PRAYAS

JEE 2026

Mathematics

Quadratic Equations

Lecture - 03

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Topics *to be covered*

- **A** General polynomial equation
- **B** Newton's formula
- **C** Practice problems

Recap *of previous lecture*

1. Factors of $ax^2 + bx + c \begin{cases}\alpha \\ \beta\end{cases}$ is $a(x-\alpha)(x-\beta)$

2. $ax^2 + bx + c$ is a perfect square if $D=0$

3. $ax^2 + bx + c$ is square of a real linear expression if $D=0$ & $a>0$

4. $(\alpha - \beta)^2 = \frac{D}{a^2}$, where α, β are roots of $ax^2 + bx + c = 0$.

5. If a, b, c $\in$ Q & $D \geq 0$, also 'D' is a perfect square then roots of $ax^2 + bx + c = 0$ are Rational

Recap *of previous lecture*

6. If a quadratic p(x) takes the value 0 at $\underbrace{x = 2 \text{ only}}$ & P(3) = 4 then the value of p(4) is ___16___

$P(x) = a(x-2)^2$ also $P(3) = 4 \Rightarrow 4 = a(3-2)^2$

$a = 4$

$P(x) = 4(x-2)^2$

$\Rightarrow P(4) = 4 \cdot 4 = 16$

7. $\alpha^5 - \beta^5 =$ $\underline{(\alpha^2-\beta^2)(\alpha^3+\beta^3)}$ $+ \alpha^2\beta^2(\beta-\alpha)$

8. If coefficient of a quadratic are all odd integers, then roots of the quadratic cannot be ___Rational___

9. If product of roots of a quadratic ^over coefficients^ is negative, then roots of the quadratic cannot be ___imaginary___

Recap of previous lecture

11. If $a, b, c \in R, D < 0$ then roots of $ax^2 + bx + c = 0$ are Imaginary hence if one root is $i - 3$ then other root $-i-3$

12. If $a + b + c = 0$ then roots of $ax^2 + bx + c = 0$ then roots are $1, \frac{c}{a}$

13. If $a - b + c = 0$ then roots of $ax^2 + bx + c = 0$ then roots are $-1, -c/a$

14. If $a = 1, b, c \in I$ & D is a perfect square. ⇒ roots are Integers.

15. If α, β are roots of $x^2 - x + 7 = 0$ then equation of with roots $2\alpha - 1$ & $2\beta - 1$ is $x^2+27=0$

$S=(2\alpha-1)+2\beta-1=2(\alpha+\beta)-2=0$

$P=(2\alpha-1)(2\beta-1)=4\alpha\beta-2(\alpha+\beta)+1=27$

Homework Discussion

QUESTION [JEE Mains 2023 (25 Jan)]

★★KCLS★★

Let $\alpha \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is

$$(\alpha^2+\beta^2)^2 - 2\alpha^2\beta^2 = -30$$

$$\left((\alpha+\beta)^2 - 2\alpha\beta\right)^2 - 2(\alpha\beta)^2 = -30$$

Ans. 45

Aao Machaay Dhamaal
Deh Swaal pe Deh Swaal

QUESTION

Let λ_1 and λ_2 be two values of λ for which the expression $x^2 + (2-\lambda)x + \lambda - \frac{3}{4}$ becomes a perfect square. The value of $(\lambda_1^2 + \lambda_2^2)$ equals

A 8

B 25

[✓ C] 50

D 100

$$D = 0$$

$$(2-\lambda)^2 - 4\left(\lambda - \frac{3}{4}\right) = 0$$

$$4 + \lambda^2 - 4\lambda - 4\lambda + 3 = 0$$

$$\lambda^2 - 8\lambda + 7 = 0 \begin{cases} \lambda_1 \\ \lambda_2 \end{cases}$$

$$\lambda_1^2 + \lambda_2^2 = (\lambda_1 + \lambda_2)^2 - 2\lambda_1\lambda_2$$

$$= 8^2 - 14 = 50 \ \underline{\text{Ans}}$$

QUESTION [AIEEE 2011]

★★KCLS★★

Let for $a \neq a_1 \neq 0$, $f(x) = ax^2 + bx + c$, $g(x) = a_1x^2 + b_1x + c_1$ and $p(x) = f(x) - g(x)$. If $p(x) = 0$ only for $x = -1$ and $p(-2) = 2$, then the value of $p(2)$ is:

[✓ A] 18

B 3

C 9

D 6

$P(x) = f(x) - g(x) =$ quadratic

↳ zere at only $x = -1$

$P(x) = A(x+1)^2$

$P(-2) = A(-1)^2 = A = 2$

$A = 2$

$P(x) = 2(x+1)^2$

$P(2) = 2(3)^2 = 18.$

QUESTION

Tah01

Let p & q be the two roots of the equation, $mx^2 + x(2-m) + 3 = 0$. Let m_1, m_2 be the two values of m satisfying $\frac{p}{q} + \frac{q}{p} = \frac{2}{3}$. Determine the numerical value of $\frac{m_1}{m_2^2} + \frac{m_2}{m_1^2}$.

$$mx^2 + x(2-m) + 3 = 0 \begin{matrix} \nearrow p \\ \searrow q \end{matrix}$$

$$S.O.R = p+q = \frac{m-2}{m}$$

$$P.O.R = pq = \frac{3}{m}$$

$$\frac{p}{q} + \frac{q}{p} = \frac{2}{3}$$

$$\frac{p^2+q^2}{pq} = \frac{2}{3}$$

$$\frac{(p+q)^2 - 2pq}{pq} = \frac{2}{3}$$

QUESTION

Kallu and Lallu solve a quadratic equation. Kallu reads its constant term wrongly and finds its roots as 8 and 2 where as Lallu reads the coefficients of x wrongly and finds its roots as –11, 1. The correct root of the equation are

A 11, 1

B –11, 1

C 11, –1

D None of these

$ax^2+bx+c=0 \to x^2+\frac{b}{a}x+\frac{c}{a}=0$

Kallu : reads wrong value of c.
- P.O.R = Galat aayaygaa
- S.O.R = $-\frac{b}{a}$ = does not depend on c

⇓

S.O.R will be correct.

⇓

$\text{S.O.R}|_{\text{correct}} = 10$

↓

roots = 8, 2

Lallu reads wrong value of b
- S.O.R = Galat aayaygaa
- P.O.R = $\frac{c}{a}$ = Indp of b

↓

roots = –11, 1

⇓

P.O.R will be correct

$\text{POR}|_{\text{correct}} = -11$

Eqn: $x^2-10x-11=0$

$(x-11)(x+1)=0 \to x=-1, 11.$

QUESTION

★★KCLS★★

Given that the quadratic equation $ax^2 + bx + c = 0$ has no real roots, but Mr. X got two roots 2 and 4 since he wrote down a wrong value of 'a'. Mr. Y also got two roots –1 and 4 because he wrote the sign of a term wrongly. Then the value of $\frac{2b+3c}{a}$ is equal to

$ax^2+bx+c=0$ has no real roots $\Rightarrow D = b^2 - 4ac < 0 \Rightarrow 0 \le b^2 < 4ac \Rightarrow 4ac = +ve$

$\Downarrow$

a, c are of same sign

Mr X: Reads wrong value of a

$a'x^2 + bx + c = 0 \langle^{2}_{4}$

$S\cdot O\cdot R = 6 = -\frac{b}{a'}$

$P\cdot O\cdot R = 8 = \frac{c}{a'}$

$\Rightarrow -\frac{b}{c} = \frac{3}{4}$

$4b = -3c$.

b, c are of opp sign.

Mr Y: wrote sign of a term wrongly

$\downarrow$

roots $-1, 4$

$\boxed{b = -3a}$ $-\frac{b}{a} = 3$ $S\cdot O\cdot R = 3$ ← a, b sahi sign hai

$P\cdot O\cdot R = -4$ → wrong sign of c.

$\frac{(-c)}{a} = -4$

$\boxed{c = 4a}$

$P\cdot O\cdot R = \frac{c}{a} = +ve$. ← a, c have same sign

b, c have opp sign

$S\cdot O\cdot R = -\frac{b}{a} = +ve$ ← a, b have opp sign.

$$\frac{2b+3c}{a} = \frac{-6a+12a}{a} = 6. \underline{\text{Ans}}$$

QUESTION [JEE Advanced 2020]

★★ASRQ★★

Suppose a, b denote the distinct real roots of the quadratic polynomial $x^2 + 20x - 2020$ and suppose c, d denote the distinct complex roots of the quadratic polynomial $x^2 - 20x + 2020$. Then the value of $ac(a - c) + ad(a - d) + bc(b - c) + bd(b - d)$ is

A 0

B 8000

C 8080

D 16000

$x^2+20x-2020 \begin{cases} a \\ b \end{cases}$

$x^2-20x+2020 \begin{cases} c \\ d \end{cases}$

$a+b=-20, \quad ab=-2020$

$c+d=20, \quad cd=2020$

$E = a^2c - ac^2 + a^2d - ad^2 + b^2c - bc^2 + b^2d - bd^2$

$= a^2(c+d) + b^2(c+d) - c^2(a+b) - d^2(a+b)$

$= (a^2+b^2)(c+d) - (a+b)(c^2+d^2)$

$= ((a+b)^2 - 2ab)(c+d) - (a+b)((c+d)^2 - 2cd)$

$$ax^2+bx+c=0 \begin{cases} \alpha \\ \beta \end{cases}$$

$$(\alpha-\beta)^2 = \frac{D}{a^2}$$ ✓

$$\cancel{|\alpha-\beta| = \frac{\sqrt{D}}{|a|}}$$

$$|a+ib| = \sqrt{a^2+b^2}$$

$$|\alpha-\beta| = \frac{\sqrt{D}}{|a|}$$

only for real roots

$$x^2+x+1=0 \begin{cases} \dfrac{-1-\sqrt{3}i}{2} = \beta \\ \dfrac{-1+\sqrt{3}i}{2} = \alpha \end{cases}$$

$$D = 1-4 = -3$$

$$|\alpha-\beta| = |\sqrt{3}i| = \sqrt{3} \neq \frac{\sqrt{-3}}{|1|}$$

QUESTION [JEE Advanced 2016]

★★ASRQ★★

Let S be the set of all non-zero numbers α such that the quadratic equation $\alpha x^2 - x + \alpha = 0$ has two distinct real roots x_1 and x_2 satisfying the inequality $|x_1 - x_2| < 1$. Which of the following intervals is(are) a subset(s) of S?

