Most Repeated PYQ's

For Class – 12th Session (2025_2026)

Exercise

Relations And Function

1. A function $f : A \to B$ defined as $f(x) = 2x$ is both one-one and onto. If $A = \{1, 2, 3, 4\}$, then find the set B. **(2023)**

2. A relation R is defined on a set of real numbers $\mathbb{R}$ as $R = \{(x, y) : x \cdot y \text{ is an irrational number}\}$. Check whether R is reflexive, symmetric and transitive or not. **(2023)**

3. A function $f : R_+ \to R$ (where R_+ is the set of all non-negative real numbers) defined by $f(x) = 4x + 3$ is : **(2024)**

(1) One-one but not onto
(2) Onto but not one-one
(3) Both one-one and onto
(4) Neither one-one nor onto

4. A relation R on set $A = \{1, 2, 3, 4, 5\}$ is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$.

Check whether the relation R is reflexive, symmetric and transitive. **(2024)**

5. A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function $f(x)$. Hence, check whether function $f(x)$ is one-one and onto or not. **(2024)**

6. Assertion (A) : Let Z be the set of integers. A function $f : Z \to Z$ defined as $f(x) = 3x - 5, \forall\, x \in Z$ is a bijective.

Reason (R) : A function is a bijective if it is both surjective and injective. **(2025)**

(1) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(2) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(3) Assertion (A) is true, but Reason (R) is false.
(4) Assertion (A) is false, but Reason (R) is true.

7. A class-room teacher is keen to assess the learning of her students the concept of "relations" taught to them. She writes the following five relations each defined on the set $A = \{1, 2, 3\}$: **(2025)**

$R_1 = \{(2, 3), (3, 2)\}$

$R_2 = \{(2, 2), (2, 3), (3, 2)\}$

$R_3 = \{(1, 2), (2, 1), (1, 1)\}$

$R_4 = \{(1, 1), (1, 2), (3, 3), (2, 2)\}$

$R_5 = \{(1, 1), (1, 2), (3, 3), (2, 2), (2, 1), (2, 3), (3, 2)\}$

The students are asked to answer the following questions about the above relations :

(i) Identify the relation which is reflexive, transitive but not symmetric.

8. Define the relation R in the set $N \times N$ as follows: For $(a, b), (c, d) \in N \times N$, $(a, b)\, R(c, d)$, iff $ad = bc$. Prove that R is an equivalence relation in $N \times N$. **[SQP(2023)]**

9. Given a non-empty set X, define the relation R in P(X) as follows:

For $A, B \in P(X), (A, B) \in R$, iff $A \subset B$.

Prove that R is reflexive, transitive and not symmetric. **[SQP(2023)]**

10. Assertion (A) : The relation $f:\{1,2,3,4\}\to\{x,y,z,p\}$ defined by $f=\{(1,x),(2,y),(3,z)\}$ is a bijective function. **[SQP(2024)]**

Reason (R) : The function $f:\{1,2,3\}\to\{x,y,z,p\}$ such that $f=\{(1,x),(2,y),(3,z)\}$ is one-one.

(1) Both (A) and (R) are true and (R) is the correct explanation of (A).
(2) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(3) (A) is true but (R) is false.
(4) (A) is false but (R) is true.

11. Let $\mathbb{N}$ be the set of all natural numbers and R be a relation on $\mathbb{N}\times\mathbb{N}$ defined by $(a, b)\mathrm{R}(c, d) \Leftrightarrow ad = bc$ for all $(a, b), (c, d) \in \mathbb{N}\times\mathbb{N}$.
Show that R is an equivalence relation on $\mathbb{N}\times\mathbb{N}$. Also, find the equivalence class of (2, 6), i.e., [(2, 6)]. **[SQP(2024)]**

12. Assertion (A): The function $f:R-\left\{(2n+1)\frac{\pi}{2}:n\in Z\right\}\to(-\infty,-1]\cup[1,\infty)$ defined by $f(x)=\sec x$ is not one-one function in its domain.
Reason (R) : The line $y = 2$ meets the graph of the function at more than one point. **[SQP(2025)]**

(1) Both (A) and (R) are true and (R) s the correct explanation of (A).
(2) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(3) (A) is true but (R) is false.
(4) (A) is false but (R) is true.

Paragarph (3 Ques.) **[SQP(2025)]**

An organization conducted bike race under 2 different categories-boys and girls. In all, there were 250 participants. Among all of them finally three from Category 1 and two from Category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.
Let B = $\{b_1, b_2, b_3\}$, G = $\{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race.
Ravi decides to explore these sets for various types of relations and functions.
On the basis of the above information, answer the following questions:

13. Ravi wishes to form all the relations possible from B to G. How many such relations are possible?

14. Write the smallest equivalence relation on G.

15. Ravi defines a relation from B to B as $\mathrm{R}_1=\{(b_1,b_2),(b_2,b_1)\}$. Write the minimum ordered pairs to be added in R_1 so that it becomes (A) reflexive but not symmetric, (B) reflexive and symmetric but not transitive.

16. If the track of the final race (for the biker b_1) follows the curve $x^2 = 4y$; (where $0 \le x \le 20\sqrt{2}$ & $0 \le y \le 200$), then state whether the track represents a one-one and onto function or not. (Justify).

Inverse Trigonometric Function

1. Assertion (A) : The range of the function $f(x) = 2\sin^{-1}x + \frac{3\pi}{2}$, where $x \in [-1, 1]$, is $\left[\frac{\pi}{2},\frac{5\pi}{2}\right]$.
Reason (R) : The range of the principal value branch of $\sin^{-1}x$ is $[0,\pi]$. **(2023)**

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(2) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(3) Assertion (A) is true and Reason (R) is false.
(4) Assertion (A) is false and Reason (R) is true.

2. If a function $f:\mathrm{X}\to\mathrm{Y}$ defined as $f(x)=y$ is one-one and onto, then we can define a unique function $g:\mathrm{Y}\to\mathrm{X}$ such that $g(y)=x$, where $x\in\mathrm{X}$ and $y=f(x), y\in\mathrm{Y}$. Function g is called the inverse of function f.
The domain of sine function is R and function sine : $\mathrm{R}\to\mathrm{R}$ is neither one-one nor onto. The following graph shows the sine function.
Let sine function be defined from set A to [–1, 1] such that inverse of sine function exists, i.e., $\sin^{-1}x$ is defined from [–1, 1] to A.
On the basis of the above information, answer the following questions:

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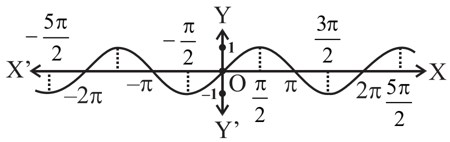
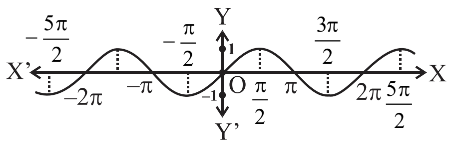
(i) If $\sin^{-1}(x)$ is defined from [–1, 1] to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. **(2024)**



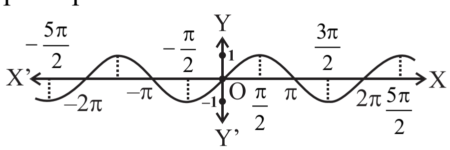
$y = \sin x$

(ii) If A is the interval other than principal value branch, give an example of one such interval.



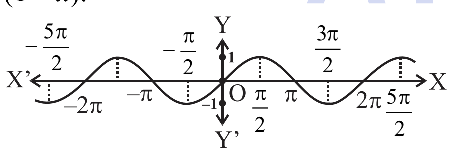
$y = \sin x$

(iii) Draw the graph of $\sin^{-1}x$ from [–1, 1] to its principal value branch.



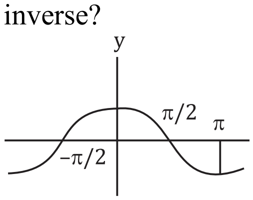
$y = \sin x$

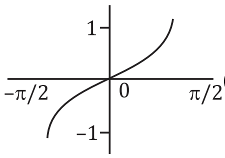
(iv) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$.



$y = \sin x$

3. The graph of a trigonometric function is as shown. Which of the following will represent graph of its inverse? **(2025)**



(1)


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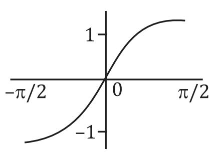

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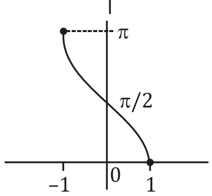
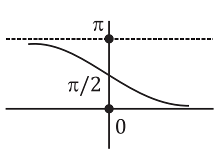


(4)


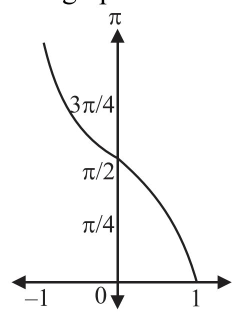
4. Evaluate : $\tan^{-1}\left[2\sin\left(2\cos^{-1}\frac{\sqrt{3}}{2}\right)\right]$ **(2025)**

5. Find the value of $\sin^{-1}\left[\sin\left(\frac{13\pi}{7}\right)\right]$ **[SQP(2023)]**

6. Find the value of $\sin^{-1}\left(\cos\left(\frac{33\pi}{5}\right)\right)$. **[SQP(2024)]**

7. Find the domain of $\sin^{-1}(x^2 - 4)$. **[SQP(2024)]**

8. The graph drawn below depicts **[SQP(2025)]**



(1) $y = \sin^{-1}x$ (2) $y = \cos^{-1}x$
(3) $y = \text{cosec}^{-1}x$ (4) $y = \cot^{-1}x$

9. If $\cot^{-1}(3x+5) > \frac{\pi}{4}$, then find the range of the values of x. **[SQP(2025)]**

Matrices

1. If for a square matrix $A, A^2 - 3A + I = O$ and $A^{-1} = xA + yI$, then the value of $x + y$ is : **(2023)**

(1) –2 (2) 2
(3) 3 (4) –3

2. If $A = \begin{bmatrix} 3 & 4 \\ 5 & 2 \end{bmatrix}$ and 2A + B is a null matrix, then B is equal to : **(2023)**

(1) $\begin{bmatrix} 6 & 8 \\ 10 & 4 \end{bmatrix}$ (2) $\begin{bmatrix} -6 & -8 \\ -10 & -4 \end{bmatrix}$

(3) $\begin{bmatrix} 5 & 8 \\ 10 & 3 \end{bmatrix}$ (4) $\begin{bmatrix} -5 & -8 \\ -10 & -3 \end{bmatrix}$

3. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ and $(3I + 4A)(3I - 4A) = x^2 I$, then the value(s) x is/are : **(2023)**

(1) $\pm\sqrt{7}$ (2) 0
(3) ± 5 (4) 25

4. If a matrix has 36 elements, the number of possible orders it can have is: **(2024)**

(1) 13 (2) 3
(3) 5 (4) 9

5. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is : **(2024)**

(1) 7 (2) 6
(3) 8 (4) 18

6. If A and B are two non-zero square matrices of same order such that $(A+B)^2 = A^2 + B^2$, then : **(2024)**

(1) $AB = O$ (2) $AB = -BA$
(3) $BA = O$ (4) $AB = BA$

7. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is **(2025)**

(1) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(3) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (4) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8. Let $A = \begin{bmatrix} 1 & -2 & -1 \\ 0 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}, C = \begin{bmatrix} 9 & 8 & 7 \end{bmatrix}$, which of the following is defined? **(2025)**

(1) Only AB (2) Only AC
(2) Only BA (4) All AB, AC and BA

9. If $A = \begin{bmatrix} 7 & 0 & x \\ 0 & 7 & 0 \\ 0 & 0 & y \end{bmatrix}$ is a scalar matrix, then y^x is equal to **(2025)**

