PRAYAS

JEE 2026

Mathematics

Quadratic Equations

Lecture - 07

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Topics *to be covered*

A Range of Rational Functions

B Practice problems

Recap of previous lecture

1. If $x \in [-1, 5]$ then

a. $2x + 3 \in$ [-2,10] → [1,13]

b. $5 - 6x \in$ [-6,30] → [-25,11]

c. $\frac{1}{x} \in$ $x \to 0^+$: $x \in [-1,0] \cup [0,5]$ → $(-\infty,-1] \cup [\frac{1}{5},\infty)$

d. $x^2 \in$ $x \in [-1,0] \cup [0,5]$ → $[0,1] \cup [0,25] = [0,25]$

e. $\frac{1}{x^2-1} \in$ $x^2-1 \in [0,25]$ → $[-1,24] = [-1,0] \cup [0,24]$

Recap of previous lecture

2. $y = f(x) = x^2 - 5x + 4, x \in R$. Then find range of

(i) $y = x^2 - 5x + 4$ where $x \in R$ is $\left[-\frac{D}{4a}, \infty\right) = \left[-\frac{(25-16)}{4}, \infty\right) = \left[-\frac{9}{4}, \infty\right)$

(ii) $y = x^2 - 5x + 4$ where $x \in [0, 2]$ is $[-2, 4]$

(iii) $y = x^2 - 5x + 4$ where $x \in [2, 5]$ is $[-9/4, 4]$

(iv) $y = x^2 - 5x + 4$ where $x \in (-1, 3)$ is $[-9/4, 10)$

$\left(\frac{x \text{ ke coeff}}{2}\right)^2$ ← add & sub.

3. $y = ax^2 + bx + c \geq 0 \ \forall \ x \in R$ if $a > 0$ & $D \leq 0$

4. $y = ax^2 + bx + c \leq 0 \ \forall \ x \in R$ if $a < 0, D \leq 0$.

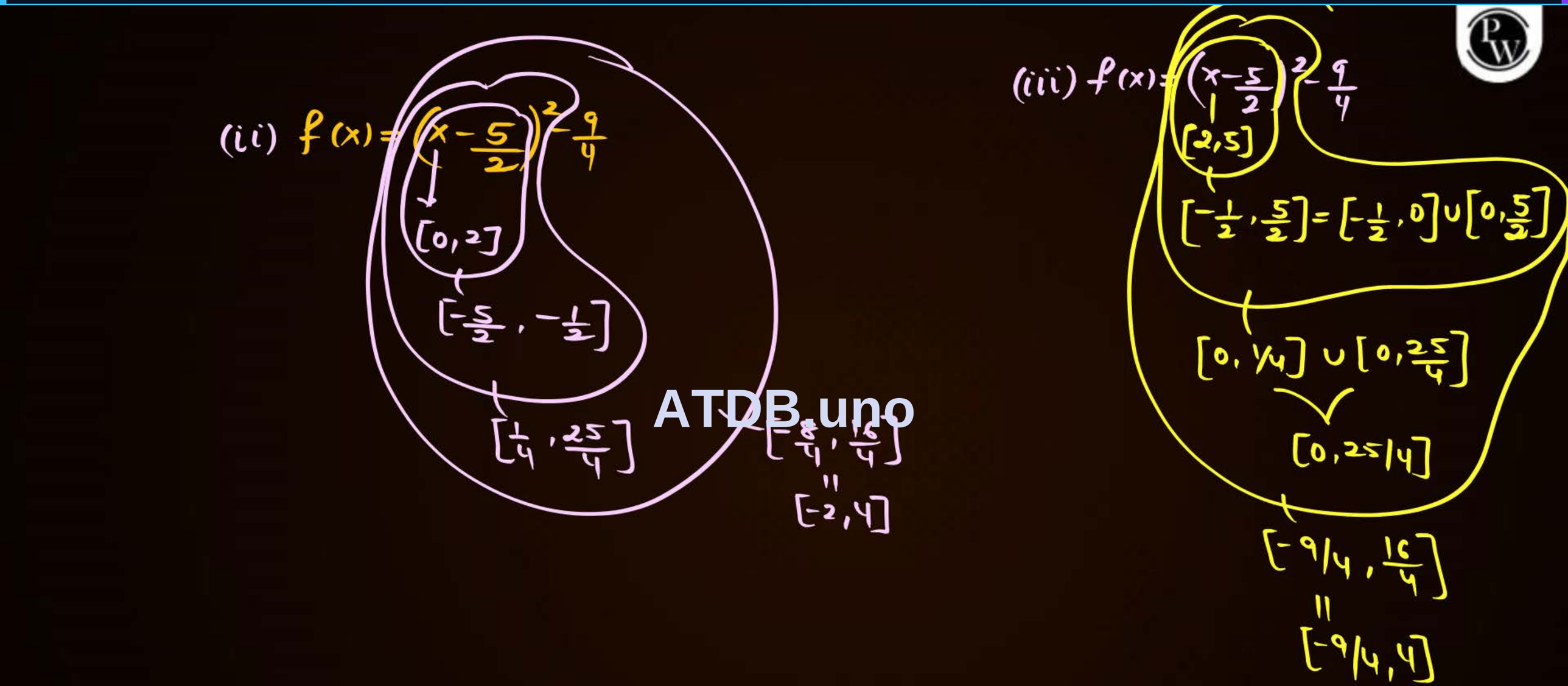
5. $y = ax^2 + bx + c > 0 \ \forall \ x \in R$ if $a > 0, D < 0$

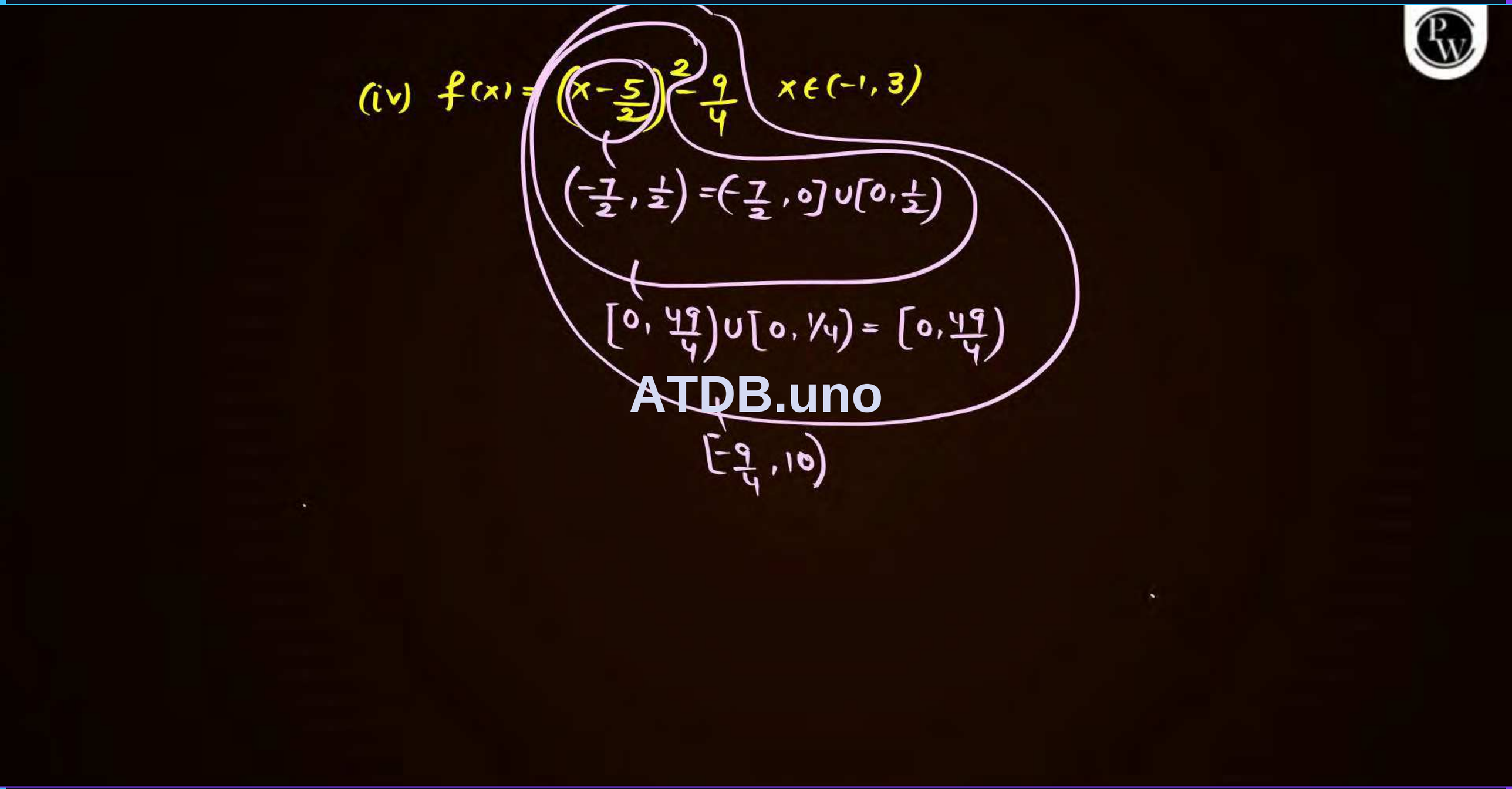
6. $y = ax^2 + bx + c < 0 \ \forall \ x \in R$ if $a < 0, D < 0$

$f(x) = x^2 - 5x + 4$

$= x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 4$

$= \left(x - \frac{5}{2}\right)^2 + 4 - 25/4 = \left(x - \frac{5}{2}\right)^2 - \frac{9}{4}$





