

# PRAAYAS

## JEE 2026

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Mathematics

# Quadratic Equations

Lecture - 06

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# Topics *To be covered*



- A** Graph of a Quadratic polynomial
- B** Range of Rational Functions
- C** Practice problems

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# Homework Discussion

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QUESTION



If two roots of the equation  $(x - 1)(2x^2 - 3x + 4) = 0$  coincide with roots of the equation  $x^3 + (a + 1)x^2 + (a + b)x + b = 0$  where  $a, b \in \mathbb{R}$  then  $2(a + b)$  equals

- A 4
- B 2
- C 1
- D 0

$$(x-1)(2x^2-3x+4)=0 \leftarrow \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$x^3 + x^2 + ax^2 + ax + bx + b = 0$$

$$x^2(x+1) + ax(x+1) + b(x+1) = 0$$

$$(x^2+0x+b)(x+1)=0 \leftarrow \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$2x^2 - 3x + 4 = 0 \leftarrow \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$x^2 + ax + b = 0 \leftarrow \begin{matrix} \alpha \\ \beta \end{matrix}$$

$$\frac{1}{a} = -\frac{a}{3} = \frac{b}{4}$$

$$a = -3/2, b = 2$$

$$2(a+b) = -3+4 = 1$$

## QUESTION

(KTK 6)



Let 'p' is a root of the equation  $x^2 - x - 3 = 0$ . Then the value of  $\frac{p^3+1}{p^5-p^4-p^3+p^2}$  is equal to

**A**  $\frac{4}{3}$

~~**B**~~  $\frac{4}{9}$

**C**  $\frac{2}{9}$

**D**  $\frac{2}{3}$

$$p^2 - p - 3 = 0$$

$$p(p-1) = 3.$$

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$$\frac{(p+1)(p^2-p+1)}{(p^4-p^2)(p-1)}$$

$$\frac{(p+1)(p^2-p+1)}{p(p^2-1) \cdot p(p-1)} = \frac{p^2-p+1}{p(p-1) \cdot p(p-1)}$$

$$= \frac{p(p-1)+1}{3 \cdot 3} = \frac{3+1}{9} = \frac{4}{9}$$

QUESTION

(KTK 8)



If  $\alpha, \beta, \gamma$  are roots  $x^3 + 2x^2 - 3x + 1 = 0$ , then value of  $\frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$  is less than

- ~~A~~ 2
- ~~B~~ 3
- ~~C~~ 4
- ~~D~~ 5

M(1)

$x^3 + 2x^2 - 3x + 1 = 0$  ←  $\begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$

$\alpha\beta\gamma = -1$   
 $\alpha + \beta + \gamma = -2$

$E = \frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$

$E = \frac{-1/\gamma}{-2-\gamma} + \frac{-1/\beta}{-2-\beta} + \frac{-1/\alpha}{-2-\alpha}$

$E = \frac{1}{\alpha(\alpha+2)} + \frac{1}{\beta(\beta+2)} + \frac{1}{\gamma(\gamma+2)}$   
 $f(\alpha) \quad f(\beta) \quad f(\gamma)$   
 $f(x) = \frac{1}{x(x+2)}$

let  $y = \frac{1}{x(x+2)}$

$x \cdot x(x+2) - 3x + 1 = 0$

$\frac{x}{y} - 3x + 1 = 0$

$x(\frac{1}{y} - 3) + 1 = 0$

$x(1 - 3y) + y = 0$

$x(3y - 1) = y$

$x = \frac{y}{3y-1} \Rightarrow x+1 = \frac{4y-1}{3y-1}$

$x(x+2) = \frac{1}{y}$

$x^2 + 2x = \frac{1}{y}$

$x^2 + 2x + 1 = \frac{1}{y} + 1$

$(x+1)^2 = \frac{1}{y} + 1$

SBS  $(x+1)^2 = \left(\frac{4y-1}{3y-1}\right)^2$

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$$\frac{1}{y} + 1 = \frac{16y^2 - 8y + 1}{9y^2 - 6y + 1}$$

$$(y+1)(9y^2 - 6y + 1) = y(16y^2 - 8y + 1)$$

$$9y^3 - 6y^2 + y + 9y^2 - 6y + 1 = 16y^3 - 8y^2 + y$$

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$$7y^3 - 11y^2 + 6y - 1 = 0$$

$$\frac{\alpha\beta}{\alpha+\beta}$$

$$\frac{\beta\gamma}{\beta+\gamma}$$

$$\frac{\alpha\gamma}{\alpha+\gamma}$$

$$S_1 = E = \frac{11}{7} = 1.5714 \dots$$

## QUESTION

(KTK 8)



If  $\alpha, \beta, \gamma$  are roots  $x^3 + 2x^2 - 3x + 1 = 0$ , then value of  $\frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$  is less than

~~A~~ 2

M (2)

$$x^3 + 2x^2 - 3x + 1 = 0 \left\{ \begin{array}{l} \alpha \\ \beta \\ \gamma \end{array} \right.$$

$$\alpha\beta\gamma = -1$$

$$\alpha + \beta + \gamma = -2$$

~~B~~ 3

~~C~~ 4

~~D~~ 5

$$E = \frac{1}{2} \left( \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} - \left( \frac{1}{\alpha+2} + \frac{1}{\beta+2} + \frac{1}{\gamma+2} \right) \right)$$

$$y = \frac{1}{x+2} \Rightarrow x+2 = \frac{1}{y}$$

$$x = \frac{1}{y} - 2$$

$$E = \frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$$

$$E = \frac{-1}{-2-\gamma} + \frac{-1}{-2-\beta} + \frac{-1}{-2-\alpha}$$

$$E = \frac{1}{\alpha(\alpha+2)} + \frac{1}{\beta(\beta+2)} + \frac{1}{\gamma(\gamma+2)}$$

$$E = \frac{\alpha+2-\alpha}{2\alpha(\alpha+2)} + \frac{\beta+2-\beta}{2\beta(\beta+2)} + \frac{\gamma+2-\gamma}{2\gamma(\gamma+2)}$$

$$E = \frac{1}{2} \left( \frac{1}{\alpha} - \frac{1}{\alpha+2} + \frac{1}{\beta} - \frac{1}{\beta+2} + \frac{1}{\gamma} - \frac{1}{\gamma+2} \right)$$

$$E = \frac{1}{2} \left( \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} - \left( \frac{1}{\alpha+2} + \frac{1}{\beta+2} + \frac{1}{\gamma+2} \right) \right)$$

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# Aao Machaay Dhamaal Deh Swaal pe Deh Swaal

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# Analysis of a Quadratic Polynomial



$$y = ax^2 + bx + c, (a > 0)$$

↓  
\* upward opening parabola

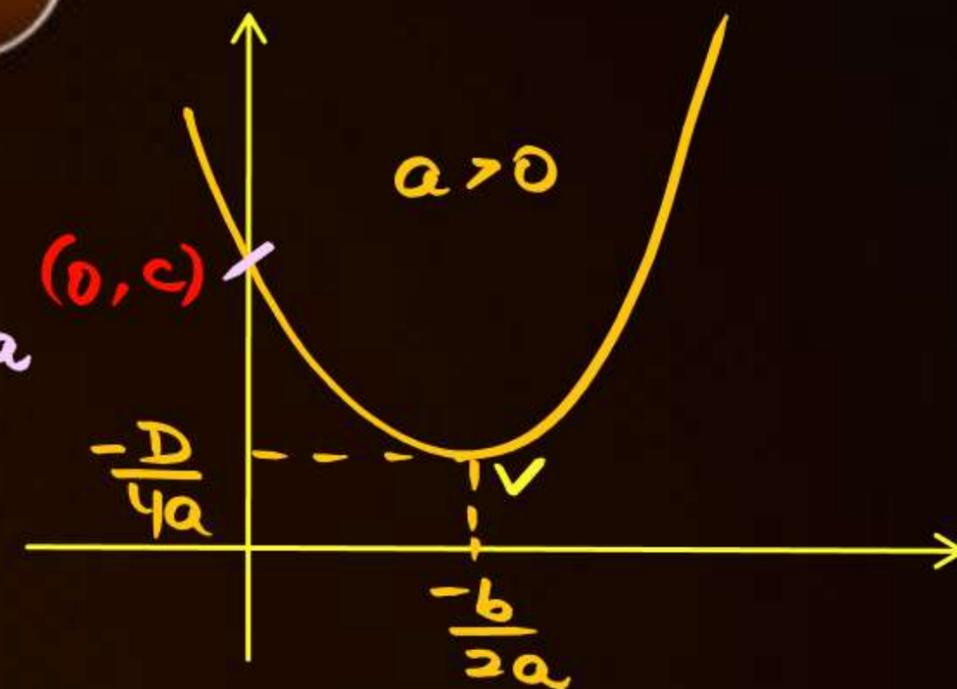
\* Symmet about  $x = -\frac{b}{2a}$

\*  $y_{\min} = -\frac{D}{4a}$  at  $x = -\frac{b}{2a}$

\*  $y_{\max}$  D.N.E but  $\rightarrow \infty$

\* Parabola intersects y axis at  $(0, c)$

\* Vertex  $V(-\frac{b}{2a}, -\frac{D}{4a})$





# Analysis of a Quadratic Polynomial



$$y = ax^2 + bx + c, (a < 0)$$

↓

\* Downward opening parabola

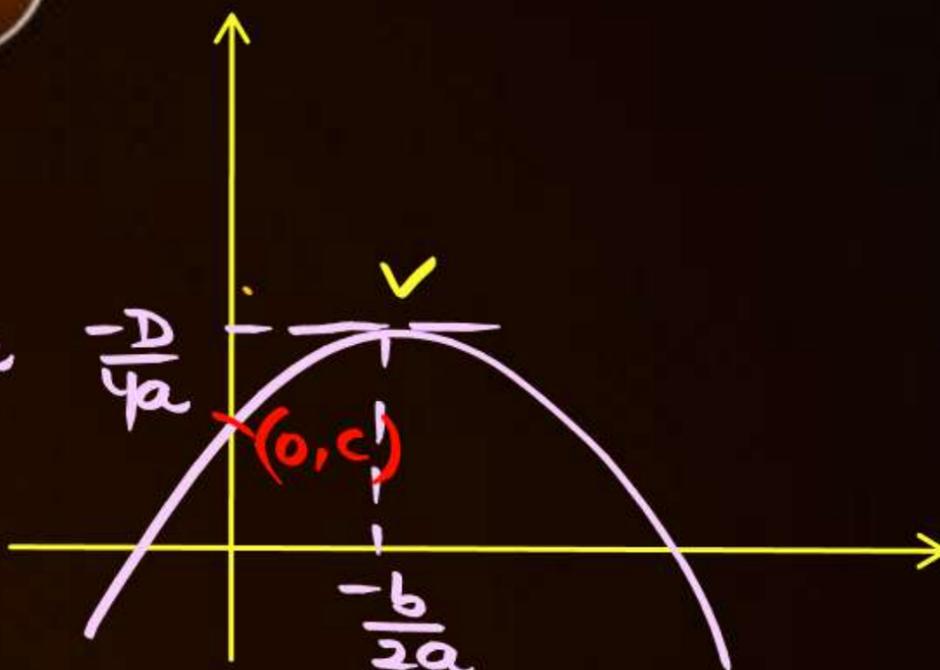
\* Symmt about  $x = -\frac{b}{2a}$

\*  $y_{\max} = \frac{-D}{4a}$  at  $x = -\frac{b}{2a}$

\*  $y_{\min}$  D.N.E but  $\rightarrow -\infty$

\* Parabola intersects y axis at  $(0, c)$

\* vertex  $(-\frac{b}{2a}, \frac{-D}{4a})$





Ex:  $y = x^2 - 3x + 4$

\* Parabola Type ~ upward opening

\* Symmt about  $x = -\frac{(-3)}{2 \cdot 1} = \frac{3}{2}$

\*  $y_{\min} = -\frac{D}{4a} = \frac{-((-3)^2 - 4 \cdot 1 \cdot 4)}{4 \cdot 1} = \frac{7}{4}$  — M②

\* Intersects y-axis at (0, 4)

$$\begin{aligned} y &= \left(\frac{3}{2}\right)^2 - 3 \cdot \frac{3}{2} + 4 \\ &= \frac{9}{4} - \frac{9}{2} + 4 \\ &= -\frac{9}{4} + 4 = \underline{\underline{\frac{7}{4}}} \end{aligned}$$

Ex:  $y = -x^2 + 2x - 5$

\* Parabola Type: Downward opening Parabola.

