
CLASS 12
ENDGAME MARATHON
COMPLETE
MATHS
CONCEPT WITH
QUESTION

Basic Concepts

If $A = \{d, 0, e\}$, then the number of relations on $A \times A$ are

$2^{n(A)\times n(A)} \Rightarrow 2^{3\times 3} = \boxed{2^9}$

Total no of reflexive relation is $2^{n(n-1)}$

total no of equivalence → Bell number

1
$1 \quad 2$
$2 \quad 3 \quad 5$
$5 \quad 7 \quad 10 \quad 15$

Total no of symmetric relation is $2^{\frac{n(n+1)}{2}}$

Number of functions from A to B $= n^m$

$n(A) = m$ $\quad n(B) = n$

Total no of bijective functions from A to B $= \begin{cases} m!, \text{ if } m = n \\ 0, \text{ if } m \neq n \end{cases}$

one-one
onto

QUESTIONS

0

Let S be the set of all the real number and let R be a relation in S defined by

$R = \{(a, b) : (1 + ab) > 0\}$

Show that R is reflexive and symmetric but not transitive

example

Reflexive

If $(a,a) \in R \ \forall a \in S$

as $1 + a.a$

$\boxed{1+a^2>0}$

$\Rightarrow (a,a) \in R \ \forall a \in S$

Hence it is Reflexive.

Symmetric

If $(a,b) \in R$ then $(b,a) \in R$

let $(a,b) \in R$

$1+ab>0$

as $ab = ba$

$1+ba>0$

$(b,a) \in R$

Hence it is Symmetric

Transitive

If $(a,b) \in R$ and $(b,c) \in R$

$\Rightarrow (a,c) \in R$

$(2,0) \in R$ and $(0,-3) \in R$

$(2,-3) \notin R$

Hence it is not transitive

QUESTIONS

Domain ↗ Co-domain ✓ 3 marks ✓✓ Onto → Range ↓ $\boxed{\text{Co-domain}}$

Let $\boxed{A = \mathbb{R} - \{3\}}$ and $\boxed{B = \mathbb{R} - \{1\}}$.

Consider the function $\boxed{f : A \to B}$ defined by $\boxed{f(x) = \left(\frac{x-2}{x-3}\right)}$.

$\boxed{\text{Is } f \text{ one-one and onto?}}$ Justify your answer. It is one-one + onto = $\boxed{\text{Bijective}}$

(i) $\boxed{\text{one-one}}$

$f(x_1) = f(x_2)$

$$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$$

$(x_1-2)(x_2-3) = (x_1-3)(x_2-2)$

$x_1(x_2-3) - 2(x_2-3) = x_1(x_2-2) - 3(x_2-2)$

$\cancel{x_1x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_1x_2} - 2x_1 - 3x_2 + \cancel{6}$

$-2x_2 + 3x_2 = -2x_1 + 3x_1$

$\boxed{x_2 = x_1}$

$\boxed{\text{onto}}$

$$y = \frac{x-2}{x-3}$$

$y(x-3) = x-2$

$yx - 3y = x - 2$

$yx - x = 3y - 2$

$x(y-1) = 3y - 2$

$$\boxed{x = \frac{3y-2}{y-1}}$$

$\boxed{y \in \mathbb{R} - \{1\}}$

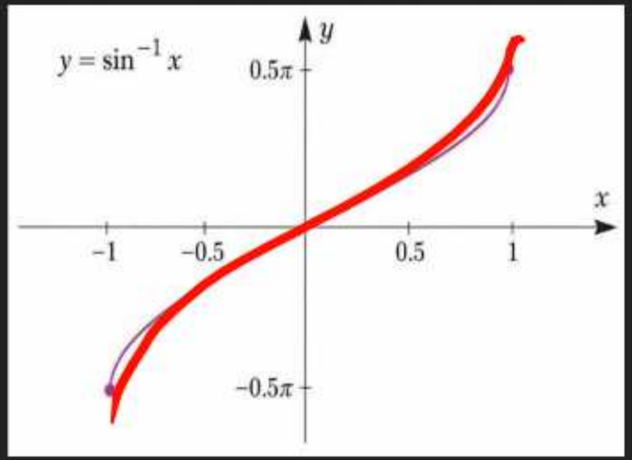
$\boxed{\text{Rang} = \mathbb{R} - \{1\}}$

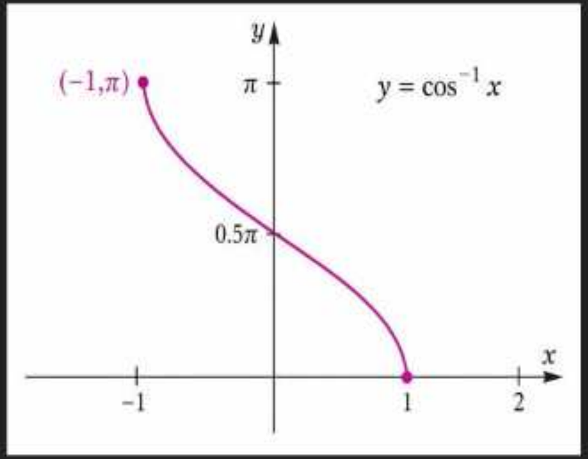
Co-domain $= \mathbb{R} - \{1\}$

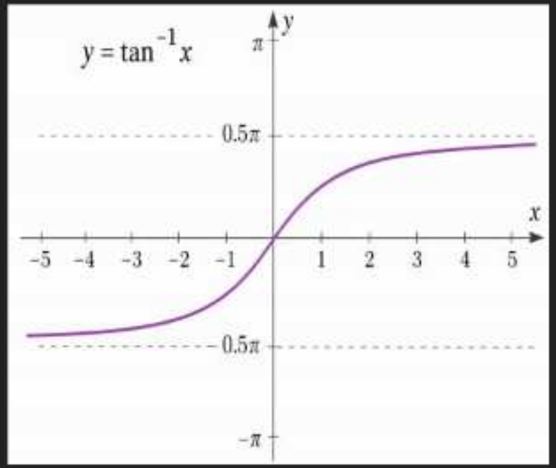
$\therefore$ Range = Co-domain

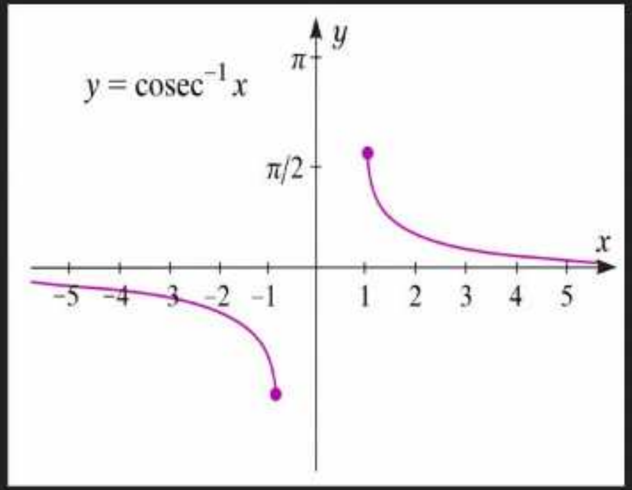
Hence it is onto

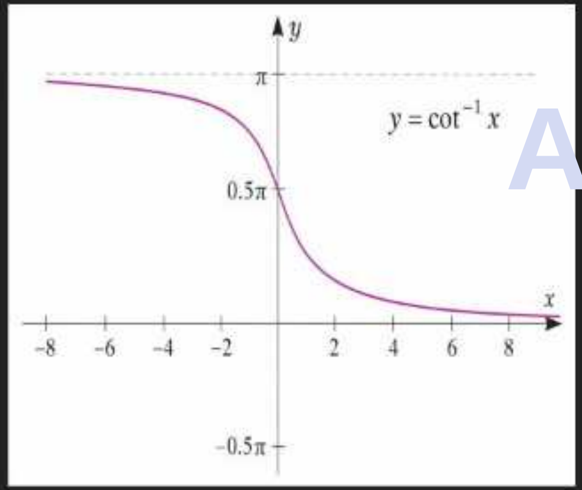
Basic Concepts

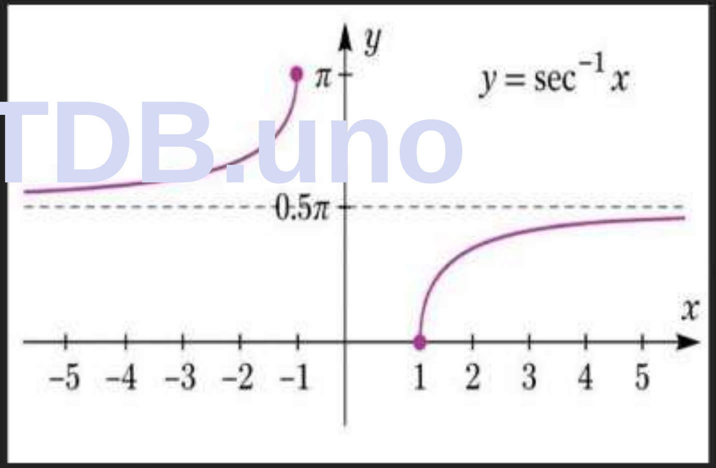












QUESTIONS

Evaluate : $\boxed{\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]}$

$\tan^{-1}\left[2\sin\left(2\cos^{-1}\cos\frac{\pi}{6}\right)\right]$

$\tan^{-1}\left[2\sin\left(\frac{\pi}{3}\right)\right]$

$\tan^{-1}\left[2\times\frac{\sqrt{3}}{2}\right]$

$\tan^{-1}\sqrt{3}$

$\tan^{-1}\tan\frac{\pi}{3} = \boxed{\frac{\pi}{3}}$

QUESTIONS

Find the values of each of the following:

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2}+\cos^{-1}\frac{1-y^2}{1+y^2}\right],\ |x|<1, y>0 \text{ and } xy<1$$

$\boxed{\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}}$ $\boxed{\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}}$

$\boxed{\tan(\theta+\phi)}$

$\therefore \boxed{\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A\tan B}}$

Putting $x = \tan\theta$, $y = \tan\phi$

$$\tan\frac{1}{2}\left[\sin^{-1}\frac{2\tan\theta}{1+\tan^2\theta}+\cos^{-1}\frac{1-\tan^2\phi}{1+\tan^2\phi}\right]$$

$$\tan\frac{1}{2}\left[\cancel{\sin^{-1}}\cancel{\sin}2\theta+\cancel{\cos^{-1}}\cancel{\cos}2\phi\right]$$

$$\tan(\theta+\phi) = \frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi} = \boxed{\frac{x+y}{1-xy}}$$

QUESTIONS

Identify the function shown in the graph

1. $\sin^{-1}(2x)$
2. $\sin^{-1}(x)$
3. $\sin^{-1}\left(\frac{x}{2}\right)$
4. $2\sin^{-1} x$

if $x = \frac{1}{2}$, $y = \frac{\pi}{2}$

$x = 0$, $y = 0$

$x = -\frac{1}{2}$, $y = -\frac{\pi}{2}$

$y = \sin^{-1}(2x)$

$y = \sin^{-1}\left(\cancel{2} \times \frac{1}{\cancel{2}}\right)$

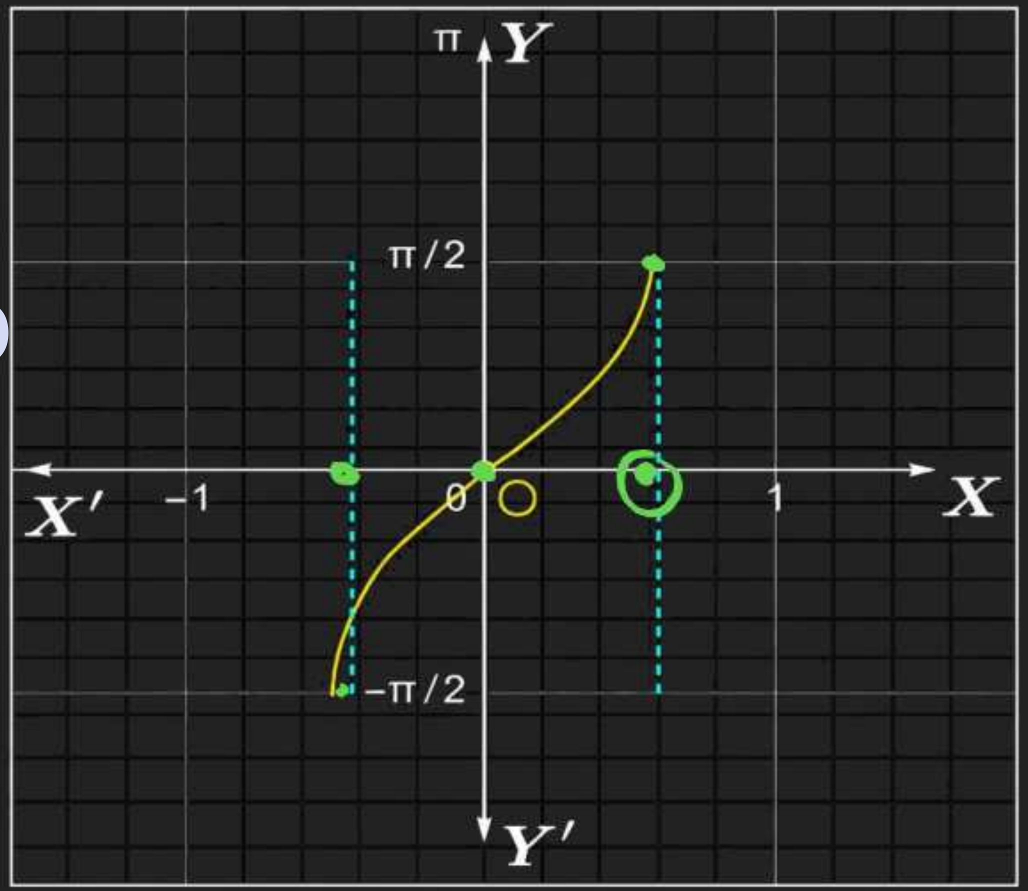
$y = \sin^{-1}\sin\frac{\pi}{2}$

$y = \frac{\pi}{2}$

(ii)

