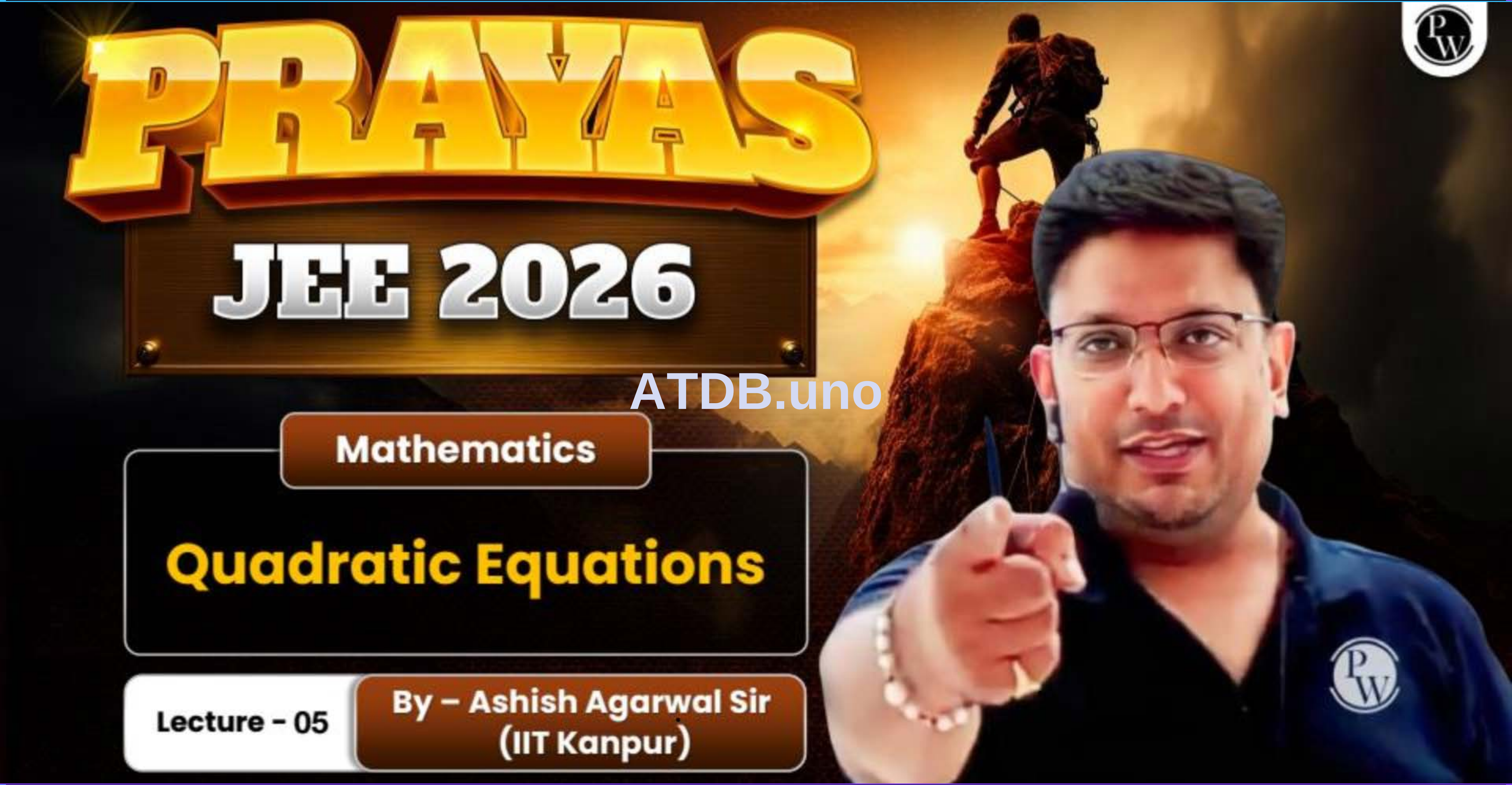

PRAYAS

JEE 2026

Mathematics

Quadratic Equations

Lecture - 05

By – Ashish Agarwal Sir
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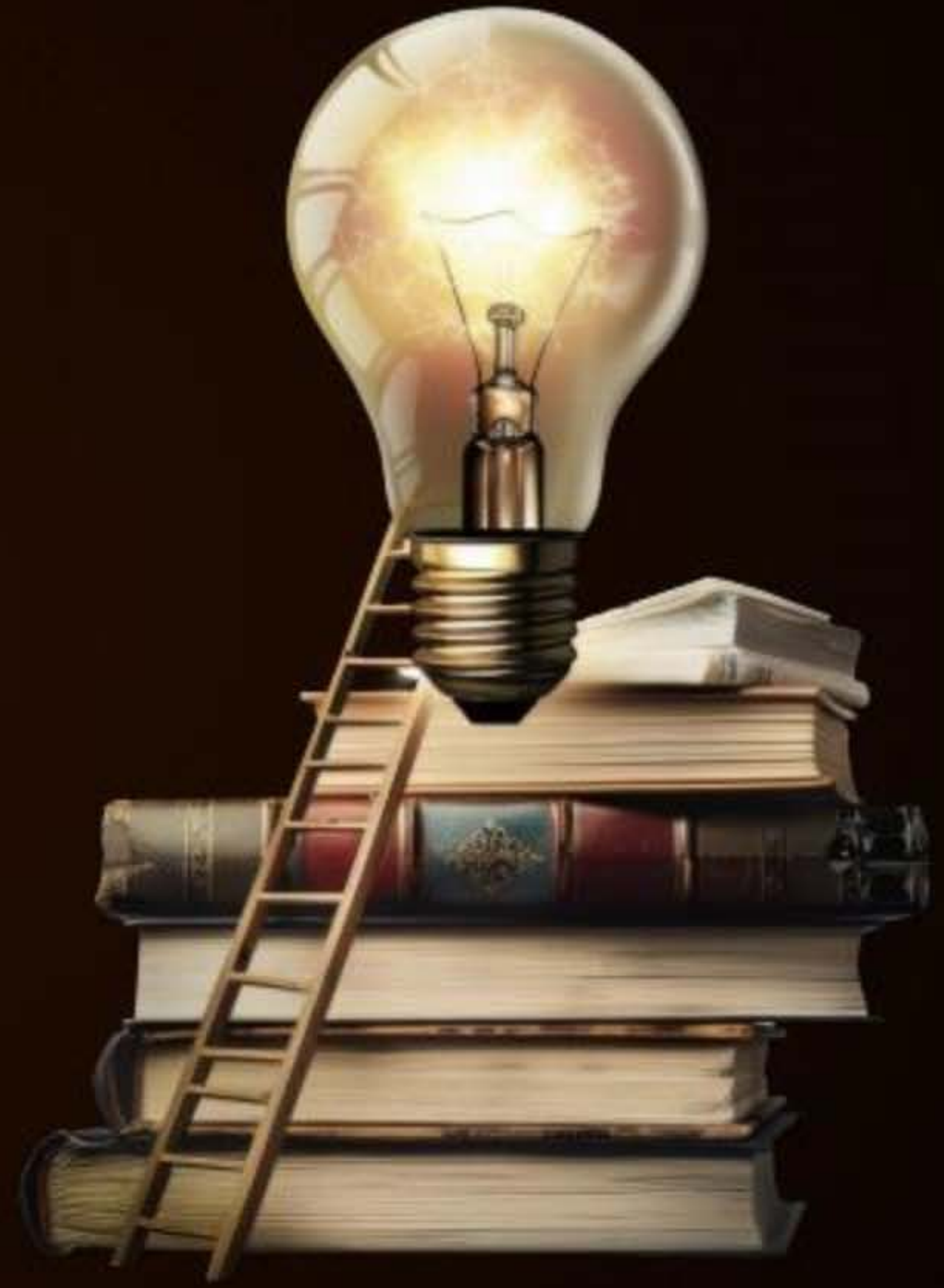
Topics *to be covered*

- **A** Question Practice on Common Root
- **B** Graph of a Quadratic polynomial
- **C** Practice problems



Homework Discussion

QUESTION [JEE Mains 2022 (27 July)]

If α, β are the roots of the equation

$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right)x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$$

0

then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is :

A $3x^2 - 20x - 12 = 0$

B $3x^2 - 10x - 4 = 0$

C $3x^2 - 10x + 2 = 0$

D $3x^2 - 20x + 16 = 0$

$$a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}.$$

$$x^2 - \left(5 + \cancel{3^{\sqrt{\log_3 5}}} - \cancel{5^{\sqrt{\log_5 3}}}\right)x - 3 = 0$$

$$x^2 - 5x - 3 = 0$$

$$3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{1-\frac{1}{3}}}$$

$$= 3^{(\log_3 5)^{\frac{1}{3}}} - 5^{\log_5 3 \cdot \frac{1}{(\log_5 3)^{\frac{1}{3}}}}$$

$$= 3^{(\log_3 5)^{\frac{1}{3}}} - \left(5^{\log_5 3}\right)^{(\log_3 5)^{\frac{1}{3}}}$$

$$= \cancel{3^{(\log_3 5)^{1/3}}} - \cancel{3^{(\log_3 5)^{1/3}}}$$

$$= 0$$

Ans. B

QUESTION [JEE Mains 2021]

Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then $\alpha^8 + \beta^8$ is equal to

- A 10
- B 100
- [✓ C] 50
- D 160

M① $x^2+(20)^{1/4}x+5^{1/2}=0$

$\alpha^2+(20^{1/4})\alpha+5^{1/2}=0$

$\alpha^2+\sqrt{5}=-(20^{1/4}\cdot\alpha)$

S.B.S $\alpha^4+5+\cancel{2\sqrt{5}\alpha^2}=20^{1/2}\alpha^2=\sqrt{20}\cdot\alpha^2=\cancel{2\sqrt{5}\alpha^2}$

$\alpha^4=-5.$

$\alpha^8=25.$

$\text{lly } \beta^8=25 \Big\} \alpha^8+\beta^8=50.$

M② $\alpha+\beta=-(20)^{1/4}$ $\alpha\beta=\sqrt{5}$

$\alpha^2+\beta^2+\cancel{2\sqrt{5}}=\cancel{2\sqrt{5}}.$

$\alpha^4+\beta^4+2\alpha^2\beta^2=0$

$\alpha^4+\beta^4=-10.$

$\alpha^8+\beta^8+2\alpha^4\beta^4=100.$

$\alpha^8+\beta^8+50=100 \rightsquigarrow \alpha^8+\beta^8=50.$

Ans. C

M③ $S_n = \alpha^n + \beta^n$

$$S_{n+2} + 20^{1/4} S_{n+1} + \sqrt{5} S_n = 0$$

$n=6$ $$S_8 + 20^{1/4} S_7 + \sqrt{5} S_6 = 0$$

| |

| |

lengthy !!

QUESTION [JEE Mains 2023]

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to

(A) -64

(B) $-64\sqrt{2}$

(C) $-128\sqrt{2}$

(D) -128

$\alpha^2 + 2 = \sqrt{2}\alpha$

$\alpha^4 + 4 + 4\alpha^2 = 2\alpha^2$

$\alpha^4 + 4 = -2\alpha^2$

$\alpha^8 + 16 + 8\alpha^4 = 4\alpha^4$

$\alpha^8 + 16 = -4\alpha^4 \longrightarrow \alpha^8 + 4\alpha^4 = -16$

multiply by α^4

$\alpha^{12} + 16\alpha^4 = -4\alpha^8$

$\alpha^{12} = -4(\alpha^8 + 4\alpha^4)$

$\alpha^{12} = 64$

$\alpha^{14} = 64\alpha^2$

lly $\beta^{14} = 64\beta^2$

$\alpha^{14} + \beta^{14} = 64(\alpha^2 + \beta^2)$

$= 64((\alpha+\beta)^2 - 2\alpha\beta)$

$= 64(2 - 2 \cdot 2)$

$= -128$

Ans. D

QUESTION

If $x^2 + 3x + 3 = 0$ and $ax^2 + bx + 1 = 0, a, b \in Q$ have a common root, then value of $(3a + b)$ is equal to

- **A** 1/3
- **B** 1
- [✓ **C** 2]
- **D** 4

$x^2 + 3x + 3 = 0$ — $D < 0$

$ax^2 + bx + 1 = 0$ — both roots common.

$$\frac{a}{1} = \frac{b}{3} = \frac{1}{3}$$

$a = 1/3$

$b = 1$

$3a + b = 2$.

Ans. C

Home Challenge - 07

$\overset{-ve}{x-3} = -ve$

$18-\underset{-ve}{x}$

Let n be the number of integers satisfying the inequality $\dfrac{(x-3)^{\frac{-x}{|x|}}\cdot\sqrt{(x-5)^2}\cdot(18-x)}{\sqrt{-x}(-x^2+x-1)(|x|-37)}<0$

then value of n is ____________

$-x>0 \Rightarrow \underbrace{x<0}$

$$\frac{\cancel{(x-3)^{\frac{-x}{-x}}}\cdot|x-5|(18-x)}{\sqrt{-x}\,\underbrace{(-x^2+x-1)}_{\substack{D<0\\a<0}}\,(-x-37)}<0$$

always -ve

$$\frac{\overset{-ve}{\cancel{(x-3)}}\cdot\overset{-ve}{\cancel{(x-5)}}\cdot\underset{+ve}{(18-x)}}{\cancel{-}(x+37)}>0$$

$\dfrac{1}{(x+37)}>0 \quad x\neq 3,5.$

$x+37>0$

$x>-37$

$x\in(-37,0)$

No: of Integral values of $x = 0-(-37)-1 = 36$ Ans

Home Challenge - 07

$x = -ve$

$$|x-5| = -(x-5)$$

$\underset{-ve}{\checkmark}$

Let n be the number of integers satisfying the inequality $\dfrac{(x-3)^{\frac{-x}{|x|}}\cdot\sqrt{(x-5)^2}\cdot(18-x)}{\sqrt{-x}(-x^2+x-1)(|x|-37)} < 0$ then value of n is ________

$$-x > 0 \Rightarrow \underset{\smile}{x<0}$$

$$\frac{(x-3)^{\cancel{\frac{-x}{-x}}}\cdot|x-5|(18-x)}{\sqrt{-x}\,(-x^2+x-1)\,(-x-37)} < 0$$

$(-x^2+x-1)$ — $D<0$, $a<0$ → always -ve

$$\frac{(x-3)\cdot\cancel{-}(x-5)\cdot(18-x)}{\cancel{-}(x+37)} > 0$$

$$-\frac{(x-3)(x-5)(x-18)}{x+37} > 0$$

$$\frac{(x-3)(x-5)(x-18)}{x+37} < 0$$

$x \in (-37,3) \cup (5,18)$

But $x<0$

$x \in (-37,0)$

QUESTION

If α, β, γ are roots of the cubic $2011x^3 + 2x^2 + 1 = 0$, then find

(i) $(\alpha\beta)^{-1} + (\beta\gamma)^{-1} + (\gamma\alpha)^{-1}$;

(ii) $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$

Ans. (i) 2 ; (ii) –4

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

QUESTION [JEE Mains 2022 (25 June)]

Let a, b ∈ R be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to:

- (A) 37
- ~~(B)~~ 58
- (C) 68
- (D) 92

$ax^2 - 2bx + 15 = 0 \begin{smallmatrix} \alpha \\ \alpha \end{smallmatrix}$

$x^2 - 2bx + 21 = 0 \begin{smallmatrix} \alpha \\ \beta \end{smallmatrix}$

S.O.R $= \alpha + \alpha = +\frac{2b}{a}$

$D = 0 \Rightarrow 4b^2 - 4\cdot 15a = 0$

$b^2 = 15a.$

$\alpha = \frac{b}{a}$

$\frac{b^2}{a^2} - 2b\cdot\frac{b}{a} + 21 = 0$

$\frac{b^2}{a^2} - \frac{2b^2}{a} + 21 = 0$

$\frac{15}{a} - 30 + 21 = 0$

$9a = 15.$

$a = 5/3$

$b^2 = 15a = 15\cdot 5/3$

$b^2 = 25.$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (2b)^2 - 2\cdot 21$

$= 4b^2 - 42 = 4\cdot 25 - 42$

$= \underline{\underline{58}}$

$\Downarrow$

we want b??

