| = |-7| = 7$.

$\Rightarrow |a| + |b| = |a-b|$.

$\Rightarrow (x-3)(x+4) \leq 0$

$\Rightarrow$ 

$\Rightarrow x \in [-4, 3]$ Ans.

krish keshri
jharkhand

QUESTION [JEE Mains 2019 (8 April)]

The sum of the solutions of the equation $|\sqrt{x}-2|+\sqrt{x}(\sqrt{x}-4)+2=0\ (x>0)$ is equal to:

(A) 9

(B) 12

(C) 4

(D) 10

Ans. D

Tab-10.

Sum of the soln of the eqn :

$|\sqrt{x}-2| + \sqrt{x}(\sqrt{x}-4) + 2 = 0 \quad (x>0)$ is equal to :

$\Rightarrow$ Case-(1) $\sqrt{x}-2 \geq 0$

$\Rightarrow \sqrt{x} \geq 2 \Rightarrow x \geq 4.$

$\Rightarrow \sqrt{x} - 2 + x - 4\sqrt{x} + 2 = 0$

$\Rightarrow x - 3\sqrt{x} = 0$

$\Rightarrow \sqrt{x}(\sqrt{x}-3) = 0$

$\Rightarrow \sqrt{x} = 0 \quad | \quad \sqrt{x} - 3 = 0$

$x = 0 \quad | \quad x = 9.$

$\times \quad | \quad \checkmark$

$\Rightarrow x = 1, 9$

$\Rightarrow$ Sum = 10 Ans.

$\Rightarrow$ Case-(2) $\sqrt{x} - 2 < 0.$

$\Rightarrow x < 4.$

$\Rightarrow -\sqrt{x} + 2 + x - 4\sqrt{x} + 2 = 0$

$\Rightarrow x - 5\sqrt{x} + 4 = 0.$

$\Rightarrow (\sqrt{x}-1)(\sqrt{x}-4) = 0$

$\Rightarrow \sqrt{x} = 1 \quad | \quad \sqrt{x} = 4$

$x = 1 \quad | \quad x = 16$

$\checkmark \quad | \quad \times$

Q-8! Sum of solns of $|\sqrt{x}-2| + \sqrt{x}(\sqrt{x}-4) + 2 = 0$. $(x>0)$

Soln: $|\sqrt{x}-2| + \sqrt{x}(\sqrt{x}-4) + 2 = 0$.

TAH 10
BY REED
FROM WB

Case-1: $(\sqrt{x}-2) \geq 0 \Rightarrow x \geq 4$.

$\therefore \sqrt{x} - 2 + x - 4\sqrt{x} + 2 = 0$

$\Rightarrow x - 3\sqrt{x} = 0$

$\Rightarrow \sqrt{x}(\sqrt{x}-3) = 0$

$\Rightarrow x = 0$ (Discarded) | $\sqrt{x} = 3 \Rightarrow x = 9$

$x = 9$

Case-2: $\sqrt{x} - 2 < 0 \Rightarrow x < 4$

$\therefore 2 - \sqrt{x} + x - 4\sqrt{x} + 2 = 0$

$\Rightarrow x - 5\sqrt{x} + 4 = 0$

$\Rightarrow x - 4\sqrt{x} - \sqrt{x} + 4 = 0$

$\Rightarrow (\sqrt{x}-1)(\sqrt{x}-4) = 0$

$\Downarrow$

$x = 1$ OR $x = 16$

accepted ✓ Rejected ✗

$\therefore x = 1$

$\therefore$

$\therefore x = 1, 9$ $\therefore$ Sum of roots $= 1 + 9 = 10$ (Ans = (D))