(1) 0 (2) 1
(3) 7 (4) ± 7

10. If A and B are invertible matrices, then which of the following is not correct ? **(2025)**

(1) $(A+B)^{-1} = B^{-1} + A^{-1}$

(2) $(AB)^{-1} = B^{-1} A^{-1}$

(3) $\text{adj}(A) = |A| A^{-1}$

(4) $|A|^{-1} = |A^{-1}|$

11. If $A = [a_{ij}]$ is a skew-symmetric matrix of order n, then **[SQP(2023)]**

(1) $a_{ij} = \frac{1}{ji} \forall i, j$

(2) $a_{ij} \neq 0 \forall i, j$

(3) $a_{ij} = 0$, where $i = j$

(4) $a_{ij} \neq 0$ where $i = j$

12. If A, B are non-singular square matrices of the same order, then $(AB^{-1})^{-1} =$ **[SQP(2023)]**

(1) $A^{-1}B$ (2) $A^{-1}B^{-1}$
(3) BA^{-1} (4) AB

13. If $A = [a_{ij}]$ is a square matrix of order 2 such that $a_{ij} = \begin{cases} 1, & \text{when } i \neq j \\ 0, & \text{when } i = j \end{cases}$, then A^2 is **[SQP(2024)]**

(1) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}_{2\times 2}$ (2) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}_{2\times 2}$

(3) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}_{2\times 2}$ (4) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$

14. If A and B are invertible square matrices of the same order, then which of the following is not correct? **[SQP(2024)]**

(1) $\left|AB^{-1}\right| = \dfrac{|A|}{|B|}$

(2) $\left|(AB)^{-1}\right| = \dfrac{1}{|A||B|}$

(3) $(AB)^{-1} = B^{-1}A^{-1}$

(4) $(A+B)^{-1} = B^{-1} + A^{-1}$

15. Assume X, Y, Z, W and P are matrices of order $2 \times n, 3 \times k, 2 \times p, n \times 3$ and $p \times k$, respectively. Then the restriction on n, k and p so that $PY + WY$ will be defined are: **[SQP(2025)]**

(1) $k = 3, p = n$

(2) k is arbitrary, $p = 2$

(3) p is arbitrary, $k = 3$

(4) $k = 2, p = 3$

16. If $A = \begin{bmatrix} 0 & 1 & c \\ -1 & a & -b \\ 2 & 3 & 0 \end{bmatrix}$ is a skew-symmetric matrix then the value of $a + b + c =$ **[SQP(2025)]**

(1) 1 (2) 2

(3) 3 (4) 4

17. The equation of the path traversed by the ball headed by the footballer is $y = ax^2 + bx + c$; (where $0 \le x \le 14$ and $a, b, c \in R$ and $a \ne 0$) with respect to a XY-coordinate system in the vertical plane. The ball passes through the points (2, 15), (4, 25) and (14, 15). Determine the values of a, b and c by solving the system of linear equations in a, b and c, using matrix method. Also find the equation of the path traversed by the ball. **[SQP(2025)]**

Determinants

1. If $|A| = 2$, where A is a 2×2 matrix, then $\left|4A^{-1}\right|$ equals: **(2023)**

(1) 4 (2) 2

(3) 8 (4) $\dfrac{1}{32}$

2. Let A be a 3×3 matrix such that $|\text{adj}A| = 64$. Then $|A|$ is equal to : **(2023)**

(1) 8 only (2) –8 only

(3) 64 (4) 8 or –8

3. If $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$, find $(AB)^{-1}$. **(2023)**

4. Solve the following system of equations by matrix method : **(2023)**

$x + 2y + 3z = 6$

$2x - y + z = 2$

$3x + 2y - 2z = 3$

5. $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to : **(2024)**

(1) $2x^3$ (2) 2

(3) 0 (4) $2x^3 - 2$

6. Assertion (A) : For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $|A| \in [2, 4]$

Reason (R) : $\cos\theta \in [-1, 1], \forall\ \theta \in [0, 2\pi]$. **(2024)**

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(2) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(3) Assertion (A) is true, but Reason (R) is false.

(4) Assertion (A) is false, but Reason (R) is true.

7. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$x - 2y = 10, 2x - y - z = 8, -2y + z = 7$ **(2024)**

8. If $A=\begin{bmatrix}-1 & a & 2\\ 1 & 2 & x\\ 3 & 1 & 1\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & -1 & 1\\ -8 & 7 & -5\\ b & y & 3\end{bmatrix}$, find the value of $(a+x)-(b+y)$. **(2024)**

9. If A is a square matrix of order 2 such that det(A) = 4, then det(4 adjA) is equal to : **(2025)**

(1) 16 (2) 64
(3) 256 (4) 512

10. A school wants to allocate students into three clubs : Sports, Music and Drama, under following conditions :

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.
- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method. **(2025)**

11. If A is a square matrix of order 3, $|A'|=-3$, then $|AA'|=$ **[SQP(2023)]**

(1) 9 (2) –9
(3) 3 (4) –3

12. If $\begin{vmatrix}2 & 4\\ 5 & 1\end{vmatrix}=\begin{vmatrix}2x & 4\\ 6 & x\end{vmatrix}$, then the possible value(s) of 'x' is / are **[SQP(2023)]**

(1) 3 (2) $\sqrt{3}$
(3) $-\sqrt{3}$ (4) $\sqrt{3},-\sqrt{3}$

13. If A is a square matrix of order 3 and $|A|=5$, then $|\text{adj}A|=$ **[SQP(2023)]**

(1) 5 (2) 25
(3) 125 (4) $\frac{1}{5}$

14. If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}. Use A^{-1} to solve the following system of equations $2x-3y+5z=11, 3x+2y-4z=-5, x+y-2z=-3$ **[SQP(2023)]**

15. If the area of the triangle with vertices (–3, 0), (3, 0) and (0, k) is 9 sq. units, then the value/s of k will be **[SQP(2024)]**

(1) 9 (2) ±3
(3) –9 (4) 6

16. The value of $|A|$, if $A=\begin{bmatrix}0 & 2x-1 & \sqrt{x}\\ 1-2x & 0 & 2\sqrt{x}\\ -\sqrt{x} & -2\sqrt{x} & 0\end{bmatrix}$, where $x\in\mathbb{R}^+$, is **[SQP(2024)]**

(1) $(2x+1)^2$ (2) 0
(3) $(2x+1)^3$ (4) $(2x-1)^2$

17. Given that A is a square matrix of order 3 and $|A|=-2$, then $|\text{adj}(2A)|$ is equal to **[SQP(2024)]**

(1) -2^6 (2) +4
(3) -2^8 (4) 2^8

18. Using the matrix method, solve the following system of linear equations :

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.$$

[SQP(2024)]

19. If for a square matrix A, A.(adjA) $=\begin{bmatrix}2025 & 0 & 0\\ 0 & 2025 & 0\\ 0 & 0 & 2025\end{bmatrix}$, then the value of $|A|+|\text{adj}A|$ is equal to: **[SQP(2025)]**

(1) 1 (2) $2025+1$
(3) $(2025)^2+45$ (4) $2025+(2025)^2$

20. If A and B are non-singular matrices of same order with $\det(A) = 5$, then $\det\left(B^{-1}AB\right)^2$ is equal to **[SQP(2025)]**

(1) 5 (2) 5^2
(3) 5^4 (4) 5^5

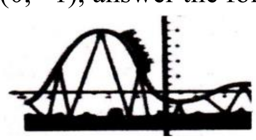
Continuity And Differentiability

1. The value of k for which $f(x) = \begin{cases} 3x+5, & x \ge 2 \\ kx^2, & x < 2 \end{cases}$ is a continuous function, is: **(2023)**

(1) $-\frac{11}{4}$ (2) $\frac{4}{11}$
(3) 11 (4) $\frac{11}{4}$

2. If $y = \left(x + \sqrt{x^2 - 1}\right)^2$, then show that $\left(x^2 - 1\right)\left(\frac{dy}{dx}\right)^2 = 4y^2$. **(2023)**

3. The equation of the path traced by a roller-coaster is given by the polynomial $f(x) = a(x + 9)(x + 1)(x - 3)$. If the roller-coaster crosses y-axis at a point (0, –1), answer the following : **(2023)**



(i) Find the value of 'a'.
(ii) Find $f''(x)$ at $x = 1$

4. The derivative of $\sin(x^2)$ w.r.t. x, at $x = \sqrt{\pi}$ is : **(2024)**

(1) 1 (2) –1
(3) $-2\sqrt{\pi}$ (4) $2\sqrt{\pi}$

5. Check whether the function $f(x) = x^2|x|$ is differentiable at $x = 0$ or not. **(2024)**

6. If $y = \sqrt{\tan\sqrt{x}}$, prove that $\sqrt{x}\frac{dy}{dx} = \frac{1+y^4}{4y}$. **(2024)**

7. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. **(2024)**

8. If $y = (\tan x)^x$, then find $\frac{dy}{dx}$. **(2024)**

9. If $f(x) = |x| + |x - 1|$, then which of the following is correct? **(2025)**

(1) $f(x)$ is both continuous and differentiable, at $x = 0$ and $x = 1$.
(2) $f(x)$ is differentiable but not continuous, at $x = 0$ and $x = 1$.
(3) $f(x)$ is continuous but not differentiable, at $x = 0$ and $x = 1$.
(4) $f(x)$ is neither continuous nor differentiable, at $x = 0$ and $x = 1$.

10. Assertion (A) : $f(x) = \begin{cases} 3x-8, & x \le 5 \\ 2k, & x > 5 \end{cases}$ is continuous at $x = 5$ for $k = \frac{5}{2}$.

Reason (R) : For a function f to be continuous at $x = a$, $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$. **(2025)**

(1) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).
(2) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(3) Assertion (A) is true, but Reason (R) is false.
(4) Assertion (A) is false, but Reason (R) is true.

11. Differentiate $2^{\cos^2 x}$ w.r.t $\cos^2 x$. **(2025)**

12. If $\tan^{-1}\left(x^2 + y^2\right) = a^2$, then find $\frac{dy}{dx}$. **(2025)**

13. If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, then prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$. **(2025)**

14. If $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right)$ and $y = \sin\theta$, then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{4}$. **(2025)**

15. Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1, & x = 0 \end{cases}$? **(2024)**

(1) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R}$
(2) $f(x)$ is continuous $\forall x \in \mathbb{R}$
(3) $f(x)$ is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$
(4) $f(x)$ is discontinuous at infinitely many points

16. The value of 'k' for which the function

$$f(x) = \begin{cases} \frac{1 - \cos 4x}{8x^2}, \text{if } x \neq 0 \\ k, \text{if } x = 0 \end{cases}$$

is continuous at $x = 0$ is **[SQP(2023)]**

(1) 0 (2) –1
(3) 1 (4) 2

17. If $y = \sin^{-1} x$, then $(1 - x^2)y_2$ is equal to **[SQP(2023)]**

(1) xy_1 (2) xy
(3) xy_2 (4) x^2

18. If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$, then prove that $\frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$ **[SQP(2023)]**

19. If $f(x) = \begin{cases} \frac{kx}{|x|}, & \text{if } x < 0 \\ 3, & \text{if } x \geq 0 \end{cases}$ is continuous at $x = 0$, then the value of k is

(1) –3 (2) 0
(3) 3 (4) any real number

20. If $(a + bx)e^{\frac{y}{x}} = x$, then prove that $x\frac{d^2y}{dx^2} = \left(\frac{a}{a + bx}\right)^2$. **[SQP(2024)]**

21. The function $f : \mathrm{R} \to \mathrm{Z}$ defined by $f(x) = [x]$; where [.] denotes the greatest integer function, is **[SQP(2025)]**

(1) Continuous at x = 2.5 but not differentiable at x = 2.5
(2) Not Continuous at x = 2.5 but differentiable at x = 2.5
(3) Not Continuous at x = 2.5 and not differentiable at x = 2.5
(4) Continuous as well as differentiable at x = 2.5

22. Assertion (A) : Consider the function defined as $f(x) = |x| + |x - 1|$, $x \in \mathrm{R}$. Then $f(x)$ is not differentiable at x = 0 and x = 1.
Reason (R) : Suppose f be defined and continuous on (a, b) and $c \in (a, b)$, then $f(x)$ is not differentiable at $x = c$, if

$$\lim_{h\to 0^-} \frac{f(c+h) - f(c)}{h} \neq \lim_{h\to 0^-} \frac{f(c+h) - f(c)}{h}.$$

[SQP(2025)]

(1) Both (A) and (R) are true and (R) is the correct explanation of (A).
(2) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(3) (A) is true but (R) is false.
(4) (A) is false but (R) is true.