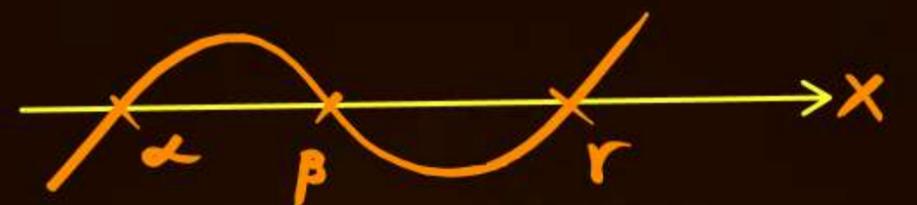
\* Symmt about:  $x = -\frac{(1)}{2 \cdot (-1)} = \frac{1}{2}$

\*  $y_{\max} = -\frac{D}{4a} = \frac{-(1^2 - 4 \cdot (-1) \cdot (-5))}{4 \cdot (-1)} = -\frac{19}{4}$

\* Intersects y-axis at (0, -5)



$y = f(x)$  intersects  $x$  axis at points where  $y = 0$



$$f(x) = 0 \rightarrow \text{roots} = \alpha, \beta, \gamma$$

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Real Roots of  $f(x) = 0$  are nothing but  $x$ -coord of POI of curve  $y = f(x)$  &  $x$  Axis.

$ax^2 + bx + c = 0$  ke roots are nothing but  $x$ -coord of POI of parabola  $y = ax^2 + bx + c$  with  $x$  Axis.



# Graph of a Quadratic Polynomial vs D



$$y = ax^2 + bx + c$$

$$ax^2 + bx + c = 0$$

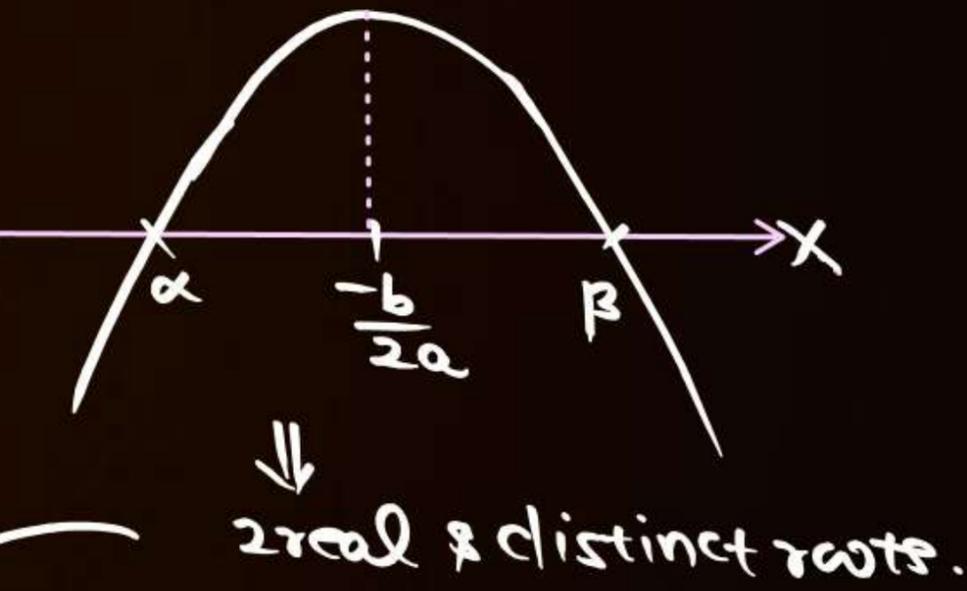
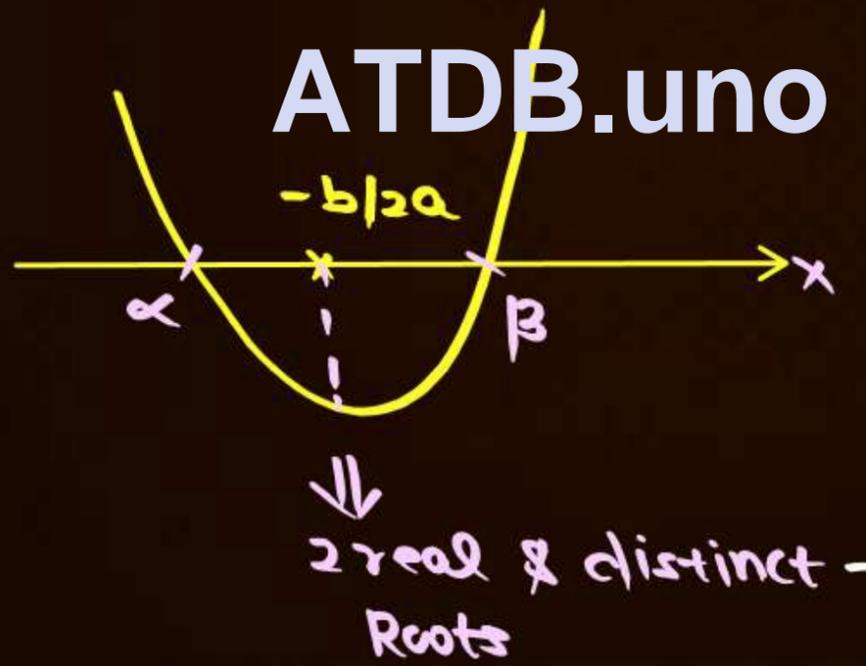
Case 1 if  $D > 0$

(i) if  $a > 0$   $y_{min} = -\frac{D}{4a} = -ve$

+ve  
-ve  
+ve

(ii) if  $a < 0$   $y_{max} = -\frac{D}{4a} = +ve$

+ve  
-ve  
+ve



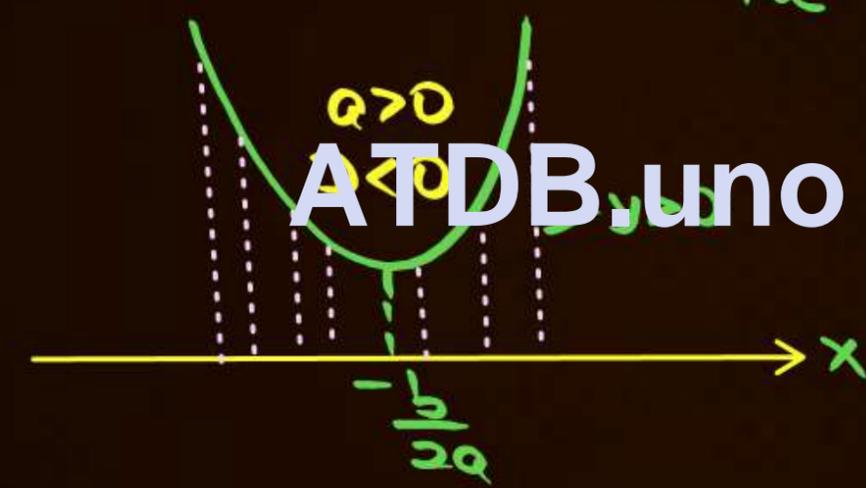
if  $D > 0 \Rightarrow$  2 real & distinct roots.



Case (ii) if  $D < 0$

(i) if  $a > 0$   $y_{min} = -\frac{D}{4a} = +ve$

-ve  
+ve

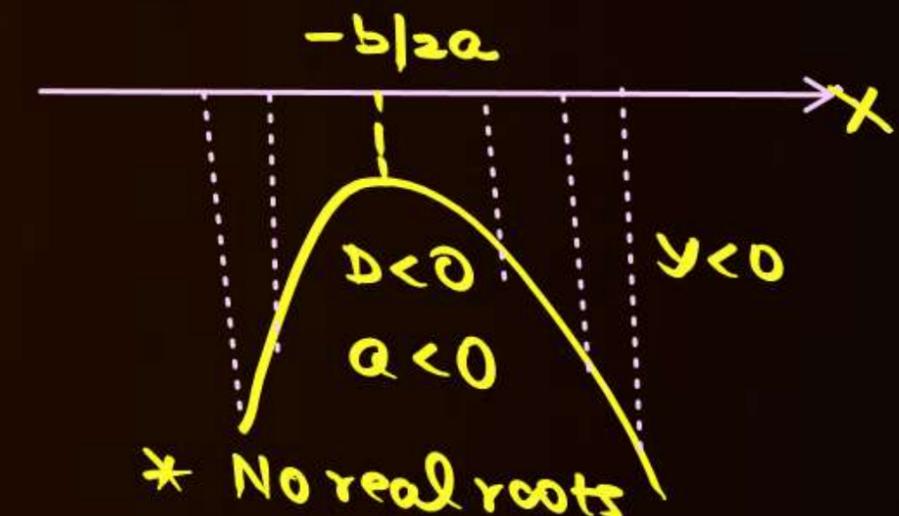


- ⇓
- \* No real roots.
  - \*  $y = ax^2 + bx + c > 0 \forall x \in \mathbb{R}$

$y = ax^2 + bx + c$   
 $ax^2 + bx + c = 0$

(ii) if  $a < 0$   $y_{max} = -\frac{D}{4a} = -ve$

-ve  
-ve



- \* No real roots
- \*  $y = ax^2 + bx + c < 0 \forall x \in \mathbb{R}$



NICHOD

$$a > 0 \ \& \ D < 0 \Rightarrow y = ax^2 + bx + c > 0 \ \forall x \in \mathbb{R}$$

$$a < 0 \ \& \ D < 0 \Rightarrow y = ax^2 + bx + c < 0 \ \forall x \in \mathbb{R}$$

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# Graph of a Quadratic Polynomial vs D



$$y = ax^2 + bx + c$$

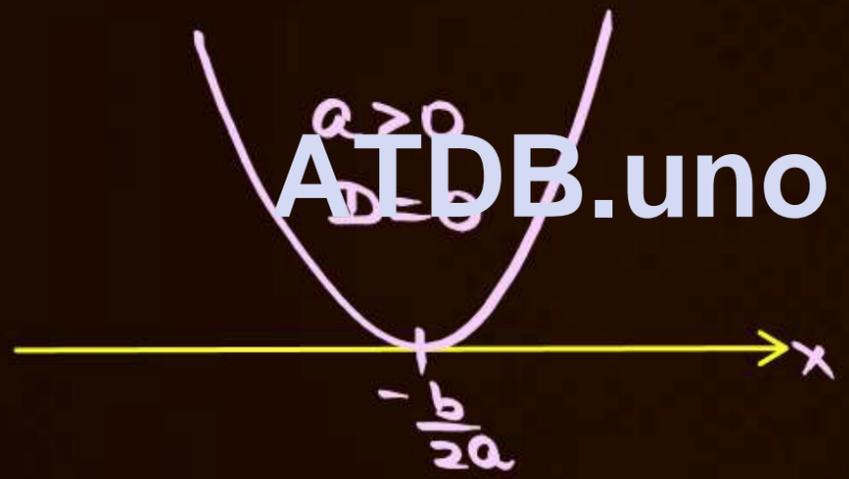
$$ax^2 + bx + c = 0$$

Case III if D = 0

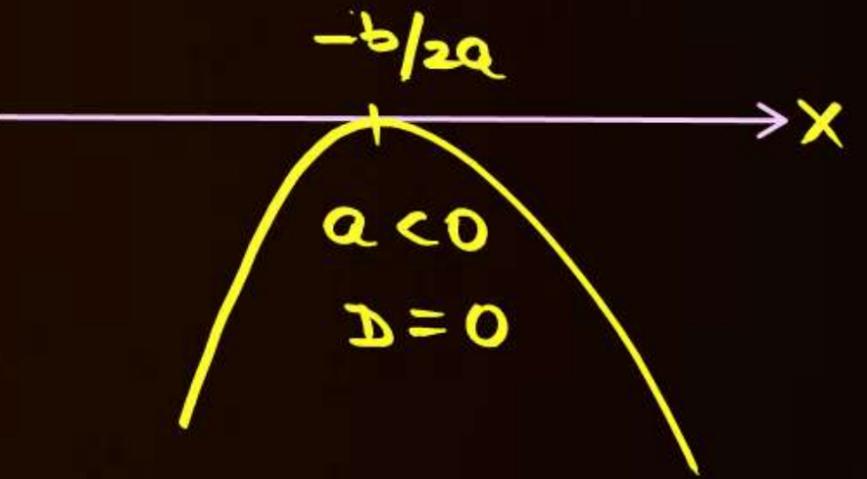
(i) if a > 0  $y_{min} = \frac{-D}{4a} = 0$

(ii) if a < 0

$$y_{max} = \frac{-D}{4a} = 0$$

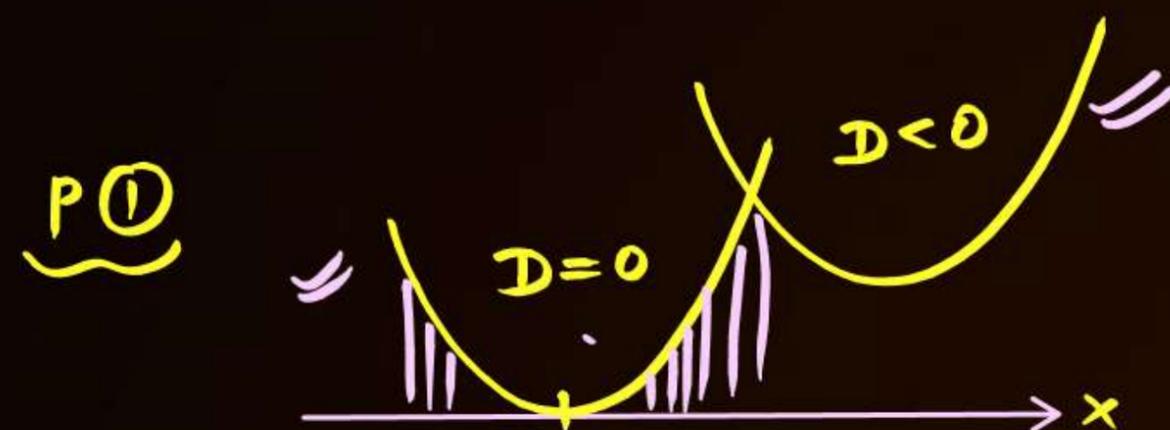


\* Two real & Equal roots



\* Two real & Equal roots.

