
PRAYAS
JEE 2025
Lecture-02
Mathematics
Relation & Functions
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Topics *to be covered*

1. Types of Relations
2. Practice Problems

Recap *of previous lecture*

1. If $n(A) = p$ & $n(B) = q$ then $n(A \times B) =$ pq, $n(B \times A) =$ pq
also if $n(A \cap B) = r$ then $n((A \times B) \cap (B \times A)) =$ r^2

2. $A \times (B \cup C) =$ $(A\times B)\cup(A\times C)$, $A \times (B \cap C) =$ $(A\times B)\cap(A\times C)$, $A \times (B - C) =$ $(A\times B)-(A\times C)$

3. If $n(A \cap C) = 3$, $n(B \cap D) = 4$ then $n((A \times B) \cap (C \times D)) =$ 12.

$n(A\cap C)\cdot n(B\cap D)$

Recap of previous lecture

4. A × B is not equal to B × A in general.

5. R × R × R = R^3 & it denotes the entire 3D Space

6. R × R = R^2 & it denotes the entire 2D Space.

QUESTION

If P, Q and R subsets of a set A then $R \times (P' \cup Q')' =$ $R \times ((P')' \cap (Q')')$

$= R \times (P \cap Q)$

$= (R \times P) \cap (R \times Q)$

(A) $(R \times P) \cap (R \times Q)$

(B) $(R \times Q) \cap (R \times P)$

(C) $(R \times P) \cup (R \times Q)$

(D) none of these

Relation: Any subset of $A \times B$ is said to be a relation from A to B.

★ If $n(A) = n$, $n(B) = m \Rightarrow n(A \times B) = nm \Rightarrow$ No: of Relation from A to B $= 2^{mn}$

★ If $n(A) = n$, $n(B) = m \Rightarrow$ No: of non empty relations from A to B $= 2^{mn} - 1$.

Ex: $A = \{1, 2, 3\}$, $B = \{2, 4\}$ $\Rightarrow A \times B = \{(1,2)\,(1,4)\,(2,2)\,(2,4)\,(3,2),\,(3,4)\}$

$\Downarrow$

No: of Relations from A to B $= 2^6$

$R_3 = \phi$ (void Relation)

$R_1 = \{(1,2), (1,4), (2,4)\}$

$R_2 = \{(3,2)\ (1,4)\}$

$(1,2) \in R_1 \Leftrightarrow 1 R_1 2$

$(3,2) \in R_2 \Leftrightarrow 3 R_2 2$

$(a,b) \in R \Leftrightarrow aRb$

'b' is called image of 'a' under R

'a' is called preimage of 'b' under R

Domain of a Relation: set of all 1st coord in ordered pairs in R is said to be Domain of R

$R_1 = \{(1,2),(1,4),(2,4)\}$ Domain $= \{1,2\}$.

$R_2 = \{(3,2),(1,4)\}$ Domain: $\{3,1\}$

Range of Relation: set of all 2nd coordinates in ordered pairs in R is said to be Range of R.

Range of $R_1 = \{2,4\}$

Range of $R_2 = \{2,4\}$.

if R is a relation from A to B then B is called codomain of R

Ex: $A = \{1,2,3,4,5,6\}$.

$B = \{3,4,9,10\}$

$R_1 : A \to B$ — codomain

Ex: $R_1 = \{(1,3)\ (1,9)\ (3,9)\ (4,10)\}$

Domain = $\{1,3,4\}$

Range = $\{3,9,10\}$

Image of 1 = 3, 9

preimage of 9 = 1, 3.

$4 R_1 10$

Any subset of $A \times A$ is called a Relation from A to A or it is said to be a Relation on A

$R : A \to A$

If $n(A) = n \Rightarrow$ No: of Relations on A $= 2^{n^2}$

Ex: $A = \{1,2,3\}$

$R_1 = \{(1,1)(1,2)\}$

Domain = $\{1\}$

Range = $\{1,2\}$

Image of 1 = 1, 2

preimage of 2 = 1

codomain = A

$R_2 = \{(2,2)(1,1)(3,3)\}$

Relation

Every subset of $A \times B$ defined a relation from set A to set B. If R is relation from $A \rightarrow B$

NOTE :

If $(a\ b) \in R$ then

(i) 'b' is called image of 'a' under R.

(ii) 'a' is called pre-image of 'b'

(iii) $(a\ b) \in R \Leftrightarrow a\,R\,b$

Relation

Domain of Relation

Set of all first entries of all ordered pairs that occur in R

Range of Relation

Set of all second entries in R.