Ans. B

QUESTION [JEE Mains 2022 (25 June)]

Let a, b ∈ R be such that the equation $ax^2 - 2bx + 15 = 0$ has a repeated root α. If α and β are the roots of the equation $x^2 - 2bx + 21 = 0$, then $\alpha^2 + \beta^2$ is equal to:

- (A) 37
- (B) ~~58~~
- (C) 68
- (D) 92

$$ax^2 - 2bx + 15 = 0 \begin{matrix} \alpha \\ \alpha \end{matrix}$$

$$x^2 - 2bx + 21 = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$\begin{vmatrix} a & -2b \\ 1 & -2b \end{vmatrix} \times \begin{vmatrix} -2b & 15 \\ -2b & 2 \end{vmatrix} = \begin{vmatrix} 15 & a \\ [illegible] & [illegible] \end{vmatrix}^2$$

$$(-2ab + 2b)(-12b) = (15 - 21a)^2$$

$$b^2(-12)(2 - 2a) = (15 - 21a)^2$$

$$\overset{5}{\cancel{15}}a \cdot (\overset{4}{\cancel{12}})(2)(a-1) = \cancel{9}(5 - 7a)^2 .$$

$$40a(a-1) = 25 + 49a^2 - 70a$$

$$9a^2 - 30a + 25 = 0$$

$$(3a-5)^2 = 0 \Rightarrow a = 5/3$$

$$D = 0 \Rightarrow 4b^2 - 4 \cdot 15a = 0$$

$$b^2 = 15a .$$

$$b^2 = 15 \cdot 5/3 = 25 .$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= (2b)^2 - 2 \cdot 21$$

$$= 4b^2 - 42$$

$\Downarrow$ we want b??

$$\alpha^2 + \beta^2 = 100 - 42$$

$$= \underline{58} .$$

Ans. B

QUESTION [JEE Mains 2020 (9 Jan)]

Tah01

Let $a, b \in R, a \neq 0$ be such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation, $x^2 - 2bx - 10 = 0$. If β is the other root of this equation, then $\alpha^2 + \beta^2$ is equal to:

- (A) 28
- (B) 24
- (C) 26
- (D) 25

Ans. D

QUESTION

Tah 02

If the equation $x^2 - 4x + 5 = 0$ and $x^2 + ax + b = 0$ have a common root, find a and b.

QUESTION [JEE Mains 2013]

Tah 03

If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0, a, b, c \in R$, have a common root, then a : b : c is

- A 1 : 2 : 3
- B 3 : 2 : 1
- C 1 : 3 : 2
- D 3 : 1 : 2

Ans. A

QUESTION [AIEEE 2008]

The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3. Then the common root is

- (A) 1
- (B) 4
- (C) 3
- [✓ (D) 2]

$x^2-6x+a=0 \begin{cases} \alpha \\ 4\beta \end{cases}$

$x^2-cx+6=0 \begin{cases} \alpha \\ 3\beta \end{cases}$

Integers.

$\alpha+4\beta=6$

$\alpha\cdot3\beta=6 \Rightarrow \alpha=\frac{2}{\beta}$

$\frac{2}{\beta}+4\beta=6$

$\frac{1}{\beta}+2\beta=3$

$2\beta^2+1=3\beta$

$2\beta^2-3\beta+1=0$

$2\beta^2-2\beta-\beta+1=0$

$(2\beta-1)(\beta-1)=0$

$\beta=\cancel{1/2}, 1$ ✓

$\boxed{\alpha=2}$

Ans. D

QUESTION [JEE Mains 2021 (26 Aug)]

Let $\lambda \neq 0$ be in R. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to

$3 \times \quad x^2 - x + 2\lambda = 0 < {\alpha \atop \beta}$ ✓ — ①

$3x^2 - 10x + 27\lambda = 0 < {\alpha \atop \gamma}$ ✓

$7x - 21\lambda = 0 < \alpha$

$x = 3\lambda = \alpha$.

Put in ① $\quad 9\lambda^2 - 3\lambda + 2\lambda = 0$

$\lambda(9\lambda - 1) = 0$

$\lambda = \cancel{0}, 1/9 \Rightarrow \quad x^2 - x + 2/9 = 0 < {1/3 \atop 2/3 = \beta}$

$3x^2 - 10x + 3 = 0$

$3x^2 - 9x - x + 3 = 0$

$(3x-1)(x-3) = 0 \quad x = 1/3, 3 = \gamma$

$\frac{\beta\gamma}{\lambda} = \frac{2/3 \cdot 3}{1/9} = 18$ Ans

Ans. 18

QUESTION [IIT-JEE 2006]

★★KCLS★★

Let a and b be the roots of the equation $x^2 - 10cx - 11d = 0$ and those $x^2 - 10ax - 11b = 0$ are c, d then the value of $a + b + c + d$, when $a \neq b \neq c \neq d$, is

$$x^2 - 10cx - 11d = 0 <^{a}_{b} \Rightarrow a+b = 10c$$

$$x^2 - 10ax - 11b = 0 <^{c}_{d} \Rightarrow c+d = 10a$$

$$b-d = 11(c-a)$$

$$a+b+c+d = 10(a+c)$$

$$a^2 - 10ca - 11d = 0$$

$$c^2 - 10ac - 11b = 0$$

$$a^2 - c^2 + 11(b-d) = 0$$

$$(a-c)(a+c) = -11(b-d) = -11 \cdot 11(c-a)$$

$$\cancel{(a-c)}(a+c) = 121\cancel{(a-c)}$$

$$a+c = 121$$

$$\underline{a+b+c+d = 1210}$$

Ans. 1210

QUESTION

★★ASRQ★★

The equations $kx^2 + x + k = 0$ and $kx^2 + kx + 1 = 0$ have exactly one root in common for

(A) $k = -1/2, 1$

(B) $k = 1$

(~~C~~) $k = -1/2$

(D) $k = 1/2$

$kx^2 + x + k = 0$

$kx^2 + kx + 1 = 0$

$$\begin{vmatrix} k & 1 \\ k & k \end{vmatrix} \times \begin{vmatrix} 1 & k \\ k & 1 \end{vmatrix} = \begin{vmatrix} k & k \\ 1 & k \end{vmatrix}^2$$

$(k^2-k)(1-k^2) = (k^2-k)^2$

$(k^2-k)(1-k^2-k^2+k)=0$

$k(k-1)(-2k^2+k+1)=0$

$k=0, k=1$, $2k^2-k-1=0$

$2k^2-2k+k-1=0$

$(2k+1)(k-1)=0$

$k = \cancel{0}, \cancel{1}, -1/2$.

Both roots are common.

if Both roots are common.

$\frac{k}{k} = \frac{1}{k} = \frac{k}{1}$

$1 = \frac{1}{k} = \frac{k}{1}$

$\Downarrow$

$k=1$

QUESTION

★★ASRQ★★

The equations $kx^2 + x + k = 0$ and $kx^2 + kx + 1 = 0$ have exactly one root in common for

- **A** $k = -1/2, 1$
- **B** $k = 1$
- **~~C~~** $k = -1/2$
- **D** $k = 1/2$

$$kx^2 + x + k = 0$$
$$kx^2 + kx + 1 = 0$$
$$(1-k)x + k-1 = 0$$
$$(1-k)x - (1-k) = 0$$
$$(1-k)(x-1) = 0$$
$$k=1 \text{ or } x=1$$

$k=1$ Both roots are common

$\underbrace{\text{if } x=1}$

$$k+k+1=0$$
$$k=-1/2$$

if Both roots are common.

$$\underbrace{\frac{k}{k} = \frac{1}{k}} = \frac{k}{1}$$
$$1 = \frac{1}{k} = \frac{k}{1}$$
$$\Downarrow$$
$$k=1$$

QUESTION

★★KCLS★★

Tan04

If the equations $ax^3 + x + 2 = 0$ and $x^3 + ax + 2 = 0$ have exactly one common root, find the value of $|a|$.

QUESTION

★★KCLS★★

If the quadratic equation $x^2 + bx + c = 0$ & $x^2 + cx + b = 0 (b \neq c)$ have a common root then prove that their uncommon roots are the roots of the equation $x^2 + x + bc = 0$.

① $x^2+bx+c=0 \begin{smallmatrix}\alpha\\ \beta\end{smallmatrix}$

② $x^2+cx+b=0 \begin{smallmatrix}\alpha\\ \gamma\end{smallmatrix}$

To show: β, γ are roots of $x^2+x+bc=0$

$(b-c)x+c-b=0$ — α

$x-1=0$ — α

$x=1=\alpha$

$\Rightarrow$ from ① $1\cdot\beta=c \Rightarrow \beta=c$

$1\cdot\gamma=b \Rightarrow \gamma=b$

Q·Eqn with β, γ as roots.

$x^2-(b+c)x+bc=0$

Since $\alpha=1$ is a root of ①

$1+b+c=0 \Rightarrow b+c=-1$

$x^2+x+bc=0 \begin{smallmatrix}\beta\\ \gamma\end{smallmatrix}$

QUESTION

★★ASRQ★★

If the quadratic equations $x^2 + ax + 12 = 0$ and $x^2 + bx + 15 = 0$ and $x^2 + (a + b) x + 36 = 0$ have a common positive root find 'a' and 'b' and the root of the equation.

$\underbrace{\alpha > 0}$,

$x^2+ax+12=0 \begin{matrix}\alpha\\ \beta\end{matrix}$

$x^2+bx+15=0 \begin{matrix}\alpha\\ \gamma\end{matrix}$

$x^2+(a+b)x+36=0 \begin{matrix}\alpha\\ \delta\end{matrix}$

$\oplus \Rightarrow 2x^2+(a+b)x+27=0 \;\text{—}\; \alpha$

$\ominus \Rightarrow x^2-9=0 \;\text{—}\; \alpha$

$x=-3,3$

Since α is +ve $\boxed{\alpha=3}$

$\alpha\beta=12 \Rightarrow \beta=4 \qquad \alpha+\beta=-a$

$3+4=-a \Rightarrow a=-7$

$\alpha\gamma=15 \Rightarrow \gamma=5 \qquad \alpha+\gamma=-b \Rightarrow b=-8$

[Ans. a = –7, b = –8; roots are (3, 4), (3, 5) and (3, 12)]

QUESTION

★★ASRQ★★

If the equations $x^3 + x^2 - 4x = 4$ and $x^2 + px + 2p = 0 (p \in R)$ have two roots common, then the value of p is

A –2

~~**B**~~ –1

C 1

D 3

$x^3 + x^2 - 4x - 4 = 0$ γ, α, β , $x^2 + px + 2p = 0$ α, β

2 roots common

$x^2(x+1) - 4(x+1) = 0$

$(x^2 - 4)(x+1) = 0$

$x = 2, -1, -2$

$\alpha + \beta = -p$

$\alpha\beta = 2p$

$\alpha = 2$
$\beta = -1$

$\alpha + \beta = 1 = -p \Rightarrow p = -1$

$\alpha\beta = 2 \cdot -1 = 2p \Rightarrow p = -1$

$p = -1$

QUESTION

★★KCLS★★

The equations $x^3 + 4x^2 + px + q = 0$ and $x^3 + 6x^2 + px + r = 0$ have two common roots, where $p, q, r \in \mathbb{R}$. If their uncommon roots are the roots of equation $x^2 + 2ax + 8c = 0$, then

(A) $a + c = 8$

(B) $a + c = 2$

(C) $3q = 2r$

(D) $3r = 2q$

$$x^3+4x^2+px+q=0 < \begin{matrix}\alpha\\ \beta\\ \gamma\end{matrix}$$

$$x^3+6x^2+px+r=0 < \begin{matrix}\alpha\\ \beta\\ \delta\end{matrix}$$

γ, δ are roots of $x^2+2ax+8c=0$

$$-2x^2+q-r=0 < \begin{matrix}\alpha\\ \beta\end{matrix}$$

$$2x^2+r-q=0 < \begin{matrix}\alpha\\ \beta\end{matrix}$$

clearly: $\alpha+\beta=0$

Now $\alpha+\beta+\gamma=-4$ $\alpha+\beta+\delta=-6$

$\gamma=-4$ $\delta=-6.$

Eqn with γ & δ as roots.

$x^2+10x+24=0$

$\Rightarrow a=5, c=3$

$\alpha\beta\gamma=-q$

$\alpha\beta\delta=-r$

$$\frac{\alpha\beta\gamma}{\alpha\beta\delta}=\frac{q}{r}$$

$$\frac{2}{3}=\frac{q}{r}$$

$3q=2r.$

QUESTION

$(x-2)$ $x^2-3x+2 \frown 2$

$x^2-6x+8 \frown 2$

If $x^2 + 3x + 5$ is the greatest common divisor of $(x^3 + ax^2 + bx + 1)$ and $(2x^3 + 7x^2 + 13x + 5)$ then find the value of $[a + b]$.

[Note: $[k]$ denotes greatest integer less than or equal to k.]

$\left.\begin{matrix}\alpha\\ \beta\end{matrix}\right\rangle x^3+ax^2+bx+1=0 \times 2$ Both are divisible by $x^2+3x+5 \left\langle\begin{matrix}\alpha\\ \beta\end{matrix}\right. = 0$

$\left.\begin{matrix}\alpha\\ \beta\end{matrix}\right\rangle 2x^3+7x^2+13x+5=0$

$(2a-7)x^2+(2b-13)x-3=0 \left\langle\begin{matrix}\alpha\\ \beta\end{matrix}\right.$.

$$\frac{2a-7}{1}=\frac{2b-13}{3}=\frac{-3}{5}.$$

$2a=7-3/5$, $2b=13-9/5$

$a=16/5$ $b=28/5$.

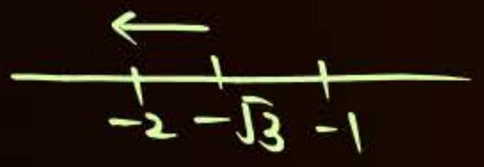
$[a+b]=\left[\frac{44}{5}\right]=8$ Ans

$f(x)$ — root α — $f(\alpha)=0$

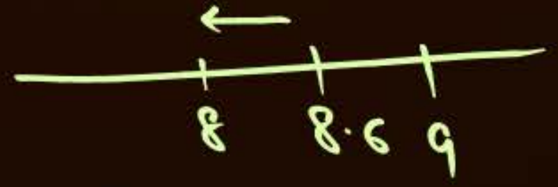
$g(x)$ — root α — $g(\alpha)=0$

$$h(x) = A f(x) \pm B g(x)$$ — root α.

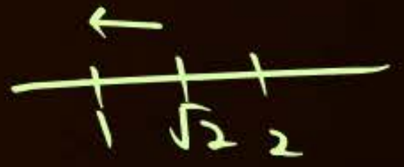
$h(\alpha) = A f(\alpha) \pm B g(\alpha) = 0$



$[-\sqrt{3}] = -2$.



$[8.6] = 8$



$[\sqrt{2}] = 1$

QUESTION

★★ASRQ★★

If $Q_1(x) = x^2 + (k - 29)x - k$ and $Q_2(x) = 2x^2 + (2k - 43)x + k$ both are factors of cubic polynomial, then largest value of k is (where $Q_1(x)$ & $Q_2(x)$ are not perfect squares)

(A) 0

(B) 33

(C) 23

~~(D)~~ 30

$Q_1(x) = x^2 + (k-29)x - k < \begin{matrix}\alpha \\ \beta\end{matrix}$

$Q_2(x) = 2x^2 + (2k-43)x + k < \begin{matrix}\gamma \\ \delta\end{matrix}$

Both are factors of a cubic

$ax^3 + bx^2 + cx + d < \begin{matrix}\alpha \\ \beta \\ \gamma \\ \delta\end{matrix}$

↓

N.P.

