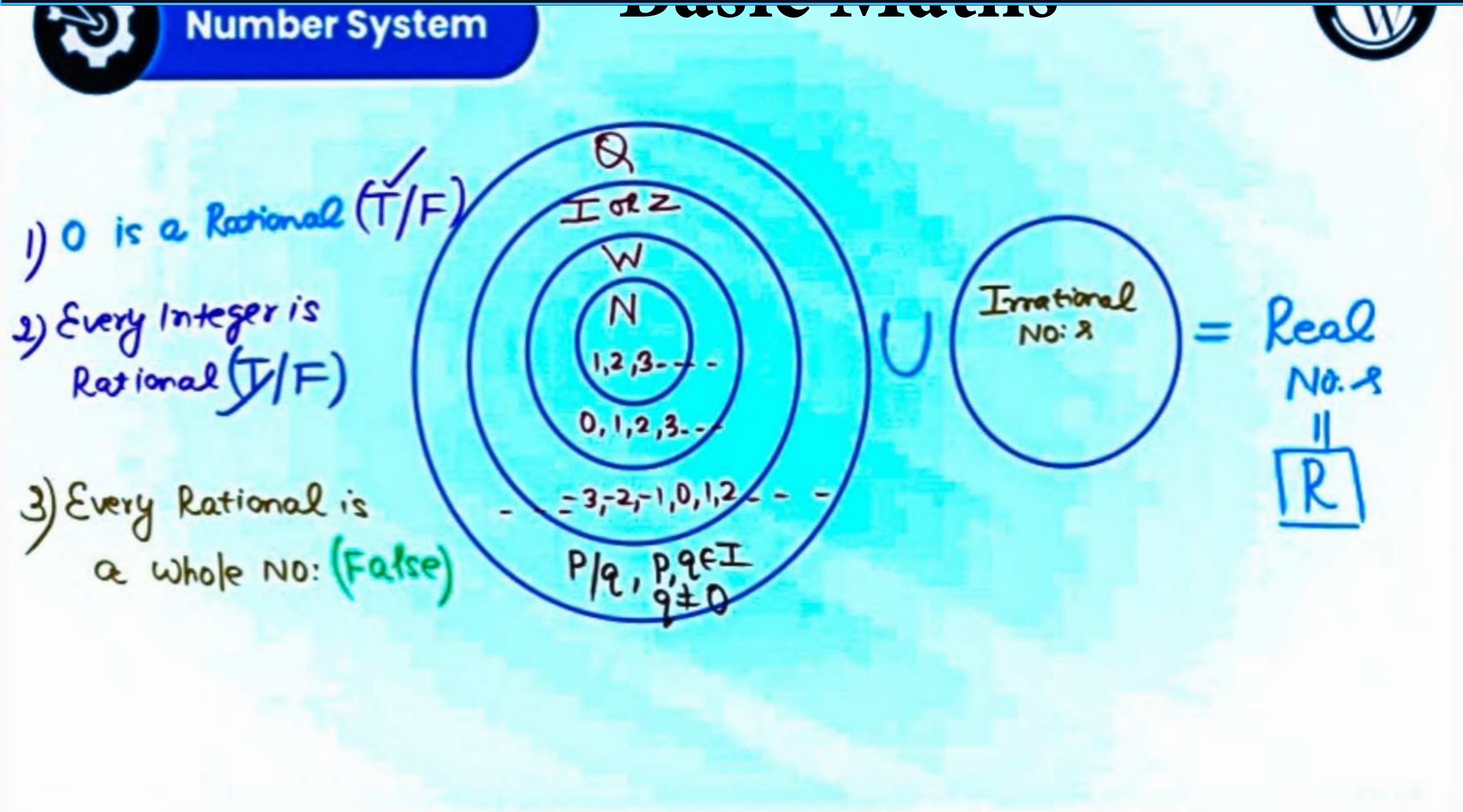
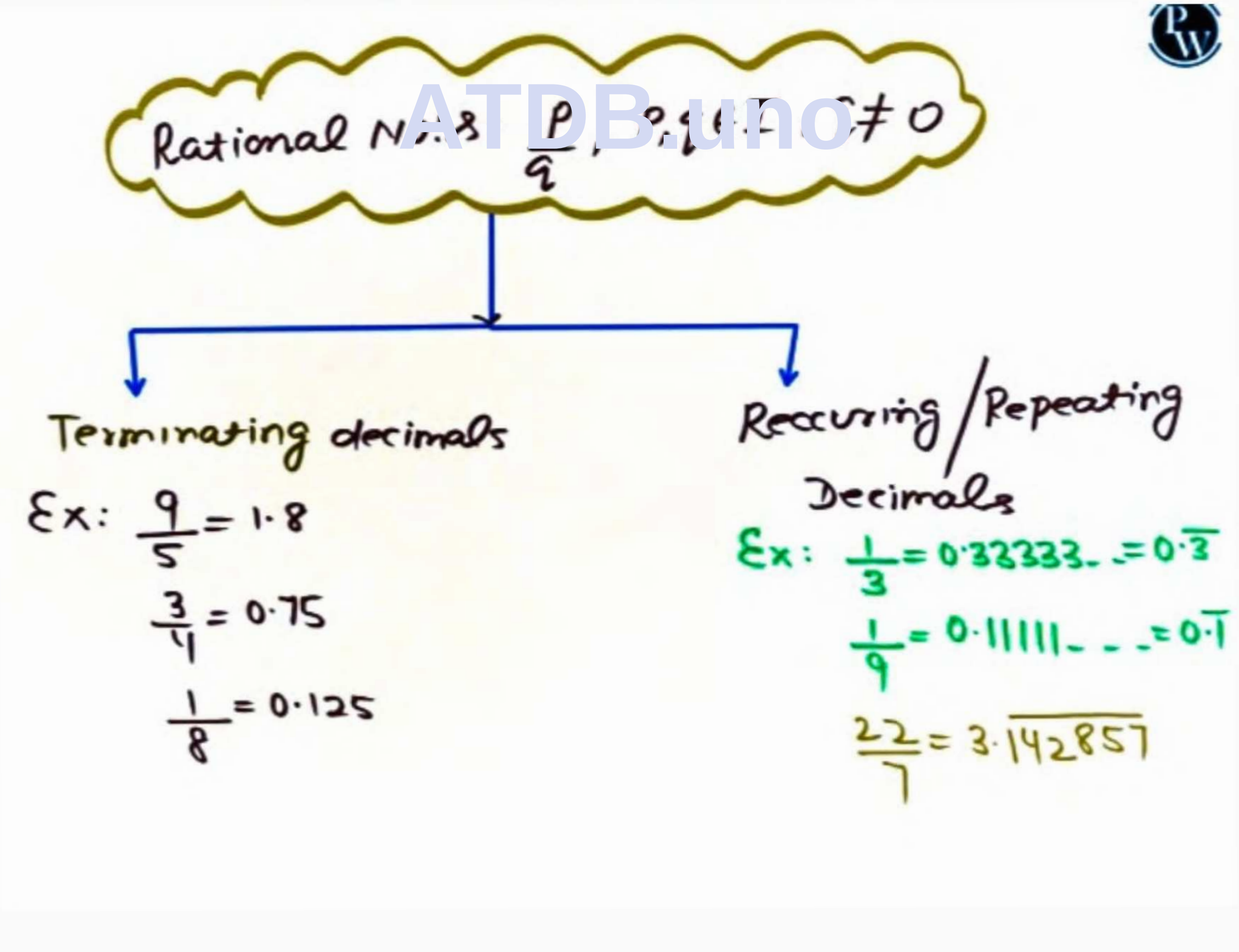


1) 0 is a Rational (T/F) ✓

2) Every Integer is Rational (T/F) ✓

3) Every Rational is a whole No: (False)



Rational No:s $\frac{P}{q}$, p,q ∈ I, q ≠ 0

Terminating decimals

Ex: $\frac{9}{5} = 1.8$

$\frac{3}{4} = 0.75$

$\frac{1}{8} = 0.125$

Reccuring/Repeating Decimals

Ex: $\frac{1}{3} = 0.33333\ldots = 0.\overline{3}$

$\frac{1}{9} = 0.11111\ldots = 0.\overline{1}$

$\frac{22}{7} = 3.\overline{142857}$

How know if a rational no: $\frac{p}{q}$, $p, q \in I$, $q \neq 0$ is repeating or Terminating without dividing

Pechaan!!

q is of the form $2^m \times 5^n$

where $m, n \in W$.