A $\left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right)$

B $\left(-\frac{1}{\sqrt{5}}, 0\right)$

C $\left(0, \frac{1}{\sqrt{5}}\right)$

D $\left(\frac{1}{\sqrt{5}}, \frac{1}{2}\right)$

$$\alpha x^2 - x + \alpha = 0 < \begin{matrix} x_1 \\ x_2 \end{matrix}$$

$$|x_1 - x_2| < 1$$

$$\frac{\sqrt{D}}{|a|} < 1$$

$$\frac{\sqrt{1-4\alpha^2}}{|\alpha|} < 1.$$

$$\sqrt{1-4\alpha^2} < |\alpha|$$

$$1 - 4\alpha^2 < \alpha 2$$

$$5\alpha^2 > 1$$

$$(\sqrt{5}\alpha - 1)(\sqrt{5}\alpha + 1) > 0 \nearrow \alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup (1/\sqrt{5}, \infty)$$

since $x_1, x_2 \in R$

$$D = 1 - 4\alpha^2 > 0$$

$$4\alpha^2 < 1$$

$$(2\alpha - 1)(2\alpha + 1) < 0$$

$$\alpha \in (-1/2, 1/2)$$

$$\cap \rightarrow \alpha \in \left(-\frac{1}{2}, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, 1/2\right)$$

Ans. A, D

QUESTION

Tah02

Let α, β are the roots of the equation $x^2 + x - 3 = 0$. Then the value of $\alpha^3 - 4\beta^2 + 19$ is equal to

Ans. 0

QUESTION

★★ASRQ★★

If α, β are the roots of $ax^2 + bx + c = 0, (a \neq 0)$ and $\alpha + \delta, \beta + \delta$ are the roots of $Ax^2 + Bx + C = 0, (A \neq 0)$ for some constant δ, then prove that $\frac{b^2 - 4ac}{a^2} = \frac{B^2 - 4AC}{A^2}$

$$ax^2+bx+c=0 < \begin{matrix}\alpha\\ \beta\end{matrix} > (D \cdot O \cdot R)^2 = (\alpha-\beta)^2 = \frac{D_1}{a^2} \text{ --- } ①$$

$$Ax^2+Bx+C=0 < \begin{matrix}\alpha+\delta\\ \beta+\delta\end{matrix} > (D \cdot O \cdot R)^2 = (\alpha+\delta-\beta-\delta)^2 = (\alpha-\beta)^2 = \frac{D_2}{A^2} \text{ --- } ⑪$$

$\underbrace{\text{from ① \& ⑪}}$

$$\frac{D_1}{a^2} = \frac{D_2}{A^2}$$

$$\frac{b^2-4ac}{a^2} = \frac{B^2-4AC}{A^2}$$

General Polynomial Equation

$$\begin{tikzcd}
 & \alpha \\
a_0x^3 + a_1x^2 + a_2x + a_3 = 0 \arrow[ur] \arrow[r] \arrow[dr] & \beta \\
 & \gamma
\end{tikzcd}$$

$$a_0x^3+a_1x^2+a_2x+a_3 = a_0(x-\alpha)(x-\beta)(x-\gamma)$$

$$= a_0(x^2-(\alpha+\beta)x+\alpha\beta)(x-\gamma)$$

$$= a_0(x^3-(\alpha+\beta+\gamma)x^2+(\alpha\beta+\beta\gamma+\gamma\alpha)x-\alpha\beta\gamma$$

$$a_0x^3+a_1x^2+a_2x+a_3 = a_0x^3-a_0(\alpha+\beta+\gamma)x^2+a_0(\alpha\beta+\beta\gamma+\gamma\alpha)-a_0\alpha\beta\gamma$$

$$-a_0(\alpha+\beta+\gamma)=a_1 \longrightarrow S_1=\alpha+\beta+\gamma=-\frac{a_1}{a_0}$$

$$a_0(\alpha\beta+\beta\gamma+\gamma\alpha)=a_2 \longrightarrow S_2=\alpha\beta+\beta\gamma+\gamma\alpha=\frac{a_2}{a_0}$$

$$-a_0\alpha\beta\gamma=a_3 \longrightarrow S_3=\alpha\beta\gamma=-\frac{a_3}{a_0}$$

$$ax^3 \overset{-}{+} bx^2 \overset{+}{+} cx \overset{-}{+} d = 0 \begin{matrix}\nearrow \alpha \\ \rightarrow \beta \\ \searrow \gamma\end{matrix}$$

$$S_1=\alpha+\beta+\gamma=-\frac{b}{a}$$

$$S_2=\alpha\beta+\beta\gamma+\gamma\alpha=c/a$$

$$S_3=\alpha\beta\gamma=-\frac{d}{a}$$

$$ax^4 \overset{-}{+} bx^3 \overset{+}{+} cx^2 \overset{-}{+} dx \overset{+}{+} e = 0 \begin{cases}\alpha\\ \beta\\ \gamma\\ \delta\end{cases}$$

$$S_1 = \alpha+\beta+\gamma+\delta = -\frac{b}{a}$$

$$S_2 = \alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = \frac{c}{a}$$

$$S_3 = \alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta = -\frac{d}{a}$$

$$S_4 = \alpha\beta\gamma\delta = \frac{e}{a}$$

$\underset{\smile}{Ex}$: $$x^3 \overset{-}{+} 6x^2 \overset{+}{-} 11x \overset{-}{+} 5 = 0 \begin{cases}\alpha\\ \beta\\ \gamma\end{cases}$$

$$\alpha+\beta+\gamma = -\frac{6}{1} = -6$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = +\frac{(-11)}{1} = -11$$

$$\alpha\beta\gamma = -\frac{(5)}{1} = -5$$

Ex: $$x^4 - 5x^2 + 6x + 7 = 0 \begin{cases}\alpha\\ \beta\\ \gamma\\ \delta\end{cases}$$

$$x^4 \overset{-}{+} 0\cdot x^3 \overset{+}{-} 5x^2 \overset{-}{+} 6x \overset{+}{+} 7 = 0$$

$$\alpha+\beta+\gamma+\delta = 0$$

$$\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = +\frac{(-5)}{1} = -5.$$

QUESTION

Given that the equation $x^3 \overset{-}{-} px^2 \overset{+}{+} qx \overset{-}{-} r = 0$ has roots α, β and γ, find

(i) $\alpha^2+\beta^2+\gamma^2$ $= (\alpha+\beta+r)^2 - 2(\alpha\beta+\beta r+r\alpha)$

$= p^2 - 2q$

(ii) $\alpha^3+\beta^3+\gamma^3$

$\alpha^3+\beta^3+r^3-3\alpha\beta r = (\alpha+\beta+r)(\underbrace{\alpha^2+\beta^2+r^2}-\alpha\beta-\beta r-r\alpha)$

$= p\cdot(p^2-2q-(\alpha\beta+\beta r+r\alpha))$

$= p\cdot(p^2-2q-q)$

$\alpha^3+\beta^3+r^3-3(r) = p\cdot(\underline{p^2-3q})$

$\alpha^3+\beta^3+r^3 = p^3-3pq+3r.$

QUESTION

Tah03

$$P(x) = x^3 + 33x^2 + 327x + 935$$

Let P(x) be a polynomial as described above with a, b, c the roots of P(x).
Find $a^2 + b^2 + c^2$ without solving $P(x) = 0$.

QUESTION

Jan04

Let r_1, r_2 and r_3 be the roots of the polynomial $5x^3 - 11x^2 + 7x + 3$.
Evaluate $r_1(1 + r_2 + r_3) + r_2(1 + r_3 + r_1) + r_3(1 + r_1 + r_2)$.

QUESTION

Let α, β, γ be roots of $x^3 + 2x^2 - 4x + 5 = 0$ find value of $\frac{(\alpha^3+5)(\beta^3+5)(\gamma^3+5)}{13\alpha\beta\gamma}$.

$$\alpha^3+2\alpha^2-4\alpha+5=0$$

$$\alpha^3+5=4\alpha-2\alpha^2$$

$$\alpha^3+5=2\alpha(2-\alpha)$$

lly $\beta^3+5=2\beta(2-\beta)$

$$\gamma^3+5=2\gamma(2-\gamma)$$

$$E=\frac{(\alpha^3+5)(\beta^3+5)(\gamma^3+5)}{13\alpha\beta\gamma}$$

$$E=\frac{2\cancel{\alpha}(2-\alpha)\,2\cancel{\beta}(2-\beta)\,2\cancel{\gamma}(2-\gamma)}{13\cancel{\alpha\beta\gamma}}$$

$$E=\frac{8}{13}(2-\alpha)(2-\beta)(2-\gamma)$$

Now $x^3+2x^2-4x+5=(x-\alpha)(x-\beta)(x-\gamma)$

put x=2 $8+8-8+5=(2-\alpha)(2-\beta)(2-\gamma)$

$(2-\alpha)(2-\beta)(2-\gamma)=13 \longrightarrow E=\frac{8}{13}\cdot 13=8$ Ans

QUESTION

If α, β, γ are roots of $5x^3 - qx - 1 = 0$, $(q \in R)$ find the value of $\frac{\alpha^2-3}{\beta\gamma} + \frac{\beta^2-3}{\alpha\gamma} + \frac{\gamma^2-3}{\alpha\beta}$.