$Q_1(x)$ & $Q_2(x)$ should have a common root.

⇓

$x^2 + (k-29)x - k = 0 < \begin{matrix}\alpha \\ \beta\end{matrix} \quad \times 2$ — ①

$2x^2 + (2k-43)x + k = 0 < \begin{matrix}\alpha \\ \delta\end{matrix}$

$-15x - 3k = 0 \swarrow \alpha$

$x = -k/5$ put in ①

$$\left(-\frac{k}{5}\right)^2 - \frac{k}{5}(k-29) - k = 0$$

$$k\left[\frac{k}{25} - \frac{(k-29)}{5} - 1\right] = 0$$

$$k = 0 \quad , \quad k - 5k + 145 - 25 = 0$$

$$4k = 120$$

$$k = 30.$$

QUESTION

Ta h05

If two roots of the equation $(x-1)(2x^2-3x+4)=0$ coincide with roots of the equation $x^3+(a+1)x^2+(a+b)x+b=0$ where $a, b \in R$ then $2(a+b)$ equals

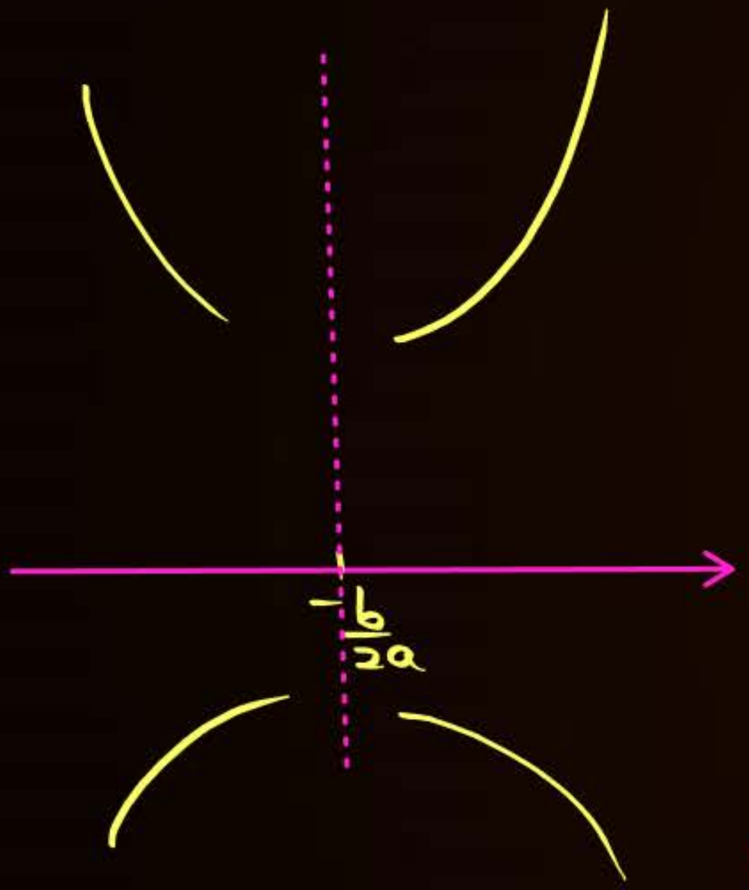
(A) 4

(B) 2

(C) 1

(D) 0

Analysis of a Quadratic Polynomial



$$P(x) = ax^2 + bx + c \quad , a \neq 0$$

$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c .$$

$$= a\left(x^2 + \left(\frac{b}{a}\right)x + \frac{b^2}{4a^2}\right) - \frac{b^2}{4a} + c .$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2}{4a} - c\right)$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2 - 4ac}{4a}\right)$$

$$P(x) = a(x + b/2a)^2 - \frac{D}{4a}$$

$$P\left(-\frac{b}{2a} + t\right) = at^2 - \frac{D}{4a}$$

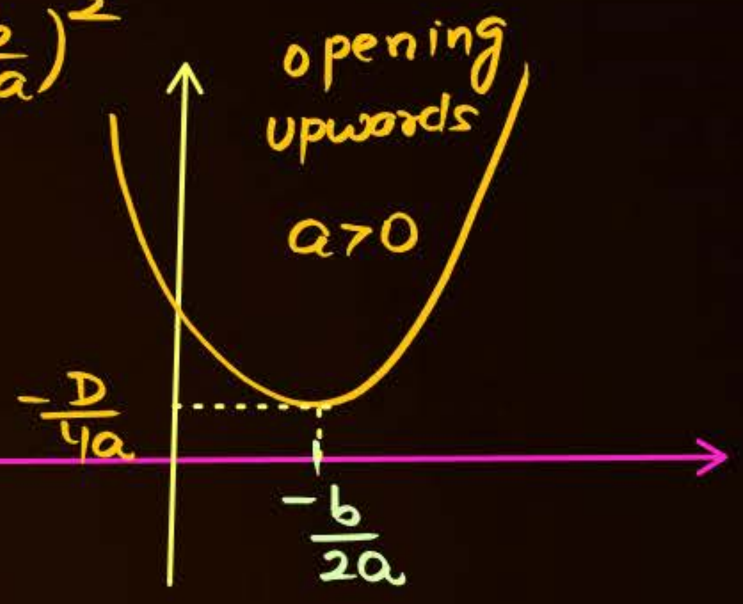
$$P\left(-\frac{b}{2a} - t\right) = at^2 - \frac{D}{4a} .$$

$$P(-b/2a + t) = P(-b/2a - t)$$

$$\Downarrow$$

graph of $y = ax^2 + bx + c$ is symm about $x = -b/2a$.

$$P(x) = a\underbrace{\left(x+\frac{b}{2a}\right)^2}_{\geq 0} - \frac{D}{4a} = -\frac{D}{4a} + a\underbrace{\left(x+\frac{b}{2a}\right)^2}_{\geq 0}$$



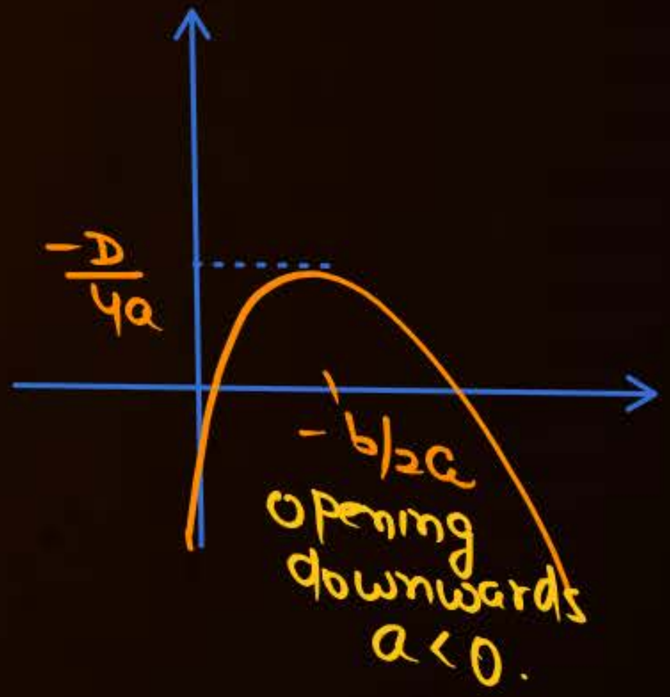
if a>0 $\quad P(x)\Big|_{\min} = -\frac{D}{4a}$ at $x = -\frac{b}{2a}$

$$P(x)\Big|_{MAX} \longrightarrow \infty$$

if a<0 $\quad P(x) = a\left(x+\frac{b}{2a}\right)^2 - \frac{D}{4a} = -\frac{D}{4a} + \underbrace{a\overset{\geq 0}{\left(x+\frac{b}{2a}\right)^2}}_{\leq 0}$

$$P(x)\Big|_{MAX} = -\frac{D}{4a} \text{ at } x = -\frac{b}{2a}$$

$$P(x)\Big|_{\min} \longrightarrow -\infty$$



Sabse Important Baat

Sabhi Class Illustrations Retry Karnay hai...

Home Challenge - 08

The ordered pair (x, y) satisfying the equation $x^2 = 1 + 6\log_4 y$ and $y^2 = 2^x y + 2^{2x+1}$ are (x_1, y_1) and (x_2, y_2), then find the value of $\log_2|x_1 x_2 y_1 y_2|$.

(Ans:7)

Today's KTK

$$\text{No Selection} \xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}} \text{Selection with Good Rank}$$

QUESTION

(KTK 1)

If α, β are the root of a quadratic equation $x^2 - 3x + 5 = 0$ then the equation whose roots are $(\alpha^2 - 3\alpha + 7)$ and $(\beta^2 - 3\beta + 7)$ is

A $x^2 + 4x + 1 = 0$

B $x^2 - 4x + 4 = 0$

C $x^2 - 4x - 1 = 0$

D $x^2 + 2x + 3 = 0$

Ans. B

QUESTION

(KTK 2)

The equations $ax^2 + bx + a = 0 (a, b \in R)$ and $x^3 - 2x^2 + 2x - 1 = 0$ have 2 roots common. Then $a + b$ must be equal to

Ⓐ 1

Ⓑ –1

Ⓒ 0

Ⓓ None of these

Ans. C

QUESTION

(KTK 3)

The value of m for which the equation $\frac{a}{x+a+m}+\frac{b}{x+b+m}=1$ has roots equal in magnitude and opposite in signs is

(A) $\frac{a-b}{a+b}$

(B) –1

(C) 0

(D) $\frac{a+b}{a-b}$

Ans. C

QUESTION

(KTK 4)

Find the values of 'k' so that the equation $x^2 + kx + (k + 2) = 0$ and $x^2 + (1 - k)x + 3 - k = 0$ have exactly one common root.

Ans. No possible value of k

QUESTION

(KTK 5)

Given a, b are two distinct real number satisfying $a^2 - 5a + 2 = 0$ and $b^2 - 5b + 2 = 0$ then $(1 - ab + a^2b + b^2a)$

QUESTION

(KTK 6)

Let 'p' is a root of the equation $x^2 - x - 3 = 0$. Then the value of $\frac{p^3+1}{p^5-p^4-p^3+p^2}$ is equal to

A $\frac{4}{3}$

B $\frac{4}{9}$

C $\frac{2}{9}$

D $\frac{2}{3}$

QUESTION

(KTK 7)

If α and β are the roots of $ax^2 + bx + c = 0$, then the equation whose roots are $\frac{\alpha+1}{\alpha-2}$ and $\frac{\beta+1}{\beta-2}$ is

(A) $a(x+1)^2 + b(x+1)(x-2) + c(x-2)^2 = 0$

(B) $a(x-2)^2 + b(x+1)(x-2) + c(x+1)^2 = 0$

(C) $a(2x+3)^2 + b(x+1)(x+2) + c(x+2)^2 = 0$

(D) $a(2x+1)^2 + b(2x+1)(x-1) + c(x-1)^2 = 0$

Ans. D

QUESTION

(KTK 8)

If α, β, γ are roots $x^3 + 2x^2 - 3x + 1 = 0$, then value of $\frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$ is less than

- **A** 2
- **B** 3
- **C** 4
- **D** 5

Homework From Module

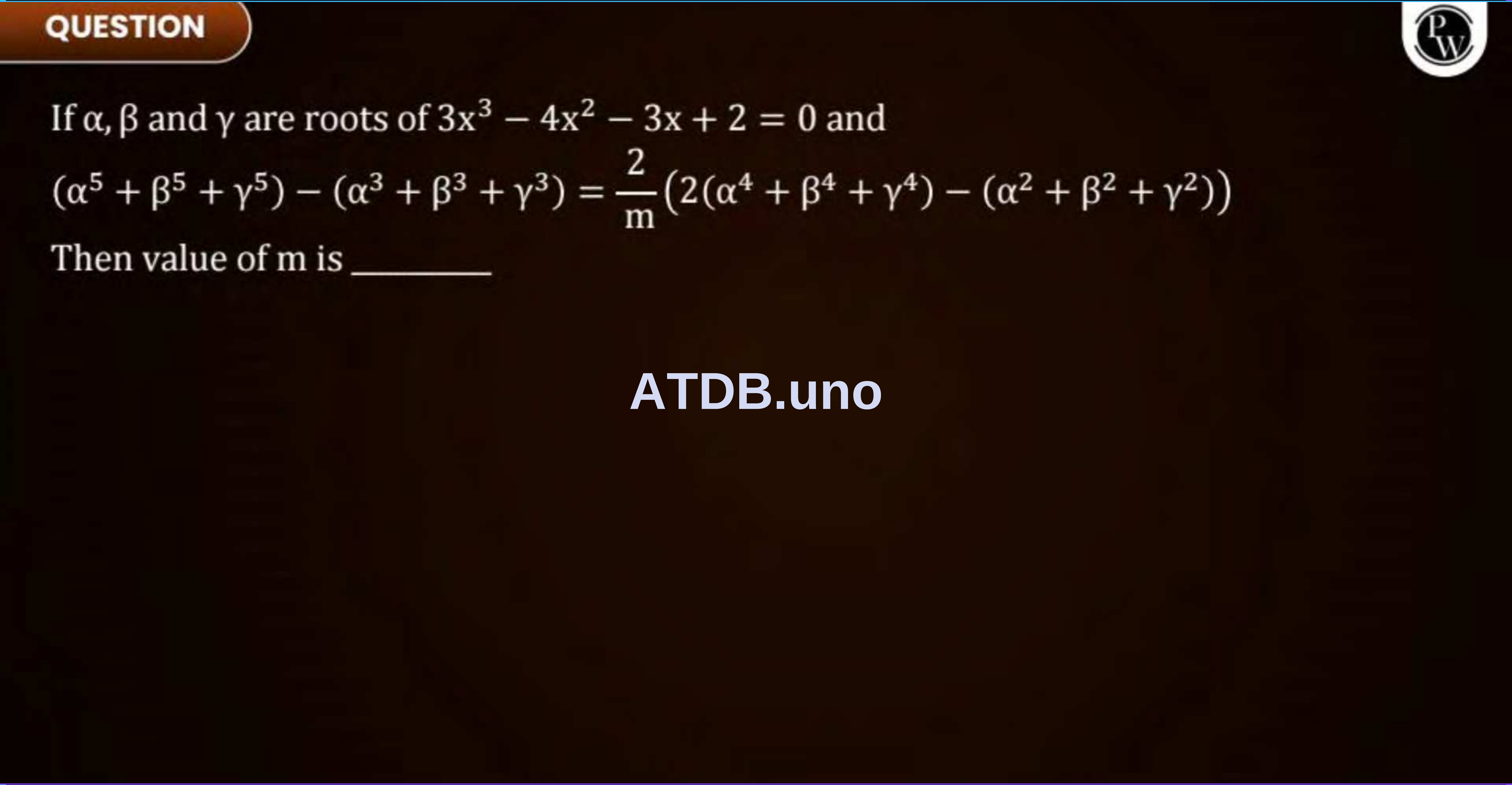
Quadratic Equations

Prarambh (Topicwise) : Q1 to Q27

Prabal (JEE Main Level) : Q1,Q2,Q6 to Q9

Parikshit (JEE Advanced Level) : Abhi Ruko

Solution to Previous TAH



QUESTION

If α, β and γ are roots of $3x^3 - 4x^2 - 3x + 2 = 0$ and

$$(\alpha^5+\beta^5+\gamma^5) - (\alpha^3+\beta^3+\gamma^3) = \frac{2}{m}\left(2(\alpha^4+\beta^4+\gamma^4) - (\alpha^2+\beta^2+\gamma^2)\right)$$

Then value of m is ________

Q-1 (TAH-1): If α, β, γ are roots of $3x^3 - 4x^2 - 3x + 2 = 0$ and $(\alpha^5 + \beta^5 + \gamma^5) - (\alpha^3 + \beta^3 + \gamma^3) = \frac{2}{m}\left(2(\alpha^4 + \beta^4 + \gamma^4) - (\alpha^2 + \beta^2 + \gamma^2)\right)$

Then $m = ?$

Sol^n: $3x^3 - 4x^2 - 3x + 2 = 0 \begin{matrix} \nearrow \alpha \\ \rightarrow \beta \\ \searrow \gamma \end{matrix}$ $\Bigg\}$ $S_n = \alpha^n + \beta^n + \gamma^n$

$$\underbrace{(\alpha^5 + \beta^5 + \gamma^5)}_{S_5} - \underbrace{(\alpha^3 + \beta^3 + \gamma^3)}_{S_3} = \frac{2}{m}\Big(2\underbrace{(\alpha^4 + \beta^4 + \gamma^4)}_{S_4} - \underbrace{(\alpha^2 + \beta^2 + \gamma^2)}_{S_2}\Big)$$

$$\Rightarrow S_5 - S_3 = \frac{2}{m}[2S_4 - S_2]$$

By N.F.: $3S_{n+3} - 4S_{n+2} - 3S_{n+1} + 2S_n = 0$.

