

# PRAAYAS

## JEE 2026

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Mathematics

# Quadratic Equations

Lecture - 04

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# Topics *To be covered*



- A** Transformation of Equation
- B** Condition for Common Root
- C** Practice problems

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# Homework Discussion

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## QUESTION

(KTK 5)



Number of integral values of 'a' for which the quadratic equation,  
 $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$  possesses roots of opposite sign is,

- A** 1
- B** 2
- C** 3
- D** 4

$$2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$$

Since roots are of  
opp sign

$$P \cdot O \cdot R = -ve$$

$$\frac{a^2 - 4a}{2} < 0$$

$$a(a - 4) < 0$$

$$a \in (0, 4)$$

Ans. C

## QUESTION

(KTK 6)



If  $a, b, c$  are real numbers satisfying the condition  $a + b + c = 0$  then the roots of the quadratic equation  $3ax^2 + 5bx + 7c = 0$  are

$$\downarrow \\ a \neq 0$$

$$D = 25b^2 - 84ac$$

$$\Downarrow$$

$$b = -(a+c)$$

- A** positive
- B** negative
- C** real and distinct
- D** imaginary

$$D = 25(a+c)^2 - 84ac$$

$$= 25a^2 + 25c^2 + 50ac - 84ac$$

$$= 25a^2 + 25c^2 - 34ac$$

$$= 8a^2 + 8c^2 + 17a^2 + 17c^2 - 34ac$$

$$= 8(a^2 + c^2) + 17(a^2 + c^2 - 2ac)$$

$$= 8(a^2 + c^2) + 17(a-c)^2 > 0$$

$\begin{matrix} >0 & \geq 0 & \geq 0 \end{matrix}$

Ans. C

## QUESTION



If  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$  are roots of equation  $x^5 - 5x^4 - 1 = 0$ , then

~~A~~  $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -\frac{1}{20}$

~~B~~  $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -20$

C  $\prod_{r=1}^{r=5} \left( \frac{1}{\alpha_r^4} + 5 \right) = 1$

D  $\prod_{r=1}^{r=5} \left( \frac{1}{\alpha_r^4} + 5 \right) = \frac{1}{5}$

$$\alpha_1^5 - 5\alpha_1^4 - 1 = 0$$

$$\alpha_1^5 - 5\alpha_1^4 = 1$$

$$\alpha_1 - 5 = \frac{1}{\alpha_1^4}$$

By  $\alpha_2 - 5 = \frac{1}{\alpha_2^4}$

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$$\alpha_5 - 5 = \frac{1}{\alpha_5^4}$$

~~A~~, ~~B~~  $\frac{(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5) - 25}{5} = \sum_{i=1}^5 \frac{1}{\alpha_i^4}$

$$5 - 25 = \sum_{i=1}^5 \frac{1}{\alpha_i^4}$$



# Aao Machaay Dhamaal Deh Swaal pe Deh Swaal

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$ax^3 + bx^2 + cx + d = 0$ 
 $\left\{ \begin{array}{l} \alpha \\ \beta \\ \gamma \end{array} \right.$ 
 $S_n = p\alpha^n + q\beta^n + t\gamma^n$ 
 $p, q, t \text{ are constants.}$

By NF

$$aS_{n+3} + bS_{n+2} + cS_{n+1} + dS_n = 0$$

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Newton's formula can be applied to polynomial Eqn of any degree.

Ex:  $x^5 - 4x^3 + 6x^2 + 7x - 5 = 0$ 
 $\left\{ \begin{array}{l} \alpha \\ \beta \\ \gamma \\ \delta \\ \Delta \end{array} \right.$ 
 $S_n = \alpha^n + \beta^n + \gamma^n + \delta^n + \Delta^n$

By NF

$$S_{n+5} - 4S_{n+4} + 6S_{n+3} + 7S_{n+2} - 5S_{n+1} = 0 \rightarrow \text{Gadhe / Gadhiyaa aisaay lithegay}$$

Phadne waala :

$$S_{n+5} - 4S_{n+3} + 6S_{n+2} + 7S_{n+1} - 5S_n = 0$$

## QUESTION

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If  $\alpha, \beta, \gamma$  are roots of equation  $x^3 - 2x^2 - 1 = 0$  and  $T_n = \alpha^n + \beta^n + \gamma^n$ , then value of  $\frac{T_{11} - T_8}{T_{10}}$  is equal to

$$T_{n+3} - 2T_{n+2} - T_n = 0$$

$$\underbrace{n=8}_{\text{}} T_{11} - 2T_{10} - T_8 = 0$$

$$\frac{T_{11} - T_8}{T_{10}} = 2$$

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 A 1 B 2 C -1 D 3

## QUESTION



Tahol

If  $\alpha$ ,  $\beta$  and  $\gamma$  are roots of  $3x^3 - 4x^2 - 3x + 2 = 0$  and

$$(\alpha^5 + \beta^5 + \gamma^5) - (\alpha^3 + \beta^3 + \gamma^3) = \frac{2}{m} (2(\alpha^4 + \beta^4 + \gamma^4) - (\alpha^2 + \beta^2 + \gamma^2))$$

Then value of  $m$  is \_\_\_\_\_

$$3x^3 - 4x^2 - 3x + 2 = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$$

$$S_n = \alpha^n + \beta^n + \gamma^n$$

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## QUESTION

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If  $a, b, c$  are roots of  $x^3 - x^2 + 1 = 0$  then find the value of  $a^{-2} + b^{-2} + c^{-2}$ .

M①  $x^3 - x^2 + 1 = 0$   $\left\{ \begin{array}{l} a \\ b \\ c \end{array} \right.$

$$a + b + c = 1$$

$$ab + bc + ca = 0 \quad \text{--- SBS}$$

$$abc = -1$$

$$S = \frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = \frac{b^2c^2 + c^2a^2 + a^2b^2}{(abc)^2}$$

$$a^2b^2 + b^2c^2 + c^2a^2 + 2(ab^2c + ac^2b + a^2bc) = 0$$

$$a^2b^2 + b^2c^2 + c^2a^2 + 2abc(a + b + c) = 0$$

$$a^2b^2 + b^2c^2 + c^2a^2 + 2(-1)(1) = 0$$

$$a^2b^2 + b^2c^2 + c^2a^2 = 2$$

$$\Rightarrow S = \frac{2}{(-1)^2} = 2 \quad \underline{\text{Ans}}$$

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## QUESTION

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If  $a, b, c$  are roots of  $x^3 - x^2 + 1 = 0$  then find the value of  $a^{-2} + b^{-2} + c^{-2}$ .

M(2)  $x^3 - x^2 + 1 = 0$  ←  $\begin{matrix} a \\ b \\ c \end{matrix}$

$$S_n = a^n + b^n + c^n$$

we want  $S_{-2}$

$$abc = -1 \Rightarrow a, b, c \neq 0$$

By NF

$$S_{n+3} - S_{n+2} + S_n = 0$$

put  $n = -2$

$$S_1 - S_0 + S_{-2} = 0$$

$$S_{-2} = S_0 - S_1$$

$$= 3 - (1) = 2 \text{ Ans.}$$

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## QUESTION

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If  $a, b, c$  are roots of  $x^3 - x^2 + 1 = 0$  then find the value of  $a^{-2} + b^{-2} + c^{-2}$ .

M (3)  $x^3 - x^2 + 1 = 0$  ←  $\begin{matrix} a \\ b \\ c \end{matrix}$

$$S_n = a^{-2} + b^{-2} + c^{-2}$$

$$a^3 - a^2 + 1 = 0$$

$$a^3 - a^2 = -1$$

$$a - 1 = -\frac{1}{a^2}$$

$$\frac{1}{a^2} = 1 - a$$

lly  $\frac{1}{b^2} = 1 - b$

$$\frac{1}{c^2} = 1 - c$$

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$$\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2} = 3 - (a + b + c) = 3 - 1 = 2 \underline{\text{Ans}}$$

## QUESTION

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Tah02



Let  $\alpha$  and  $\beta$  are two real roots of  $x^2 + 10x - 7 = 0$ . Then

**A** 
$$\frac{\alpha^{20} + \beta^{20} - 7(\alpha^{18} + \beta^{18})}{\alpha^{19} + \beta^{19}} = -10$$

**B** 
$$\frac{\alpha\beta^{18} - 7\alpha\beta^{16} - 10\alpha^{17}\beta}{\alpha^{16} + \beta^{16}} = 70$$
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**C** 
$$\sqrt{\left(\alpha - \frac{7}{\alpha}\right)\left(\beta - \frac{7}{\beta}\right)} = 10$$

**D** 
$$\frac{\alpha^3 + 9\alpha^2 - 17\alpha + 14}{\beta^3 + 11\beta^2 + 3\beta - 8} = 7$$

**QUESTION [JEE Mains 2025 (23 Jan)]**

Tah04



If the equation  $a(b - c)x^2 + b(c - a)x + c(a - b) = 0$  has equal roots, where  $a + c = 15$  and  $b = \frac{36}{5}$ , then  $a^2 + c^2$  is equal to

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Ans. 117

**QUESTION**

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Let  $\frac{-\pi}{6} < \theta < \frac{-\pi}{12}$ . Suppose  $\alpha_1$  and  $\beta_1$  are the roots of the equation  $x^2 - 2x \sec \theta + 1 = 0$  and  $\alpha_2$  and  $\beta_2$  are the roots of the equation  $x^2 + 2x \tan \theta - 1 = 0$ . If  $\alpha_1 > \beta_1$  and  $\alpha_2 > \beta_2$ , then  $\alpha_1 + \beta_2$  equals

- A  $2(\sec \theta - \tan \theta)$
- B  $2 \sec \theta$
- C  $-2 \tan \theta$
- D  $0$

$$x^2 - 2x \sec \theta + 1 = 0 \begin{cases} \alpha_1 \\ \beta_1 \end{cases}$$

$$x = \frac{2 \sec \theta \pm \sqrt{4 \sec^2 \theta - 4}}{2}$$

$$x = \sec \theta \pm \tan \theta$$

$$\begin{aligned} \pm |x| &= \pm x & x \geq 0 \\ \pm |x| &= \pm(-x) & x < 0 \\ &= \mp x \end{aligned}$$

$$x^2 + 2x \tan \theta - 1 = 0 \begin{cases} \alpha_2 \\ \beta_2 \end{cases}$$

$$x = \frac{-2 \tan \theta \pm \sqrt{4 \tan^2 \theta + 4}}{2}$$

$$x = -\tan \theta \pm \sec \theta$$

$$\beta_2 = -\tan \theta - \sec \theta$$

$$\alpha_2 = -\tan \theta + \sec \theta$$

$\theta \in (-30^\circ, -15^\circ)$

$\tan \theta = -ve$   
 $\sec \theta = +ve$

$$\alpha_1 = \sec \theta - \tan \theta$$

$$\beta_1 = \sec \theta + \tan \theta$$

$$\alpha_1 + \beta_2 = -2 \tan \theta$$



# Quadratic Equation v/s Identity



Identity: a mathematical relation which holds for possible values of variable for which it is defined