$y = \sin^{-1}\sin 0$

$y = 0$



QUESTIONS

Find the domain of $\cos^{-1}(3x-2)$

$x \in$

$\cos^{-1}(x)$

$\boxed{-1 \le x \le 1}$

$\boxed{-1 \le 3x-2 \le 1}$

$-1+2 \le 3x-\cancel{2}+\cancel{2} \le 1+2$

$1 \le 3x \le 3$

$\frac{1}{3} \le \frac{3x}{3} \le \frac{3}{3}$

$\boxed{\frac{1}{3} \le x \le 1}$

$\boxed{x \in \left[\frac{1}{3}, 1\right]}$

QUESTIONS

The domain of the function $y = \sin^{-1}(-x^2)$ is

f^{-1} imp

$\sin^{-1}x$

$\boxed{-1 \le x \le 1}$

1. $[0, 1]$
2. $(0, 1)$
3. $\boxed{[-1, 1]}$
4. ϕ

$\boxed{-1 \le -x^2 \le 1}$

$1 \ge x^2 \ge -1$

$\boxed{-1 \le x^2 \le 1}$

$\boxed{0 \le x^2 \le 1}$

$\boxed{x \in [-1, 1]}$

Case 1

$\boxed{x^2 \ge 0}$

Case 2

$x^2 \le 1$

$x^2 - 1 \le 0$

$x^2 - 1^2 \le 0$

$(x+1)(x-1) \le 0$

+ +

-1 1

$x \in [-1, 1]$

Basic Concepts

If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then $\left(\frac{24}{x} + \frac{24}{y}\right)$ is : 18

If $A = \begin{bmatrix} 1 & 12 & 4y \\ 6x & 5 & 2x \\ 8x & 4 & 6 \end{bmatrix}$ is a symmetric matrix, then $(2x+y)$ is $= 8$

$6x = 12$
$x = 2$
$8x = 4y$
$16 = 4y$
$y = 4$

Total number of possible matrices of order 3×3 with each entry 2 or 0 is $= 2^9 = 512$

If A is a square matrix such that $A^2 = A$, then $(A + I)^2 - 3A$ is equal to

$A^2 + I^2 + 2AI - 3A = \cancel{A} + I + 2\cancel{A} - 3\cancel{A} = I$

If $A = \begin{pmatrix} 2y-7 & 0 & 0 \\ 0 & x-3 & 0 \\ 0 & 0 & 7 \end{pmatrix}$ scalar matrix, then $(x+y)$ 17

$x - 3 = 7$ | $2y - 7 = 7$
$x = 10$ | $2y = 14$
$y = 7$

If $A = \begin{pmatrix} 0 & x^2+6 & 1 \\ -5x & x^2-9 & 7 \\ -1 & -7 & 0 \end{pmatrix}$ skew-symmetric matrix, then x equals

$x^2 - 9 = 0$
$x^2 = 9$
$\boxed{x = \pm 3}$

QUESTIONS

If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ and $A + A^T = I$, then find α

$$A + A^T = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2\cos\alpha = 1$$

$$\cos\alpha = \frac{1}{2}$$

$$\boxed{\cos\alpha = \cos\frac{\pi}{3}}$$

$$\alpha = \frac{\pi}{3}$$

QUESTIONS

If $A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$, then the value of $I - A + A^2 - A^3 + \cdots$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4-4 & 2-2 \\ -8+8 & -4+4 \end{bmatrix}$$

$$\boxed{A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$$

$$A^3 = A \cdot A^2 = A \cdot 0 = 0$$

$$I - A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & -2 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}}$$

QUESTIONS

Correct

If $A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}$, show that $A^T A^{-1} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$.

$$\boxed{A^T = \begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}}$$

$$\boxed{A^{-1} = \frac{\text{adj}A}{|A|}}$$

$$|A| = \begin{vmatrix} 1 & \tan x \\ -\tan x & 1 \end{vmatrix}$$

$$\boxed{|A| = 1+\tan^2 x}$$

adj A

$A_{11} = 1$

$A_{12} = -(-\tan x) = \tan x$

$A_{21} = -(\tan x) = -\tan x$

$A_{22} = 1$

$\text{adj}A = \begin{bmatrix} 1 & \tan x \\ -\tan x & 1 \end{bmatrix}^T = \boxed{\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}}$

$$A^{-1} = \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$A^T A^{-1} = \frac{1}{1+\tan^2 x}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan x \\ \tan x & 1 \end{bmatrix}$$

$$\frac{1}{1+\tan^2 x}\begin{bmatrix} 1-\tan^2 x & -\tan x-\tan x \\ \tan x+\tan x & 1-\tan^2 x \end{bmatrix}$$

$$\begin{bmatrix} \frac{1-\tan^2 x}{1+\tan^2 x} & \frac{-2\tan x}{1+\tan^2 x} \\ \frac{2\tan x}{1+\tan^2 x} & \frac{1-\tan^2 x}{1+\tan^2 x} \end{bmatrix} = \begin{bmatrix} \cos 2x & -\sin 2x \\ \sin 2x & \cos 2x \end{bmatrix}$$

H.P

QUESTIONS

Find the values of x and y, if $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ satisfies the equation $A^2 + xA + yI = 0$.

$A^2 + xA + yI = 0$

$A^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(A)^2 = \begin{bmatrix} 1+1 & 1+1 \\ 1+1 & 1+1 \end{bmatrix}$

$\boxed{A^2 = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}}$

$\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} x & x \\ x & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$\begin{bmatrix} 2+x+y & 2+x+0 \\ 2+x & 2+x+y \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$2+(-2)+y=0$

$y=0$

$x+2=0$

$x=-2$

$\boxed{\begin{matrix} x=-2 \\ y=0 \end{matrix}}$

QUESTIONS

Home work

Let $\boxed{A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}}$, $\boxed{B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}}$, $\boxed{C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}}$. Find $\boxed{\text{a matrix } D}$ such that $\boxed{CD - AB = O}$.

order

$$D = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$CD - AB = O$$

$$\begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix} = O$$

$$\begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} = \begin{bmatrix} 10-7 & 4-4 \\ 15+28 & 6+16 \end{bmatrix}$$

$$\begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$$

QUESTIONS

If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x)F(y) = F(x+y)$.

QUESTIONS

If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2,

Show that $I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Basic Concepts

If $\boxed{\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}}$, then $\boxed{x}$

$x^2-36=36-36$

$x^2-36=0$

$x^2=36$

$x=\pm 6$

$\boxed{|A|\neq 0}$

$\lambda-2\neq 0$

$\lambda\neq 2$

$R-\{2\}$

If the points $\boxed{(3,-2),(x,2),(8,8)}$ are collinear, then $\boxed{x=}$

comment

If $\boxed{A=\begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}}$, then $\boxed{A^{-1} \text{ exists if}}$

$3\begin{vmatrix} 2 & 1 \\ 8 & 1 \end{vmatrix} + 2\begin{vmatrix} x & 1 \\ 8 & 1 \end{vmatrix} + 1\begin{vmatrix} x & 2 \\ 8 & 8 \end{vmatrix} = 0$

$3(2-8)+2(x-8)+1(8x-16)=0$

$3(-6)+2x-16+8x-16=0$

$-18+\underbrace{2x+8x}-32=0$

$10x-50=0$

$10x=50$

$x=5$

$\frac{1}{2}\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$

$\begin{vmatrix} 3 & -2 & 1 \\ x & 2 & 1 \\ 8 & 8 & 1 \end{vmatrix} = 0$

$\boxed{\text{Area}=0}$

QUESTIONS

V.V.Imp

If A is a square matrix of order 3, such that $\boxed{|A| = 5}$, then find the value of

(a) $|3A|$ (b) $|-2A^T|$ (c) $|4A^{-1}|$

(d) $|\mathrm{Adj}A|$ (e) $A.\,\mathrm{Adj}A$ (f) $|A \cdot \mathrm{Adj}A|$

(g) $|A^3|$

a) $|3A| = 3^3|A| = 27 \times 5 = \boxed{135}$

b) $|-2A^T| = (-2)^3|A^T| = -8 \times |A| = -8 \times 5 = -40$

c) $|4A^{-1}| = 4^3|A^{-1}| = 4^3 \times \frac{1}{|A|} = \frac{64}{5}$

d) $|\mathrm{Adj}A| = |A|^2 = 25$

$\hookrightarrow |A|^{n-1}$

e) $A.\,\mathrm{Adj}A = |A|I = 5\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} = \begin{bmatrix}5&0&0\\0&5&0\\0&0&5\end{bmatrix}$

f) $|A \cdot \mathrm{Adj}A| = |A|^n$

$= 5^3$

$\Rightarrow \boxed{125}$

QUESTIONS

Let $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \le \theta \le 2\pi$. Then

1. $\operatorname{Det}(A) = 0$
2. $\operatorname{Det}(A) \in (2, \infty)$
3. $\operatorname{Det}(A) \in (2,4)$
4. $\operatorname{Det}(A) \in [2,4]$

$$1\begin{vmatrix} 1 & \sin\theta \\ -\sin\theta & 1 \end{vmatrix} - \sin\theta\begin{vmatrix} -\sin\theta & \sin\theta \\ -1 & 1 \end{vmatrix} + 1\begin{vmatrix} -\sin\theta & 1 \\ -1 & -\sin\theta \end{vmatrix}$$

$$1(1+\sin^2\theta) - \sin\theta(-\sin\theta+\sin\theta) + 1(\sin^2\theta+1)$$

$$1+\sin^2\theta+\sin^2\theta+1$$

$$\boxed{2\sin^2\theta+2}$$

$$\boxed{-1 \le \sin\theta \le 1}$$

$$0 \le \sin^2\theta \le 1$$

$$0 \le 2\sin^2\theta \le 2$$

$$0+2 \le |A| \le 2+2$$

$$\boxed{2 \le |A| \le 4}$$

QUESTIONS

H.W

✓✓ 5 Marks

For the matrix $\boxed{A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}}$. Show that $\boxed{A^3 - 6A^2 + 9A - 4I = 0}$.

Hence, find $\boxed{A^{-1}}$.

$\boxed{A^{-1}}$

$\star\ A^{-1}A = I$

$\boxed{A^3 - 6A^2 + 9A - 4I = 0}$ Premultiply

$\boxed{A^{-1}A^3 - 6A^{-1}A^2 + 9A^{-1}A - 4A^{-1}I}$

$A^2 - 6A + 9I - 4A^{-1} = 0$

$A^2 - 6A + 9I = 4A^{-1}$

$\boxed{\frac{1}{4}[A^2 - 6A + 9I] = A^{-1}}$

QUESTIONS

Solve the system of equations

① Simple
②
③ word problem 2024 | 2025
(case-Based)

Practice

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4$$

$$\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1$$

$$\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

$\frac{1}{x}=a, \frac{1}{y}=b, \frac{1}{z}=c$

$$\boxed{\begin{aligned}2a+3b+10c&=4\\4a-6b+5c&=1\\6a+9b-20c&=2\end{aligned}}$$

$a=\frac{1}{2}$

$\frac{1}{x}=\frac{1}{2}$

$x=2$

$A=\begin{bmatrix}2&3&10\\4&-6&5\\6&9&-20\end{bmatrix}, B=\begin{bmatrix}4\\1\\2\end{bmatrix}, X=\begin{bmatrix}a\\b\\c\end{bmatrix}$

$\boxed{X=A^{-1}B}$

$\boxed{A^{-1}=\frac{\text{adj}\,A}{|A|}}$

QUESTIONS

If $\boxed{\cos 2\theta = 0}$, then $\boxed{\begin{vmatrix} 0 & \cos\theta & \sin\theta \\ \cos\theta & \sin\theta & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}^2 =}$

$\cos 2\theta = 0$

$\cos 2\theta = \cos\frac{\pi}{2}$

$2\theta = \frac{\pi}{2}$

$\boxed{\theta = \frac{\pi}{4}}$

$\sqrt{2}\times\sqrt{2}\times\sqrt{2}$

$\left(\frac{1}{\sqrt{2}}\right)^3 + \left(\frac{1}{\sqrt{2}}\right)^3$

$\frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\boxed{\text{expanding along } R_1}$

$-\cos\theta\begin{vmatrix} \cos\theta & 0 \\ \sin\theta & \cos\theta \end{vmatrix} + \sin\theta\begin{vmatrix} \cos\theta & \sin\theta \\ \sin\theta & 0 \end{vmatrix}$