Solution to Previous BPPs

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Bumper Practice Problems

1. $\sqrt[3]{\frac{x-2}{x-1}} < \sqrt[3]{\frac{1}{x-1}}$

2. $\sqrt{3x-2} < \sqrt{x+4}$

3. $\sqrt{4x-3} < \sqrt{2x+5}$

4. $\sqrt[3]{(3x-5)} < (x-1)$

5. $\sqrt[3]{3x-2} < x$

6. $\sqrt{(x+14)} < (x+2)$

7. $\sqrt{2x-2} < x-1$

8. $\sqrt{-x^2+4x-3} > (6-2x)$

9. $\sqrt{x-2} + \sqrt{x-1} > 2$

10. $7\sqrt{x} + 8\sqrt{-x} + \frac{15}{x^3} = 98$

11. $\sqrt{2x-4} - \sqrt{x+5} = 1$

12. $\sqrt{x-1} + \sqrt{2x+6} = 6$

Answers

1. $x \in (1, 3)$

2. $x \in \left[\frac{2}{3}, 6\right)$

3. $x \in \left[\frac{3}{4}, 1\right)$

4. $x \in (-1, \infty) - \{2\}$

5. $x \in (-2, 1) \cup (1, \infty)$

6. $x \in (2, \infty)$

7. $x \in (3, \infty)$

8. $x \in \left(\frac{13}{5}, 3\right)$

9. $x \in \left(\frac{41}{16}, \infty\right)$

BPP

1. $\sqrt[3]{\frac{x-2}{x-1}} < \sqrt[3]{\frac{1}{x-1}}$

$\Rightarrow$ on cubing both sides $\rightarrow$

$\Rightarrow \left(\frac{x-2}{x-1}\right)^{\frac{1}{3}\times 3} < \left(\frac{1}{x-1}\right)^{\frac{1}{3}\times 3}$