If a quadratic equation has more than 2 distinct roots then it becomes an identity.

let if possible have more than 2 say 3 distinct roots.

$$ax^2 + bx + c = 0 \begin{cases} \alpha \\ \beta \\ \gamma \end{cases}$$

$$\begin{aligned} a\alpha^2 + b\alpha + c &= 0 \\ a\beta^2 + b\beta + c &= 0 \\ a\gamma^2 + b\gamma + c &= 0 \end{aligned}$$

$$\begin{aligned} a(\alpha^2 - \beta^2) + b(\alpha - \beta) &= 0 \\ a(\alpha + \beta) + b &= 0 \end{aligned}$$

$$\begin{aligned} a(\beta^2 - \gamma^2) + b(\beta - \gamma) &= 0 \\ a(\beta + \gamma) + b &= 0 \end{aligned}$$

$$\begin{aligned} a(\alpha - \gamma) &= 0 \\ \Downarrow \\ a &= 0 \end{aligned}$$

$$\begin{aligned} c &= 0 \\ b &= 0 \\ \Rightarrow a = 0, b = 0, c = 0 \end{aligned}$$

Ex:  $\sin^2\theta + \cos^2\theta = 1$   
 Ex:  $1 + \tan^2\theta = \sec^2\theta$   
 where  $\theta \neq (2n+1)\frac{\pi}{2}$



If a quad is satisfied by more than 2 distinct values of  $x$  then it becomes an identity  $0 \cdot x^2 + 0 \cdot x + 0 = 0$

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## NOTE :

- (1) In any polynomial equation, if the number of roots  $>$  degree of equation then it becomes an identity.
- (2) If any polynomial equation becomes an identity, then its all coefficients are simultaneously zero.

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## QUESTION



For what values of  $p$ , the equation  $(p + 2)(p - 1)x^2 + (p - 1)(2p + 1)x + p^2 - 1 = 0$  has more than two roots.

$$\begin{aligned} & (p+2)(p-1)=0 \rightarrow p=1, -2 \\ \& \quad (p-1)(2p+1)=0 \rightarrow p=1, -1/2 \\ \& \quad p^2-1=0 \rightarrow p=-1, 1 \end{aligned}$$

∩

P=1

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**QUESTION**



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Let  $\alpha, \beta, \gamma$  be distinct real number and  $f(x)$  is a quadratic polynomial such that

$$f(2)\alpha^2 + f(3)\alpha + f(4) = 4\alpha^2 + 4\alpha + 8 \quad \text{---} \quad (f(2)-4)\alpha^2 + (f(3)-4)\alpha + f(4)-8 = 0$$

$$f(2)\beta^2 + f(3)\beta + f(4) = 4\beta^2 + 4\beta + 8 \quad \text{---} \quad (f(2)-4)\beta^2 + (f(3)-4)\beta + f(4)-8 = 0$$

$$f(2)\gamma^2 + f(3)\gamma + f(4) = 4\gamma^2 + 4\gamma + 8 \quad \text{---} \quad (f(2)-4)\gamma^2 + (f(3)-4)\gamma + f(4)-8 = 0$$

then find the value of  $f(7)$ .

$f(2)-4=0$   
 $f(3)-4=0$   
 $g(x) = f(x)-4$  *quad*  
 roots of  $g(x) = 2, 3$ .  
 $g(x) = a(x-2)(x-3)$   
 $f(x)-4 = a(x-2)(x-3)$   
 $f(x) = a(x-2)(x-3)+4$

(Aaram Zindagi)  
Mazdoori :

$(f(2)-4)x^2 + (f(3)-4)x + f(4)-8 = 0$

$\swarrow \alpha$   
 $\searrow \beta$   
 $\searrow \gamma$

It is an identity

$f(2)=4, f(3)=4, f(4)=8$

$f(x) = ax^2 + bx + c$   
 $f(2) = 4a + 2b + c = 4$   
 $f(3) = 9a + 3b + c = 4$   
 $f(4) = 16a + 4b + c = 8$

Solve.

(Mentos Zindagi)

$f(x) = a(x-2)(x-3)+4$   
 $f(4) = a(2)(1)+4 = 8$   
 $a = 2$   
 $f(x) = 2(x-2)(x-3)+4$   
 $f(7) = 2(5)(4)+4 = 44$

**QUESTION**

$$\frac{(x - a)(x - b)}{(c - a)(c - b)} + \frac{(x - b)(x - c)}{(a - b)(a - c)} + \frac{(x - c)(x - a)}{(b - c)(b - a)} = 1$$

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# Transformation of Equation

$$x^2 - 5x + 6 = 0 \left\{ \begin{array}{l} \alpha = 2 \\ \beta = 3 \end{array} \right.$$

form an Eqn whose roots are

- (a)  $\frac{1}{\alpha}, \frac{1}{\beta}$   $f(x) = \frac{1}{x}$
- (b)  $-\alpha, -\beta$   $f(x) = -x$
- (c)  $\alpha - 2, \beta - 2$   $f(x) = x - 2$

Soln (a)

$$y = \frac{1}{x} = f(x)$$

$$x = \frac{1}{y} \text{ put in } \textcircled{1}$$

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$$6y^2 - 5y + 1 = 0 \left\{ \begin{array}{l} \frac{1}{\alpha} \\ \frac{1}{\beta} \end{array} \right.$$

$$6x^2 - 5x + 1 = 0 \left\{ \begin{array}{l} \frac{1}{\alpha} = \frac{1}{2} \\ \frac{1}{\beta} = \frac{1}{3} \end{array} \right.$$

(b)  $y = -x \Rightarrow x = -y$   
put in  $\textcircled{1}$

$$(-y)^2 - 5(-y) + 6 = 0$$

$$y^2 + 5y + 6 = 0$$

$$y^2 + 5y + 6 = 0$$

$$x^2 + 5x + 6 = 0 \left\{ \begin{array}{l} -\alpha = 2 \\ -\beta = 3 \end{array} \right.$$

(c)  $y = x - 2$   
 $x = y + 2$

$$(y+2)^2 - 5(y+2) + 6 = 0$$

$$y^2 + 4y + 4 - 5y - 10 + 6 = 0$$

$$y^2 - y = 0$$

$$x^2 - x = 0 \left\{ \begin{array}{l} \alpha - 2 = 0 \\ \beta - 2 = 1 \end{array} \right.$$



## Transformation of Equation



Suppose we have a polynomial equation of degree  $n$  given by

$a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n = 0$  with roots  $\alpha_1, \alpha_2, \dots, \alpha_n$  and we want to form a polynomial equation whose roots are  $f(\alpha_1), f(\alpha_2), \dots, f(\alpha_n)$  then follow the following steps

**Step 1** Put  $y = f(x)$  ✓

**Step 2** Find  $x$  in terms of  $y$  ✓

**Step 3** Put the value obtained above in given Equation

**Step 4** Replace  $y$  by  $x$  to get the desired equation

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## QUESTION



Tah 05

Given, the cubic equation  $x^3 - 5x^2 + 6x - 3 = 0$  has roots  $\alpha, \beta, \gamma$ . Find the cubic having roots

(i)  $\alpha + 1, \beta + 1, \gamma + 1$   $\rightarrow y = f(x) = x + 1$  (ii)  $\alpha - 1, \beta - 1, \gamma - 1$

(iii)  $-\alpha, -\beta, -\gamma$

(iv)  $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

(v)  $\frac{3\alpha-2}{\alpha+1}, \frac{3\beta-2}{\beta+1}, \frac{3\gamma-2}{\gamma+1}$

$\rightarrow y = f(x) =$

$x = y - 1$  put in (i)

$(y-1)^3 - 5(y-1)^2 + 6(y-1) - 3 = 0$

$y^3 - 1 - 3y^2 + 3y - 5y^2 + 10y + 6y - 6 - 3 = 0$

$y^3 - 8y^2 + 19y - 15 = 0$

$x^3 - 8x^2 + 19x - 15 = 0$

## QUESTION

Tah06



Let  $\alpha$ ,  $\beta$  and  $\gamma$  are the roots of the cubic  $x^3 - 3x^2 + 1 = 0$ . Find a cubic whose roots are  $\frac{\alpha}{\alpha-2}$ ,  $\frac{\beta}{\beta-2}$  and  $\frac{\gamma}{\gamma-2}$ . Hence or otherwise find the value of  $(\alpha - 2)(\beta - 2)(\gamma - 2)$ .

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$$\text{Ans. } 3y^3 - 9y^2 - 3y + 1 = 0; (\alpha - 2)(\beta - 2)(\gamma - 2) = 3$$

**QUESTION**



Jah06@

$\alpha\beta\gamma = -1$

If  $\alpha, \beta, \gamma$  are roots of the cubic  $2011x^3 + 2x^2 + 1 = 0$ , then find

(i)  $(\alpha\beta)^{-1} + (\beta\gamma)^{-1} + (\gamma\alpha)^{-1}$ ;  $\rightarrow \frac{1}{\alpha\beta}, \frac{1}{\beta\gamma}, \frac{1}{\gamma\alpha} \leftarrow \frac{\gamma}{\alpha\beta\gamma}, \frac{\alpha}{\alpha\beta\gamma}, \frac{\beta}{\alpha\beta\gamma}$

$-2011\gamma, -2011\alpha, -2011\beta$

~~(ii)~~  $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$   
 By 4 methods

M(1)

M(2)

$2011x^3 + 2x^2 + 1 = 0 \leftarrow \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$

We want an Eqn with roots  $-2011\alpha, -2011\beta, -2011\gamma$ .