$-\cos\theta\left[\cos^2\theta\right] + \sin\theta\left[0 - \sin^2\theta\right]$

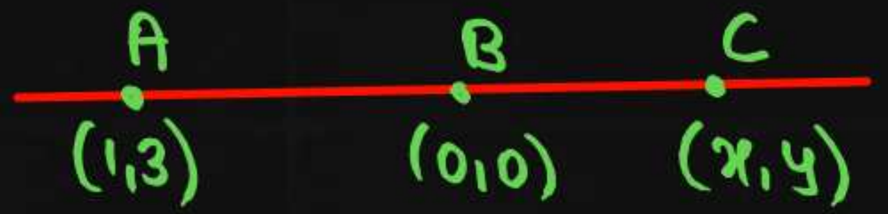
$-\cos^3\theta - \sin^3\theta$

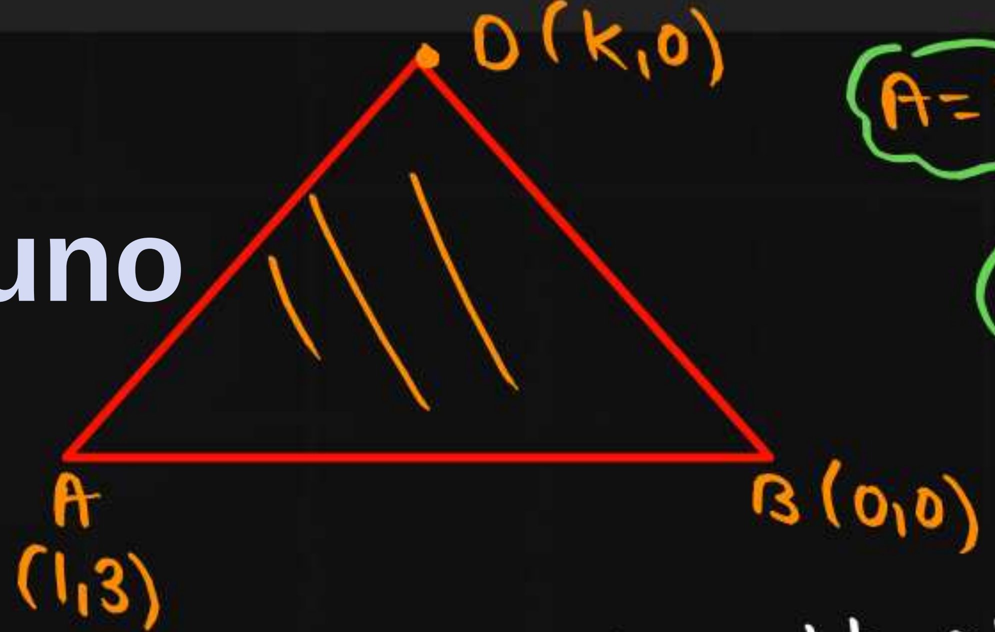
$\boxed{-\left[\cos^3\theta + \sin^3\theta\right]} = -\left[\cos^3\frac{\pi}{4} + \sin^3\frac{\pi}{4}\right] = \boxed{-\frac{1}{\sqrt{2}}}$

$\left(-\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$

QUESTIONS

Find the equation of the line joining A(1, 3) and B(0, 0) using determinants and find k, if D(k, 0) is a point such that area of triangle ABD is 3 squre units.





$A = 3$

$A = \pm 3$

equation

$A=0$

$$\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ x & y & 1 \end{vmatrix} = 0$$

$$-1\begin{vmatrix} 1 & 3 \\ x & y \end{vmatrix} = 0$$

$$-1(y-3x) = 0$$

$$\boxed{y = 3x}$$

$$y - 3x = 0$$

$$\pm 3 = \frac{1}{2}\begin{vmatrix} k & 0 & 1 \\ 1 & 3 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$\pm 6 = 1\begin{vmatrix} k & 0 \\ 1 & 3 \end{vmatrix}$$

$$\boxed{3k = \pm 6}$$

$3k = 6$, $k = 2$ | $3k = -6$, $k = -2$

Basic Concepts

$\boxed{\text{at } x = a}$

Continuous

$\boxed{LHL = RHL = f(a)}$

$$\boxed{\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)}$$

$\boxed{\text{Differentiable}}$

LHD = RHD

$$\boxed{\lim_{h \to 0} \frac{f(x-h) - f(x)}{-h}}$$

RHD

$$\boxed{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Every Differentiable function is a continuous.

But Converse not need to be true

QUESTIONS

v.v.imp 3-5

Show that the function $\boxed{f(x) = |x+1| + |x-1|}$ for all $x \in R$, is not differentiable at $\boxed{x = -1 \text{ and } x = 1.}$

$-2 \quad 0 \quad 0 \neq 2$

$f(x) = 2$

$-2x \qquad 2 \qquad 2x$

$f(x) = -2x$

$-1 \qquad 1$

at $x = -1$

$F(-1-h) = -2(-1-h)$
$= 2+2h$

$f(x) = x + \cancel{1} + x - \cancel{1}$

$\boxed{f(x) = 2x}$

LHD

$$\boxed{\lim_{h\to 0} \frac{f(x-h) - f(x)}{-h}}$$

$$\lim_{h\to 0} \frac{f(-1-h) - f(-1)}{-h}$$

$$\lim_{h\to 0} \frac{\cancel{2} + 2h - \cancel{2}}{-h} = \boxed{-2}$$

RHD

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h}$$

$$\lim_{h\to 0} \frac{2-2}{h} = \boxed{0}$$

if $x < -1$ $\quad f(x) = -(x+1) - (x-1)$
$= -x - 1 - x + 1$
$= \boxed{-2x}$

as LHD ≠ RHD
Hence
It is not
Differentiable

if $\boxed{-1 < x < 1}$

$f(x) = x + 1 - (x-1)$
$= \cancel{x} + 1 - \cancel{x} + 1$
2

QUESTIONS

If $y = \left(x + \sqrt{x^2 - 1}\right)^2$, then show that $(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2$.

$\sqrt{y} = x + \sqrt{x^2 - 1}$

d.w.r.to x

$\frac{1}{2\sqrt{y}} \times \frac{dy}{dx} = \frac{1}{1} + \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2 - 1}}$

$\frac{dy}{dx} \times \frac{1}{2\sqrt{y}} = \frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}$

$\boxed{\frac{dy}{dx} \times \frac{1}{2\sqrt{y}} = \frac{\sqrt{y}}{\sqrt{x^2 - 1}}}$

$\sqrt{x^2 - 1}\,\frac{dy}{dx} = 2y$

Squaring both side

$\boxed{(x^2 - 1)\left(\frac{dy}{dx}\right)^2 = 4y^2}$

HP

QUESTIONS

If $\boxed{y = cosec\,(cot^{-1}x)}$, then prove that $\boxed{\sqrt{1+x^2}\,\frac{dy}{dx} - x = 0}$

$a = \cot^{-1}x$

$\cot a = \frac{x}{1} = \frac{b}{p}$

$\boxed{h = \sqrt{1+x^2}}$

$\operatorname{cosec} a = \sqrt{1+x^2}$

$\boxed{a = \operatorname{cosec}^{-1}\sqrt{1+x^2}}$

\# $1 + \cot^2 a = \operatorname{cosec}^2 a$

$1 + x^2 = \operatorname{cosec}^2 a$

$y = \cancel{\operatorname{cosec}}(\cancel{\operatorname{cosec}^{-1}}\sqrt{1+x^2})$

$\boxed{y = \sqrt{1+x^2}}$

$\frac{dy}{dx} = \frac{1}{\cancel{2}\sqrt{1+x^2}} \times \cancel{2}x$

$\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$

$\sqrt{1+x^2}\,\frac{dy}{dx} = x$

H.P.

$\boxed{\sqrt{1+x^2}\,\frac{dy}{dx} - x = 0}$

QUESTIONS

3 marks

Find the value of k so that the function f is continuous at the indicated point:

$$\boxed{f(x) = \begin{cases} \dfrac{1-\cos kx}{x \sin x}, & x \neq 0 \\ \dfrac{1}{2} & x = 0 \end{cases}, x = 0}$$

$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$$

$1 - \cos x = 2\sin^2 \frac{x}{2}$

$$\boxed{1 - \cos kx = 2\sin^2 \frac{kx}{2}}$$

$\boxed{\lim_{x \to 0} f(x) = \frac{1}{2}}$

$$\boxed{\lim_{x \to 0} \frac{1-\cos kx}{x \sin x}}$$

$$\lim_{x \to 0} \frac{2\sin^2 \frac{kx}{2}}{x^2 \frac{\sin x}{x}}$$

$$\lim_{x \to 0} \frac{2\sin^2 \frac{kx}{2}}{x^2} \div \lim_{x \to 0} \frac{\sin x}{x}$$

$$\lim_{x \to 0} k^2 \cancel{2} \times \frac{\sin \frac{kx}{2} \times \sin \frac{kx}{2}}{\frac{4}{2} \times \frac{kx}{2} \cdot \frac{kx}{2}}$$

$$\frac{k^2}{\cancel{2}} = \frac{1}{\cancel{2}}$$

$$\boxed{k = \pm 1}$$

QUESTIONS

3 marks   1 marks   #

Is it true that the function $f(x)$ given by $f(x) = \begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & \text{when } x \neq 0 \\ \boxed{0}, & \boxed{\text{when } x = 0} \end{cases}$ is $\boxed{\text{continuous at}}$ $x = 0$.

$x = 0-h$, $x = 0+h$

L'Hospital rule

$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\cos x}{1}$

$\cos(0)$

1

$\lim_{x \to 0} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$

$$\lim_{x \to 0} \frac{\cancel{e^{\frac{1}{x}}}\left[1 - \frac{1}{e^{\frac{1}{x}}}\right]}{\cancel{e^{\frac{1}{x}}}\left[1 + \frac{1}{e^{\frac{1}{x}}}\right]} \Rightarrow \frac{1 - \frac{1}{e^{\frac{1}{0}}}}{1 + \frac{1}{e^{\frac{1}{0}}}} = \frac{1 - \frac{1}{e^{\infty}}}{1 + \frac{1}{e^{\infty}}} = \frac{1 - \left(\frac{1}{\infty}\right)}{1 + \left(\frac{1}{\infty}\right)} = \frac{1-0}{1+0} = \frac{1}{1} = \boxed{1}$$

QUESTIONS

2024 + 2025 → Ncert exemplar

If $\boxed{\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)}$, prove that $\boxed{\dfrac{dy}{dx}=\sqrt{\dfrac{1-y^2}{1-x^2}}}$.

$\boxed{x=\sin\theta,\ y=\sin\phi}$

$\sqrt{1-\sin^2\theta}+\sqrt{1-\sin^2\phi}=a[\sin\theta-\sin\phi]$

$\boxed{\cos\theta+\cos\phi=a(\sin\theta-\sin\phi)}$

$\cancel{2}\cos\cancel{\dfrac{\theta+\phi}{2}}\cos\dfrac{\theta-\phi}{2}=a\times\cancel{2}\cos\cancel{\dfrac{\theta+\phi}{2}}\sin\dfrac{\theta-\phi}{2}$

$$\frac{\cos\frac{\theta-\phi}{2}}{\sin\frac{\theta-\phi}{2}}=a$$

$$\cot\frac{\theta-\phi}{2}=a$$

$$\frac{\theta-\phi}{2}=\cot^{-1}a$$

$$\boxed{\theta-\phi=2\cot^{-1}a}$$

$$\sin^{-1}x-\sin^{-1}y=2\cot^{-1}a$$

$$\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\times\frac{dy}{dx}=0$$

$$\frac{1}{\sqrt{1-x^2}}=\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}}$$

$$\boxed{\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}}$$

H.P.

QUESTIONS

$* \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$

If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$.

$x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$

$\frac{dx}{d\theta} = a\left[-\sin\theta + \frac{1}{\tan\frac{\theta}{2}} \times \sec^2\frac{\theta}{2} \times \frac{1}{2}\right]$

$\frac{dx}{d\theta} = a\left[-\sin\theta + \frac{\cancel{\cos\frac{\theta}{2}}}{\sin\frac{\theta}{2}} \times \frac{1}{\cos^2\frac{\theta}{2}} \times \frac{1}{2}\right]$

$\frac{dx}{d\theta} = a\left[-\sin\theta + \frac{1}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]$

$\frac{dx}{d\theta} = a\left[-\frac{\sin\theta}{1} + \frac{1}{\sin\theta}\right]$

$\# \frac{dx}{d\theta} = a\left[\frac{-\sin^2\theta + 1}{\sin\theta}\right]$

$\boxed{\frac{dx}{d\theta} = a\frac{\cos^2\theta}{\sin\theta}}$

$\# \boxed{\frac{d\theta}{dx} = \frac{\sin\theta}{a\cos^2\theta}}$

$\# y = \sin\theta$

$\boxed{\frac{dy}{d\theta} = \cos\theta}$

$\# \frac{dy}{dx} = \frac{\cancel{\cos\theta} \times \sin\theta}{a\cos^{\cancel{2}}\theta}$

$\# \boxed{\frac{dy}{dx} = \frac{1}{a}\tan\theta}$

$\# \frac{d^2y}{dx^2} = \frac{1}{a} \times \sec^2\theta \times \frac{d\theta}{dx}$

$\frac{d^2y}{dx^2} = \frac{1}{a\cos^2\theta} \times \frac{\sin\theta}{a\cos^2\theta}$

$\boxed{\frac{d^2y}{dx^2} = \frac{\sin\theta}{a^2\cos^4\theta}}$

$\left.\frac{d^2y}{dx^2}\right]_{\theta=\frac{\pi}{4}} \Rightarrow$ Comment

Basic Concepts

QUESTIONS

A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y-coordinate is changing 8 times as fast as the x-coordinate.