$\Rightarrow \frac{x-2}{(x-1)} < \frac{1}{(x-1)}$

$\Rightarrow \frac{x-2}{x-1} - \frac{1}{(x-1)} < 0$

$\Rightarrow \frac{(x-2)-(1)}{(x-1)^2} < 0$

$\Rightarrow \frac{x-3}{x-1} < 0$

$\Rightarrow x \in (1,3)$

2. $\sqrt{3x-2} < \sqrt{x+4}$

on Sq. both sides,

$3x-2 \geq 0$

$x \geq \frac{2}{3}$

$x+4 \geq 0$

$x \geq -4$

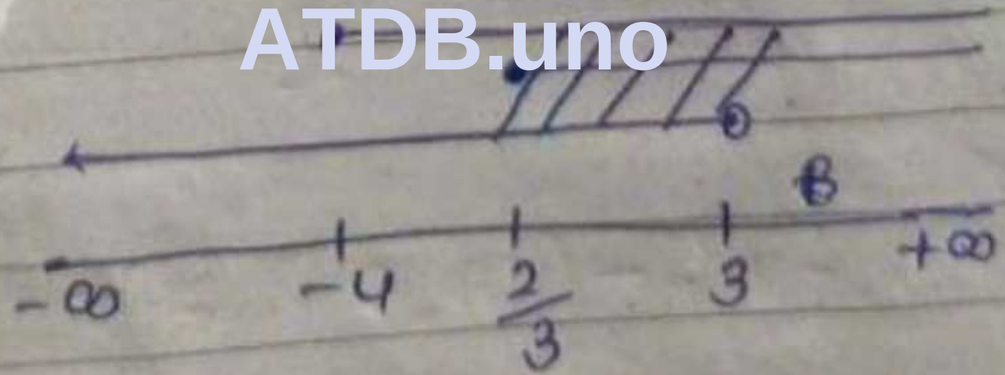
$3x-2 < x+4$

$2x-2-4 < 0$

$x < 3$

$2x-4 < 0$

$x < 2$



$x \in \left[\frac{2}{3}, 3\right)$

3. $\sqrt{4x-3} < \sqrt{2x+5}$

$\sqrt{4x-3} < \sqrt{2x+5}$

SBS

$4x-3 < 2x+5$

$2x < 8$

$x < 4$

$4x-3 \geq 0$

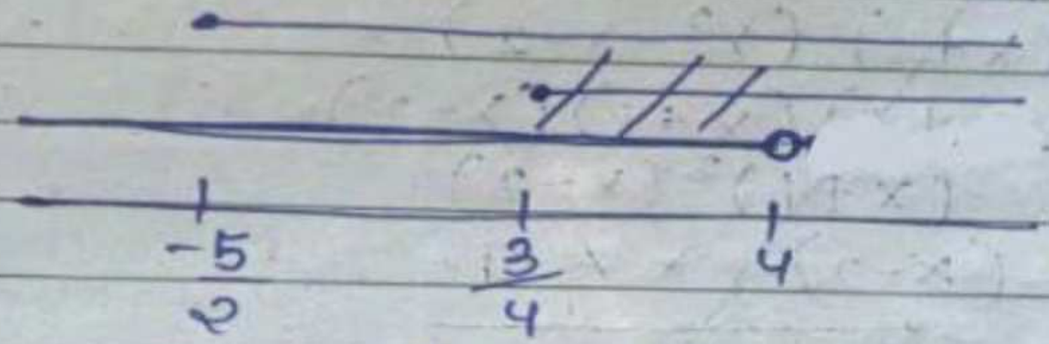
$4x \geq 3$

$x \geq 3/4$

$2x+5 \geq 0$

$2x \geq -5$

$x \geq -5/2$



$x \in \left[\frac{3}{4}, 4\right)$

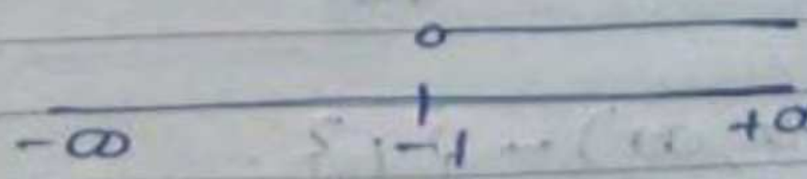
4.

$\sqrt[3]{3x-5} < (x-1)$ ⇒

cubing both sides.

$$3x-5 < (x-1)^3$$
$$3x-5 < x^3 - 1^3 - 3x(x-1)$$
$$0 < x^3 - 3x^2 + 3x - 1 - 3x + 5$$
$$0 < x^3 - 3x^2 + 4$$
$$0 < x^2(x+1) - 4x(x+1) + 4(x+1)$$
$$0 < (x+1)(x^2 - 4x + 4)$$
$$0 < (x+1)(x-2)^2$$

$$(x+1)(x-2)^2 > 0$$
$$(x+1) > 0, \quad x \neq 2$$



$$x \in (-1, \infty) - \{2\}$$

5:

$\sqrt[3]{3x-2} < x$

cubing →

$3x-2 < x^3$

$0 < x^3 - 3x + 2$

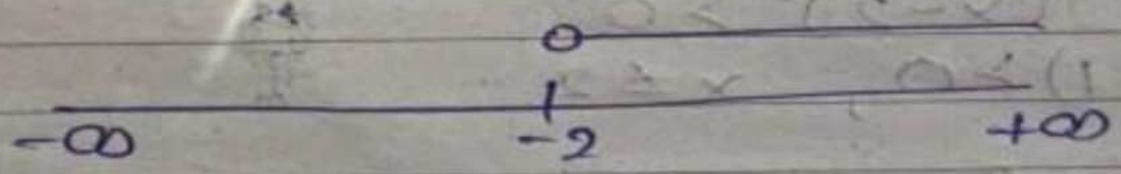
$x^3 - 3x + 2 > 0$

$x^2(x-1) + x(x-1) - 2(x-1) > 0$

$(x-1)(x^2+x-2) > 0$

$(x-1)(x-1)(x+2) > 0$

$(x+2) > 0, \quad x \neq 1$



$x \in (-2, \infty) - \{1\}$

BPP - 6

$\sqrt{(x+14)} < (x+2)$

Case (I)

$x+1 \geq 0$

$x+2 \geq 0 \Rightarrow x \geq -2$

By S.B.S, $x+14 < x^2+4x+4$

$x^2+3x-10 > 0$.