$2011x^3 + 2x^2 + 1 = 0 \leftarrow \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$

Let  $y = -2011x, x = \frac{-y}{2011}$

$-\frac{2011y^3}{2011^3} + \frac{2y^2}{2011^2} + 1 = 0$

$-y^3 + 2y^2 + 2011^2 = 0$

$y^3 - 2y^2 - 2011^2 = 0 \leftarrow \begin{matrix} 1/\alpha\beta \\ 1/\beta\gamma \\ 1/\gamma\alpha \end{matrix} \rightarrow S = -\frac{(-2)}{1} = 2$

$\frac{1}{\alpha\beta}, \frac{1}{\beta\gamma}, \frac{1}{\gamma\alpha} = -2011\alpha, -2011\beta, -2011\gamma$

$\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} = -2011(\alpha + \beta + \gamma)$   
 $= -2011 \cdot \frac{-(-2)}{2011}$   
 $= 2$  Ans

Ans. (i) 2 ; (ii) -4

## QUESTION



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Tah 05 (b)

Let roots of the equation  $x^3 + 3x^2 + 4x = 11$  are  $\alpha, \beta, \gamma$  and the roots of equation  $x^3 + lx^2 + mx + n = 0$  ( $l, m, n \in \mathbb{R}$ ) are  $\alpha + \beta, \beta + \gamma, \gamma + \alpha$ .

## Column-I

- (A)  $l$  is equal to  
 (B)  $m$  is equal to  
 (C)  $n$  is equal to  
 (D)  $(l + m + n)$  is equal to

## Column-II

- (P)  $-6$   
 (Q)  $6$   
 (R)  $13$   
 (S)  $23$

$x^3 + 3x^2 + 4x - 11 = 0$

$\alpha + \beta + \gamma = -3$

To form an Eqn with root  $\alpha + \beta, \beta + \gamma, \gamma + \alpha$

$\downarrow \quad \downarrow \quad \downarrow$   
 $-3 - \gamma, -3 - \alpha, -3 - \beta$

$y = f(x) = -3 - x$

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## QUESTION

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The length of sides of a triangle and the 3 distinct roots of the equation  $4x^3 - 24x^2 + 47x - 30 = 0$  is area of triangle is  $\Delta$ , find  $100\Delta$ .

M①  $4x^3 - 24x^2 + 47x - 30 = 0$   $\left\{ \begin{array}{l} a \\ b \\ c \end{array} \right.$

$$s = \frac{a+b+c}{2} = \frac{24}{2} = 12$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Delta = \sqrt{3(3-a)(3-b)(3-c)}$$

$$4x^3 - 24x^2 + 47x - 30 = 4(x-a)(x-b)(x-c)$$

put  $x=3$   $108 - 216 + 141 - 30 = 4(3-a)(3-b)(3-c)$

$$\frac{3}{4} = (3-a)(3-b)(3-c)$$

$$\Delta = \sqrt{3 \cdot \frac{3}{4}} = \frac{3}{2} \Rightarrow 100\Delta = 150$$

M② we form an Eqn with roots  $3-a, 3-b, 3-c$   $y = f(x) = 3-x$

$$x = 3-y$$

$$4(3-y)^3 - 24(3-y)^2 + 47(3-y) - 30 = 0$$

$$-4y^3 + y^2(\quad) + y(\quad) + 108 - 216 + 141 - 30 = 0$$

$$-4y^3 + y^2(\quad) + y(\quad) + 3 = 0 \left\{ \begin{array}{l} 3-a \\ 3-b \\ 3-c \end{array} \right.$$

$$(3-a)(3-b)(3-c) = -\frac{3}{-4} = \frac{3}{4}$$

## QUESTION



Tahot

Let  $\alpha_1, \alpha_2, \alpha_3$  and  $\alpha_4$  are the roots of equation  $x^4 - 7x + 1 = 0$ , then

**A** 
$$\sum_{i=1}^4 \frac{\alpha_i}{1 + \alpha_i} = \frac{25}{9}$$

**B** 
$$\prod_{i=1}^4 \frac{\alpha_i}{1 + \alpha_i} = 1$$

**C** 
$$\prod_{i=1}^4 \frac{\alpha_i}{1 + \alpha_i} = \frac{1}{9}$$

**D** 
$$\sum_{i=1}^4 \frac{\alpha_i}{1 + \alpha_i} = \frac{23}{9}$$

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## QUESTION



Tahor

If the roots of  $p(x) = x^3 + 3x^2 + 4x - 8$  are  $a, b$  and  $c$ , what is the value of  $a^2(1 + a^2) + b^2(1 + b^2) + c^2(1 + c^2)$ ?

$S_4 + S_2$

M(1)

$$S_n = a^n + b^n + c^n$$

(NF)

M(2)

$$a + b + c = -3$$

$$ab + bc + ca = 4$$

$$abc = +8$$

using Manipulation

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lengthy  
Hogaa

M(3)

$$p(1) = 0$$

$$p(x) = x^2(x-1) + 4x(x-1) + 8(x-1)$$

$$= (x^2 + 4x + 8)(x-1)$$



## Solution of equations in two variables



$$a_1x + b_1y + c_1 = 0 \times b_2$$

$$a_2x + b_2y + c_2 = 0 \times b_1$$

$$(a_1b_2 - b_1a_2)x + b_2c_1 - b_1c_2 = 0$$

$$a_1x + b_1y + c_1 = 0 \times a_2$$

$$a_2x + b_2y + c_2 = 0 \times a_1$$

$$(b_1a_2 - a_1b_2)y + a_2c_1 - a_1c_2 = 0$$

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$$x = \frac{b_1c_2 - b_2c_1}{a_1b_2 - b_1a_2} = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$y = \frac{a_2c_1 - a_1c_2}{a_1b_2 - b_1a_2} = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Determinant of 2<sup>nd</sup> order:  $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$



# NICHOD

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

①

②

$$\text{Then } x = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

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## Condition for Common Root



$$\left. \begin{array}{l} a_1x^2 + b_1x + c_1 = 0 \\ a_2x^2 + b_2x + c_2 = 0 \end{array} \right\} \text{Have a common root } \alpha$$

$$a_1\alpha^2 + b_1\alpha + c_1 = 0$$

$$a_2\alpha^2 + b_2\alpha + c_2 = 0$$

$$\alpha^2 = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\alpha = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

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$$\alpha^2 = \frac{\begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}^2} = \frac{\begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2$$



$$\text{If } a_1x^2 + b_1x + c_1 = 0$$

$$a_2x^2 + b_2x + c_2 = 0$$

have a common  
root

then

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2$$

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$$(b_1c_2 - b_2c_1) \cdot (a_1b_2 - a_2b_1) = (a_2c_1 - a_1c_2)^2$$

Condition for common root

↕ In determinant form

$$a_1x^2 + b_1x + c_1 = 0$$

$$a_2x^2 + b_2x + c_2 = 0$$

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$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \times \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = \begin{vmatrix} c_1 & a_1 \\ c_2 & a_2 \end{vmatrix}^2$$

Condition for  
at least common root



## NOTE :

If both roots of the given equations are common then  $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ .

$$a_1x^2 + b_1x + c_1 = 0 \quad \left\langle \begin{array}{l} \alpha \\ \beta \end{array} \right.$$

$$a_2x^2 + b_2x + c_2 = 0 \quad \left\langle \begin{array}{l} \alpha \\ \beta \end{array} \right.$$

S.O.R =  $-\frac{b_1}{a_1} = -\frac{b_2}{a_2} \rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

P.O.R =  $\frac{c_1}{a_1} = \frac{c_2}{a_2} \rightarrow \frac{a_1}{a_2} = \frac{c_1}{c_2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$



Condition for a common root

$$(b_1c_2 - b_2c_1)(a_1b_2 - a_2b_1) = (c_1a_2 - a_1c_2)^2 \quad \text{--- (1)}$$

is satisfied even when both roots are common.

Since

condition for both roots common

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \lambda$$

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$$a_1 = \lambda a_2$$

$$b_1 = \lambda b_2$$

$$c_1 = \lambda c_2$$

put in (1)

$$\lambda(b_1c_1 - b_1c_1) \lambda \cdot (a_1b_1 - a_1b_1) = \lambda^2(c_1a_1 - a_1c_1)^2$$

$$0 = 0$$



**NOTE:**

#  $a_1x^2 + b_1x + c_1 = 0$   $\begin{cases} p + iq \\ p - iq \end{cases}$

$a_2x^2 + b_2x + c_2 = 0$   $\begin{cases} p + iq \\ p - iq \end{cases}$

$a_1x^2 + b_1x + c_1 = 0$   $\begin{cases} p + iq \\ p - iq \end{cases}$

$a_2x^2 + b_2x + c_2 = 0$   $\begin{cases} p + iq \\ p - iq \end{cases}$

Given Both have a common root & roots of one are imaginary  
 $\Downarrow$   
 Both roots will be common.

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Have a common roots & the roots of one of the equations are imaginary then both roots will be common.

# If the equation

$f(x) = 0 \rightarrow \alpha \curvearrowright f(\alpha) = 0$   
 $g(x) = 0 \rightarrow \alpha \curvearrowright g(\alpha) = 0$

put  $x = \alpha$   $Af(\alpha) \pm Bg(\alpha) = A \cdot 0 \pm B \cdot 0 = 0$   
 $\Downarrow$   
 $x = \alpha$  is also a root of  $Af(x) \pm Bg(x) = 0$

Have a common root say  $\alpha$  then  $x = \alpha$  is also  $Af(x) \pm Bg(x) = 0 \rightarrow \alpha$ .