$6y = x^3 + 2$

$\boxed{\frac{dy}{dx} = 8}$

$(4, 11), \left(-4, -\frac{31}{3}\right)$

$6\frac{dy}{dx} = 3x^2$

$6 \times 8 = 3x^2$

$3x^2 = 48$

$x^2 = \frac{48}{3}$

$x^2 = 16$

$\boxed{x = \pm 4}$

if $\boxed{x = 4}$

$6y = 4^3 + 2$

$6y = 64 + 2$

$6y = 66$

$y = \frac{66}{6}$

$\boxed{y = 11}$

if $x = -4$

$6y = (-4)^3 + 2$

$6y = -64 + 2$

$6y = -62$

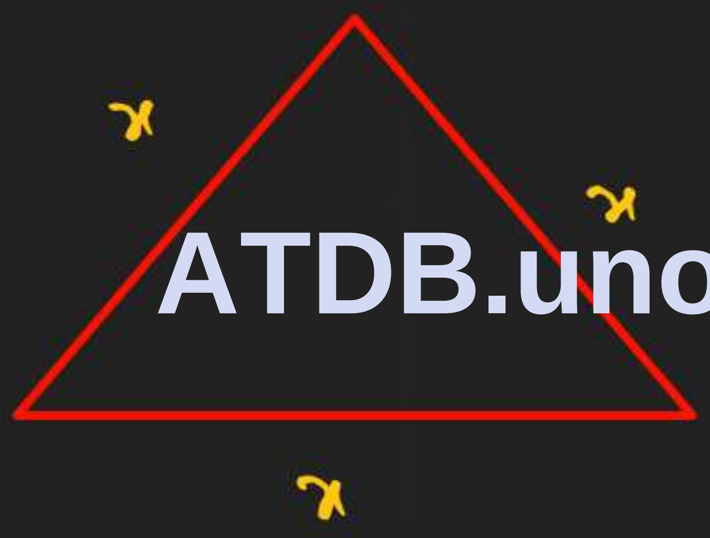
$y = -\frac{62}{6}$

$\boxed{y = -\frac{31}{3}}$

QUESTIONS

The sides of an equilateral triangle are increasing at the rate of 2 cm/sec. The rate at which its area increases, when its side is 10 cm is :

1. $10\ \text{cm}^2/\text{sec}$
2. $10\sqrt{3}\ \text{cm}^2/\text{sec}$
3. $\frac{10}{3}\ \text{cm}^2/\text{sec}$
4. $\sqrt{3}\ \text{cm}^2/\text{sec}$



$$\boxed{\frac{dx}{dt} = \frac{2\,cm}{sec}}$$

$$A = \frac{\sqrt{3}}{4}x^2$$

$$\frac{dA}{dt} = \frac{\sqrt{3}}{\cancel{4}} \times \cancel{2}x \times \frac{dx}{dt}$$

$$= \frac{\sqrt{3}}{\cancel{2}} x \times \cancel{2}$$

$$\frac{dA}{dt} = \sqrt{3}x$$

$$\left.\frac{dA}{dt}\right]_{x=10} = 10\sqrt{3}\,\frac{cm^2}{s}$$

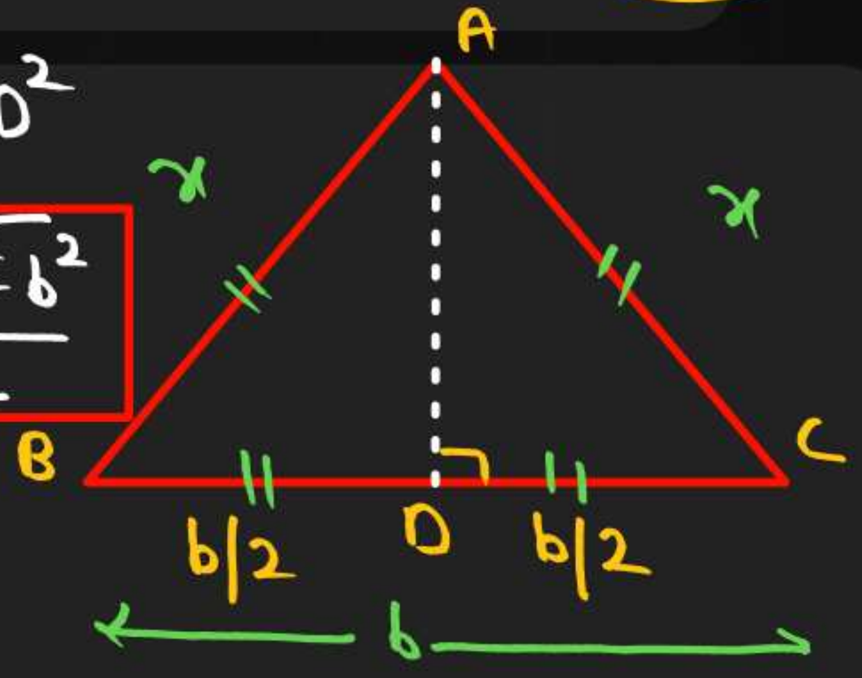
QUESTIONS

3 marks

The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3 cm per second. How fast is the area decreasing when the two equal sides are equal to the base ?

1. $\sqrt{3}b\ \text{cm}^2/\text{sec}$
2. $\sqrt{2}b\ \text{cm}^2/\text{sec}$
3. $2.5b\ \text{cm}^2/\text{sec}$
4. *none of these*

$x = b$



$x^2 = AD^2 + CD^2$

$x^2 = AD^2 + \frac{b^2}{4}$

$x^2 - \frac{b^2}{4} = AD^2$

$\frac{4x^2 - b^2}{4} = AD^2$

$\boxed{AD = \frac{\sqrt{4x^2 - b^2}}{2}}$

$A = \frac{1}{2} \times b \times h$

$\boxed{A = \frac{b}{2} \times \frac{1}{2}\sqrt{4x^2 - b^2}}$

$\frac{dA}{dt} = \frac{b}{\cancel{4}} \times \frac{1}{\cancel{2}\sqrt{4x^2 - b^2}} \times \cancel{4} \times \cancel{2}x \times \frac{dx}{dt}$

$\left.\frac{dA}{dt}\right\}_{x=b} = \frac{b \times b}{\sqrt{3b^2}} \times (-3) = \frac{b^2 \times -\cancel{3}\sqrt{3}}{\cancel{\sqrt{3}}b}$

$= \boxed{-\sqrt{3}b\ \frac{cm^2}{s}}$

QUESTIONS

For what values of a the function f given by $f(x) = x^2 + ax + 1$ is increasing on $[1, 2]$?

$x = 1, x = 2$

$\boxed{f(x) = x^2 + ax + 1}$

for increasing

$\boxed{f'(x) \geq 0}$

$\boxed{f'(x) = 2x + a}$

$2x + a \geq 0$

$\boxed{a \geq -2x}$

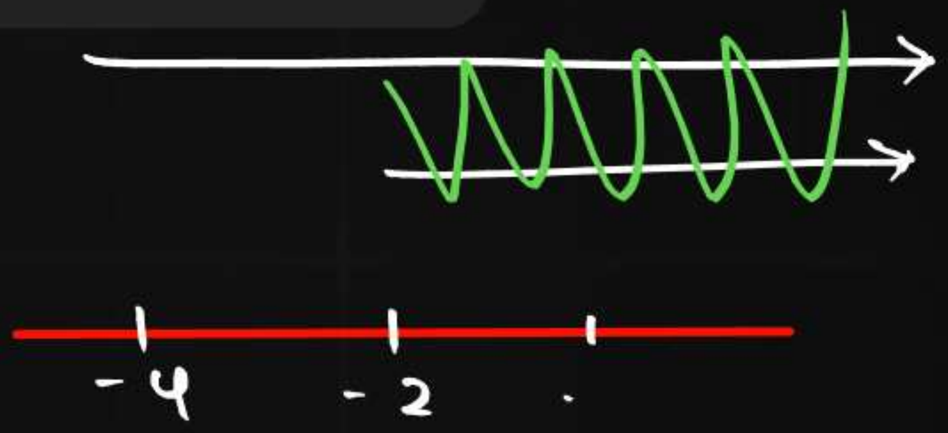
if $x = 1$

$\boxed{a \geq -2}$

if $x = 2$

$\boxed{a \geq -4}$

$\boxed{a \geq -2}$



QUESTIONS

pyq's

5 marks

Imp

It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at $x = 1$. Find the value of 'a', hence obtain all other points where the given function $f(x)$ attains local maximum or local minimum values.

$$\boxed{f(x) = x^4 - 62x^2 + ax + 9}$$

$$\boxed{f'(x) = 4x^3 - 124x + a}$$

$f'(1) = 0$

$4(1)^3 - 124(1) + a = 0$

$4 - 124 + a = 0$

$-120 + a = 0$

$$\boxed{a = 120}$$

$$\boxed{f'(x) = 4x^3 - 124x + 120}$$

$f'(x) = 4[x^3 - 31x + 30]$

$f'(x) = 4(x-1)(x^2 + x - 30)$

$f'(x) = 4(x-1)(x^2 + 6x - 5x - 30)$

$f'(x) = 4(x-1)(x(x+6) - 5(x+6))$

$= \boxed{4(x-1)(x+6)(x-5)}$

$+ \quad + \quad +$

$$\boxed{x = 1, 5, -6}$$

$- \quad + \quad - \quad +$

-6: point of local minima

1: point of local maxima

5: point of local minima.

QUESTIONS

Cond ✓ 5 Marks

Find both the maximum value and the minimum value of

$3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$.

$$\boxed{f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25}$$

$$f'(x) = 12x^3 - 24x^2 + 24x - 48$$

$$= \boxed{12[x^3 - 2x^2 + 2x - 4]}$$

$$= 12[x^2(x-2) + 2(x-2)]$$

$$= 12(x-2)[x^2+2]$$

$$f'(x) = 0$$

$$12(x-2)(x^2+2) = 0$$

$x = 2$ $\qquad x^2 = -2$ ✗

$$\boxed{x = 0, x = 2, x = 3}$$

$f(0) = \sim 25$

$f(2) = \sim$

$f(3) = \sim$ absolute mini

QUESTIONS

Find intervals in which the function given by

$$f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$$

is (a) increasing (b) decreasing.

QUESTIONS

An Apache helicopter of enemy is flying along the curve given by $y = x^2 + 7$. A soldier, placed at (3, 7), wants to shoot down the helicopter when it is nearest to him. Find the nearest distance. — Function

$$AB^2 = (x-3)^2 + (x^2+\cancel{7}-\cancel{7})^2$$

$$\boxed{AB^2 = x^2 + 9 - 6x + x^4}$$

$$\boxed{P = x^4 + x^2 - 6x + 9}$$

$$P' = 4x^3 + 2x - 6$$

$$P' = (x-1)(\underbrace{4x^2+4x+6}_{\times})$$

$$x - 1 = 0$$

$$\boxed{x = 1}$$

$$P'' = 12x^2 + 2 = 12 + 2 = 14 > 0$$

point of minima.

$$AB^2 = (1-3)^2 + (1)^4$$

$$\Rightarrow 4 + 1$$

$$\boxed{AB = \sqrt{5}} \text{ unit}$$

$$4x^2 + 4x + 6$$

$$x-1)\,4x^3 + 2x - 6\,($$

$$4x^3 - 4x^2$$

$$- \quad +$$

$$4x^2 + 2x - 6$$

$$4x^2 - 4x$$

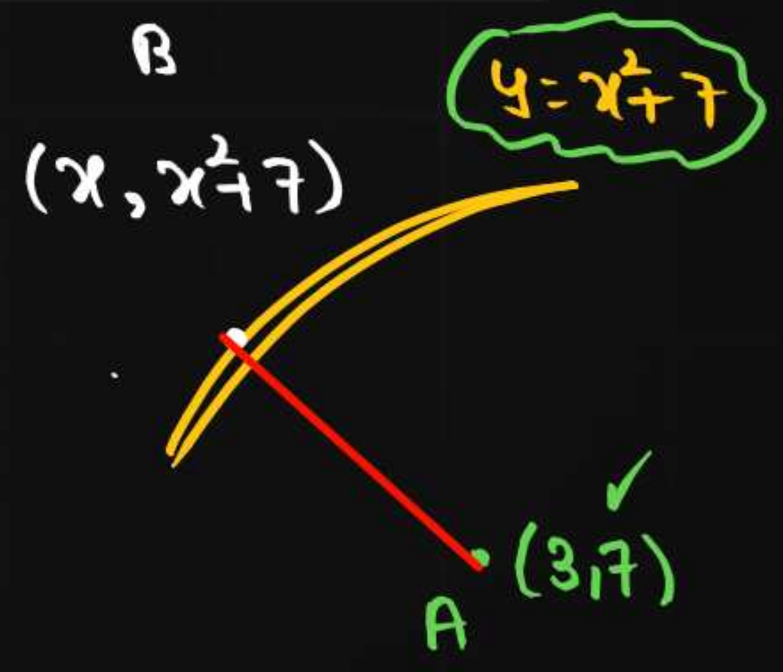
$$- \quad +$$

$$6x - 6$$

$$6x - 6$$

$$- \quad +$$

$$0$$



QUESTIONS

r.v.imp

$\pi r^2 = \frac{1}{2}\pi r^2$ $\quad 2\pi r$

A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

Area → function

$P = 10m$

$x + y + \frac{\pi x}{2} + y = 10$

$2y = 10 - x - \frac{\pi x}{2}$

$$\boxed{y = 5 - \frac{x}{2} - \frac{\pi x}{4}}$$

$y = \frac{10}{\pi+4}$

$A = \frac{1}{2}\pi \times \frac{x^2}{4} + x\left[5 - \frac{x}{2} - \frac{\pi x}{4}\right]$

$$\boxed{A = \frac{\pi x^2}{8} + 5x - \frac{x^2}{2} - \frac{\pi x^2}{4}}$$

$\frac{dA}{dx} = \frac{2\pi x}{8} + 5 - \frac{2x}{2} - \frac{2\pi x}{4}$

$\frac{\pi x}{4} + 5 - x - \frac{\pi x}{2} = 0$

$-\frac{\pi x}{4} - x + 5 = 0$

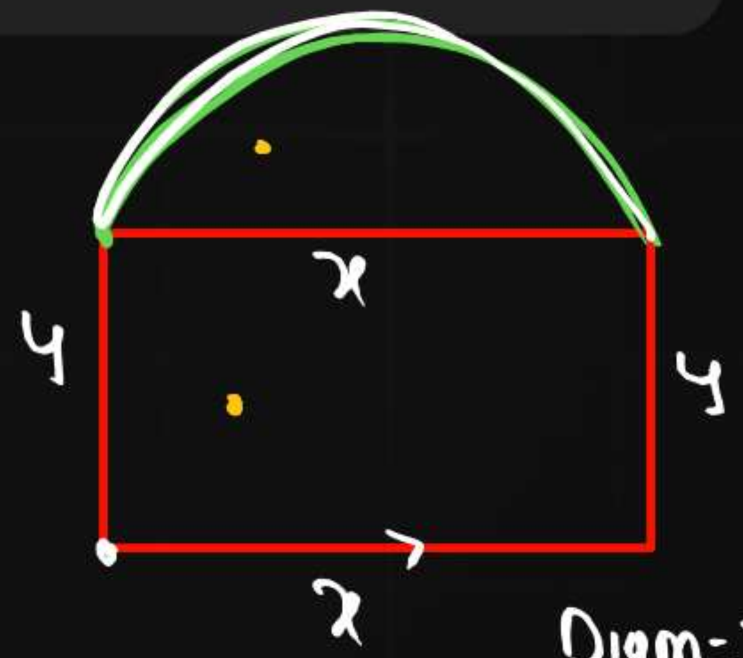
$5 = \frac{x}{1} + \frac{\pi x}{4}$

$4 \times 5 = 4x + \pi x$

$4x + \pi x = 20$

$x(\pi + 4) = 20$

$$\boxed{x = \frac{20}{\pi+4}}$$



$A''(x)$

Diam $= x$

$r = \frac{x}{2}$

QUESTIONS

A square piece of tin of side 18 cm is to be made into a box without top, by cutting a square from each corner and folding up the flaps to form the box. What should be the side of the square to be cut off so that the volume of the box is the maximum possible.