Relation

NOTE :

If $n(A) = p$ $n(B) = q$ then

(i) number of relations from A to B = 2^{pq}

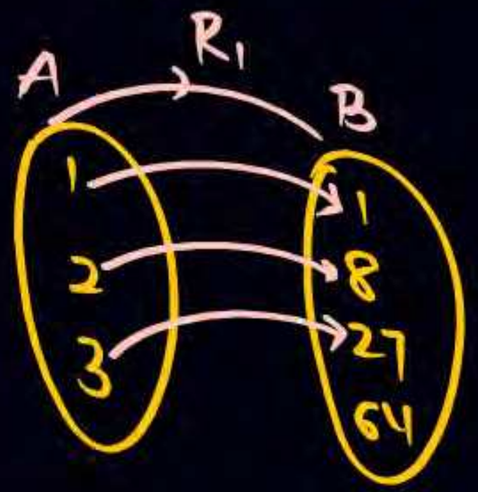
(ii) number of non empty relations from A to B = $2^{pq} - 1$

Representation of a Relation

$A = \{1, 2, 3\}$ $\quad$ $B = \{1, 8, 27, 64\}$

$R_1 = \{(1,1), (2,8), (3,27)\}$ $\longrightarrow$ All elements are listed separated by commas $\Rightarrow$ Roster form.

$R_1 = \{(a,b) \mid b = a^3, a \in A, b \in B\}$ $\longrightarrow$ Property satisfied by all elements only is mentioned $\Rightarrow$ Set Builder form.



Arrow Diagram

Representation of a Relation

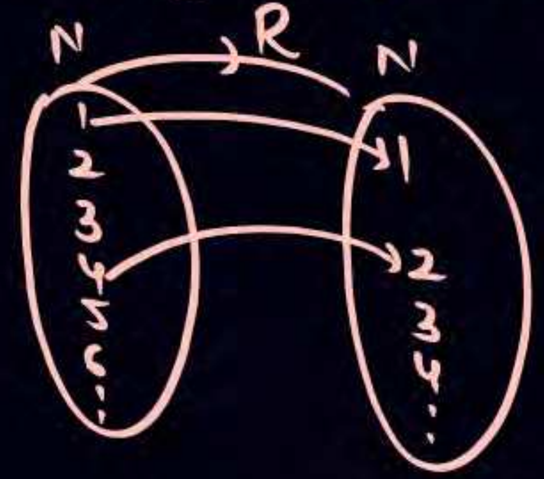
- Roster Form
- Set Builder Form
- Arrow Diagram

Ex: $aRb \iff a = b^2 \quad a, b \in N$.

① R is Relation from $\underline{N}$ to $\underline{N}$

② $R = \{(a,b) \mid a = b^2,\ a, b \in N\}$ Set Buider form

$R = \{(1,1),(4,2),(9,3),(16,4)\ldots\}$ Roster form.



(Arrow Diagram)

QUESTION

A relation R is defined on a set A = {1, 2, 3, 4, 5} defined by

$R = \{(x, y) : |x^2 - y^2| < 20$ then find :

R is a subset of A×A

(a) R

(b) Domain of R = {1, 2, 3, 4, 5}

(c) Range of R = {1, 2, 3, 4, 5}.

R = {(1,1) (1,2) (2,1) (1,3) (3,1) (1,4) (4,1)
(2,2) (2,3) (3,2) (2,4) (4,2)
(3,3) (3,4) (4,3) (3,5) (5,3)
(4,4) (4,5) (5,4)
(5,5)}

QUESTION

Tah 1

If $R = \{(x, y) | x^2 + y^2 \leq 4|$ where $x, y \in Z\}$ is a relation on Z then

(A) Domain of R is $\{0, 1, 2\}$

(B) Domain of R is $\{-2, -1, 0, 1, 2\}$

(C) Domain of R = range of R

(D) $n(R) = 13$

QUESTION

Let X = {1 2 3 4} and Y = {1 3 5 7 9}. Which of the following is relation from X to Y -

$R_1 = \{(1,3)\ (3,5)\}$

(A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$

(B) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$ – Not a Relation $X \longrightarrow Y$ bcoz $R_2 \not\subseteq X \times Y$.

(C) $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$ $R_3 \not\subseteq X \times Y$

(D) $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$ $R_4 \not\subseteq X \times Y$

QUESTION [JEE Mains 2024 (1 Feb)]

Let $A = \{1, 2, 3, \ldots, 20\}$. Let R_1 and R_2 two relation on A such that

$R_1 = \{(a, b) : b \text{ is divisible by } a\}$

$R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$.

a is divisible by b.

Then, number of elements in $R_1 - R_2$ is equal to

$R_1 = \{(1,1)\ (1,2)\ (1,3) - - - (1,20) \rightarrow$ 20 elements

$(2,2)\ (2,4)\ (2,6) - - - (2,20) \rightarrow$ 10 elements.

$(3,3)\ (3,6)\ (3,9) - - - (3,18) \rightarrow$ 6 elements.

$(4,4)\ (4,8) - - - - (4,20) \rightarrow$ 5 elements.

$(5,5)\ (5,10)\ (5,15)\ (5,20) \rightarrow$ 4 elements

$(6,6)\ (6,12)\ (6,18) \rightarrow$ 3 elements.