Ex: $\frac{29}{25}$
- Repeating ✗
- Terminating ✓

$25 \to 2^0 \cdot 5^2$

Ex: $\frac{37}{18}$
- Repeating ✓
- Terminating ✗

$18 \to 2 \times 3^2$

Ex: $\frac{91}{14}$
- Repeating ✗
- Terminating ✓

$\Rightarrow \frac{\overset{13}{\cancel{91}}}{\cancel{14}\,2} = \frac{13}{2} = 6.5$ ✓ 2!.50

KAAM KI BAAT : Every Integer is a Rational Number

B'coz :

$2 = \frac{2}{1}$

$-3 = \frac{-3}{1}$

$7 = \frac{7}{1}$

$0 = \frac{0}{1}$

$\to \frac{p}{q}$ form $p, q \in I, q \neq 0$.

IRRATIONAL NO:s

↓

Neither Repeating nor Terminating Decimals.

Ex: $\sqrt{2} = 1.414\ldots$

$\sqrt{3} = 1.73205\ldots$

$\sqrt{5} = 2.236\ldots$

$e \approx 2.718\ldots$

$\pi \approx 3.14\ldots$

Division By 0 is not defined In Mathematics

Remainder Theorem

Let P(x) be a polynomial of degree ≥ 1 and 'a' is any real number. If P(x) is divided by (x – a), then the remainder is P(a).

$P(x) = x^3 - 3x^2 + 3x + 5.$

$$\begin{array}{r|l} & x^2 - 2x + 1 \\ \hline x-1 & x^3 - 3x^2 + 3x + 5. \\ & x^3 - x^2 \\ \hline & -2x^2 + 3x \\ & -2x^2 + 2x \\ \hline & x + 5. \\ & x - 1 \\ \hline & 6 \end{array}$$

Quotient

Dividend.

divisor.

Remainder.

by Remainder Theorem

$P(x) = x^3 - 3x^2 + 3x + 5.$

Remainder when divided by x-1

$P(1) = 1 - 3 + 3 + 5 = 6.$

Dividend = Quotient × Divisor + Remainder

$x^3 - 3x^2 + 3x + 5 = (x-1)(x^2 - 2x + 1) + 6.$

$x = 0$

$5 = (-1)\cdot 1 + 6 = 5$

$x = 1 \quad 1 - 3 + 3 + 5 = 0 + 6$

$6 = 6$

Trick

$x - a = 0$

$x = a$

$x + a = 0$

$x = -a$

$2x + 3 = 0$

$x = -3/2$

	Divisor	Remainder
P(x)	x-a	P(a)
	x+a	P(-a)
	2x+3	P(-3/2)
	3x-5	P(5/3)

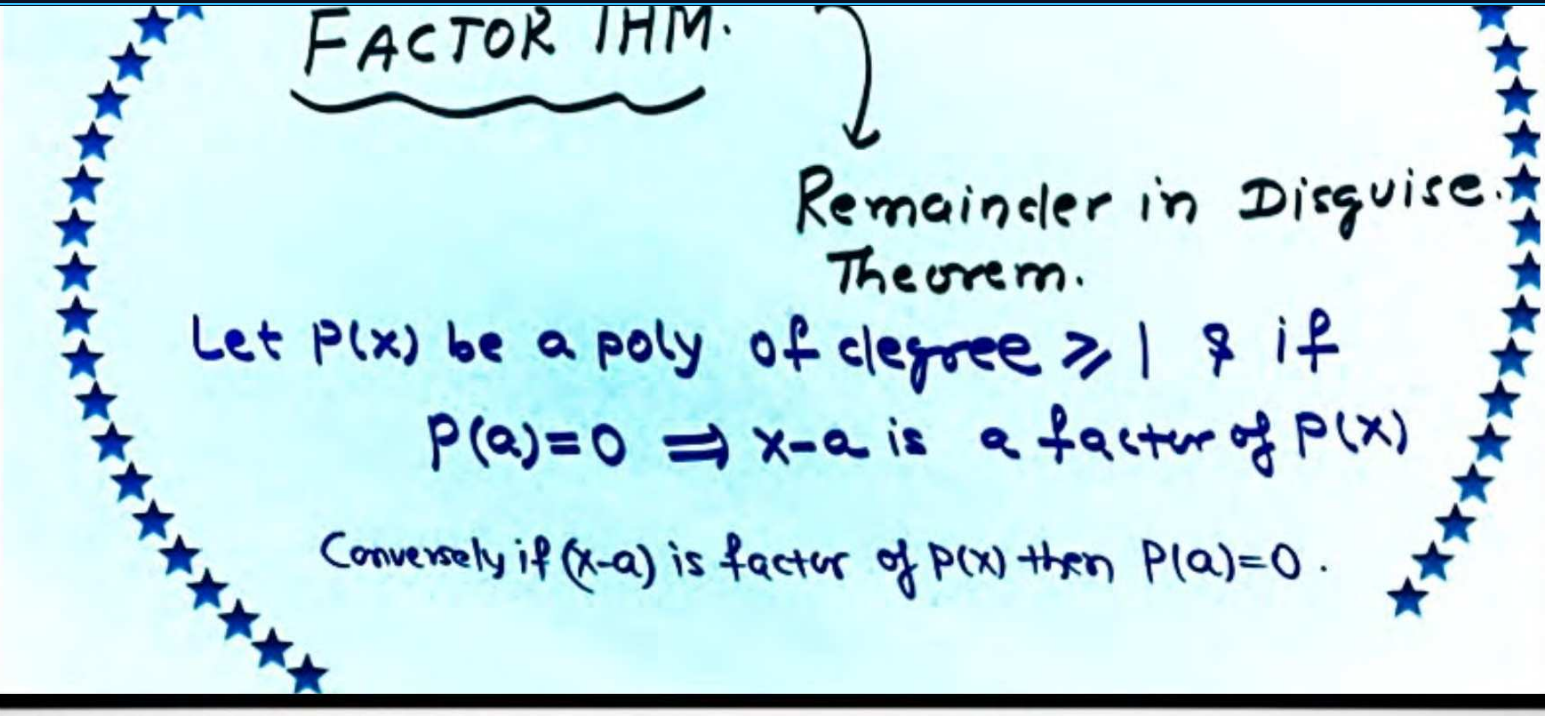
Remark

i. p(−a) is remainder on dividing p(x) by (x + a) $[\because x + a = 0 \Rightarrow x = -a]$

ii. $p\left(\frac{b}{a}\right)$ is remainder on dividing p(x) by (ax − b) $\left[\because ax - b = 0 \Rightarrow x = \frac{b}{a}\right]$

iii. $p\left(\frac{-b}{a}\right)$ is remainder on dividing p(x) by (ax + b) $\left[\because ax + b = 0 \Rightarrow x = -\frac{b}{a}\right]$

iv. $p\left(\frac{b}{a}\right)$ is remainder on dividing p(x) by (b − ax) $\left[\because b - ax = 0 \Rightarrow x = \frac{b}{a}\right]$