Basic Concepts

Integration

Integration Of some Particular Function

(1) $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + C$

(2) $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + C$

(3) $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$

(4) $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left| x + \sqrt{x^2 - a^2} \right| + C$

(5) $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C$

(6) $\int \frac{dx}{\sqrt{x^2 + a^2}} = \log\left| x + \sqrt{x^2 + a^2} \right| + C$

Integration by partial fraction

S. No.	Form of the rational function	Form of the partial fraction
1.	$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
2.	$\frac{px+q}{(x-a)^2}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}$
3.	$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
4.	$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
5.	$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
	where x^2+bx+c cannot be factorised further	

Properties of definite integrals

P_1: $\int_a^b f(x)dx = -\int_b^a f(x)dx$. In particular, $\int_a^a f(x)dx = 0$

P_2: $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

P_3: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

P_4: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

P_5: $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

P_6: $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, if $f(2a-x) = f(x)$ and 0, if $f(2a-x) = -f(x)$

Properties of definite integrals

P₃: $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

P₄: $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

P₅: $\int_0^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(2a-x)dx$

P₆: $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$, if $f(2a-x) = f(x)$ and 0, if $f(2a-x) = -f(x)$

QUESTIONS

Find $\int \frac{x^2 + x + 1\, dx}{(x+2)(x^2+1)}$

$$\frac{x^2+x+1}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1}$$

$$x^2+x+1 = A(x^2+1)+(x+2)(Bx+C)$$

$$x^2+x+1 = Ax^2+A+Bx^2+Cx+2Bx+2C$$

$$x^2+x+1 = x^2(A+B)+x(2B+C)+A+2C$$

On Comparing

$A+B=1,\quad 2B+C=1,\quad A+2C=1$

$A=1-B$

$4C+C=1$ $\cancel{1}-B+2C=\cancel{1}$

$5C=1$ $B=2C$

$\boxed{C=\frac{1}{5}}$

$\boxed{B=\frac{2}{5}}$

$A = 1-\frac{2}{5}$

$\boxed{A=\frac{3}{5}}$

$$\frac{3}{5}\int\frac{1}{x+2}dx + \frac{1}{5}\int\frac{2x+1}{x^2+1}dx$$

$$\frac{3}{5}\log|x+2| + \frac{1}{5}\int\frac{2x}{x^2+1}+\frac{1}{x^2+1}dx$$

$$\frac{3}{5}\log|x+2| + \frac{1}{5}\int\frac{2x}{x^2+1}dx + \frac{1}{5}\int\frac{1}{x^2+1}dx$$

$$\boxed{\frac{3}{5}\log|x+2| + \frac{1}{5}\log|x^2+1| + \frac{1}{5}\tan^{-1}x + C}$$

let $x^2+1=t$

$2x\,dx = dt$

$\frac{1}{5}\int\frac{1}{t}dt$

$\frac{1}{5}\log|x^2+1|$

QUESTIONS

Find $\int \frac{x^2+1}{x^2-5x+6}\,dx$

$x^2-3x-2x+6$
$x(x-3)-2(x-3)$
$(x-3)(x-2)$

$x^2-5x+6\,)\,\cancel{x^2}+1\,(\,1$
$\cancel{x^2}-5x+6$
$-\quad+\quad-$
$\boxed{5x-5}$

$\frac{x-1}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$

$\boxed{x-1 = A(x-2)+B(x-3)}$

$\int \frac{x^2+1}{x^2-5x+6}dx = \int 1 + \frac{5x-5}{(x-3)(x+2)}\,dx$

$\Rightarrow 1\int dx + 5\underbrace{\int\frac{x-1}{(x-3)(x+2)}dx}_{I_1}$

$x+$

QUESTIONS

Find $\int \frac{x^2}{(x^2+1)(x^2+4)}\,dx$

let $\boxed{x^2 = t}$

$$\frac{t}{(t+1)(t+4)} = \frac{A}{t+1} + \frac{B}{t+4}$$

$$\boxed{t = A(t+4) + B(t+1)}$$

(i) $t = -4$

$-4 = B(-4+1)$

$-4 = B(-3)$

$\boxed{B = \frac{4}{3}}$

(ii) $t = -1$

$-1 = A(-1+4)$

$-1 = 3A$

$\boxed{A = -\frac{1}{3}}$

$$I = -\frac{1}{3}\int \frac{1}{x^2+1}\,dx + \frac{4}{3}\int \frac{1}{x^2+4}\,dx$$

$$I = -\frac{1}{3}\int \frac{1}{x^2+1^2}\,dx + \frac{4}{3}\int \frac{1}{x^2+2^2}\,dx$$

$$-\frac{1}{3}\times\frac{1}{1}\tan^{-1}x + \frac{4}{3}\times\frac{1}{2}\times\tan^{-1}\frac{x}{2}$$

$$\boxed{-\frac{1}{3}\tan^{-1}x + \frac{2}{3}\tan^{-1}\frac{x}{2}} + C$$

QUESTIONS

5 marker

Find $\displaystyle\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta}\,d\theta$

$$\int \frac{(3\sin\theta-2)\cos\theta}{5-(1-\sin^2\theta)-4\sin\theta}\,d\theta$$

$$\int \frac{(3\sin\theta-2)\cos\theta}{5-1+\sin^2\theta-4\sin\theta}\,d\theta$$

$$\boxed{\int \frac{(3\sin\theta-2)\cos\theta}{\sin^2\theta-4\sin\theta+4}\,d\theta}$$

let $\sin\theta = t$

$$\boxed{\cos\theta\, d\theta = dt}$$

$$\int \frac{3t-2}{t^2-4t+4}\,dt$$

$t^2-2t-2t+4$

$t(t-2)-2(t-2)$

$(t-2)^2$

$$\int \frac{3t-2}{(t-2)^2}\,dt$$

$$\boxed{\frac{3t-2}{(t-2)^2} = \frac{A}{t-2} + \frac{B}{(t-2)^2}}$$

QUESTIONS

$$\boxed{\frac{(x-3)e^x}{(x-1)^3}}$$

$\int e^x [f(x) + f'(x)]\, dx$

$$\Rightarrow \boxed{e^x f(x) + C}$$

$$\int e^x \left[\frac{x-3}{(x-1)^3}\right] dx$$

$$\int e^x \left[\frac{x-1-2}{(x-1)^3}\right] dx$$

$$\int e^x \left[\frac{\cancel{x-1}}{(x-1)^{\cancel{3}}} - \frac{2}{(x-1)^3}\right] dx$$

$$\boxed{\int e^x \left[\underbrace{\frac{1}{(x-1)^2}}_{f(x)} \underbrace{- \frac{2}{(x-1)^3}}_{f'(x)}\right] dx}$$

$$e^x \left[\frac{1}{(x-1)^2}\right] + C$$

$$\boxed{\frac{e^x}{(x-1)^2} + C}$$

× 2026 ×

2025 + 2024

QUESTIONS

$$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$$

ILATE

$x = \tan\theta$ $\quad \theta = \tan^{-1}x$

$$\int \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)dx$$

$$\int \sin^{-1}\sin 2\theta \, dx$$

$$\boxed{\int \underset{\text{II}}{2}\cdot \underset{\text{I}}{\tan^{-1}x}\, dx}$$

$$\tan^{-1}x\int 2\,dx - \int\left[\frac{d}{dx}(\tan^{-1}x)\int 2\,dx\right]dx$$

$$2x\tan^{-1}x - \int \frac{2x}{1+x^2}\,dx$$

$$\boxed{2x\tan^{-1} - \log|1+x^2| + C}$$

QUESTIONS

By using the properties of definite integrals, $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)dx$

$$I = \int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$$

$$I = \int_0^{\frac{\pi}{4}} \log\left(1+\tan\left(\frac{\pi}{4}-x\right)\right)$$

$$I = \int_0^{\frac{\pi}{4}} \log\left[1+\frac{\tan\frac{\pi}{4}-\tan x}{1+\tan\frac{\pi}{4}\tan x}\right]$$

$$I = \int_0^{\frac{\pi}{4}} \log\left[\frac{1}{1}+\frac{1-\tan x}{1+\tan x}\right]dx$$

$$I = \int_0^{\frac{\pi}{4}} \log\left[\frac{1+\cancel{\tan x}+1-\cancel{\tan x}}{1+\tan x}\right]dx$$

$$I = \int_0^{\frac{\pi}{4}} \log\left[\frac{2}{1+\tan x}\right]dx$$

$$I = \int_0^{\frac{\pi}{4}} \log 2 - \log(1+\tan x)]\,dx$$

$$I = \int_0^{\frac{\pi}{4}} \log 2\,dx - \boxed{\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx}$$

$$2I = \log 2\int_0^{\frac{\pi}{4}} dx$$

$$2I = \log 2\,[x]_0^{\frac{\pi}{4}}$$

$$2I = \log 2\,\frac{\pi}{4}$$

$$\boxed{I = \frac{\pi}{8}\log 2}$$

QUESTIONS

Evaluate $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

$$\boxed{I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx} \quad -(i)$$

$$I = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx$$

$$\boxed{I = \int_0^{\pi} \frac{\pi \sin x - x \sin x}{1+\cos^2 x} dx} \quad -(ii)$$

adding (i) and (ii)

$\int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx$ if $f(2a-x) = f(x)$

$$2I = \int_0^{\pi} \frac{\pi \sin x}{1+\cos^2 x} dx$$

$$\boxed{2I = \pi \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx}$$

$$\cancel{2}I = \cancel{2}\pi \int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$$

$$I = \pi \int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x} dx$$

if $x = \frac{\pi}{2}$, $t = 0$

if $x = 0$, $t = 1$

let $\cos x = t$

$-\sin x\,dx = dt$

$$\boxed{\sin x\,dx = -dt}$$

$$I = -\pi \int_1^0 \frac{dt}{1+t^2}$$

$$I = -\pi \left[\tan^{-1} t\right]_1^0$$

$$I = -\pi \left[\tan^{-1} 0 - \tan^{-1} 1\right]$$

$$I = -\pi \left[0 - \cancel{\tan^{-1}\tan}\frac{\pi}{4}\right]$$

$$\boxed{I = \frac{\pi^2}{4}}$$

QUESTIONS

Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1+\sqrt{\tan x}}$

QUESTIONS

Evaluate $\boxed{\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x}}$

$$I = \int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2\sin^2 x} \quad \text{—(i)}$$

$$I = \int_0^{\pi} \frac{\pi - x\ dx}{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)}$$

$$\boxed{I = \int_0^{\pi} \frac{\pi - x\,dx}{a^2\cos^2 x + b^2\sin^2 x}} \quad \text{—(ii)}$$

$$2I = \int_0^{\pi} \frac{\pi\,dx}{a^2\cos^2 x + b^2\sin^2 x}$$

$$\boxed{2I = \pi\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}}$$

$$2I = 2\pi\int_0^{\frac{\pi}{2}} \frac{\sec^2 x\,dx}{a^2 + b^2\tan^2 x}$$

$$\boxed{\cancel{2}I = \frac{\cancel{2}\pi}{b^2}\int_0^{\frac{\pi}{2}} \frac{\sec^2 x}{\frac{a^2}{b^2} + \tan^2 x}\,dx}$$

let $\tan x = t$ $\quad$ if $x = \frac{\pi}{2}$, $t = \infty$

$\boxed{\sec^2 x\,dx = dt}$ $\quad$ if $x = 0$, $t = 0$

$$\frac{\pi}{b^2}\int_0^{\infty} \frac{dt}{\left(\frac{a}{b}\right)^2 + t^2}$$

$$\frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$\frac{\pi}{b^2}\left[\frac{b}{a}\tan^{-1}\frac{bt}{a}\right]_0^{\infty}$$

$$\frac{\pi}{b^2}\times\frac{b}{a}\left[\tan^{-1}\frac{bt}{a}\right]_0^{\infty}$$

$$\frac{\pi}{ab}\left[\tan^{-1}\infty - \tan^{-1}0\right]$$

$$\frac{\pi}{ab}\left[\cancel{\tan^{-1}}\tan\frac{\pi}{2}\right]$$

$$\frac{\pi}{ab}\times\frac{\pi}{2} = \boxed{\frac{\pi^2}{2ab}}$$

QUESTIONS

Evaluate : $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx$

QUESTIONS

Evaluate the definite integrals $\int_0^{\frac{\pi}{2}} \frac{\cos^2 x\, dx}{\cos^2 x + 4\sin^2 x}$