$D = 0 \Rightarrow$  2 real & Equal roots.



$$y = ax^2 + bx + c > 0 \quad \forall x \in \mathbb{R} \quad \text{if } a > 0 \text{ \& } D \leq 0$$

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$$a \leq b$$

$$\downarrow$$

$$a < b \text{ वा } a = b$$



$$y = ax^2 + bx + c \leq 0 \quad \text{if } a < 0 \text{ \& } D \leq 0.$$



## Important Points



(1) If  $a > 0$  and  $D < 0$  then  $y = ax^2 + bx + c > 0$  for all  $x \in \mathbb{R}$

(2) If  $a < 0$  and  $D < 0$  then  $y = ax^2 + bx + c < 0 \forall x \in \mathbb{R}$

(3)  $ax^2 + bx + c \geq \forall x \in \mathbb{R}$

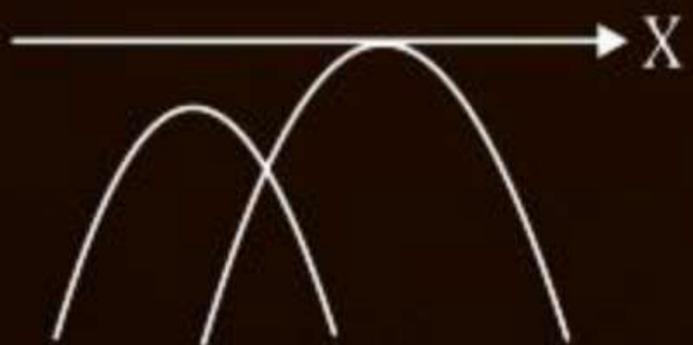
$D < 0$

$D = 0$  if  $a > 0$  &  $D \leq 0$

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(4)  $ax^2 + bx + c \leq 0 \forall x \in \mathbb{R}$



## QUESTION



Consider graph of  $y = ax^2 + bx + c$  as shown above, comment on signs of

**A**  $a < 0$

**B**  $b > 0$

**C**  $c > 0$

**D**  $D > 0$

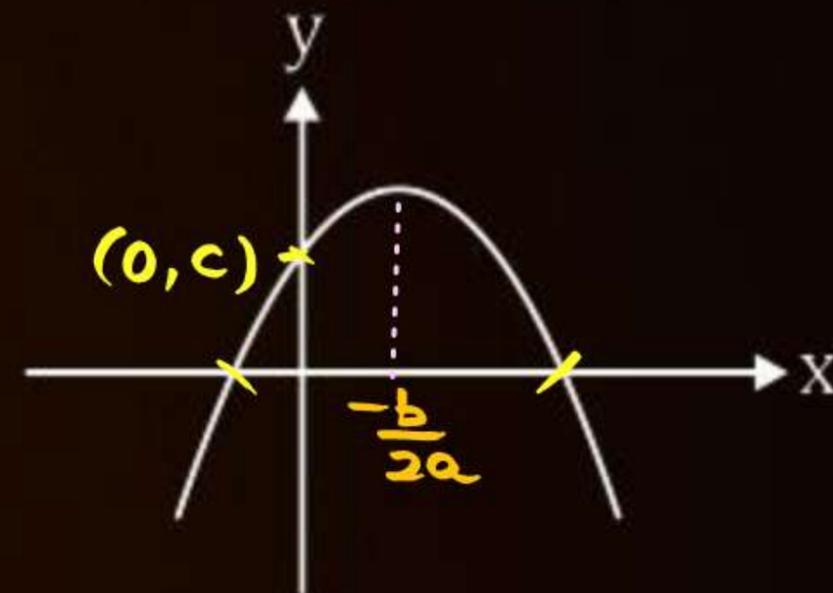
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M①  $-\frac{b}{2a} > 0$   
 $-b < 0$   
 $b > 0$

$a$  is -ve

M②  $S.O.R > 0$   
 $-\frac{b}{a} > 0$   
 $-b < 0$   
 $b > 0$

$a$  is -ve





# Approach Banao

If  $f(x) = ax^2 + bx + c$  then remember

(a)  $f(1) = a + b + c$

(b)  $f(2) = 4a + 2b + c$

(c)  $f(3) = 9a + 3b + c$

(d)  $f(-1) = a - b + c$

(e)  $f(-2) = 4a - 2b + c$

(f)  $f(-3) = 9a - 3b + c$

(g)  $f(1/2) = \frac{a}{4} + \frac{b}{2} + c = \frac{a + 2b + 4c}{4}$

(h)  $f(-1/2) = \frac{a - 2b + 4c}{4}$

(i)  $f(1/3) = \frac{a + 3b + 9c}{9}$

(j)  $f(-1/3) = \frac{a - 3b + 9c}{9}$

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**QUESTION**



Consider the graph of quadratic polynomial  $y = ax^2 + bx + c$  as shown below. Which of the following is(are) correct?

~~A~~  $\frac{a - b + c}{abc} = 0$   $\frac{f(-1)}{abc} = \frac{0}{abc} = 0$

~~B~~  $abc(9a + 3b + c) < 0$

~~C~~  $\frac{a + 3b + 9c}{abc} < 0$   $\frac{9f(1/3)}{abc} < 0$

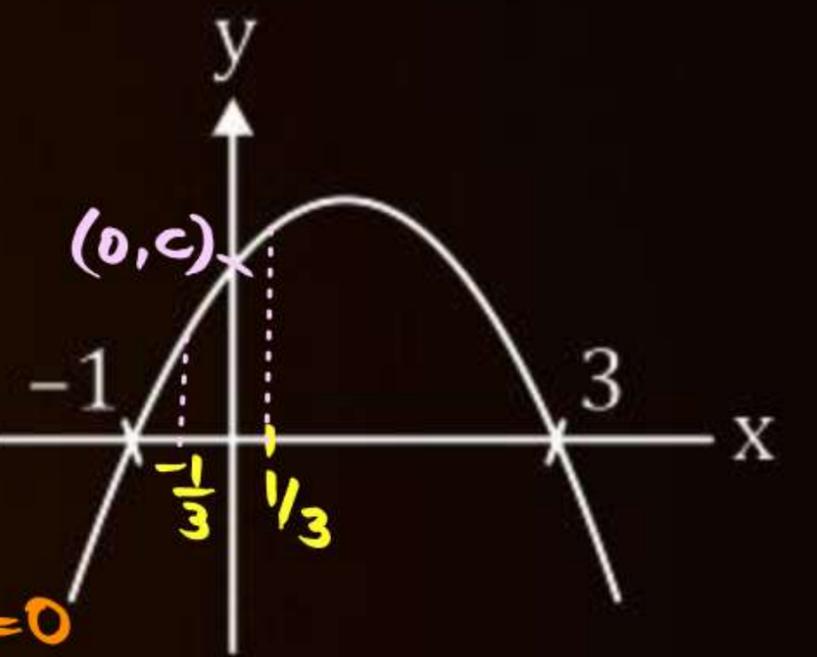
~~D~~  $ab(a - 3b + 9c) > 0$

$a \cdot b \cdot (9f(-1/3)) < 0$

$y = f(x) = ax^2 + bx + c$

$a < 0$   
 $c > 0$

$-\frac{b}{a} > 0 \rightarrow -b < 0$   
 $b > 0$



$f(-1) = a - b + c = 0$   
 $f(3) = 9a + 3b + c = 0$

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## QUESTION



Tan 01

The graph of  $y = ax^2 + bx + c$  is shown in the figure, then which of the following is(are) correct?

**A**  $ab^2c^3 < 0$

**B**  $ab < 0$

**C**  $bc(4a + 2b + c) > 0$

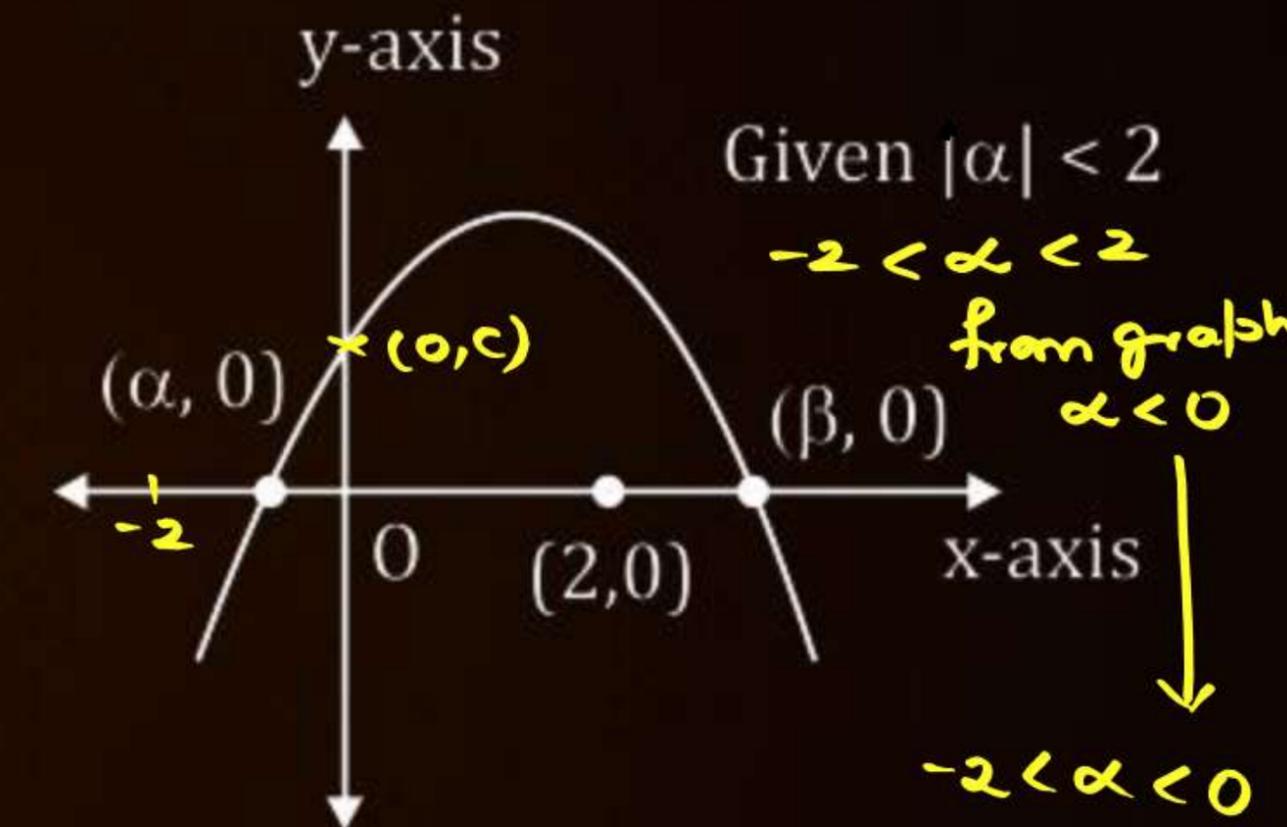
**D**  $ab(4a - 2b + c) > 0$

①  $a < 0$

②  $c > 0$

③  $b$

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## QUESTION



Tah 02

The graph of quadratic polynomial  $f(x) = ax^2 + bx + c$  is shown in below. Which of the following are correct?