$(7,7)\ (7,14) \rightarrow$ 2 elements

$(8,8)\ (8,16) \rightarrow$ 2 elements

$(9,9)\ (9,18) \rightarrow$ 2 elements

$(10,10)\ (10,20) \rightarrow$ 2 elements

$\left.(11,11)\ (12,12) - - (20,20)\right\}$ 10 elements

$n(R_1) = 66$

If $(x,y) \in R_1$ then $(y,x) \in R_2 \Rightarrow n(R_2) = 66$.

Ans. 46

$$R_1 - R_2 = R_1 - (R_1 \cap R_2)$$

$$(R_1 \cap R_2) = \{(1,1)\ (2,2)\ (3,3) - (20,20)\}$$

$$n(R_1 \cap R_2) = 20$$

$$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$$

$$= 66 - 20 = 46 \text{ Ans}$$

QUESTION [JEE Mains 2022]

Let R be a relation from the set {1, 2, 3, ..., 60} to itself such that R = {(a, b) : b = pq, where p, q ≥ 3 are prime numbers}. Then the number of elements in R is :

(A) 600

(B) 660

(C) 540

(D) 720

$b = 3\times3,\ 5\times5,\ 7\times7$

$3\times5,\ 3\times7,\ 3\times11,\ 3\times13,\ 3\times17,\ 3\times19$

$5\times7,\ 5\times11$

11 options for b.

$(\ \_\ ,\ \_\ )$

60 ways 11 ways $= 60\times11 = 660$ Ans.

QUESTION [JEE Mains 2023 (6 April)]

Tan2

Let $A = \{1, 2, 3, 4, ..., 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times B : 2(a - b)^2 + 3(a - b) \in B\}$ is ______

Ans. 18

★ If R is a Relation on $A \Rightarrow R \subseteq A \times A$ — $(a,b) \in R$

★ If R is a Relation from A to $B \Longrightarrow R \subseteq A \times B$ — $(a,b) \in R$

★ If R is a Relation on $A \times B \Longrightarrow R \subseteq (A \times B) \times (A \times B)$

$((a,b),(c,d)) \in R$

★ If R is a relation on $A \times A \Rightarrow R \subseteq (A \times A) \times (A \times A)$

Ex: $A = \{1,2\}$

$R = \{(1,2)\,(2,1),(2,2)\}$

$A \times A = \{(1,1)\,(1,2)\,(2,1)\,(2,2)\}$

16 elements

$(A \times A) \times (A \times A) = \{((1,1),(1,1))\,((1,1),(1,2))\,((1,1),(2,1))\,((1,1),(2,2))$
$((1,2),(1,1))\,((1,2)(1,2))\,((1,2),(2,1))\,((1,2)(2,2))\,\ldots\}$

QUESTION [JEE Mains 2024 (9 April)]

Let A = {2, 3, 6, 7} and B = {4, 5, 6, 8). Let R be a relation defined on A × B by (a_1, b_1) R (a_2, b_2) if and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is

$2+6+3+2+8+4=25$ elements

$((a_1, b_1)(a_2, b_2))$

m . n

$((a_1, b_1), (a_2, b_2)) \in R$ if $a_1+a_2 = b_1+b_2$.

$2+2=4$
$2+3=5=3+2$
$2+6=8=6+2$
$2+7=9=7+2$
$3+3=6$
$3+6=9=6+3$
$3+7=10=7+3$
$6+7=13=7+6$
$7+7=14$

$4+4=8$
$4+5=9=5+4$
$4+6=10=6+4$.
$4+8=12=8+4$
$5+5=10$
$5+6=11=6+5$.
$5+8=13=8+5$.
$6+6=12$
$6+8=14=8+6$.
$8+8=16$.

$((2, \underline{4}), (6, \underline{4}))$

a_1+a_2	b_1+b_2
$8 = 2+6 =$	$4+4$
$6+2$	

2×1

$((7, -), (3, -))$

$10 = 7+3$	$4+6$
$3+7$	$6+4$
	$5+5$

2×3 6 elements.

$12 = 6+6$	$6+6$
	$4+8$
	$8+4$

1×3 = 3 elements.

$14 = 7+7$	$6+8$
	$8+6$

1×2 = 2 elements.

8 elements.
4×2

$9 = 2+7 = 4+5$
$7+2 \quad 5+4$
$3+6$
$6+3$

a_1+a_2	b_1+b_2
$13 = 6+7$	$8+5$
$7+6$	$5+8$

4 elements.
$= 2\times2$.