23. The cost (in rupees) of producing x items in factory, each day is given by

$$\mathrm{C}(x) = 0.00013x^3 + 0.002x^2 + 5x + 2200$$

Find the marginal cost when 150 items are produced. **[SQP(2025)]**

24. Find the derivative of $\tan^{-1}x$ with respect to $\log x$; (where $x \in (1, \infty)$). **[SQP(2025)]**

25. Differentiate the following function with respect to $x : (\cos x)^x$; (where $x \in \left(0, \frac{\pi}{2}\right)$). **[SQP(2025)]**

26. If $f : \mathrm{R} \to \mathrm{R}$ is defined by $f(x) = |x|^3$, show that $f''(x)$ exists for all real x and find it. **[SQP(2025)]**

27. If $(x-a)^2+(y-b)^2=c^2$, for some $c>0$, prove that $\dfrac{\left[1+\left(\dfrac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\dfrac{d^2y}{dx^2}}$ is a constant independent of a and b. **[SQP(2025)]**

Application Of Derivatives

1. If $f(x)=a(x-\cos x)$ is strictly decreasing in $\mathbb{R}$, then 'a' belongs to **(2023)**

(1) $\{0\}$ (2) $(0,\infty)$

(3) $(-\infty,0)$ (4) $(-\infty,\infty)$

2. Show that the function $f(x)=\dfrac{16\sin x}{4+\cos x}-x$, is strictly decreasing in $\left(\dfrac{\pi}{2},\pi\right)$. **(2023)**

3. A tank, as shown in the figure below, formed using a combination of a cylinder and a cone, offers better drainage as compared to a flat bottomed tank. **(2023)**

A tap is connected to such a tank whose conical part is full of water. Water is dripping out from a tap at the bottom at the uniform rate of $2\text{ cm}^3/\text{s}$. The semi-vertical angle of the conical tank is 45°. On the basis of given information, answer the following questions:

(i) Find the volume of water in the tank in terms of its radius r.

(ii) Find rate of change of radius at an instant when $r=2\sqrt{2}$ cm.

(iii) Find the rate at which the wet surface of the conical tank is decreasing at an instant when radius $r=2\sqrt{2}$ cm.

4. Let $f(x)$ be a continuous function on $[a, b]$ and differentiable on (a, b). Then, this function $f(x)$ is strictly increasing in (a, b), if **(2024)**

(1) $f'(x)<0, \forall x\in(a,b)$

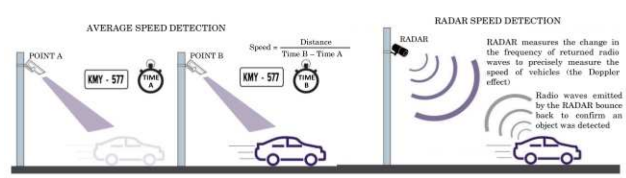
(2) $f'(x)>0, \forall x\in(a,b)$

(3) $f'(x)=0, \forall x\in(a,b)$

(4) $f(x)>0, \forall x\in(a,b)$

5. Show that the function $f(x)=4x^3-18x^2+27x-7$ has neither maxima nor minima. **(2024)**

6. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark. A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ. On the basis of the above information, answer the following questions: **(2024)**



(i) Express θ in terms of height of the camera installed on the pole and x.

(ii) Find $\dfrac{d\theta}{dx}$.

(iii) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole.

(iv) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\dfrac{3}{101}$ rad/s, then find the speed of the car.

7. The absolute maximum value of function $f(x) = x^3 - 3x + 2$ in [0, 2] is : **(2025)**

(1) 0 (2) 2
(3) 4 (4) 5

8. Find the intervals in which function $f(x) = 5x^{\frac{3}{2}} - 3x^{\frac{5}{2}}$ is **(2025)**

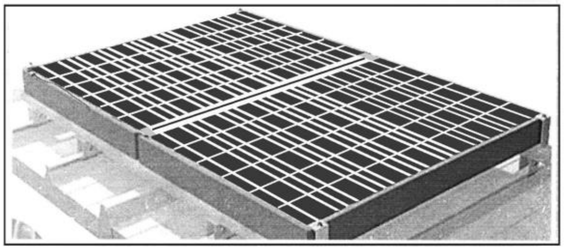
(i) increasing (ii) decreasing.

9. The side of an equilateral triangle is increasing at the rate of 3 cm / s . At what rate its area increasing when the side of the triangle is 15 cm? **(2025)**

10. Find the absolute maximum and absolute minimum of function $f(x) = 2x^3 - 15x^2 + 36x + 1$ on $[1, 5]$. **(2025)**

11. A technical company is designing a rectangular solar panel installation on a roof using 300 metres of boundary material. The design includes a partition running parallel to one of the sides dividing the area (roof) into two sections.

Let the length of the side perpendicular to the partition be x metres and with parallel to the partition be y metres. **(2025)**



(i) Write the equation for the total boundary material used in the boundary and parallel to the partition in terms of x and y.

(ii) Write the area of the solar panel as a function of x.

(iii) (a) Find the critical points of the area function. Use second derivative test to determine critical points at the maximum area. Also, find the maximum area.

OR

(iii) (b) Using first derivative test, calculate the maximum area the company can enclose with the 300 metres of boundary material, considering the parallel partition.

12. A man 1.6 m tall walks at the rate of 0.3 m / sec away from a street light that is 4 m above the ground. At what rate is the tip of his shadow moving? At what rate is his shadow lengthening? **[SQP (2023)]**

13. Read the following passage and answer the questions given below. The temperature of a person during an intestinal illness is given by $f(x) = -0.1x^2 + mx + 98.6$, $0 \leq x \leq 12$, m being a constant, where $f(x)$ is the temperature in °F at x days. **[SQP (2023)]**

(i) Is the function differentiable in the interval (0, 12)? Justify your answer.

(ii) If 6 is the critical point of the function, then find the value of the constant m.

(iii)(a)Find the intervals in which the function is strictly increasing / strictly decreasing.

OR

(iii)(b)Find the points of local maximum/local minimum, if any, in the interval (0, 12) as well as the points of absolute maximum/absolute minimum in the interval [0, 12]. Also, find the corresponding local maximum/local minimum and the absolute maximum/absolute minimum values of the function.

14. Read the following passage and answer the questions given below.

In an elliptical sport field the authority wants to design a rectangular soccer field with the maximum possible area. The sport field is given by the graph of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **[SQP (2023)]**

(i) If the length and the breadth of the rectangular field be $2x$ and $2y$ respectively, then find the area function in terms of x.

(ii) Find the critical point of the function.

(iii) (a) Use First derivative Test to find the length $2x$ and width $2y$ of the soccer field (in terms of a and b) that maximize its area.

OR

(iii) (b) Use Second Derivative Test to find the length $2x$ and width $2y$ of the soccer field (in terms of a and b) that maximize its area.

15. The set of all points where the function $f(x) = x + |x|$ is differentiable, is **[SQP (2024)]**

(1) $(0,\infty)$ (2) $(-\infty,0)$

(3) $(-\infty,0)\cup(0,\infty)$ (4) $(-\infty,\infty)$

16. Let $f(x)$ be a polynomial function of degree 6 such that $\frac{d}{dx}\left(f(x)\right)=(x-1)^3(x-3)^2$, then

Assertion (A) : $f(x)$ has a minimum at $x = 1$.

Reason (R) : When $\frac{d}{dx}\left(f(x)\right)<0, \forall x\in(a-h,a)$ and $\frac{d}{dx}\left(f(x)\right)>0, \forall\ x\in(a,a+h)$; where '$h$' is an infinitesimally small positive quantity, then $f(x)$ has a minimum at $x = a$, provided $f(x)$ is continuous at $x = a$.

(1) Both (A) and (R) are true and (R) is the correct explanation of (A).

(2) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(3) (A) is true but (R) is false.

(4) (A) is false but (R) is true.

17. Find the interval/s in which the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=xe^x$, is increasing. **[SQP (2024)]**

18. If $f(x)=\frac{1}{4x^2+2x+1}; x\in\mathbb{R}$, then find the maximum value of $f(x)$. **[SQP (2024)]**

19. Find the maximum profit that a company can make, if the profit function is given by $P(x) = 72 + 42x - x^2$, where x is the number of units and P is the profit in rupees. **[SQP (2024)]**

20. Check whether the function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=x^3+x$, has any critical point/s or not ? If yes, then find the point/s. **[SQP (2024)]**

21. Read the following passage and answer the questions given below:

The relation between the height of the plant ('y' in cm) with respect to its exposure to the sunlight is governed by the following equation $y=4x-\frac{1}{2}x^2$, where 'x' is the number of days exposed to the sunlight, for $x\le 3$. **[SQP (2024)]**

(i) Find the rate of growth of the plant with respect to the number of days exposed to the sunlight.

(ii) Does the rate of growth of the plant increase or decrease in the first three days?

What will be the height of the plant after 2 days?

22. The interval in which the function f defined by $f(x) = e^x$ is strictly increasing, is **[SQP (2025)]**

(1) $[1,\infty)$ (2) $(-\infty, 0)$

(3) $(-\infty, \infty)$ (4) $(0,\infty)$

23. A kite is flying at a height of 3 metres and 5 metres of string is out. If the kite is moving away horizontally at the rate of 200 cm / s, find the rate at which the string is being released. **[SQP (2025)]**

24. According to a psychologist, the ability of a person to understand spatial concepts is given by $A=\frac{1}{3}\sqrt{t}$, where t is the age in years, $t\in[5,18]$. Show that the rate of increase of the ability to understand spatial concepts decreases with age in between 5 and 18. **[SQP (2025)]**

25. Ramesh, the owner of a sweet selling shop, purchased some rectangular card board sheets of dimension 25 cm by 40 cm to make container packets without top. Let x cm be the length of the side of the square to be cut out from each corner to give that sheet the shape of the container by folding up the flaps. **[SQP (2025)]**

(i) Express the volume (V) of each container as function of x only.

(ii) Find $\frac{dV}{dx}$

(iii) For what value of x, the volume of each container is maximum?

(iv) Check whether V has a point of inflection at $x=\frac{65}{6}$ or not?