$$5x^3 - qx - 1 = 0 \begin{matrix}\alpha \\ \beta \\ \gamma\end{matrix}$$

$$5x^3 + 0\cdot x^2 - qx - 1 = 0$$

$$\alpha + \beta + \gamma = 0$$

$$\Downarrow$$

$$\alpha^3 + \beta^3 + \gamma^3 = 3\alpha\beta\gamma$$

$$E = \frac{\alpha^3 - 3\alpha + \beta^3 - 3\beta + \gamma^3 - 3\gamma}{\alpha\beta\gamma}$$

$$E = \frac{\alpha^3 + \beta^3 + \gamma^3 - 3(\alpha + \beta + \gamma)}{\alpha\beta\gamma}$$

$$E = \frac{3\alpha\beta\gamma - 3\overset{0}{(\alpha + \beta + \gamma)}}{\alpha\beta\gamma}$$

$E = 3$ Ans

QUESTION

Jah05

If α, β and γ are roots of cubic equation $x^3 + 3x - 1 = 0$ then find value of :

(i) $(2-\alpha)(2-\beta)(2-\gamma)$

(ii) $(3+\alpha)(3+\beta)(3+\gamma)$

(iii) $(4-\alpha^2)(4-\beta^2)(4-\gamma^2)$

(iv) $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$

QUESTION [JEE Mains 2024 (6 April)]

Tan06

Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$ and $(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2) = \frac{125}{16}m$. Then the value of m is

Ans. 221

QUESTION

If $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are roots of equation $x^5 - 5x^4 - 1 = 0$, then

Home Challenge

(A) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -\frac{1}{20}$

(B) $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -20$

(C) $\prod_{r=1}^{r=5} \left(\frac{1}{\alpha_r^4} + 5\right)^5 = 1$

$$\left(\frac{1}{\alpha_1^4}+5\right)^5 \cdot \left(\frac{1}{\alpha_2^4}+5\right)^5 \cdot \left(\frac{1}{\alpha_3^4}+5\right)^5 \left(\frac{1}{\alpha_4^4}+5\right)^5 \left(\frac{1}{\alpha_5^4}+5\right)^5 .$$

(D) $\prod_{r=1}^{r=5} \left(\frac{1}{\alpha_r^4} + 5\right)^3 = \frac{1}{5}$

Newton's Formula

$S_n = p\alpha^n + q\beta^n$ p, q are constants.

$ax^2+bx+c=0 \begin{smallmatrix}\alpha \\ \beta\end{smallmatrix}$

$a\alpha^2+b\alpha+c=0$, $a\beta^2+b\beta+c=0$

Multiply both sides by $p\alpha^n$

multiply both sides by $q\beta^n$

$ap\alpha^{n+2}+bp\alpha^{n+1}+cp\alpha^n=0$, $aq\beta^{n+2}+bq\beta^{n+1}+q\beta^n.c=0$

$\oplus$

$$a(p\alpha^{n+2}+q\beta^{n+2})+b(p\alpha^{n+1}+q\beta^{n+1})+c(p\alpha^n+q\beta^n)=0$$

$$\boxed{aS_{n+2}+bS_{n+1}+cS_n=0} \rightarrow \boxed{\text{where } S_n=p\alpha^n+q\beta^n}$$

if P.O.R $\neq 0$, $n \in I$

if P.O.R $= 0$, $n \in N$.

$S_n = p\alpha^n + q\beta^n$

$\left.\begin{matrix}\alpha\\ \beta\end{matrix}\right\rangle x^2 - 5x + 7 = 0$ M① $V = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{5}{7}$, $V' = \alpha^5 + \beta^5 \rightarrow S_5$

$S_1 = \alpha + \beta = 5$

$S_0 = \alpha^0 + \beta^0 = 1 + 1 = 2$

M② $S_n = \alpha^n + \beta^n \longrightarrow$ we want S_{-1}

By NF

$S_{n+2} - 5S_{n+1} + 7S_n = 0$ → $n=3$, $n=2$, $n=1$, $n=0$

put $n=-1$

$S_1 - 5S_0 + 7S_{-1} = 0$

$5 - 5(2) + 7S_{-1} = 0$

$S_{-1} = 5/7$

$S_5 - 5S_4 + 7S_3 = 0$

$S_4 - 5S_3 + 7S_2 = 0$

$S_3 - 5S_2 + 7S_1 = 0$

$S_2 - 5S_1 + 7S_0 = 0$

$\Downarrow$

$S_2 - 5(5) + 7\cdot 2 = 0$

$S_2 = 11.$

$S_3 - 5\cdot 11 + 7(5) = 0$

$S_3 = 55 - 35 = 20$

$S_4 - 5(20) + 7\cdot 11 = 0$

$S_4 = 100 - 77 = 23.$

$S_5 - 5(23) + 7\cdot 20 = 0$

$S_5 = 115 - 140 = -25.$

QUESTION [JEE Mains 2024 (27 Jan)]

If α, β are the roots of the equation, $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then

(A) $2S_{12} = S_{11} + S_{10}$

[✓ (B)] $S_{12} = S_{11} + S_{10}$

(C) $S_{11} = S_{10} + S_{12}$

(D) $2S_{11} = S_{12} + S_{10}$

$x^2 - x - 1 = 0$

$S_n = 2023\alpha^n + 2024\beta^n$

$\underbrace{\text{By NF}}$

$S_{n+2} - S_{n+1} - S_n = 0$

$\underbrace{\text{put } n=10}$

$S_{12} - S_{11} - S_{10} = 0$

$S_{12} = S_{11} + S_{10}$

Ans. B

QUESTION [JEE Mains 2020]

Let α and β be the roots of the equations, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1, 2, 3, ... then

By NF $5S_{n+2} + 6S_{n+1} - 2S_n = 0$

n=4 $5S_6 + 6S_5 - 2S_4 = 0$

$5S_6 + 6S_5 = 2S_4$

A $6S_6 + 5S_5 = 2S_4$

[✓ B] $5S_6 + 6S_5 = 2S_4$

C $6\,S_6 + 5\,S_5 + 2\,S_4 = 0$

D $5\,S_6 + 6\,S_5 + 2\,S_4 = 0$

QUESTION [IIT-JEE 2011]

Let α, β be the roots of $x^2 - 6x - 2 = 0$ with $\alpha > \beta$ if $a_n = \alpha^n - \beta^n, n \geq 1$. Then find the value of $\frac{a_{10}-2a_8}{2a_9}$.

M ① N.F $a_{n+2} - 6a_{n+1} - 2a_n = 0$

put $n = 8$

$a_{10} - 6a_9 - 2a_8 = 0$

$a_{10} - 2a_8 = 6a_9$

$\frac{a_{10}-2a_8}{2a_9} = 3.$

M ②

$$\frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$\frac{\alpha^{10} - 2\alpha^8 - \beta^{10} + 2\beta^8}{2(\alpha^9 - \beta^9)}$$

$$\frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$\alpha^2 - 6\alpha - 2 = 0$

$\alpha^2 - 2 = 6\alpha$

lly $\beta^2 - 2 = 6\beta$

$$\frac{\alpha^8 \cdot 6\alpha - \beta^8 \cdot 6\beta}{2(\alpha^9 - \beta^9)}$$

$$\frac{6\cancel{(\alpha^9 - \beta^9)}}{2\cancel{(\alpha^9 - \beta^9)}} = 3.$$

QUESTION [JEE Mains 2021]

★★ASRQ★★

If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0, \alpha > \beta$ and $P_n = \alpha^n - \beta^n$ each positive integer n, then the value of $\left(\frac{P_{17}P_{20}+5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19}+5\sqrt{2}P_{18}^2}\right)$ is equal to

$x^2+5\sqrt{2}\,x+10=0$, $\alpha>\beta$

$P_n=\alpha^n-\beta^n$

By NF

$P_{n+2}+5\sqrt{2}P_{n+1}+10P_n=0$

$n=18$ $P_{20}+5\sqrt{2}P_{19}+10P_{18}=0$

$P_{20}+5\sqrt{2}P_{19}=-10P_{18}$.

$n=17$ $P_{19}+5\sqrt{2}P_{18}+10P_{17}=0$

$P_{19}+5\sqrt{2}P_{18}=-10P_{17}$

$$E=\frac{P_{17}\left(P_{20}+5\sqrt{2}P_{19}\right)}{P_{18}\left(P_{19}+5\sqrt{2}P_{18}\right)}$$

$$E=\frac{P_{17}(-10\cdot P_{18})}{P_{18}(-10P_{17})}=1.$$

QUESTION [JEE Mains 2024 (9 April)]

★★ASRQ★★

Jah 07

Let $\alpha, \beta; \alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$.