Ans. 25

Inverse of a Relation

$A = \{1, 2, 3\}$, $B = \{2, 4, 5\}$

$R_1 : A \to B$ $\quad R_1 = \{(1,2)(2,4)(2,5)(3,2)\}$ $\quad R_1^{-1} = \{(2,1)(4,2),(5,2),(2,3)\}$

$R_2 : A \to B$ $\quad R_2 = \{(1,5)(2,4)(3,5)\}$ $\quad R_2^{-1} = \{(5,1)(4,2)(5,3)\}$.

if $(a,b) \in R$ then $(b,a) \in R^{-1}$

Domain of $R_1 = \{1, 2, 3\}$ = Range of R_1^{-1}

Range of $R_1 = \{2, 4, 5\}$ = Domain of R_1^{-1}

Domain of R = Range of R^{-1}

Range of R = Domain of R^{-1}

Inverse of a Relation

Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R denoted by R^{-1} is a relation from B to A and is defined by $R^{-1} = \{(b\ a) : (a\ b) \in R\}$

Clearly $(a\ b) \in R \Leftrightarrow (b\ a) \in R^{-1}$.

Range $(R) = \text{Dom}(R^{-1})$.

Domain of R = Range of R^{-1}

QUESTION

Tah3

The relation R defined in $A = \{1, 2, 3\}$ by a R b if $|a^2 - b^2| \le 5$. Which of the following is false?

(A) $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)\}$

(B) $R^{-1} = R$

(C) Domain of $R = \{1, 2, 3\}$

(D) Range of $R = \{5\}$

Types of Relation

Let R be relation on A i.e. $R : A \rightarrow A$ then

- **Identity Relation :**
  A relation defined on a set A is said to be an identity relation if each & every element of A is related to itself & only to itself.

- **Reflexive :**
  A relation defined on a set A is said to be reflexive relation if each & every element of A is related to itself.

Let R be a Relation on $A \Rightarrow R \subseteq A \times A$ then R is said to be

1) Identity Relation : If each & every element of A is related to itself and only to itself by R.

$A = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (3,3)\}$ → Identity Relation.

$R = \{(1,1)\ (2,2)\}$. → Not an identity Relation. b'coz $(3,3)$ is missing

$R = \{(1,1), (2,2), (3,3), (1,3)\}$ → Not an identity Relation → b'coz $(1,3) \in R$

2) Reflexive Relation : If each & every element of A is related to itself by R

(a, a), a ∈ A type ke saaray elements R mai honay chahiyay phir kuch aur elements hoo yaa nahoo

Ex: $A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3)\}$ → Reflexive = Identity

$R = \{(1,1)\ (2,2)\ (1,3)\}$ → Not Reflexive → (3,3) is missing

$R = \{(1,1)\ (2,2)\ (3,3)\ (2,3)\}$ — Reflexive

$R = \{(1,1)\ (2,2)\ (3,2)\}$ — Not Reflexive.

Every identity relation is Reflexive but not the converse.

Identity relation on a set is unique.

(3) Symmetric Relation : If $(a,b) \in R$ then (b,a) also lies in R

$A = \{1, 2, 3\}$

$R_1 = \{(1,1), (1,2), (1,3), (3,1)\}$ → Not symmt.
$(1,2) \in R$ but $(2,1) \notin R$

$R_2 = \{(1,1)(1,2)(2,1)(3,3)\}$. → symmt.

$R_3 = \{(1,3)(1,2)(2,1)(3,1)(2,2)\}$ → symmt.

④ Transitive Relation : If (a,b) & $(b,c) \in R$ then (a,c) should also lie in R.

Agar (a,b) & (b,c) belong to R then (a,c) should also belong to R.

$A = \{1, 2, 3\}$.

$R = \{(1,1)\ (1,2)\ (2,3)\ (1,3)\}$ (Transitive)

$R = \{(1,1)\ (2,3)\}$

$(a,a)\ (a,b)$ check karne ki zaroorat nahi

Empty Relation on a Set A.
- Reflexive ✗
- Symmt. ✓✓
- Transitive ✓✓

⑤ Antisymmetric Relation

: If (a,b) & $(b,a) \in R$ then $a = b$.

$A = \{1, 2, 3\}$

$R = \{(1,2)(1,3)(3,3)\} \rightarrow$ Antisymmt

$R = \{(1,1)(2,2)(3,3)\} \rightarrow$ Symmt ✓✓, Antisymmt ✓✓

$R = \{(1,1)(2,3)(3,2),(3,3)\}$ → Symmt ✓, Antisymmt ✗

Kisi bhi element kaa reverse R mai nahi honaa chahiyay except for elements of type (a,a)

⭐ If a relation is symmetric then it can not be antisymmt (False)

⭐ If a relation is antisymmt. then it can not be symmt. False

(6) Equivalence Relation : If it is Reflexive, Symmt as well as transitive

Types of Relation

➢ **Symmetric :**

A relation defined on a set is said to be symmetric if $a\ R\ b \Rightarrow b\ R\ a$.

If $(a\ b) \in R$ then $(b\ a)$ must be necessarily there in the same relation.