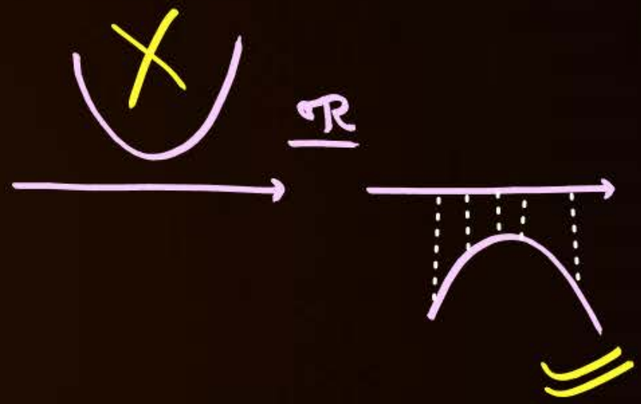
Recap of previous lecture

7. If $D < 0$ then graph of $y = ax^2 + bx + c$ lies entirely above x-axis or entirely below x-axis.

8. If $ax^2 + bx + c$ has non real roots & $a + 2b + 4c < 0$ then

$a - b + c$ < 0, $4a - 2b + c$ < 0

$f(-1)$ $f(-2)$ $f(x)$ $4f(1/2)$ $f(1/2) < 0$



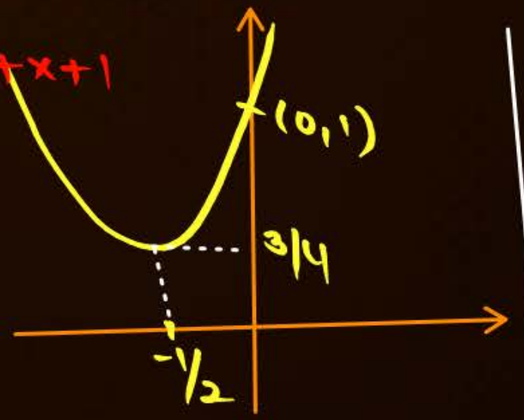
9. Graph of 1. $y = x^2 + x + 1$ 2. $y = x^2 + 2x + 1$

* upward opening parabola
* $D < 0, a > 0$ ✓ lies above x axis.
* $y_{min} = \frac{-D}{4a} = -\frac{(1-4)}{4} = 3/4$ at $x = -1/2$

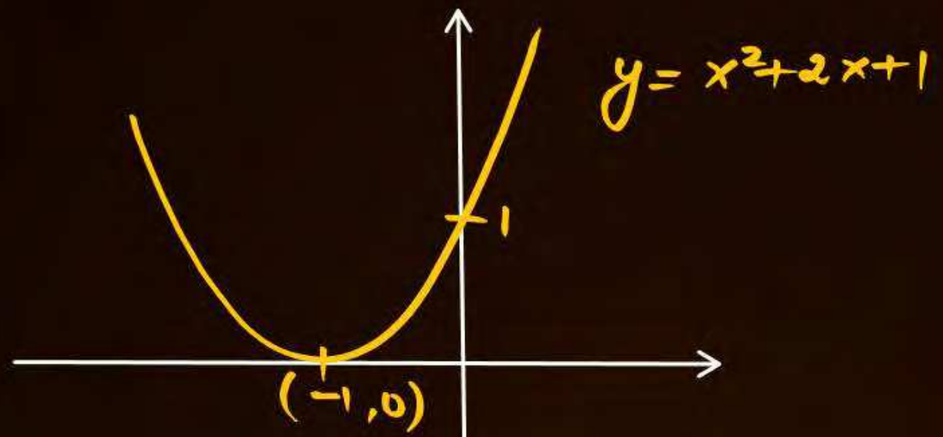
$V(-1/2, 3/4)$

* POI y axis $(0,1)$

$y = x^2 + x + 1$



* upward opening parabola
* Roots $-1, -1$ $D = 0$ ⇒ touches x axis.
* $V(-\frac{2}{2}, 0) = (-1, 0)$
* POI y axis $(0,1)$

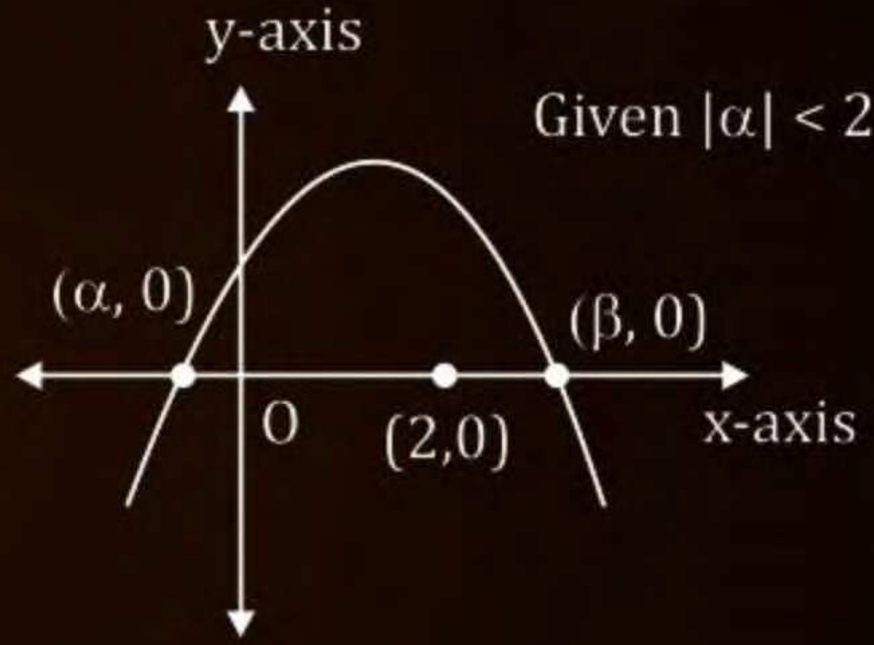


Homework Discussion

QUESTION

(ADBST)

The graph of $y = ax^2 + bx + c$ is shown in the figure, then which of the following is(are) correct?



A $ab^2c^3 < 0$

B $ab < 0$

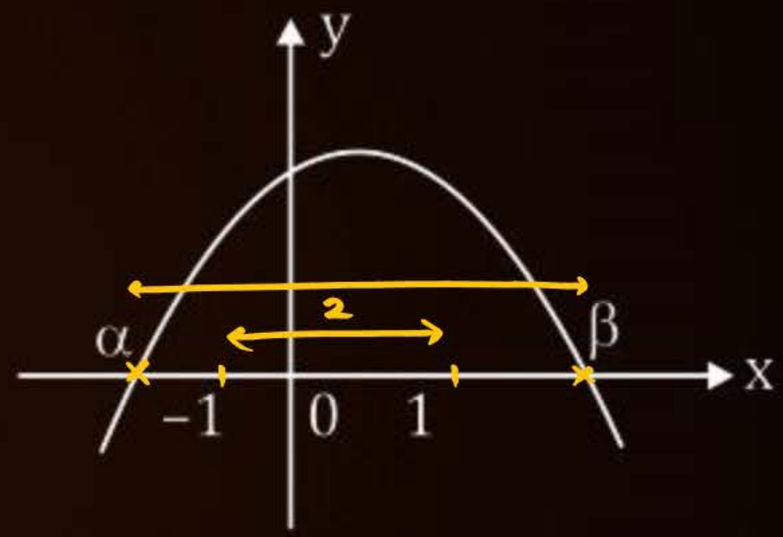
C $bc(4a + 2b + c) > 0$

D $ab(4a - 2b + c) > 0$

QUESTION

(ADBST)

The graph of quadratic polynomial $f(x) = ax^2 + bx + c$ is shown in below. Which of the following are correct?



(A) $\frac{c}{a} < -1$

(B) $|\beta - \alpha| > 2$

(C) $f(x) > 0 \ \forall \ x \in (0, \beta)$

(D) $abc < 0$

(B) $|\beta - \alpha| > 2$

(A) $\alpha < -1$

$\beta > 1$

$\alpha\beta < -1$

$\frac{c}{a} < -1$

$\alpha = -1.5$

$\beta = 2 \rightarrow \alpha\beta = -3 < -1$

$S.O.R = -\frac{b}{a} > 0$

$-b < 0$

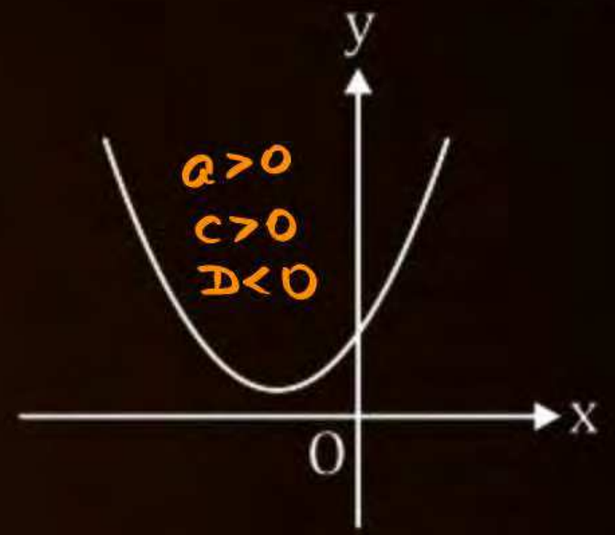
$b > 0$

-ve >0 +ve

QUESTION

(ADBST)

The curve of the quadratic expression $y = ax^2 + bx + c$ is shown in the figure and α, β be the roots of the equation $ax^2 + bx + c = 0$ then correct option is
[D is the discriminant]



A $a > 0,\ b > 0, c > 0, D > 0, \alpha + \beta > 0, \alpha\beta > 0$

B $a > 0, b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta < 0$

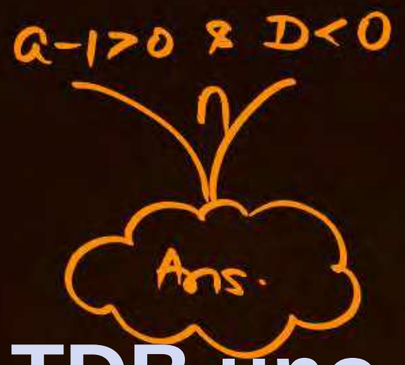
C $a > 0,\ b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta > 0$

D $a > 0, b < 0, c > 0, D < 0, \alpha + \beta > 0, \alpha\beta > 0$

Ans. C

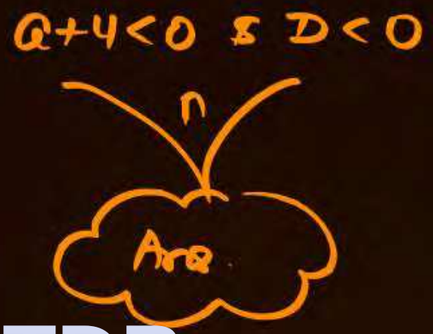
QUESTION

Find the set of values of a for which $(a-1)x^2-(a+1)x+a+1>0$ for all $x \in R$.