## QUESTION



If the quadratic equation  $3x^2 + ax + 1 = 0$  &  $2x^2 + bx + 1 = 0$  have a common root find the value of  $5ab - 2a^2 - 3b^2$ ,  $(a, b \in \mathbb{R})$ .

$$\begin{array}{l} 3x^2 + ax + 1 = 0 \\ 2x^2 + bx + 1 = 0 \end{array} \left. \vphantom{\begin{array}{l} 3x^2 + ax + 1 = 0 \\ 2x^2 + bx + 1 = 0 \end{array}} \right\} \text{have a common root}$$

$$\Downarrow$$

$$\begin{vmatrix} 3 & a \\ 2 & b \end{vmatrix} x = \begin{vmatrix} a & 1 \\ 1 & 3 \end{vmatrix}^2$$

$$(3b - 2a)(a - b) = (2 - 3)^2$$

$$3ab - 2a^2 + 2ab - 3b^2 = 1$$

$$5ab - 2a^2 - 3b^2 = 1 \text{ Ans.}$$

## QUESTION [JEE Advanced 2011]



A value of  $b$  for which the equations  $x^2 + bx - 1 = 0$ ,  $x^2 + x + b = 0$  have one root in common is

- A**  $-\sqrt{2}$
- ~~**B**  $-i\sqrt{3}$~~
- C**  $i\sqrt{5}$
- D**  $\sqrt{2}$

$$x^2 + bx - 1 = 0$$

$$x^2 + x + b = 0$$

$$\begin{vmatrix} 1 & b & -1 \\ 1 & 1 & b \end{vmatrix} \times \begin{vmatrix} b & -1 \\ 1 & b \end{vmatrix} = \begin{vmatrix} -1 & 1 \\ b & 1 \end{vmatrix}^2$$

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$$(-1)(b+1) = (-1-b)^2$$

~~$$b^2 + 1 - b^3 - b = 1 + b^2 + 2b$$~~

$$b^3 + 3b = 0$$

$$b(b^2 + 3) = 0$$

$$b = 0, b = \pm\sqrt{3}i$$

**QUESTION [JEE Mains 2023 (30 Jan)]**

Tah09



If the value of real number  $a > 0$  for which  $x^2 - 5ax + 1 = 0$  and  $x^2 - ax - 5 = 0$  have a common real root is  $\frac{3}{\sqrt{2\beta}}$  then  $\beta$  is equal to

$$x^2 - 5ax - 1 = 0$$

$$x^2 - ax - 5 = 0$$

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Ans. 13



**Sabse Important Baat**



**Sabhi Class Illustrations <sup>ATDB.uno</sup> Retry Karnay hai...**



## Home Challenge - 07



Let  $n$  be the number of integers satisfying the inequality 
$$\frac{(x-3)^{|x|} \cdot \sqrt{(x-5)^2 \cdot (18-x)}}{\sqrt{-x}(-x^2+x-1)(|x|-37)} < 0$$
 then value of  $n$  is \_\_\_\_\_

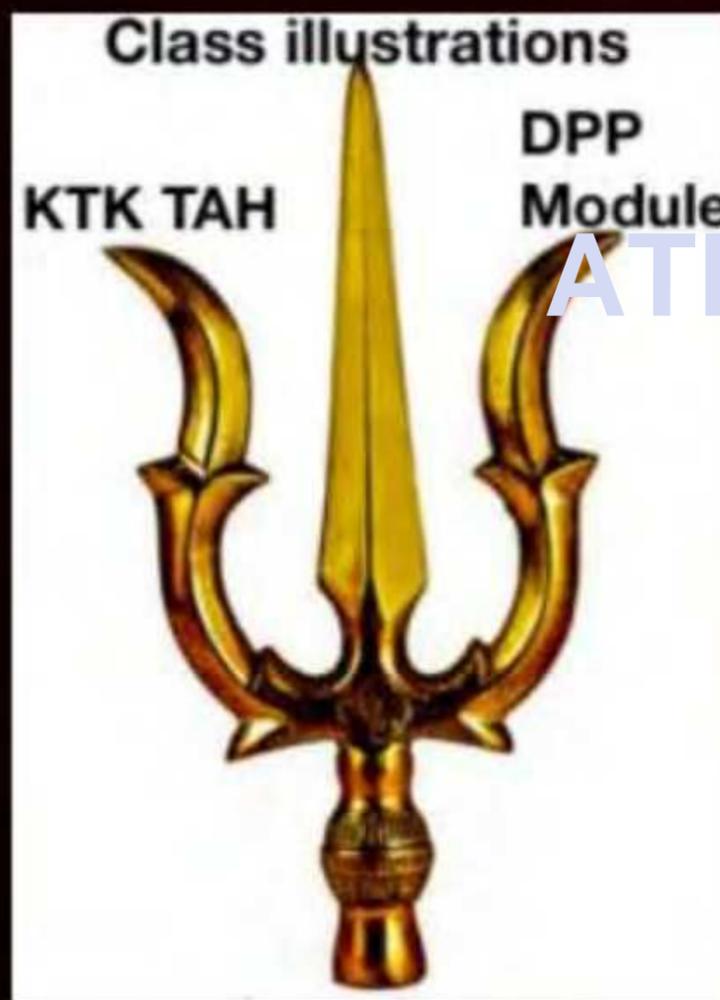
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## Today's KTK



No Selection  $\xrightarrow{\text{TRISHUL Apnao IIT Jao}}$  Selection with Good Rank



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## QUESTION [JEE Mains 2019 (10 April)]



KTKOI

If  $\alpha$  and  $\beta$  are the roots of the quadratic equation,  $x^2 + x \sin \theta - 2 \sin \theta = 0, \theta \in \left(0, \frac{\pi}{2}\right)$ ,

then  $\frac{\alpha^{12} + \beta^{12}}{(\alpha^{-12} + \beta^{-12}) \cdot (\alpha - \beta)^{24}}$  is equal to :

**A**  $\frac{2^{12}}{(\sin \theta - 8)^6}$

**B**  $\frac{2^6}{(\sin \theta + 4)^{12}}$

**C**  $\frac{2^{12}}{(\sin \theta + 8)^{12}}$

**D**  $\frac{2^{12}}{(\sin \theta - 4)^{12}}$

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Ans. C

**QUESTION [JEE Mains 2022 (27 July)]**

KTR02

If  $\alpha, \beta$  are the roots of the equation

$$x^2 - \left(5 + 3\sqrt{\log_3 5} - 5\sqrt{\log_5 3}\right)x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$$

then the equation, whose roots are  $\alpha + \frac{1}{\beta}$  and  $\beta + \frac{1}{\alpha}$ , is :

**A**  $3x^2 - 20x - 12 = 0$

**B**  $3x^2 - 10x - 4 = 0$

**C**  $3x^2 - 10x + 2 = 0$

**D**  $3x^2 - 20x + 16 = 0$

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Ans. B

**QUESTION [JEE Mains 2021]**

KTK03



Let  $\alpha, \beta$  be two roots of the equation  $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$ . Then  $\alpha^8 + \beta^8$  is equal to

- A** 10
- B** 100
- C** 50
- D** 160

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Ans. C

## QUESTION [JEE Mains 2023]

KTR04



Let  $\alpha, \beta$  be the roots of the equation  $x^2 - \sqrt{2}x + 2 = 0$ . Then  $\alpha^{14} + \beta^{14}$  is equal to

- A** -64
- B**  $-64\sqrt{2}$
- C**  $-128\sqrt{2}$
- D** -128

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Ans. D

## QUESTION

KTK 5



If  $x^2 + 3x + 3 = 0$  and  $ax^2 + bx + 1 = 0$ ,  $a, b \in \mathbb{Q}$  have a common root, then value of  $(3a + b)$  is equal to

- A**  $1/3$
- B**  $1$
- C**  $2$
- D**  $4$

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Ans. C



# Solution to Previous TAH

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## QUESTION



Let  $p$  &  $q$  be the two roots of the equation,  $mx^2 + x(2 - m) + 3 = 0$ . Let  $m_1, m_2$  be the two values of  $m$  satisfying  $\frac{p}{q} + \frac{q}{p} = \frac{2}{3}$ . Determine the numerical value of  $\frac{m_1}{m_2^2} + \frac{m_2}{m_1^2}$ .

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# Homework

Mathematics ✨  
 Tah: 01



$$mx^2 + x(2-m) + 3 = 0 \begin{matrix} \curvearrowright p \\ \curvearrowright q \end{matrix} \text{ and } m_1 \text{ and } m_2 \rightarrow \frac{p}{q} + \frac{q}{p} = \frac{2}{3} \rightarrow \frac{m_1}{m_2^2} + \frac{m_2}{m_1^2}$$

$$\frac{p}{q} + \frac{q}{p} = \frac{2}{3}$$

$$\Rightarrow \frac{p^2 + q^2}{pq} = \frac{2}{3}$$

$$\Rightarrow 3(p^2 + q^2) = 2pq$$

$$pq = \frac{3}{m} \text{ and } p+q = \frac{m-2}{m}$$

$$p^2 + q^2 = (p+q)^2 - 2pq$$

$$= \left(\frac{m-2}{m}\right)^2 - \frac{6}{m}$$

$$3(p^2 + q^2) = 2pq$$

$$3\left(\frac{(m-2)^2}{m^2} - \frac{6}{m}\right) = \frac{6}{m}$$

$$\frac{3(m-2)^2}{m^2} - 3\left(\frac{6}{m}\right) = \frac{6}{m}$$

$$\Rightarrow \frac{3(m-2)^2}{m^2} = 4\left(\frac{6}{m}\right)$$

$$\Rightarrow 3m^2 - 12m + 12 = 24m$$

$$\Rightarrow 3m^2 - 36m + 12 = 0$$

$$\Rightarrow m^2 - 12m + 4 = 0 \begin{matrix} \curvearrowright m \\ \curvearrowright m_2 \end{matrix}$$

$$\frac{m_1^3 + m_2^3}{(m_1 m_2)^2}$$

$$m_1 + m_2 = 12 \rightarrow m_1^2 + m_2^2 = 136$$

$$m_1 m_2 = 4$$

$$m_1^3 + m_2^3 = (m_1 + m_2)(m_1^2 + m_2^2 - m_1 m_2)$$

$$= 12(136 - 4)$$

$$= 12 \times 132$$

$$\frac{m_1^3 + m_2^3}{m_1^2 m_2^2}$$

$$= \frac{12 \times 132}{4 \times 4} = 3 \times 33$$

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AKASHI



Let  $p$  &  $q$  be the two roots of the eqn  $mx^2 + x(2-m) + 3 = 0$ . Let  $m_1, m_2$  be the two values of  $m$  satisfying  $\frac{p}{q} + \frac{q}{p} = \frac{2}{3}$ . Determine the numerical value of  $\frac{m_1}{m_2^2} + \frac{m_2}{m_1^2}$ .