**A**  $\frac{c}{a} < -1$

**B**  $|\beta - \alpha| > 2$

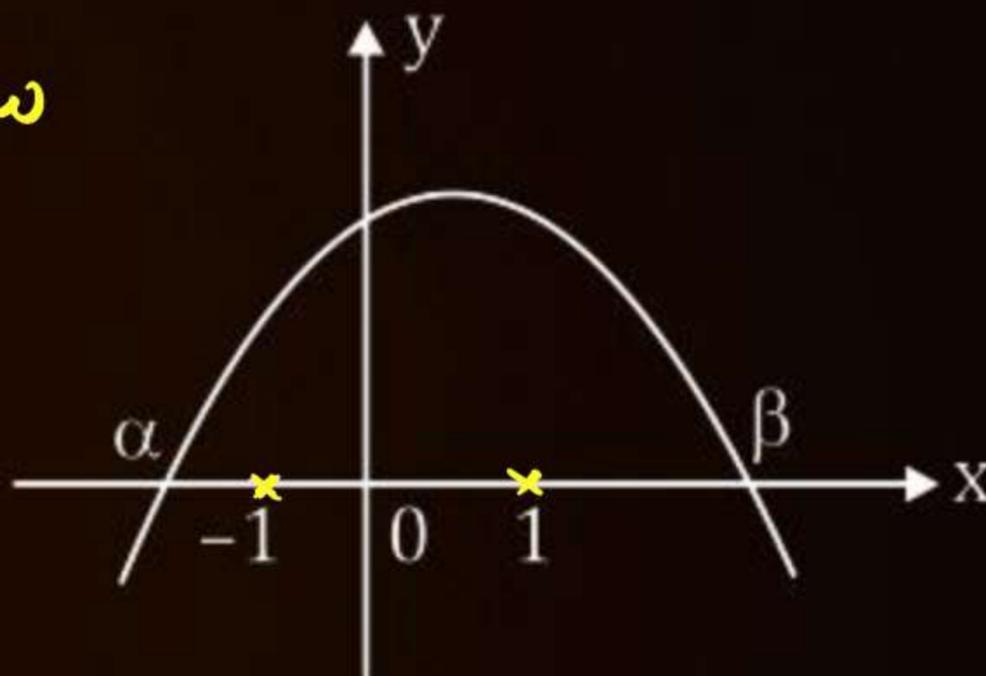
**C**  $f(x) > 0 \forall x \in (0, \beta)$

**D**  $abc < 0$

$|a - b| = \text{distance b/w } a \text{ \& } b.$

Ex:  $|-2 - 3| = 5$

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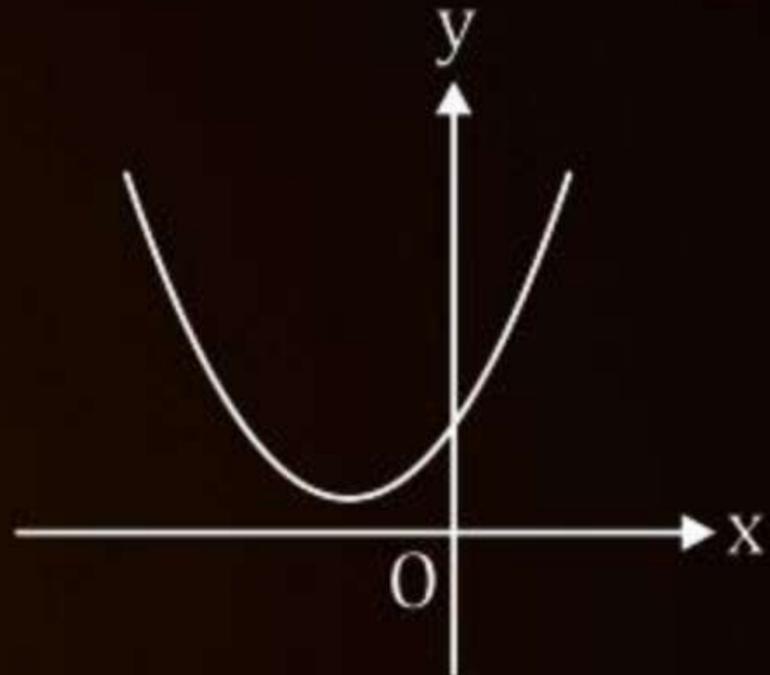


## QUESTION

Tan 03



The curve of the quadratic expression  $y = ax^2 + bx + c$  is shown in the figure and  $\alpha, \beta$  be the roots of the equation  $ax^2 + bx + c = 0$  then correct option is [D is the discriminant]



- A**  $a > 0, b > 0, c > 0, D > 0, \alpha + \beta > 0, \alpha\beta > 0$
- B**  $a > 0, b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta < 0$
- C**  $a > 0, b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta > 0$
- D**  $a > 0, b < 0, c > 0, D < 0, \alpha + \beta > 0, \alpha\beta > 0$

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Ans. C

**QUESTION**

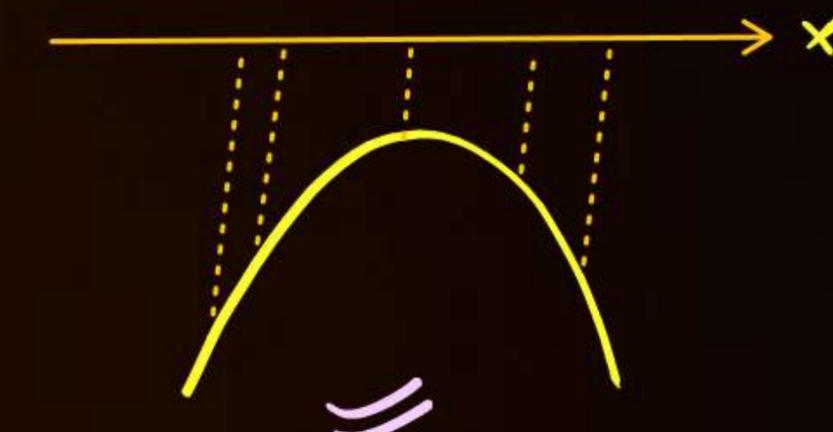
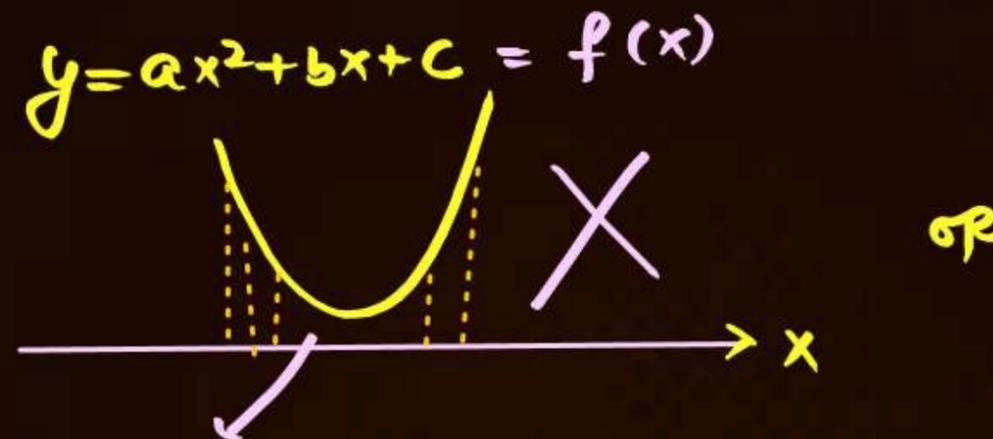


★★★★KCLS★★★★

If  $ax^2 + bx + c = 0$  has no real root and  $a + b + c < 0$  then

- ~~A~~  $4a - 2b + c > 0$
- ~~B~~  $4a - 2b + c < 0$
- ~~C~~  $13a + 5b + 2c < 0$
- ~~D~~  $5b - 25a - c > 0$

$y = ax^2 + bx + c = f(x)$



$f(1) = a + b + c < 0$

$f(-2) = 4a - 2b + c$

$13a + 5b + 2c = 9a + 3b + c + 4a + 2b + c$   
 $= f(3) + f(2) < 0$   
 $< 0$        $< 0$

$f(5) = 25a + 5b + c$

$f(-5) = 25a - 5b + c < 0 \rightarrow -25a + 5b - c > 0$

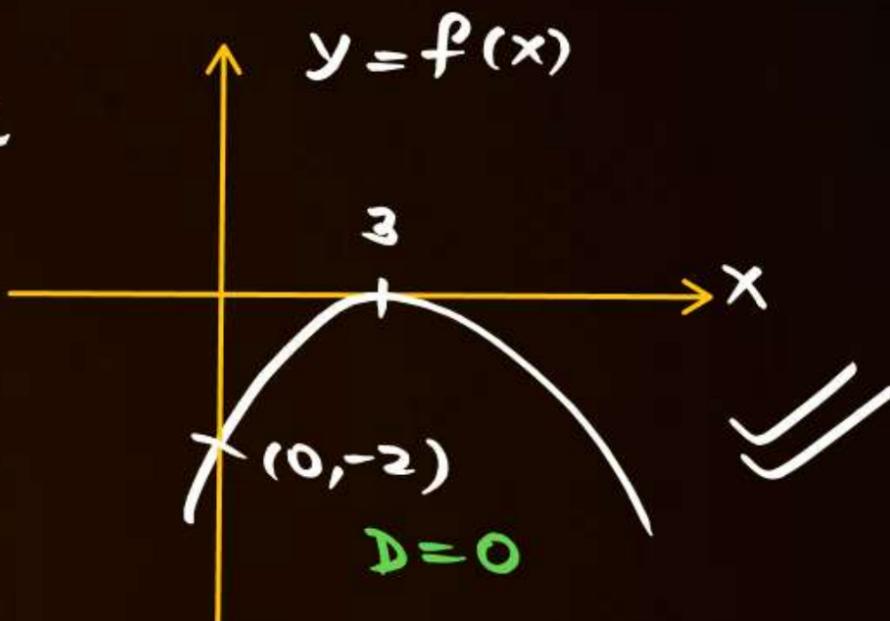
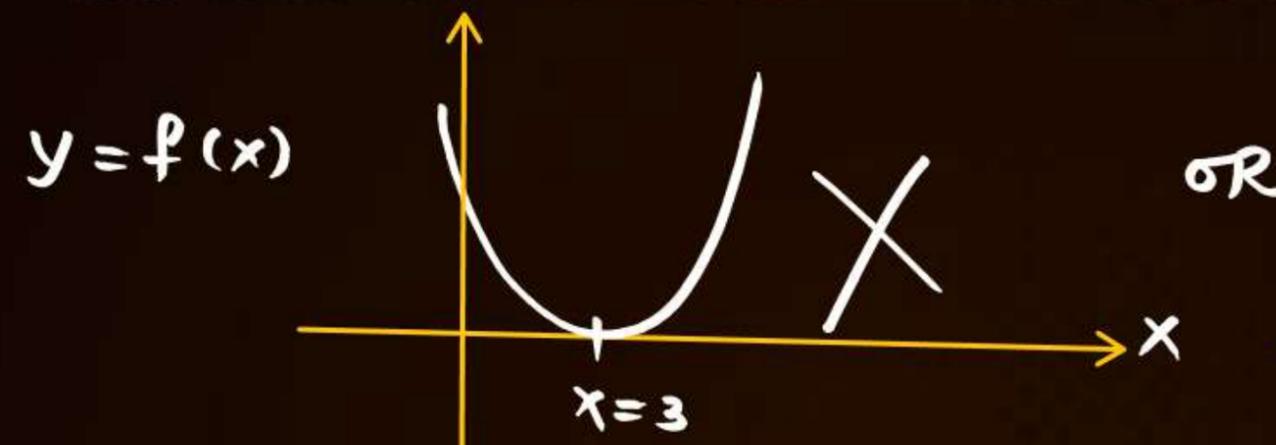
Ans. B, C, D

## QUESTION

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If quadratic function  $f(x)$  touches  $x$ -axis at  $x = 3$  and crosses  $y$ -axis at  $(0, -2)$ , then find  $f(7)$ .