QUESTION

Find the set of values of a for which $(a + 4)x^2 - 2ax + 2a - 6 < 0$ for all $x \in R$.

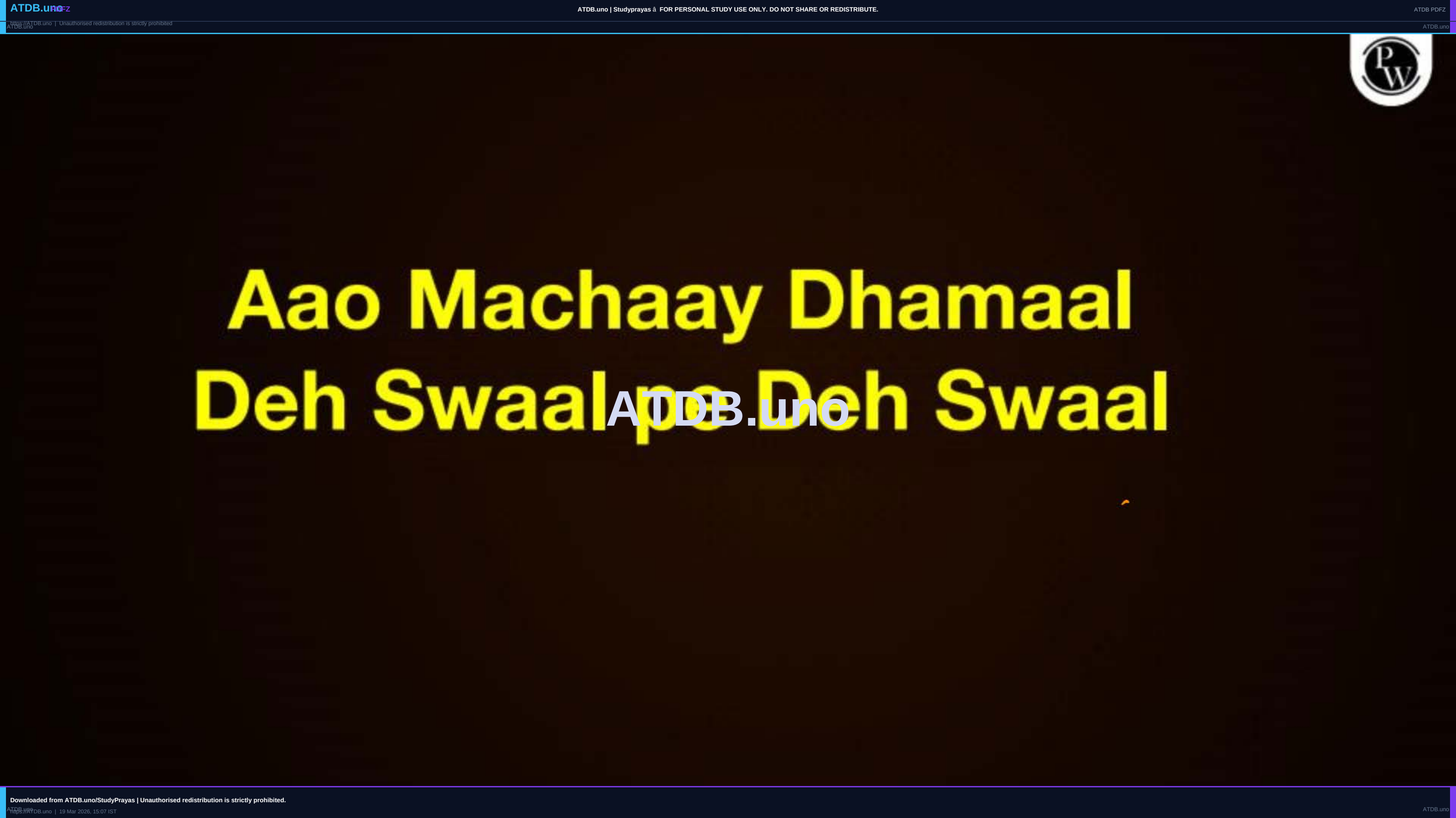


QUESTION

For what values of p the vertex of $x^2 + px + 13$ lies at a distance 5 unit from origin.

$V\left(-\frac{p}{2}, -\left(\frac{p-52}{4}\right)\right)$

$O(0, 0)$



QUESTION

Find the range of $f(x) = 2x^2 - 3x + 2$ in $[0, 2]$ Ans. : $\left[\frac{7}{8}, 4\right]$

Find the range of $f(x) = -x^2 + 6x - 1$ in $[0, 4]$ Ans.: $[-1, 8]$

$y = 2x^2 - 3x + 2$

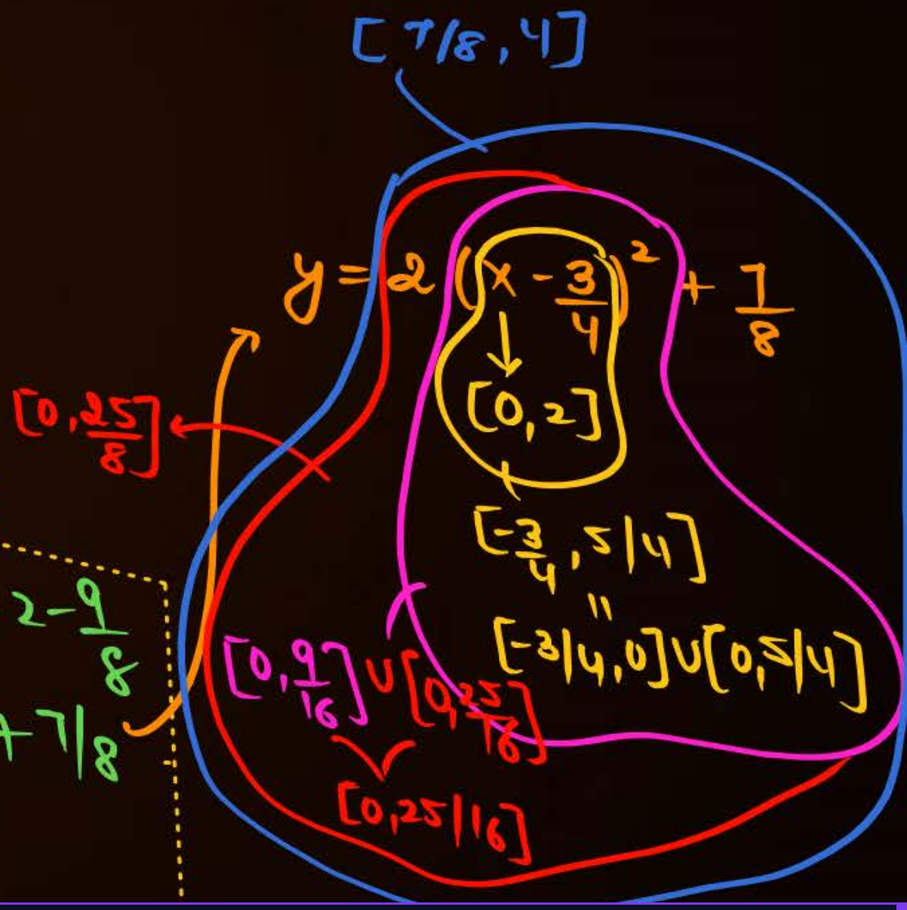
$= 2\left(x^2 - \frac{3}{2}x\right) + 2$

$= 2\left(x^2 - \frac{3}{2}x + \frac{9}{16} - \frac{9}{16}\right) + 2$

$= 2\left((x - 3/4)^2 - \frac{9}{16}\right) + 2$

$= 2(x - 3/4)^2 - \frac{9}{8} + 2 = 2(x - 3/4)^2 + 2 - \frac{9}{8}$

$= 2(x - 3/4)^2 + 7/8$



QUESTION

Find the range of $f(x) = 2x^2 - 3x + 2$ in $[0, 2]$

Ans. : $\left[\frac{7}{8}, 4\right]$

Find the range of $f(x) = -x^2 + 6x - 1$ in $[0, 4]$

Ans.: $[-1, 8]$

$f(x) = -x^2 + 6x - 1$

$= -1 - (x^2 - 6x + 9 - 9)$

$= -1 - (x-3)^2 + 9$

$= 8 - (x-3)^2$

$[-3, 1] = [-3, 0] \cup [0, 1]$

$[0, 9] \cup [0, 1] = [0, 9]$

$[-1, 8]$

QUESTION

Tanoi

For $x \in [1, 5], y = x^2 - 5x + 3$ has-

A Least value $= -1.5$

B Greatest value $= 3$

C Least value $= -3.25$

D Greatest value $= \frac{5+\sqrt{13}}{2}$

QUESTION

Tahoz

Find range of following functions:

(i) $f(x) = 3x^2 - 2x - 7$

(ii) $f(x) = 3x^2 - 2x - 7, x \in (0, 5]$

(iii) $f(x) = 3x^2 - 2x - 7, x \in [-6, -1]$

Ans:[22/3,∞)

Ans: [22/3,58]

Ans: [-2,113]

QUESTION

Tah03

Find the range of f(x) :