Soln

$$mx^2 + x(2-m) + 3 = 0 \begin{cases} p \\ q \end{cases}$$

$$\Rightarrow \frac{p}{q} + \frac{q}{p} = \frac{2}{3}$$

$$\Rightarrow \frac{p^2 + q^2}{pq} = \frac{2}{3}$$

$$\Rightarrow \frac{(p+q)^2 - 2pq}{pq} = \frac{2}{3}$$

$$\Rightarrow \frac{(2-m)^2 - 2(3)}{3} = \frac{2}{3}$$

$$\Rightarrow 4 + m^2 - 4m - 6 = 0$$

$$\Rightarrow m^2 - 4m - 2 = 0 \begin{cases} m_1 \\ m_2 \end{cases}$$

then

$$\frac{m_1}{m_2^2} + \frac{m_2}{m_1^2}$$

$$\boxed{\begin{aligned} m_1 + m_2 &= 4 \\ m_1 \cdot m_2 &= -2 \end{aligned}}$$

$$= \frac{(m_1)^3 + (m_2)^3}{(m_1 \cdot m_2)^2} = \frac{(m_1 + m_2)(m_1^2 + m_2^2 - m_1 m_2)}{(m_1 \cdot m_2)^2}$$

$$= \frac{4((4)^2 + 2 \times 4 + 2)}{(-2)^2}$$

Y29

litish Adalpur Bihar May 15, 2025, 09:22

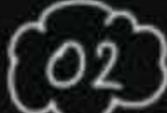
**QUESTION**

Let  $\alpha, \beta$  are the roots of the equation  $x^2 + x - 3 = 0$ . Then the value of  $\alpha^3 - 4\beta^2 + 19$  is equal to

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Ans. 0

# Homework

Mathematics   
Tah:  02



$$x^2 + x - 3 = 0 \begin{matrix} \curvearrowright \alpha \\ \curvearrowleft \beta \end{matrix} \rightarrow \alpha^3 - 4\beta^2 + 19 = ?$$

$$\alpha^2 + \alpha - 3 = 0 \rightarrow -\alpha^2 = \alpha - 3$$

$$\alpha^3 + \alpha^2 - 3\alpha = 0 \rightarrow \alpha^3 = 3\alpha - \alpha^2$$

$$\beta^2 + \beta - 3 = 0$$

$$-4\beta^2 - 4\beta + 12 = 0 \rightarrow -4\beta^2 = 4\beta - 12$$

$$\alpha + \beta = -1$$

$$\alpha\beta = -3$$

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*Adding both we get*

$$\alpha^3 - 4\beta^2 = 3\alpha - \alpha^2 + 4\beta - 12$$

$$\alpha^3 - 4\beta^2 + 19 = 3\alpha - \alpha^2 + 4\beta + 7$$

$$\alpha^3 - 4\beta^2 + 19 = 3\alpha + 3\beta - \alpha^2 + \beta + 7$$

$$= -3 - \alpha^2 + \beta + 7$$

$$= \beta - \alpha^2 + 4$$

$$= \beta + \alpha - 3 + 4$$

$$= -4 + 4$$

$$\alpha^3 - 4\beta^2 + 19 = 0 \text{ Ans}$$

AKASHI

\* Ques-023-

Let  $\alpha, \beta$  are the roots of the equation  $x^2 + x - 3 = 0$ .  
Then the value of  $\alpha^3 - 4\beta^2 + 19$  is equal to.

$$x^2 + x - 3 = 0 \quad \left[ \begin{array}{l} \alpha \\ \beta \end{array} \right]$$

$$\left\{ \begin{array}{l} \alpha + \beta = -1, \alpha\beta = -3 \end{array} \right\}$$

$$\alpha^2 + \alpha - 3 = 0$$

$$\alpha \times \rightarrow \alpha^3 + \alpha^2 - 3\alpha = 0$$

$$\alpha^3 = 3\alpha - \alpha^2$$

$$\text{Similarly: } \beta^2 + \beta - 3 = 0$$

$$\beta^2 = 3 - \beta$$

Then:-

$$\Rightarrow \alpha^3 - 4\beta^2 + 19$$

$$\Rightarrow 3\alpha - \alpha^2 - 12 + 4\beta + 19$$

$$\Rightarrow -\alpha^2 + 3\alpha + 4\beta + 7$$

$$\Rightarrow -\alpha^2 - \alpha + 4\alpha + 4\beta + 7$$

$$\Rightarrow -\alpha^2 - \alpha + 4(\alpha + \beta) + 7$$

$$\Rightarrow -\alpha^2 - \alpha + 4(-1) + 7$$

$$\Rightarrow -\alpha^2 - \alpha - 4 + 7$$

$$\Rightarrow -\alpha^2 - \alpha + 3 = 0$$

$$\Rightarrow \alpha^2 + \alpha - 3 = 0$$

**QUESTION**

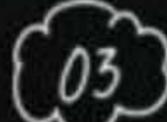
$$P(x) = x^3 + 33x^2 + 327x + 935$$

Let  $P(x)$  be a polynomial as described above with  $a, b, c$  the roots of  $P(x)$ .

Find  $a^2 + b^2 + c^2$  without solving  $P(x) = 0$ .

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# Homework

Mathematics   
Tah :  03

$$P(x) = x^3 + 33x^2 + 327x + 935 \begin{matrix} \curvearrowright a \\ \longrightarrow b \\ \curvearrowleft c \end{matrix}$$

$$a^2 + b^2 + c^2 = ?$$

$$a^2 + b^2 + c^2 = (a+b+c)^2 - 2(ab+bc+ca)$$

$$\begin{aligned} a+b+c &= -33 & \Rightarrow a^2 + b^2 + c^2 &= 33 \times 33 - 2 \times 327 \\ ab+bc+ca &= 327 & &= 99 - 654 \\ & & &+ 990 \\ & & &= 1089 - 654 \\ & & &= \boxed{435} \text{ Ans} \end{aligned}$$

$$P(x) = x^3 + 33x^2 + 327x + 935$$

let  $P(x)$  be a polynomial as described above with  $a, b, c$  the roots of  $P(x)$   
find  $a^2 + b^2 + c^2$  without sol<sup>n</sup> ( $P(x) = 0$ )

Sol<sup>n</sup>

$$P(x) = x^3 + 33x^2 + 327x + 935 \begin{matrix} \swarrow a \\ \rightarrow b \\ \searrow c \end{matrix}$$

$$a + b + c = -33$$

$$ab + bc + ca = 327$$

$$a \cdot b \cdot c = -935$$

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Tah-03

$$\begin{aligned} \Rightarrow a^2 + b^2 + c^2 &= (a + b + c)^2 - 2(ab + bc + ca) \\ &= (-33)^2 - 2(327) \end{aligned}$$

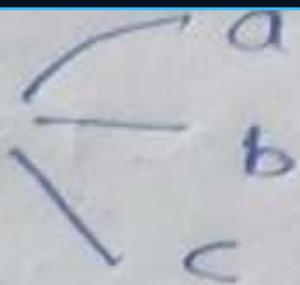
$$= 1089 - 654$$

$$= \underline{435} \quad \text{Ans}$$

$$\begin{array}{r} 33 \\ 33 \\ \hline 99 \\ 99 \\ \hline 198 \end{array}$$

Q.11 - 05

Given,  $P(x) = x^3 + 33x^2 + 327x + 935$



$$a + b + c = -33$$

$$ab + bc + ca = 327$$

$$abc = 935$$

vandana  
from Bihar

Now,  $a + b + c =$  **ATDB.uno**

S.B.S

$$a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1089$$

$$a^2 + b^2 + c^2 + 2 \times 327 = 1089$$

$$a^2 + b^2 + c^2 = 1089 - 654$$

$$\boxed{a^2 + b^2 + c^2 = 435} \quad \underline{\underline{\text{Ans}}}$$

## QUESTION



Let  $r_1, r_2$  and  $r_3$  be the roots of the polynomial  $5x^3 - 11x^2 + 7x + 3$ .  
Evaluate  $r_1(1 + r_2 + r_3) + r_2(1 + r_3 + r_1) + r_3(1 + r_1 + r_2)$ .

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# Homework

Mathematics ✨  
Tah: 04

$$5x^3 - 11x^2 + 7x + 3 \begin{matrix} \nearrow \pi_1 \\ \rightarrow \pi_2 \\ \searrow \pi_3 \end{matrix} \rightarrow \boxed{\pi_1(1 + \pi_2 + \pi_3) + \pi_2(1 + \pi_3 + \pi_1) + \pi_3(1 + \pi_1 + \pi_2)} = ? \quad \xrightarrow{E}$$

$$\begin{aligned} & \pi_1(1 + \pi_2 + \pi_3) + \pi_2(1 + \pi_3 + \pi_1) + \pi_3(1 + \pi_1 + \pi_2) \\ &= \pi_1 + \pi_2 + \pi_3 + \pi_1\pi_2 + \pi_1\pi_3 + \pi_2\pi_3 + \pi_1\pi_2 + \pi_1\pi_3 + \pi_2\pi_3 \\ &= \pi_1 + \pi_2 + \pi_3 + 2(\pi_1\pi_2 + \pi_2\pi_3 + \pi_3\pi_1) \end{aligned}$$

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$$\pi_1 + \pi_2 + \pi_3 = \frac{11}{5} ; \pi_1\pi_2 + \pi_2\pi_3 + \pi_3\pi_1 = \frac{7}{5}$$

$$\Rightarrow E = \frac{11}{5} + \frac{14}{5} = 5 \text{ Ans}$$



Let,  $\alpha_1, \alpha_2$  and  $\alpha_3$  be the roots of the Polynomial  $5x^3 - 11x^2 + 7x + 3$ . Evaluate -  
 $\alpha_1(1 + \alpha_2 + \alpha_3) + \alpha_2(1 + \alpha_3 + \alpha_1) + \alpha_3(1 + \alpha_1 + \alpha_2)$

Sol<sup>n</sup>

$$P(x) = 5x^3 - 11x^2 + 7x + 3 \begin{matrix} \swarrow \alpha_1 \\ \swarrow \alpha_2 \\ \swarrow \alpha_3 \end{matrix}$$

$$\begin{aligned} \alpha_1 + \alpha_2 + \alpha_3 &= 11 \\ \alpha_1 \cdot \alpha_2 + \alpha_2 \cdot \alpha_3 + \alpha_3 \cdot \alpha_1 &= 7 \\ \alpha_1 \cdot \alpha_2 \cdot \alpha_3 &= -3 \end{aligned}$$

Tak-04

$$E = \alpha_1(1 + \alpha_2 + \alpha_3) + \alpha_2(1 + \alpha_3 + \alpha_1) + \alpha_3(1 + \alpha_1 + \alpha_2)$$

$$= \alpha_1 + \alpha_1\alpha_2 + \alpha_1\alpha_3 + \alpha_2 + \alpha_2\alpha_3 + \alpha_1\alpha_2 + \alpha_3 + \alpha_1\alpha_3 + \alpha_2\alpha_3$$

$$= (\alpha_1 + \alpha_2 + \alpha_3) + (\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_1\alpha_3) + (\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_3\alpha_1)$$

$$= 11 + 7 + 7$$



**QUESTION**

If  $\alpha$ ,  $\beta$  and  $\gamma$  are roots of cubic equation  $x^3 + 3x - 1 = 0$  then find value of :