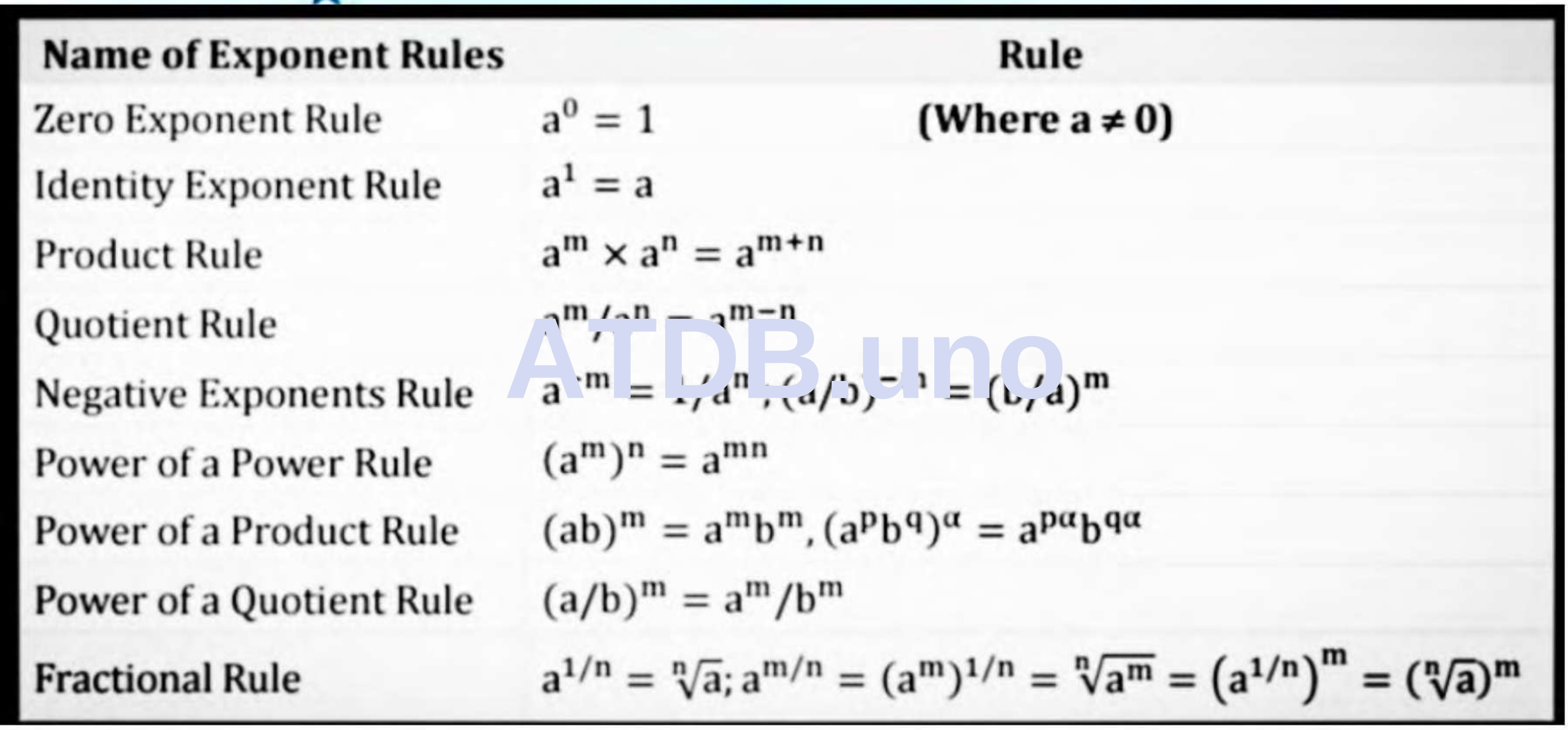


FACTOR THM. → Remainder in Disguise Theorem.

Let P(x) be a poly of degree ≥ 1 & if

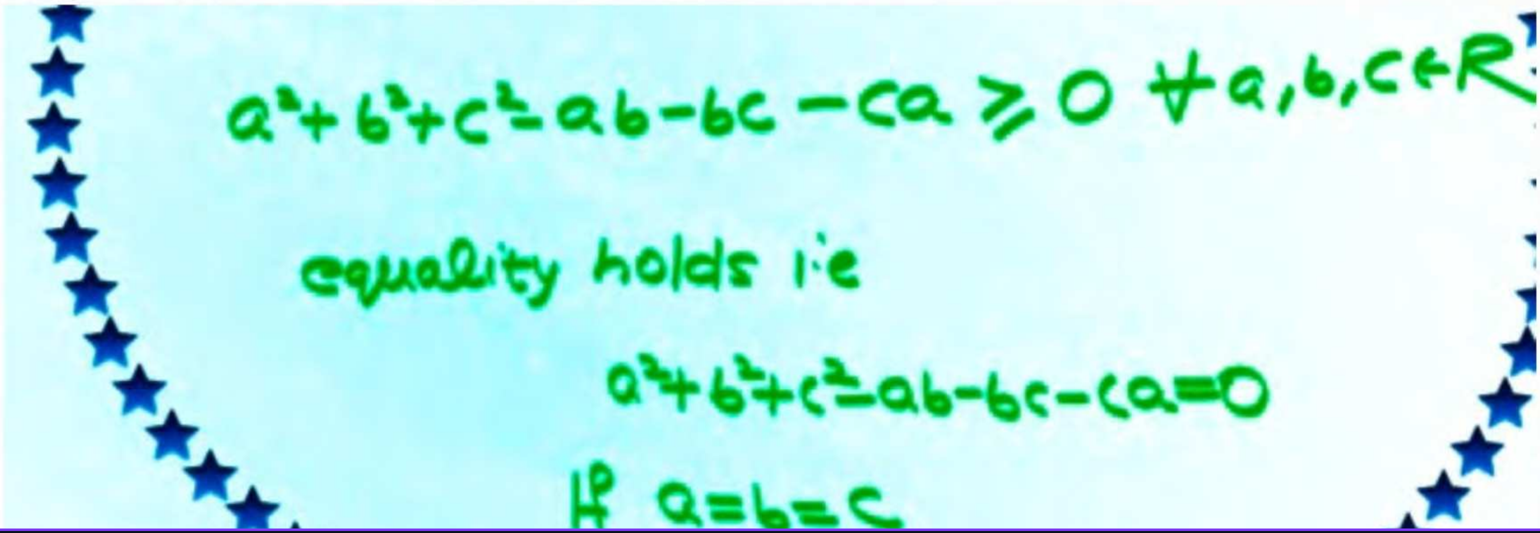
$P(a)=0 \Rightarrow x-a$ is a factor of P(x)

Conversely if $(x-a)$ is factor of P(x) then $P(a)=0$.