(i) $f(x) = 2x^2 - 3x + 2$ Ans. $\left[\frac{7}{8}, \infty\right)$

(ii) $f(x) = 2x^2 - 3x + 2, x \in [0, 2]$ Ans. $\left[\frac{7}{8}, 4\right]$

(iii) $f(\theta) = 2\cos^2\theta - 6\sin\theta + 1$ Ans. $[-5, 7]$

QUESTION

Tanoy

Find the maximum and Minimum values If they exist

(i) $f(x) = x^2 + 2x + 4$

(ii) $f(x) = x^2 + 4x + 4$

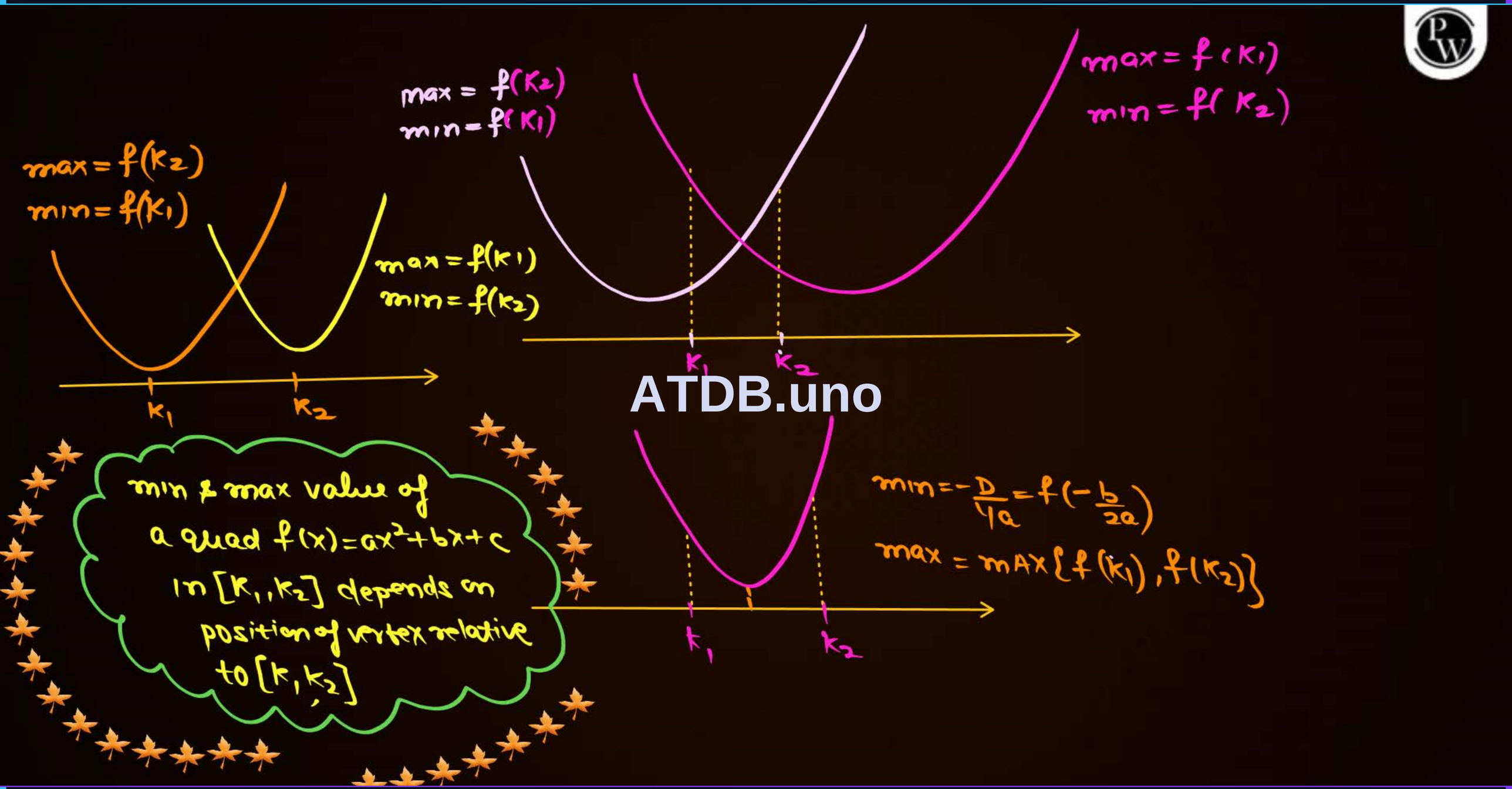
(iii) $f(x) = x^2 - 5x + 4$

(iv) $f(x) = -x^2 + x - 4$

(v) $f(x) = -x^2 + 6x - 9$

(vi) $f(x) = -x^2 + 6x - 8$

Ans. (i) Min value = 3; (ii) Min value = 0
(iii) Min value = –9/4, (iv) Max value = –15/4
(v) Max value = 0, (vi) Max value = –1



QUESTION

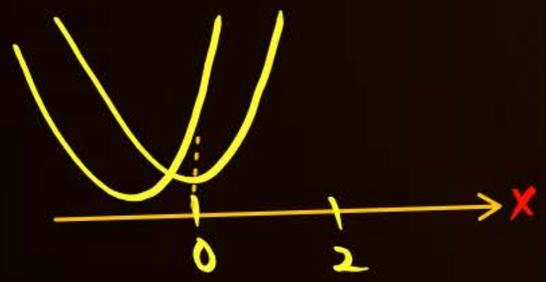
★★★KCLS★★★

Find all numbers p for each of which the least value of the quadratic trinomial $4x^2 - 4px + p^2 - 2p + 2$ on the interval $0 \le x \le 2$ is equal to 3

Given $f(x) = 4x^2 - 4px + p^2 - 2p + 2$ has min value = 3 in $x \in [0,2]$

$$x_V = -\frac{(-4p)}{2 \cdot 4} = \frac{p}{2}$$

Case① $\frac{p}{2} \le 0 \rightarrow p \le 0$



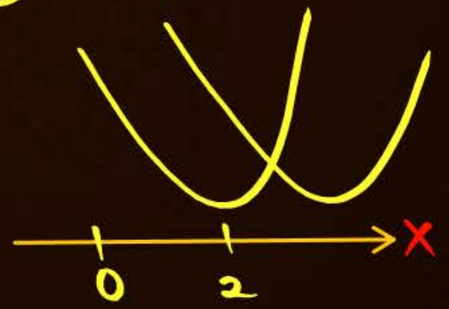
$\min = f(0) = 3 \Rightarrow p^2 - 2p + 2 = 3$

$p^2 - 2p - 1 = 0$

$p = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

$p = 1 - \sqrt{2}$

Case② if $\frac{p}{2} \ge 2 \rightarrow p \ge 4$



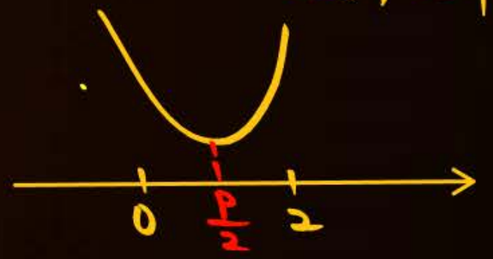
$\min = f(2) = 3$

$16 - 8p + p^2 - 2p + 2 = 3$

$p^2 - 10p + 15 = 0$

Case③ if $0 < \frac{p}{2} < 2$

$0 < p < 4$



$\min = f\left(\frac{p}{2}\right) = 3$

$p^2 - 2p^2 + p^2 - 2p + 2 = 3$

$p = -1/2$ (rejected)

Ans. $P = 1 - \sqrt{2}$ or $5 + \sqrt{10}$

$p^2 - 10p + 15 = 0$

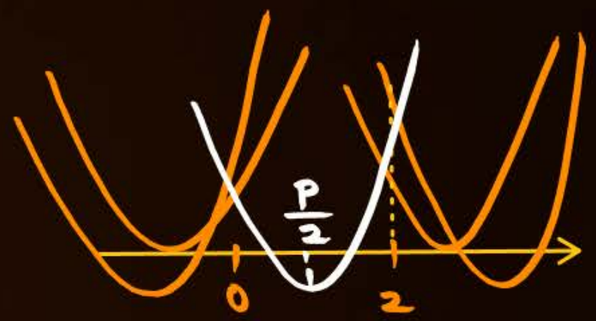
$p = \frac{10 \pm \sqrt{40}}{2}$

$p = 5 \pm \sqrt{10}$

$p \geq 4$ — n

$p = 5 + \sqrt{10}$

Ans $p = 5 + \sqrt{10}, 1 - \sqrt{3}$



Range of Rational functions

$f(x) = \frac{(\text{Polynomial})_1}{(\text{Polynomial})_2}$

Type 1:

$$f(x) = \frac{\text{linear}_1}{\text{linear}_2} \text{ i.e } f(x) = \frac{ax+b}{cx+d}\left(\frac{a}{c} \neq \frac{b}{d}\right)$$

Ex: $y = \frac{3x+2}{4x+5}$

Range mat lab y ka domain kya hoga

$4xy + 5y = 3x + 2$

$4xy - 3x = 2 - 5y$

$x(4y-3) = 2-5y$

$x = \frac{2-5y}{4y-3}$ ~ $y \in R - \{3/4\}$

* $y = \frac{ax+b}{cx+d}$, $\frac{a}{c} \neq \frac{b}{d}$

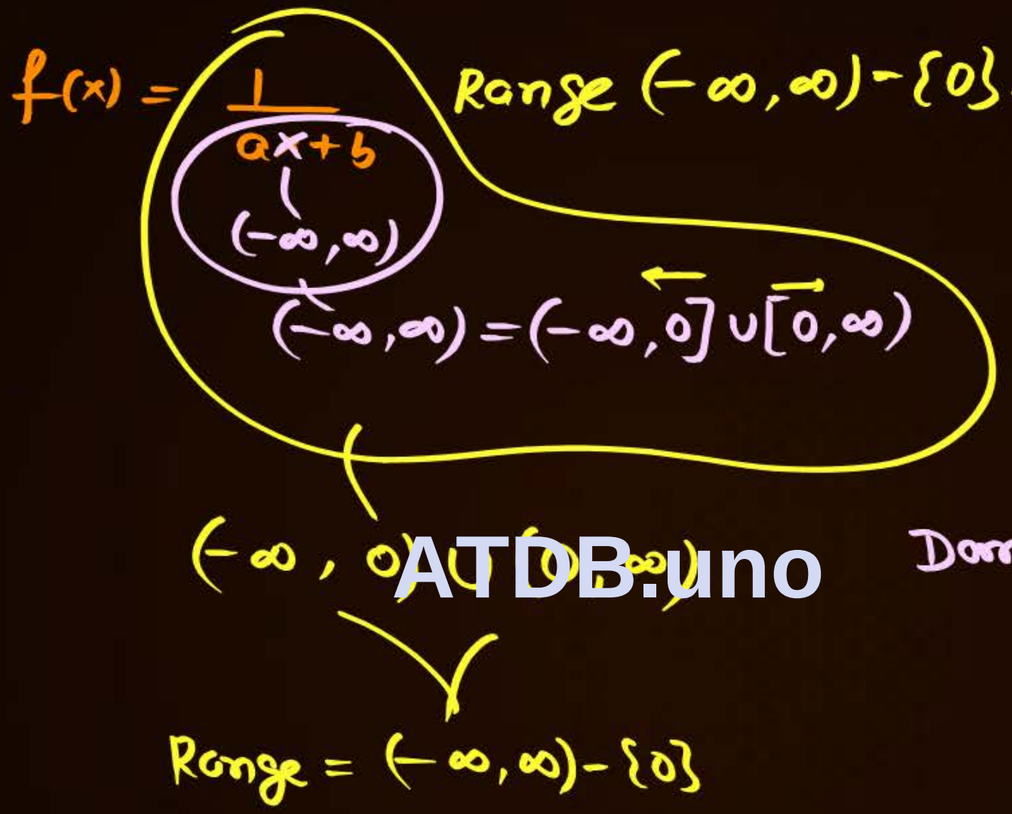
↓

Range: $R - \{a/c\}$.