$n = 2 \rightarrow$ $3S_5 - 4S_4 - 3S_3 + 2S_2 = 0$

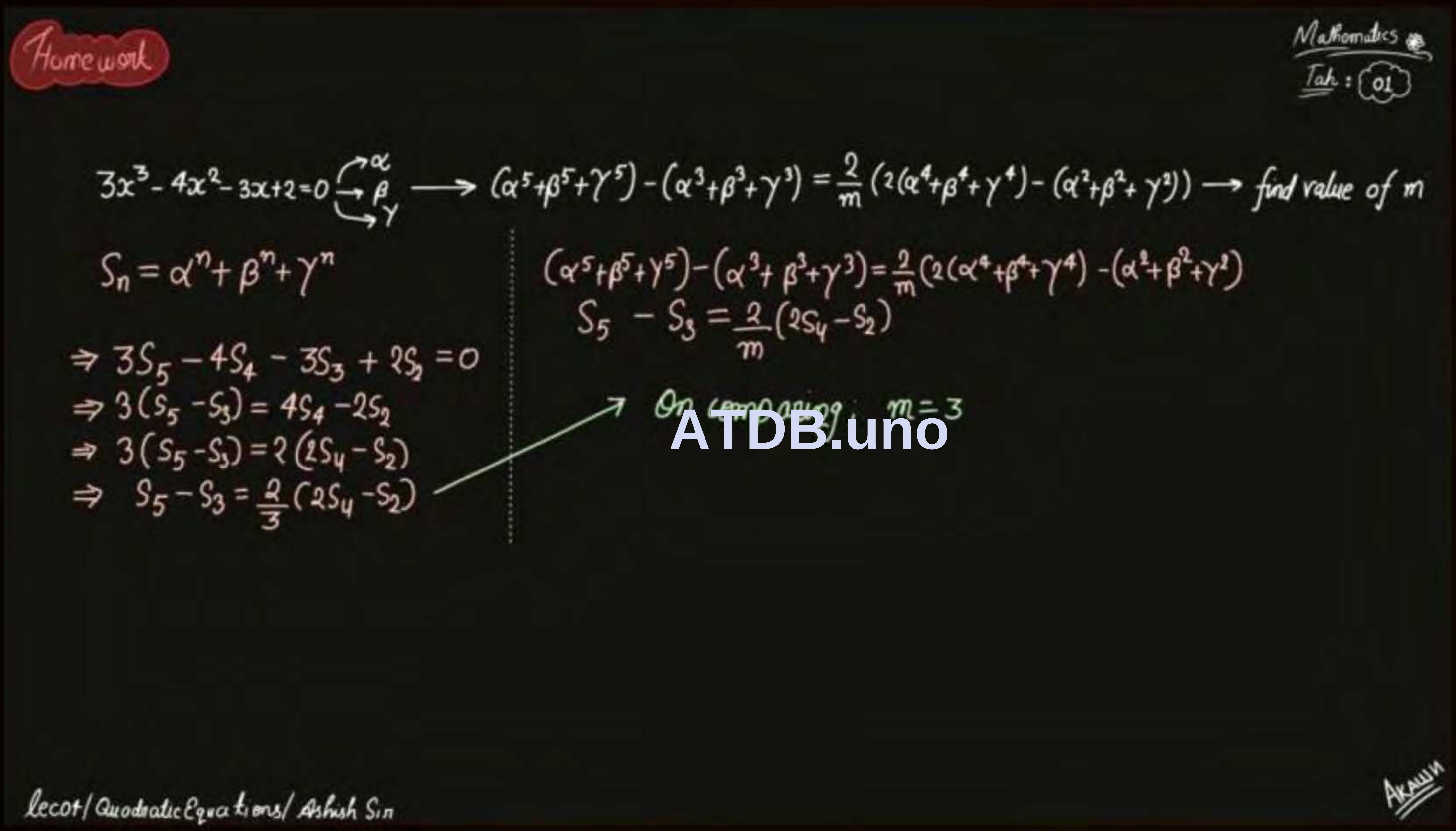
$\Rightarrow 3(S_5 - S_3) - 2(2S_4 - S_2) = 0$.

TAH 1
BY REED

$$\Rightarrow S_5 - S_3 = \frac{2(2S_4 - S_2)}{3}$$

$$\therefore S_5 - S_3 = \frac{2}{\underset{m}{\boxed{3}}}(2S_4 - S_2)$$

$\therefore \underline{\underline{m = 3}}$ (Ans.)



Homework

Mathematics

Tah : 01

$3x^3-4x^2-3x+2=0 \begin{matrix}\to\alpha\\ \to\beta\\ \to\gamma\end{matrix} \longrightarrow (\alpha^5+\beta^5+\gamma^5)-(\alpha^3+\beta^3+\gamma^3)=\frac{2}{m}(2(\alpha^4+\beta^4+\gamma^4)-(\alpha^2+\beta^2+\gamma^2)) \longrightarrow$ find value of m

$$S_n=\alpha^n+\beta^n+\gamma^n$$

$\Rightarrow 3S_5-4S_4-3S_3+2S_2=0$

$\Rightarrow 3(S_5-S_3)=4S_4-2S_2$

$\Rightarrow 3(S_5-S_3)=2(2S_4-S_2)$

$\Rightarrow S_5-S_3=\frac{2}{3}(2S_4-S_2)$

$(\alpha^5+\beta^5+\gamma^5)-(\alpha^3+\beta^3+\gamma^3)=\frac{2}{m}(2(\alpha^4+\beta^4+\gamma^4)-(\alpha^2+\beta^2+\gamma^2)$

$S_5-S_3=\frac{2}{m}(2S_4-S_2)$

On comparing : $m=3$

lec01/Quadratic Equations/ Ashish Sir

Akash

QUESTION

★★KCLS★★

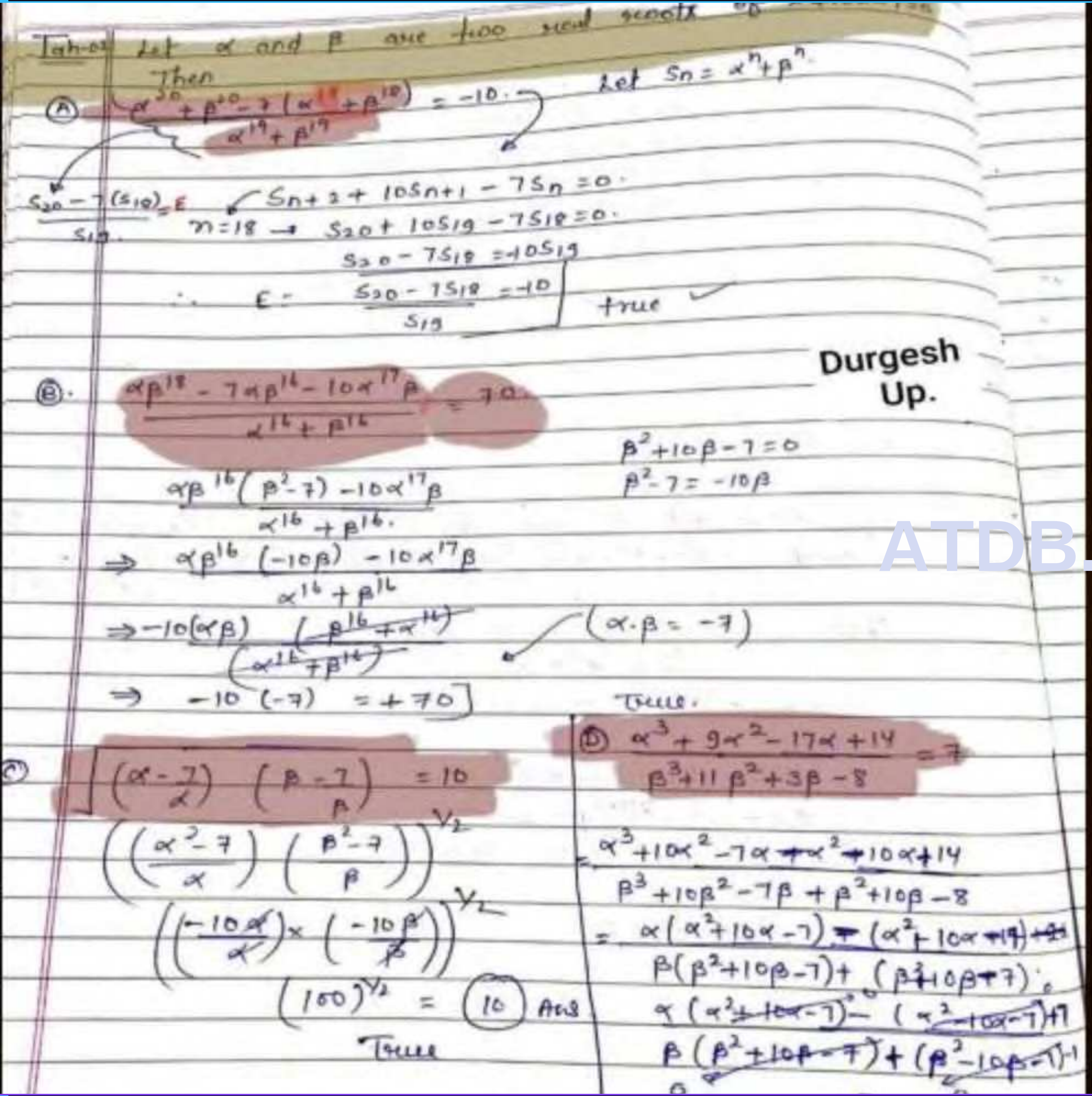
Let α and β are two real roots of $x^2 + 10x - 7 = 0$. Then

A $\dfrac{\alpha^{20} + \beta^{20} - 7(\alpha^{18} + \beta^{18})}{\alpha^{19} + \beta^{19}} = -10$

B $\dfrac{\alpha\beta^{18} - 7\alpha\beta^{16} - 10\alpha^{17}\beta}{\alpha^{16} + \beta^{16}} = 70$

C $\sqrt{\left(\alpha - \dfrac{7}{\alpha}\right)\left(\beta - \dfrac{7}{\beta}\right)} = 10$

D $\dfrac{\alpha^3 + 9\alpha^2 - 17\alpha + 14}{\beta^3 + 11\beta^2 + 3\beta - 8} = 7$



Tah-02 Let α and β are two real roots of
Then

Ⓐ $\frac{\alpha^{20}+\beta^{20}-7(\alpha^{18}+\beta^{18})}{\alpha^{19}+\beta^{19}} = -10.$

Let $S_n = \alpha^n+\beta^n$

$\frac{S_{20}-7(S_{18})}{S_{19}} = E$

$S_{n+2}+10S_{n+1}-7S_n=0.$

$n=18 \rightarrow S_{20}+10S_{19}-7S_{18}=0.$

$S_{20}-7S_{19}=-10S_{19}$

$\therefore\ E = \frac{S_{20}-7S_{18}}{S_{19}} = -10$ true ✓

Durgesh
Up.

Ⓑ. $\frac{\alpha\beta^{18}-7\alpha\beta^{16}-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}} = 70.$

$\beta^2+10\beta-7=0$
$\beta^2-7=-10\beta$

$\frac{\alpha\beta^{16}(\beta^2-7)-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}.$

$\Rightarrow \frac{\alpha\beta^{16}(-10\beta)-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}$

$\Rightarrow -10(\alpha\beta)\frac{\cancel{(\beta^{16}+\alpha^{16})}}{\cancel{(\alpha^{16}+\beta^{16})}}$ $(\alpha.\beta=-7)$

$\Rightarrow -10(-7) = +70$

True.

Ⓒ $\sqrt{\left(\alpha-\frac{7}{\alpha}\right)\left(\beta-\frac{7}{\beta}\right)} = 10$

$\left(\left(\frac{\alpha^2-7}{\alpha}\right)\left(\frac{\beta^2-7}{\beta}\right)\right)^{1/2}$

$\left(\left(\frac{-10\cancel{\alpha}}{\cancel{\alpha}}\right)\times\left(\frac{-10\cancel{\beta}}{\cancel{\beta}}\right)\right)^{1/2}$

$(100)^{1/2} = \boxed{10}$ Ans

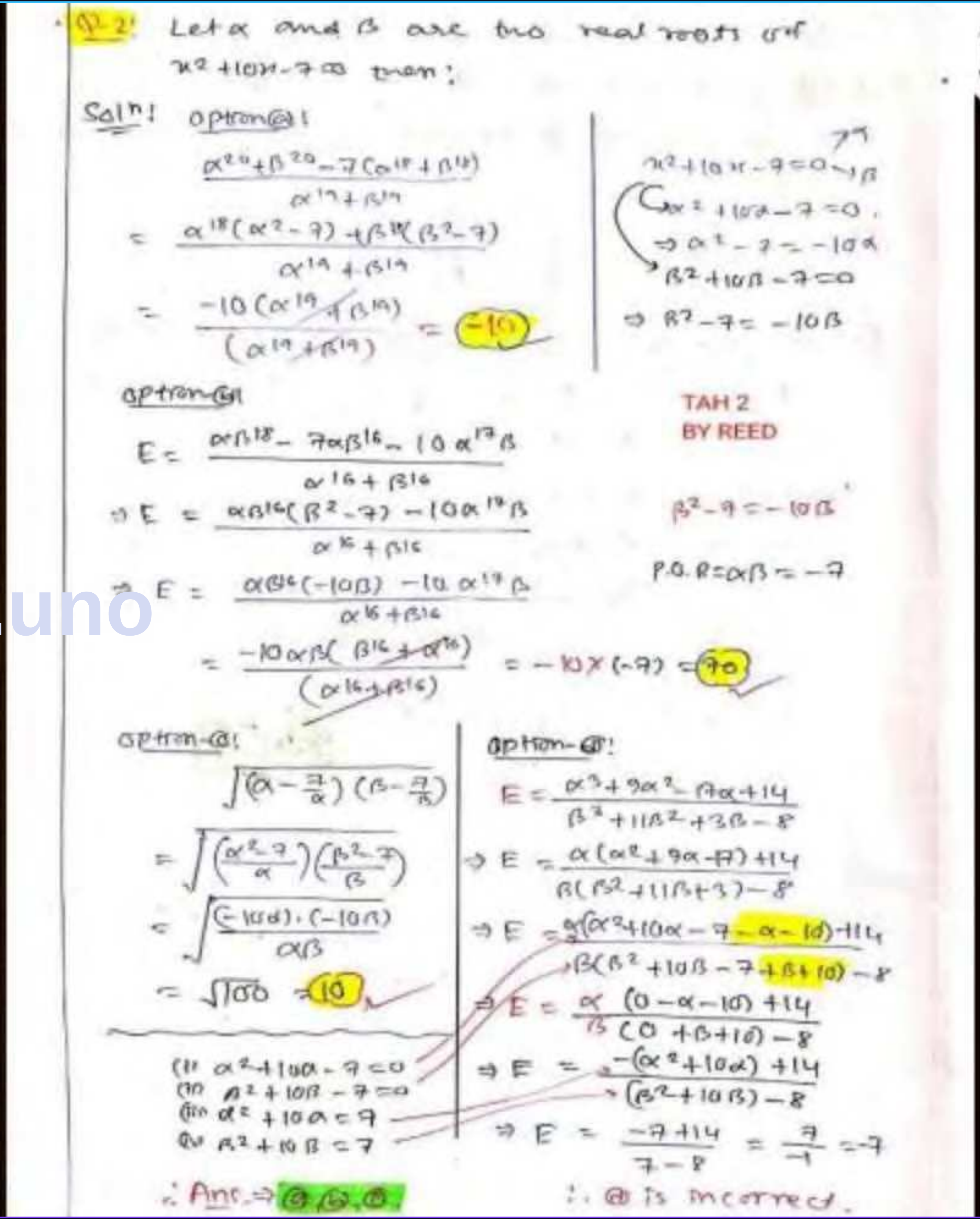
True

Ⓓ $\frac{\alpha^3+9\alpha^2-17\alpha+14}{\beta^3+11\beta^2+3\beta-8} = 7$

$= \frac{\alpha^3+10\alpha^2-7\alpha-\alpha^2-10\alpha+14}{\beta^3+10\beta^2-7\beta+\beta^2+10\beta-8}$