Let $P_n = \alpha^n - \beta^n, n \in N$. Then $(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$ is equal to

(A) $10\sqrt{3}P_9$

(B) $11\sqrt{3}P_9$

(C) $11\sqrt{2}P_9$

(D) $10\sqrt{2}P_9$

Ans. A

QUESTION [JEE Mains 2020]

★★ASRQ★★

Let α and β be the roots of the equations $x^2 - x - 1 = 0$. If $P_k = (\alpha)^k + (\beta)^k$, $k \geq 1$, then which one of the following statements is not true,

(A) $P_3 = P_5 - P_4$ ✓

(B) $(P_1 + P_2 + P_3 + P_4 + P_5) = 26$ ✓

(C) $P_5 = 11$ ✓

~~(D)~~ $P_5 = P_2 \cdot P_3$ ✗

$x^2 - x - 1 = 0$

$P_K = \alpha^K + \beta^K \quad K \geq 1$

$P_{K+2} - P_{K+1} - P_K = 0$

$K=3 \quad P_5 - P_4 - P_3 = 0$

$P_3 = P_5 - P_4$

$P_5 = P_4 + P_3 = 7 + 4 = 11$

$K=2 \quad P_4 = P_3 + P_2 = 4 + 3 = 7$

$K=1 \quad P_3 = P_2 + P_1 = 3 + 1 = 4$

$P_2 = \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 1 + 2 = 3$

$P_1 = \alpha + \beta = 1$

$\Downarrow$

$P_1 + P_2 + P_3 + P_4 + P_5 = 26$

QUESTION [JEE Mains 2025 (3 April)]

Tah 08

Let α and β be the roots of $x^2 + \sqrt{3}x - 16 = 0$, and γ and δ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n$ and $Q_n = \gamma^n + \delta^n$, then $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$ is equal to

- **A** 4
- **B** 3
- **C** 5
- **D** 7

Ans. C

QUESTION [JEE Mains 2025 (2 April)]

Jan09

Let $P_n = \alpha^n + \beta^n, n \in N$. If $P_{10} = 123, P_9 = 76, P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

$\alpha + \beta = 1 = -\frac{b}{a}$

(A) $x^2 + x - 1 = 0$

(B) $x^2 - x + 1 = 0$

(C) $x^2 + x + 1 = 0$

(D) $x^2 - x - 1 = 0$

Ans. A

QUESTION [JEE Mains 2023 (11 April)]

★★ ASRQ ★★★

If a and b are the roots of equation $x^2 - 7x - 1 = 0$, then the value of $\dfrac{\overbrace{a^{21}+b^{21}}^{S_{21}}+\overbrace{a^{17}+b^{17}}^{S_{17}}}{\underbrace{a^{19}+b^{19}}_{S_{19}}}$ is equal to

$S_n = a^n + b^n$

$x^2 - 7x - 1 = 0 \begin{smallmatrix} a \\ b \end{smallmatrix}$

$\underbrace{\text{By NF}}$ $S_{n+2} - 7S_{n+1} - S_n = 0$

$\underbrace{\text{put } n = 19}$

$S_{21} - 7\cancel{S_{20}} - S_{19} = 0$

⊕

$S_{21} - 50S_{19} - 7\cancel{S_{18}} = 0$

$\underbrace{\text{put } n = 18}$

$\cancel{S_{20}} - 7S_{19} - S_{18} = 0 \quad \times 7$

⊖

$\underbrace{\text{put } n = 17}$

$S_{19} - 7\cancel{S_{18}} - S_{17} = 0$

$S_{21} - 51S_{19} + S_{17} = 0$

$S_{21} + S_{17} = 51 \cdot S_{19}$

$\dfrac{S_{21} + S_{17}}{S_{19}} = 51$

Ans. 51

QUESTION [JEE Mains 2023 (11 April)]

★★ ASRQ ★★★

If a and b are the roots of equation $x^2 - 7x - 1 = 0$, then the value of $\dfrac{\overbrace{a^{21}+b^{21}}^{S_{21}}+\overbrace{a^{17}+b^{17}}^{S_{17}}}{\underbrace{a^{19}+b^{19}}_{S_{19}}}$ is equal to

$\underline{M②}$

$x^2-7x-1=0 < \begin{matrix} a \\ b \end{matrix}$

$a^2-7a-1=0 \longrightarrow a^2-1=7a$ → SBS

$a^4+1-2a^2=49a^2$

$a^4+1=51a^2$

lly $b^4+1=51b^2$

$$\frac{\overbrace{a^{21}+a^{17}}+b^{21}+b^{17}}{a^{19}+b^{19}} = \frac{a^{17}(a^4+1)+b^{17}(b^4+1)}{a^{19}+b^{19}}$$

$$= \frac{a^{17}\cdot 51a^2+b^{17}\cdot 51b^2}{a^{19}+b^{19}}$$

$$= \frac{51\cancel{(a^{19}+b^{19})}}{\cancel{a^{19}+b^{19}}} = 51$$

Ans. 51

Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with Good Rank

QUESTION

(KTK 1)

If α, β are the roots of the equation $x^2 + px - r = 0$ and $\frac{\alpha}{3}, 3\beta$ are the roots of the equation $x^2 + qx - r = 0$, then r equals

(A) $\frac{3}{8}(p - 3q)(3p + q)$

(B) $\frac{3}{8}(p + 3q)(3p - q)$

(C) $\frac{3}{64}(3p - q)(p - 3q)$

(D) $\frac{3}{64}(3q - p)(p - q)$

Ans. C

QUESTION

(KTK 2)

If one of the root of the equation $4x^2 - 15x + 4p = 0$ is the square of the other then the value of p is

(A) $\frac{125}{64}$

(B) $-\frac{27}{8}$

(C) $-\frac{125}{8}$

(D) $\frac{27}{8}$

Ans. C, D

QUESTION

(KTK 3)

If $b \in R^{+}$ then roots of the equation $(2+b)x^2+(3+b)x+(4+b)=0$ is

- (A) Real and distinct
- (B) Real and equal
- (C) Imaginary
- (D) Cannot be predicted

Ans. C

QUESTION

(KTK 4)

If x satisfies $|x-1| + |x-2| + |x-3| \geq 6$, then

(A) $0 \leq x \leq 4$

(B) $x \leq -2$ or $x \geq 4$

(C) $x \leq 0$ or $x \geq 4$

(D) none

Ans. C

QUESTION

(KTK 5)

Number of integral values of 'a' for which the quadratic equation, $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$ possesses roots of opposite sign is,

(A) 1

(B) 2

(C) 3

(D) 4

Ans. C

QUESTION

(KTK 6)

If a, b, c are real numbers satisfying the condition $a + b + c = 0$ then the roots of the quadratic equation $3ax^2 + 5bx + 7c = 0$ are

- (A) positive
- (B) negative
- (C) real and distinct
- (D) imaginary

Ans. C

Homework From Module

Prarambh (Topicwise) : Q1 to Q32

Prabal (JEE Main Level) : Q1 to Q46

Parikshit (JEE Advanced Level) : Q1 to Q42

Solution to Previous BPPs

Bumper Practice Problems

1. For what values of a does the equation $9x^2 - 2x + a = 6 - ax$ posses equal roots ?

2. Find the values of a for which the roots of the equation $(2a - 5)x^2 - 2(a - 1)x + 3 = 0$ are equal.

3. For what values of m does the equation $x^2 - x + m = 0$ possess no real roots ?

4. For what values of m does the equation $mx^2 - (m + 1)x + 2m - 1 = 0$ possess no real roots?

5. Find integral values of k for which the equation $(k - 12)x^2 + 2(k - 12)x + 2 = 0$ possess no real roots?

6. For what values of 'a' does the equation $x^2 + 2a\sqrt{a^2 - 3}x + 4 = 0$ possess equal roots?

Bumper Practice Problems

7. Form a quadratic equation whose roots are the numbers $\frac{1}{10-\sqrt{72}}$ and $\frac{1}{10+6\sqrt{2}}$.

8. Find the least integral value of k for which the equation $x^2 - 2(k+2)x + 12 + k^2 = 0$ has two different real roots.

9. For what values of a is the sum of the roots of the equation $x^2 + (2 - a - a^2)x - a^2 = 0$ equal to zero?

10. For what values of a do the graphs of the functions $y = 2ax + 1$ and $y = (a-6)x^2 - 2$ not intersect?

$2ax+1=(a-6)x^2-2$ should have no real roots.

$(a-6)x^2-2ax-3=0$

if $a \neq 6$ $D<0$

$\cancel{4}a^2-\cancel{4}(-3)(a-6)<0$.

$a^2+3a-18<0$

$(a+6)(a-3)<0$

$a \in (-6,3)$

if $a=6$ $-12x-3=0$

$x=-1/4$ $a=6$ (rejected)

11. For what values of a is the ratio of the roots of the equation $x^2 + ax + a + 2 = 0$ equal to 2?

Bumper Practice Problems

12. For what values of a do the roots x_1 and x_2 of the equation $x^2 - (3a + 2)x + a^2 = 0$ satisfy the relation $x_1 = 9x_2$?

13. Find a such that one of the roots of the equation $x^2 - \frac{15}{4}x + a = 0$ is the square of the other.

14. The roots x_1 and x_2 of the equation $x^2 + px + 12 = 0$ are such that $x_2 - x_1 = 1$. Find p.

15. Find k in the equation $5x^2 - kx + 1 = 0$ such that the difference between the roots of the equation is unity.

Answers

1. $a = 20 \pm 6\sqrt{5}$
2. $a = 4$
3. $m \in \left(\frac{1}{4}, \infty\right)$
4. $m \in \left(-\infty, -\frac{1}{7}\right) \cup (1, \infty)$
5. $k = 13$
6. $a = \pm 2$
7. $28x^2 - 20x + 1 = 0$
8. $k = 3$
9. $a_1 = -2, a_2 = 1$
10. $a \in (-6,3)$
11. $a_1 = -\frac{3}{2}, a_2 = 6$
12. $a = 6, -\frac{6}{19}$
13. $a_1 = -\frac{125}{8}, a_2 = \frac{27}{8}$
14. $p = \pm 7$
15. $k = \pm 3\sqrt{5}$

Bumper Practice problems

1. $9x^2 - 2x + a = 6 - ax$, what values of a has roots equal.

$D = 0$

$9x^2 - x(2-a) + a - 6 = 0$

$D = b^2 - 4ac = 0 \Rightarrow (2-a)^2 - 36(a-6) = 0$

$\Rightarrow 4 + a^2 - 4a - 36a + 216 = 0$

$\Rightarrow a^2 - 40a + 220 = 0$

$$a = \frac{40 \pm \sqrt{720}}{2}$$

Aadya
Jharkhand

$$= \frac{40 \pm 12\sqrt{5}}{2} = \boxed{20 \pm 6\sqrt{5}}$$ Ans.