$y = \frac{3x+5}{6x+10}$ $\quad \frac{3}{6} = \frac{5}{10}$

$y = \frac{3x+5}{2(3x+5)} = 1/2$, $x \neq -5$.

$y = 1/2$ Range $= \{1/2\}$.



QUESTION

Find range of :

(1) $f(x) = \frac{2x-3}{x-1}$ — $R-\{\frac{2}{1}\} = R-\{2\}$

(2) $f(x) = \frac{x+3}{2-5x}$ → $R-\{-\frac{1}{5}\}$

(3) $f(x) = \frac{6}{4x+7} = 6 \cdot \frac{1}{4x+7}$

$(-\infty, \infty) - \{0\}$

$(-\infty, \infty) - \{0\}$

(4) $f(x) = \frac{7x+5}{3}$ — $(-\infty, \infty)$

$(-\infty, \infty)$

Type 2 :

$f(x) = \frac{(quad)_1}{(quad)_2}$ where numerator & denominator contain a common factor

$f(x) = \frac{x^2-7x+12}{x-10x+21} = \frac{(x-3)(x-4)}{(x-3)(x-7)}$

Steps:

(i) Factorize numerator & Denominator

(ii) Say $(ax - b)$ is a common factor, cancel $(ax - b)$ from numerator & denominator and write $x \neq b/a$.

$f(x) = \frac{x-4}{x-7}, x \neq 3$

$f(3) = \frac{3-4}{3-7} = \frac{-1}{-4}$

Range $= R - \{1, 1/4\}$

(iii) Now $f(x) = \frac{px+q}{rx+s}, x \neq b/a$ & range $= R - \left\{\frac{P}{r}, f\left(\frac{b}{a}\right)\right\}$

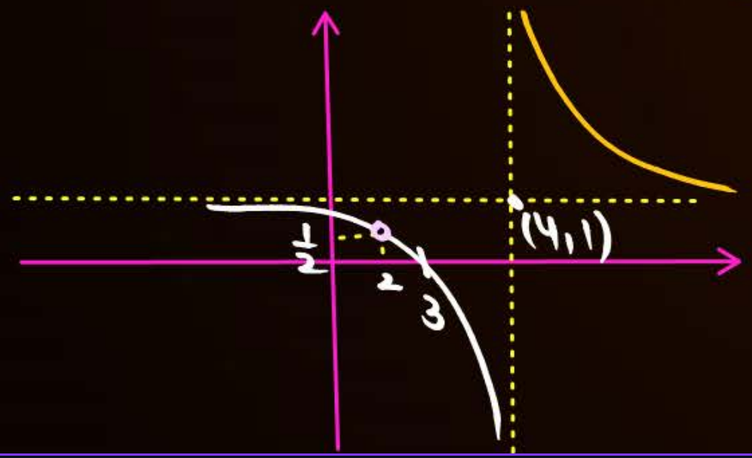
QUESTION

Find range of $f(x) = \dfrac{x^2 - 5x + 6}{x^2 - 6x + 8}$.

$$y = \frac{(x-2)(x-3)}{(x-2)(x-4)} \quad — \quad y(2) = \frac{2-3}{2-4} = \frac{-1}{-2} = \frac{1}{2}$$

$$y = \frac{1 \cdot x - 3}{1 \cdot x - 4}, \quad x \neq 2 \quad — \quad \text{Range } R - \{f(2)\}$$

Range $R - \{1, 1/2\}$.



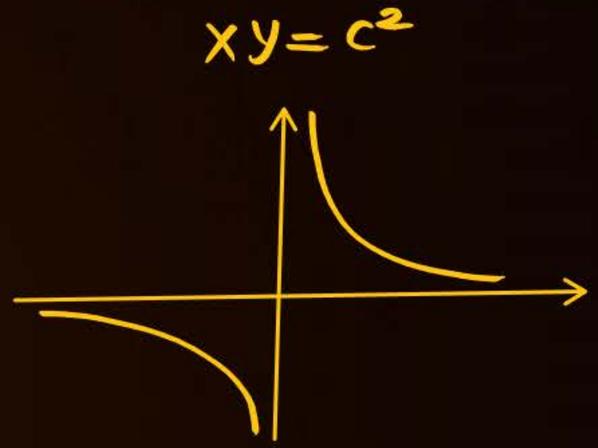
$$y = \frac{x-3}{x-4}$$

$$xy - 4y = x - 3$$

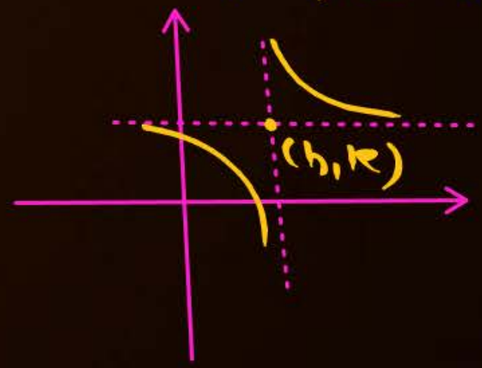
$$xy - x - 4y + 4 = -3 + 4$$

$$x(y-1) - 4(y-1) = 1$$

$$(x-4)(y-1) = 1$$



$$(x-h)(y-k) = c^2$$



QUESTION

Find the range of $f(x) = \frac{x^2 - 5x + 4}{x^2 + 2x - 3}$.

Tan05

Also Draw its graph.

Type 3 :

$f(x) = \frac{\text{quad}}{\text{quad}}, \frac{\text{linear}}{\text{quad}}, \frac{\text{quad}}{\text{linear}}$ where numerator & denominator contain no common factor.

Steps:

(i) Equate given expression to y

(ii) Cross multiply & make quad in X

(iii) Since $x \in R$, put $D \geq 0$ & set range of y.

(vi) Equate coefficient of $x^2 = 0$ to get $y = y_1$ put expression $= y_1$ & solve for x

If we get real x answer of step 3 is final

If we do not get real x exclude y_1 from answer.

QUESTION

Find domain & range of $f(x) = \frac{x^2+x+1}{x^2-x+1}$.

$x^2-x+1 \neq 0 \rightarrow x \in R$

$D<0, a>0$

always +ve

$y = \frac{x^2+x+1}{x^2-x+1}$

$x^2y - xy + y = x^2 + x + 1$

$x^2(y-1) - x(y+1) + y - 1 = 0$

Case ① $y-1 \neq 0$ i.e $y \neq 1$ Since $x \in R$, $D \geq 0$

$D = (y+1)^2 - 4(y-1)(y-1) \geq 0$

$\Rightarrow (y+1)^2 - (2(y-1))^2 \geq 0$

$(y+1+2y-2)(y+1-2y+2) \geq 0$

$(3y-1)(3-y) \geq 0$

$(3y-1)(y-3) \leq 0 \Rightarrow y \in [1/3, 3] - \{1\}$

Ex: $\frac{x^2+x+1}{x^2-x+1} = 3 \in$ Range

$x^2+x+1 = 3x^2-3x+3$

$2x^2-4x+2=0$

$x^2-2x+1=0$

$x=1$

Ex: $\frac{x^2+x+1}{x^2-x+1} = 4 \notin$ Range

$x^2+x+1 = 4x^2-4x+4$

$3x^2-5x+3=0$

$D<0$ No real x

Case (ii) if $y=1$

$$-2x = 0$$

$$x = 0$$

$\Downarrow$

$y=1$ is also possible

$$y \in \left[\frac{1}{3}, 3\right] - \{1\} \cup \{1\}.$$

$$y \in \left[\frac{1}{3}, 3\right]$$

QUESTION

M① TaH 06

Find domain & range of

$$f(x) = \frac{2x^2 + 2x + 3}{x^2 + x + 1}$$

M②

$$y = \frac{2x^2+2x+3}{x^2+x+1} = \frac{2x^2+2x+2-2+3}{x^2+x+1}$$

$$= \frac{2(x^2+x+1)+1}{x^2+x+1} = 2 + \frac{1}{x^2+x+1}$$

Domain : R

$x^2+x+1 \neq 0$

$D<0$

$a>0$

always +ve

$x \in R$

$$y = 2 + \frac{1}{x^2+x+1}$$

$$= 2 + \frac{1}{x^2+x+1/4-1/4+1}$$

$$= 2 + \frac{1}{(x+1/2)^2 + 3/4}$$

$(x+1/2)^2 \in [0,\infty)$

$(x+1/2)^2 + 3/4 \in [3/4,\infty)$

$\frac{1}{(x+1/2)^2+3/4} \in (0, \frac{4}{3}]$

$2 + \frac{1}{(x+1/2)^2+3/4} \in (2, \frac{10}{3}]$

Range $= (2, \frac{10}{3}]$

M③

$y = 2 + \frac{1}{x^2+x+1}$

$x^2+x+1 \in [-\frac{D}{4a}, \infty) = [\frac{3}{4}, \infty)$

$\frac{1}{x^2+x+1} \in (0, \frac{4}{3}]$

$2 + \frac{1}{x^2+x+1} \in (2, \frac{10}{3}]$

$(2, \frac{10}{3}]$ = Range.

QUESTION

Find domain & range of $f(x) = \frac{2x}{1+x^2}$.