(i)  $(2 - \alpha)(2 - \beta)(2 - \gamma)$

(ii)  $(3 + \alpha)(3 + \beta)(3 + \gamma)$

(iii)  $(4 - \alpha^2)(4 - \beta^2)(4 - \gamma^2)$

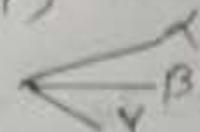
(iv)  $(1 + \alpha^2)(1 + \beta^2)(1 + \gamma^2)$

# ATDB.uno

cubic eqn  $x^3 + 3x - 1 = 0$  then find value of

(i)  $(2-\alpha)(2-\beta)(2-\gamma)$

Sol<sup>n</sup>

$x^3 + 3x - 1 = 0$   Tak-05

$P(x) = x^3 + 3x - 1 = (x-\alpha)(x-\beta)(x-\gamma)$

$P(2) = (2)^3 + 3(2) - 1 = (2-\alpha)(2-\beta)(2-\gamma)$

$= 8 + 6 - 1 = (2-\alpha)(2-\beta)(2-\gamma)$

$= 13 = (2-\alpha)(2-\beta)(2-\gamma)$

(ii)  $(3+\alpha)(3+\beta)(3+\gamma)$

$$\begin{aligned} \alpha + \beta + \gamma &= 0 \\ \alpha\beta + \beta\gamma + \gamma\alpha &= 3 \\ \alpha \cdot \beta \cdot \gamma &= -1 \end{aligned}$$

$= (9 + 3\beta + 3\alpha + \alpha\beta)(3+\gamma)$

$= (27 + 9\gamma + 9\beta + 3\beta\gamma + 9\alpha + 3\alpha\gamma + 3\alpha\beta + \alpha\beta\gamma)$

$= (27 + 9(\alpha + \beta + \gamma) + 3(\alpha\beta + \beta\gamma + \gamma\alpha) + 1)$

$\Rightarrow (27 + 9(0) + 3(3) + 1)$

$= 27 + 9 + 1$

$= \underline{37}$



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(iii)  $(4-\alpha^2)(4-\beta^2)(4-\gamma^2)$

$\Rightarrow (16 - 4\beta^2 - 4\alpha^2 + 4\alpha^2\beta^2)(4-\gamma^2)$  Tak-05

$\Rightarrow (64 - 16\gamma^2 - 16\beta^2 + 4\beta^2\gamma^2 - 16\alpha^2 + 4\alpha^2\gamma^2 + 4\alpha^2\beta^2 - \alpha^2\beta^2\gamma^2)$

$\Rightarrow (64 - 16(\alpha^2 + \beta^2 + \gamma^2) + 4(\alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2) - \alpha^2\beta^2\gamma^2)$

$\Rightarrow (64 - 16\{(0 - 2(3))\} + 4(9) - (1)^2)$

$\Rightarrow 64 - 16(6) + 36 - 1$

$= 64 - 96 + 36 - 1$

$\Rightarrow 100 - 97$

$\Rightarrow \underline{3}$

$= \underline{3}$  Ans



(iv)  $(1+\alpha^2)(1+\beta^2)(1+\gamma^2)$

$\Rightarrow (1 + \gamma^2 + \beta^2 + \beta^2\gamma^2 + \alpha^2 + 2\alpha^2\gamma^2 + \alpha^2\beta^2 + \alpha^2\beta^2\gamma^2)$

$\Rightarrow (1 + (\alpha^2 + \beta^2 + \gamma^2) + 2(\alpha^2\beta^2 + \beta^2\gamma^2 + \alpha^2\gamma^2) + (\alpha\beta\gamma)^2)$

$\Rightarrow (1 + (6) + 2(9) + (1)^2)$

$\Rightarrow \underline{17}$  Ans

VO Y29

la Nitish Adalpur Bihar May 15, 2025, 09:26



# Homework

Mathematics ✨  
Tah: 05

$$x^3 + 3x - 1 = 0 \begin{matrix} \nearrow \alpha \\ \rightarrow \beta \\ \searrow \gamma \end{matrix} \rightarrow \begin{matrix} (a) (2-\alpha)(2-\beta)(2-\gamma) & (c) (4-\alpha^2)(4-\beta^2)(4-\gamma^2) \\ (b) (3+\alpha)(3+\beta)(3+\gamma) & (d) (1+\alpha^2)(1+\beta^2)(1+\gamma^2) \end{matrix}$$

$$\alpha + \beta + \gamma = 0 ; \alpha\beta + \beta\gamma + \alpha\gamma = 3 ; \alpha\beta\gamma = 1$$

$$\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha) = -6$$

$$\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 = (\alpha\beta + \beta\gamma + \gamma\alpha)^2 - 2\alpha\beta\gamma(\alpha + \beta + \gamma) = 9$$

$$(c) (4-\alpha^2)(4-\beta^2)(4-\gamma^2) = (2+\alpha)(2+\gamma)(2+\beta)(2-\alpha)(2-\beta)(2-\gamma)$$

$$= 13(2+\alpha)(2+\beta)(2+\gamma)$$

for this we can replace -ve with +ve in (1)  
 $= 13(8 + 4(\alpha + \beta + \gamma) + 2(\alpha\beta + \beta\gamma + \gamma\alpha) + \alpha\beta\gamma)$   
 $= 13(9 + 6) = 13 \times 15 = 195$  Ans

ATDB.uno

$$(a) (2-\alpha)(2-\beta)(2-\gamma) = (4 - 2\alpha - 2\beta + \alpha\beta)(2-\gamma)$$

$$= 8 - 4\alpha - 4\beta + 2\alpha\beta - 4\gamma + 2\alpha\gamma + 2\beta\gamma - \alpha\beta\gamma$$

$$= 8 - 4(\alpha + \beta + \gamma) + 2(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma$$

$$= 7 - 4(0) + 2(3)$$

$$= 13$$
 Ans

$$(d) (1+\alpha^2)(1+\beta^2)(1+\gamma^2) = (1 + \beta^2 + \alpha^2 + \alpha^2\beta^2)(1+\gamma^2)$$

$$= 1 + \beta^2 + \alpha^2 + \alpha^2\beta^2 + \gamma^2 + \gamma^2\beta^2 + \gamma^2\alpha^2 + \gamma^2\beta^2\alpha^2$$

$$= 2 + (-6) + 9$$

$$= 5$$
 Ans

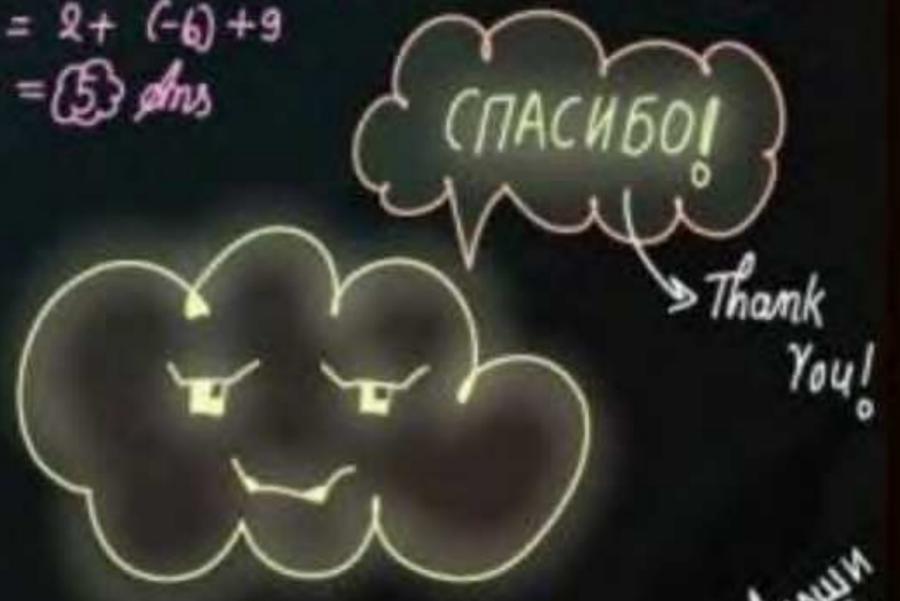
$$(b) (3+\alpha)(3+\beta)(3+\gamma) = (9 + 3\beta + 3\alpha + \alpha\beta)(3+\gamma)$$

$$= 27 + 9\beta + 9\alpha + 3\alpha\beta + 9\gamma + 3\beta\gamma + 3\alpha\gamma + \alpha\beta\gamma$$

$$= 27 + 9(\alpha + \beta + \gamma) + 3(\alpha\beta + \beta\gamma + \gamma\alpha) + \alpha\beta\gamma$$

$$= 28 + 9(0) + 3(3)$$

$$= 28 + 9 = 37$$
 Ans



АКАШИ

## QUESTION



If  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$  are roots of equation  $x^5 - 5x^4 - 1 = 0$ , then

**A**  $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -\frac{1}{20}$

**B**  $\sum_{r=1}^{r=5} \frac{1}{\alpha_r^4} = -20$

**C**  $\prod_{r=1}^{r=5} \left( \frac{1}{\alpha_r^4} + 5 \right)^5 = 1$

**D**  $\prod_{r=1}^{r=5} \left( \frac{1}{\alpha_r^4} + 5 \right)^3 = \frac{1}{5}$

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# Home challenge

# if  $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$  are roots of -  
 eqn  $x^5 - 5x^4 - 1 = 0$  then.

So  $x^5 - 5x^4 - 1 = 0$

$$\alpha_1^5 - 5\alpha_1^4 - 1 = 0$$

$$\alpha_1^4(\alpha_1 - 5) = 1$$

$$\alpha_1^4 = \frac{1}{\alpha_1 - 5}$$

(i)  $\sum_{r=1}^5 \frac{1}{\alpha_r^4} = \frac{1}{20}$

$$\Rightarrow \frac{1}{\alpha_1^4} + \frac{1}{\alpha_2^4} + \frac{1}{\alpha_3^4} + \frac{1}{\alpha_4^4} + \frac{1}{\alpha_5^4}$$

$$\Rightarrow \alpha_1 - 5 + \alpha_2 - 5 + \alpha_3 - 5 + \alpha_4 - 5 + \alpha_5 - 5$$

$$\Rightarrow \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 - 25$$

$$= 5 - 25$$

$$= -20$$

1st option  $\rightarrow$  wrong.