26. The cost (in rupees) of producing x items in factory, each day is given by $C(x) = 0.00013x^3 + 0.002x^2 + 5x + 2200$. Find the marginal cost when 150 items are produced. **[SQP(2025)]**

Integrals

1. If $\frac{d}{dx}\left(f(x)\right)=\log x$, then $f(x)$ equals : **(2023)**

(1) $-\frac{1}{x}+C$ (2) $x(\log x-1)+C$

(3) $x(\log x+x)+C$ (4) $\frac{1}{x}+C$

2. $\int_0^{\frac{\pi}{6}}\sec^2\left(x-\frac{\pi}{6}\right)dx$ is equal to : **(2023)**

(1) $\frac{1}{\sqrt{3}}$ (2) $-\frac{1}{\sqrt{3}}$

(3) $\sqrt{3}$ (4) $-\sqrt{3}$

3. Evaluate : **(2023)**

$$\int_0^{\frac{\pi}{2}}\left[\log(\sin x)-\log(2\cos x)\right]dx$$

4. Evaluate: **(2023)**

$$\int_0^{\frac{\pi}{2}}e^x\sin x\,dx$$

5. Find : $\int\frac{1}{\cos(x-a)\cos(x-b)}dx$ **(2023)**

6. $\int_a^b f(x)dx$ is equal to : **(2024)**

(1) $\int_a^b f(a-x)dx$

(2) $\int_a^b f(a+b-x)dx$

(3) $\int_a^b f(x-(a+b))dx$

(4) $\int_a^b f((a-x)+(b-x))dx$

7. Find : $\int x\sqrt{1+2x}\,dx$ **(2024)**

8. Evaluate : $\int_0^{\frac{\pi^2}{4}}\frac{\sin\sqrt{x}}{\sqrt{x}}dx$ **(2024)**

9. Find : $\int\frac{x^2}{(x^2+4)(x^2+9)}dx$ **(2024)**

10. Evaluate : $\int_1^3\left(|x-1|+|x-2|+|x-3|\right)dx$ **(2024)**

11. $\int_{-1}^{1}\frac{|x|}{x}dx, x\neq 0$ is equal to **(2025)**

(1) -1 (2) 0

(3) 1 (4) 2

12. Find : $\int\frac{x+\sin x}{1+\cos x}dx$ **(2025)**

13. Evaluate : $\int_0^{\frac{\pi}{4}}\frac{dx}{\cos^3 x\sqrt{2\sin 2x}}$ **(2025)**

14. If $\int\frac{2^{\frac{1}{x}}}{x^2}dx=k\cdot 2^{\frac{1}{x}}+C$, then k is equal to **(2025)**

(1) $\frac{-1}{\log 2}$ (2) $-\log 2$

(3) -1 (4) $\frac{1}{2}$

15. If $f'(x) = x + \frac{1}{x}$, then $f(x)$ is **[SQP(2023)]**

(1) $x^2 + \log|x| + C$ (2) $\frac{x^2}{2} + \log|x| + C$

(3) $\frac{x}{2} + \log|x| + C$ (4) $\frac{x}{2} - \log|x| + C$

16. The value of $\int_2^3 \frac{x}{x^2+1} dx$ is **[SQP(2023)]**

(1) $\log 4$ (2) $\log \frac{3}{2}$

(3) $\frac{1}{2}\log 2$ (4) $\log \frac{9}{4}$

17. Find : $\int \frac{dx}{\sqrt{3-2x-x^2}}$ **[SQP(2023)]**

18. Evaluate: $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ **[SQP(2023)]**

19. Evaluate: $\int_0^4 |x-1| dx$ **[SQP(2023)]**

20. For any integer n, the value of

$\int_{-\pi}^{\pi} e^{\cos^2 x} \sin^3(2n+1)x\, dx$ is **[SQP(2024)]**

(1) -1 (2) 0

(3) 1 (4) 2

21. Evaluate : $\int_{-1}^{1} \log\left(\frac{2-x}{2+x}\right) dx$. **[SQP(2024)]**

22. Find : $\int \frac{2x^2+3}{x^2(x^2+9)} dx$; $x \neq 0$. **[SQP(2024)]**

23. Find : $\int \sqrt{\frac{x}{1-x^3}} dx$; $x \in (0,1)$. **[SQP(2024)]**

24. Evaluate: $\int_0^{\frac{\pi}{4}} \log(1+\tan x) dx$. **[SQP(2024)]**

25. $\int \frac{dx}{x^3(1+x^4)^{\frac{1}{2}}}$ equals

(1) $-\frac{1}{2x^2}\sqrt{1+x^4} + c$ (2) $\frac{1}{2x}\sqrt{1+x^4} + c$

(3) $-\frac{1}{4x}\sqrt{1+x^4} + c$ (4) $\frac{1}{4x^2}\sqrt{1+x^4} + c$

26. $\int_0^{2\pi} \text{cosec}^7 x\, dx =$

(1) 0 (2) 1

(3) 4 (4) 2π

27. Evaluate: $\int \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}\right\} dx$; (where $x > 1$).

[SQP(2025)]

28. Evaluate : $\int_0^1 x(1-x)^n dx$; (where $n \in \mathrm{N}$).

[SQP(2025)]

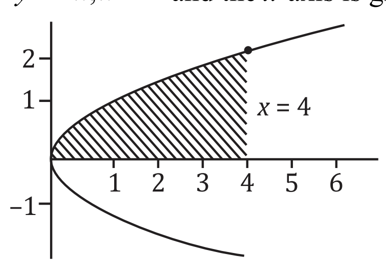
Application Of Integrals

1. Find the area of the region bounded by the curves $x^2 = y, y = x+2$ and x-axis, using integration.

(2023)

2. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$. **(2024)**

3. The area of the shaded region bounded by the curves $y^2 = x, x = 4$ and the x-axis is given by **(2025)**



(1) $\int_0^4 x\, dx$ (2) $\int_0^2 y^2 dy$

(3) $2\int_0^4 \sqrt{x}\, dx$ (4) $\int_0^4 \sqrt{x}\, dx$

4. Sketch the graph of $y = |x+3|$ and find the area of the region enclosed by the curve, x-axis, between $x = -6$ and $x = 0$, using integration. **(2025)**

5. A student observes an open-air Honeybee nest on the branch of a tree, whose plane figure is parabolic shape given by $x^2 = 4y$. Then the area (in sq. units) of the region bounded by parabola $x^2 = 4y$ and the line $y = 4$ is **[SQP(2025)]**

(1) $\frac{32}{3}$ (2) $\frac{64}{3}$

(3) $\frac{128}{3}$ (4) $\frac{256}{3}$

6. Draw the rough sketch of the curve $y = 20\cos 2x$; (where $\frac{\pi}{6} \le x \le \frac{\pi}{3}$). Using integration, find the area of the region bounded by the curve $y = 20\cos 2x$ from the ordinates $x = \frac{\pi}{6}$ to $x = \frac{\pi}{3}$ and the x-axis.

[SQP(2025)]

Differential Equations

1. The sum of the order and the degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = \sin y$ is :

(2023)

(1) 5 (2) 2

(3) 3 (4) 4

2. The general solution of the differential equation $xdy - (1+x^2)dx = dx$ is : **(2023)**

(1) $y = 2x + \frac{x^3}{3} + C$ (2) $y = 2\log x + \frac{x^3}{3} + C$

(3) $y = \frac{x^2}{2} + C$ (4) $y = 2\log x + \frac{x^2}{2} + C$

3. Find the particular solution of the differential equation $\frac{dy}{dx} + \sec^2 x \cdot y = \tan x \cdot \sec^2 x$, given that $y(0) = 0$. **(2023)**

4. Solve the differential equation given by **(2023)**

$$xdy - ydx - \sqrt{x^2 + y^2}\,dx = 0$$

5. The integrating factor of the differential equation $(1-x^2)\frac{dy}{dx} + xy = ax$, $-1 < x < 1$, is : **(2024)**

(1) $\frac{1}{x^2-1}$ (2) $\frac{1}{\sqrt{x^2-1}}$

(3) $\frac{1}{1-x^2}$ (4) $\frac{1}{\sqrt{1-x^2}}$

6. The order and degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \frac{d^2y}{dx^2}$ respectively are : **(2024)**

(1) 1, 2 (2) 2, 3

(3) 2, 1 (4) 2, 6

7. Find the particular solution of the differential equation given by $x^2\frac{dy}{dx} - xy = x^2\cos^2\left(\frac{y}{2x}\right)$, given that when $x = 1, y = \frac{\pi}{2}$. **(2024)**

8. Which of the following is not a homogeneous function of x and y ? **(2025)**

(1) $y^2 - xy$ (2) $x - 3y$

(3) $\sin^2\frac{y}{x} + \frac{y}{x}$ (4) $\tan x - \sec y$

9. If $\int \frac{2^{\frac{1}{x}}}{x^2}dx = k \cdot 2^{\frac{1}{x}} + C$, then k is equal to **(2025)**

(1) $\frac{-1}{\log 2}$ (2) $-\log 2$

(3) -1 (4) $\frac{1}{2}$

10. The integrating factor of differential equation $\left(x+2y^3\right)\frac{dy}{dx}=2y$ is **(2025)**

(1) $e^{\frac{y^2}{2}}$ (2) $\frac{1}{\sqrt{y}}$

(3) $\frac{1}{y^2}$ (4) $e^{-\frac{1}{y^2}}$

11. If m and n, respectively, are the order and the degree of the differential equation $\frac{d}{dx}\left[\left(\frac{dy}{dx}\right)\right]^4=0$, then $m+n=$ **[SQP(2023)]**

(1) 1 (2) 2

(3) 3 (4) 4

12. The general solution of the differential equation $ydx-xdy=0$ is **[SQP(2023)]**

(1) $xy=\text{C}$ (2) $x=\text{C}y^2$

(3) $y=\text{C}x$ (4) $y=\text{C}x^2$

13. Solve the differential equation: $ydx+\left(x-y^2\right)dy=0$ **[SQP(2023)]**

14. Solve the differential equation: $xdy-ydx=\sqrt{x^2+y^2}\,dx$ **[SQP(2023)]**

15. The degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=\left(\frac{d^2y}{dx^2}\right)^2$ is

(1) 4 (2) $\frac{3}{2}$

(3) 2 (4) Not defined

16. The general solution of the differential equation $ydx-xdy=0$;
(Given $x,y>0$), is of the form **[SQP(2024)]**

(1) $xy=c$ (2) $x=cy^2$

(3) $y=cx$ (4) $y=cx^2$

(Where 'c' is an arbitrary positive constant of integration)

17. Solve the differential equation:

$ye^{\frac{x}{y}}dx=\left(xe^{\frac{x}{y}}+y^2\right)dy,\left(y\neq 0\right)$. **[SQP(2024)]**

18. Solve the differential equation:

$\left(\cos^2 x\right)\frac{dy}{dx}+y=\tan x;\left(0\leq x<\frac{\pi}{2}\right)$. **[SQP(2024)]**

19. The value of 'n', such that the differential equation $x^n\frac{dy}{dx}=y\left(\log y-\log x+1\right)$; (where $x,y\in\text{R}^+$) is homogeneous, is **[SQP(2025)]**

(1) 0 (2) 1

(3) 2 (4) 3

20. What is the general solution of the differential equation $e^{y'}=x$? **[SQP(2025)]**

(1) $y=x\log x+c$

(2) $y=x\log x-x+c$

(3) $y=x\log x+x+c$

(4) $y=x+c$

Vector Algebra

1. The value of p for which the vectors $2\hat{i}+p\hat{j}+\hat{k}$ and $-4\hat{i}-6\hat{j}+26\hat{k}$ are perpendicular to each other, is: **(2023)**

(1) 3 (2) –3

(3) $-\frac{17}{3}$ (4) $\frac{17}{3}$

2. The value of $\left(\hat{i}\times\hat{j}\right)\cdot\hat{j}+\left(\hat{j}\times\hat{i}\right)\cdot\hat{k}$ is : **(2023)**

(1) 2 (2) 0

(3) 1 (4) –1

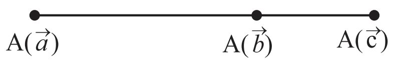
3. If $\vec{a}+\vec{b}=\hat{i}$ and $\vec{a}=2\hat{i}-2\hat{j}+2\hat{k}$, then $\left|\vec{b}\right|$ equals: **(2023)**

(1) $\sqrt{14}$ (2) 3

(3) $\sqrt{12}$ (4) $\sqrt{17}$

4. Find all the vectors of magnitude $3\sqrt{3}$ which are collinear to vector $\hat{i}+\hat{j}+\hat{k}$. **(2023)**

5. Position vectors of the points A, B and C as shown in the figure below are $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. If $\overrightarrow{AC} = \frac{5}{4}\overrightarrow{AB}$, express $\vec{c}$ in terms of $\vec{a}$ and $\vec{b}$. **(2023)**



6. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin\theta = \frac{3}{5}$. Then, $\hat{a}\cdot\hat{b}$ is equal to: **(2024)**

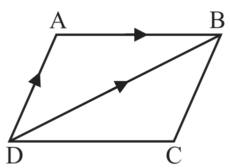
(1) $\pm\frac{3}{5}$ (2) $\pm\frac{3}{4}$
(3) $\pm\frac{4}{5}$ (4) $\pm\frac{4}{3}$

7. The vector with terminal point $A(2,-3,5)$ and initial point $B(3,-4,7)$ is : **(2024)**

(1) $\hat{i}-\hat{j}+2\hat{k}$ (2) $\hat{i}+\hat{j}-2\hat{k}$
(3) $-\hat{i}-\hat{j}-2\hat{k}$ (4) $-\hat{i}+\hat{j}-2\hat{k}$

8. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a}+\vec{b})\perp\vec{a}$ and $(2\vec{a}+\vec{b})\perp\vec{b}$, then prove that $|\vec{b}| = \sqrt{2}|\vec{a}|$. **(2024)**

9. In the given figure, ABCD is a parallelogram. If $\overrightarrow{AB} = 2\hat{i}-4\hat{j}+5\hat{k}$ and $\overrightarrow{DB} = 3\hat{i}-6\hat{j}+2\hat{k}$, then find $\overrightarrow{AD}$ and hence find the area of parallelogram ABCD **(2024)**



10. If vector $\vec{a} = 3\hat{i}+2\hat{j}-\hat{k}$ and vector $\vec{b} = \hat{i}-\hat{j}+\hat{k}$, then which of the following is correct ? **(2025)**

(1) $\vec{a} \parallel \vec{b}$ (2) $\vec{a}\perp\vec{b}$
(3) $|\vec{b}| > |\vec{a}|$ (4) $|\vec{a}| = |\vec{b}|$

11. If $\vec{a}+\vec{b}+\vec{c} = \vec{0}, |\vec{a}| = \sqrt{37}, |\vec{b}| = 3$ and $|\vec{c}| = 4$, then angle between $\vec{b}$ and $\vec{c}$ is **(2025)**

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$
(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{2}$

12. The diagonals of a parallelogram are given by $\vec{a} = 2\hat{i}-\hat{j}+\hat{k}$ and $\vec{b} = \hat{i}+3\hat{j}-\hat{k}$. Find the area of the parallelogram. **(2025)**

13. Two friends while flying kites from different locations, find the strings of their kites crossing each other. The strings can be represented by vectors $\vec{a} = 3\hat{i}+\hat{j}+2\hat{k}$ and $\vec{b} = 2\hat{i}-2\hat{j}+4\hat{k}$. Determine the angle formed between the kite strings. Assume there is no slack in the strings. **(2025)**

14. Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$ where A and B are the points $A(2,1,3)$ and $B(8,-1,0)$ respectively. **(2025)**

15. The area of a triangle with vertices A, B, C is given by **[SQP(2023)]**

(1) $|\overrightarrow{AB}\times\overrightarrow{AC}|$ (2) $\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AC}|$
(3) $\frac{1}{4}|\overrightarrow{AC}\times\overrightarrow{AB}|$ (4) $\frac{1}{8}|\overrightarrow{AC}\times\overrightarrow{AB}|$

16. The scalar projection of the vector $3\hat{i}-\hat{j}-2\hat{k}$ on the vector $\hat{i}+2\hat{j}-3\hat{k}$ is **[SQP(2023)]**

(1) $\frac{7}{\sqrt{14}}$ (2) $\frac{7}{14}$
(3) $\frac{6}{13}$ (4) $\frac{7}{2}$

17. If two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}\cdot\vec{b} = 4$, then $|\vec{a}-2\vec{b}|$ is equal to **[SQP(2023)]**

(1) $\sqrt{2}$ (2) $2\sqrt{6}$
(3) 24 (4) $2\sqrt{2}$

18. If $\vec{a}=\hat{i}-\hat{j}+7\hat{k}$ and $\vec{b}=5\hat{i}-\hat{j}+\lambda\hat{k}$, then find the value of λ so that the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are orthogonal. **[SQP(2023)]**

19. Find $|\vec{x}|$ if $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=12$, where $\vec{a}$ is a unit vector. **[SQP(2023)]**

20. If $\vec{a}=4\hat{i}+6\hat{j}$ and $\vec{b}=3\hat{j}+4\hat{k}$, then the vector form of the component of $\vec{a}$ along $\vec{b}$ is **[SQP(2024)]**

(1) $\frac{18}{5}(3\hat{i}+4\hat{k})$ (2) $\frac{18}{25}(3\hat{j}+4\hat{k})$

(3) $\frac{18}{5}(3\hat{i}+4\hat{k})$ (4) $\frac{18}{25}(4\hat{i}+6\hat{j})$

21. The value of λ for which two vectors $2i-\hat{j}+2\hat{k}$ and $3\hat{i}+\lambda\hat{j}+\hat{k}$ are perpendicular is **[SQP(2024)]**

(1) 2 (2) 4

(3) 6 (4) 8

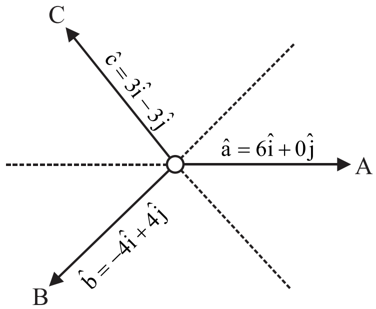
22. Read the following passage and answer the question given below:

Teams A, B, C went for playing a tug of war game. Teams A, B, C have attached a rope to a metal ring and is trying to pull the ring into their own area. **[SQP(2024)]**

Team A pulls with force $F_1=6\hat{i}+0\hat{j}kN$,

Team B pulls with force $F_2=-4\hat{i}+4\hat{j}kN$,

Team C pulls with force $F_3=-3\hat{i}-3\hat{j}kN$,



(i) What is the magnitude of the force of Team A ?

(ii) Which team will win the game?

(iii) Find the magnitude of the resultant force exerted by the teams.

(iv) In what direction is the ring getting pulled?

23. The value of α if the angle between $\vec{p}=2\alpha^2\hat{i}-3\alpha\hat{j}+\hat{k}$ and $\vec{q}=\hat{i}+\hat{j}+\alpha\hat{k}$ is obtuse, is **[SQP(2025)]**

(1) $R-[0,1]$ (2) $(0,1)$

(3) $[0,\infty)$ (14 $[1,\infty)$

24. If $|\vec{a}|=3, |\vec{b}|=4$ and $|\vec{a}+\vec{b}|=5$, then $|\vec{a}-\vec{b}|=$ **[SQP(2025)]**

(1) 3 (2) 4

(3) 5 (4) 8

25. The two co-initial adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}-5\hat{k}$ and $2\hat{i}+2\hat{j}+3\hat{k}$. Find its diagonals and use them to find the area of the parallelogram. **[SQP(2025)]**

3d Geometry

1. Direction cosines of the line $\frac{x-1}{2}=\frac{1-y}{3}=\frac{2z-1}{12}$ are : **(2023)**

(1) $\frac{2}{7},\frac{3}{7},\frac{6}{7}$ (2) $\frac{2}{\sqrt{157}},-\frac{3}{\sqrt{157}},\frac{12}{\sqrt{157}}$

(3) $\frac{2}{7},-\frac{3}{7},-\frac{6}{7}$ (4) $\frac{2}{7},-\frac{3}{7},\frac{6}{7}$

2. Assertion (A) : Equation of a line passing through the points (1, 2, 3) and $(3,-1,3)$ is

$$\frac{x-3}{2}=\frac{y+1}{3}=\frac{z-3}{0}.$$

Reason (R) : Equation of a line passing through points (x_1,y_1,z_1), (x_2,y_2,z_2) is given by

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1}.$$ **(2023)**

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(2) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(3) Assertion (A) is true and Reason (R) is false.

(4) Assertion (A) is false and Reason (R) is true.

3. Find the vector and the Cartesian equations of a line passing through the point $(1, 2, -4)$ and parallel to the line joining the points $A(3, 3, -5)$ and $B(1, 0, -11)$. Hence, find the distance between the two lines. **(2023)**

4. Find the equations of the line passing through the points $A(1, 2, 3)$ and $B(3, 5, 9)$. Hence, find the coordinates of the points on this line which are at a distance of 14 units from point B. **(2023)**

5. If the direction cosines of a line are $\sqrt{3}k, \sqrt{3}k, \sqrt{3}k$, then the value of k is: **(2024)**

(1) ± 1 (2) $\pm\sqrt{3}$

(3) ± 3 (4) $\pm\frac{1}{3}$

6. The distance of point $P(a,b,c)$ from y-axis is: **(2024)**

(1) b (2) b^2

(3) $\sqrt{a^2+c^2}$ (4) a^2+c^2

7. Assertion (A) : A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$. **(2024)**

(1) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(2) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(3) Assertion (A) is true, but Reason (R) is false.

(4) Assertion (A) is false, but Reason (R) is true.

8. The image of point $P(x,y,z)$ with respect to line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ is $P'(1, 0, 7)$. Find the coordinates of point P. **(2024)**

9. Verify that lines given by

$\vec{r} = (1-\lambda)\hat{i} + (\lambda-2)\hat{j} + (3-2\lambda)\hat{k}$ and

$\vec{r} = (\mu+1)\hat{i} + (2\mu-1)\hat{j} - (2\mu+1)\hat{k}$ are skew lines. Hence, find shortest distance between the lines. **(2025)**

10. During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are in a line given by $\vec{B} = 2\hat{i} + 8\hat{j}$, $\vec{W} = 6\hat{i} + 12\hat{j}$ and $\vec{F} = 12\hat{i} + 18\hat{j}$ respectively. Calculate the ratio in which the wicketkeeper divides the line segment joining the bowler and the leg slip fielder. **(2025)**

11. Find the image A' of the point $A(1, 6, 3)$ in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, find the equation of the line joining A and A'. **(2025)**

12. Find a point P on the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ such that its distance from point $Q(2, 4, -1)$ is 7 units. Also, find the equation of line joining P and Q. **(2025)**

13. P is a point on the line joining the points $A(0, 5, -2)$ and $B(3, -1, 2)$. If the x-coordinate of P is 6, then its z-coordinate is **[SQP(2023)]**

(1) 10 (2) 6

(3) -6 (4) -10

14. Assertion (A): The acute angle between the line $\vec{r} = \hat{i} + \hat{j} + 2\hat{k} + \lambda(\hat{i} - \hat{j})$ and the x-axis is $\frac{\pi}{4}$

Reason(R): The acute angle θ between the lines $\vec{r} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k} + \lambda(a_1\hat{i} + b_1\hat{j} + c_1\hat{k})$ and

$\vec{r} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} + \mu\left(a_2\hat{i} + b_2\hat{j} + c_2\hat{k}\right)$ is given by

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

[SQP(2023)]

(1) Both A and R are true and R is the correct explanation of A.
(2) Both A and R are true but R is not the correct explanation of A.
(3) A is true but R is false.
(4) A is false but R is true.