$1+x^2 \neq 0 \rightarrow x \in R$ = Domain

$1+x^2 \neq 0$ → $a>0, D<0$ → always +ve

M① $y = \frac{2x}{1+x^2}$ Tan θ

M② $y = \frac{2x}{1+x^2} = \begin{cases} \frac{2}{x+\frac{1}{x}} & x>0 \\ 0 & x=0 \\ \frac{2}{x+1/x} & x<0 \end{cases}$

$x>0$ → $y = \frac{2}{x+\frac{1}{x}} \rightarrow [2,\infty)$ → $2(0,\frac{1}{2}] = (0,1]$

$x=0 \rightarrow y=0$

$x<0$ → $y = \frac{2}{x+\frac{1}{x}} \rightarrow (-\infty,-2]$ → $2[-\frac{1}{2},0) = [-1,0)$

M③ $y = \frac{2x}{1+x^2}$ $x = \tan\theta$

$y = \frac{2\tan\theta}{1+\tan^2\theta}$

$y = \sin 2\theta$.

$y \in [-1,1]$

Range: $(0,1] \cup \{0\} \cup [-1,0)$

Range $= [-1,1]$

QUESTION

Tan 08

If x be real, then prove that $\frac{x}{x^2 - 5x + 9}$ must lie between $-\frac{1}{11}$ and 1.

QUESTION

If x is real, then maximum value of $\frac{3x^2 + 9x + 17}{3x^2 + 9x + 7}$ is

Tan09

(A) 1

(B) 17/7

(C) 1/4

(D) 4

Domain

$3x^2+9x+7 \neq 0$

$x \in R$

$y = \frac{3x^2+9x+7+10}{3x^2+9x+7}$

$y = 1 + \frac{10}{3x^2+9x+7}$

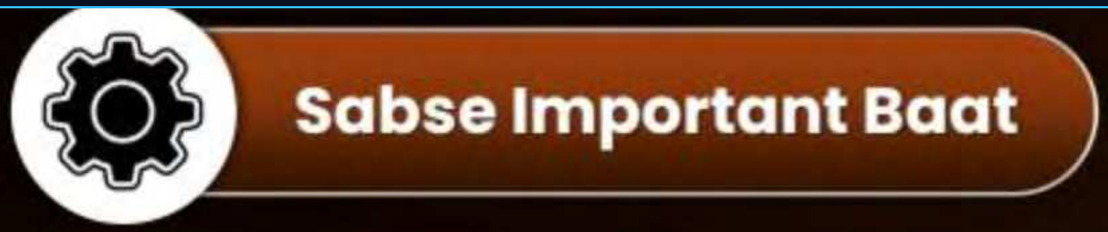
$\left[\frac{-D}{4a}, \infty\right)$

Ans. 4

Ex: $\frac{1}{x^2-5x+6}$ $x \neq 2,3$

$\left[-\frac{D}{4a}, \infty\right) = \left[-\frac{1}{4}, \infty\right) = \left[-\frac{1}{4}, 0\right] \cup [0, \infty)$

Range $\left(-\infty, -\frac{1}{4}\right] \cup (0, \infty)$



Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection — TRISHUL Apnao IIT Jao → **Selection with Good Rank**

QUESTION

(KTK 01)

The value of 'a' for which the equation $x^7 + ax^2 + 3 = 0$ and $x^8 + ax^3 + 3 = 0$ have a common root, can be

A 1

B −2

C −3

D −4

Ans. D

QUESTION

(KTK 02)

If α, β are roots of $Ax^2 + Bx + C = 0$ and α^2, β^2 are roots of $x^2 + px + q = 0$ then p is equal to

A $\frac{B^2 - 4AC}{A^2}$

B $\frac{2AC - B^2}{A^2}$

C $\frac{4AC - B^2}{A^2}$

D None of these

Ans. B

QUESTION

(KTK 03)

Find the values of 'k' so that the equation

$x^2 + kx + (k + 2) = 0$ and $x^2 + (1 - k)x + 3 - k = 0$ have exactly one common root.

Ans. No possible value of k

QUESTION

(KTK 04)

In a triangle PQR, $\angle R = \frac{\pi}{2}$, if $\tan\left(\frac{P}{2}\right)$ and $\tan\left(\frac{Q}{2}\right)$ are the roots of $ax^2 + bx + c = 0$, $a \neq 0$ then

A $a = b + c$

B $c = a + b$

C $b = c$

D $b = a + c$

Ans. B

QUESTION

(KTK 05)

The equations $ax^2 + bx + a = 0 (a, b \in R)$ and $x^3 - 2x^2 + 2x - 1 = 0$ have 2 roots common. Then $a + b$ must be equal to

A 1

B -1

C 0

D None of these

Ans. C

Homework From Module

Quadratic Equations

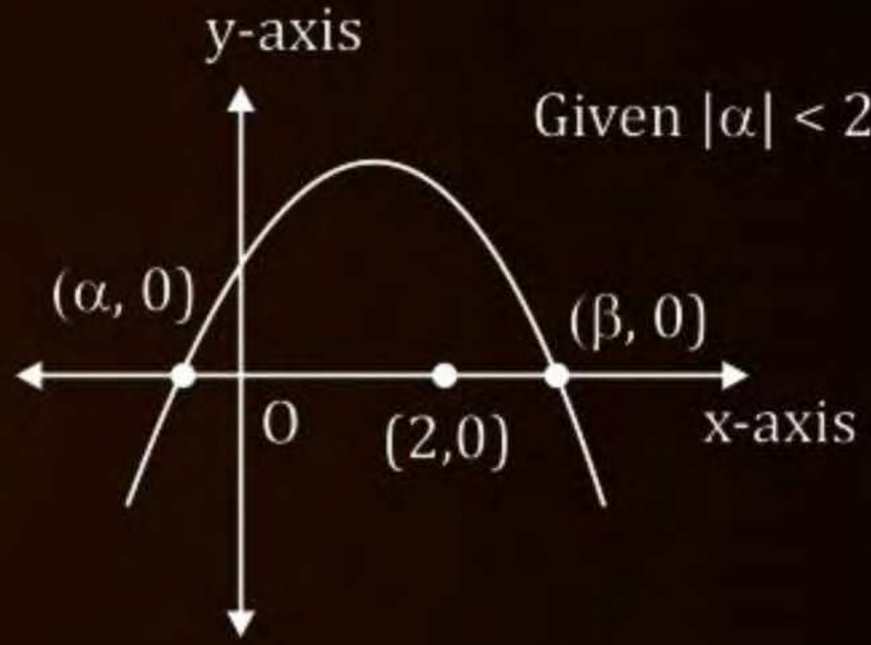
Prarambh (Topicwise)	: Q1 to Q27
Prabal (JEE Main Level)	: Q1,Q2,Q6 to Q9
Parikshit (JEE Advanced Level)	: Abhi Ruko

Solution to Previous TAH

ATDB.uno

QUESTION

The graph of $y = ax^2 + bx + c$ is shown in the figure, then which of the following is(are) correct?



A $ab^2c^3 < 0$

B $ab < 0$

C $bc(4a + 2b + c) > 0$

D $ab(4a - 2b + c) > 0$

Lecture-06.

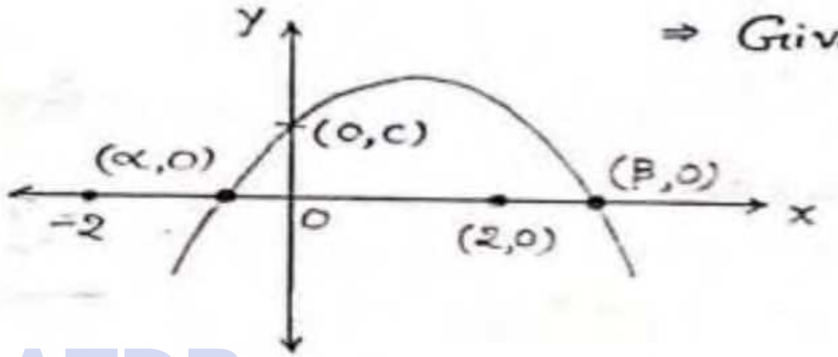
Tah-01 · The graph of $y = ax^2 + bx + c$ is shown in the figure, then which of the following is (are) correct ?

(A) $ab^2c^3 < 0$

(B) $ab < 0$

(C) $bc(4a+2b+c) > 0$

(D) $ab(4a-2b+c) > 0$.



⇒ Given $|a| < 2$.

$-2 < \alpha < 2$.

from graph

$\alpha < 0$

$-2 < \alpha < 0$.

$a < 0$

$c > 0$.

$b > 0$.

krish

(A) $ab^2c^3 < 0$.

-ve +ve +ve ⇒ -ve

(B) $ab < 0$.

-ve +ve ⇒ -ve.

(C) $bc(4a+2b+c) > 0$.

+ve +ve $f(2)$ ⇒ +ve.

+ve

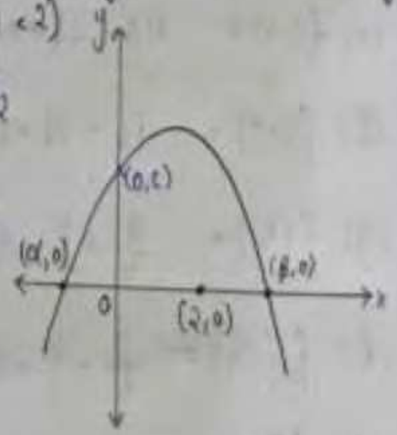
(D) $ab(4a-2b+c) > 0$

-ve +ve $f(-2)$ ⇒ +ve

-ve

(Q. The graph of $y = ax^2 + bx + c$ is shown in the figure. Then which of the following is (are) correct? (given $|\alpha| < 2$)

TAH 1-A



a) $ab^2c^3 < 0$

$a < 0$

$c > 0$

$-\frac{b}{2a} > 0$

$-2 < \alpha < 2$

from graph

$\alpha < 0$

$-2 < \alpha < 0$

$b > 0$

b) $ab < 0$

c) $bc(4a + 2b + c) > 0$

$f(2)$

$bc\,[f(2)] > 0$

d) $ab(4a - 2b + c) > 0$

$ab\,[f(-2)] > 0$

in the figure, then which of the following is (are) correct?