Types of Relation

- **Antisymmetric Relation :**
  A relation on a set A is said to be antisymmetric if (a b) $\in$ R & (b a) $\in$ R then a = b

- **Equivalence Relation :**
  If a relation is Reflexive symmetric and transitive then it is said to be an equivalence relation.

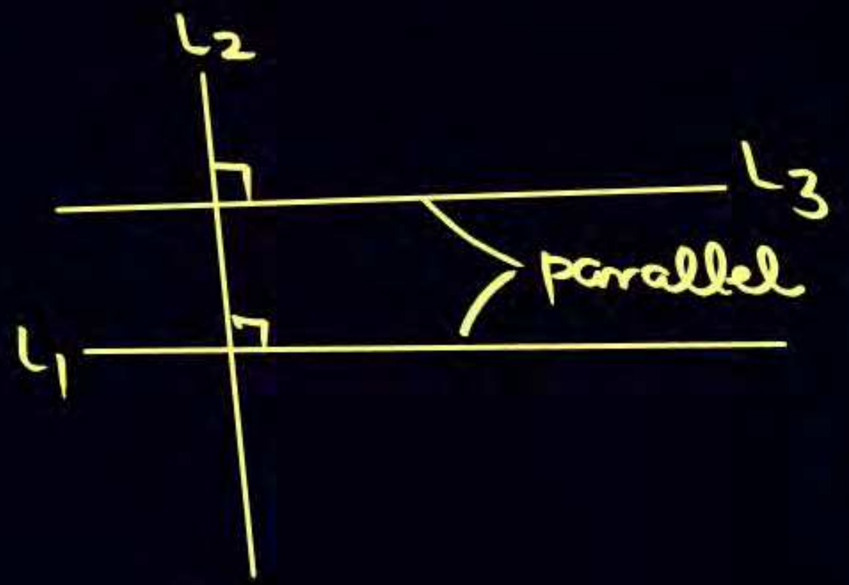
QUESTION

Let L denote the set of all straight lines in a plane. Let a relation R be defined $\alpha R \beta \Leftrightarrow \alpha \perp \beta, \alpha\, \beta \in L$. Then R is

$L = \{ L_\alpha, L_\beta, L_\gamma \ldots \}$

$R : L \to L$ s.t $L_\alpha R L_\beta \Leftrightarrow L_\alpha \perp L_\beta$

Reflexive Relation ✗
since $(L_\alpha, L_\alpha) \notin R$
b'coz no line is ⊥ar to itself



symmt ✓
If $(L_\alpha, L_\beta) \in R \Rightarrow L_\alpha \perp L_\beta$
$\Downarrow$
$(L_\beta, L_\alpha) \Leftarrow L_\beta \perp L_\alpha$
also lies in R.

Transitive.

(L_1, L_2) & $(L_2, L_3) \in R$

$L_1 \perp L_2,\ L_2 \perp L_3 \Rightarrow L_1 \parallel L_3 \Rightarrow (L_1, L_3) \notin R$

Not transitive

- ⭐ To say some statement is true we have to prove it
- ⭐ To say some statement is false we just need one counter example to say it is false

QUESTION

Tah

Relation R in the set of A of human beings in a town at a particular time given by

(A) R = {(x y) : x and y work at the same place} Reflex ✓ symm ✓ Transt ✓

(B) R = {(x y) : x and y live in the same locality} Ref ✓ symm ✓ Trans ✓

same locality

$(x, y), (y, z)$

(x, z)

(C) R = {(x y) : x is exactly 7 cm taller than y}

(D) R = {(x y) : x is wife of y}

(E) R = {(x y) : x is father of y}

QUESTION [AIEEE 2006]

Let W denote the words in the English dictionary. Define the relation R by: $R = \{(x,y) \in W \times W \mid$ the words x and y have at least one letter in common$\}$. Then R is

(A) reflexive symmetric and not transitive

(B) reflexive symmetric and transitive

(C) reflexive not symmetric and transitive

(D) not reflexive symmetric and transitive

$(x, y) \in R$ if x & y have atleast one alphabet common

word

Reflexive ✓ $(x,x) \in R \ \forall x \in W$

symmt ✓ if $(x,y) \in R \Rightarrow x$ & y have atleast one common alphabet $\Downarrow$ $(y,x) \in R$

✗ Transitive:

(pen, egg), (egg, gut) $\in R$

but (pen, gut) $\notin R$

QUESTION [JEE Mains 2022 (28 June)]

Let $R_1 = \{(a\ b) \in N \times N : |a - b| \leq 13\}$ and $R_2 = \{(a\ b) \in N \times N : |a - b| \neq 13\}$. Then on N :