15. An insect is crawling along the line $\vec{r} = 6\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$ and another insect is crawling along the line $\vec{r} = -4\hat{i} - \hat{k} + \mu\left(3\hat{i} - 2\hat{j} - 2\hat{k}\right)$. At what points on the lines should they reach so that the distance between them is the shortest? Find the shortest possible distance between them. **[SQP(2023)]**

16. The lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda\left(2\hat{i} + 3\hat{j} - 6\hat{k}\right)$ and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(6\hat{i} + 9\hat{j} - 18\hat{k}\right)$; (where λ & μ are scalars) are **[SQP(2024)]**

(1) coincident (2) skew
(3) intersecting (4) parallel

17. ABCD is a rhombus whose diagonals intersect at E. Then $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$ equals to **[SQP(2024)]**

(1) $\vec{0}$ (2) $\overrightarrow{AD}$
(3) $2\overrightarrow{BD}$ (4) $2\overrightarrow{AD}$

18. If the direction cosines of a line are $\left\langle \frac{1}{c}, \frac{1}{c}, \frac{1}{c} \right\rangle$, then **[SQP(2024)]**

(1) $0 < c < 1$ (2) $c > 2$
(3) $c = \pm\sqrt{2}$ (4) $c = \pm\sqrt{3}$

19. Find the coordinates of the image of the point $(1, 6, 3)$ with respect to the line $\vec{r} = \left(\hat{j} + 2\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right)$; where '$\lambda$' is a scalar. Also, find the distance of the image from the y-axis. **[SQP(2024)]**

20. An aeroplane is flying along the line $\vec{r} = \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$; where '$\lambda$' is a scalar and another aeroplane is flying along the line $\vec{r} = \hat{i} - \hat{j} + \mu\left(-2\hat{j} + \hat{k}\right)$; where '$\mu$' is a scalar. At what points on the lines should they reach, so that the distance between them is the shortest? Find the shortest possible distance between them. **[SQP(2024)]**

21. If vectors $\vec{a} = 2\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ are such that $\vec{b} + \lambda\vec{c}$ is perpendicular to $\vec{a}$, then find the value of λ. **[SQP(2025)]**

22. A person standing at $O(0, 0, 0)$ is watching an aeroplane which is at the coordinate point $A(4, 0, 3)$. At the same time he saw a bird at the coordinate point $B(0, 0, 1)$. Find the angles which $\overrightarrow{BA}$ makes with the x, y and z-axes. **[SQP(2025)]**

23. An ant is moving along the vector $\vec{l_1} = \hat{i} - 2\hat{j} + 3\hat{k}$. Few sugar crystals are kept along the vector $\vec{l_2} = 3\hat{i} - 2\hat{j} + \hat{k}$ which is inclined at an angle θ with the vector $\vec{l_1}$. Then find the angle θ. Also find the scalar projection of $\vec{l_1}$ on $\vec{l_2}$. **[SQP(2025)]**

24. Find the vector and the cartesian equation of the line that passes through $(-1, 2, 7)$ and is perpendicular to the lines $\vec{r} = 2\hat{i} + \hat{j} - 3\hat{k} + \lambda\left(\hat{i} + 2\hat{j} + 5\hat{k}\right)$ and $\vec{r} = 3\hat{i} + 3\hat{j} - 7\hat{k} + \mu\left(3\hat{i} - 2\hat{j} + 5\hat{k}\right)$. **[SQP(2025)]**

25. Find the shortest distance between the lines l_1 and l_2 whose vector equations are $\vec{r} = \left(-\hat{i} - \hat{j} - \hat{k}\right) + \lambda$

$\left(7\hat{i}-6\hat{j}+\hat{k}\right)$ and $\vec{r}=\left(3\hat{i}+5\hat{j}+7\hat{k}\right)+\mu\left(\hat{i}-2\hat{j}+\hat{k}\right)$ where λ and μ are parameters. **[SQP(2025)]**

26. Find the image of the point $(1, 2, 1)$ with respect to the line $\frac{x-3}{1}=\frac{y+1}{2}=\frac{z-1}{3}$. Also find the equation of the line joining the given point and its image. **[SQP(2025)]**

Linear Programming

1. The corner points of the feasible region in the graphical representation of a linear programming problem are (2, 72), (15, 20) and (40, 15). If $z = 18x + 9y$ be the objective function, then : **(2023)**
(1) z is maximum at (2, 72), minimum at (15, 20)
(2) z is maximum at (15, 20), minimum at (40, 15)
(3) z is maximum at (40, 15), minimum at (15, 20)
(4) z is maximum at (40, 15), minimum at (2, 72)

2. The number of corner points of the feasible region determined by the constraints $x - y \geq 0, 2y \leq x + 2$, $x \geq 0, y \geq 0$ is : **(2023)**
(1) 2 (2) 3
(3) 4 (4) 5

3. Solve graphically the following linear programming problem :
Maximise $z = 6x + 3y$, subject to the constraints **(2023)**
$4x + y \geq 80$
$3x + 2y \leq 150$
$x + 5y \geq 115$
$x \geq 0, y \geq 0$

4. A linear programming problem deals with the optimization of a / a : **(2024)**
(1) logarithmic function
(2) linear function
(3) quadratic function
(4) exponential function

5. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is : **(2024)**
(1) 0 (2) 1
(3) 2 (4) 3

6. Solve the following linear programming problem graphically :
Maximise $z = 500x + 300y$, subject to constraints
$x + 2y \leq 12$ **(2024)**
$2x + y \leq 12$
$4x + 5y \geq 20$
$x \geq 0, y \geq 0$

7. The corner points of the feasible region in graphical representation of a L.P.P. are $(2, 72), (15, 20)$ and $(40, 15)$. If $Z = 18x + 9y$ be the objective function, then **(2025)**
(1) Z is maximum at (2, 72), minimum at (15, 20)
(2) Z is maximum at (15, 20) minimum at (40, 15)
(3) Z is maximum at (40, 15), minimum at (15, 20)
(4) Z is maximum at (40, 15), minimum at (2, 72)

8. If the feasible region of a linear programming problem with objective function Z = $ax + by$, is bounded, then which of the following is correct ? **(2025)**
(1) It will only have a maximum value.
(2) It will only have a minimum value.
(3) It will have both maximum and minimum values.
(4) It will have neither maximum nor minimum value.

9. Solve the following linear programming problem graphically :
Maximise $Z = x + 2y$
Subject to the constraints : **(2025)**
$x - y \geq 0$
$x - 2y \geq -2$
$x \geq 0, y \geq 0$

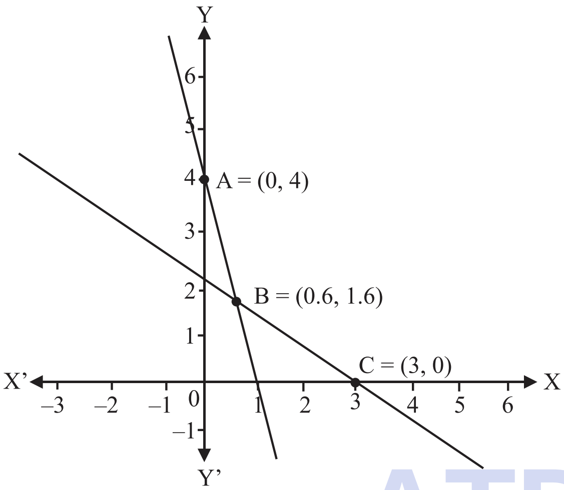
10. The solution set of the inequality $3x + 5y < 4$ is **[SQP(2023)]**
(1) an open half-plane not containing the origin.
(2) an open half-plane containing the origin.

(3) the whole XY-plane not containing the line $3x + 5y = 4$.

(4) a closed half plane containing the origin.

11. The corner points of the shaded unbounded feasible region of an LPP are $(0, 4)$, $(0.6, 1.6)$ and $(3, 0)$ as shown in the figure.

The minimum value of the objective function $Z = 4x + 6y$ occurs at



(1) $(0.6, 1.6)$ only

(2) $(3, 0)$ only

(3) $(0.6, 1.6)$ and $(3, 0)$ only

(4) at every point of the line-segment joining the points $(0.6, 1.6)$ and $(3, 0)$

12. Solve the following Linear Programming Problem graphically:

Maximize $Z = 400x + 300y$ subject to

$x + y \le 200, x \le 40, x \ge 20, y \ge 0$ **[SQP(2023)]**

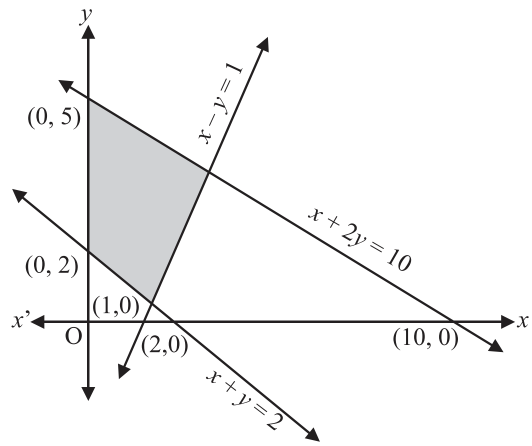
13. The corner points of the bounded feasible region determined by a system of linear constraints are $(0, 3), (1, 1)$ and $(3, 0)$. Let $Z = px + qy$, where $p, q > 0$. The condition on p and q so that the minimum of Z occurs at (3, 0) and (1, 1) is **[SQP(2024)]**

(1) $p = 2q$ (2) $p = \frac{q}{2}$

(3) $p = 3q$ (4) $p = q$

14. The feasible region corresponding to the linear constraints of a Linear Programming Problem is given below.

Which of the following is not a constraint to the given Linear Programming Problem? **[SQP(2024)]**



(1) $x + y \ge 2$ (2) $x + 2y \le 10$

(3) $x - y \ge 1$ (4) $x - y \le 1$

15. Solve the following Linear Programming Problem graphically:

Minimize : $z = x + 2y$, subject to the constraints : $x + 2y \ge 100, 2x - y \le 0, 2x + y \le 200, x, y \ge 0$.

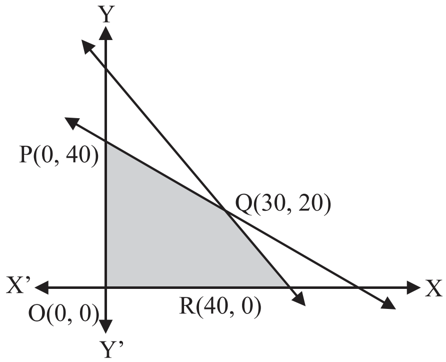
[SQP(2024)]

16. Solve the following Linear Programming Problem graphically:

Maximize : $z = -x + 2y$, subject to the constraints : $x \ge 3, x + y \ge 5, x + 2y \ge 6, y \ge 0$. **[SQP(2024)]**

17. For the linear programming problem (LPP), the objective function is $Z = 4x + 3y$ and the feasible region determined by a set of constraints is shown in the graph. **[SQP(2025)]**

Which of the following statements is true?



(1) Maximum value of Z is at $R(40, 0)$.

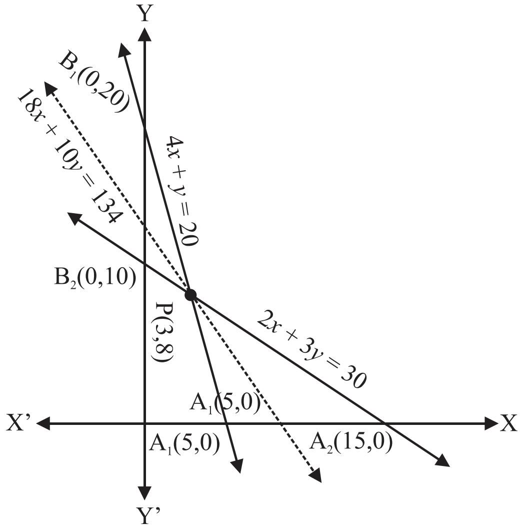
(2) Maximum value of Z is at $Q(30, 20)$.

(3) Value of Z at $R(40, 0)$ is less than the value at $P(0, 40)$.

(4) The value of Z at $Q(30, 20)$ is less than the value at $R(40, 0)$.