Name of Exponent Rules	Rule	
Zero Exponent Rule	$a^0 = 1$	**(Where $a \neq 0$)**
Identity Exponent Rule	$a^1 = a$	
Product Rule	$a^m \times a^n = a^{m+n}$	
Quotient Rule	$a^m/a^n = a^{m-n}$	
Negative Exponents Rule	$a^{-m} = 1/a^m, (a/b)^{-n} = (b/a)^m$	
Power of a Power Rule	$(a^m)^n = a^{mn}$	
Power of a Product Rule	$(ab)^m = a^m b^m, (a^p b^q)^\alpha = a^{p\alpha} b^{q\alpha}$	
Power of a Quotient Rule	$(a/b)^m = a^m/b^m$	
Fractional Rule	$a^{1/n} = \sqrt[n]{a}; a^{m/n} = (a^m)^{1/n} = \sqrt[n]{a^m} = \left(a^{1/n}\right)^m = (\sqrt[n]{a})^m$	

An Important Result



$a^2+b^2+c^2-ab-bc-ca \geq 0 \ \forall a,b,c \in R$

equality holds i.e

$a^2+b^2+c^2-ab-bc-ca=0$

iff $a=b=c$

An Important Result

★ If $x, y \in R$ & $x^2 + y^2 = 0 \Longrightarrow x = 0$ & $y = 0$

Generalization:

If $a_1, a_2, \dots a_n \in R$ then $a_1^2 + a_2^2 + \cdots + a_n^2 = 0$ then $a_1 = a_2 = \cdots = a_n = 0$

Ex: find x & y $4x^2 + 4x + 1 + y^2 - 6y + 9 = 0$, $x, y \in R$

$$(2x)^2 + 2 \cdot 2x \cdot 1 + 1^2 + y^2 - 2 \cdot 3 \cdot y + 3^2 = 0$$

$$\underset{\geq 0}{(2x+1)^2} + \underset{\geq 0}{(y-3)^2} = 0.$$

$$2x + 1 = 0 \ \& \ y - 3 = 0$$

$$x = -1/2 \ \& \ y = 3.$$

$$(\text{Number or Expression})^{\text{Even power}} \geq 0$$

$$\left(\begin{matrix}\text{Any positive} \\ \text{real number}\end{matrix}\right)^{\text{Any real power}} > 0$$

Algebraic Identities

I_1: $(a \pm b)^2 = a^2 + b^2 \pm 2ab$

I_2: $a^2 - b^2 = (a - b)(a + b)$

I_3: $a^3 + b^3 = (a + b)\left(a^2 - ab + b^2\right)$

I_4: $a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$

I_5: $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

I_6: $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

I_7: $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + bc)$ $= a^2 + b^2 + c^2 + 2abc\left(\frac{ab+bc+ca}{abc}\right)$

$= a^2 + b^2 + c^2 + 2abc(1/a + 1/b + 1/c)$ $\quad ||$

$a^2+b^2+c^2+2abc\left(\frac{ab}{abc}+\frac{bc}{abc}+\frac{ca}{abc}\right)$

I_8: $(a + b + c)^3 = a^3 + b^3 + c^3 + 3(a + b)(b + c)(c + a)$ $\quad a^2+b^2+c^2+2abc\left(\frac{1}{c}+\frac{1}{a}+\frac{1}{b}\right)$

I_9: $a^3 + b^3 + c^3 - 3abc = (a + b + c)\left(a^2 + b^2 + c^2 - ab - bc - ca\right)$

Yaad Rakhe !!

★ $\sqrt{x^2} = |x|$ ★ $\sqrt[3]{x^3} = x$

★ $\sqrt[2n]{x^{2n}} = |x|$ ★ $\sqrt[2n+1]{x^{2n+1}} = x.$

Some Golden Points

$\forall$ for all / for every

1. If a > 0 and D < 0 then $y = ax^2 + bx + c > 0$ for all $x \in R$

2. If a < 0 and D < 0 then $y = ax^2 + bx + c < 0$ for all $x \in R$

Ex: $P(x) = x^2 + x + 3$

$D = 1^2 - 4 \cdot 1 \cdot 3 = -11 < 0$

$a = 1 > 0$

$x^2 + x + 3 > 0 \ \forall \ x \in R$

for all or for every

$P(-2) = (-2)^2 - 2 + 3 = 5 > 0$

$P(0) = 0^2 + 0 + 3 = 3 > 0$

Ex: $P(x) = -x^2 + x - 3$

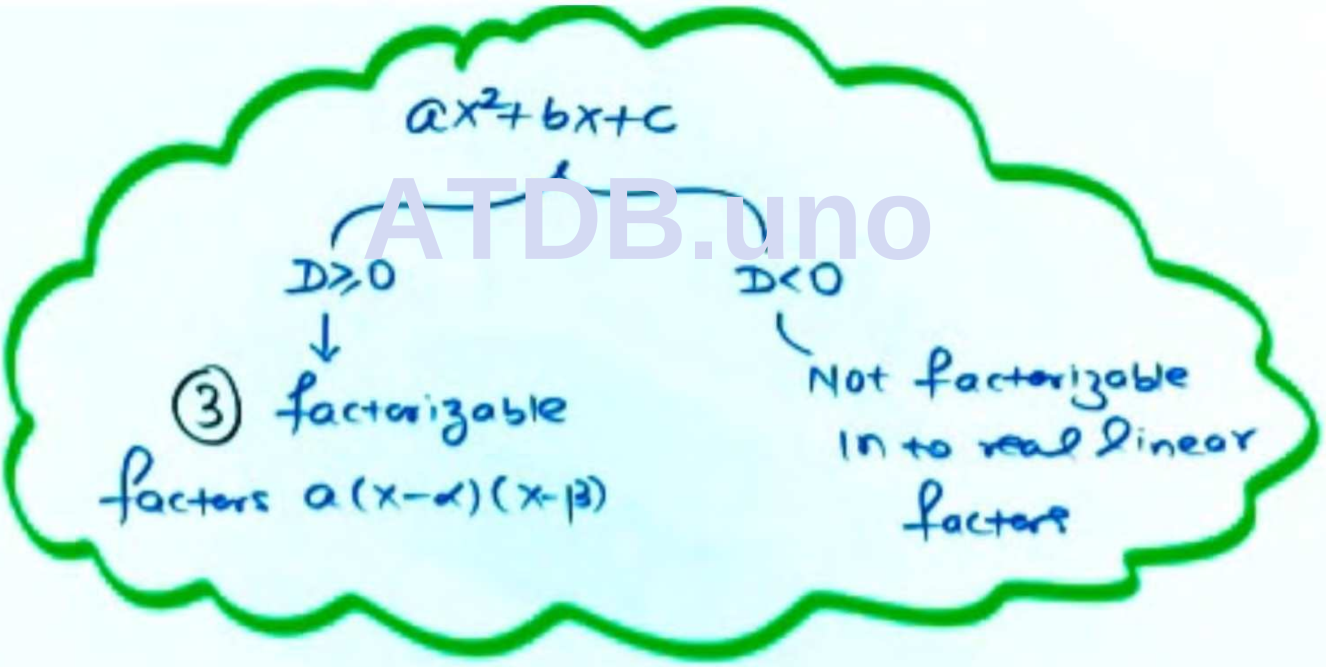
$D = 1^2 - 4(-1)(-3) = 1 - 12 = -11 <$

$a = -1 < 0$

$\Rightarrow -x^2 + x - 3 < 0 \ \forall \ x \in R$

$P(2) = -(2)^2 + 2 - 3 = -4 + 2 - 3 = -$

$P(0) = -0 + 0 - 3 = -3 < 0$



Inequalities

B1 : We can add (or subtract) any number 'k' on both sides of inequality. Doing this will not change the sign of inequality.

B2 : We can multiply (or divide) any non-zero number 'k' on both sides of inequality and sign of inequality will change according to sign of 'k' that is

- If $k > 0$ then sign of inequality will remains same,
- If $k < 0$ then sign of inequality will get reversed.