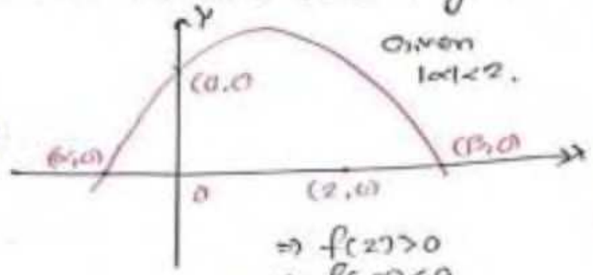
TAH 1
BY REED
FROM WB

(a) $ab^2c^3 < 0$

(b) $ab < 0$

(c) $bc(4a + 2b + c) > 0$

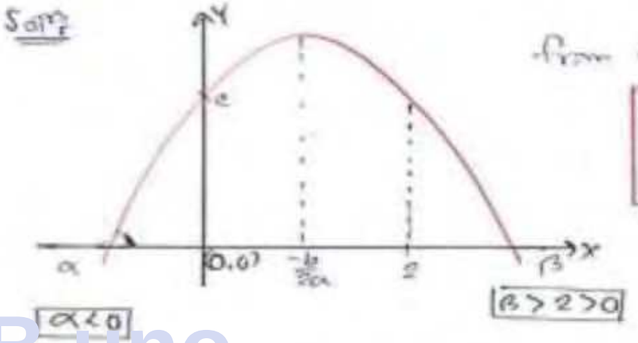
(d) $ab(4a - 2b + c) > 0$



$\Rightarrow f(2) > 0$

$\Rightarrow f(-2) < 0$

Soln:



$\alpha < 0$

$\beta > 2 > 0$

from figure,

$D > 0$
$c > 0$
$a < 0$
$b > 0$

$\because -\frac{b}{2a} > 0$

$\Rightarrow -b < 0$

$\Rightarrow b > 0$

given $|\alpha| < 2$

$\Downarrow$

$-2 < \alpha < 2$.

& $\alpha < 0$

$-2 < \alpha < 0$

- option-A: $a(b^2)(c^3)$ → (−ve, +ve, +ve) $\equiv$ −ve.

∴ $ab^2c^3 < 0$, is correct.

- option-B: ab → (−ve, +ve) $\equiv$ −ve

∴ $ab < 0$, is correct.

- option-C: $bc(4a + 2b + c)$ → (+ve, +ve, $f(2)$ = +ve) $\equiv$ +ve.

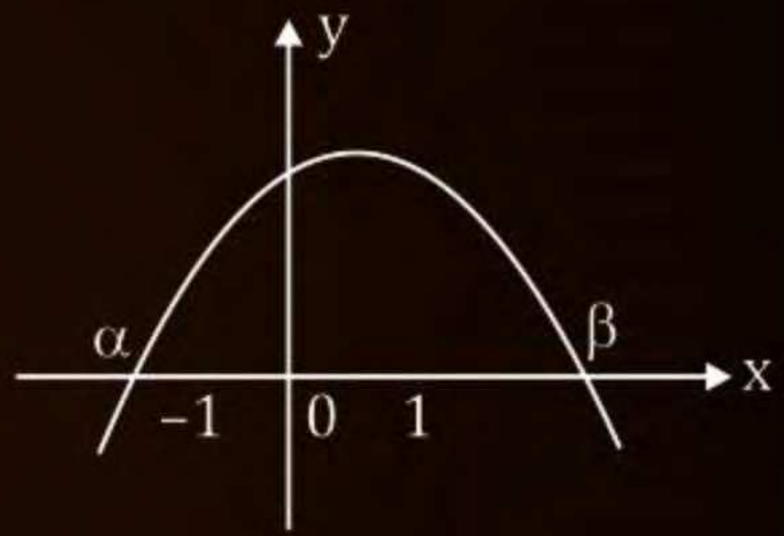
∴ $bc(4a + 2b + c) > 0$, is correct.

- option-D: $ab(4a - 2b + c)$ → (ab = −ve, $f(-2)$ = −ve) $\equiv$ +ve.

∴ $ab(4a - 2b + c) > 0$, is also correct.

QUESTION

The graph of quadratic polynomial $f(x) = ax^2 + bx + c$ is shown in below. Which of the following are correct?



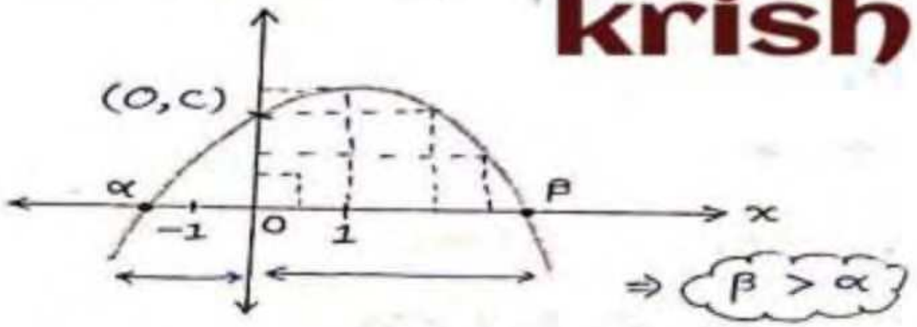
A $\frac{c}{a} < -1$

B $|\beta - \alpha| > 2$

C $f(x) > 0 \; \forall \; x \in (0, \beta)$

D $abc < 0$

Ex-02: The graph of Quadratic polynomial $f(x) = ax^2+bx+c$ is shown in below. which of the following are correct ?



(A) $\frac{c}{a} < -1$.

(B) $|\beta - \alpha| > 2$.

(C) $f(x) > 0 \ \forall \ x \in (0, \beta)$.

(D) $abc < 0$.

krish

$\Rightarrow \beta > \alpha$

$a < 0$.
$b > 0$.
$c > 0$.

$\alpha < -1$
$\beta > 1$.

} from graph.

(A) $\frac{c}{a} < -1$ ($\frac{c}{a}$: +ve / -ve)

(A) $\frac{c}{a} = \alpha\beta \Rightarrow \frac{c}{a} < -1$ ✓
($\alpha\beta$: -ve, +ve; $\alpha < -1$, $\beta > 1$)

(B) $|\beta - \alpha| > 2$ (β: +ve, α: -ve)

Ex: $\Rightarrow \beta > 1, \alpha < -1$

$\Rightarrow |2 - (-2)|$

$\Rightarrow 4 > 2$ ✓

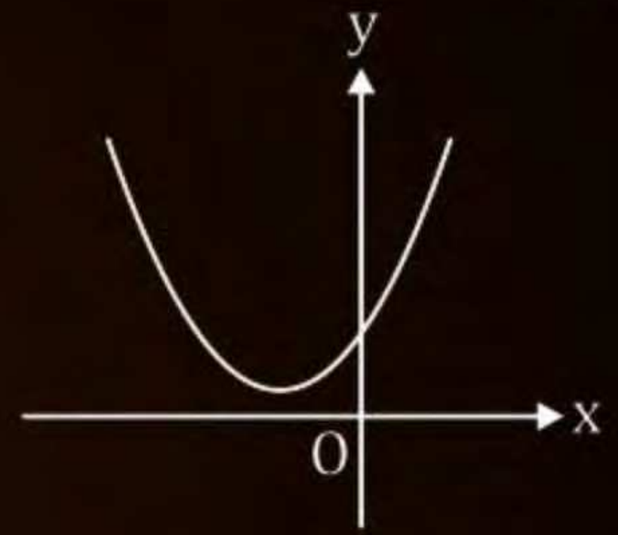
(C) $f(x) > 0 \ \forall \ x \in (0, \beta)$

$\Rightarrow$ yes it is true..

(D) $abc < 0$.
(-ve, +ve, +ve) $\Rightarrow$ -ve

QUESTION

The curve of the quadratic expression $y = ax^2 + bx + c$ is shown in the figure and α, β be the roots of the equation $ax^2 + bx + c = 0$ then correct option is
[D is the discriminant]



A $a > 0,\ b > 0, c > 0, D > 0, \alpha + \beta > 0, \alpha\beta > 0$

B $a > 0, b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta < 0$

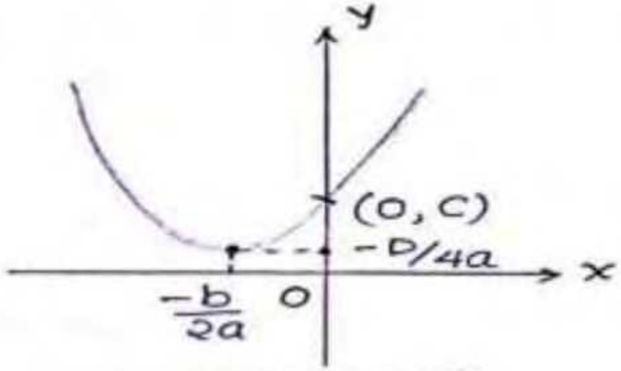
C $a > 0,\ b > 0, c > 0, D < 0, \alpha + \beta < 0, \alpha\beta > 0$

D $a > 0, b < 0, c > 0, D < 0, \alpha + \beta > 0, \alpha\beta > 0$

Ans. C

Jah-03. The curve of the Quadratic expression $y = ax^2 + bx + c$ is shown in figure and α, β be the roots of the equation $ax^2 + bx + c = 0$ then correct option is:

[D is the dicriminant].



(A) $a>0, b>0, c>0, D>0, \alpha+\beta>0, \alpha\beta>0$.

(B) $a>0, b>0, c>0, D<0, \alpha+\beta<0, \alpha\beta<0$.

(C) $a>0, b>0, c>0, D<0, \alpha+\beta<0, \alpha\beta>0$.

(D) $a>0, b<0, c>0, D<0, \alpha+\beta>0, \alpha\beta>0$

$\Rightarrow$ $a>0$, $c>0$, $b>0$.

$\Rightarrow \frac{-D}{4a} > 0$ (+ve)

$\Rightarrow -D > 0$

$\Rightarrow \boxed{D<0}$

$\Rightarrow -\frac{b}{2a} < 0$ (+ve)

$\Rightarrow -b < 0$

$\Rightarrow \boxed{b>0}$.

$\alpha+\beta = \frac{-b}{a}$ (-ve, +ve)

$\Rightarrow \boxed{\alpha+\beta<0}$

$\alpha\beta = \frac{c}{a}$ (+ve, +ve)

$\boxed{\alpha\beta>0}$

$\Rightarrow$ option (C)

QUESTION

Find the set of values of a for which $(a-1)x^2-(a+1)x+a+1>0$ for all $x \in R$.

Tah-04: Find the set of values of a for which $(a-1)x^2 - (a+1)x + a + 1 > 0$, for all $x \in R$.