$= \frac{\alpha(\alpha^2+10\alpha-7)-(\alpha^2+10\alpha-7)+7}{\beta(\beta^2+10\beta-7)+(\beta^2+10\beta-7)-1}$

$\frac{\alpha(\cancel{\alpha^2+10\alpha-7})-(\cancel{\alpha^2+10\alpha-7})+7}{\beta(\cancel{\beta^2+10\beta-7})+(\cancel{\beta^2+10\beta-7})-1}$



Q-2: Let α and β are two real roots of $x^2+10x-7=0$ then:

Soln: option(a):

$\frac{\alpha^{20}+\beta^{20}-7(\alpha^{18}+\beta^{18})}{\alpha^{19}+\beta^{19}}$

$= \frac{\alpha^{18}(\alpha^2-7)+\beta^{18}(\beta^2-7)}{\alpha^{19}+\beta^{19}}$

$= \frac{-10(\alpha^{19}+\beta^{19})}{(\alpha^{19}+\beta^{19})} = \boxed{-10}$ ✓

$x^2+10x-7=0 \rightarrow \alpha, \beta$
$\alpha^2+10\alpha-7=0.$
$\Rightarrow \alpha^2-7=-10\alpha$
$\beta^2+10\beta-7=0$
$\Rightarrow \beta^2-7=-10\beta$

option(b)

$E = \frac{\alpha\beta^{18}-7\alpha\beta^{16}-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}$

$\Rightarrow E = \frac{\alpha\beta^{16}(\beta^2-7)-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}$

$\Rightarrow E = \frac{\alpha\beta^{16}(-10\beta)-10\alpha^{17}\beta}{\alpha^{16}+\beta^{16}}$

$= \frac{-10\alpha\beta\cancel{(\beta^{16}+\alpha^{16})}}{\cancel{(\alpha^{16}+\beta^{16})}} = -10\times(-7) = \boxed{70}$ ✓

TAH 2
BY REED

$\beta^2-7=-10\beta$

P.O.R = αβ = −7

option-(c):

$\sqrt{\left(\alpha-\frac{7}{\alpha}\right)\left(\beta-\frac{7}{\beta}\right)}$

$= \sqrt{\left(\frac{\alpha^2-7}{\alpha}\right)\left(\frac{\beta^2-7}{\beta}\right)}$

$= \sqrt{\frac{(-10\alpha)\cdot(-10\beta)}{\alpha\beta}}$

$= \sqrt{100} = \boxed{10}$ ✓

(i) $\alpha^2+10\alpha-7=0$
(ii) $\beta^2+10\beta-7=0$
(iii) $\alpha^2+10\alpha=7$
(iv) $\beta^2+10\beta=7$

∴ Ans ⇒ (a),(b),(c).

option-(d):

$E = \frac{\alpha^3+9\alpha^2-17\alpha+14}{\beta^3+11\beta^2+3\beta-8}$

$\Rightarrow E = \frac{\alpha(\alpha^2+9\alpha-17)+14}{\beta(\beta^2+11\beta+3)-8}$

$\Rightarrow E = \frac{\alpha(\alpha^2+10\alpha-7-\alpha-10)+14}{\beta(\beta^2+10\beta-7+\beta+10)-8}$

$\Rightarrow E = \frac{\alpha(0-\alpha-10)+14}{\beta(0+\beta+10)-8}$

$\Rightarrow E = \frac{-(\alpha^2+10\alpha)+14}{(\beta^2+10\beta)-8}$

$\Rightarrow E = \frac{-7+14}{7-8} = \frac{7}{-1} = -7$

∴ (d) is incorrect.

QUESTION [JEE Mains 2025 (23 Jan)]

If the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, where $a + c = 15$ and $b = \frac{36}{5}$, then $a^2 + c^2$ is equal to

Ans. 117

Q-3) If the equation $a(b-c)x^2 + b(c-a)x + c(a-b) = 0$ has equal roots, where $a+c = 15$ and $b = \frac{36}{5}$, then $a^2+c^2 = ??$

Soln

$$\underbrace{a(b-c)}_{A}x^2 + \underbrace{b(c-a)}_{B}x + \underbrace{c(a-b)}_{C} = 0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix} \quad (\alpha = \beta)$$

$$A+B+C = \cancel{ab} - \cancel{ac} + \cancel{bc} - \cancel{ab} + \cancel{ac} - \cancel{bc} = 0.$$

$\therefore$ roots are $1, \beta$

but since roots are equal $\Rightarrow \boxed{\beta = 1}$

$$S.O.R = 2 = \frac{-b(c-a)}{a(b-c)}$$

$\Rightarrow 2ab - 2ac = ab - bc$

$\Rightarrow ab + bc = 2ac.$

$\Rightarrow b(a+c) = 2ac.$

$\Rightarrow \frac{36}{\cancel{5}} \times \overset{3}{\cancel{15}} = 2ac$

$\Rightarrow \boxed{ac = 54}$

$\therefore a^2+c^2$

$= (a+c)^2 - 2ac$

$= 225 - 2\times 54$

$= 225 - 108$

$= 117$ (Ans)

TAH 3
BY REED

Q-4) $$\frac{(x-a)(x-b)}{(c-a)(c-b)} + \frac{(x-b)(x-c)}{(a-b)(a-c)} + \frac{(x-c)(x-a)}{(b-c)(b-a)} = 1$$

TAH 4
BY REED

Soln put $x = a$, $L.H.S. = 0 + \frac{(a-b)(a-c)}{(a-b)(a-c)} + 0 = 0+1+0$
$= 1 = R.H.S.$

$\therefore x = a$ is a root.

a, b, c should be distinct !!

put $x = b$, $L.H.S. = 0 + 0 + \frac{(b-c)(b-a)}{(b-c)(b-a)}$
$= 0 + 0 + 1 = R.H.S.$ $\therefore x = b$ is a root.

put $x = c$, $L.H.S. = 1 + 0 + 0 = 1 = R.H.S.$ $\therefore x = c$ is a root.

$\therefore$ This quadratic has more than **2** roots, hence it is an identity. $\Rightarrow$ ∞ly many solns of x.

QUESTION

Given, the cubic equation $x^3 - 5x^2 + 6x - 3 = 0$ has roots α, β, γ. Find the cubic having roots

(i) $\alpha + 1, \beta + 1, \gamma + 1$

(ii) $\alpha - 1, \beta - 1, \gamma - 1$

(iii) $-\alpha, -\beta, -\gamma$

(iv) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$

(v) $\frac{3\alpha-2}{\alpha+1}, \frac{3\beta-2}{\beta+1}, \frac{3\gamma-2}{\gamma+1}$

Q-5 Given the cubic eqn $x^3-5x^2+6x-3=0$ has roots α, β, γ. Find the cubic having roots:

Soln) **(i)** $\alpha-1, \beta-1, \gamma-1$ are roots of:

$f(x) = x-1$

$\Rightarrow y = x-1 \Rightarrow \boxed{x = y+1}$

Put $x=y+1$ in eqn $\Rightarrow (y+1)^3 - 5(y+1)^2 + 6(y+1) - 3 = 0$

replace $y \to x$

$\Rightarrow y^3+1+3y^2+3y-5y^2-10y-5+6y+6-3=0$

$x^3-2x^2-x-1=0$ (Ans)

$\Rightarrow y^3-2y^2-y-1=0$

(ii) $-\alpha, -\beta, -\gamma$ are roots of:

$f(x) = -x \Rightarrow y = -x \Rightarrow x = -y.$

Put $x=-y$, $-y^3-5y^2-6y-3=0$

$\Rightarrow y^3+5y^2+6y+3=0$

replace y by x

$\Rightarrow x^3+5x^2+6x+3=0.$

(iii) $\frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ are roots of:

$\frac{1}{x} = y \Rightarrow x = \frac{1}{y}.$

Put $x=\frac{1}{y}$ in eqn $\Rightarrow \left(\frac{1}{y}\right)^3 - 5\left(\frac{1}{y^2}\right) + 6\left(\frac{1}{y}\right) - 3 = 0$

$\Rightarrow 1-5y+6y^2-3y^3=0$

replace y by x $\Rightarrow 3x^3-6x^2+5x-1=0.$ (Ans.)

(iv) $\frac{3\alpha-2}{\alpha+1}, \frac{3\beta-2}{\beta+1}, \frac{3\gamma-2}{\gamma+1}$ are roots of:

put ~~x=~~ $\frac{3x-2}{x+1} = y$

$\Rightarrow 3x-2 = xy+y$

$\Rightarrow x(3-y) = y+2$

$\Rightarrow x = \frac{y+2}{\;}$

Put $x = \frac{y+2}{3-y}$ in the main eqn,

$$\left(\frac{y+2}{3-y}\right)^3 - 5\left(\frac{y+2}{3-y}\right)^2 + 6\left(\frac{y+2}{3-y}\right) - 3 = 0.$$

or, $(y+2)^3 - 5(y+2)^2(3-y) + 6(y+2)(3-y)^2 - 3(3-y)^3 = 0.$

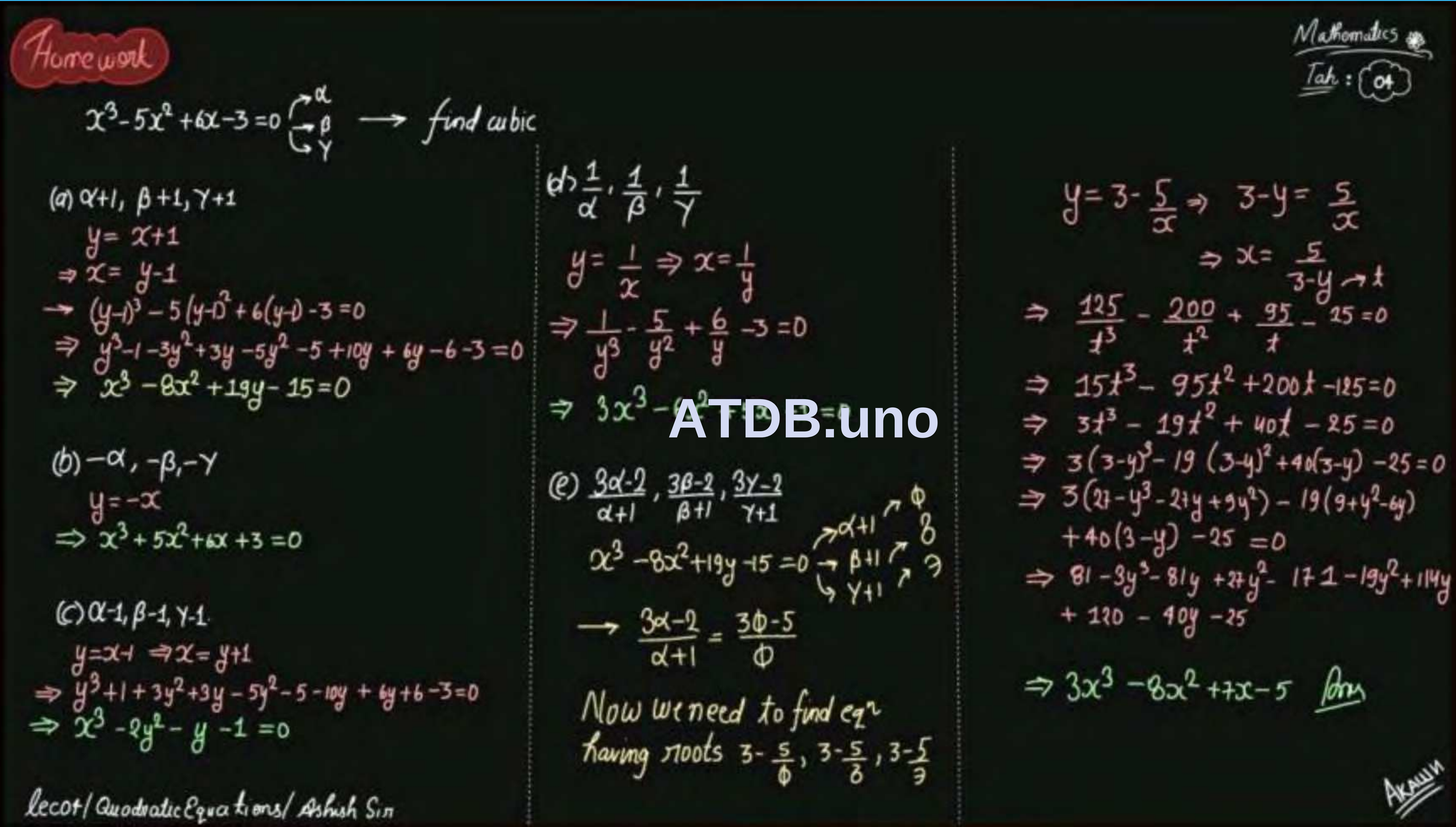
or, $y^3+8+6y^2+12y-5(y^2+2y+4)(3-y)+6(y+2)(9-y^2+6y)$
$-3(27-y^3-27y+9y^2)=0$

or, $y^3+8+6y^2+12y-5(3y^2-y^2+12-4y+12y-4y^2)$
$+6(y^3+9y-6y^2+2y^2+18-12y)$
$-3(27-y^3-27y+9y^2)=0$

or, $15y^3-40y^2+35y-25=0$

or, $3y^3-8y^2+7y-5=0$

replace $y \to x$ $\Rightarrow \underset{\text{Required Eqn.}}{3x^3-8x^2+7x-5=0}$



Homework

Mathematics

Tah : 04

$$x^3-5x^2+6x-3=0 \begin{cases}\alpha\\ \beta\\ \gamma\end{cases} \longrightarrow \text{find cubic}$$

(a) $\alpha+1,\ \beta+1,\ \gamma+1$

$y = x+1$

$\Rightarrow x = y-1$

$\rightarrow (y-1)^3-5(y-1)^2+6(y-1)-3=0$

$\Rightarrow y^3-1-3y^2+3y-5y^2-5+10y+6y-6-3=0$

$\Rightarrow x^3-8x^2+19y-15=0$

(b) $-\alpha,\ -\beta,\ -\gamma$

$y=-x$

$\Rightarrow x^3+5x^2+6x+3=0$

(c) $\alpha-1,\ \beta-1,\ \gamma-1$

$y=x-1 \Rightarrow x=y+1$

$\Rightarrow y^3+1+3y^2+3y-5y^2-5-10y+6y+6-3=0$

$\Rightarrow x^3-2y^2-y-1=0$

(d) $\frac{1}{\alpha},\ \frac{1}{\beta},\ \frac{1}{\gamma}$

$y=\frac{1}{x} \Rightarrow x=\frac{1}{y}$

$\Rightarrow \frac{1}{y^3}-\frac{5}{y^2}+\frac{6}{y}-3=0$

$\Rightarrow 3x^3-6$ [illegible] $=0$

(e) $\frac{3\alpha-2}{\alpha+1},\ \frac{3\beta-2}{\beta+1},\ \frac{3\gamma-2}{\gamma+1}$

$$x^3-8x^2+19y-15=0 \begin{cases}\alpha+1 \nearrow \phi\\ \beta+1 \nearrow \delta\\ \gamma+1 \nearrow \backepsilon\end{cases}$$

$\longrightarrow \frac{3\alpha-2}{\alpha+1}=\frac{3\phi-5}{\phi}$

Now we need to find eqⁿ having roots $3-\frac{5}{\phi},\ 3-\frac{5}{\delta},\ 3-\frac{5}{\backepsilon}$

$y=3-\frac{5}{x} \Rightarrow 3-y=\frac{5}{x}$

$\Rightarrow x=\frac{5}{3-y} \rightarrow t$

$\Rightarrow \frac{125}{t^3}-\frac{200}{t^2}+\frac{95}{t}-25=0$

$\Rightarrow 15t^3-95t^2+200t-125=0$

$\Rightarrow 3t^3-19t^2+40t-25=0$

$\Rightarrow 3(3-y)^3-19(3-y)^2+40(3-y)-25=0$

$\Rightarrow 3(27-y^3-27y+9y^2)-19(9+y^2-6y)$
$+40(3-y)-25=0$

$\Rightarrow 81-3y^3-81y+27y^2-171-19y^2+114y$
$+120-40y-25$

$\Rightarrow 3x^3-8x^2+7x-5$ Ans

lecot/QuadraticEquations/Ashish Sir

Akash

QUESTION

Let α, β and γ are the roots of the cubic $x^3 - 3x^2 + 1 = 0$. Find a cubic whose roots are $\frac{\alpha}{\alpha-2}, \frac{\beta}{\beta-2}$ and $\frac{\gamma}{\gamma-2}$. Hence or otherwise find the value of $(\alpha - 2)(\beta - 2)(\gamma - 2)$.