B3 : Squaring (raising even power both side) is only allowed when both sides of inequality are non negative.

$3 > 2$
$9 > 4$) S.B.S

Ex: $-2 > -3$
$4 > 9$ ✗) S.B.S

Ex: $5 > -2$
$25 > 4$) S.B.S

Ex: $3 > -7$
$9 > 49$ ✗) S.B.S

B4 : Raising both sides to odd power is fine.

Ex: $3 > 2$
$27 > 8$) C.B.S

Ex: $-2 > -3$
$-8 > -27$) C.B.S

Ex: $5 > -2$
$125 > -8$) C.B.S

Ex: $3 > -7$
$27 > -343$) C.B.S

Inequalities can be added provided they have same sign of inequality. But inequalities can not be subtracted.

Inequalities can be multiplied provided both sides are positive and have same sign of inequality, but they can not be divided.

Method of Intervals

Steps Involved

1. Make one side of inequality 0.
2. Factorize the non zero side in to linear factors
3. Put each linear factor equal to zero & find value of x.
4. Plot all values of x on a number line.
5. Start with a positive sign on the extreme right part & then place negative, positive signs alternately.

Important Point to Note

1. Values of x corresponding to denominator are never included in answer.
2. Coefficient of x in every linear factor should be positive if not then make it positive.

Case of Repeated Factors

B_1 : **Every odd integral power of a linear factor is treated as 1.**

B_2 : **In case of even power of any factor, first we assume that it is always positive. So we delete it from the inequality but in the end we make a direct check at that value of x where the deleted factor is zero.**

In case if a factor is eliminated

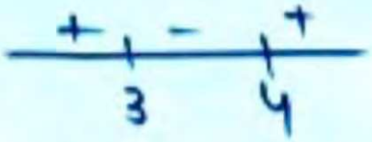
In this case the factor that is cancelled in the Numerator & Denominator, it is put not equal tozero & it's roots are never included in answer

Ex: $\frac{x^2-5x+6}{x^2-6x+8} > 0 \rightarrow \frac{x^2-3x-2x+6}{x^2-2x-4x+8} > 0$

$$\frac{\cancel{(x-2)}(x-3)}{\cancel{(x-2)}(x-4)} > 0$$

$\frac{x-3}{x-4} > 0 \rightarrow x-2 \neq 0$

i.e $x \neq 2$



$x \in (-\infty,3) \cup (4,\infty) - \{2\}$

* $x+\frac{1}{x} \geq 2$, $x \in R^+$
* $x+\frac{1}{x} \leq -2$, $x \in R^-$

* $\sqrt{x^2}=|x|$
* $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

* consists of two or more variables try to make perfect squares

*→ $a^2 \geq 0, a \in R$

* If $a_1, a_2, \dots, a_n \in R$ then $a_1^2+a_2^2+\dots+a_n^2=0$ then $a_1=a_2=\dots=a_n=0$

*→ In an inequality we can add or subtract any no. from both sides

→ If we multiply or divide both sides by any ~~no.~~ +ve no: sign of inequality remains same

→ If we multiply or divide both sides by any -ve no: sign of inequality reversed

* coeff of x in each linear factor should +ve

* ax^2+bx+c
  - → if $D<0, a>0$ → $ax^2+bx+c > 0 \ \forall x \in R$
  - → if $D<0, a<0$ → $ax^2+bx+c < 0 \ \forall x \in R$
  - → $D \geq 0$ then quad is factorizable into real linear factor
    $ax^2+bx+c = a(x-\alpha)(x-\beta)$
    $\alpha, \beta = \frac{-b \pm \sqrt{b^2-4a}}{2a}$

* $a^0 = 1 \ (a \neq 0)$ | 0^0 → not define

* $5>4 \xrightarrow{(\ )^2} 25>16$
Agar dono sides non-+ive ho then we can square

* $(3-x)^{\textcircled{2}} = (x-3)^{\textcircled{2}}$
but $(4-x)^{\textcircled{3}} \neq (x-4)^{\textcircled{3}}$

* $\log_a N$ denotes power to which should be raised in order to get

* $\log_a b = \frac{\log_c b}{\log_c a}$
* $\log_a b = \frac{1}{\log_b a}$

* $\log_a N$ is defined if $a>0, a \neq 1, N>0$

* $a^{\log_a N} = N, \log_a a = 1, \log_a \frac{1}{a} = -1, \log_{\frac{1}{a}} a = -1$ | * $a^{\log_c b} = b^{\log_c a}$

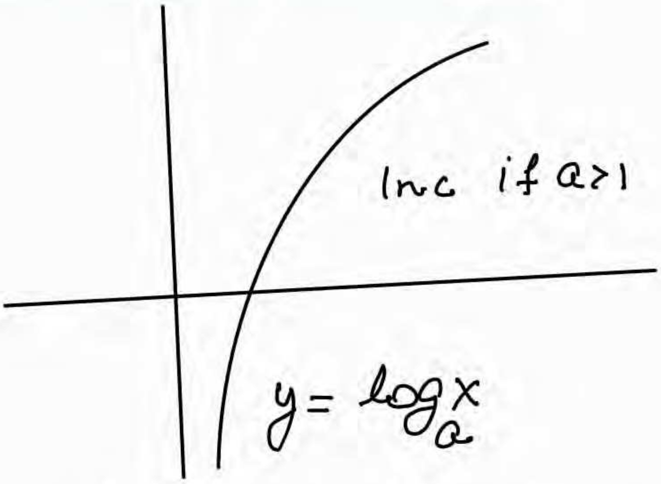
* $\log_a 1 = 0$ * $\log_1 1$ is not defined becoz it can not give definite value

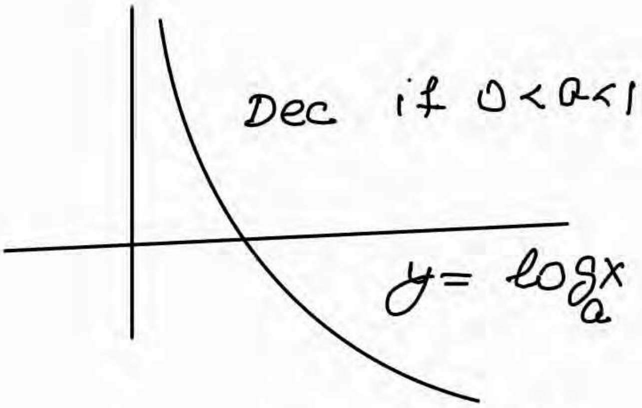
* $\log_{a^y} m^x = \frac{x}{y} \log_a m$

* $\log_{a_1} a_2 \cdot \log_{a_2} a_3 \cdot \log_{a_3} a_4 \dots \dots \log_{a_{n-1}} a_n = \log_{a_1} a_n$

* $\log_a(m \cdot n) = \log_a(m) + \log_a(n)$
* $\log_a\left(\frac{m}{n}\right) = \log_a(m) - \log_a(n)$
* $\log_a m^x = x \cdot \log_a m$