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$$y = f(x) = a(x-3)^2$$

M① passes  $(0, -2)$

$$-2 = a(0-3)^2$$

$$a = -2/9$$

$$f(x) = -2/9(x-3)^2$$

$$f(7) = -\frac{2}{9} \cdot 16$$

$$f(7) = -32/9$$

M②  $f(x) = a(x^2 - 6x + 9)$

$$f(x) = ax^2 - 6ax + 9a$$

$$c = 9a = -2$$

$$a = -2/9$$

# QUESTION



Draw graph of following quadratic :

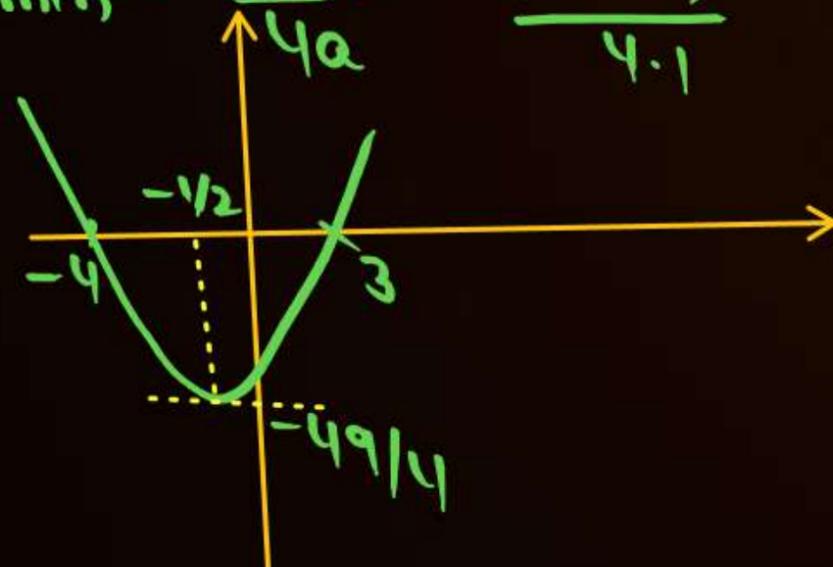
(i)  $f(x) = x^2 + x - 12 = y$

$a > 0 \rightarrow$  upward opening

$(0, -12) \sim$  intersects y axis

Roots  $x = \frac{-1 \pm \sqrt{1+48}}{2} = \frac{-1 \pm 7}{2}$   
 $x = -4, 3$

$y_{min} = -\frac{D}{4a} = -\frac{(1+48)}{4 \cdot 1} = -\frac{49}{4}$  at  $x = -\frac{1}{2}$



Range:  $[-\frac{49}{4}, \infty)$

(ii)  $f(x) = x - (1 + x^2) \quad y = -x^2 + x - 1$

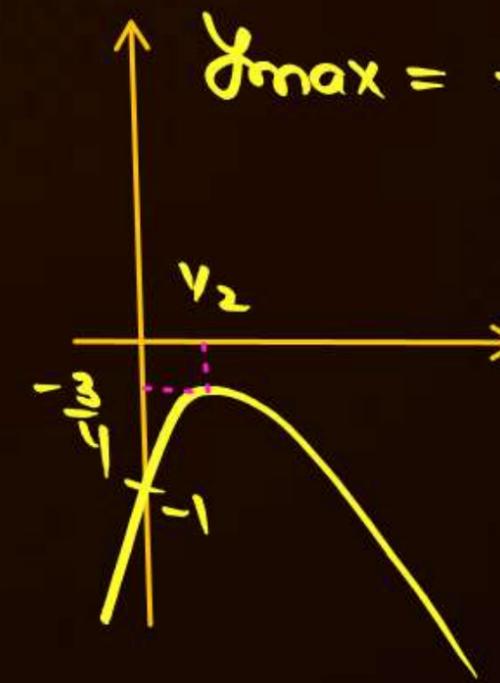
1)  $a < 0 \rightarrow$  down ward opening

2)  $(0, -1) \sim$  POI y axis.

3) No real roots.

4)  $y_{max} = -\frac{D}{4a} = -\frac{(1-4(-1)(-1))}{4(-1)}$

$y_{max} = -\frac{3}{4}$  at  $x = -\frac{1}{2(-1)} = \frac{1}{2}$



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QUESTION

★★★★KCLS★★★★



If  $a + b + c > \frac{9c}{4}$  and quadratic equation  $ax^2 + 2bx - 5c = 0$  has non-real roots, then

- ~~A~~  $a > 0, c > 0$
- ~~B~~  $a > 0, c < 0$
- ~~C~~  $a < 0, c < 0$
- ~~D~~  $a < 0, c > 0$

$f(x) = ax^2 + 2bx - 5c = 0$  — No real roots.

$a + b + c > \frac{9c}{4}$

ATDB.uno

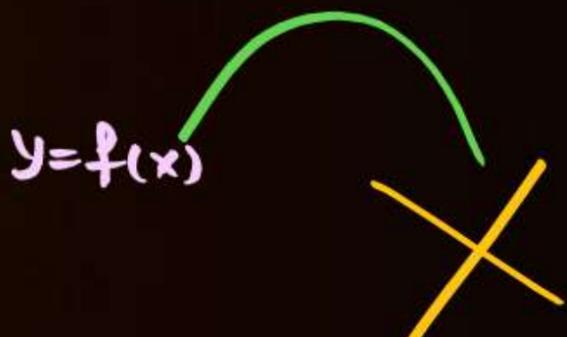
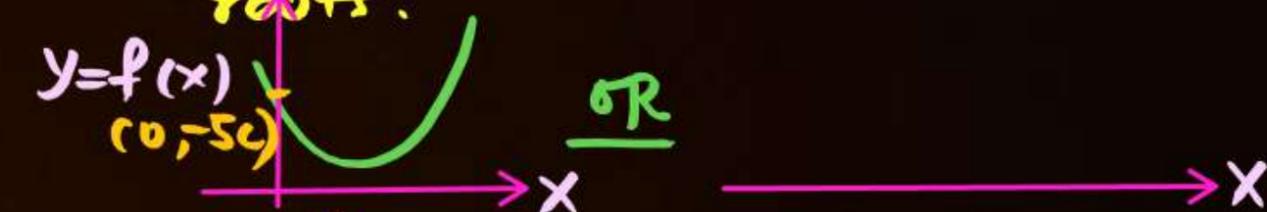
$4a + 4b + 4c - 9c > 0$

$4a + 4b - 5c > 0$

$f(2) > 0$

$y = ax^2 + 2bx - 5c$  parabola is upward opening

$a > 0, -5c > 0$   
 $c < 0.$



## QUESTION



Tahoy

Find the set of values of  $a$  for which  $(a - 1)x^2 - (a + 1)x + a + 1 > 0$  for all  $x \in \mathbb{R}$ .

$$a - 1 > 0 \quad \& \quad D < 0$$



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**QUESTION**

Tah 05



Find the set of values of  $a$  for which  $(a + 4)x^2 - 2ax + 2a - 6 < 0$  for all  $x \in \mathbb{R}$ .

**ATDB.uno**

## QUESTION



Tahob

For what values of  $p$  the vertex of  $x^2 + px + 13$  lies at a distance 5 unit from origin.

$$y = x^2 + px + 13$$

$$\text{Vertex } \left( -\frac{p}{2a}, -\frac{(p^2 - 52)}{4} \right)$$

origin (0,0)

distance = 5

 $p = ?$ 

ATDB.uno

## QUESTION



If  $y = x^2 - 3x - 4$  then find the range of  $y$  when

(i)  $x \in \mathbb{R}$

(ii)  $x \in [0, 3]$

(iii)  $x \in [-2, 0]$

$$y = x^2 - 3x - 4$$

\* Range:  $[-\frac{D}{4a}, \infty)$

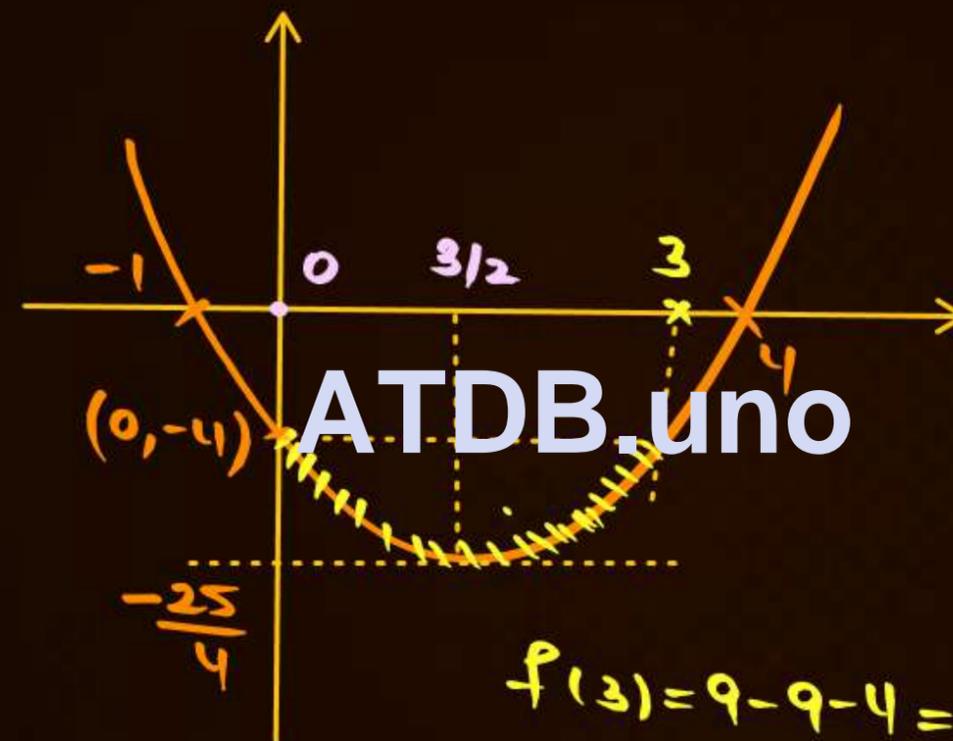
$$= [-\frac{(9+16)}{4}, \infty)$$

$$= [-\frac{25}{4}, \infty)$$

\* Roots  $x = \frac{3 \pm \sqrt{25}}{2} = 4, -1$

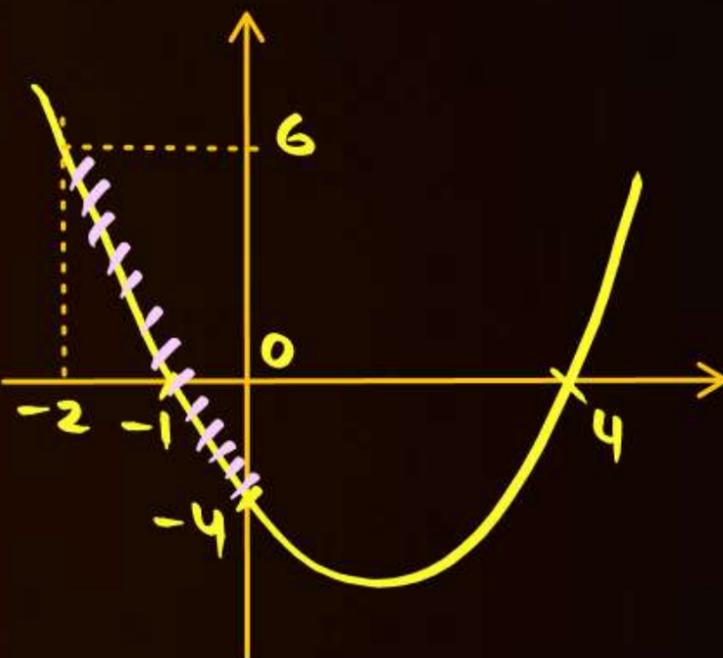
\*  $y_{\min} = -\frac{25}{4}$  at  $x = -\frac{(-3)}{2 \cdot 1} = \frac{3}{2}$