$\Rightarrow (a-1)x^2 - (a+1)x + a + 1 > 0.$

$\Rightarrow (a-1) > 0$ and $D < 0.$

$\Downarrow$

$\Rightarrow a > 1$ and $(-(a+1))^2 - 4(a-1)(a+1) < 0$

$\Rightarrow a > 1$ and $a^2 + 2a + 1 - 4(a^2 - 1) < 0.$

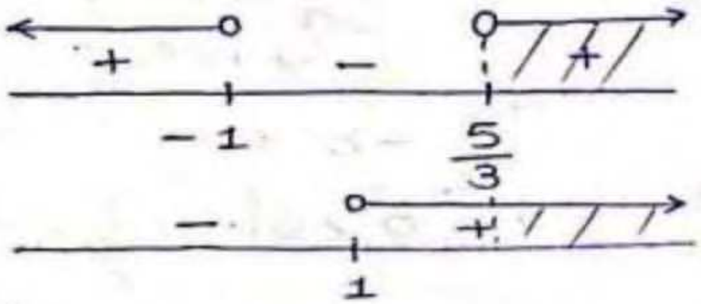
$\Rightarrow a^2 + 2a + 1 - 4a^2 + 4 < 0.$

$\Rightarrow -3a^2 + 2a + 5 < 0$

$\Rightarrow 3a^2 - 2a - 5 > 0$

$\Rightarrow 3a^2 - 5a + 3a - 5 > 0$

$\Rightarrow (a+1)(3a-5) > 0$



$\Rightarrow$

$\Rightarrow \boxed{a \in \left(\frac{5}{3}, \infty\right)}$ Ans.

Q-4(TAH-4): Find the set of values of a for which

$(a-1)x^2 - (a+1)x + a + 1 > 0$ for all $x \in R$,

Sol^n: $(a-1)x^2 - (a+1)x + (a+1) > 0 \ \forall x \in R$,

$\Downarrow$

$D < 0$ & $(a-1) > 0$

TAH 4
BY REED

$(a+1)^2 - 4(a-1)(a+1) < 0$

$\Rightarrow (a+1)[a+1-4a+4] < 0$

$\Rightarrow (a+1)(3a-5) > 0$

$\therefore a \in (-\infty, -1) \cup (\frac{5}{3}, \infty)$

$a > 1$

$\Downarrow$

$a \in (1, \infty)$

'n'

$a \in \frac{5}{3}, \infty$ (Ans.)

QUESTION

Find the set of values of a for which $(a + 4)x^2 - 2ax + 2a - 6 < 0$ for all $x \in R$.

Tah-05: Find the set of value of a for which $(a+4)x^2 - 2ax + 2a - 6 < 0$ for all $x \in R$.

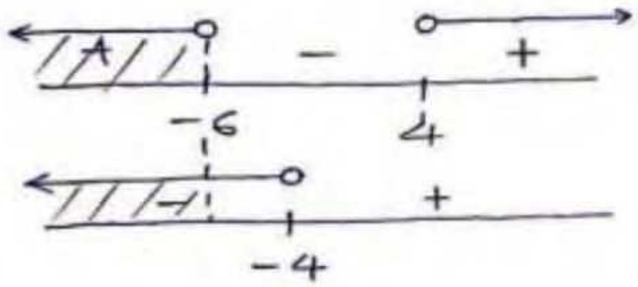
$\Rightarrow (a+4)x^2 - 2ax + 2a - 6 < 0.$

$\Rightarrow (a+4) < 0$ and $D < 0.$

$\Downarrow$

$\Rightarrow a < -4$ and $\Rightarrow (-2a)^2 - 4(a+4)(2a-6) < 0$

$\Rightarrow a < -4$ and $\Rightarrow 4a^2 - 4(2a^2 - 6a + 8a - 24) < 0$



$\Rightarrow 4a^2 - 8a^2 + 24a - 32a + 96 < 0$

$\Rightarrow -4a^2 - 8a + 96 < 0.$

$\Rightarrow 4a^2 + 8a - 96 > 0.$

$\Rightarrow a^2 + 2a - 24 > 0.$

$\Rightarrow (a-4)(a+6) > 0.$

$\Rightarrow \boxed{a \in (-\infty, -6)}$ Ans.

krish

Eu. Find the set of a for which $(a+4)x^2 - 2ax + 2a - 6 < 0 \ \forall x \in R$

$D < 0$

$a + 4 < 0 \ \cap \ D < 0$

$a < -4$

$4a^2 - 4(a+4)(2a-6) < 0$

$4a^2 - (4a+16)(2a-6) < 0$

$4a^2 - (8a^2 - 24a + 32a - 96) < 0$

$4a^2 - 8a^2 - 8a + 96 < 0$

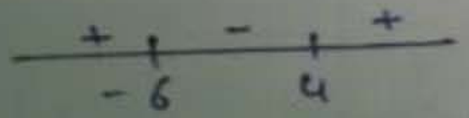
$4a^2 + 8a - 96 > 0$

$a^2 + 2a - 24 > 0$

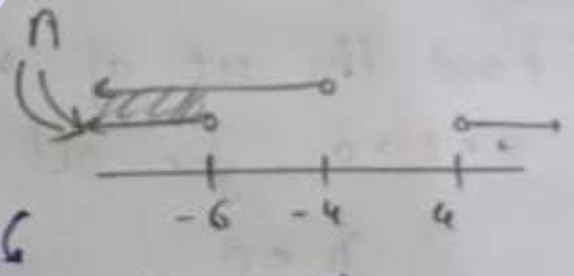
$a^2 + 6a - 4a - 24 > 0$

$a(a+6) - 4(a+6) > 0$

$(a-4)(a+6) > 0$



$a \in (-\infty, -6) \cup (4, \infty)$



$a \in (-\infty, -6)$

QUESTION

For what values of p the vertex of $x^2 + px + 13$ lies at a distance 5 unit from origin.

TDH-06 · For what values of p the vertex of $x^2+px+13$ lies at a distance 5 unit from origin :

$\Rightarrow x^2+px+13=0$. $\Rightarrow$ vertex $= \left(-\frac{b}{2a}, \frac{-D}{4a}\right)$.

$\Rightarrow$ vertex at $\Rightarrow \left(-\frac{p}{2}, -\frac{p^2-52}{4}\right)$.

$\Rightarrow 5 = \sqrt{\left(0+\frac{p}{2}\right)^2 + \left(0+\frac{p^2-52}{4}\right)^2}$.

$\Rightarrow 5 = \sqrt{\frac{p^2}{4} + \frac{(p^2-52)^2}{16}}$.

$\Rightarrow 25 = \frac{p^2}{4} + \frac{p^4-104p^2+2704}{16}$

$\Rightarrow 25 = \frac{4p^2+p^4-104p^2+2704}{16}$.

$\Rightarrow 4p^2+p^4-104p^2+2704-400=0$.

$\Rightarrow p^4-100p^2+2304=0$.

$\Rightarrow p^4-64p^2-36p^2+2304=0$.

$\Rightarrow p^2(p^2-64)-36(p^2-64)=0$.

$\Rightarrow (p^2-64)(p^2-36)$.

$\Rightarrow p^2=64, \quad p^2=36$

$\Rightarrow$ **$p=\pm 8$, $p=\pm 6$** Ans.

—x—

krish

A) Tah-06

vertex $= \left(-\frac{b}{2a}, -\frac{D}{4a}\right) = \left(\frac{-p}{2}, \frac{-(p^2-52)}{4}\right)$

origin $= (0,0)$

According to distance formula:

$$D = \sqrt{\left(0+\frac{p}{2}\right)^2 + \left(0+\frac{p^2-52}{4}\right)^2}$$

$$\Rightarrow 5 = \sqrt{\frac{p^2}{4} + \frac{(p^2-52)^2}{16}} = \sqrt{\frac{4p^2+(p^2-52)^2}{16}}$$

$$\Rightarrow 16\times 25 = 4p^2 + p^4 + (52)^2 - 104p^2$$

$$\Rightarrow p^4 - 100p^2 + (52)^2 - 25\times 16 = 0$$

$$\Rightarrow p^4 - 100p^2 + 4^2\times 13^2 - 4^2\times 25 = 0$$

$$\Rightarrow p^4 - 100p^2 + 4^2(169-25) = 0$$

$$\Rightarrow p^4 - 100p^2 + 4^2\times 144 = 0$$

$$\Rightarrow p^4 - 100p^2 + 2304 = 0$$

$$\Rightarrow p^4 - 64p^2 - 36p^2 + 2304 = 0$$

$$\Rightarrow (p^2-64)(p^2-36) = 0$$

$$\Rightarrow p^2 = 64, 36 \Rightarrow p = \pm\sqrt{64}, \pm\sqrt{36}$$

$$\Rightarrow p = +8, -8, +6, -6$$

Ans: values of p : $+8, -8, +6, -6$

Q-6 (TAH-6): For what values of p the vertex of $x^2+px+13$ lies at a distance 5 unit from origin?

Soln: $\because$ $5 = \sqrt{\left(-\frac{p}{2}-0\right)^2 + \left(\frac{p^2-52}{4}-0\right)^2}$

$$\Rightarrow 25 = \frac{p^2}{4} + \frac{(p^2-52)^2}{16}$$

$$\Rightarrow 25 = \frac{4p^2+p^4-104p^2+52^2}{16}$$

$$\Rightarrow p^4 - 100p^2 + 2704 - 400 = 0$$

$$\Rightarrow p^4 - 100p^2 + 2304 = 0$$

$$\Rightarrow (p^2-64)(p^2+36) = 0$$

$$\Rightarrow p^2 = 64 \text{ OR, } p^2 = 36$$

$$\Rightarrow \boxed{p = \pm 8} \text{ OR, } \boxed{p = \pm 6} \quad \therefore \boxed{p = -8, -6, 6, 8.} \text{ (Ans)}$$

$f(x) = x^2+px+13$

$\downarrow$

$D = p^2 - 52.$

vertex $= \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

$\Rightarrow V = \left(\frac{-p}{2}, \frac{-(p^2-52)}{4}\right)$

$\downarrow$

distance from (0,0) is 5 units,

4 | 2304	
4 | 576	
4 | 144	
4 | 36	
	9

64 = 4 × 4 × 4; 36 = 4 × 9

TAH 6
BY REED