18. A linear programming problem (LPP) along with the graph of its constraints is shown below.
The corresponding objective function is:
Z = 18x + 10y, which has to be minimized. The smallest value of the objective function Z is 134 and is obtained at the corner point $(3, 8)$.

(**Note :** The figure is not to scale.)
The optimal solution of the above linear programming problem. **[SQP(2025)]**



(1) Does not exist as the feasible region is unbounded.

(2) Does not exist as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region.

(3) Exists as the inequality $18x + 10y > 134$ has infinitely many points in common with the feasible region.

(4) Exists as the inequality $18x + 10y < 134$ does not have any point in common with the feasible region.

19. Consider the following Linear Programming Problem:
Minimise $Z = x + 2y$
Subject to $2x + y \geq 3, x + 2y \geq 6, x, y \geq 0$.
Show graphically that the minimum of Z occurs at more than two points

Probability

1. If $P\left(\frac{A}{B}\right) = 0.3, P(A) = 0.4$ and $P(B) = 0.8$, then $P\left(\frac{B}{A}\right)$ is equal to : **(2023)**

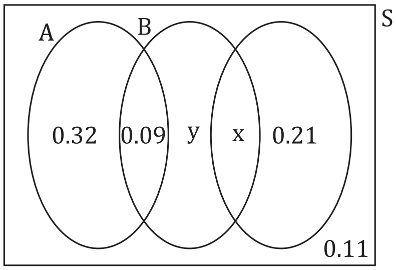
(1) 0.6 (2) 0.3
(3) 0.06 (4) 0.4

2. A and B are independent events such that $P(A \cap \bar{B}) = \frac{1}{4}$ and $P(\bar{A} \cap B) = \frac{1}{6}$. Find $P(A)$ and $P(B)$. **(2023)**

3. There are different types of Yoga which involve the usage of different poses of Yoga Asanas, Meditation and Pranayam as shown in the figure below : **(2023)**

Types of Yoga

The Venn diagram below represents the probabilities of three different types of Yoga, A, B and C performed by the people of a society. Further, it is given that probability of a member performing type C Yoga is 0.44. On the basis of the above information, answer the following questions:



(i) Find the value of x.

(ii) Find the value of y.

(iii) Find $P\left(\frac{C}{B}\right)$.

(iv) Find the probability that a randomly selected person of the society does Yoga of type A or B but not C

4. If $P(A|B) = P(A'|B)$, then which of the following statements is true ? **(2024)**

(1) $P(A) = P(A')$

(2) $P(A) = 2P(B)$

(3) $P(A \cap B) = \frac{1}{2}P(B)$

(4) $P(A \cap B) = 2P(B)$

5. E and F are two independent events such that $P(\bar{E}) = 0.6$ and $P(E \cup F) = 0\cdot6$. Find $P(F)$ and $P(\bar{E} \cup \bar{F})$. **(2024)**

6. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights. **(2024)**

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.

On the basis of the above information, answer the following questions :

(i) Find the probability that an airplane reached its destination late.

(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence.

7. If E and F are two independent events such that $P(E) = \frac{2}{3}, P(F) = \frac{3}{7}$, then $P(E/\bar{F})$ is equal to :

(2025)

(1) $\frac{1}{6}$ (2) $\frac{1}{2}$

(3) $\frac{2}{3}$ (4) $\frac{7}{9}$

8. For the vacancy advertised in the newspaper, 3000 candidates submitted their applications. From the data it was revealed that two third of the total applicants were females and other were males. The selection for the job was done through a written test. The performance of the applicants indicates that the probability of a male getting a distinction in written test is 0.4 and that a female getting a distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

(2025)

9. A bank offers loan to its customers on different types of interest namely, fixed rate, floating rate and variable rate. From the past data with the bank, it is known that a customer avails loan on fixed rate, floating rate or variable rate with probabilities $10\%, 20\%$ and 70% respectively. A customer after availing loan can pay the loan or default on loan repayment. The bank data suggests that the probability that a person defaults on loan after availing it at fixed rate, floating rate and variable rate is $5\%, 3\%$ and 1% respectively.

Based on the above information, answer the following : **(2025)**

(i) What is the probability that a customer after availing the loan will default on the loan repayment?

(ii) A customer after availing the loan, defaults on loan repayment.

What is the probability that he availed the loan at a variable rate of interest?

10. Given two independent events A and B such that $P(A)=0.3, P(B)=0.6$ and $P(A' \cap B')$ is **[SQP(2023)]**

(1) 0.9 (2) 0.18

(3) 0.28 (4) 0.1

11. Three friends go for coffee. They decide who will pay the bill, by each tossing a coin and then letting the "odd person" pay. There is no odd person if all three tosses produce the same result. If there is no odd person in the first round, they make a second round of tosses and they continue to do so until there is an odd person. What is the probability that exactly three rounds of tosses are made? **[SQP(2023)]**

12. Find the mean number of defective items in a sample of two items drawn one-by-one without replacement from an urn containing 6 items, which include 2 defective items. Assume that the items are identical in shape and size. **[SQP(2023)]**

13. Read the following passage and answer the questions given below. There are two antiaircraft guns, named as A and B. The probabilities that the shell fired from them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time. **[SQP(2023)]**

(i) What is the probability that the shell fired from exactly one of them hit the plane?

(ii) If it is known that the shell fired from exactly one of them hit the plane, then what is the probability that it was fired from B ?

14. A problem in Mathematics is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. If the events of their solving the problem are independent then the probability that the problem will be solved, is **[SQP(2024)]**

(1) $\frac{1}{4}$ (2) $\frac{1}{3}$

(3) $\frac{1}{2}$ (4) $\frac{3}{4}$

15. Read the following passage and answer the questions given below :

In an Office three employees Jayant, Sonia and Oliver process incoming copies of a certain form. Jayant processes 50% of the forms, Sonia processes 20% and Oliver the remaining 30% of the forms. Jayant has an error rate of 0.06, Sonia has an error rate of 0.04 and Oliver has an error rate of 0.03.

Based on the above information, answer the following questions. **[SQP(2024)]**

(i) Find the probability that Sonia processed the form and committed an error.

(ii) Find the total probability of committing an error in processing the form.

(iii) The manager of the Company wants to do a quality check. During inspection, he selects a form at random from the days output of processed form. If the form selected at random has an error, find the probability that the form is not processed by Jayant.

(iv) Let E be the event of committing an error in processing the form and let E_1, E_2 and E_3 be the events that Jayant, Sonia and Oliver processed the form. Find the value o $\sum_{i=1}^{3} P(E_i | E)$.

16. For any two events A and B, if $P(\vec{A})=\frac{1}{2}, P(\vec{B})=\frac{2}{3}$ and $P(A \cap B)=\frac{1}{4}$, then $P(\vec{A}/\vec{B})$ equals :

[SQP(2025)]

(1) $\frac{3}{8}$ (2) $\frac{8}{9}$

(3) $\frac{5}{8}$ (4) $\frac{1}{4}$

17. Arka bought two cages of birds: Cage-I contains 5 parrots and 1 owl and Cage -II contains 6 parrots. One day Arka forgot to lock both cages and two birds flew from Cage-I to Cage-II (simultaneously). Then two birds flew back from cage-II to cage-I(simultaneously). **[SQP(2025)]**

Assume that all the birds have equal chances of flying.

On the basis of the above information, answer the following questions:

(i) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I then find the probability that the owl is still in Cage-I.

(ii) When two birds flew from Cage-I to Cage-II and two birds flew back from Cage-II to Cage-I, the owl is still seen in Cage-I, what is the probability that one parrot and the owl flew from Cage-I to Cage-II?

Answer

Relations & Function

1. B = {2, 4, 6, 8}

2. R is symmetric

3. (1)

4. R is reflexive & symmetric.

5. f is onto.

6. (D)
Assertion (A) is false, but Reason (R) is true.

7. (i) R_4

8. Prove

9. Prove

10. (4)

11. $\{(1,3),(2,6),(3,9),\ldots\ldots\}$

12. (1)

13. 64

14. $\{(b_1,b_1),(b_2,b_2),(b_3,b_3)\}$

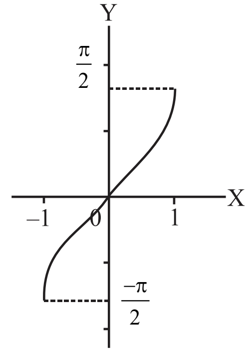
15. Reflexive but not symmetric
$$=\begin{Bmatrix}(b_1,b_2),(b_2,b_1),(b_1,b_1),(b_2,b_2),\\(b_3,b_3),(b_2,b_3)\end{Bmatrix}$$

Inverse Trigonometric Function

1. (3)

2. (i) $\frac{-4\pi}{6}$ or $\frac{-2\pi}{3}$

(ii) $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$ or any other interval corresponding to the domain [–1, 1]

(iii)


(iv) Domain = [0, 2] range $=[-\pi,\pi]$

3. (3)

4. $\frac{\pi}{3}$

5. $-\frac{\pi}{7}$

6. $-\frac{\pi}{10}$

7. $\left[-\sqrt{5},-\sqrt{3}\right]\cup\left[\sqrt{3},\sqrt{5}\right]$

8. (2)

9. $\therefore x\in\left(-\infty,-\frac{4}{3}\right)$

Matrices

1. (2)

2. (2)

3. (3)

4. (4)

5. (4)

6. (2)

7. (4)

8. (1)

9. (2)

10. (1)

11. (3)

12. (3)

13. (4)

14. (4)

15. (1)

16. (1)

17. $\therefore a = -\frac{1}{2}, b = 8, c = 1$

So, the equation becomes $y = -\frac{1}{2}x^2 + 8x + 1$

Determinants

1. (3)

2. (4)

3. The value of $(AB)^{-1}$ is $\begin{bmatrix} 10 & 7 & 21 \\ -49 & -34 & -103 \\ 17 & 12 & 36 \end{bmatrix}$

4. $x = 1\ y = 1\ z = 1$

5. (2)

6. (1)

7. $\begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}$

8. 3

9. (2)

10. Number of students allocated in sports, music and drama are 90, 65 and 25 respectivelv.

11. (1)

12. (4)

13. (2)

14. $A^{-1} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$

15. (2)

16. (2)

17. (4)

18. $x = 2, y = 3, z = 5$.

19. (4)

20. (2)

Continuity & Differentiability

1. (4)

2. Prove

3. $a = \frac{1}{27}, f''(1) = \frac{20}{27}$

4. (3)

5. $f(x)$ is differentiable at $x = 0$

6. Prove

7. Prove

8. $\frac{dy}{dx} = (\tan x)^x \left[\left(\frac{x \sec^2 x}{\tan x} \right) + \log(\tan x) \right]$

9. (3)

10. (4)

11. $2^{\cos^2 x} \log 2$

12. $\frac{dy}{dx} = -\frac{x}{y}$

13. Prove

14. $\frac{2\sqrt{2}}{a^2}$

15. (3)

16. (3)

17. (1)

18. Prove

19. (1)

20. Prove

21. (4)

22. (1)

23. ₹14.375

24. $\frac{x}{1+x^2}$

25. $(\cos x)^x \left(\log_e \cos x - x \tan x \right)$

26. Prove

27. Prove

Application Of Derivatives

1. (3)

2. Prove

3. (i) $\frac{1}{3}\pi r^3$ (ii) $\pi r^2 \frac{dr}{dt}$ (iii) $-2\ \text{cm}^2/\text{sec}$

4. (2)

5. Prove

6. (i) $\tan^{-1}\left(\frac{5}{x}\right)$ (ii) $\frac{-5}{5^2 + x^2}$ (iii) $\frac{-100}{2525}$ or $\frac{-4}{101}$ rad / s

   (iv) 15 m / s

7. (3)

8. (i) (0, 1), [0, 1) or (0, 1] (ii) $(1, \infty)$

9. $\frac{45\sqrt{3}}{2}\text{cm}^2/\text{s}$

10. maximum value is 56 and the absolute minimum value is 24.