\* POI with y-axis  $(0, -4)$



$$f(3) = 9 - 9 - 4 = -4$$

Range:  $[-\frac{25}{4}, -4]$



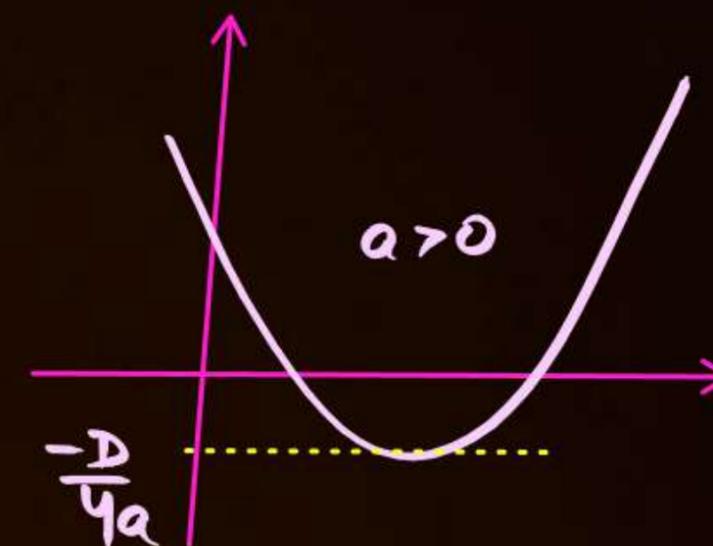
$$f(-2) = 4 + 6 - 4 = 6$$

Range =  $[-4, 6]$



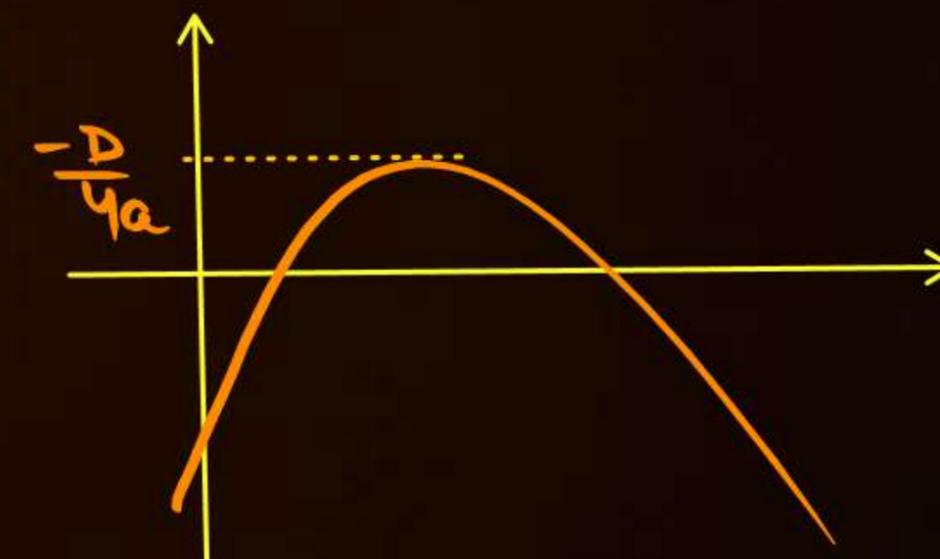
\*  $y = ax^2 + bx + c$

\* if  $a > 0$  Range:  $[-\frac{D}{4a}, \infty)$



\* if  $a < 0$

ATDB.uno  
Range:  $(-\infty, -\frac{D}{4a}]$





if  $x \in [2, 3]$

$x^2 \in [4, 9]$

$x+2 \in [4, 5]$

$x-5 \in [-3, -2]$

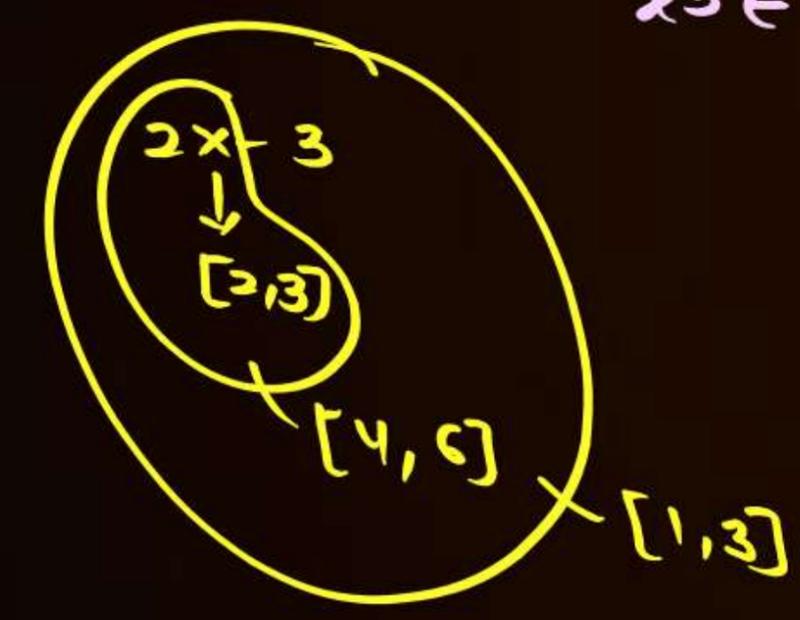
$2x-3 \in [1, 3]$

$\frac{1}{x} \in [\frac{1}{3}, \frac{1}{2}]$

$x^3 \in [8, 27]$

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$\frac{1}{0^+} \rightarrow \infty$   
 $\frac{1}{0^-} \rightarrow -\infty$



Ex:  $x \in [-2, 1]$   $x^2 \in [0, 4] \cup [1, 4]$

$\downarrow$   
 $x \in [-2, 0] \cup [0, 1]$

$x^2 \in [0, 4] \cup [0, 1]$

$x^2 \in [0, 4]$

Ex:  $x \in [-2, 1] = [-2, 0] \cup [0, 1]$

$\frac{1}{x} \in (-\infty, -\frac{1}{2}] \cup [1, \infty)$



Ex:  $x \in [2, 4]$  — find Range

$$(x-3)^2$$

$$[-1, 1] = [-1, 0] \cup [0, 1]$$

$$[0, 1] \cup [0, 1]$$

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$$(x-3)^2 \in [0, 1]$$

Ex:  $x \in (-\infty, \infty)$      $x^2 \in [0, \infty)$

$$\downarrow$$

$$x \in (-\infty, 0] \cup [0, \infty)$$

$$x^2 \in [0, \infty) \cup [0, \infty)$$

**QUESTION**



If  $y = x^2 - 3x - 4$  then find the range of  $y$  when

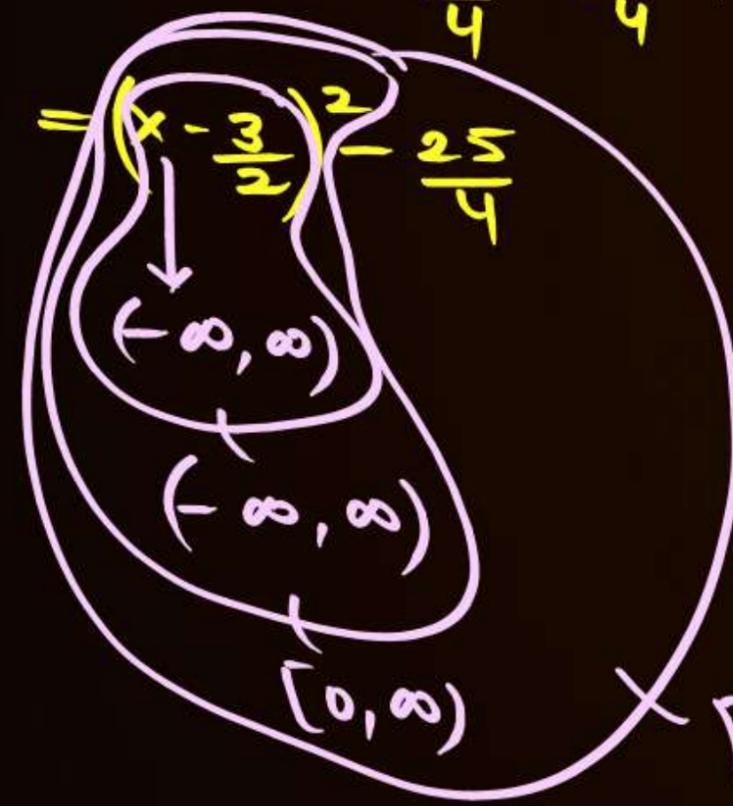
(i)  $x \in \mathbb{R}$

(ii)  $x \in [0, 3]$

(iii)  $x \in [-2, 0]$

$$y = x^2 - 3x - 4$$

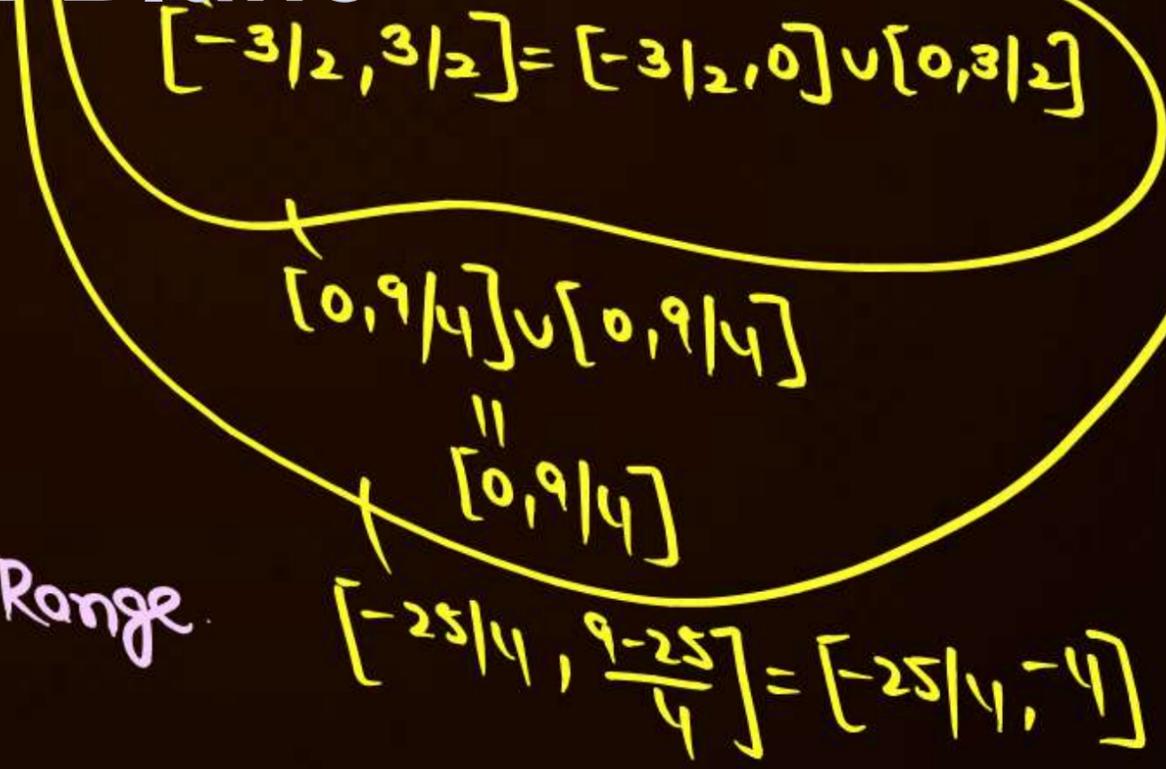
$$= x^2 - 3x + \frac{9}{4} - \frac{9}{4} - 4$$



$[-25/4, \infty) = \text{Range}$

$$y = \left(x - \frac{3}{2}\right)^2 - \frac{25}{4}$$

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$$x \in [0, 3]$$

find range:  $y = x^2 - 2x + 7$ .

$$y = (x-1)^2 + 6$$

$$[-1, 2] = [-1, 0] \cup [0, 2]$$

$$[0, 1] \cup [0, 4]$$

$$[0, 4]$$

$$\text{Range: } [6, 10]$$

Gadhe / Gadhi ne yeh  
kiya

$$y = (x-1)^2 + 6$$

$$[-1, 2]$$

$$[1, 4]$$

$$[7, 11]$$



**Sabse Important Baat**



**Sabhi Class Illustrations **ATDB.uno** Retry Karnay hai...**



# Solution to Previous TAH

## ATDB.uno

**QUESTION [JEE Mains 2020 (9 Jan)]**

Let  $a, b \in \mathbb{R}$ ,  $a \neq 0$  be such that the equation,  $ax^2 - 2bx + 5 = 0$  has a repeated root  $\alpha$ , which is also a root of the equation,  $x^2 - 2bx - 10 = 0$ . If  $\beta$  is the other root of this equation, then  $\alpha^2 + \beta^2$  is equal to:

- A** 28
- B** 24
- C** 26
- D** 25

**ATDB.uno**

Ans. D



TAH 01

$$ax^2 - 2bx + 5 = 0 \quad \begin{matrix} \alpha \\ \alpha \end{matrix}$$

$$2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

$$\alpha^2 = \frac{5}{a} \Rightarrow \boxed{b = a\alpha}$$

Put  $b = a\alpha, \beta = -\frac{10}{\alpha}$  in ①

$$x - \frac{10}{x} = 2ax$$

$$x^2 - 10 = 2ax^2$$

$$(1 - 2a)x^2 = 10$$

$$x^2 = \frac{10}{1 - 2a} \quad \text{and} \quad x^2 = \frac{5}{a}$$

$$\frac{10^2}{1 - 2a} = \frac{5}{a}$$

$$\Rightarrow 2a = 1 - 2a \Rightarrow a = \frac{1}{4}$$

$$\boxed{x^2 = 20}$$

$$\beta^2 = \frac{100}{x^2} = \frac{100}{20} = 5$$

$$\boxed{\beta^2 = 5}$$

$$\boxed{x^2 + \beta^2 = 25} \quad \underline{\underline{Ans}}$$

**TAH-1**  
**By Nikita**

ATDB.uno

**QUESTION**

If the equation  $x^2 - 4x + 5 = 0$  and  $x^2 + ax + b = 0$  have a common root, find a and b.