Date ___ / ___ / _____

2. $(2a-5)x^2 - 2(a-1)x + 3 = 0$, for what values of a roots are equal.

Sol : $D = 0$

$b^2 - 4ac \Rightarrow (2a-2)^2 - 12(2a-5) = 0$

$\Rightarrow 4a^2 + 4 - 8a - 24a + 60 = 0$

$\Rightarrow 4a^2 - 32a + 64 = 0$

$\Rightarrow a^2 - 8a + 16 = 0$

$\Rightarrow (a-4)^2 = 0$

$\Rightarrow \boxed{a = 4}$

3. Value of m for which $x^2 - x + m = 0$, no real roots

Sol : $D < 0 \Rightarrow b^2 - 4ac \Rightarrow 1 - 4m < 0$

$\Rightarrow$ ~~$m = \frac{1}{4}$~~

$\Rightarrow 4m - 1 > 0$

$\Rightarrow \boxed{m \in \left(\frac{1}{4}, \infty\right)}$

4) $mx^2-(m+1)x+2m-1=0$

$D<0$

$(m+1)^2-4(2m-1)m<0$

$m^2+1+2m-8m^2+4m<0$

$-7m^2+6m+1<0$

$7m^2-6m-1>0$

$7m^2-7m+m-1>0$

$7m(m-1)+(m-1)>0$

$(7m+1)(m-1)>0$

$+ \quad - \quad +$

$-\frac{1}{7} \quad 1$

$m\in\left(-\infty,-\frac{1}{7}\right)\cup(1,\infty)$

5) $(k-12)x^2+2(k-12)x+2=0$

$D<0$

$4(k-12)^2-8(k-12)<0$

$(k-12)(4k-48-8)<0$

$(k-12)(4k-56)<0$

$(k-12)(k-14)<0$

$+ \quad - \quad +$

$12 \quad 14$

$k\in(12,14)$

integral value of $k=13$

7.

Solⁿ $\alpha = \frac{1}{10-\sqrt{72}}$, $\beta = \frac{1}{10+6\sqrt{2}}$

$$S.O.R = \frac{1}{10-6\sqrt{2}} + \frac{1}{10+6\sqrt{2}}$$

$$= \frac{10+6\sqrt{2}+10-6\sqrt{2}}{100-72}$$

$$= \frac{20}{28} \quad \frac{5}{7}$$

$$P.O.R = \frac{1}{10-6\sqrt{2}}\left(\frac{1}{10+6\sqrt{2}}\right)$$

$$= \frac{1}{28}$$

$$\therefore \text{Equation} = x^2 - \frac{5}{7}x + \frac{1}{28} = 0$$

$$\frac{28x^2-20x+1}{28} = 0$$

$$28x^2-20x+1=0$$

⑧

Solⁿ $x^2 - 2(k+2)x + 12 + k^2 = 0$

$$D = (2k+4)^2 - 4.1.(12+k^2) > 0$$

$$\cancel{4k^2} + 16k + 16 - 48 - \cancel{4k^2} > 0$$

$$16k - 32 > 0$$

$$k - 2 > 0$$

$$k > 2$$

$$k \in (2, \infty)$$

$\therefore$ least integral value of $k = 3$

Sakshi

(9) $x^2+(2-a-a^2)x-a^2=0$ $\qquad a=?$

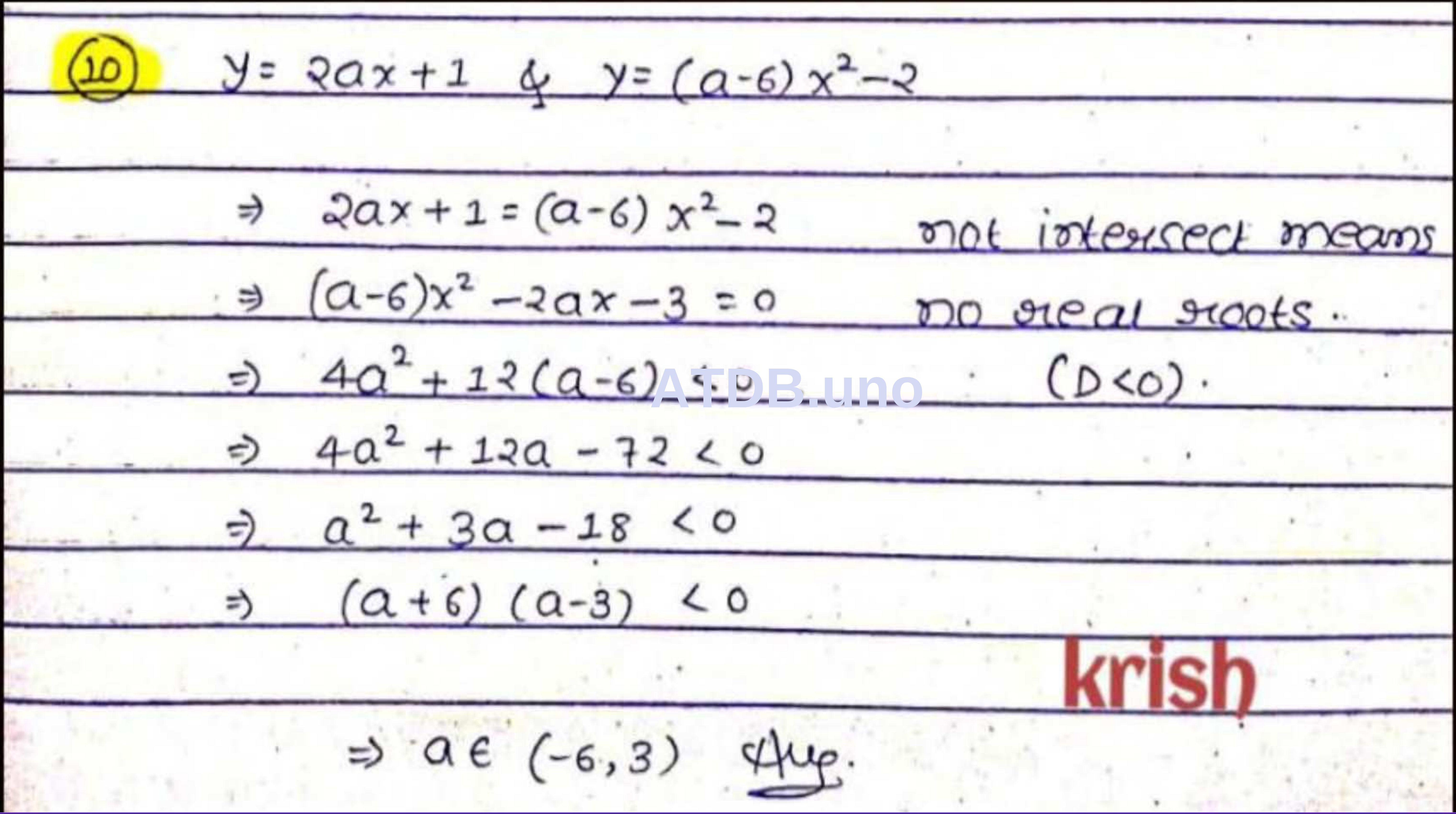
$S.O.R=0$

$$\alpha+\beta=-\frac{b}{a}=\frac{-(2-a-a^2)}{1}=0$$

$$\Rightarrow a^2+a-2=0$$

$$(a+2)(a-1)=0$$

$$a=\underset{\alpha}{\underset{\downarrow}{1}},\underset{\beta}{\underset{\downarrow}{-2}}$$



⑩ $y = 2ax + 1$ & $y = (a-6)x^2 - 2$

$\Rightarrow 2ax + 1 = (a-6)x^2 - 2$

not intersect means
no real roots.
$(D<0)$.

$\Rightarrow (a-6)x^2 - 2ax - 3 = 0$

$\Rightarrow 4a^2 + 12(a-6) < 0$

$\Rightarrow 4a^2 + 12a - 72 < 0$

$\Rightarrow a^2 + 3a - 18 < 0$

$\Rightarrow (a+6)(a-3) < 0$

$\Rightarrow a \in (-6, 3)$ Ans.

⑪ $x^2 + ax + (a+2) = 0 \begin{cases} \rightarrow \alpha \\ \rightarrow \beta \end{cases} \Rightarrow$ given : $\boxed{\dfrac{\alpha}{\beta} = 2}$