$(x+5)(x-2) > 0$.

$\therefore x \in (2, \infty)$

Case (II)

$x+2 < 0$.

$x < -1$

Not possible

On taking union of case (I) & (II).

$x \in (2, \infty)$ — (A)

$x+14 \geq 0$.

$\Rightarrow x \geq -14$

$x \in [-14, \infty)$ — (B).

$\therefore A \cap B \Rightarrow x \in (2, \infty)$

6) $\sqrt{(x+14)} < (x+2)$

Case-01

$x+2 \geq 0 \longrightarrow x \geq -2$

$\sqrt{(x+14)} < (x+2)$ ∵ always +ve

always +ve.

SBS

$\therefore x+14 < x^2+4+4x$

$x^2+3x-10 > 0$

$(x+5)(x-2) > 0$

$x \in (-\infty, -5) \cup (2, \infty)$

$\therefore x \in (2, \infty)$

Case-02

$x+2 < 0$

$x < -2$

$\therefore \sqrt{(x+14)} < (x+2)$

+ve ↓ -ve

Not possible

$\therefore x \in \phi$

$x+14 \geq 0$

$\therefore x \geq -14$

$\therefore x \in (2, \infty)$

7. $\sqrt{2x-2} < x-1$

Case - (1)

$x-1 \geq 0$

$x \geq 1 \Rightarrow x \in [1, \infty)$

SBS

$2x-2 < x^2-2x+1$

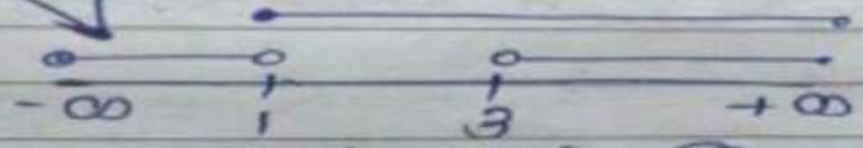
$0 < x^2-4x+3$

$0 < x^2-3x-x+3$

$0 < x(x-3)-1(x-3)$

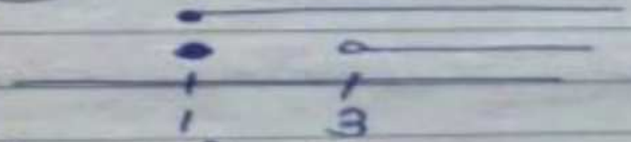
$(x-3)(x-1) > 0$

$x \in (-\infty, 1) \cup (3, \infty)$



$x \in (3, \infty)$ (B)

$A \cap B :-$



$x \in (3, \infty)$

$2x-2 \geq 0$

$2x \geq 2$

$x \geq 1, x \in [1, \infty)$ (A)

Case - (2)