# ATDB.uno



TAH 02

$$x^2 - 4x + 5 = 0.$$

$$\Downarrow \\ D = 0.$$

$\Downarrow$   
imaginary roots

$$x^2 + ax + b = 0.$$

Both roots are common.

ATDB.uno

So, using condition,

$$\frac{1}{1} = \frac{-4}{a} = \frac{5}{b} \Rightarrow$$

$a = -4$
$b = 5$

Ans

**TAH-2**  
**By Nikita**

**QUESTION [JEE Mains 2013]**

If the equations  $x^2 + 2x + 3 = 0$  and  $ax^2 + bx + c = 0$ ,  $a, b, c \in \mathbb{R}$ , have a common root, then  $a : b : c$  is

- A** 1 : 2 : 3
- B** 3 : 2 : 1
- C** 1 : 3 : 2
- D** 3 : 1 : 2

**ATDB.uno****Ans. A**



TAH 03

$$x^2 + 2x + 3 = 0$$



$$D < 0$$



imaginary roots

$$ax^2 + bx + c = 0.$$



Both roots are common.

**TAH-3**  
**By Nikita**

$$\frac{1}{a} = \frac{2}{b} = \frac{3}{c} \Rightarrow a : b : c :: 1 : 2 : 3.$$

Ans

## QUESTION

★★★KCLS★★★



If the equations  $ax^3 + x + 2 = 0$  and  $x^3 + ax + 2 = 0$  have exactly one common root, find the value of  $|a|$ .

$$\begin{array}{r} ax^3 + x + 2 = 0 \quad \swarrow \alpha \\ x^3 + ax + 2 = 0 \quad \swarrow \alpha \\ \hline (1-a^2)x + 2(1-a) = 0 \quad \swarrow \alpha \end{array}$$

$$(1-a) \left( (1+a)x + 2 \right) = 0$$

$$a = 1 \quad \text{or} \quad x = \frac{-2}{1+a} = \alpha$$

(N.P)

$$\begin{array}{l} \text{Eqn (1)} \quad x^3 + x + 2 = 0 \\ \text{Eqn (2)} \quad x^3 + x + 2 = 0 \end{array} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} \text{All 3} \\ \text{roots} \\ \text{Common.} \end{array}$$



$$ax^3 + x + 2 = 0 \quad \begin{matrix} x \\ \swarrow \\ \beta \\ \searrow \\ \gamma \end{matrix} \quad x^3 + ax + 2 = 0 \quad \begin{matrix} x \\ \swarrow \\ \delta \\ \searrow \\ \theta \end{matrix}$$

As,  $\alpha$  is common root in both eq<sup>n</sup>.

$$a\alpha^3 + \alpha + 2 = 0 \quad \text{--- (1)}$$

$$\alpha^3 + a\alpha + 2 = 0 \quad \times a$$

$$\hline (1-a^2)\alpha + (2-2a) = 0.$$

$$(1-a) [(1+a)\alpha + 2] = 0.$$

$$\alpha = \frac{-2}{1+a} \checkmark$$

$\boxed{a=1}$   
↓  
N.P

Put in eq<sup>n</sup> (1)

$$\Rightarrow a \left(\frac{-2}{1+a}\right)^3 + \left(\frac{-2}{1+a}\right) + 2 = 0.$$

$$\Rightarrow -8a + (-2)(1+a)^2 + 2(1+a)^3 = 0.$$

$$\Rightarrow -8a - 2(1+a)^2 + 2(1+a)^3 = 0$$

$$\Rightarrow -8a - 2a^2 - 2 - 4a + 2 + 2a^3 + 6a + 6a^2 = 0.$$

$$\Rightarrow 2a^3 + 4a^2 - 6a = 0.$$

$$\Rightarrow 2a(a^2 + 2a - 3) = 0.$$

$$a^2 + 2a - 3 = 0 \Rightarrow (a+3)(a-1) = 0.$$

$\boxed{a=0}$ ,  
↓  
N.P.

$\boxed{a=-3}$  ✓  $\boxed{a=1}$  rejected.

So,  $|a| = |-3| = 3$  Any

**TAH-4**  
**By Nikita**  
**Raj.**



\* Tan 043-

$$ax^3 + x + 2 = 0$$

$$x^3 + ax + 2 = 0$$

$$\frac{84}{14} = 4$$

$$\begin{vmatrix} a & 1 & 1 & 2 \\ 1 & a & a & 2 \end{vmatrix} = \begin{vmatrix} 2 & a & 2 \\ 2 & 1 & 1 \end{vmatrix}$$

Ankush!!

$$ax^3 + x + 2 = 0$$

$$x^3 + ax + 2 = 0$$

$$(a^4 - 1)(2 - 2a) = (2 - 2a)^2$$

$$2a^2 - 2a^3 - 2 + 2a = 4 + 4a^4 - 8a$$

$$-2a^3 - 2a^4 + 10a - 6 = 0$$

$$2a^3 + 2a^4 - 10a + 6 = 0 \Rightarrow a^3 + a^4 - 5a + 3 = 0$$

$$a^2(a-1) + 2a(a-1) - 3(a-1) = 0$$

$$(a-1)(a^2 + 2a - 3) = 0$$

$$a^2 + 3a + a - 3 = 0$$

$$(a-1) = 0 \quad a(a+3) \Rightarrow (a+3) = 0$$

$$a = 1 \quad (a+1)(a+3) = 0$$

$$a = 1, -3$$

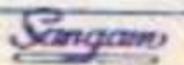
$a = -3$  or  $(a=1) \Rightarrow$  Not possible

$$a = -3 \Rightarrow |a| = |-3| \Rightarrow \boxed{|a| = 3}$$

$$a^3 = \frac{\begin{vmatrix} a & 2 \\ 1 & a \end{vmatrix}}{\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}}$$

$$a^3 = \frac{\begin{vmatrix} 2 & a \\ a & 1 \end{vmatrix} - 5a + 3 = 0}{\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}}$$

$$\frac{\begin{vmatrix} 1 & 2 \\ a & 2 \end{vmatrix}}{\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}} = \frac{\begin{vmatrix} 2 & a \\ 2 & 1 \end{vmatrix}^3}{\begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix}^3}$$



## QUESTION



If two roots of the equation  $(x - 1)(2x^2 - 3x + 4) = 0$  coincide with roots of the equation  $x^3 + (a + 1)x^2 + (a + b)x + b = 0$  where  $a, b \in \mathbb{R}$  then  $2(a + b)$  equals

**A** 4

**B** 2

**C** 1

**D** 0

ATDB.uno



TqHOS

$$(x-1)(2x^2-3x+4)=0 \begin{cases} \alpha \\ \alpha \\ \beta \end{cases}$$

$$\Downarrow \\ x-1=0.$$

$$\& 2x^2-3x+4=0. \begin{cases} \alpha \\ \alpha \end{cases}$$

$\Downarrow$   
 $D < 0$   
 $\Downarrow$   
imaginary roots.

$$x^3+(a+1)x^2+(a+b)x+b=0 \begin{cases} \alpha \\ \alpha \\ \beta \end{cases}$$

$$\Downarrow \\ x^2(x+1)+ax(x+1)+b(x+1)=0.$$

$$(x+1)(x^2+ax+b)=0.$$

$$x+1=0. \\ x^2+ax+b=0. \begin{cases} \alpha \\ \alpha \end{cases}$$

Both roots are common.

So, using condition  $\Rightarrow$

$$\frac{2}{1} = -\frac{3}{a} = \frac{4}{b} \Rightarrow a = -\frac{3}{2}, b = 2$$

$$2(a+b) = 2\left(-\frac{3}{2} + 2\right) = \underline{\underline{1}} \underline{\underline{Ans}}$$

**TAH-5**  
**By Nikita, Raj.**



# Solution to Previous KTKs

## ATDB.uno

## QUESTION

(KTK 1)



If  $\alpha, \beta$  are the root of a quadratic equation  $x^2 - 3x + 5 = 0$  then the equation whose roots are  $(\alpha^2 - 3\alpha + 7)$  and  $(\beta^2 - 3\beta + 7)$  is

**A**  $x^2 + 4x + 1 = 0$

**B**  $x^2 - 4x + 4 = 0$

**C**  $x^2 - 4x - 1 = 0$

**D**  $x^2 + 2x + 3 = 0$

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Ans. B



- **Q-7!** If  $\alpha, \beta$  are the roots of a quadratic equation  $x^2 - 3x + 5 = 0$  then the equation whose roots are  $(\alpha^2 - 3\alpha + 7), (\beta^2 - 3\beta + 7)$  is:

Soln!

$$x^2 - 3x + 5 = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$$

↓

$$\alpha^2 - 3\alpha = -5$$

$$\beta^2 - 3\beta = -5$$

KTK 1

BY Reed

From WB

for new eqn, roots =  $\alpha^2 - 3\alpha + 7, \beta^2 - 3\beta + 7$   
 $= -5 + 7, -5 + 7$   
 $= 2, 2$

∴ The eqn is:  $(x - 2)^2 = 0$



Shoyo  
Page: \_\_\_\_\_  
Date: \_\_\_\_\_  
**KTK 1**

**KTK-01** If  $\alpha, \beta$  are the roots of a quadratic equation  $x^2 - 3x + 5$ , then equation whose roots are  $\alpha^2 - 3\alpha + 7$  and  $\beta^2 - 3\beta + 7$ , is?

Sol.

$x^2 - 3x + 5$   $\xrightarrow{\alpha, \beta}$

$\alpha^2 - 3\alpha + 5 = 0 \Rightarrow \alpha^2 - 3\alpha = -5$

lly  $\beta^2 - 3\beta = -5$

New roots  $\alpha' = \alpha^2 - 3\alpha + 7$   
 $= -5 + 7$   
 $= 2$

lly  $\beta' = 2$

req. equation  
 $\Rightarrow x^2 - (\alpha' + \beta')x + \alpha'\beta' = 0$   
 $\Rightarrow x^2 - 4x + 4 = 0$

## QUESTION

(KTK 2)



The equations  $ax^2 + bx + a = 0$  ( $a, b \in \mathbb{R}$ ) and  $x^3 - 2x^2 + 2x - 1 = 0$  have 2 roots common. Then  $a + b$  must be equal to

- A** 1
- B** -1
- C** 0
- D** None of these

ATDB.uno

Ans. C



KTR 12

Q. The Eqn  $ax^2 + bx + a = 0$ , ( $a, b \in \mathbb{R}$ ) and  $x^3 - 2x^2 + 2x - 1 = 0$  have 2 roots common. Then  $a + b$  must be.

$$x^3 - 2x^2 + 2x - 1 = 0$$

~~$$x^2(x-1)$$~~

$$x^3 - 2x^2 + 2x - 1 = 0$$

$$x^2(x-1) - x(x-1) + 1(x-1) = 0$$

$$(x-1)(x^2 - x + 1) = 0$$

$$\rightarrow D < 0$$

Both have Two common  
Roots.