Ans. $3y^3 - 9y^2 - 3y + 1 = 0; (\alpha - 2)(\beta - 2)(\gamma - 2) = 3$

Q-61 If α, β, γ are roots of $x^3 - 3x^2 + 1 = 0$. Find a cubic whose roots are $\frac{\alpha}{\alpha-2}, \frac{\beta}{\beta-2}, \frac{\gamma}{\gamma-2}$. Hence or otherwise find value of $(\alpha-2)\cdot(\beta-2)\cdot(\gamma-2)$:

Soln

$$\frac{\alpha}{\alpha-2} = f(\alpha), \quad \frac{\beta}{\beta-2} = f(\beta), \quad \frac{\gamma}{\gamma-2} = f(\gamma)$$

$$\Downarrow$$

$$f(x) = \frac{x}{x-2}$$

$$\Rightarrow y = \frac{x}{x-2}$$

$$\Rightarrow x = xy - 2y$$

$$\Rightarrow x(1-y) = -2y$$

$$\Rightarrow \boxed{x = \frac{2y}{y-1}}$$

put $x = \frac{2y}{y-1}$

in the main equn

$$\downarrow$$

$$\left(\frac{2y}{y-1}\right)^3 - 3\left(\frac{2y}{y-1}\right)^2 + 1 = 0$$

$$\Rightarrow 8y^3 - 3\cdot 4y^2(y-1) + 1(y-1)^3 = 0$$

$$\Rightarrow 8y^3 - 12y^3 + 12y^2 + y^3 - 1 - 3y^2 + 3y = 0$$

$$\Rightarrow -4y^3 + y^3 + 9y^2 + 3y - 1 = 0$$

$$\Rightarrow 3y^3 - 9y^2 - 3y + 1 = 0. \Longrightarrow \boxed{y \to x} \boxed{3x^3 - 9x^2 - 3x + 1 = 0}$$

2nd: $x^3 - 3x^2 + 1 = (x-\alpha)(x-\beta)(x-\gamma)$

$$\Rightarrow x^3 - 3x^2 + 1 = -(\alpha - x)(\beta - x)(\gamma - x)$$

$$\downarrow$$

put $x = 2 \Rightarrow 8 - 3\cdot 4 + 1 = -(\alpha-2)(\beta-2)(\gamma-2)$

$$\Rightarrow (\alpha-2)(\beta-2)(\gamma-2) = -(-3) = \boxed{3}$$ Ans.

* Tah Ob:-

$x^3 - 3x^2 + 1 = 0$ has α, β, γ are roots.

$$y = f(x) = \frac{x}{x-2} \Rightarrow x = \frac{2y}{y-1}$$ put in eqn.

$$\left(\frac{2y}{y-1}\right)^3 - 3\left(\frac{2y}{y-1}\right)^2 + 1 = 0$$

$$\frac{8y^3}{(y-1)^3} - \frac{12y^2}{(y-1)^2} + 1 = 0$$

$$8y^3 - 12y^2(y-1) + (y-1)^3 = 0$$

$$8y^3 - 12y^3 + 12y^2 + y^3 - 1 - 3y^2 + 3y = 0$$

$$-3y^3 + 9y^2 + 3y - 1 = 0$$

$$3y^3 - 9y^2 - 3y + 1 = 0$$

$$\boxed{3x^3 - 9x^2 - 3x + 1 = 0}$$ Ans

Date ____
Page ____

$\Rightarrow \quad y = f(x) = x-2 \Rightarrow x = y+2$ put in eqⁿ.

$$(y+2)^3 - 3(y+2)^2 + 1 = 0$$

$$y^3 + 8 + 3y^2\cdot 2 + 3\cdot 4\cdot y - 3(y^2+4+4y) + 1 = 0$$

$$y^3 + 6y^2 + \cancel{12y} + 8 - 3y^2 - \cancel{12y} - 12 + 1 = 0$$

$$y^3 + 3y^2 - 3 = 0$$

$\hookrightarrow \left[x^3 + 3x^2 - 3 = 0\right]$ Ans

Ankush!!

QUESTION

If α, β, γ are roots of the cubic $2011x^3 + 2x^2 + 1 = 0$, then find

(i) $(\alpha\beta)^{-1} + (\beta\gamma)^{-1} + (\gamma\alpha)^{-1}$;

(ii) $\alpha^{-2} + \beta^{-2} + \gamma^{-2}$

Ans. (i) 2 ; (ii) −4

Q-7: α, β, γ are roots of $2011x^3+2x^2+1=0$. then find:

(i) $\alpha^{-2}+\beta^{-2}+\gamma^{-2}$

$2011x^3+2x^2+1=0 \rightarrow \alpha, \beta, \gamma$ ①

$S_1 = \frac{-2}{2011}$

$S_2 = 0$.

$S_3 = \frac{-1}{2011}$

Sol^n: • **M-1:** Normal way!

$$E = \frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2} = \frac{\alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2}{(\alpha\beta\gamma)^2}$$

from ① $\alpha\beta+\beta\gamma+\alpha\gamma = 0$.

S.B.S.

$$\Rightarrow \alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2+2\alpha\beta\gamma(\alpha+\beta+\gamma) = 0$$

$$\Rightarrow \alpha^2\beta^2+\beta^2\gamma^2+\alpha^2\gamma^2 = -2\alpha\beta\gamma(\alpha+\beta+\gamma)$$

$$\therefore E = \frac{-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{(\alpha\beta\gamma)^2} = \frac{-2(\alpha+\beta+\gamma)}{\alpha\beta\gamma} = \frac{-2\times\left(\frac{-2}{2011}\right)}{\left(\frac{-1}{2011}\right)}$$

$$\Rightarrow \boxed{E = -4}$$

• **M-2:** Newton's formula!

let, $S_n = \alpha^n+\beta^n+\gamma^n$. $E = \alpha^{-2}+\beta^{-2}+\gamma^{-2}$

$$\underbrace{S_{-2}} = \alpha^{-2}+\beta^{-2}+\gamma^{-2}$$

from $2011x^3+2x^2+1=0$

By N.F. $2011\,S_{n+3}+2S_{n+2}+S_n = 0$

put $n=-2$, $2011\,S_1+2S_0+S_{-2} = 0$

$$\Rightarrow \cancel{2011}\left(\frac{-2}{\cancel{2011}}\right)+(2\times3)+S_{-2} = 0.$$

$$\Rightarrow S_{-2} = 2-6 = -4$$

$$\Rightarrow \boxed{S_{-2} = -4 = E}$$

TAH 6A
BY REED
FROM WB

- **M-3:** By manipulation:

$$2011x^3 + 2x^2 + 1 = 0 \quad \begin{matrix} \nearrow \alpha \\ \to \beta \\ \searrow \gamma \end{matrix}$$

$$\alpha \quad 2011\alpha^3 + 2\alpha^2 = -1$$

$$\Rightarrow \alpha^2(2011\alpha + 2) = -1$$

$$\Rightarrow \frac{1}{\alpha^2} = -(2011\alpha + 2)$$

Illy, $\frac{1}{\beta^2} = -(2011\beta + 2)$

$\frac{1}{\gamma^2} = -(2011\gamma + 2)$

adding $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = -2011(\alpha+\beta+\gamma) - 6 = E$

$$\Rightarrow E = -2011 \times \left(\frac{-2}{2011}\right) - 6$$

$$\Rightarrow E = 2 - 6 = \boxed{-4}$$ (Ans.)

- **M-4:** By transformation:

we have to find a cubic whose roots are $\frac{1}{\alpha^2}, \frac{1}{\beta^2}, \frac{1}{\gamma^2}$ $\Rightarrow f(x) = \frac{1}{x^2}$

$$\Rightarrow y = \frac{1}{x^2}$$

$$\Rightarrow \boxed{x = \pm\frac{1}{\sqrt{y}}}$$

Put $x = \pm\frac{1}{\sqrt{y}}$ in ①,

$$2011\left(\frac{\pm 1}{\sqrt{y}}\right)^3 + \left(\frac{\pm 1}{\sqrt{y}}\right)^2 \cdot 2 + 1 = 0$$

$$\Rightarrow \pm 2011 + 2\sqrt{y} + y\sqrt{y} = 0.$$

$$\Rightarrow \pm 2011 = -\sqrt{y}(2+y)$$

S.B.S.

$$\Rightarrow (2011)^2 = y(4 + y^2 + 4y)$$

$$\Rightarrow y^3 + 4y^2 + 4y - (2011)^2 = 0 \quad \begin{matrix} \nearrow \alpha^{-2} \\ \to \beta^{-2} \\ \searrow \gamma^{-2} \end{matrix}$$

$$\therefore \text{S.O.R} = \alpha^{-2} + \beta^{-2} + \gamma^{-2} = -\frac{4}{1} = \boxed{-4}$$

Mathematics
Tah : 06(a)

Homework

$2011x^3+2x^2+1=0 \to \alpha, \beta, \gamma$

$\to \alpha^{-2}+\beta^{-2}+\gamma^{-2}=?$

Method - 01

$2011S_1+2S_0+S_{-2}=0$

$S_1=\alpha+\beta+\gamma=\frac{-2}{2011}$

$S_0=3$

$\Rightarrow -2+6+S_{-2}=0$

$S_{-2}=-4$

$\Rightarrow \alpha^{-2}+\beta^{-2}+\gamma^{-2}=\boxed{-4}$ Ans

Method 02

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}$

$=\frac{\beta^2\gamma^2+\alpha^2\gamma^2+\alpha^2\beta^2}{(\alpha\beta\gamma)^2} \nearrow E$

$\alpha\beta\gamma=-1/2011$

$\alpha\beta+\beta\gamma+\gamma\alpha=0$

$\alpha+\beta+\gamma=-2/2011$

$\beta^2\gamma^2+\alpha^2\beta^2+\alpha^2\gamma^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)$

$\Rightarrow \beta^2\gamma^2+\alpha^2\beta^2+\alpha^2\gamma^2=\frac{-4}{(2011)}$

$\Rightarrow E=\frac{-4\times(2011)^2}{(2011)}=\boxed{-4}$ Ans

Method 03

$y=\frac{1}{x^2}\Rightarrow x=\frac{1}{\sqrt{y}}$

$\Rightarrow \frac{2011}{y^{3/2}}+\frac{2}{y}+1=0$

$\Rightarrow y^{3/2}+2y^{1/2}+2011=0$

$\Rightarrow y^{3/2}+2y^{1/2}=-2011$

$\Rightarrow y^3+4y+4y^2=(2011)^2$

$\Rightarrow x^3+4x^2+4x-(2011)^2=0$

Sum of roots $=\boxed{-4}$ Ans

Method 04

Method 2.5

$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)^2-2\left(\overset{\nearrow 0}{\frac{1}{\alpha\beta}+\frac{1}{\beta\gamma}+\frac{1}{\alpha\gamma}}\right)=-2\left(\frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}\right)=\boxed{-4}$ Ans

lecot/QuadraticEquations/Ashish Sin

Akash

QUESTION

★★ASRQ★★

Let roots of the equation $x^3 + 3x^2 + 4x = 11$ are α, β, γ and the roots of equation $x^3 + lx^2 + mx + n = 0$ $(l, m, n \in R)$ are $\alpha + \beta, \beta + \gamma, \gamma + \alpha$.

Column-I	Column-II
(A) l is equal to	(P) –6
(B) m is equal to	(Q) 6
(C) n is equal to	(R) 13
(D) $(l + m + n)$ is equal to	(S) 23

AH-06 (b)

$$x^3 + 3x^2 + 4x - 11 = 0 \begin{matrix} \nearrow \alpha \\ \rightarrow \beta \\ \searrow \gamma \end{matrix} \quad \text{——} \quad ①$$

$$\alpha + \beta + \gamma = -3$$

$$\begin{matrix} \alpha + \beta & \beta + \gamma & \gamma + \alpha \\ \downarrow & \downarrow & \downarrow \\ -3 - \gamma & -3 - \alpha & -3 - \beta \end{matrix}$$

$$\therefore \; y = -3 - x$$

$$x = -3 - y$$

$$= -(3 + y) \text{ put in } ①$$

$$-(3+y)^3 + 3(3+y)^2 - 4(3+y) - 11 = 0$$

$$-(y^3 + 9y^2 + 27y + 27) + 3(9 + y^2 + 6y)$$

$$- 12 - 4y - 11 = 0$$

$$\Rightarrow -y^3 - 9y^2 - 27y - \cancel{27} + \cancel{27} + 3y^2 + 18y$$

$$- 12 - 4y - 11 = 0$$

$$\Rightarrow -y^3 - 6y^2 - 13y - 23 = 0$$

$$\Rightarrow y^3 + 6y^2 + 13y + 23 = 0$$

<u>Given</u> $x^3 + lx^2 + mx + n = 0$

$$\therefore \; \underset{(Q)}{l = 6} \; / \; \underset{(R)}{m = 13} \; / \; \underset{(S)}{n = 23}$$

$$l + m + n = 6 + 13 + 23$$

$$= 42$$

Q-8: Let roots of the equation $x^3+3x^2+4x=11$ are α,β,γ and the roots of equation $x^3+lx^2+mx+n=0$ $(l,m,n\in R)$ are $\alpha+\beta,\ \beta+\gamma,\ \gamma+\alpha$.

Soln:

$$x^3+3x^2+4x-11=0 \begin{matrix}\nearrow\alpha\\ \rightarrow\beta\\ \searrow\gamma\end{matrix} \quad -① \left.\right\} \quad \alpha+\beta+\gamma=-3$$

$\Rightarrow \alpha+\beta=-3-\gamma$

$\Rightarrow \beta+\gamma=-3-\gamma$

$\Rightarrow \gamma+\alpha=-3-\beta$

to form an equation with roots $\alpha+\beta,\ \beta+\gamma,\ \gamma+\alpha$.

$$\underset{-3-\gamma}{\overset{}{\alpha+\beta}},\quad \underset{-3-\gamma}{\beta+\gamma},\quad \underset{-3-\beta}{\gamma+\alpha}$$

$y=f(x)=-3-x \Rightarrow x=-3-y$

So, put $\boxed{x=-3-y}$ in ①

$$x^3+3x^2+4x-11=0$$

$\Rightarrow (-3-y)^3+3(-3-y)^2+4(-3-y)-11=0$

$\Rightarrow -(3+y)^3+3(3+y)^2-4(3+y)-11=0$

$\Rightarrow -27-y^3-27y-9y^2+27+18y+3y^2-4y-23=0$

$\Rightarrow y^3+13y+6y^2+23=0.$ replace $y\to x$.