$x-1 < 0$

$x < 1$

$\sqrt{2x-2} < x-1$

↳ Not possible

$x \in \phi$

7) $\sqrt{2x-2} < x-1$

case-01

$x+1 \geq 0 \Rightarrow x \geq -1$

$\sqrt{2x-2} < x-1$

+ve +ve

$\therefore 2x-2 < x^2+1-2x$

$x^2-4x+3 > 0$

$(x+3)(x-1) > 0$

$\therefore x \in (-\infty,1) \cup (3,\infty)$

$\therefore x \in (3,\infty)$

case-02

$x-1 < 0$

$x < 1$

$\sqrt{2x-2} < x-1$

$x \in \phi$

$2x-2 \geq 0 \Rightarrow x \geq 1$

$x \in (3,\infty)$

Case-01

$6-2x \geq 0$

$-2x \leq 6$

$\boxed{x \leq 3}$

Case-02

$6-2x < 0$

$2x > 6$

$x > 3$

$\boxed{x^2+4x-3 > 6-2x}$

+ve — −ve

always True

$\therefore x \in (3, \infty)$

SBS

$-x^2+4x-3 > 36+4x^2-24x$

$5x^2-28x+39 < 0$

$5x^2-15x-13x+39 < 0$

$5x(x-3)-13(x-3) < 0$

$(5x-13)(x-3) < 0$

$\therefore x \in (13/5, 3)$

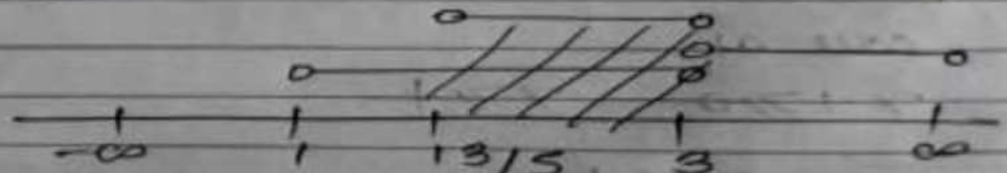
$\therefore x \in (13/5, 3) \cup (3, \infty)$

$-x^2+4x-3 \geq 0$

$x^2-4x+3 \leq 0$

$(x-3)(x-1) \leq 0$

$\therefore x \in [1, 3]$



$\therefore x \in (13/5, 3)$

BPP - 11

$$\sqrt{8x-4} - \sqrt{x+5} = 1$$

S.B.S

$$2x - 4 + x + 5 - 2\sqrt{2(x-2)(x+5)} = 1$$

$$\Rightarrow \quad 3x + 1 - 1 = 2\sqrt{2(x-2)(x+5)}$$

$$\Rightarrow \quad 3x = 2\sqrt{2(x-2)(x+5)}$$

S.B.S

$$\Rightarrow \quad (3x)^2 = 8(x^2 + 3x - 10)$$

$$\Rightarrow \quad 9x^2 - 8x^2 - 24x + 80 = 0$$

$$\Rightarrow \quad x^2 - 24x + 80 = 0.$$

$$\Rightarrow \quad (x-20)(x-4) = 0$$

$$\therefore \quad x \in \{4, 20\}. \quad \text{— (A)}$$

$\therefore A \cap B \Rightarrow x \in \{4, 20\}$.

$2x - 4 \geq 0$ & $x + 5$

$x \geq 2$ $\quad x \geq -5$ — (B)

8×10

11) $\sqrt{2x-4} - \sqrt{x+5} = 1$

$2x-4 \geq 0$ & $x+5 \geq 0$

$x \geq 2$ $x \geq -5$

$\therefore x \in [2, \infty)$

SBS

$2x-4 + (x+5) - 2\sqrt{2x-4}\cdot\sqrt{x+5} = 1$

$3x+1 - 2\sqrt{2x-4}\cdot\sqrt{x+5} = 1$

$3x = 2\sqrt{2x-4}\cdot\sqrt{x+5}$

SBS

$9x^2 = 4[(2x-4)(x+5)]$

$9x^2 = 4[2x^2+6x-20]$

$9x^2 = 8x^2+24x-80$

$x^2 - 24x + 80 = 0$

$(x-20)(x-4) = 0$

$x = 4, 20$

$\therefore$ Ans $\rightarrow$ $x = 4, 20$

BPP - 12

$$\sqrt{x-1} + \sqrt{2x+6} = 6.$$

So B.S,

$$x - 1 + 2x + 6 + 2\sqrt{2(x-1)(x+3)} = 36.$$

$$\Rightarrow 3x + 5 + 2\sqrt{2(x-1)(x+3)} = 36$$

$$\Rightarrow 2\sqrt{2(x-1)(x+3)} = 31 - 3x.$$

Again S. B. S,

$$8(x-1)(x+3) = 961 - 186x + 9x^2$$

$$\Rightarrow 8x^2 + 16x - 24 = 961 - 186x + 9x^2$$

$$\Rightarrow x^2 - 202x + 1185 = 0$$

$$\Rightarrow (x-197)(x-5) = 0$$

$$x = 5, 197$$

$$\therefore x \in \{5, 197\} \quad \text{— (A)}$$

$x - 1 \geq 0$ & $2x + 6 \geq 0$

$x \geq 1$ $\quad x + 3 \geq 0$

$x \geq -3$. — (B).