$\downarrow$

$\alpha = 2\beta$

$S.O.R = -a$

$P.O.R = a+2$

Then: $\alpha + \beta = -a$

$\Rightarrow 2\beta + \beta = -a$

$\Rightarrow 3\beta = -a$

$\Rightarrow \beta = -a/3$

$\alpha\beta = a+2$ $\quad [\beta = -a/3]$

$\Rightarrow 2\beta^2 = a+2$

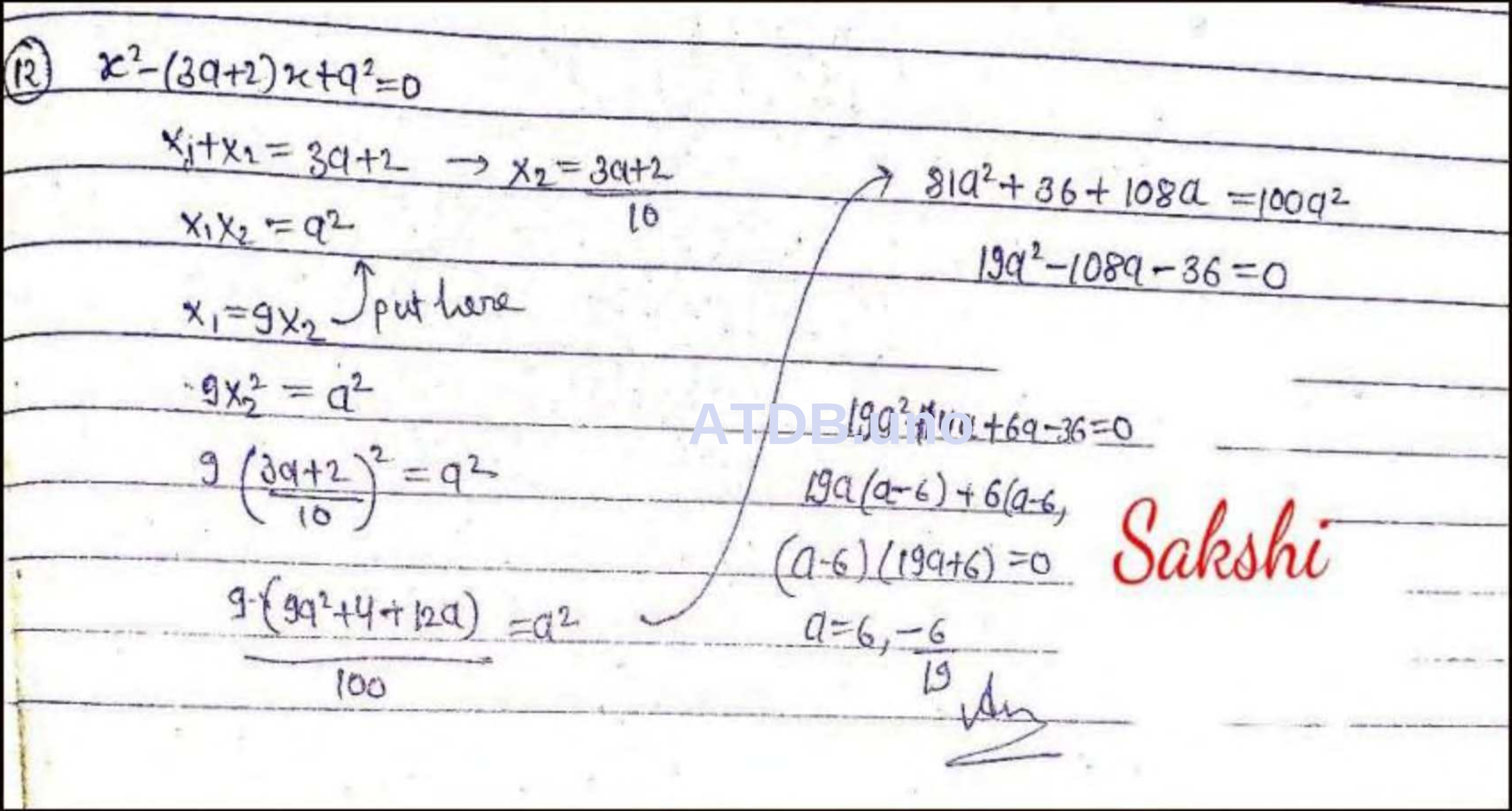
$\Rightarrow 2\left(\dfrac{-a}{3}\right)^2 = a+2$

$\Rightarrow 2a^2 = 9a + 18$

$\Rightarrow 2a^2 - 9a - 18 = 0$

$\Rightarrow (2a+3)(a-6) = 0$

$\Rightarrow a = -\dfrac{3}{2}, 6$. Ans.



⑫ $x^2-(3a+2)x+a^2=0$

$x_1+x_2=3a+2 \rightarrow x_2=\frac{3a+2}{10}$

$x_1x_2=a^2$

$x_1=9x_2$ ↑ put here

$9x_2^2=a^2$

$9\left(\frac{3a+2}{10}\right)^2=a^2$

$\frac{9\cdot(9a^2+4+12a)}{100}=a^2$

$81a^2+36+108a=100a^2$

$19a^2-108a-36=0$

$19a^2-114a+6a-36=0$

$19a(a-6)+6(a-6)$

$(a-6)(19a+6)=0$

$a=6,-\frac{6}{19}$

Ans

Sakshi

⑫ $x^2-(3a+2)x+a^2=0 \longrightarrow$ Satisfy the relation

$\Rightarrow x_1 = 9x_2$.

#S.O.R ⇒ $x_1+x_2=(3a+2)$.

#P.O.R ⇒ $x_1\cdot x_2 = a^2$.

$$9x_2^2=a^2$$

$$x_2=\sqrt{\frac{a^2}{9}}=\pm\frac{a}{3}$$

$\Rightarrow -\frac{10a}{3}=3a+2$

$\Rightarrow -10a=9a+6$

$\Rightarrow -19a=6$

$\Rightarrow \boxed{a=\frac{-6}{19}}$ Ans.

$9x_2+x_2=3a+2$

$\Rightarrow 10x_2=3a+2$

$\Rightarrow \frac{10a}{3}=3a+2$

$\Rightarrow 10a=9a+6$

$\Rightarrow \boxed{a=6}$ Ans.

⑬ $x^2 - \frac{15}{4}x + a = 0 \begin{cases} \to \alpha. \\ \to \alpha^2. \end{cases}$

$$\# \ \underline{S \cdot O \cdot R} = \alpha + \alpha^2 = \frac{15}{4}$$

$$\Rightarrow \alpha^2 + \alpha - \frac{15}{4} = 0$$

$$\# \ \underline{P \cdot O \cdot R} \Rightarrow \alpha^3 = a$$

$$a = \left(\frac{3}{2}\right)^3 \ \& \ \left(\frac{-5}{2}\right)^3$$

$$= \frac{27}{8}, \frac{-125}{8} \ \text{Ans.}$$

$$\Rightarrow 4\alpha^2 + 4\alpha - 15 = 0$$

$$\Rightarrow 4\alpha^2 + 10\alpha - 6\alpha - 15 = 0$$

$$\Rightarrow 2\alpha(2\alpha + 5) - 3(2\alpha + 5) = 0$$

$$\Rightarrow (2\alpha - 3)(2\alpha + 5) = 0$$

$$\Rightarrow \alpha = \frac{3}{2}, \frac{-5}{2}.$$

⑭ $x^2 + Px + 12 = 0 \begin{matrix} \nearrow x_1 \\ \searrow x_2 \end{matrix}$; $\boxed{x_2 - x_1 = 1}$

S.O.R : $x_1 + x_2 = -P$

$x_1 + 1 + x_1 = -P$

$2x_1 + 1 = -P$ — ①

P.O.R : $x_1 \cdot x_2 = 12$

$x_1(1 + x_1) = 12$

$\Rightarrow x_1^2 + x_1 - 12 = 0$

$\Rightarrow (x_1 + 4)(x_1 - 3) = 0$

$\Rightarrow \boxed{x_1 = -4, 3}$

$\Rightarrow 2(-4) + 1 = -P$

$\Rightarrow \boxed{P = 7}$ Ay.

$\Rightarrow 2(3) + 1 = -P$

$= \boxed{P = -7}$ Ay.

⑮. $5x^2 - Kx + 1 = 0$, $\alpha - \beta = 1$.

$\alpha + \beta = K/5$, # $\alpha\beta = 1/5$

↓ sq.

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \frac{K^2}{25}$

यहाँ $\alpha^2+\beta^2$ को $(\alpha+\beta)^2 - 2\alpha\beta$ BCZ फिर K के term मे आ जाता !

$\Rightarrow (\alpha-\beta)^2 + 2\alpha\beta + 2\alpha\beta = \frac{K^2}{25}$

$\Rightarrow 1 + \frac{4}{5} = \frac{K^2}{25}$

$\Rightarrow \frac{9}{5} = \frac{K^2}{25} \Rightarrow K^2 = 45$

$K = \pm 3\sqrt{5}$ Ans.

Solution to Previous TAH

QUESTION [JEE Mains 2019]

The number of integral value of m for which the equation $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is

Ⓐ 1

Ⓑ infinitely many

Ⓒ 3

Ⓓ 2

Ans. B

Q-1! $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real roots

$m = ?$ $(m \in \mathbb{Z})$

Soln. for no real roots

$D < 0$

TAH-1
By Reed
From WB

$\Rightarrow \cancel{4}(1+3m) - \cancel{4}(1+m^2)(1+8m) < 0$

$\Rightarrow \cancel{1} + 6m + 9m^2 - (\cancel{1} + 8m + m^2 + 8m^3) < 0$

$\Rightarrow -8m^3 + 8m^2 - 2m < 0$

$\Rightarrow 4m^3 - 4m^2 + m > 0$

$\Rightarrow m(4m^2 - 4m + 1) > 0$

$\Rightarrow m(2m-1)^2 > 0$

$\Rightarrow m > 0 \; ; \; m \neq \frac{1}{2}$

$\therefore m \in (0, \infty) - \{\frac{1}{2}\}$ $\Rightarrow$ infinite integral solutions of m. (Ans: (b))

Q(Tah-1) The number of integral value of m for which the eqn $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$ has no real root is

Soln:- for no real root, D < 0

$$(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$$

$$D \Rightarrow (2(1+3m))^2 - 4(1+m^2)(1+8m) < 0$$

$$\Rightarrow 4(1+9m^2+6m) - 4(1+8m+m^2+8m^3) < 0$$

$$\Rightarrow \cancel{4} + 36m^2 + 24m \cancel{-4} - 32m - 4m^2 - \cancel{8}\,32m^3 < 0$$

$$\Rightarrow 32m^2 - 8m - 32m^3 < 0$$

$$\Rightarrow m(32m - 8 - 32m^2) < 0$$

$$\Rightarrow -8m(4m^2 - 4m + 1) < 0 \;;\; m > 0$$

$\Rightarrow 4m^2 - 4m + 1 > 0$

$m > \dfrac{4 \pm \sqrt{16-16}}{2\times 4} \Rightarrow m > \dfrac{4}{8} \Rightarrow \dfrac{1}{2}.$