(A) ~~Both R_1 and R_2 are equivalence relations~~

(B) ~~Neither R_1 nor R_2 is an equivalence relation~~

(C) ~~R_1 is an equivalence relation but R_2 is not~~

(D) R_2 is an equivalence relation but R_1 is not

$R_1 : N \to N$

R_1:
- Reflexive ✓ $\because (a, a) \in R$ since $|a-a| \leq 13, \forall a \in N$
- symmt ✓ If $(a, b) \in R \Rightarrow |a-b| \leq 13 \Downarrow |b-a| \leq 13 \Rightarrow (b, a) \in R$
- Transitivity: $(1,7), (7,15) \in R$. But $(1,15) \notin R$ b'coz $|1-15| = 14 \not\leq 13$

R_2:
- Reflexive ✓ $\because |a-a| = 0 \neq 13 \Rightarrow (a,a) \in R_2\ \forall a \in N$
- symmt ✓ $\because$ If $(a,b) \in R \Rightarrow |a-b| \neq 13 \Rightarrow |b-a| \neq 13 \Downarrow (b,a) \in R$
- ✗ Transitive: $(1,7), (7,14) \in R$ But $(1,14) \notin R$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP

KTK,TAH
CHALLENGER

QUESTION

Paragraph

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1}), n \in N$ and I is an identity matrix of order 3 then answer the following questions.

det. $(A_0 + A_0^2 B_0^2 + A_0^3 + A_0^4 B_0^4 + \ldots$ 10 terms) is equal to

(A) 1000

(B) -800

(C) 0

(D) -8000

Ans. C

QUESTION

PW

Paragraph

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1}), n \in N$ and I is an identity matrix of order 3 then answer the following questions.

$B_1 + B_2 + \ldots\ldots + B_{49}$ is equal to

(A) B_0

(B) $7B_0$

(C) $49B_0$

(D) $49I$

Ans. C

QUESTION

Paragraph

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1}), n \in N$ and I is an identity matrix of order 3 then answer the following questions.

For a variable matrix X the equation $A_0X = B_0$ will have

(A) unique solution

(B) infinite solution

(C) finitely many solution

(D) no solution

Ans. D

QUESTION

Consider a system of linear equation $3x + y - z = 0$, $x - \frac{py}{4} + z = 2$ and $2x - y + 2z = q$ where $p, q \in I$ and $p, q \in [1, 10]$, then identify the correct statement(s).

List-I		List-II	
(I)	Number of ordered pairs (p, q) for which system of equation has unique solution is	(P)	1
(II)	Number of ordered pairs (p, q) for which system of equation has no solution is	(Q)	9
(III)	Number of ordered pairs (p, q) for which system of equation has infinite solution is	(R)	91
(IV)	Number of ordered pairs (p, q) for which system of equation has atleast one solution is	(S)	90

QUESTION

Which one of the following option is correct?

(A) I → P, II → R, III → S, IV → R

(B) I → Q, II → S, III → P, IV → R

(C) I → S, II → Q, III → P, IV → R

(D) I → Q, II → P, III → S, IV → P

Ans. C

Homework from Module

Chapter: Matrices

Prarambh: COMPLETE

Prabal : Complete

(Revision Practice Problems)

QUESTION

RPP 1

Let $a, b, c, d \in R$; $a + b + c + d = 10$, the minimum value of $a^2 \cot 9^\circ + b^2 \cot 27^\circ + c^2 \cot 63^\circ + d^2 \cot 81^\circ$ is $\sqrt{n}$; $n \in N$, then 'n' is

(A) even

(B) odd

(C) prime

(D) divisible by 5

Ans. B, D

QUESTION

RPP 2

If the number of solutions of the equation

$\cos^2\left(\frac{\pi}{4}(\cos x + \sin x)\right) - \tan^2\left(x + \frac{\pi}{4}\tan^2 x\right) = 1$ in $[-2\pi, 2\pi]$ is 'k', then $\frac{3k}{25}$ equals

Ans. 0.48

QUESTION

RPP 3

Let $f_n(\theta) = \sum_{r=0}^{n} \frac{1}{4^r} \cdot \sin^4(2^r\theta)$, then

(A) $f_2\left(\frac{\pi}{4}\right) = \frac{\pi}{\sqrt{2}}$

(B) $f_3\left(\frac{\pi}{8}\right) = \frac{2+\sqrt{2}}{4}$

(C) $f_4\left(\frac{3\pi}{2}\right) = 1$

(D) $f_5(\pi) = 0$

Ans. C, D

Previous TAH

Solutions

QUESTION [JEE Mains 2022 (26 June)]

Let $A = \{n \in N : \text{H.C.F.}\ (n, 45) = 1\}$ and let $B = \{2k : k \in \{1, 2, \ldots\ldots, 100\}\}$.
Then the sum of all the elements of $A \cap B$ is ____

Ans. 5264

KASHYAP

Q1 $A = \{n \in N : HCF(n,45) = 1\}$
$B = \{2k : k \in \{1,2,\ldots,100\}\}$
Then the sum of all the elements of $A \cap B$ is ___