$\therefore A \cap B \Rightarrow x \in \{5, 197\}$

12) $\sqrt{x-1} + \sqrt{2x+6} = 6$

SBS

$x-1 \geq 0$ $\quad 2x+6 \geq 0$

$x \geq 1$ $\quad x \geq -3$

$\therefore x \in [1, \infty)$

$(x-1) + (2x+6) + 2\sqrt{x-1}\cdot\sqrt{2x+6} = 36$

$3x + 5 + 2\sqrt{x-1}\cdot\sqrt{2x+6} = 36$

$2\sqrt{x-1}\cdot\sqrt{2x+6} = 31 - 3x$

SBS

$\therefore 4(x-1)(2x+6) = 961 + 9x^2 - 186x$

$4(2x^2 + 4x - 6) = 11$

$8x^2 + 16x - 24 = 961 + 9x^2 - 186x$

$x^2 - 202x + 985 = 0$

$x^2 - 197x - 5x + 985 = 0$

$x(x-197) - 5(x-197) = 0$

$\therefore (x-5)(x-197) = 0.$

$\therefore x = 197, 5$

Home Challenge-08

Let $f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)}$. If $\prod_{r=-10}^{10} f(\ln 2^r) = e^{\lambda}$, then find the value of $(2\lambda - 15)$. [Ans. 6]

H.C. – 8: Let $f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)}$. If $\prod_{r=-10}^{10} f(\ln 2^r) = e^{\lambda}$, then find the value of $(2\lambda - 15)$.

Sol:

$$f(x) = e^{\left(\frac{2^{-x}}{1+2^{-x}}\right)} = e^{\left(\frac{1/2^x}{1+1/2^x}\right)} = e^{\left(\frac{1}{2^x+1}\right)}$$

$$f(\ln 2^r) = f(r\ln 2) = e^{\frac{1}{2^{r\ln 2}+1}}$$

$$E = \prod_{r=-10}^{10} f(r\ln 2) = e^{\frac{1}{1+2^{-10\ln 2}}} \times e^{\frac{1}{1+2^{-9\ln 2}}} \times \ldots \times e^{\frac{1}{1+2^{0\ln 2}}} \times \ldots$$

$$\ldots e^{\frac{1}{1+2^{10\ln 2}}}$$

$$\Rightarrow E = e^{\frac{1}{1+2^{-10\ln 2}} + \frac{1}{1+2^{-9\ln 2}} + \ldots + \frac{1}{1+2^{0\ln 2}} + \ldots + \frac{1}{1+2^{10\ln 2}}}$$

Observe: $\frac{1}{1+2^{-r\ln 2}} + \frac{1}{1+2^{r\ln 2}}$ ($2^{-r\ln 2} = 1/t$, $2^{r\ln 2} = t$)

$$= \frac{1}{1+\frac{1}{t}} + \frac{1}{1+t} = \frac{t}{1+t} + \frac{1}{1+t} = 1$$