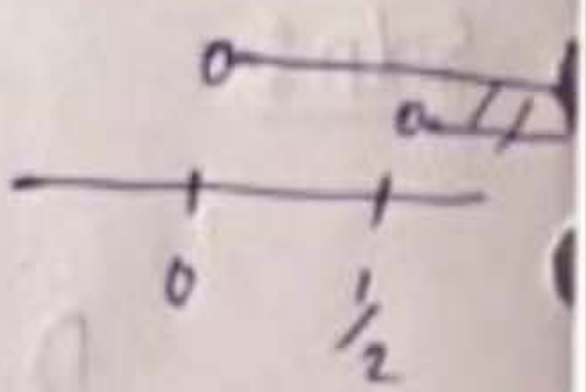
the no. of integral solution is infinitly many solution.

if $m > 0 \, , \, m > \frac{1}{2}.$

$\cup$

$\cap$

$m > \frac{1}{2}.$



Tah-0

The no. of integral value of m for which of the Eqn $(1+m^2)x^2 - 2(1+3m)x + (1+8m) = 0$; has no real root is :

$\Rightarrow$ $D<0$ $\Rightarrow (-(2+6m))^2 - 4(1+m^2)(1+8m) < 0.$

$\Rightarrow (-(2+6m))^2 - (4+4m^2)(1+8m) < 0.$

$\Rightarrow 4 + 36m^2 + 24m - 4 - 32m - 4m^2 - 32m^3 < 0$

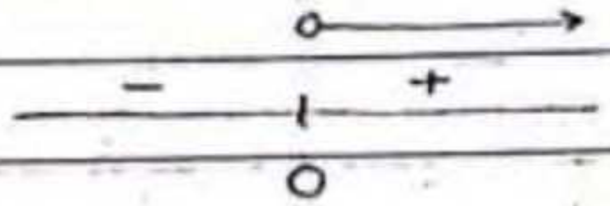
$\Rightarrow -32m^3 + 32m^2 - 8m < 0.$

$\Rightarrow -8m(4m^2 - 4m + 1) < 0.$

$\Rightarrow 8m(4m^2 - 4m + 1) > 0.$

$\Rightarrow 8m(2m-1)^2 > 0.$

$\Rightarrow$



($m = \frac{1}{2}$ is not possible).

$\Rightarrow m \in (0, \infty) - \{1/2\}$

$\Rightarrow$ Option (B) infinite soln. Ans.

QUESTION

If α and β be the roots of the equation $x^2 + 3x + 1 = 0$ then the value of $\left(\frac{\alpha}{1+\beta}\right)^2 + \left(\frac{\beta}{\alpha+1}\right)^2$ is equal to

- (A) 15
- (B) 18
- (C) 21
- (D) none

Ans. B

Q-2! If α and β be the roots of the equation $x^2+3x+1=0$ then the value of $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{1+\alpha}\right)^2$ is equal to:

TAH-2 **By Reed**

ⓐ 15 ⓑ 18 ⓒ 21 ⓓ none.

Soln

$$x^2+3x+1=0 \begin{array}{l}\nearrow \alpha \\ \searrow \beta\end{array} \qquad \alpha+\beta=-3 \quad \alpha\beta=1.$$

$$\left.\begin{array}{l}\alpha^2+3\alpha+1=0 \\ \text{or, } \alpha^2+2\alpha+1=-\alpha\end{array}\right\} \text{lly } \beta^2+2\beta+1=-\beta.$$

$$E=\frac{\alpha^2}{(1+\beta)^2}+\frac{\beta^2}{(1+\alpha)^2}=\frac{\alpha^2}{1+\beta^2+2\beta}+\frac{\beta^2}{1+\alpha^2+2\alpha}$$

$$\alpha^3+\beta^3$$
$$=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$$
$$=-27-3(-3)$$
$$=-27+9=-18$$

$$\Rightarrow E=\frac{\alpha^2}{-\beta}+\frac{\beta^2}{-\alpha}$$

$$\Rightarrow E=\frac{-(\alpha^3+\beta^3)}{\alpha\beta}$$

$$\Rightarrow E=\frac{-(-18)}{1}=\boxed{18}$$

Ans.

Tah: 2 - if α and β be the roots of the equation $x^2+3x+1=0$ then the value of $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{\alpha+1}\right)^2$ is equal to

Soln:- $x^2+3x+1=0 <^{\alpha}_{\beta}$ $\boxed{\begin{array}{l}\alpha+\beta=-3\\ \alpha\beta=1.\end{array}}$

$\alpha^2+3\alpha+1=0$, $\beta^2+3\beta+1=0$

$\Rightarrow \boxed{\begin{array}{l}\alpha^2+1=-3\alpha\\ \Rightarrow \alpha^2+2\alpha+1=-\alpha.\end{array}}$ $\boxed{\begin{array}{l}\Rightarrow \beta^2+1=-3\beta.\\ \Rightarrow \beta^2+2\beta+1=-\beta\end{array}}$

Now, $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{\alpha+1}\right)^2$

$\Rightarrow \frac{\alpha^2}{1+\beta^2+2\beta}+\frac{\beta^2}{\alpha^2+1+2\alpha} \Rightarrow \frac{\alpha^2}{-\beta}+\frac{\beta^2}{-\alpha}.$

$\Rightarrow \frac{-\alpha^3-\beta^3}{\alpha\beta}.$

$\Rightarrow -\frac{(\alpha^3+\beta^3)}{\alpha\beta} = -\frac{(\alpha+\beta)(\alpha^2+\beta^2\mp\alpha\beta)}{(\alpha\beta)}$

$\Rightarrow \frac{-(-3)\left((\alpha+\beta)^2-3\alpha\beta\right)}{\alpha\beta}$

$\Rightarrow \frac{3\left((3)^2-3\right)}{1} \Rightarrow 3(6)$

$\Rightarrow 18.$

Tah-02. If α, β be the roots of the equation $x^2+3x+1=0$ then the value of $\left(\frac{\alpha}{1+\beta}\right)^2+\left(\frac{\beta}{1+\alpha}\right)^2$ is equal to.

$$\Rightarrow x^2+3x+1=0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$$\left.\begin{matrix} \Rightarrow \alpha+\beta=-3. \\ \Rightarrow \alpha\beta = 1. \end{matrix}\right\}$$

$$\Rightarrow \alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$

$$= 9-2$$

$$= 7$$

$$\Rightarrow \left(\frac{\alpha^2+\alpha+\beta^2+\beta}{\underbrace{(\alpha+1)(\beta+1)}_{\alpha\beta+\alpha+\beta+1}}\right)^2-\frac{2\alpha\beta}{(\alpha+1)(\beta+1)}.$$

$$\Rightarrow \left(\frac{7-3}{1-3+1}\right)^2-\frac{2}{(1-3+1)}.$$

$$\Rightarrow \left(\frac{4}{-1}\right)^2-\frac{2}{-1} \Rightarrow 16+2.$$

$$\Rightarrow 18 \text{ Ans.}$$

QUESTION

If α, β be the roots of $x^2 - a(x-1) + b = 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$ is

(A) $\frac{4}{(a+b)}$

(B) $\frac{1}{(a+b)}$

(C) 0

(D) $\frac{2}{(a+b)}$

Q-3! α, β be roots of $x^2 - a(x-1) + b = 0$ then value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$ is:

Soln:

$$x^2 - a(x-1) + b = 0 \begin{array}{l} \nearrow \alpha \\ \searrow \beta \end{array} \qquad \alpha + \beta = a \qquad \alpha\beta = a+b$$

$\alpha^2 - a\alpha + a + b = 0$

$\Rightarrow \alpha^2 - a\alpha = -(a+b)$

| lly, $\beta^2 - a\beta = -(a+b)$

TAH 3

BY REED

$$\therefore E = \frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$$

$$\Rightarrow E = \cancel{\frac{1}{-(a+b)}} + \cancel{\frac{1}{-(a+b)}} + \cancel{\frac{2}{a+b}} = 0 \text{ (Ans)}$$

Tah-03 :- If α, β are roots of $x^2 - a(x-1) + b = 0$ then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$

Soln:- $x^2 - a(x-1) + b = 0 < \begin{matrix}\alpha \\ \beta\end{matrix}$

$\alpha^2 - a(\alpha - 1) + b = 0$	$\beta^2 - a(\beta - 1) + b = 0$
$\alpha^2 - a\alpha + a + b = 0$	$\beta^2 - a\beta = -(a+b)$
$\Rightarrow \alpha^2 - \alpha a = -(a+b)$	

Now, $\frac{1}{-(a+b)} - \frac{1}{(a+b)} + \frac{2}{(a+b)}$

$\Rightarrow -\frac{2}{a+b} + \frac{2}{a+b} = 0$

Tah-03. If α, β be the roots of $x^2 - a(x-1) + b = 0$, then the value of $\frac{1}{\alpha^2 - a\alpha} + \frac{1}{\beta^2 - a\beta} + \frac{2}{a+b}$ is :

$$\Rightarrow \quad x^2 - a(x-1) + b = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

$$\Rightarrow \quad x^2 - ax + (a+b) = 0$$

Put $\boxed{x = \alpha}$ $\Rightarrow \alpha^2 - a\alpha = -(a+b).$

Similarly

$$\Rightarrow \beta^2 - a\beta = -(a+b).$$

$$\Rightarrow -\frac{1}{a+b} - \frac{1}{a+b} + \frac{2}{a+b}$$

$$\Rightarrow \frac{-2}{a+b} + \frac{2}{a+b}$$

$\Rightarrow 0$ Ans.

QUESTION [JEE Mains 2020]

If α and β be two roots of the equation $x^2 - 64x + 256 = 0$.

Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$ is

- **A** 1
- **B** 3
- **C** 4
- **D** 2

Q-4! α, β roots of $x^2 - 64x + 256 = 0$. Then find value of : $\left(\frac{\alpha^3}{\beta^5}\right)^{\frac{1}{8}} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}} = ?$

Soln:

$$x^2 - 64x + 256 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \qquad \begin{matrix} \alpha + \beta = 64 \\ \alpha\beta = 256 \end{matrix}$$

$$E = \left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}} = \frac{\alpha^{\frac{3}{8}}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} = \frac{\alpha + \beta}{(\alpha \cdot \beta)^{5/8}}$$

$$\Rightarrow E = \frac{64}{(256)^{5/8}} = \frac{64}{(2^8)^{5/8}} = \frac{64}{32} = \boxed{2} \text{ Ans.}$$

TAH 4
BY REED
FROM WB

Soln:- $x^2 - 64x + 256 = 0 \left\langle \begin{matrix} \alpha \\ \beta \end{matrix} \right.$

$$\alpha + \beta = 64, \quad \alpha\beta = 256$$

$$\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{\frac{1}{8}}$$

$$\Rightarrow \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}} \Rightarrow \frac{\alpha^{3/8}\cdot\alpha^{5/8} + \beta^{3/8}\cdot\beta^{5/8}}{(\alpha\beta)^{5/8}}$$

$$\Rightarrow \frac{\alpha+\beta}{(\alpha\beta)^{5/8}} \Rightarrow \frac{64}{(2^8)^{5/8}} \Rightarrow \frac{64}{32} = 2.$$

Tah-04 : If α and β be two roots of the Eqⁿ : $x^2 - 64x + 256 = 0$; Then the value of $\left(\frac{\alpha^3}{\beta^5}\right)^{1/8} + \left(\frac{\beta^3}{\alpha^5}\right)^{1/8}$ is:

$\Rightarrow \alpha + \beta = 64$.

$\Rightarrow \alpha\beta = 256$.

$$\Rightarrow \frac{\alpha^{3/8}}{\beta^{5/8}} + \frac{\beta^{3/8}}{\alpha^{5/8}}$$

$$\Rightarrow \frac{\alpha + \beta}{(\alpha\beta)^{5/8}}$$

$$\begin{array}{r|l} 2 & 256 \\ \hline 2 & 128 \\ \hline 2 & 64 \\ \hline & 32 \\ \hline \end{array}$$

$\alpha^{3/8} \cdot \alpha^{5/8}$

$\alpha^{8/8} = \alpha$

$\beta^{3/8} \cdot \beta^{5/8}$

$\beta^{8/8} = \beta$

$$\Rightarrow \frac{64}{(2)^5} = \frac{64}{32}$$

$= 2^3 \times 2^5 = 2^8$

$= 2$ Ans.

QUESTION [JEE Mains 2023 (25 Jan)]

★★KCLS★★

Let $\alpha \in \mathbb{R}$ and let α, β be the roots of the equation $x^2 + 60^{\frac{1}{4}}x + a = 0$. If $\alpha^4 + \beta^4 = -30$, then the product of all possible values of a is

Ans. 45

Jan-0

. Let $\alpha \in R$ and let α, β be the roots of the eq^n $x^2 + 60^{1/4}x + a = 0$, If $\alpha^4 + \beta^4 = -30$ then the product of all possible value of a is : .

$\Rightarrow \alpha + \beta = -60^{1/4}$.

$\Rightarrow \alpha\beta = a$.

$\rightarrow$ SBS : $\alpha^2 + \beta^2 = (-60^{1/4})^2 - 2a$.

$= 60^{1/2} - 2a$

$= \sqrt{60} - 2a$.

$\Rightarrow$ given : $\alpha^4 + \beta^4 = -30$.

$\Rightarrow (\alpha^2+\beta^2)^2 - 2\cdot\alpha^2\beta^2 = -30$

$\Rightarrow (\sqrt{60} - 2a)^2 - 2a^2 = -30$

$\Rightarrow 60 + 4a^2 - 4\sqrt{60}\,a - 2a^2 = -30$

$\Rightarrow 2a^2 - 4\sqrt{60}\,a + 60 + 30 = 0$

$\Rightarrow 2a^2 - 4\sqrt{60}\,a + 90 = 0$.

$D = (4\sqrt{60})^2 - 4\cdot 2\cdot 90$

$= 960 - 720$

$= 240$.

$*$ $a = \dfrac{4\sqrt{60} \pm \sqrt{240}}{4}$

$= \dfrac{\cancel{4}\sqrt{60} \pm \cancel{4}\sqrt{15}}{\cancel{4}}$

$= 2\sqrt{15} \pm \sqrt{15}$

$= 3\sqrt{15}, \sqrt{15}$.

Product of value $= (3\sqrt{15}) \times (\sqrt{15})$

$= 45$ Ans.

QUESTION

If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :

- (A) 18
- (B) 24
- (C) 36
- (D) 96

- **Q-6!** If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2-\lambda x-1=0$ is 15, then $6(\alpha^3+\beta^3)^2$ is equal to:

Sol^n: $\left(\frac{1}{\alpha}\right)^2+\left(\frac{1}{\beta}\right)^2=15.$

$3x^2-\lambda x-1=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$

$\alpha+\beta=\lambda/3$

$\alpha\beta=-1/3$

$\Rightarrow \dfrac{\alpha^2+\beta^2}{(\alpha\beta)^2}=15$

$\Rightarrow \dfrac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}=15$

$\Rightarrow \dfrac{\lambda^2}{9}+\dfrac{2}{3}=\overset{5}{\cancel{15}}\times\dfrac{1}{\underset{3}{\cancel{9}}}.$

$\Rightarrow \dfrac{\lambda^2}{9}=\dfrac{5}{3}-\dfrac{2}{3}=1$

$\Rightarrow \boxed{\lambda=\pm 3}$

$\therefore \alpha+\beta=\dfrac{\lambda}{3}=\dfrac{\pm 3}{3}=\pm 1$

$\alpha\beta=-\dfrac{1}{3}$

TAH 6
BY REED
FROM WB

$E=6(\alpha^3+\beta^3)^2$

$E=6\left[(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)\right]^2$

$E=6\left[(\pm1)^3-\cancel{3}\left(-\frac{1}{\cancel{3}}\right)(\pm1)\right]^2$

$E=6\left[\pm1+(\pm1)\right]^2$

$6[1+1]^2 = 24$ | $6[-1-1]^2 = 24$

$E=\boxed{24}$ Ans.

Tak-

If the sum of the sq. of the reciprocals of the roots α and β of the eq^n : $3x^2+\lambda x-1=0$ is 15, then $6(\alpha^3+\beta^3)^2$. .

\# Ques उल्टा पढ़ो Hint मिल जाएगा !.

$$\# \ \alpha+\beta = \frac{-\lambda}{3}$$

$$\# \ \alpha\beta = -1/3$$

$$\Rightarrow \left(\frac{1}{\alpha}\right)^2 + \left(\frac{1}{\beta}\right)^2 = 15$$

$$\Rightarrow \alpha^2+\beta^2 = 15\alpha^2\beta^2$$

$$\Rightarrow \left(-\frac{\lambda}{3}\right)^2 + \frac{2}{3} = \frac{15}{9}$$

$$\Rightarrow \frac{\lambda^2}{9} + \frac{2}{3} = \frac{15}{9}$$

$$\Rightarrow \frac{\lambda^2}{\cancel{9}} + \frac{6}{\cancel{9}} = \frac{15}{\cancel{9}}$$

$$\Rightarrow \lambda^2 = 9$$

$$\Rightarrow \boxed{\lambda = \pm 3}$$

$$\Rightarrow \alpha+\beta = -1, 1$$

find : $6(\alpha^3+\beta^3)^2$

$$\Rightarrow 6\left[(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)\right]^2$$

$$\Rightarrow 6\left[(1)^3 + \frac{3}{3}(1)\right]^2$$

$$\Rightarrow 6 \times 4 = 24 \ \text{Ans.}$$

+1 या -1 कुछ भी Put करो Ans same ही आएगा !

QUESTION

Let f(x) be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is –1, then the sum of the roots of $f(x) = 0$ is equal to:

A $\frac{11}{3}$

B $\frac{7}{3}$

C $\frac{13}{3}$

D $\frac{14}{3}$

Ans. A

Tah-06:

(b) Let f(x) be a quad. Poly. such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1, then the sum of the roots of $f(x) = 0$ is equal to :

$\Rightarrow$ let ; $f(x) = ax^2 + bx + c = 0$ $\begin{cases} -1 \\ -c/a \end{cases}$

$-1 \cdot \beta = \frac{c}{a} \Rightarrow \beta = -\frac{c}{a}$

$S \cdot OR = -1 - \frac{c}{a} = -\frac{b}{a}$

$\frac{b}{a} = 1 + \frac{c}{a}$

such that :

$\Rightarrow f(-2) + f(3) = 0$

$\Rightarrow 4a - 2b + c + 9a + 3b + c = 0$

$\Rightarrow 13a + b + 2c = 0$

$\Rightarrow 13 + \frac{b}{a} + \frac{2c}{a} = 0$

$\Rightarrow 13 + 1 + \frac{c}{a} + \frac{2c}{a} = 0$

$\Rightarrow 3\frac{c}{a} = -14$

$\Rightarrow \boxed{\frac{c}{a} = -\frac{14}{3}}$

$\Rightarrow \alpha + \beta = -\frac{b}{a}$

$\Rightarrow -1 - \frac{c}{a} = -\frac{b}{a}$

$\Rightarrow -\frac{b}{a} = -\left(1 + \frac{c}{a}\right)$.

Sum of root $= -1 + \frac{14}{3}$

$= \frac{11}{3}$ Ans.


THANK
YOU