QUESTIONS

Evaluate : $\boxed{\int_1^3 (|x-1|+|x-2|+|x-3|)dx}$

$$\boxed{\int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx}$$

$$f(x) = x-1-(x-2)-(x-3)$$
$$= \cancel{x}-1-\cancel{x}+2-x+3$$
$$= -x+4$$

$$f(x) = (x-1)+(x-2)-(x-3)$$
$$= x-1+\cancel{x}-2-\cancel{x}+3$$
$$= x-3+3$$
$$= x$$

$$\int_1^2 -x+4\,dx + \int_2^3 x\,dx$$

$$\left[-\frac{x^2}{2}+4x\right]_1^2 + \left[\frac{x^2}{2}\right]_2^3$$

= Comment section

5

Basic Concepts

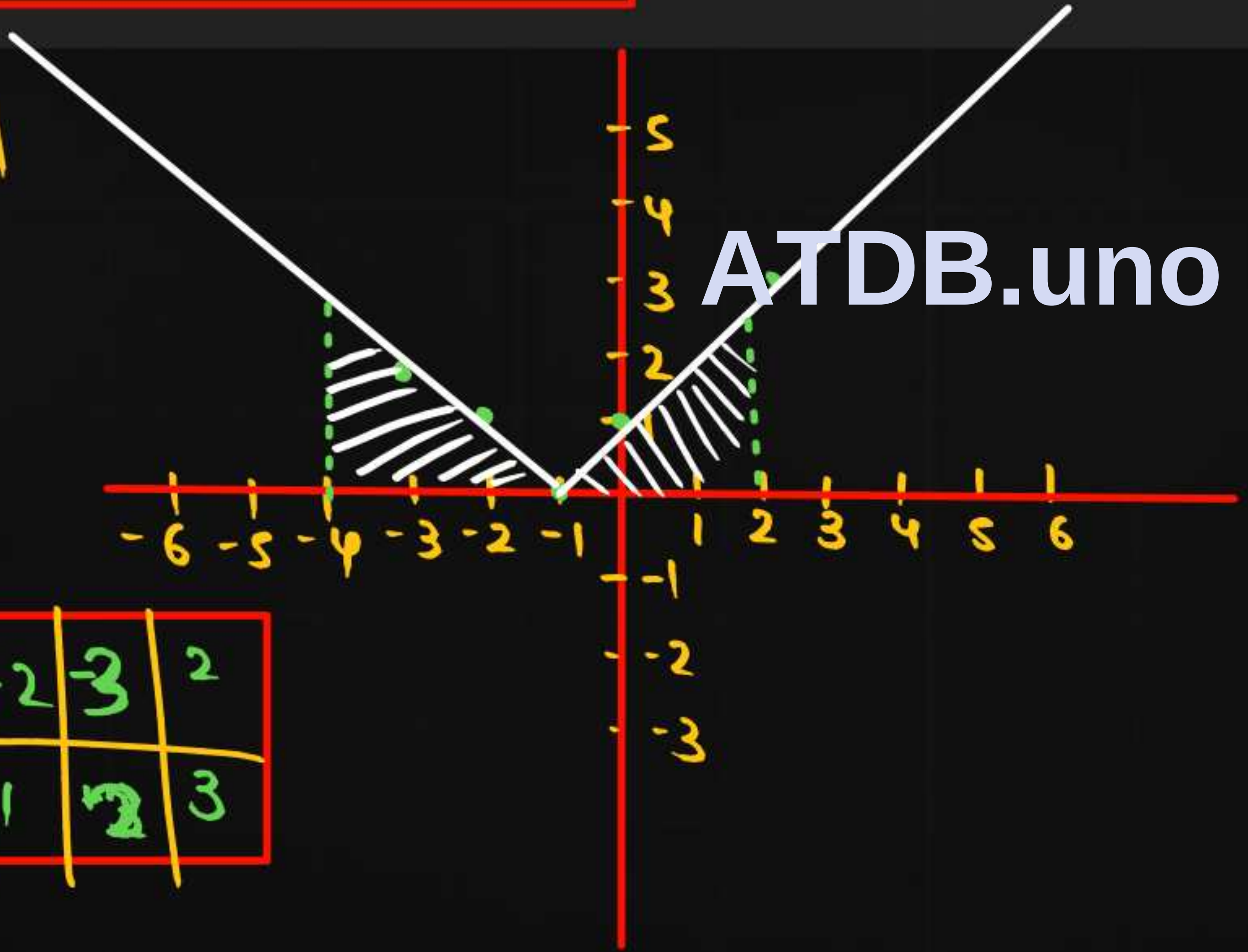
QUESTIONS

$y = x + 1$

5 marks

Sketch the graph $y = |x+1|$. Evaluate $\int_{-4}^{2} |x+1|\,dx$. What does the value of this integral represent on the graph?

$y = |x+1|$



x	0	-1	-2	-3	2
y	1	0	1	2	3

$$\int_{-4}^{-1} f(x)\,dx + \int_{-1}^{2} f(x)\,dx$$

$$\int_{-4}^{-1} -x-1\,dx + \int_{-1}^{2} x+1\,dx$$

$$\boxed{\left[-\frac{x^2}{2} - x\right]_{-4}^{-1} + \left[\frac{x^2}{2} + x\right]_{-1}^{2}} = 9 \text{ sq. unit}$$

QUESTION

Smarla

Using integration, find the area of the ellipse $\frac{x^2}{16}+\frac{y^2}{4}=1$, included between the lines $x=-2$ and $x=2$.

$$\int\sqrt{a^2-x^2}\,dx=\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$$

$$\boxed{A=4\int_0^2 f(x)\,dx}$$

$$\boxed{A=4\int_0^2 y\,dx}$$

$$\boxed{A=\frac{4}{2}\int_0^2\sqrt{4^2-x^2}\,dx \quad a=4}$$

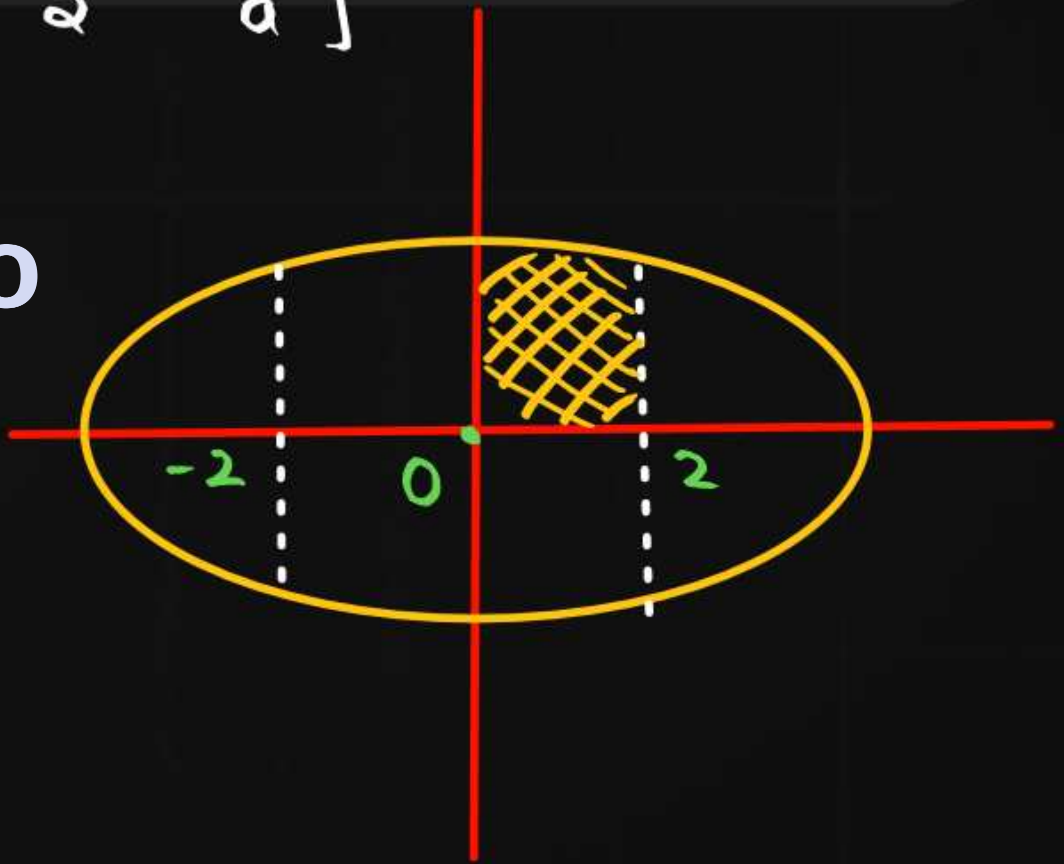
$$A=2\left[\frac{x}{2}\sqrt{4^2-x^2}+\frac{4^2}{2}\sin^{-1}\frac{x}{4}\right]_0^2$$

$$A=2\left[1\sqrt{16-4}+8\sin^{-1}\frac{1}{2}\right]$$

$$A=2\left[\sqrt{12}+\cancel{8}^{\,4}\,\cancel{\sin^{-1}}\cancel{\sin}\frac{\pi}{\cancel{6}_3}\right]$$

$$\boxed{A=2\left[2\sqrt{3}+\frac{4\pi}{3}\right]}$$

$$=4\sqrt{3}+\frac{8}{3}\pi \text{ sq unit}$$



Basic Concepts

QUESTIONS

The order and degree of $\left(1+\left(\frac{dy}{dx}\right)^2\right)^{3/2} = k\frac{d^2y}{dx^2}$ are

1. (2, 1)
2. (1, 2)
3. (2, 2)
4. order = 2 but degree not defined

$$\left(\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}\right)^2 = k^2\left(\frac{d^2y}{dx^2}\right)^2$$

$$\boxed{\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = k^2\left(\frac{d^2y}{dx^2}\right)^2}$$

Order = 2
Degree = 2

QUESTIONS

The degree of $\boxed{\sqrt{\dfrac{d^2y}{dx^2}} + y = 0 \text{ is}}$

1. 2
2. $\boxed{1}$
3. Not defined
4. 1/2

$$\sqrt{\frac{d^2y}{dx^2}} = -y$$

Squaring both side

$$\boxed{\left(\frac{d^2y}{dx^2}\right)^1 = y^2}$$

Order = 2

Degree = 1

QUESTIONS

The Degree and Order of given differential Equation :

$$\boxed{\tan^{-1}\left(\frac{dy}{dx}\right) = x + y}$$

$$\boxed{\frac{dy}{dx} = \tan(x+y)}$$

1. 1, 1
2. 1, not defined
3. 1, 3
4. None of these

QUESTIONS

The order and degree of the following differential equation are respectively :

$$\boxed{\frac{d^4y}{dx^4} + 2e^{\frac{dy}{dx}} + y^2 = 0}$$

1. $-4, 1$
2. $\boxed{4, \text{not defined}}$
3. $1, 1$
4. $4, 1$

QUESTIONS

$\frac{x}{y} \to \frac{dx}{dy}$ $\quad \frac{y}{x} = \frac{dy}{dx}$

Find the particular solution of the differential equation :

$$2ye^{x/y}dx + (y - 2xe^{x/y})dy = 0 \quad y(0) = 1$$

if $x = 0$, $y = 1$

$$2ye^{\frac{x}{y}}dx = -(y - 2xe^{\frac{x}{y}})dy$$

$$\boxed{\frac{dx}{dy} = \frac{2xe^{\frac{x}{y}} - y}{2ye^{\frac{x}{y}}}}$$

H is a homogenous

$\boxed{x = vy}$ $\quad v = \frac{x}{y}$

$$\boxed{\frac{dx}{dy} = y\frac{dv}{dy} + v}$$

$$y\frac{dv}{dy} + v = \frac{2xve^{v} - y}{2ye^{\frac{vy}{y}}}$$

$$y\frac{dv}{dy} + v = \frac{\cancel{y}[2ve^{v} - 1]}{2\cancel{y}e^{v}}$$

$$y\cdot\frac{dv}{dy} = \frac{2ve^{v} - 1}{2e^{v}} - \frac{v}{1}$$

$$y\cdot\frac{dv}{dy} = \frac{\cancel{2ve^{v}} - 1 - \cancel{2ve^{v}}}{2e^{v}}$$

$$y\cdot\frac{dv}{dy} = -\frac{1}{2e^{v}}$$

$$-2e^{v}dv = \frac{1}{y}dy$$

$$-2\int e^{v}dv = \int\frac{1}{y}dy$$

$$\boxed{-2e^{v} = \log y + C}$$

$$-2e^{\frac{x}{y}} = \log y + C$$

$$-2e^{\frac{0}{1}} = \log 1 + C$$

$$-2 = C$$

$$\boxed{-2e^{\frac{x}{y}} = \log y - 2}$$

QUESTIONS

Smark

$x = \lambda x,\ y = \lambda y$

Solve the differential equation : $\boxed{y + \frac{d}{dx}(xy) = x(\sin x + x)}$

$y + x'y + xy' = x(\sin x + x)$

$y + y + x\frac{dy}{dx} = x(\sin x + x)$

$2y + x\frac{dy}{dx} = x(\sin x + x)$

$x\frac{dy}{dx} + 2y = x(\sin x + x)$

$$\boxed{\frac{dy}{dx} + \frac{2}{x}y = \sin x + x}$$

$$\boxed{\frac{dy}{dx} + Py = Q}$$

$P = \frac{2}{x}$

$Q = \sin x + x$

$\boxed{I.f = e^{\int P dx}}$

$I.f = e^{\int \frac{2}{x} dx}$

$I.f = e^{2\int \frac{1}{x} dx}$

$= e^{2\log x}$

$= e^{\log x^2}$

$\boxed{I.f = x^2}$

$y \times I.f = \int Q \times I.f\, dx + C$

$y \times x^2 = \int (\sin x + x)x^2 dx$

$y \cdot x^2 = \int x^2 \sin x + x^3 dx$

$y \cdot x^2 = \int x^3 dx + \boxed{\int x^2 \sin x\, dx}$

2 times Integration by parts

$y x^2 = \frac{x^4}{4} + \left[x^2 \int \sin x\, dx - \int \frac{d}{dx}(x^2) \int \sin x\, dx\right]$

$= \frac{x^4}{4} + \left[x^2 - \cos x + \int 2x \cdot \cos x\, dx\right.$

Ans

QUESTIONS

Find the particular solution of the differential equation $\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x \, (x \neq 0)$ given that $y = 0$ when $x = \frac{\pi}{2}$.