(ii)  $\sum_{r=1}^5 \frac{1}{\alpha_r^4} = -20$  2nd option  $\checkmark$

(iii)  $\prod_{r=1}^5 \left( \frac{1}{\alpha_r^4} + 5 \right) = 1$

$$\Rightarrow \left( \frac{1}{\alpha_1^4} + 5 \right) \left( \frac{1}{\alpha_2^4} + 5 \right) \left( \frac{1}{\alpha_3^4} + 5 \right) \left( \frac{1}{\alpha_4^4} + 5 \right) \left( \frac{1}{\alpha_5^4} + 5 \right)$$

$$\Rightarrow (\alpha_1 - 5 + 5) (\alpha_2 - 5 + 5) (\alpha_3 - 5 + 5) (\alpha_4 - 5 + 5) (\alpha_5 - 5 + 5)$$



# Home challenge

$$\Rightarrow \alpha_1 \cdot \alpha_2 \cdot \alpha_3 \cdot \alpha_4 \cdot \alpha_5$$

$$\Rightarrow -(-1) = \underline{1}$$

option  $\rightarrow$  B and C is right

Use Newton's formula :-

$$ax^2 + bx + c = 0 \begin{cases} \alpha \\ \beta \end{cases} \quad S_m = p\alpha^m + q\beta^m$$

$p, q$  are constant

$$a\alpha^2 + b\alpha + c = 0 \quad a\beta^2 + b\beta + c = 0$$

## QUESTION [JEE Mains 2024 (6 April)]



Let  $x_1, x_2, x_3, x_4$  be the solution of the equation  $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$  and  $(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2) = \frac{125}{16}m$ . Then the value of  $m$  is

$$P(-1) = 4 - 8 - 17 + 12 + 9 = 0 \rightarrow (x+1) \text{ is a factor of } P(x)$$

M1

$$4x^3(x+1) + 4x^2(x+1) - 21x(x+1) + 9(x+1) = 0$$

$$(x+1)(4x^3 + 4x^2 - 21x + 9) = 0 \quad P(-3) = -108 + 36 + 63 + 9 = 0$$

$$(x+1)(4x^2(x+3) - 8x(x+3) + 3(x+3)) = 0$$

$$(x+1)(x+3)(4x^2 - 8x + 3) = 0$$

$$(x+1)(x+3)(4x^2 - 6x - 2x + 3) = 0$$

$$(x+1)(x+3)(2x-1)(2x-3) = 0$$

$$x = -1, -3, \frac{1}{2}, \frac{3}{2}$$

Ans. 221

## QUESTION [JEE Mains 2024 (6 April)]



$$i = \sqrt{-1}$$

$$i^2 = -1$$

Let  $x_1, x_2, x_3, x_4$  be the solution of the equation  $4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0$  and  $(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2) = \frac{125}{16}m$ . Then the value of  $m$  is

M②

$$P(x) = 4(x - x_1)(x - x_2)(x - x_3)(x - x_4)$$

put  $x = 2i$

$$P(2i) = 4(2i - x_1)(2i - x_2)(2i - x_3)(2i - x_4)$$

put  $x = -2i$

$$P(-2i) = 4(2i + x_1)(2i + x_2)(2i + x_3)(2i + x_4)$$

$$P(2i) \cdot P(-2i) = 16(-4 - x_1^2)(-4 - x_2^2)(-4 - x_3^2)(-4 - x_4^2)^2$$

$$P(2i)P(-2i) = 16(4 + x_1^2)(4 + x_2^2)(4 + x_3^2)(4 + x_4^2)$$

$$(2i - x_1)(2i + x_1)$$

$$= (2i)^2 - x_1^2$$

$$= 4i^2 - x_1^2$$

$$= -4 - x_1^2$$

Ans. 221



$$4 + x_1^2 = x_1^2 - (2i)^2 = (x - 2i)(x + 2i)$$

$$x^2 + a^2 = x^2 - (ia)^2 = (x - ia)(x + ia)$$

$$x^2 - a^2 = (x - a)(x + a)$$

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# Home work

Mathematics ✨  
Tah: 06



$$4x^4 + 8x^3 - 17x^2 - 12x + 9 = 0 \xrightarrow{\begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{matrix}} (4+x_1^2)(4+x_2^2)(4+x_3^2)(4+x_4^2) = \frac{125}{16} m \text{ find value of } m$$

$$(a^2+x^2) = (ai-x)(ai+x)$$

$$(4+x_1^2)(4+x_2^2)(4+x_3^2)(4+x_4^2) = \boxed{(2i-x_1)(2i-x_2)(2i-x_3)(2i-x_4)(-2i-x_1)(-2i-x_2)(-2i-x_3)(-2i-x_4)} \xrightarrow{M}$$

$$4(x-x_1)(x-x_2)(x-x_3)(x-x_4) = 4x^4 + 8x^3 - 17x^2 - 12x + 9$$

putting  $x = 2i$

$$E_1 = 4(2i-x_1)(2i-x_2)(2i-x_3)(2i-x_4) = 4(2i)^4 + 8(2i)^3 - 17(2i)^2 - 12(2i) + 9$$

$$= 4(16i^4) + 8(8i^3) - 17(4i^2) - 12(2i) + 9$$

$$= 64 - 64i + 68 - 24i + 9$$

$$= 141 - 88i$$

putting  $x = -2i$

$$E_2 = 4(-2-x_1)(-2-x_2)(-2-x_3)(-2-x_4) = 4(-2i)^4 + 8(-2i)^3 - 17(-2i)^2 - 12(-2i) + 9$$

$$= 64 + 64i + 68 + 24i + 9$$

$$= 141 + 88i$$

multiplying  $E_1$  and  $E_2$

$$16(m) = (141 - 88i)(141 + 88i)$$

$$= (141)^2 - (88)^2$$

$$= 141 + 5640 + 14100 + (90 - 2) 88$$

$$= 19881 + 7920 - 176$$

$$= 19881 + 7744$$

$$16m = 27625$$

$$m = \frac{27625}{16}$$

$$M = \frac{125 \times 221}{16}$$

$m = 221$  Ans

$$\begin{array}{r} 221 \\ 125 \overline{) 27625} \\ \underline{250} \phantom{00} \\ 262 \phantom{00} \\ \underline{250} \phantom{00} \\ 125 \phantom{00} \end{array}$$

## QUESTION [JEE Mains 2024 (9 April)]

★★★★ASRQ★★★★



Let  $\alpha, \beta; \alpha > \beta$ , be the roots of the equation  $x^2 - \sqrt{2}x - \sqrt{3} = 0$ .

Let  $P_n = \alpha^n - \beta^n, n \in \mathbb{N}$ . Then  $(11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12}$  is equal to

**A**  $10\sqrt{3}P_9$

**B**  $11\sqrt{3}P_9$

**C**  $11\sqrt{2}P_9$

**D**  $10\sqrt{2}P_9$

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Ans. A

# Homework

Mathematics   
Tah:  07



$$x^2 - \sqrt{2}x - \sqrt{3} = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \rightarrow \alpha > \beta \rightarrow P_n = \alpha^n - \beta^n \rightarrow (11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12} = ?$$

$$P_{12} - \sqrt{2}P_{11} - \sqrt{3}P_{10} = 0 \Rightarrow 11P_{12} - 11\sqrt{2}P_{11} - 11\sqrt{3}P_{10} = 0 \quad \text{--- (1)}$$

$$P_{11} - \sqrt{2}P_{10} - \sqrt{3}P_9 = 0 \Rightarrow 10P_{11} - 10\sqrt{2}P_{10} - 10\sqrt{3}P_9 = 0 \quad \text{--- (2)}$$

$$(2) - (1)$$

$$= 10P_{11} - 10\sqrt{2}P_{10} - 10\sqrt{3}P_9 - 11P_{12} + 11\sqrt{2}P_{11} + 11\sqrt{3}P_{10} = 0$$

$$= (11\sqrt{3} - 10\sqrt{2})P_{10} + (11\sqrt{2} + 10)P_{11} - 11P_{12} = 10\sqrt{3}P_9 \quad \text{(A) } \checkmark$$

**QUESTION [JEE Mains 2025 (3 April)]**

Let  $\alpha$  and  $\beta$  be the roots of  $x^2 + \sqrt{3}x - 16 = 0$ , and  $\gamma$  and  $\delta$  be the roots of

$x^2 + 3x - 1 = 0$ . If  $P_n = \alpha^n + \beta^n$  and  $Q_n = \gamma^n + \delta^n$ , then  $\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$

is equal to

- A** 4
- B** 3
- C** 5
- D** 7

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Ans. C

# Homework

Mathematics   
Tah: 08



$$x^2 + \sqrt{3}x - 16 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \text{ and } x^2 + 3x - 1 = 0 \begin{matrix} \nearrow \gamma \\ \searrow \delta \end{matrix} \text{ and } P_n = \alpha^n + \beta^n \text{ and } Q_n = \gamma^n + \delta^n \rightarrow \frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25} - Q_{23}}{Q_{24}}$$

$$P_{25} + \sqrt{3}P_{24} - 16P_{23} = 0$$

$$P_{25} + \sqrt{3}P_{24} = 16P_{23}$$

$$\frac{P_{25} + \sqrt{3}P_{24}}{2P_{23}} = 8$$

$$Q_{25} + 3Q_{24} - Q_{23} = 0$$

$$\frac{Q_{25} - Q_{23}}{Q_{24}} = -3$$

adding both we get

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$$8 - 3 = 5 \text{ Ans } \textcircled{e}$$

# QUESTION [JEE Mains 2025 (2 April)]



Let  $P_n = \alpha^n + \beta^n, n \in \mathbb{N}$ . If  $P_{10} = 123, P_9 = 76, P_8 = 47$  and  $P_1 = 1$ , then the quadratic equation having roots  $\frac{1}{\alpha}$  and  $\frac{1}{\beta}$  is :

**A**  $x^2 + x - 1 = 0$

**B**  $x^2 - x + 1 = 0$

**C**  $x^2 + x + 1 = 0$

**D**  $x^2 - x - 1 = 0$

let  $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$P_1 = \alpha + \beta = 1 = -\frac{b}{a}$