Logarithmic Inequalities

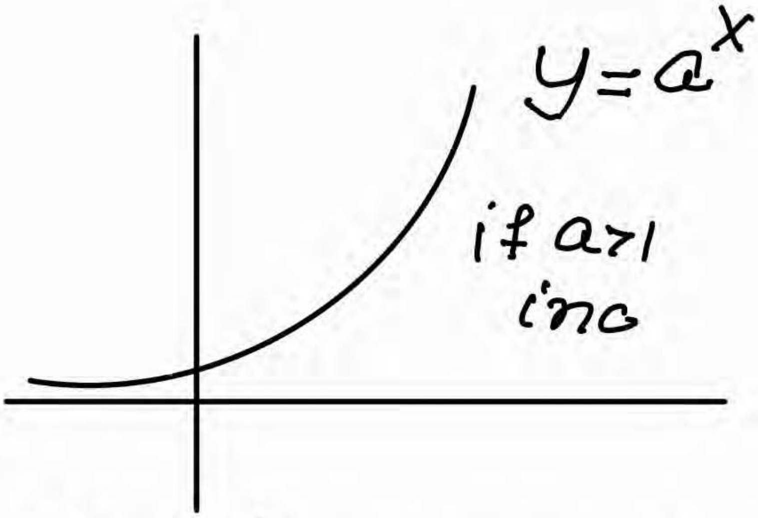


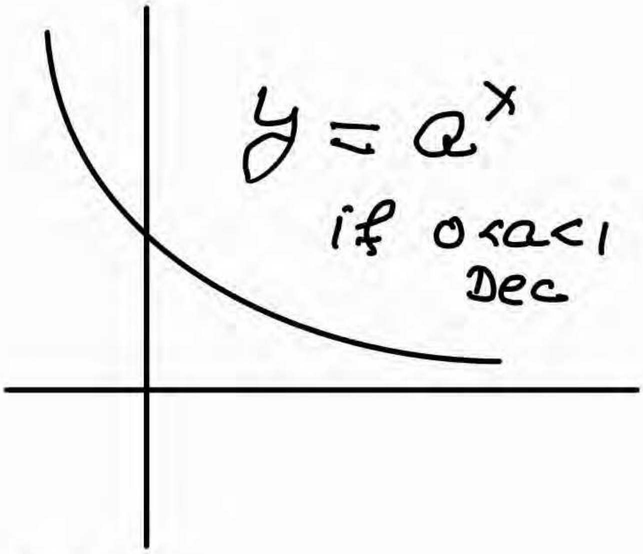


⇒ $\log_a x > \log_a y$ where $a>1$ then $x>y$

$\log_a x < \log_a y$ where $0<a<1$ then $x<y$

Exponential Inequalities





$a^x > a^y$ where $a>1$ ⇒ $x>y$

$a^x > a^y$ where $0<a<1$ ⇒ $x<y$.

या हटाओ Koi Farak nahi Padtaa

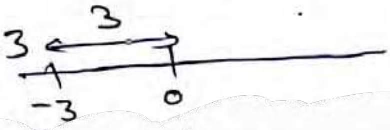
- Decrease Function kisi bhi Inequality mai lagao या हटाओ sign of inequality is reversed

Inc. Eg: a^x $(a>1)$, $\log_a(x)$ $(a>0)$

Dec. Eg: a^x $(0<a<1)$, $\log_a(x)$ $(0<a<1)$

- |x| denotes distance b/w 0 & x on numberline

eg: $|-3|=3$



$\boxed{|x| \geq 0} \rightarrow |x| \in [0, \infty)$

- $|-x| = |x|$, $|x|^2 = x^2$, $|xy| = |x|\cdot|y|$, $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ $y \neq 0$, $\sqrt{x^2} = |x|$

- $\sqrt[2n]{x^{2n}} = |x|$, $\sqrt[2n+1]{x^{2n+1}} = x$ * $|x| \geq x$ $\begin{cases} |x| = x \Leftrightarrow x \geq 0 \\ |x| > x \Leftrightarrow x < 0 \end{cases}$

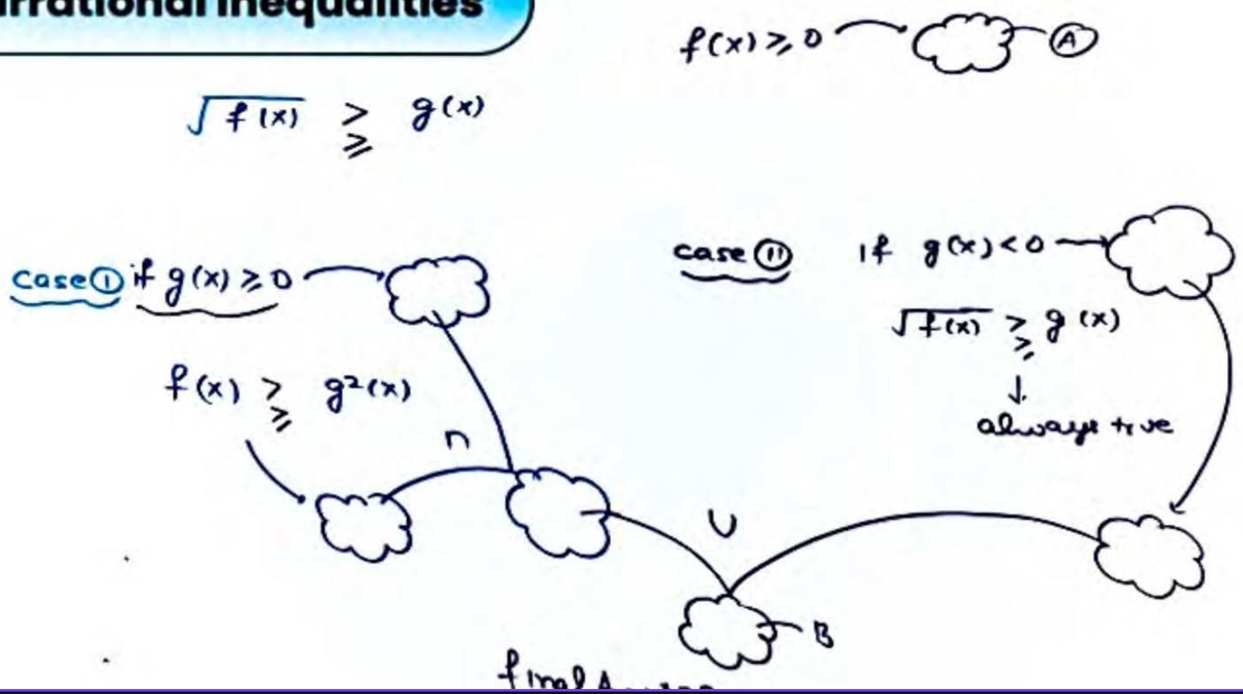
- $|x| \leq a$, $a \in R^+ \Rightarrow x \in [-a, a]$ * $|x| \geq a$, $a \in R^+$ $\Rightarrow$ $x \in (-\infty, -a] \cup [a, \infty)$