$\Rightarrow x^3+6x^2+13x+23=0$

Compare with $x^3+lx^2+mx+n=0$ → $l=6,\ m=13,\ n=23.$

$\therefore l+m+n=42.$

Ans ⇒ (A) → (Q) 6 , (C) → (S) 23

(B) → (R) 13 , (D) → ( ) 42.

QUESTION

If the roots of $p(x) = x^3 + 3x^2 + 4x - 8$ are a, b and c, what is the value of $a^2(1 + a^2) + b^2(1 + b^2) + c^2(1 + c^2)$?

Q-14: If the roots of $p(x) = x^3 + 3x^2 + 4x - 8$ are a, b, c then what is the value of $a^2(1+a^2) + b^2(1+b^2) + c^2(1+c^2) = ?$

Sol^n: $p(x) = x^3 + 3x^2 + 4x - 8 = 0 \begin{matrix} \nearrow a \\ \rightarrow b \\ \searrow c \end{matrix} \Big\}$ roots — ①

$$\begin{pmatrix} S_1 = -3 \\ S_2 = ? \end{pmatrix}$$

$E = a^2 + b^2 + c^2 + a^4 + b^4 + c^4 = S_2 + S_4$

By N.F. from ① $\Rightarrow$ $S_{n+3} + 3S_{n+2} + 4S_{n+1} - 8S_n = 0$

$\boxed{n=1} \rightarrow S_4 + 3S_3 + 4S_2 - 8S_1 = 0$.

$S_2 = (a+b+c)^2 - 2(ab+bc+ca) \nearrow 4$

$S_2 = (-3)^2 - 2 \times 4$

$S_2 = 9 - 8 = 1$

$\Rightarrow S_4 + 3S_3 + 4 - 8(-3) = 0$

$\Rightarrow S_4 + 3S_3 = -28$ — ②

put $\boxed{n=0} \rightarrow S_3 + 3S_2 + 4S_1 - 8S_0 = 0$

$\Rightarrow S_3 + 3(1) + 4(-3) - 8(3) = 0$

$\Rightarrow \boxed{S_3 = 33}$ — ② $\rightarrow S_4 = -28 - 99$

$\Rightarrow S_4 = -127$

$\therefore E = S_4 + S_2 = -127 + 1 = -126$ (Ans)

Method-2: $p(x) = x^2 + 3x^2 + 4x - 8 = 0 \begin{matrix} \nearrow a \\ \rightarrow b \\ \searrow c \end{matrix} \Big\}$ roots.

$a + b + c = -3$

$ab + bc + ac = 4$

$abc = 8$.

Now, $(ab + bc + ac)^2 = a^2b^2 + b^2c^2 + a^2c^2 - 2abc(a+b+c)$

$\Rightarrow (ab + bc + ac)^2 - 2abc(a+b+c) = a^2b^2 + b^2c^2 + c^2a^2$

$\Rightarrow 16 - 16(-3) = 64 = \underbrace{a^2b^2 + b^2c^2 + c^2a^2}_{t}$

$\therefore E = a^2 + b^2 + c^2 + a^4 + b^4 + c^4$

$\Rightarrow E = \{(a+b+c)^2 - 2(ab+bc+ca)\} + \{(a^2+b^2+c^2)^2 - 2t\}$

$\Rightarrow E = (9 - 8) + (\underbrace{(a+b+c)^2 - 2(ab+bc+ca)}_{1})^2 - 2(64)$

$\Rightarrow E = 1 + 1^2 - 128$

$\Rightarrow E = -126$ (Ans.)

$p(x) = x^3 + 3x^2 + 4x - 8 \begin{matrix} \to a \\ \to b \\ \to c \end{matrix}$

M₁

$$a^2(1+a^2) + b^2(1+b^2) + c^2(1+c^2)$$

$$= a^2 + a^4 + b^2 + b^4 + c^2 + c^4$$

$$= (a^2+b^2+c^2) + (a^4+b^4+c^4)$$

$$= S_2 + S_4$$

where,

$S_n = a^n + b^n + c^n$

By Newton's formula,

$$S_{n+3} + 3S_{n+2} + 4S_{n+1} - 8S_n = 0$$

$n = 1$

$$S_4 + 3S_3 + 4S_2 - 8S_1 = 0$$

$$S_4 + 3S_3 + 4S_2 + 24 = 0 \quad \left\{ a+b+c = -3 \right.$$

put $n = 0$

$$S_3 + 3S_2 + 4S_1 - 8S_0 = 0$$

$$S_3 + 3S_2 + 4(-3) - 8(3) = 0$$

$$S_3 + 3S_2 = 36$$

$$S_3 = 36 - 3S_2$$

$$S_4 + 108 - 9S_2 + 4S_2 + 24 = 0$$

$$S_4 = 5S_2 - 132$$

$$S_4 + S_2 = 6S_2 - 132$$

$$S_2 = a^2 + b^2 + c^2$$

$$= (a+b+c)^2 - 2(ab+bc+ca)$$

$$= (-3)^2 - 2(4)$$

$$= 9 - 8$$

$$= 1$$

$$\Rightarrow S_4 + S_2 = 6 \times 1 - 132$$

$$= -126$$

M₂

$$a+b+c = -3$$

$$ab+bc+ca = 4$$

$$abc = 8$$

S.B.S

$$a^2+b^2+c^2+2(ab+bc+ca) = 9$$

$$a^2+b^2+c^2+8 = 9$$

$$a^2+b^2+c^2 = 1$$

S.B.S

$$a^4+b^4+c^4+2(a^2b^2+b^2c^2+c^2a^2) = 1$$

$$ab+bc+ca = 4$$

S.B.S

$$a^2b^2+b^2c^2+c^2a^2+2abc(a+b+c) = 16$$

$$a^2b^2+b^2c^2+c^2a^2+2(8)(-3) = 16$$

$$a^2b^2+b^2c^2+c^2a^2 = 16+48$$

$$= 64$$

$$a^4+b^4+c^4+2 \times 64 = 1$$

$$a^4+b^4+c^4+128 = 1$$

$$a^4+b^4+c^4 = -127$$

Ans

$$\therefore (a^2+b^2+c^2) + (a^4+b^4+c^4) = 1 - 127 = \underline{-126}$$

M_3

$p(1) = 1+3+4-8 = 0$

$p(x) = x^2(x-1)+4x(x-1)+8(x-1)$

$= (x-1)(x^2+4x+8)$

Let, $a=1$ Q

then the other roots b & c is the root of

$$x^2+4x+8=0 \quad \begin{matrix} \nearrow b \\ \searrow c \end{matrix}$$

$\Rightarrow b+c=-4,\ bc=8$

$\therefore (a^2+b^2+c^2)+(a^4+b^4+c^4)$

put, $a=1$

$1+b^2+c^2+1+b^4+c^4$

$= 2+(b^2+c^2)+(b^4+c^4)$

$= 2+(b+c)^2-2bc+b^4+c^4$

$= 2+16-16+b^4+c^4$

$= 2+\overset{0}{(b^2+c^2)^2}-2(bc)^2$

$= 2-2(8)^2$

$= 2-2\times 64$

$= 2-128$

$= -126$

Ans

QUESTION

Let $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the roots of equation $x^4 - 7x + 1 = 0$, then

(A) $\displaystyle\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{25}{9}$

(B) $\displaystyle\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = 1$

(C) $\displaystyle\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{1}{9}$

(D) $\displaystyle\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{23}{9}$

Q-(3)! Let $\alpha_1, \alpha_2, \alpha_3$ and α_4 are the roots of eqn $x^4 - 7x + 1 = 0$ then:

Soln

$$x^4 - 7x + 1 = 0 \to \alpha_1, \alpha_2, \alpha_3, \alpha_4 \quad \text{———} ①$$

$$\& \ \alpha_1\alpha_2\alpha_3\alpha_4 = \frac{+1}{1}$$

for (b),(c):
$$\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{\alpha_1\alpha_2\alpha_3\alpha_4}{(1+\alpha_1)(1+\alpha_2)(1+\alpha_3)(1+\alpha_4)}$$

from ① → $x^4 - 7x + 1 = (x-\alpha_1)(x-\alpha_2)(x-\alpha_3)(x-\alpha_4)$

put $x = -1$, $1 + 7 + 1 = (1+\alpha_1)(1+\alpha_2)(1+\alpha_3)(1+\alpha_4)$

$$\Rightarrow 9 = (1+\alpha_1)(1+\alpha_2)(1+\alpha_3)(1+\alpha_4)$$

$$\therefore \prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{1}{9} \quad (\text{Ans} \Rightarrow (c))$$

for (a),(d):
$$\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{\alpha_1}{1+\alpha_1} + \frac{\alpha_2}{1+\alpha_2} + \frac{\alpha_3}{1+\alpha_3} + \frac{\alpha_4}{1+\alpha_4}$$

we have to find a 4 degree polynomial with roots, $f(x) = \frac{x}{1+x} = y$

$$\Rightarrow x(1-y) = y$$

$$\Rightarrow \boxed{x = \frac{y}{1-y}} \quad \text{put in ①}$$

TAH 07
BY REED
FROM WB

$$\left(\frac{y}{1-y}\right)^4 - 7\left(\frac{y}{1-y}\right) + 1 = 0$$

$$\Rightarrow y^4 - 7y(1-y)^3 + (1-y)^4 = 0.$$

$$\Rightarrow y^4 - 7y(1-y^3 - 3y + 3y^2) + 1 + y^4 - 4y^3 + 6y^2 - 4y = 0.$$

$$\Rightarrow y^4 - 7y + 7y^4 + 21y^2 - 21y^3 + y^4 - 4y^3 + 6y^2 - 4y = 0$$

$$\Rightarrow 9y^4 - 25y^3 + \ldots\ldots = 0$$

$$\therefore \sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{-(-25)}{9} = \frac{25}{9} \quad (\text{Ans} \Rightarrow (a)).$$

(AH-0

$$x^4 - 7x + 1 = 0 \quad \begin{cases}\alpha_1\\ \alpha_2\\ \alpha_3\\ \alpha_4\end{cases}$$

$$\sum_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \frac{\alpha_1}{\alpha_1+1} + \frac{\alpha_2}{\alpha_2+1} + \frac{\alpha_3}{1+\alpha_3} + \frac{\alpha_4}{\alpha_4+1}.$$

$$y = \frac{x}{x+1} \Rightarrow xy + y = x \Rightarrow x = \frac{y}{1-y}$$

$$4.\ \left(\frac{y}{1-y}\right)^4 - 7\left(\frac{y}{1-y}\right) + 1 = 0 \qquad (1-y)^2 \times (1-y)^2$$

$$(1-y)^4$$

$$\Rightarrow y^4 - 7(y)(1-y)^3 ~~+(1-y)^4~~ = 0$$

$$\Rightarrow y^4 - 7y(1 - y^3 - 3y + 3y^2) + (1 + y^2 - 2y)^2 = 0$$

$$+2x$$

$$\Rightarrow \cancel{y^4 - 7y + 7y^4 + 21y^2 - 21y^3 + (1 + y^4 + 9y^2 + 2(y^2 + 2y^3 + 2y}$$

$$\Rightarrow \cancel{9y^4 - 17y^3 + 27y^2}$$

$$\Rightarrow y^4 - 7y + 7y^4 - 21y^2 - 21y^3 + 1 + y^4 + 4y^2 + 2(y^2 - 2y^3 - 2y$$

$$= 0$$

$$\Rightarrow 9y^4 - 25y^3 - 15y^2 - 11y + 1 = 0$$

Sum of roots $= -\left(\frac{-25}{9}\right) = \frac{25}{9}$ (A)

$$\prod_{i=1}^{4} \frac{\alpha_i}{1+\alpha_i} = \left(\frac{\alpha_1}{1+\alpha_1}\right) \times \left(\frac{\alpha_2}{1+\alpha_2}\right)\left(\frac{\alpha_3}{1+\alpha_3}\right)\left(\frac{\alpha_4}{1+\alpha_4}\right) =$$

$$\Rightarrow \frac{(\alpha_1\alpha_2\alpha_3\alpha_4)}{(1+\alpha_1+\alpha_2+\alpha_1\alpha_2)(1+\alpha_3+\alpha_4+\alpha_3\alpha_4)}$$

$$\Rightarrow \frac{1}{1+\alpha_1+\alpha_2+\alpha_1\alpha_2+\alpha_3+\alpha_1\alpha_3+\alpha_2\alpha_3+\alpha_1\alpha_2\alpha_3+\alpha_4+\alpha_1\alpha_4+\alpha_2\alpha_4+\alpha_1\alpha_2\alpha_4 + \alpha_4 + \alpha_3\alpha_4 + \alpha_3\alpha_4\alpha_1 + \alpha_1\alpha_2\alpha_4\alpha_3 + \alpha_1\alpha_2\alpha_3\alpha_4}$$

$$\Rightarrow \frac{1}{1+(\alpha_1+\alpha_2+\alpha_3+\alpha_4)+(\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_{24}+\alpha_3\alpha_4) + (\alpha_1\alpha_2\alpha_3 + \alpha_1\alpha_2\alpha_4 + \alpha_1\alpha_3\alpha_4 + \alpha_2\alpha_3\alpha_4) + \alpha_1\alpha_2\alpha_3\alpha_4}$$

$$\Rightarrow \frac{1}{1+(0)+(0)+(7)+(1)}$$

$$= \frac{1}{9} \quad (C)$$

QUESTION [JEE Mains 2023 (30 Jan)]

If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to

Ans. 13

Homework

Mathematics
Tak : 09

$x^2-5ax+1=0$ and $x^2-ax-5=0$ ⟶ common real root → $\frac{3}{\sqrt{2\beta}}$ then $\beta=?$

$$x^2-5ax+1=0$$
$$x^2-ax-5=0$$

$$\begin{vmatrix}1&-5a\\1&-a\end{vmatrix}\times\begin{vmatrix}-5a&1\\-a&-5\end{vmatrix}=\begin{vmatrix}1&1\\-5&1\end{vmatrix}^2$$

$$(4a)(26a)=36$$
$$a^2=\frac{36}{4\times26}$$
$$a^2=\frac{9}{2\times13}\nearrow\quad\beta=13\ \underline{\text{Ans}}$$

lecot/ Quadratic Equations/ Ashish Sir

Akash

TAH-07.

$$x^2 - 5ax + 1 = 0$$

$$x^2 - ax - 5 = 0.$$

$$\Rightarrow \begin{vmatrix} 1 & -5a \\ 1 & -a \end{vmatrix} \times \begin{vmatrix} -5a & 1 \\ -a & -5 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ -5 & 1 \end{vmatrix}^2$$

$$\Rightarrow 4a \times 26a = (1+5)^2$$

$$\Rightarrow \cancel{4}a \times 26a = \cancel{36}^{9}.$$

$$\because a > 0$$

$$\Rightarrow a^2 = \frac{9}{26} \Rightarrow a = \pm\frac{3}{\sqrt{26}} = +\frac{3}{\sqrt{2\beta}} \Rightarrow \boxed{\beta = 13}$$