A = A consist of elements which have HCF = 1 with 45

$\Downarrow$

If $\alpha \in A \Rightarrow \alpha$ is not divisible by 3 or 5

$45 = 3^2 \cdot 5$

$B = \{2,4,6,8,\ldots,100\}$

$A \cap B$ elements in B s.t HCF $(\alpha,45) = 1$

$M_3 = \{6,12,18,\ldots,198\}$
$M_5 = \{10,20,30,\ldots,200\}$
$M_3 \cap M_5 = \{30,60,90,120,\ldots,180\}$

* $n(M_3)$ $\quad 6+(P-1)6 = 198$, $P = 33$

$S(M_3) = \frac{33}{2}(6+198) = 3366$

* $n(M_5)$ $\quad 10+(P-1)10 = 200$, $P = 20$

$S(M_5) = \frac{20}{2}(10+200) = 2100$

* $n(M_3 \cap M_5)$ $\quad 30+(P-1)30 = 180$, $P = 6$

$S(M_3 \cap M_5) = \frac{6}{2}(30+180) = 630$

Sum of elements divisible by 3 or 5
$= S_{M_3} + S_{M_5} - S_{M_3 \cap M_5}$
$= 3366 + 2100 - 630$
$= 4836$

Sum of all elements in set B
$= \frac{100}{2}(2+200) = (5050)2$
$= 10100$

$\therefore$ Final ans
$S_{A \cap B} = 10100 - 4836$
$= 5264$

Mains 2022 (26 June)

Ques ⇒ Let $A = \{n \in N : HCF(n,45) = 1\}$ and let $B = \{2k : k \in \{1,2,\ldots,100\}\}$. Then the sum of all the elements of $A \cap B = ?$

$B = \{2,4,6,\ldots,200\} \rightarrow S = 100 \times 101 = 10100$

A consist of elements which have HCF = 1 with 45
If $\alpha \in A \rightarrow \alpha$ is not divisible of 3 or 5.

$A \cap B$ — elements in B such that $HCF(\alpha,45) = 1$

$M_3 = \{6,12,18,\ldots,198\}$

$M_5 = \{10,20,30,\ldots,200\}$

$M_3 \cap M_5 = \{30,60,90,120,150,180\}$

Sum of elements divisible by 3 or 5
$= S_{M_3} + S_{M_5} - S_{M_3 \cap M_5}$
$= \frac{33}{2}(6+198) + \frac{20}{2}(10+200) - \frac{6}{2}(30+180)$
$= 33 \times 102 + 10 \times 210 - 3 \times 210$
$= 3366 + 210(10-3) = 4836$

Sum of elements not divisible by 3 or 5
= Total sum − Sum of elements divisible by 3 or 5
$= 10100 - 4836 = 5264$ Ans.

Shweta from UP

QUESTION [JEE Mains 2020]

A survey shows that 63% of the people in a city reads newspaper A whereas 76% read newspaper B. If x% of the people read both the newspaper, then a possible value of x can be

(A) 55

(B) 65

(C) 29

(D) 37

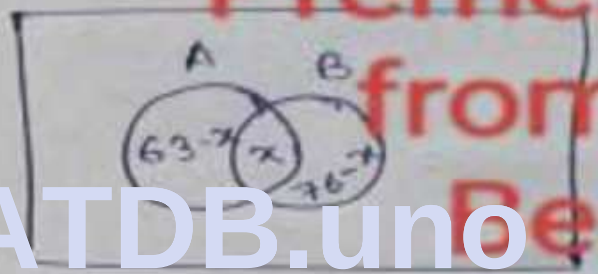
IAH - 1

A survey shows that 63% of the people in a city reads newspaper A, whereas 76% read newspaper B. If x% of the people read both the newspaper, then a possible value of x can be.

$n(U) = 100$

$n(A) = 63$

$n(B) = 76$



$63 - x \geq 0$

$x \leq 63$

$76 - x \geq 0$

$x \leq 76$

$n(A \cup B) \leq 100$

$63 + 76 - x \leq 100$

$139 - x \leq 100$

$x \geq 39$

$\therefore \; 39 \leq x \leq 63$

(A) 55 Ans.

(B) 65

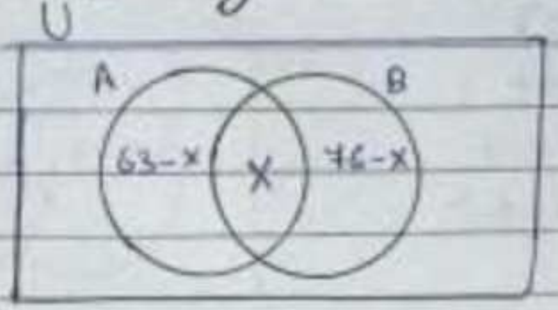
(C) 29

(D) 37

Que [JEE Main 2020] TAH-1

A Survey shows that 63% of the people in a city reads newspaper A whose 76% read newspaper B. If x% of the people read both the the newspaper, then a possible value of x can be.