$$E = e^{(1+1+\ldots \text{ 10 times}) + \frac{1}{1+2^{0\ln 2}}}$$

$$\Rightarrow E = e^{10 + \frac{1}{1+1}}$$

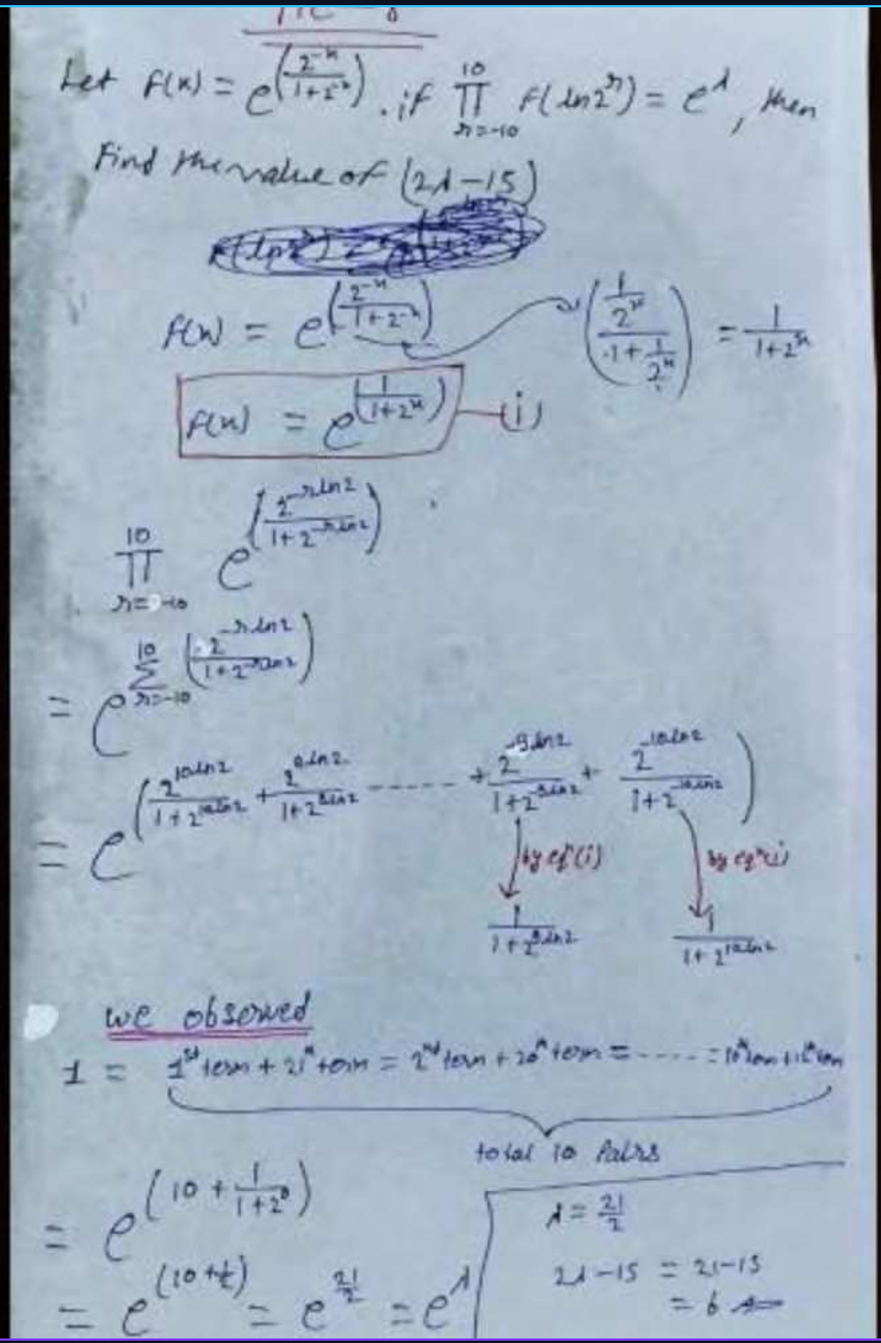
$$\Rightarrow E = e^{\left(10 + \frac{1}{2}\right)}$$

$$\Rightarrow E = e^{\frac{21}{2}} = e^{\lambda} \Rightarrow \lambda = \frac{21}{2}$$

$$\therefore 2\lambda - 15 = 2 \times \frac{21}{2} - 15 = 21 - 15 = 6$$

Ans.

HC 8
BY REED
FROM WB



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