TAH-09

$$\left.\begin{array}{l} x^2 - 5ax + 1 = 0 \\ x^2 - ax - 5 = 0 \end{array}\right\}$$ α is a common root

$$\begin{array}{l} \alpha^2 - 5a\alpha + 1 = 0 \\ \alpha^2 - a\alpha - 5 = 0 \\ \ominus \quad\ \oplus \quad\ \oplus \\ \hline \end{array}$$

$$-4a\alpha + 6 = 0$$

$$a\alpha = 3/2$$

$$\alpha^2 - a\alpha = 5$$

$$\alpha^2 - 3/2 = 5$$

$$\alpha^2 = 13/2$$

$$a^2\alpha^2 = 9/4$$

$$a^2 \times 13/\cancel{2} = 9/\cancel{4}\,2$$

$$a^2 = \frac{9}{2 \times 13}$$

$$a = \frac{3}{\sqrt{2 \times 13}}$$ $(a > 0)$

Q-11! If the value of real no. $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common root is $\frac{3}{\sqrt{2\beta}}$ then $\beta = ?$

TAH 09
BY REED
FROM WB

Sol

Soln Method-2!

$$x^2 - 5ax + 1 = 0 \text{ —— (i)}$$

$$\underset{-}{x^2} \underset{+}{-ax} \underset{+}{-5} = 0 \text{ —— (ii)}$$

$$-4ax + 6 = 0$$

$$\Rightarrow 4ax = -6$$

$$\Rightarrow \boxed{x = \frac{3}{2a}} \rightarrow x = \frac{3}{2a} = \alpha \text{ (common root)}$$

put $x = \frac{3}{2a}$ in (ii),

$$\left(\frac{3}{2a}\right)^2 - \cancel{a}\left(\frac{3}{2\cancel{a}}\right) - 5 = 0$$

$$\Rightarrow \frac{9}{4a^2} = \frac{3}{2} + 5$$

$$\Rightarrow \frac{9}{4a^2} = \frac{13}{2}$$

$$\Rightarrow a^2 = \frac{9}{26}$$

$$\Rightarrow a = \frac{3}{\sqrt{26}} = \frac{3}{\sqrt{2\beta}} \quad \therefore 2\beta = 26$$

$$\Rightarrow \boxed{\beta = 13} \text{ Ans.}$$

Solution to Previous KTKs

QUESTION [JEE Mains 2019 (10 April)]

If α and β are the roots of the quadratic equation, $x^2 + x\sin\theta - 2\sin\theta = 0, \theta \in \left(0, \frac{\pi}{2}\right)$,

then $\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\cdot(\alpha-\beta)^{24}}$ is equal to :

(A) $\frac{2^{12}}{(\sin\theta-8)^6}$

(B) $\frac{2^6}{(\sin\theta+4)^{12}}$

(C) $\frac{2^{12}}{(\sin\theta+8)^{12}}$

(D) $\frac{2^{12}}{(\sin\theta-4)^{12}}$

Ans. C

Q-131 α, β roots of $x^2 + x\sin\theta - 2\sin\theta = 0,\ \theta \in (0, \frac{\pi}{2})$

$$\frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\,(\alpha-\beta)^{24}} \text{ is } = ?$$

KTK 1
BY REED
FROM WB

Soln

$$x^2 + x\sin\theta - 2\sin\theta = 0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$$

$$\alpha+\beta = -\sin\theta$$

$$\alpha\beta = -2\sin\theta.$$

$$E = \frac{\alpha^{12}+\beta^{12}}{(\alpha^{-12}+\beta^{-12})\,(\alpha-\beta)^{24}}$$

$$(\alpha-\beta)^2 = \frac{D}{a^2}$$

$$(\alpha-\beta)^2 = \frac{\sin^2\theta + 8\sin\theta}{1}$$

$$= \frac{\alpha^{12}\cancel{+\beta^{12}}}{\frac{(\alpha^{12}\cancel{+\beta^{12}})}{(\alpha\beta)^{12}} \times (\alpha-\beta)^{24}}$$

$$= \frac{2^{12}\sin^{12}\theta}{(\sin^2\theta + 8\sin\theta)^{12}}$$

$$= \frac{2^{12}\,\cancel{\sin^{12}\theta}}{\cancel{\sin^{12}\theta}\,(\sin\theta+8)^{12}}$$

$$= \frac{2^{12}}{(\sin\theta+8)^{12}}$$ (Ans = ©)

QUESTION [JEE Mains 2022 (27 July)]

If α, β are the roots of the equation

$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right)x + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$$

then the equation, whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$, is :

(A) $3x^2 - 20x - 12 = 0$

(B) $3x^2 - 10x - 4 = 0$

(C) $3x^2 - 10x + 2 = 0$

(D) $3x^2 - 20x + 16 = 0$

Ans. B

$$E = x^2 - (5 + \underbrace{3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}}_{t})x + 3(\underbrace{3^{(\log_3 5)^{1/3}} - 5^{(\log_5 3)^{\frac{2}{3}}}}_{u} - 1) = 0$$

$$t = 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}$$

we know

$$a^{\sqrt{\log_a b}} = b^{\sqrt{\log_b a}}$$

$$\Rightarrow t = 3^{\sqrt{\log_3 5}} - 3^{\sqrt{\log_3 5}}$$

$$\Rightarrow \boxed{t = 0}$$

$$u = 3^{(\log_3 5)^{1/3}} - 5^{(\log_5 3)^{\frac{2}{3}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{1-\frac{1}{3}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{\frac{1}{3}}} - 5^{\frac{\log_5 3}{(\log_5 3)^{\frac{1}{3}}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{\frac{1}{3}}} - (5^{\log_5 3})^{\frac{1}{(\log_5 3)^{\frac{1}{3}}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{\frac{1}{3}}} - (3^{\log_3 5})^{\frac{1}{(1/\log_3 5)^{\frac{1}{3}}}}$$

$$\Rightarrow u = 3^{(\log_3 5)^{\frac{1}{3}}} - 3^{(\log_3 5)^{\frac{1}{3}}}$$

$$\Rightarrow \boxed{u = 0}$$

put $t = 0$, $u = 0$ in E,

$$E = x^2 - 5x + 3(0 - 1) = x^2 - 5x - 3 = 0.$$

∴ quad reqd eqn with roots $\alpha + \frac{1}{\beta}$, $\beta + \frac{1}{\alpha}$ is:

$\alpha + \beta = 5$
$\alpha\beta = -3$.

$$x^2 - (\alpha + \tfrac{1}{\beta} + \beta + \tfrac{1}{\alpha})x + (\alpha + \tfrac{1}{\beta})(\beta + \tfrac{1}{\alpha}) = 0$$

$$\Rightarrow x^2 - (\alpha + \beta + \frac{\alpha+\beta}{\alpha\beta})x + (\alpha\beta + \frac{1}{\alpha\beta} + 1 + 1) = 0$$

$$\Rightarrow x^2 - (5 + \frac{5}{-3})x + (-3 - \frac{1}{3} + 2) = 0$$

$$\Rightarrow x^2 - \frac{10}{3}x + (\frac{-9-1+6}{3}) = 0$$

$$\Rightarrow x^2 - \frac{10}{3}x + (-\frac{4}{3}) = 0 \Rightarrow \boxed{3x^2 - 10x - 4 = 0}.$$ Ans.

QUESTION [JEE Mains 2021]

Let α, β be two roots of the equation $x^2 + (20)^{1/4}x + (5)^{1/2} = 0$. Then $\alpha^8 + \beta^8$ is equal to

Ⓐ 10

Ⓑ 100

Ⓒ 50

Ⓓ 160

Ans. C

KTK - 3.

$$x^2 + (20)^{\frac{1}{4}} x + (5)^{\frac{1}{2}} = 0$$

$\Rightarrow \alpha + \beta = -(20)^{\frac{1}{4}}, \quad \alpha\beta = (5)^{\frac{1}{2}}$

S.B.S.

$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \left[-(20)^{\frac{1}{4}}\right]^2$

$\Rightarrow \alpha^2 + \beta^2 = (-)^2 (20)^{\frac{1}{4} \times 2} - 2(\sqrt{5}).$

$= (20)^{\frac{1}{2}} - 2\sqrt{5}$

$= 2\sqrt{5} - 2\sqrt{5} = 0$

S.B.S.

$\Rightarrow \alpha^4 + \beta^4 + 2\alpha^2\beta^2 = 0.$

$\Rightarrow \alpha^4 + \beta^4 = -2(5). = -10.$

$\Rightarrow$ S.B.S.

$\Rightarrow \alpha^8 + \beta^8 + 2\alpha^4.\beta^4 = 100$

$\Rightarrow \alpha^8 + \beta^8 = 100 - 2((5)^2) = 50$ (C)

Ans

Q-7: Let α, β be two roots of the equation $x^2+(20)^{1/4}x+(5)^{1/2}=0$. Then $\alpha^8+\beta^8$ is equal to:

(a) 10 (b) 100 (c) 50 (d) 160.

KTK 3
BY REED
FROM WB

Solⁿ: $x^2+(20)^{1/4}x+(5)^{1/2}=0 \begin{matrix}\nearrow\alpha\\ \searrow\beta\end{matrix}$ —— ①

Since; α, β are the roots of eqnⁿ ①,

$\Rightarrow \alpha^2+(20)^{1/4}\alpha+(5)^{1/2}=0$

or, $\alpha^2+5^{\frac{1}{2}}=-20^{1/4}\alpha$

S.B.S. → or, $\alpha^4+5+(2)\cdot(5)^{\frac{1}{2}}\alpha^2=20^{\frac{1}{2}}\alpha^2$

or, $\alpha^4+5+\cancel{(20)^{\frac{1}{2}}\cdot\alpha^2}=\cancel{(20)^{\frac{1}{2}}\cdot\alpha^2}$

or, $\alpha^4+5=0$

or, $\alpha^4=-5$

S.B.S. → or, $\boxed{\alpha^8=25}$ —(A) Similarly $\Rightarrow \boxed{\beta^8=25}$ —(B)

(A) + (B):

$\therefore \alpha^8+\beta^8=25+25=50$. (Ans.)

QUESTION [JEE Mains 2023]

Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$. Then $\alpha^{14} + \beta^{14}$ is equal to

- A -64
- B $-64\sqrt{2}$
- C $-128\sqrt{2}$
- D -128

Ans. D

Q-61 Let α, β be the roots of the equation $x^2 - \sqrt{2}x + 2 = 0$, then $\alpha^{14} + \beta^{14}$ is equal to:

ⓐ -64 ⓑ $-64\sqrt{2}$ ⓒ $-128\sqrt{2}$ ⓓ -128

Soln: $x^2 - \sqrt{2}x + 2 = 0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix} \Big\}$ roots.

$\boxed{\alpha + \beta = \sqrt{2}}$

KTK 4
DONE BY REED
FROM WB

$\Rightarrow \alpha^2 - \sqrt{2}\alpha + 2 = 0.$

or, $\alpha^2 + 2 = \sqrt{2}\alpha \Longrightarrow \boxed{\alpha^2 = \sqrt{2}\alpha - 2}$

S.B.S or, $(\alpha^2 + 2)^2 = (\sqrt{2}\alpha)^2$

or, $\alpha^4 + 4\alpha^2 + 4 = 2\alpha^2$

or, $\alpha^4 + 4 = -2\alpha^2 \longrightarrow \boxed{\alpha^4 = (-2\alpha^2 - 4)}$

S.B.S or, $\alpha^8 + 16 + 8\alpha^4 = 4\alpha^4$

or, $\alpha^8 + 16 = -4\alpha^4 \rightarrow \boxed{\alpha^8 = -4\alpha^4 - 16}$

multiply both sides by α^4.

or, $\alpha^{12} + 16\alpha^4 = -4\alpha^8$

or, $\alpha^{12} + 16\alpha^4 = -4(-4\alpha^4 - 16)$ (put the value of α^8)

or, $\alpha^{12} + 16\alpha^4 = 16\alpha^4 + 64$

or, $\alpha^{12} = 64$

multiply both sides by α^2,

or, $\alpha^{14} = 64\alpha^2$

or, $\boxed{\alpha^{14} = 64(\sqrt{2}\alpha - 2)} \rightarrow$ Ⓐ [put the value of α^2]

Similarly $\Rightarrow \boxed{\beta^{14} = 64(\sqrt{2}\beta - 2)} \longrightarrow$ Ⓑ

Ⓐ + Ⓑ:

$\alpha^{14} + \beta^{14} = 64(\sqrt{2}\alpha - 2 + \sqrt{2}\beta - 2)$

or, $\alpha^{14} + \beta^{14} = 64[\sqrt{2}(\alpha + \beta) - 4]$ $(\because \alpha + \beta = \sqrt{2})$

or, $\alpha^{14} + \beta^{14} = 64(2 - 4) = -128$ (Ans) $\therefore$ Ans $\Rightarrow$ ⓓ -128.

QUESTION

If $x^2 + 3x + 3 = 0$ and $ax^2 + bx + 1 = 0, a, b \in Q$ have a common root, then value of $(3a + b)$ is equal to

A 1/3

B 1

C 2

D 4

Ans. C

Q-8) If $x^2+3x+3=0$ and $ax^2+bx+1=0$; $a, b \in Q$ have a common root, then value of $(3a+b)$ is equal to :

(a) $\frac{1}{3}$ (b) 1 (c) 2 (d) 4.

Soln:

$$\left.\begin{array}{l} x^2+3x+3=0 \quad (i) \\ ax^2+bx+1=0 \quad (ii) \end{array}\right\} \text{have a common root.}$$

Now, $x^2+3x+3=0$

2

$D = 9-12 = -3 \ (<0) \rightarrow$ roots are imaginary

$\Downarrow$

both roots are in pair.

Since, a root is common & roots are imaginary $\Rightarrow$ both roots are common

Condition for both roots to be common:

$$\frac{a}{1} = \frac{b}{3} = \frac{1}{3}$$

$$\boxed{a = \frac{1}{3}}, \quad \boxed{b = \frac{3}{3} = 1}$$

$\therefore \ 3a+b = \left(3\cdot\frac{1}{3}\right) + 1 = 1+1 = 2.$ (Ans)

Solution to Previous Home Challenge

Home Challenge - 07

Let n be the number of integers satisfying the inequality $\frac{(x-3)^{\frac{-x}{|x|}}\cdot\sqrt{(x-5)^2}\cdot(18-x)}{\sqrt{-x}(-x^2+x-1)(|x|-37)}<0$ then value of n is ________

Q- 121 $n \rightarrow$ no. of integers, satisfying the ineq.

$$\frac{(x-3)^{\frac{-x}{|x|}}\ \sqrt{(x-5)^2}\cdot(18-x)}{\sqrt{-x}\,(-x^2+x-1)\,(|x|-37)} < 0$$

$x \rightarrow T_2$
$(x-5) \rightarrow T_1$

Sol^n $\Rightarrow \dfrac{(x-3)^{\frac{-x}{|x|}}\cdot\overset{T_1}{\overline{|x-5|}}\,(18-x)}{\sqrt{-x}\,(-x^2+x-1)\,(\underset{T_2}{\underline{|x|}}-37)} < 0$

T_1 −ve −ve +ve
0 5
T_2 −ve +ve +ve

Case- I! $x \leq 0$

$$\frac{(x-3)^1\,(\cancel{-}(x-5)\,(x-18))}{\sqrt{-x}\,(-x^2+x-1)\,(-x-37)} < 0 \quad ; \quad x \neq 0$$

$\sqrt{-x} \hookrightarrow$ +ve ; $(-x^2+x-1) \hookrightarrow \left.\begin{matrix} D<0 \\ a<0 \end{matrix}\right\}$ −ve } 2 times sign change.

$$\Rightarrow \frac{(x-3)\,(x-5)\,(x-18)}{(x+37)} < 0$$

+ − + − +
−37 3 5 18

$x \in (-37, 3) \cup (5, 18)$ → "∩" $\boxed{x \in (-37, 0)}$ → final interval

Case-II ! $(x \in (0, 5))$

but for $\sqrt{-x}$ to be defined $x < 0$

∴ Case II rejected

Case-III ! $(x \geq 5)$ ← Case-III also rejected.

$\boxed{\therefore x \in (-37, 0)} \rightarrow$ no. of integers $= 0 - (-37) - 1$
$= 36$. (Ans)


THANK
YOU