- $a \leq |x| \leq b$, $a, b \in R^+$

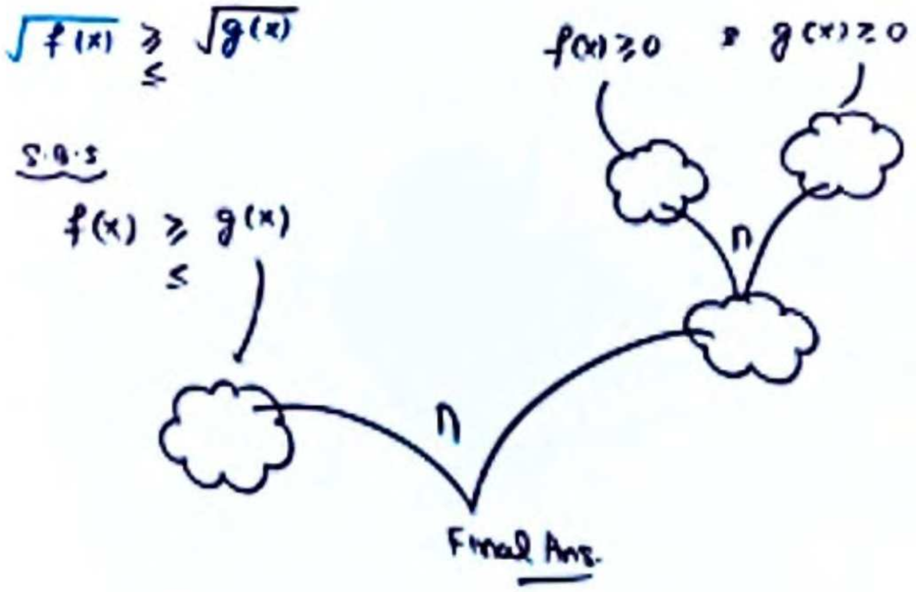
$\Rightarrow$ $a \leq x \leq b$ OR $-b \leq x \leq -a$

Irrational Inequalities

$\sqrt{f(x)} \underset{\geq}{>} g(x)$



Irrational Inequalities



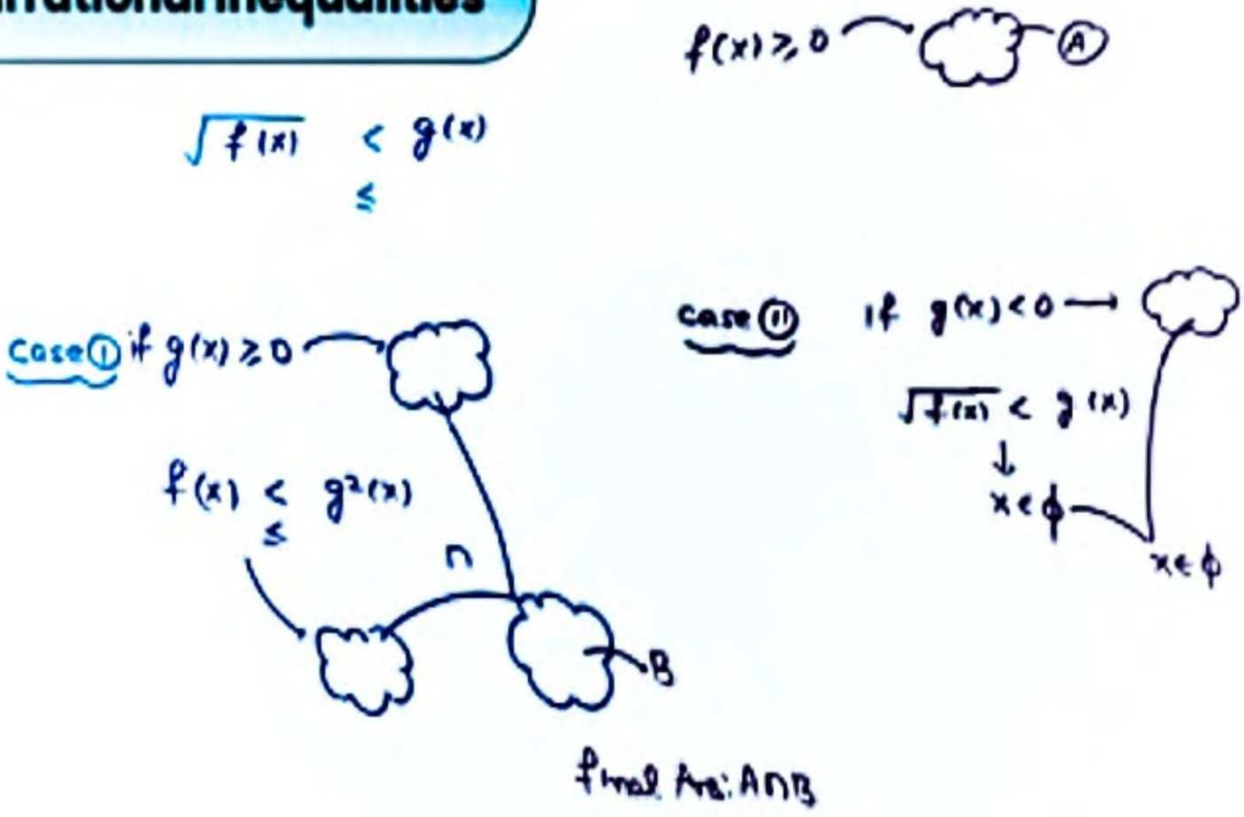
$\sqrt{f(x)} \underset{\leq}{\geq} \sqrt{g(x)}$

S.O.S

$f(x) \underset{\leq}{\geq} g(x)$

$f(x) \geq 0$ & $g(x) \geq 0$

Irrational Inequalities



$f(x) \geq 0$ → Ⓐ

$\sqrt{f(x)} < g(x)$
$\leq$

Case① if $g(x) \geq 0$

$f(x) \underset{\leq}{<} g^2(x)$

Case(II) if $g(x) < 0$ →

$\sqrt{f(x)} < g(x)$
↓
$x \in \phi$

$x \in \phi$

Final Ans: $A \cap B$

Using Triangle Inequality

P_9: $||a| - |b|| \le |a + b| \le |a| + |b|$

$|a+b| = |a| + |b| \Leftrightarrow ab \geq 0$

$|a+b| = ||a| - |b|| \Leftrightarrow ab \le 0$

Ex: $||2| - |-3|| \le |2 + (-3)| \le |2| + |-3|$

$1 \le 1 \le 5$

Ex: $||-5| - |-6|| \le |-5-6| \le |-5| + |-6|$

$1 \le 11 \le 11$

$b \to -b$

$||a| - |-b|| \le |a-b| \le |a| + |-b|$

P_{10}: $||a| - |b|| \le |a-b| \le |a| + |b|$

$|a-b| = |a| + |b| \Leftrightarrow a \cdot b \le 0$

$||a| - |b|| = |a-b| \Leftrightarrow ab \geq 0$

Very important points to Note

$||a| - |b|| = |a+b| \Leftrightarrow ab \le 0$

$||a| - |b|| \le |a+b| \le |a| + |b|$

$|a+b| = |a| + |b| \Leftrightarrow ab \geq 0$

$|a+b| < |a| + |b| \Leftrightarrow ab < 0.$

$||a| - |b|| < |a+b| \Leftrightarrow ab > 0.$

$||a| - |b|| = |a-b|$
$\Updownarrow$
$ab \geq 0$

★ $||a| - |b|| \le |a-b| \le |a| + |b|$

$|a-b| = |a| + |b| \Leftrightarrow ab \le 0$

$||a| - |b|| < |a-b| \Leftrightarrow ab < 0$

$|a-b| < |a| + |b| \Leftrightarrow ab > 0.$

Two Damdaar Properties

1. $|x + y| = |x| + |y| \iff x \cdot y \geq 0$

2. $|x - y| = |x| + |y| \iff x \cdot y \leq 0$

Ex: $\underbrace{|x^2-5x+7|}_{|a|} + \underbrace{|x^2-6x+5|}_{|b|} = |x+2|.$

$|a| + |b| = |x^2-5x+7-(x^2-6x+5)|$

$\overset{\shortparallel}{|a-b|}$

$\Downarrow$

$(x^2-5x+7)(x^2-6x+5) \leq 0.$

$D=(-5)^2-4\cdot 7<0$

$a=1>0 \Downarrow$

always +ve $\Rightarrow x^2-6x+5\leq 0$

$(x-5)(x-1)\leq 0 \rightarrow x\in[1,5]$

Ex: Solve:

$|x-3|+|2-x|=1 : |x-3+2-x|$

$|a| + |b| = |a+b|.$

$\Downarrow$

$(x-3)(2-x)\geq 0$

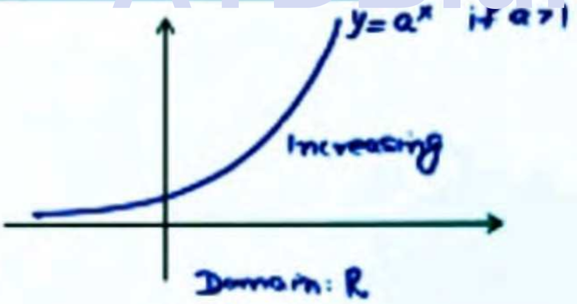
$-(x-3)(x-2)\geq 0$

$(x-3)(x-2)\leq 0$

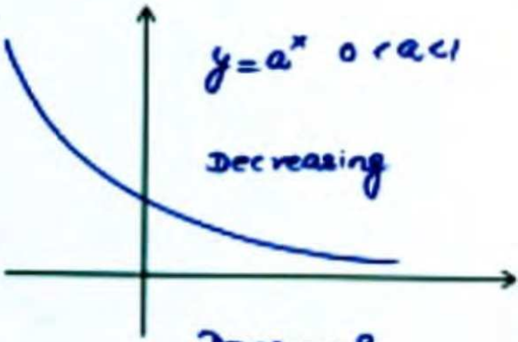
$\begin{array}{ccccc} + & & - & & + \\ \hline & 2 & & 3 & \end{array}$

$x\in[2,3].$

Exponential Inequalities



Domain: R
Range R^+



Domain : R
Range = R^+

Ex: $3^{x-2} > 3^{2x-3}$ find range of x

$x-2>2x-3$

$x<1 \Rightarrow x\in(-\infty,1)$

Ex: $\left(\frac{1}{2}\right)^{x^2-2x} > \left(\frac{1}{2}\right)^{4x-8}$ find range of x.

$x^2-2x<4x-8 \Rightarrow x^2-6x+8<0 \rightarrow (x-2)(x-4)<0$

$x\in(2,4)$

Characteristic & Mantissa

logarithm of any no: to a given base always has two parts an integral part called characteristic and a fractional part called Mantissa

★ Characteristic ∈ Integer.
Mantissa ∈ [0,1)

Ex: $\log_2 16 = 4 \cdot 0$

integral part → Characteristic = 4

fractional part → mantissa = 0

Ex: find characteristic of $\log_2 17$

clearly: $2^4 < 17 < 2^5 \rightarrow 4 < \log_2 17 < 5 \rightarrow \log_2 17 = 4\cdot$something

characteristic = 4

$\log_{10}$ N KIKAHAANI

Ex: $\log_2 1 = 0$, $\log_{10} 1 = 0$

Ex: $\log_2 0$ is not defined in reals.

$f \in [0,1)$

If $0 < N < 1$

characteristic

$N = \overrightarrow{0.689}, \overrightarrow{0.976}$ * $\frac{1}{10} \leq N < 1 \Rightarrow -1 \leq \log_{10} N < 0 \Rightarrow \log_{10} N = -1 + f$ | -1

$N = 0.\overrightarrow{078}, 0.\overrightarrow{09705}$ * $\underset{}{\overset{0.01}{\frac{1}{100}}} \leq N < \overset{0.1}{\frac{1}{10}} \Rightarrow -2 \leq \log_{10} N < -1 \Rightarrow \log_{10} N = -2 + f$ | -2

$N = \overrightarrow{.0078}, 0.\overrightarrow{00965}$ * $\underset{0.01}{\frac{1}{1000}} \leq N < \underset{0.01}{\frac{1}{100}} \Rightarrow -3 \leq \log_{10} N < -2 \Rightarrow \log_{10} N = -3 + f$ | -3

If $0<N<1$, $\log_{10} N$ has characteristic $= -$(No: of 0's immediately to right of decimal in N before a significant digit starts $+1$)

$\log_{10}$N KIKAHAANI

Ex: $\log_2 1 = 0$, $\log_{10} 1 = 0$

Ex: $\log_2 0$ is not defined in reals

$f \in [0,1)$

If $N \geq 1$

	Characteristic
$N = \overleftarrow{1}.63, \overleftarrow{9}.85$ * $1 \le N < 10 \Rightarrow 0 \le \log_{10} N < 1 \Rightarrow \log_{10} N = 0 + f$	0
$N = \overleftarrow{95}.62, \overleftarrow{88}.55$ * $10 \le N < 100 \Rightarrow 1 \le \log_{10} N < 2 \Rightarrow \log_{10} N = 1 + f$	1
$N = \overleftarrow{110}.23, \overleftarrow{999}.25$ * $100 \le N < 1000 \Rightarrow 2 \le \log_{10} N < 3 \Rightarrow \log_{10} N = 2 + f$	2

If N>1, $\log_{10} N$ has characteristic = (No: of significant digits to left of decimal in N) − 1

$$* \log_a N = \underset{\text{Integer part (characteristic)}}{I} + f \leftarrow \text{fractional part} \in [0,1)$$

(Mantissa) → (mantissa is never negative)

if $N \geq 1$ characteristic of $\log_{10} N$

= (No: of significant digits before decimal − 1)

if $0 < N < 1$

characteristic of $\log_{10} N$ =

= − (No. of zeros immediately after decimal before significant digits starts + 1)

* $\log_{10} 2 = 0.3010$
* $\log_{10} 3 = 0.4771$
* $\log_{10} 7 = 0.8451$

$\pi \simeq 3.14$

$\pi^2 \simeq g$

$\pi/2 \simeq 1.57$

$3\pi/2 \simeq 4.7$

- * $a \le x < b$ $x, a, b \in I$

  No: of possible values of $x = b-a$

- * $a < x \le b$, $x, a, b \in I$

  No: of possible values of $x = b-a$

- * $a \le x \le b$, $x, a, b \in I$

  No: of possible values of $x = b-a+1$

- * $a < x < b$, $a, b, x \in I$

  No: of values of $x = b-a-1$

b-a se eak end point answer mai shaamil hotaa hai

If $\frac{a}{b} = \frac{c}{d}$ then

* Componendo Dividendo $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ or $\frac{a-b}{a+b} = \frac{c-d}{c+d}$

* $\frac{a \pm b}{b} = \frac{c \pm d}{d}$ or $\frac{a}{a \pm b} = \frac{c}{c \pm d}$

If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \text{- - - -}$ then

* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \text{- - -} = \frac{k_1a_1 + k_2b_1 + k_3c_1 + \text{- -}}{k_1a_2 + k_2b_2 + k_3c_2 + \text{- -}}$

* $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \text{- - -} = \frac{a_1 + b_1 + c_1 \text{- -}}{a_2 + b_2 + c_2 + \text{- - -}}$

Ex: find α, β if $3\alpha + 2\beta = 13$

& $\frac{\alpha-2}{1} = \frac{\beta-1}{2}$

$$\frac{\alpha-2}{1} = \frac{\beta-1}{2} = \frac{\overset{\frown}{3\alpha-6+2\beta}-2}{3+4}$$

$$\frac{\alpha-2}{1} = \frac{\beta-1}{2} = \frac{13-8}{7} = 5/7$$

$\alpha = 19/7$, $\beta = 1 + 2 \cdot /7 = 15/7$

The End