∴ Compare  $a = 1$  @  $b = -1$ .

$$a + b = 1 - 1 = 0 \text{ Ans}$$

[KTK-02]

The equations  $ax^2 + bx + a = 0$  and  $x^3 - 2x^2 + 2x - 1 = 0$  have 2 common roots, then  $a+b = ?$

$$ax^2 + bx + a = 0 \quad \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix} \quad \text{--- (1)}$$

$$\alpha + \beta = -\frac{b}{a}$$

$$\alpha\beta = \frac{a}{a} = 1$$

Now,  $x^3 - 2x^2 + 2x - 1 = 0$   $\begin{matrix} \nearrow \alpha \\ \searrow \beta \\ \searrow \gamma \end{matrix}$

$$\alpha + \beta + \gamma = -(-2)$$

$$-\frac{b}{a} + \gamma = 2$$

$$\gamma = 2 + \frac{b}{a}$$

and,  $\alpha\beta\gamma = 1$   
 $\gamma = 1 \rightarrow$  put in (1)

$$1 = 2 + \frac{b}{a}$$

$$a = 2a + b$$

$$\boxed{a + b = 0}$$



Shoyo  
KTK 2

## QUESTION

(KTK 3)



The value of  $m$  for which the equation  $\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$  has roots equal in magnitude and opposite in signs is

**A**  $\frac{a-b}{a+b}$

**B**  $-1$

**C**  $0$

**D**  $\frac{a+b}{a-b}$

ATDB.uno

Ans. C

KTK: 3 Find value of  $m$  for which eqn  $\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$  has roots Equal in magn & opp. in sign.

$$\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$$

$$\cancel{a}x + \cancel{a}b + \cancel{a}m + \cancel{b}x + \cancel{a}b + \cancel{b}m = x^2 + bx + mx + \cancel{a}x + \cancel{a}b + \cancel{a}m + mx + \cancel{b}m + m^2$$

$$ab = x^2 + mx + mx + m^2$$

$$ab = x^2 + 2mx - ab + m^2$$

$$\Rightarrow \text{SOR} = -2m = \alpha - \alpha$$

$$\Rightarrow -2m = 0$$

$$m = 0$$





[KTK 3]

The value of  $m$  for which the equation  $\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$  has roots equal in magnitude but opposite in signs is?

Solo

$$\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1$$

$$\frac{ax + ab + am + bx + ab + bm}{x^2 + bx + mx + ax + ab + am + mx + mb + m^2} = 1$$

$$\cancel{ax} + \cancel{ab} + \cancel{am} + \cancel{bx} + \cancel{ab} + \cancel{bm} = x^2 + \cancel{bx} + \cancel{mx} + \cancel{ax} + \cancel{ab} + \cancel{am} + \cancel{mx} + \cancel{mb} + m^2$$

$$ab = x^2 + 2mx + m^2$$
$$x^2 + 2mx + (m^2 - ab) = 0$$

SOR  $\Rightarrow \alpha - \alpha = -2m$   
 $m = 0$

**Shoyo**  
**KTK 3**

Homework

Mathematics

KTK 03



$$\frac{a}{x+a+m} + \frac{b}{x+b+m} = 1 \quad \xrightarrow{m} \text{roots equal in magnitude but opposite in sign.}$$

$$\rightarrow ax + ab + am + bx + ab + bm = (x+a+m)(x+b+m)$$

$$\cancel{ax} + ab + am + \cancel{bx} + ab + bm = x^2 + (a+b+m)x + ab + am + bm + mx + mb + m^2$$

$$x^2 + 2mx + m^2 + ab = 0$$

if roots have same magnitude + opp sign

$$\text{then } -2m = 0$$

$$m = 0 \quad \textcircled{C} \checkmark$$

lec 05 / Quadratic equations / Ashish sir

AKASHI

**QUESTION****(KTK 4)**

Find the values of 'k' so that the equation  
 $x^2 + kx + (k + 2) = 0$  and  $x^2 + (1 - k)x + 3 - k = 0$  have exactly one common root.

**ATDB.uno**

Ans. No possible value of k

Homework

Mathematics  
 KTK 01



$x^2 + kx + (k+2) = 0$   $\xrightarrow{\alpha, \beta}$  have exactly one common root.  
 $x^2 + (1-k)x + 3-k = 0$   $\xrightarrow{\alpha, \gamma}$

Both have one root in common, thus

putting  $\alpha = -1$  in E,

$\alpha^2 + k\alpha + k+2 = \alpha^2 + (1-k)\alpha + 3-k$

$1 - k + k + 2 = 0$

ATDB.uno

$1 = -2$  (Not possible)

$\Rightarrow$  Thus no value of  $k$  is possible.

$\Rightarrow k\alpha - (1-k)\alpha + 2k - 1 = 0$

$\Rightarrow \alpha(k - 1 + k) + 2k - 1 = 0$

$\Rightarrow (2k - 1)(\alpha + 1) = 0$

$\Rightarrow k = 1/2$  or  $\alpha = -1$

$\rightarrow$  Both have two roots in common

AKASHI

**QUESTION****(KTK 5)**

Given  $a, b$  are two distinct real number satisfying  
 $a^2 - 5a + 2 = 0$  and  $b^2 - 5b + 2 = 0$  then  $(1 - ab + a^2b + b^2a)$

**ATDB.uno**



**KTK-05**

Given  $a, b$  are two distinct real number satisfying

$a^2 - 5a + 2 = 0$  and  $b^2 - 5b + 2 = 0$  then  $(1 - ab + a^2b + b^2a)$

$x^2 - 5x + 2 = 0$   $\begin{cases} a \\ b \end{cases}$

$a + b = 5$

$a \cdot b = 2$

**KTK-5**  
**Ayush Patel**  
**Prayagraj up**

$1 - ab + ab(a + b)$

$1 - 2 + 2(5)$

$1 - 2 + 10$

**9 Ans**



KTK=5

$$a^2 - 5a + 2 = 0$$

$$b^2 - 5b + 2 = 0$$

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$$x^2 - 5x + 2 = 0 \begin{cases} a \\ b \end{cases}$$

$$a + b = 5$$

$$ab = 2$$

$$\Rightarrow 1 - ab + ab(a + b)$$

$$= 1 - 2 + 2 \times 5 \Rightarrow 10 - 1 = \underline{\underline{9}} \underline{\underline{\text{Ans}}}$$

**KTK-5**  
**By Nikita**

## QUESTION

(KTK 6)



Let 'p' is a root of the equation  $x^2 - x - 3 = 0$ . Then the value of  $\frac{p^3 + 1}{p^5 - p^4 - p^3 + p^2}$  is equal to

**A**  $\frac{4}{3}$

**B**  $\frac{4}{9}$

**C**  $\frac{2}{9}$

**D**  $\frac{2}{3}$

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KTK = 6

$$x^2 - x - 3 = 0 \quad \swarrow P$$

$$\boxed{P(P-1) = 3}$$

**KTK-6**

$$\frac{P^3 + 1}{P^5 - P^4 - P^3 + P^2} = ?$$

$$\frac{P^3 + 1}{P^5 - P^4 - P^3 + P^2} = \frac{(P+1)(P^2 - P + 1)}{P^4(P-1) - P^2(P-1)} = \frac{(P+1)(P^2 - P + 1 + 3 - 3)}{(P-1)P^2(P^2 - 1)}$$

$$\Rightarrow \frac{(P+1)(P^2 - P - 3 + 4)}{P^2(P-1)^2(P+1)} = \frac{4}{P^2(P-1)^2} = \frac{4}{(P(P-1))^2} = \frac{4}{9}$$

Ans ✓

## QUESTION

(KTK 7)



If  $\alpha$  and  $\beta$  are the roots of  $ax^2 + bx + c = 0$ , then the equation whose roots are  $\frac{\alpha+1}{\alpha-2}$  and  $\frac{\beta+1}{\beta-2}$  is

**A**  $a(x+1)^2 + b(x+1)(x-2) + c(x-2)^2 = 0$

**B**  $a(x-2)^2 + b(x+1)(x-2) + c(x+1)^2 = 0$

**C**  $a(2x+3)^2 + b(x+1)(x+2) + c(x+2)^2 = 0$

**D**  $a(2x+1)^2 + b(2x+1)(x-1) + c(x-1)^2 = 0$

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Ans. D



KTK 07

$$ax^2 + bx + c = 0 \quad \begin{matrix} \alpha \\ \beta \end{matrix} \quad \text{--- (1)}$$

let  $y = f(x) = \frac{x+1}{x-2} \Rightarrow yx - 2y = x+1$

$$(y-1)x = 1+2y$$

$$x = \frac{2y+1}{y-1}$$

**KTK-7**  
**By Nikita, Raj.**

Put in (1)

$$a\left(\frac{2y+1}{y-1}\right)^2 + b\left(\frac{2y+1}{y-1}\right) + c = 0.$$

$$\Rightarrow a(2y+1)^2 + b(2y+1)(y-1) + c(y-1)^2 = 0.$$

$$\Rightarrow \underline{a(2x+1)^2 + b(2x+1)(x-1) + c(x-1)^2 = 0} \quad \underline{\underline{\text{Ans}}}$$

## QUESTION

(KTK 8)



If  $\alpha, \beta, \gamma$  are roots  $x^3 + 2x^2 - 3x + 1 = 0$ , then value of  $\frac{\alpha\beta}{\alpha+\beta} + \frac{\alpha\gamma}{\alpha+\gamma} + \frac{\beta\gamma}{\beta+\gamma}$  is less than

**A** 2

**B** 3

**C** 4

**D** 5

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11-8

$$x^3 + 2x^2 - 3x + 1 = 0 \quad \text{--- (1)}$$

$\alpha + \beta + \gamma = -2$   
 $\alpha\beta\gamma = -1$

$\alpha + \beta = -2 - \gamma$   
 $\beta + \gamma = -2 - \alpha$   
 If,  $\alpha + \gamma = -2 - \beta$

&  
 $\alpha\beta = -1/\gamma$   
 $\beta\gamma = -1/\alpha$   
 $\alpha\gamma = -1/\beta$

$$x^2(x+2) = 3x-1$$

$$x(x+2) = \frac{3x-1}{x}$$

$$\Rightarrow \frac{1}{x(x+2)} = \frac{x}{3x-1}$$

$$\frac{-1/\gamma}{-\gamma-2} + \frac{-1/\beta}{-\beta-2} + \frac{-1/\alpha}{-\alpha-2} \Rightarrow \frac{1}{\alpha(\alpha+2)} + \frac{1}{\beta(\beta+2)} + \frac{1}{\gamma(\gamma+2)}$$

Let  $y = \frac{x}{3x-1} \Rightarrow 3xy - y = x$   
 $(3y-1)x = y \Rightarrow x = \frac{y}{3y-1}$  Put in (1)

$$\Rightarrow \frac{y^3}{(3y-1)^3} + \frac{2y^2}{(3y-1)^2} - \frac{3y}{3y-1} + 1 = 0$$

$$\Rightarrow y^3 + 2y^2(3y-1) - 3y(3y-1)^2 + (3y-1)^3 = 0$$

$$\Rightarrow y^3 + 6y^3 - 2y^2 - 27y^3 - 3y + 18y^2 + 27y^3 - 1 - 27y^2 + 9y = 0$$

$$\Rightarrow 7y^3 - 11y^2 + 6y - 1 = 0$$

replaced y by x  $\rightarrow$

$$7x^3 - 11x^2 + 6x - 1 = 0 \quad \begin{cases} a \\ b \\ c \end{cases}$$

$a + b + c = \frac{11}{7} = 1.5$

which is less than 2, 3, 4, 5

**KTK-8**  
 By Nikita, Raj



# Solution to Previous Home Challenge

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## Home Challenge - 08



The ordered pair  $(x, y)$  satisfying the equation  $x^2 = 1 + 6 \log_4 y$  and  $y^2 = 2^x y + 2^{2x+1}$  are  $(x_1, y_1)$  and  $(x_2, y_2)$ , then find the value of  $\log_2 |x_1 x_2 y_1 y_2|$ .

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(Ans:7)



### Home challenge - 08

$$x^2 = 1 + 6 \log_4 y, \quad y > 0$$

$$\Rightarrow x^2 = 1 + 3 \log_2 y$$

$$\Rightarrow x^2 = 1 + 3 \log_2 2^{x+1}$$

$$\Rightarrow x^2 = 1 + 3(x+1)$$

$$\Rightarrow x^2 - 3x - 4 = 0$$

$$\Rightarrow (x-4)(x+1) = 0$$

$x = 4, -1$

$y = 2^5, 2^0$

$y = 32, 1$

$x_1 = 4 \quad y_1 = 2^5$   
 $x_2 = -1 \quad y_2 = 1$

$\log_2 |x_1 x_2 y_1 y_2|$

$= \log_2 |4(-1)2^5 \cdot 1|$

$\Rightarrow \log_2 (2^7) = 7 \text{ Ans}$

$y^2 = 2^x y + 2^{2x+1}$   
 $y^2 = 2^x y + 2^{2x} \cdot 2$  let  $2^x = t$

$y^2 - ty - 2t^2 = 0$   
 $y = \frac{t \pm \sqrt{t^2 + 8t^2}}{2}$

$y = \frac{t \pm 3t}{2}$

$y = 2t, -t$  since  $y > 0$   
 $y = 2 \cdot 2^x, -2^x$   
 $y = 2^{x+1}$  X N.P

**Home challenge-8**  
**By Nikita, Raj.**



**THANK**  
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**YOU**