Basic Concepts

QUESTIONS

10 questions. → practice

Find the unit vector in the direction of the sum of the vectors, $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$.

$$\vec{a}+\vec{b} = 4\hat{i}+3\hat{j}-2\hat{k}$$

$$\boxed{\vec{c} = 4\hat{i}+3\hat{j}-2\hat{k}}$$

$$\boxed{\hat{c} = \frac{\vec{c}}{|\vec{c}|}} = \frac{4\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{16+9+4}} = \frac{4\hat{i}+3\hat{j}-2\hat{k}}{\sqrt{29}}$$

$$\boxed{\hat{c} = \frac{4}{\sqrt{29}}\hat{i}+\frac{3}{\sqrt{29}}\hat{j}-\frac{2}{\sqrt{29}}\hat{k}}$$

QUESTIONS

Given $\overrightarrow{AB} = 3\hat{i} - \hat{j} - 5\hat{k}$ and co-ordinate of the terminal point is $(0, 1, 3)$.

Find the coordinate of the initial point.

QUESTIONS

Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\hat{i}+2\hat{j}-\hat{k}$ and $-\hat{i}+\hat{j}+\hat{k}$ respectively, in the ratio 2 : 1

$m = 2 : 1 = n$

(i) Internally

(ii) Externally

$$\vec{r} = \frac{m\vec{b}+n\vec{a}}{m+n}$$

$$\vec{r} = \frac{2[-\hat{i}+\hat{j}+\hat{k}]+1[\hat{i}+2\hat{j}-\hat{k}]}{3}$$

$$\vec{r} = \frac{-2\hat{i}+2\hat{j}+2\hat{k}+\hat{i}+2\hat{j}-\hat{k}}{3}$$

$$\boxed{\vec{r} = \frac{-\hat{i}+4\hat{j}+\hat{k}}{3}}$$

(ii) $\vec{r} = \dfrac{m\vec{b}-n\vec{a}}{m-n}$

QUESTIONS

If $\vec{a} = 5\hat{i} - \hat{j} - 3\hat{k}$ and $\vec{b} = \hat{i} + 3\hat{j} - 5\hat{k}$, then show that the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular.

$$\vec{a}+\vec{b} = 6\hat{i}+2\hat{j}-8\hat{k}$$

$$\vec{a}-\vec{b} = 4\hat{i}-4\hat{j}+2\hat{k}$$

$$(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b}) = 24-8-16$$
$$= 16-16$$
$$= \boxed{0}$$

Hence they are $\perp$

QUESTIONS

v.v.v Imp ③ marks

Let $\boxed{\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}}$, $\boxed{\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}}$ and $\boxed{\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 15$.

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ 3 & -2 & 7 \end{vmatrix}$$

$$\vec{a} \times \vec{b} = \hat{i}\begin{vmatrix} 4 & 2 \\ -2 & 7 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 2 \\ 3 & 7 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 4 \\ 3 & -2 \end{vmatrix}$$

$$\Rightarrow (28+4)\hat{i} - \hat{j}(7-6) + \hat{k}(-2-12)$$

$$\vec{a} \times \vec{b} = \boxed{32\hat{i} - \hat{j} - 14\hat{k}}$$

$\boxed{\vec{d} = \lambda(32\hat{i} - \hat{j} - 14\hat{k})}$

$$\boxed{\vec{d} = \frac{5}{3}(32\hat{i} - \hat{j} - 14\hat{k})}$$

$$\boxed{\vec{d} = 32\lambda\hat{i} - \lambda\hat{j} - 14\lambda\hat{k}}$$

$$2(32\lambda) - (1)(-\lambda) + 4(-14\lambda) = 15$$

$$64\lambda + \lambda - 56\lambda = 15$$

$$65\lambda - 56\lambda = 15$$

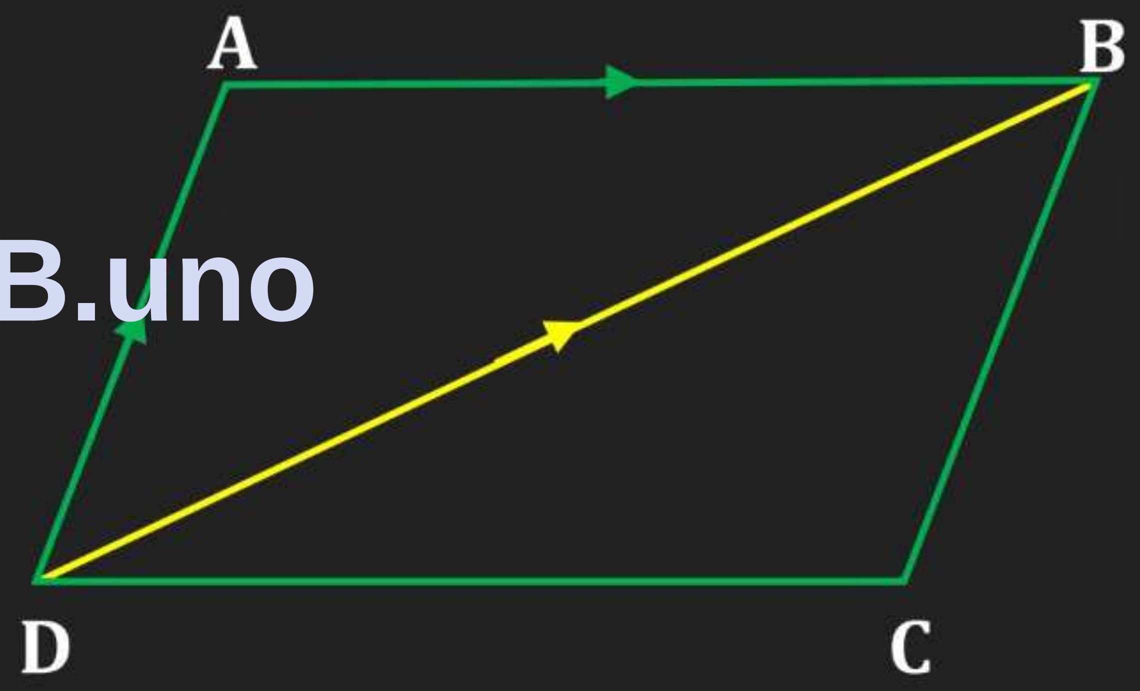
$$9\lambda = 15$$

$$\lambda = \frac{15}{9} = \frac{5}{3}$$

QUESTIONS

In the given figure, ABCD is a parallelogram.

If $\overrightarrow{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\overrightarrow{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD.



QUESTIONS

If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, $|\vec{a}|=\sqrt{37}$, $|\vec{b}|=3$ and $|\vec{c}|=4$, then angle between $\vec{b}$ and $\vec{c}$ is

1. $\frac{\pi}{6}$
2. $\frac{\pi}{4}$
3. $\frac{\pi}{3}$
4. $\frac{\pi}{2}$

$\vec{b}+\vec{c}=-\vec{a}$

$|\vec{b}+\vec{c}|^2=|\vec{a}|^2$

$|\vec{b}|^2+|\vec{c}|^2+2\vec{b}\cdot\vec{c}=|\vec{a}|^2$

$9+16+2|\vec{b}||\vec{c}|\cos\theta=37$

$25+2\times3\times4\cos\theta=37$

$24\cos\theta=37-25$

$24\cos\theta=12$

$\cos\theta=\frac{12}{24}$

$\cos\theta=\frac{1}{2}$

$\cos\theta=\cos\frac{\pi}{3}$

$\boxed{\theta=\frac{\pi}{3}}$

QUESTIONS

Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points A(2, 1, 3) and B(8, −1, 0) respectively.

$$\boxed{\overrightarrow{BA} = -6\hat{i}+2\hat{j}+3\hat{k}}$$

$$\hat{a} = \frac{-6\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{36+4+9}}$$

$$\boxed{\hat{a} = \frac{-6}{7}\hat{i}+\frac{2}{7}\hat{j}+\frac{3}{7}\hat{k}}$$

$$\vec{a} = |\vec{a}|\cdot\hat{a}$$

$$\vec{a} = 21\left[\frac{-6}{7}\hat{i}+\frac{2}{7}\hat{j}+\frac{3}{7}\hat{k}\right]$$

$$\boxed{\vec{a} = -18\hat{i}+6\hat{j}+9\hat{k}}$$

QUESTIONS

$\lambda = -3, -2,$
$|\lambda| = 0, 3$

If $\boxed{|\vec{a}| = 4}$ and $\boxed{-3 \le \lambda \le 2}$, then the range of $\boxed{|\lambda\vec{a}|}$ is

$|\lambda|$
0   3

$|\lambda\vec{a}| = \boxed{|\lambda||\vec{a}|}$

$= \boxed{4|\lambda|}$

$\boxed{0 \le |\lambda| \le 3}$

$0 \le 4|\lambda| \le 12$

$4|\lambda|$

1. [0, 8]
2. [−12, 8]
3. [0, 12]
4. [8, 12]

Basic Concepts

3 marks

Distance between two skew lines

$$\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$$
$$\vec{r} = \vec{a}_2 + \mu\vec{b}_2$$

$$d = \left|\frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|}\right|$$

Distance between parallel lines

$$\vec{r} = \vec{a}_1 + \lambda\vec{b}$$

$$\vec{r} = \vec{a}_2 + \mu\vec{b}$$

$$d = |\overrightarrow{PT}| = \left|\frac{\vec{b} \times (\vec{a}_2 - \vec{a}_1)}{|\vec{b}|}\right|$$

QUESTIONS

Find the direction cosines of the line passing through the two points $(-2, 4, -5)$ and $(1, 2, 3)$.

QUESTIONS

Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by $\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

QUESTIONS

Find the values of p, so that the lines

$$l_1: \frac{1-x}{3} = \frac{7y-14}{p} = \frac{z-3}{2} \text{ and } l_2: \frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$$

are perpendicular to each other.

$$\frac{-7[x-1]}{3p} = \frac{y-5}{1} = \frac{-(z-6)}{5}$$

$$\frac{-(x-1)}{3} = \frac{7(y-2)}{p} = \frac{z-3}{2}$$

$$\frac{x-1}{-3} = \frac{y-2}{\frac{p}{7}} = \frac{z-3}{2}$$

$$\frac{x-1}{\frac{-3p}{7}} = \frac{y-5}{1} = \frac{z-6}{-5}$$

$$-3\left(\frac{-3p}{7}\right) + \frac{p}{7} - 10 = 0$$

$$\frac{9p}{7} + \frac{p}{7} = 10$$

$$\frac{10p}{7} = 10$$

$$10p = 70$$

$$\boxed{p = 7}$$

QUESTIONS

Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines:

$$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7} \text{ and } \frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}.$$

$\# \ \vec{r} = \vec{a} + \lambda \vec{b}$

$\vec{b}_1 = 3\hat{i} - 16\hat{j} + 7\hat{k}$

$\vec{b}_2 = 3\hat{i} + 8\hat{j} - 5\hat{k}$

$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(\quad)$

$\vec{b}_1 \times \vec{b}_2 = \boxed{\quad} = \vec{b}$

QUESTIONS

Find the distance of the point $(-1, -5, -10)$ from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

QUESTIONS

Find the foot of the perpendicular from the point $(0, 2, 3)$ on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Also, find the length of the perpendicular.