11. (i) $2x + 3y = 300$

    (ii) $A = xy\ \frac{x}{3}(300 - 2x)$

    (iii) $3750\ \text{m}^2$

12. 0.2 m/s

13. (i) $f(x) = -0.1x^2 + mx + 98.6$, being a polynomial function
(ii) 1.2
(iii) (a) (b) 102.2, 98.6

In the Interval	$f'(x)$	Conclusion
(0, 6)	+ve	*f* is strictly increasing in [0, 6]
(6, 12)	–ve	*f* is strictly decreasing in [6, 12]

14. (i) $\frac{4b}{a}x\sqrt{a^2 - x^2}, x \in (0, a)$.
(ii) $\frac{a}{\sqrt{2}}$
(iii) $a\sqrt{2}, b\sqrt{2}$.

15. (3)

16. (1)

17. $[-1, \infty)$.

18. $\frac{4}{3}$

19. The maximum profit is ₹513

20. No critical point exists.

21. (i) $4 - x$ (ii) $6cm$

22. (3)

23. 160 cm/s

24. Prove

25. (i) $V = (40 - 2x)(25 - 2x)x \text{ cm}^3$
(ii) $\frac{dV}{dx} = 4(3x - 50)(x - 5)$
(iii) V is max when $x = 5$
(iv) $x = \frac{65}{6}$ is a point of inflection.

26. ₹14.375

Integrals

1. (2)

2. (1)

3. $I = \frac{\pi}{4}\log\frac{1}{4} \text{ OR } -\frac{\pi}{2}\log 2$

4. $\frac{1}{2}e^{\pi/2} + \frac{1}{2}$ or $\frac{1}{2}\left(e^{\pi/2} + 1\right)$

5. $= \frac{1}{\sin(a-b)}\left[\log\left|\sec(x-b)\right| - \log\left|\sec(x-a)\right|\right] + C$

6. (2)

7. $\frac{(1+2x)^{\frac{5}{2}}}{10} - \frac{(1+2x)^{\frac{3}{2}}}{6} + C$

8. 2

9. $\frac{-2}{5}\tan^{-1}\left(\frac{x}{2}\right) + \frac{3}{5}\tan^{-1}\left(\frac{x}{3}\right) + C$

10. 5

11. (2)

12. $x\tan\frac{x}{2} + C$

13. $\frac{6}{5}$

14. (1)

15. (2)

16. (3)

17. $\sin^{-1}\left(\frac{x}{a}\right) + C]$

18. $\frac{\pi}{12}$

19. 5

20. (2)

21. $\int_{-1}^{1}\log\left(\frac{2-x}{2+x}\right)dx=0$

22. $-\frac{1}{3x}+\frac{5}{9}\tan^{-1}\left(\frac{x}{3}\right)+c$

23. $\frac{2}{3}\sin^{-1}\left(x^{\frac{3}{2}}\right)+c$

24. $I=\frac{\pi}{8}\log_e 2$

25. (1)

26. (1)

27. $\frac{x}{\log_e x}+c$

28. $\frac{1}{(n+1)(n+2)}$

Application Of Integrals

1. $\frac{5}{6}$

2. $4\sqrt{3}+\frac{8\pi}{3}$

3. (4)

4. 9

5. (2)

6. $20\left(1-\frac{\sqrt{3}}{2}\right)$ sq. units

Differential Equations

1. (3)

2. (4)

3. $\tan x-1+e^{-\tan x}$

4. Cx^2

5. (4)

6. (3)

7. $\log|x|+2$

8. (4)

9. (1)

10. (2)

11. (3)

12. (3)

13. $xy=\frac{y^3}{3}+C$

14. Cx^2

15. (3)

16. (3)

17. $y+c$

18. $y=\tan x-1+c.\left(e^{-\tan x}\right)$

19. (2)

20. (2)

Vector Algebra

1. (1)

2. (4)

3. (2)
Unit vector along $\hat{i}+\hat{j}+\hat{k}$ is $\frac{\text{i}}{\sqrt{3}}+\frac{\text{j}}{\sqrt{3}}+\frac{\text{k}}{\sqrt{3}}$

4. Required vectors are $3\hat{i}+3\hat{j}+3\hat{k}$ and $-3\hat{i}-3\hat{j}-3\hat{k}$

5. $\vec{c}=\frac{5\,\vec{b}}{4}-\frac{\vec{a}}{4}$

6. (3)

7. (4)

8. Prove

9. $\sqrt{605}$ or $11\sqrt{5}$

10. (2)

11. (3)

12. $\frac{\sqrt{62}}{2}$

13. $\cos^{-1}\left(\frac{3}{\sqrt{21}}\right)$

14. $3\left(-6\hat{i}+2\hat{j}+3\hat{k}\right)$ or $-18\hat{i}+6\hat{j}+9\hat{k}$

15. (2)

16. (1)

17. (2)

18. $\lambda=\pm 5$

19. $|\vec{x}|=\sqrt{13}$

20. (2)

21. (4)

22. (i) A = 6kN
(ii) B will win the game
(iii) $\sqrt{2}k$N
(iv) +ve direction of the x-axis

23. (2)

24. (3)

25. $2\sqrt{101}$ sq. units.

3d Geometry

1. (4)

2. (4)

3. $d=\frac{\sqrt{293}}{7}$

4. required points are $(7,11,21)$ and $(-1,-1,-3)$

5. (4)

6. (3)

7. (1)

8. P(1, 6, 3)

9. $\frac{8}{\sqrt{29}}$

10. 2 : 3

11. $\frac{z-3}{2}$

12. $\dfrac{z+1}{2}$

13. (2)

14. (1)

15. 9

16. (4)

17. (1)

18. (4)

19. $\sqrt{50}$ units

20. $\sqrt{\dfrac{2}{3}}$ units

21. $\lambda = -\dfrac{5}{8}$

22. $\cos^{-1}\left(\dfrac{2}{\sqrt{5}}\right), \dfrac{\pi}{2}, \cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right)$

23. $\dfrac{10}{\sqrt{14}}$.

24. $\dfrac{z-7}{-4}$

25. $2\sqrt{29}$ units.

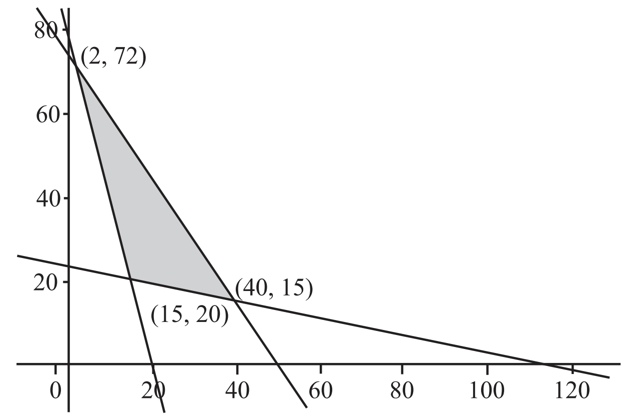
26. $\dfrac{z-1}{6}$.

Linear Programming

1. (3)

2. (1)

3.



Corner points	Value of Z
(2, 72)	(12 + 216 = 228)
(15, 20)	(90 + 60 = 150)
(40, 15)	(240 + 45 = 285)
Maximum	

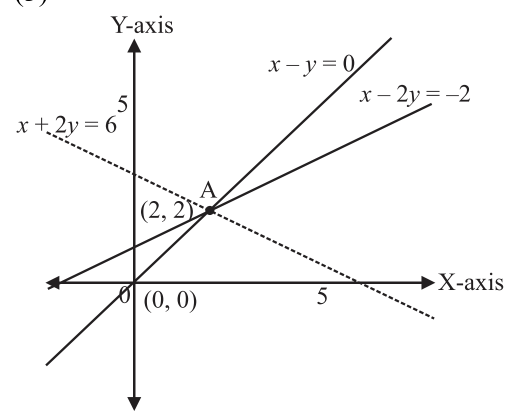
4. (2)

5. (3)

6. Max $z = 3$[illegible]00 at $x = 4, y = 4$

7. (3)

8. (3)



9.

Corner Point	Value of $Z = x + 2y$
O(0, 0)	0
A(2, 2)	6

10. (2)

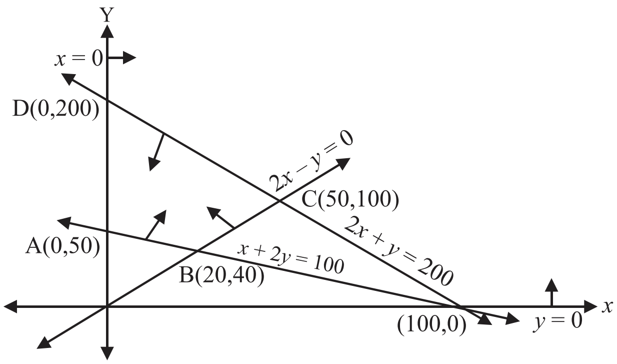
11. (4)

12. Maximum profit occurs at x = 40, y = 160 and the maximum profit = 64,000

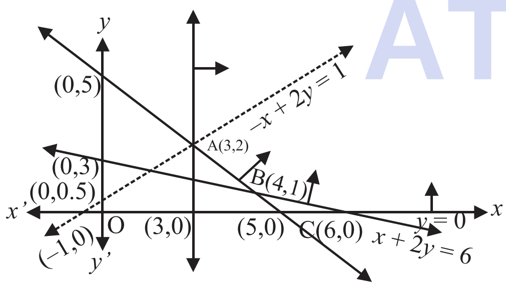
13. (2)

14. (3)

15.



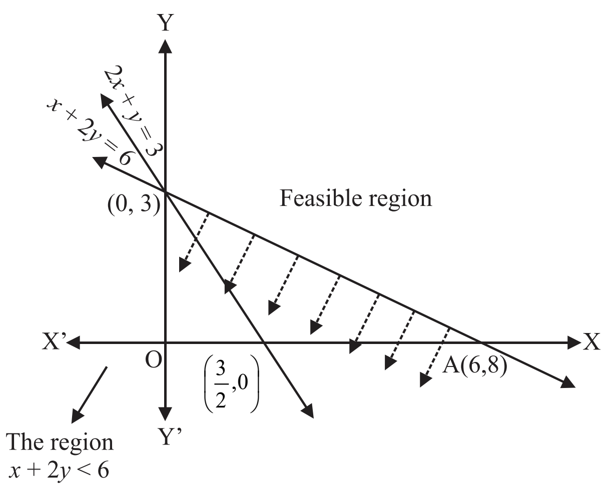
16.



17. **(2)**

18. **(4)**

19.



Probability

1. (1)

$$P(A)=\frac{1}{3}\Rightarrow P(B)=\frac{1}{4}$$

2. $P(A)=\frac{3}{4}\Rightarrow P(B)=\frac{2}{3}$

3. (i) The value of x is 0.23.

(ii) The value of y is 0.04.

(iii) $P(C|\,B)=\frac{23}{36}$ (or approximately 0.6389).

(iv) The probability that a person does Yoga of type A or B but not C is 0.45.

4. (3)

5. $P(F)=\frac{1}{3},\frac{13}{15}$

6. (i) $\frac{109}{300}$ (ii) $\frac{27}{109}$

7. (3)

8. $\frac{11}{30}$

9. (i) $\frac{9}{500}$ (ii) $\frac{7}{18}$

10. (3)

11. $\frac{1}{4},\frac{3}{4},\frac{3}{64}$

12. $\frac{2}{3}$

13. (i) 0.38 (ii) $\frac{7}{19}$ (or approx. 0.368)

14. (4)

15. (i) 0.008 (ii) 0.047

(iii) $\frac{17}{47}$ (iv) 1

16. (3)

17. (i) $\frac{3}{4}$ (ii) $\frac{1}{9}$