$\frac{b}{a} = -1$

$x^2 - x + \frac{c}{a} = 0$

NF

$P_{n+2} - P_{n+1} + \frac{c}{a}P_n = 0$

$n=8$

$P_{10} - P_9 + \frac{c}{a}P_8 = 0$

$123 - 76 + 47 \frac{c}{a} = 0$

$47 \frac{c}{a} = -47$

$\frac{c}{a} = -1$

Eqn:  $x^2 - x - 1 = 0$   $\left\{ \begin{array}{l} \alpha \\ \beta \end{array} \right\} \rightarrow \begin{array}{l} \alpha + \beta = 1 \\ \alpha\beta = -1 \end{array}$

S.O.R =  $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{1}{-1} = -1$

P.O.R =  $\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = -1$

Eqn:  $x^2 + x - 1 = 0$

Ans. A



# Solution to Previous KTKs

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## QUESTION

(KTK 1)



If  $\alpha, \beta$  are the roots of the equation  $x^2 + px - r = 0$  and  $\frac{\alpha}{3}, 3\beta$  are the roots of the equation  $x^2 + qx - r = 0$ , then  $r$  equals

- A**  $\frac{3}{8}(p - 3q)(3p + q)$
- B**  $\frac{3}{8}(p + 3q)(3p - q)$
- C**  $\frac{3}{64}(3p - q)(p - 3q)$
- D**  $\frac{3}{64}(3q - p)(p - q)$

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Ans. C

## QUESTION

(KTK 2)



If one of the root of the equation  $4x^2 - 15x + 4p = 0$  is the square of the other then the value of p is

**A**  $\frac{125}{64}$

**B**  $-\frac{27}{8}$

**C**  $-\frac{125}{8}$

**D**  $\frac{27}{8}$

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Ans. C, D



KTR-02

$$4x^2 - 15x + 4P = 0 \Rightarrow \alpha, \beta, \alpha^2 = \beta$$

$$4\alpha^2 - 15\alpha + 4P = 0$$

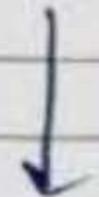
$$\alpha + \beta = \frac{15}{4}$$

$$\alpha^2 + \alpha = \frac{15}{4}$$

$$\alpha \cdot \beta = \frac{4P}{4}$$

$$\alpha^3 = P$$

Now,



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$$4\alpha^2 + 4\alpha = 15$$

$$4\alpha^2 + 4\alpha - 15 = 0$$

$$4\alpha^2 + 10\alpha - 6\alpha - 15 = 0$$

$$2\alpha(2\alpha + 5) - 3(2\alpha + 5) = 0$$

$$(2\alpha + 5)(2\alpha - 3) = 0$$

$$\alpha = \frac{3}{2}, -\frac{5}{2}$$

$$\alpha^3 = \frac{27}{8}$$

$$\alpha^3 = -\frac{125}{8}$$

$$P = \frac{27}{8}, -\frac{125}{8}$$

**Lakshya  
From Raj**

## QUESTION

(KTK 3)



If  $b \in \mathbb{R}^+$  then roots of the equation  $(2 + b)x^2 + (3 + b)x + (4 + b) = 0$  is

- A** Real and distinct
- B** Real and equal
- C** Imaginary
- D** Cannot be predicted

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Ans. C



KTK-03

Q)  $(2+b)x^2 + (3+b)x + (4+b) = 0 \quad \forall x \in \mathbb{R}^+$

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$$\begin{aligned} D &= (b+3)^2 - 4(b+2)(b+4) \\ &= b^2 + 6b + 9 - 4(b^2 + 6b + 8) \\ &= b^2 + 6b + 9 - 4b^2 - 24b - 32 \end{aligned}$$

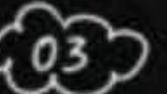


$$D = -3b^2 - 19b - 23 < 0$$

$$b \in \mathbb{R}^+, \quad b^2 \in \mathbb{R}^+$$

So, roots are imaginary.

# Homework

Mathematics   
KTK 03 



$$b \in \mathbb{R}^+ \rightarrow (2+b)x^2 + (3+b)x + (4+b) = 0$$

$$(3+b)^2 - 4(4+b)(2+b)$$

$$= 9 + b^2 + 6b - 4(8 + 6b + b^2)$$

$$= 9 + b^2 + 6b - 32 - 24b - 4b^2$$

$$= -3b^2 - 18b - 23$$

$$= -(3b^2 + 18b + 23)$$

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$b \in \mathbb{R}^+$

$$= -(\text{+ve})$$

$$= -ve$$

imaginary roots

## QUESTION

(KTK 4)



If  $x$  satisfies  $|x - 1| + |x - 2| + |x - 3| \geq 6$ , then

- A**  $0 \leq x \leq 4$
- B**  $x \leq -2$  or  $x \geq 4$
- C**  $x \leq 0$  or  $x \geq 4$
- D** none

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Ans. C



KTR-04

Lakshya  
From Raj

$$|x-1| + |x-2| + |x-3| \geq 6$$

$$|x-1| + |x-2| + |x-3| - 6 \geq 0$$

$T_1$                        $T_2$                        $T_3$

	1	2	3
$T_1$	-	+	+
$T_2$	÷	÷	+
$T_3$	÷	÷	+

$x \leq 1 \rightarrow$  Case (i)

$$-x+1 -x+2 -x+3 -6 \geq 0$$

$$-3x \geq 0$$

$$3x \leq 0$$

$$x \leq 0 \rightarrow x \in (-\infty, 0]$$

$1 < x \leq 2 \rightarrow$  Case (ii)

$$x-1 -x+2 -x+3 -6 \geq 0$$

$$-x -2 \geq 0$$

$$x+2 \leq 0$$

$$x \leq -2$$

}  $x \in \phi$

Case (III)  $2 < n < 3$

$$n-1 + n-2 - n + 3 - 6 \geq 0$$

$$n - 6 \geq 0$$

$$n \geq 6$$

$$n \rightarrow \emptyset \in \mathbb{R}$$

Case (IV)  $n \geq 3$

$$n-1 + n-2 + n-3 - 6 \geq 0$$

$$3n \geq 12$$

$$n \geq 4$$

$$n \rightarrow n \geq 4$$

(I)  $\cup$  (II)  $\cup$  (III)  $\cup$  (IV)

$$x \in (-\infty, 0] \cup [4, \infty)$$

Lakshya  
From Raj



# Home work

$$|x-1| + |x-2| + |x-3| \geq 6$$

Case I  $x \geq 3$

$$x-1 + x-2 + x-3 \geq 6$$

$$3x - 6 \geq 6$$

$$3x \geq 12$$

$$\boxed{x \geq 4}$$

Case II

$$2 \leq x < 3$$

$$x-1 + x-2 - x+3 \geq 6$$

$$x \geq 6$$

$$x \in \emptyset$$

Case III

$$1 \leq x \leq 2$$

$$x-1 - x+2 - x+3 \geq 6$$

$$-x + 4 \geq 6$$

$$-x \geq 2$$

$$x \leq -2$$

$$x \in \emptyset$$

Case IV

$$x \leq 1$$

$$-3x + 6 \geq 6$$

$$x \leq 0$$

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Union of values of  $x = x \in (-\infty, 0] \cup [4, \infty)$   $\cup \emptyset$

Mathematics 

KTK  04



## QUESTION

(KTK 5)



Number of integral values of 'a' for which the quadratic equation,  
 $2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$  possesses roots of opposite sign is,

**A** 1

**B** 2

**C** 3

**D** 4

$P \cdot O \cdot R < 0 \Rightarrow$  Root can not be imaginary

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KTK-5

$$2x^2 - (a^3 + 8a - 1)x + a^2 - 4a = 0$$

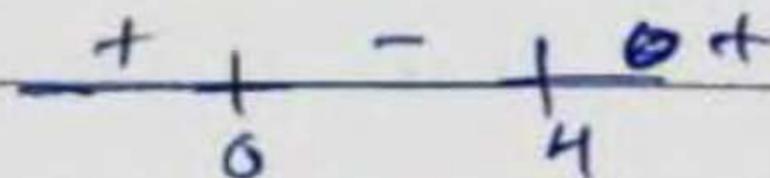
real & distinct roots  $\rightarrow D > 0$

also,

•  $\alpha, \beta$  signs are opposite  $\therefore$

$$\alpha \cdot \beta = -ve = \frac{a^2 - 4a}{2} < 0$$

$$a(a-4) < 0$$



$$a \in (0, 4)$$





So, Integral values of  $0 = 1, 2, 3$

no. of Integral values of  $0 = 3$



# Homework

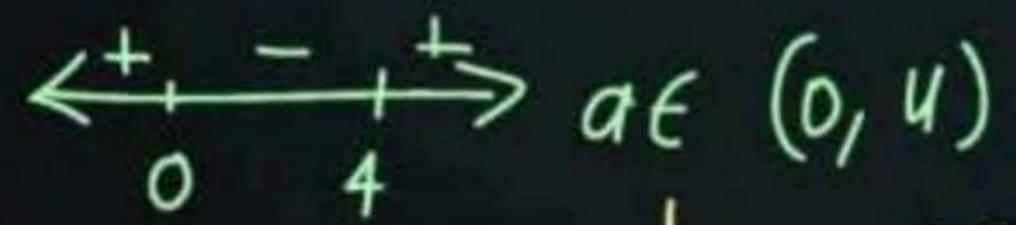
Mathematics ✨  
KTK ☁️ 05

$2x^2 - (a^2 + 8a - 1)x + a^2 + a = 0 \rightarrow$  values of  $a$  for which  $\rightarrow$  Sign of roots are opposite  
 $\rightarrow \alpha, \beta \rightarrow \alpha(+ve) \beta(-ve)$

$\alpha\beta \rightarrow (-ve)$   
 $\alpha\beta \leq 0$  }  $\frac{a^2 - 4a}{2} < 0$   
 $a(a-4) < 0$

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Two roots of opposite sign can have +ve, -ve and 0 as their sum  
So  $a$  need to satisfy only one cond<sup>n</sup>.



$\hookrightarrow a = 1, 2, 3 \Rightarrow \textcircled{3} \textcircled{0} \checkmark$

AKALI

## QUESTION

(KTK 6)



If  $a, b, c$  are real numbers satisfying the condition  $a + b + c = 0$  then the roots of the quadratic equation  $3ax^2 + 5bx + 7c = 0$  are

- A** positive
- B** negative
- C** real and distinct
- D** imaginary

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Ans. C



## Homework From Module



Basic math

**Prarambh (Topicwise) : Q1 to Q32**

**Prabal (JEE Main Level) : Q1 to Q46**

**Parikshit (JEE Advanced Level) : Q1 to Q42**

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**THANK**  
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**YOU**