$\frac{0+a}{2} = 2 \quad \Big| \quad \frac{2+b}{2} = 3 \quad \Big| \quad \frac{3+c}{2} = -1$

$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = \lambda$

$x+3 = 5\lambda \quad \Big| \quad y = 2\lambda+1 \quad \Big| \quad z = 3\lambda-4$

$x = 5\lambda - 3$

Direction ratio of line

$(5, 2, 3)$

D.r of PR

$(5\lambda-3,\ 2\lambda-1,\ 3\lambda-7)$

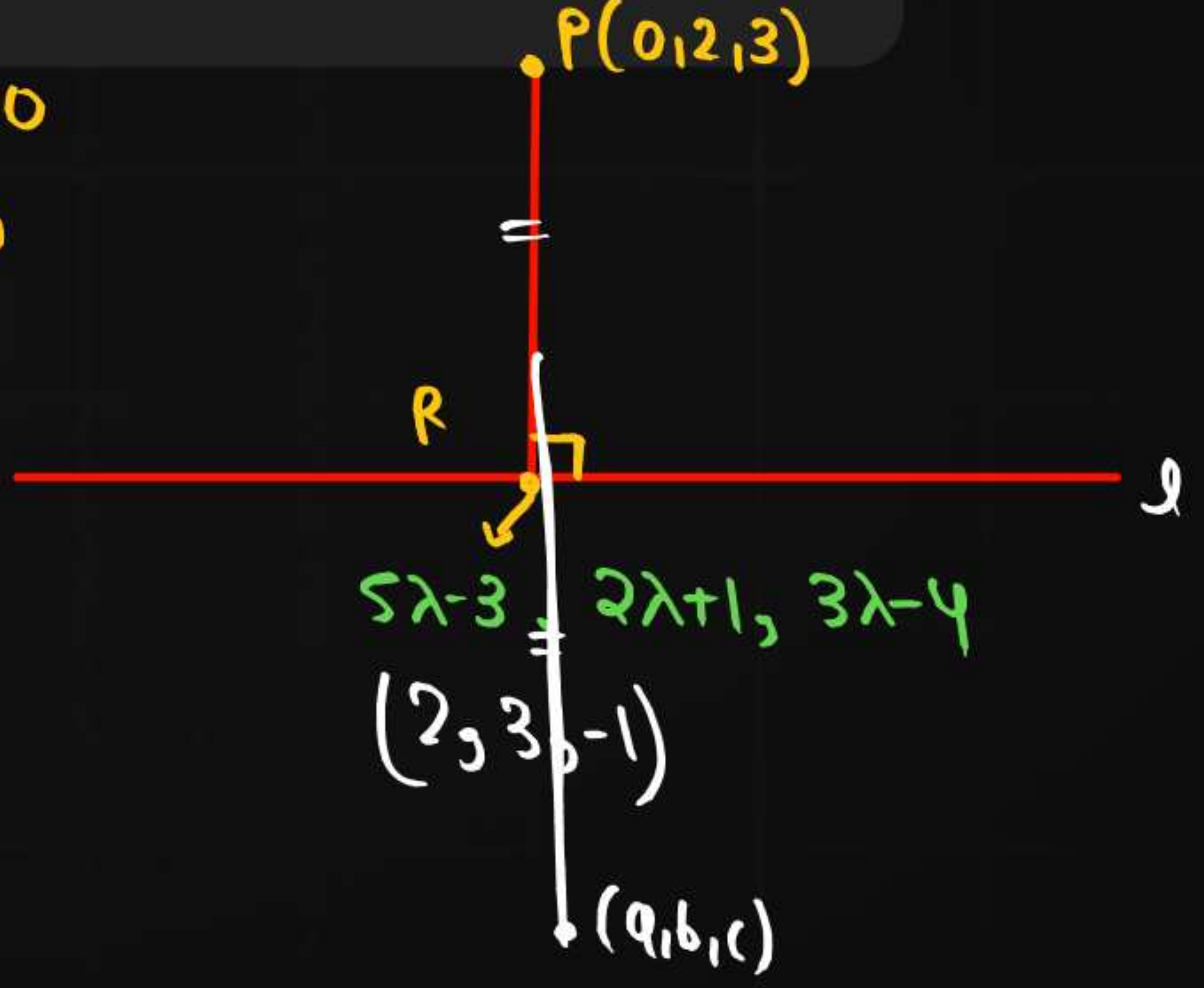
$5(5\lambda-3)+2(2\lambda-1)+3(3\lambda-7)=0$

$25\lambda-15+4\lambda-2+9\lambda-21=0$

$38\lambda-38=0$

$38\lambda=38$

$\boxed{\lambda=1}$



QUESTIONS

The acute angle between the line joining the points $(2, 1, -3), (-3, 1, 7)$ and a line parallel to $\frac{x-1}{3} = \frac{y}{4} = \frac{z+3}{5}$ through the point $(-1, 0, 4)$ is

Basic Concepts

QUESTIONS

lpp → 2 hour ✗ vipeta   1 hour   4 type → 5 marks
2 mcq's 1 3 marks

Solve graphically the following linear programming problem :

Maximise $z = 6x + 3y$, subject to the constraints (CBSE 2023,2024)

$$4x + y \geq 80$$
$$3x + 2y \leq 150$$
$$x + 5y \geq 115$$
$$x \geq 0, y \geq 0$$

Corner point	$z = 6x + 3y$
30,20	180 ✓
10,2	120
0,10	110 ✓

QUESTIONS

10+28

$\vec{a} \times \vec{b} = \square$ vector
$\vec{a} \cdot \vec{b} =$ Number

Determine graphically the minimum value of the objective function

Minimize $\boxed{Z = 5x + 7y}$

Subject to $\boxed{2x + y \geq 8}$, $\boxed{x + 2y \geq 10}$ and $\boxed{x, y \geq 0}$

0+0 ≥ 8 0+

$2x+y=8$

x	0	4
y	8	0

$x+2y=10$

x	0	10
y	5	0

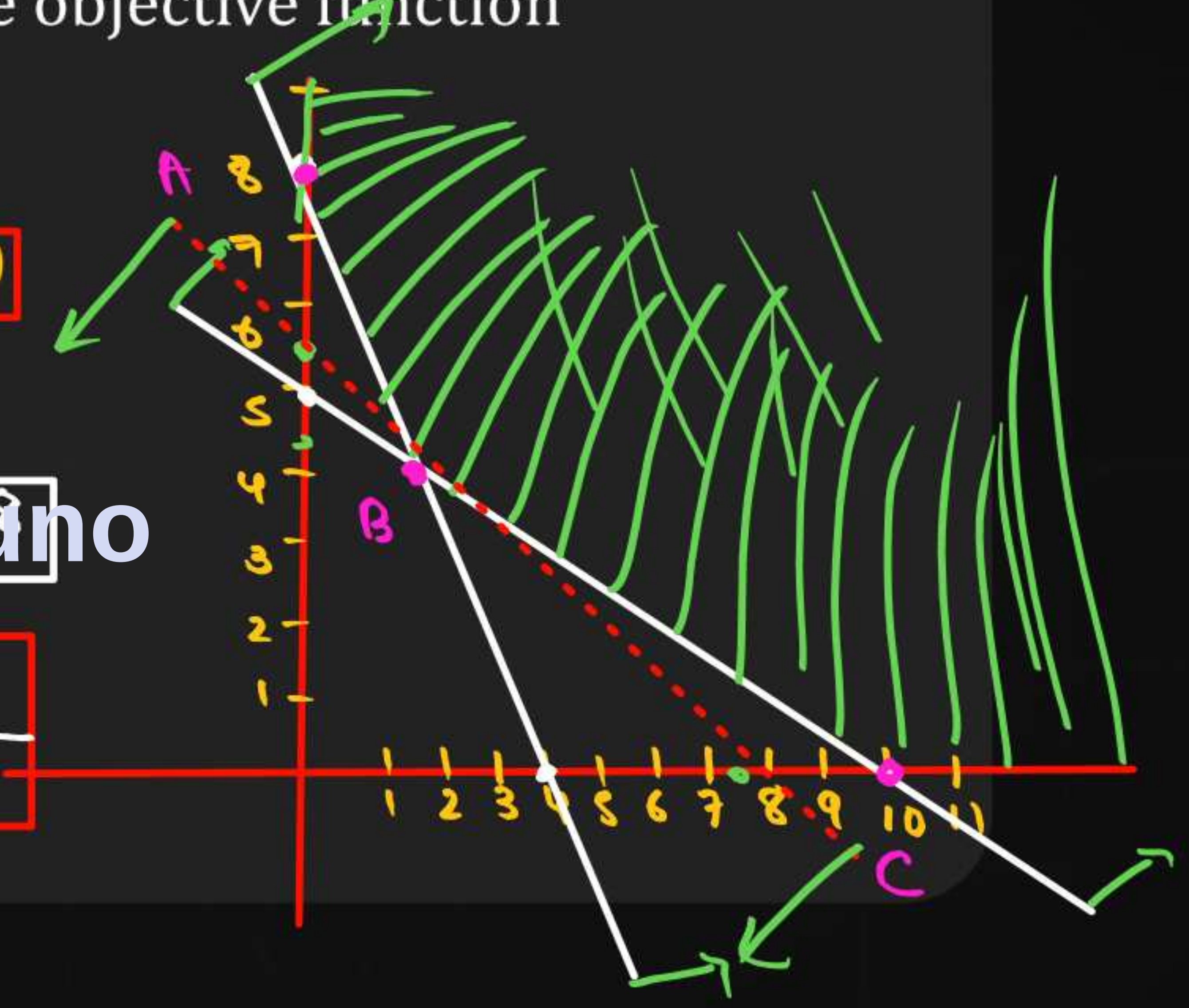
$Z < 38$

$5x+7y < 38$

x	0	7.6
y	5.4	0

0+0 < 38

0 <



Corner point	$Z = 5x+7y$
(0, 8)	56
(10, 0)	50
(2, 4)	38 — minimum

Basic Concepts

① Conditional P
② Multiplication therem
③ Independent
④ Bayes | theorem of total Probabity

QUESTIONS

A black and a red dice are rolled. Find the conditional probability of obtaining a sum greater than 9, given that the black die resulted in a 5.

QUESTIONS

Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) Both balls are red.

(ii) First ball is black and second is red.

(iii) One of them is black and other is red.

QUESTIONS

Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that

(i) The problem is solved

(ii) Exactly one of them solves the problem.

QUESTIONS

Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

Hostel $= E_1$

Day Scholar $= E_2$

$A =$ He has grade A

$P(E_1) = \frac{60}{100}$

$P(E_2) = \frac{40}{100}$

$P(A|E_1) = \frac{30}{100}$

$P(A|E_2) = \frac{20}{100}$

$$\# \; P(E_1|A) = \frac{P(E_1) \times P(A|E_1)}{P(E_1) \times P(A|E_1) + P(E_2) \times P(A|E_2)}$$

$$\#$$

$$\frac{\frac{60}{\cancel{100}} \times \frac{30}{\cancel{100}}}{\frac{60}{\cancel{100}} \times \frac{30}{\cancel{100}} + \frac{40}{\cancel{100}} \times \frac{20}{\cancel{100}}} \Rightarrow \frac{1800}{1800+800} = \frac{1800}{2600} = \frac{9}{13}$$

QUESTIONS

In answering a question on a multiple choice test, a student either knows the answer or guesses. Let $\frac{3}{4}$ be the probability that he knows the answer and $\frac{1}{4}$ be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability $\frac{1}{4}$. What is the probability that the student knows the answer given that he answered it correctly?

QUESTIONS

Home work

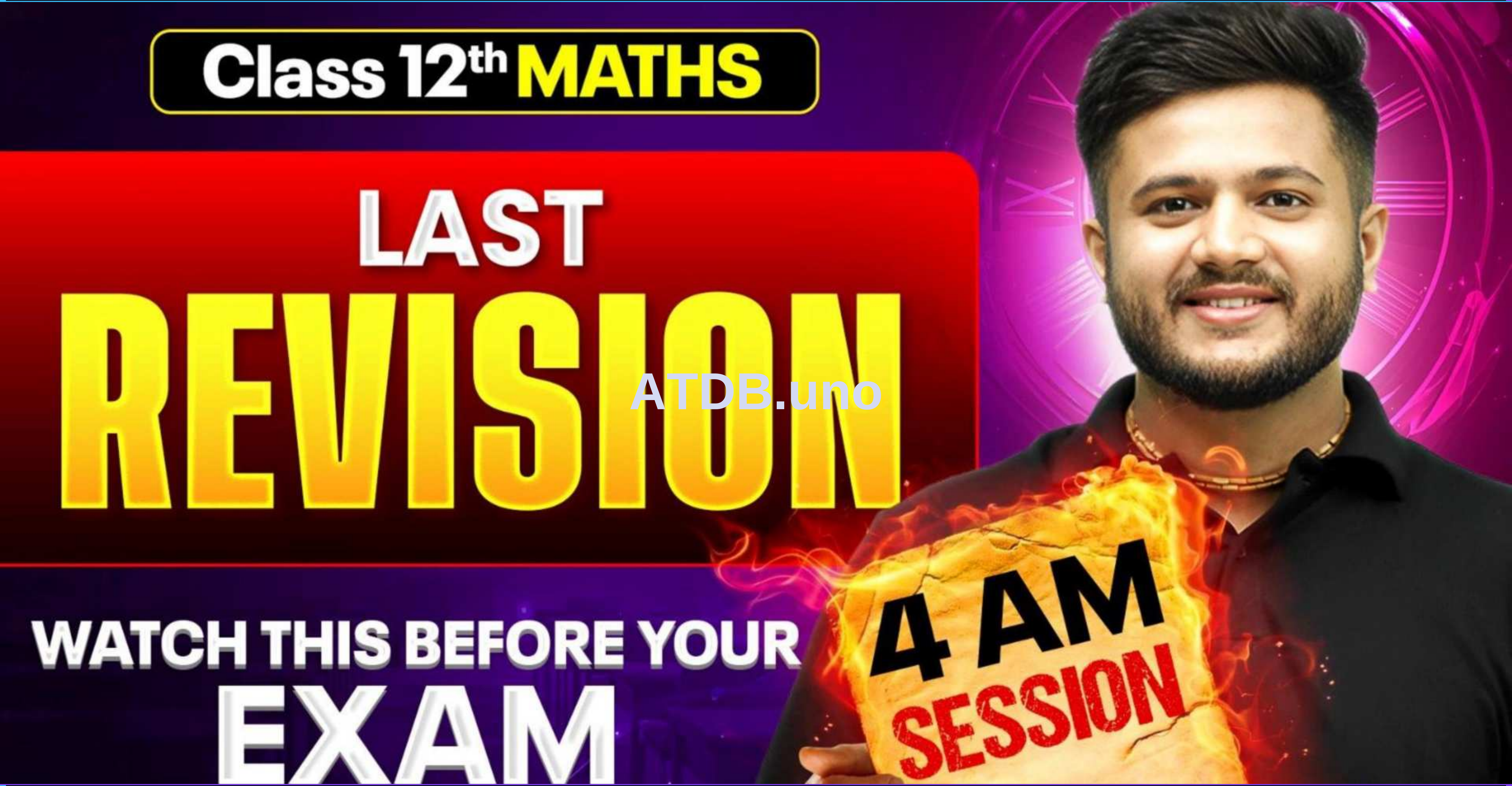
A card from a pack of 52 cards is lost. From the remaining cards of the pack, two cards are drawn and are found to be both diamonds. Find the probability of the lost card being a diamond.

HOMEWORK

6:00

1. naya x
2. Rrvision
   +
   Ncert + pyq's
   +
   Formula sheet

Thank You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