(A) 55
(B) 65
(C) 29
(D) 37.



$63 - x \geq 0 \Rightarrow x \leq 63$

$76 - x \geq 0 \Rightarrow x \leq 76$

$x \geq 0 \Rightarrow x \geq 0$

$\Downarrow$

$0 \leq x \leq 63$ (I)

$n(A \cup B) \leq 100$

$63 + 76 - x \leq 100$

$139 - x \leq 100$

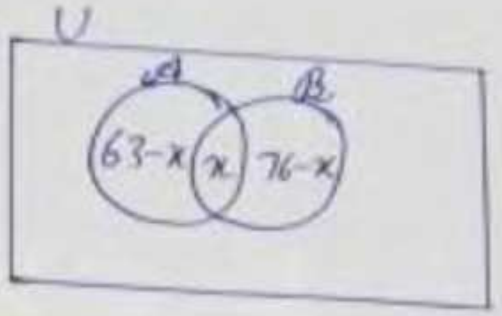
$x \geq 39$ (II)

$\Rightarrow 39 \leq x \leq 63$

$\therefore$ Possible value of $x \Rightarrow 55$ Ans

Shreya Sahu
from
Madhya Pradesh

TAH-1



Sol^n: $n(U) = 100$

$n(A) = 63$

$n(B) = 76$

We know that,

$63 - x \geq 0 \Rightarrow x \leq 63$

$76 - x \geq 0 \Rightarrow x \leq 76$

$x \geq 0$

$\Rightarrow 0 \leq x \leq 63$ — (1)

$n(A \cup B) \leq 100$

$63 + 76 - x \leq 100$

$x \geq 39$ — (2)

$\boxed{39 \leq x \leq 63}$

$\therefore$ Option (A) 55 is correct.

Name- Bhumika Sharma
From- Sri Ganganagar, Rajasthan

QUESTION [JEE Mains (July) 2021]

Out of all the patients in a hospital 89% are found to be suffering from heart aliment and 98% are suffering from lungs infection. If k% of them are suffering from both aliments, the K can not belong to the set :

(A) {80, 83, 86, 89}

(B) {84, 86, 88, 90}

(C) {79, 81, 8, 85}

(D) {84, 87, 90, 93}

JAN(2)

H, L: $89-K$, K, $98-K$

$K \le 89$
$K \le 98$
$K \ge 0$
$\Rightarrow 0 \le K \le 98$ — (i)

$n(H \cup L) \le 100$

$89 + 98 - K \le 100$

$87 \le K$ — (ii)

From (i) & (ii)

$87 \le K \le 98$

$K : \{87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98\}$

(C) $K \notin \{79, 81, 83, 85\}$

Manik kumhar

Tan2

$n(p) = 89$
$n(\text{lungs infe}) = 98$

$89-K$, K, $98-K$

$89 - K \ge 0 = x \le 89$
$98 - K \ge 0 = x \le 98$
$x \ge 0$

$0 \le x \le 89$ — (i)

$n(P \cup L) \le 100$
$89 + 98 - K \le 100$
$187 - K \le 100$
$K \ge 87$ — (ii)

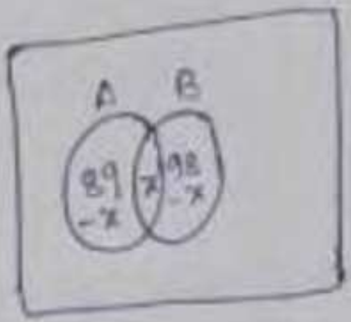
Sumit kar
from Bihar

$= 87 \le x \le 89$

$= \{80, 83, 86, 89\}$ Ans

ATDB.uno

tah - 2



$n(U) = 100$

$n(A) = 89$

$n(B) = 98$

$89 - x \geq 0 \qquad 98 - x \geq 0$

$x \leq 89 \qquad x \leq 98$

$x \geq 0$

$n(A \cup B) \leq 100 \qquad 0 \leq x \leq 89$

$89 + 98 - x \leq 100$

$187 - x \leq 100$

$x \geq 87$

$\therefore \quad 87 \leq x \leq 89$

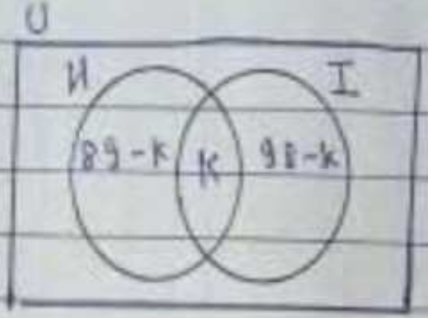
Ans. (c) {79, 81, 8.., 85}

Premendu Gain from West Bengal

Tah 2 [JEE Main (July 2021)]

Que Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set:

(a) {80, 83, 86, 89}
(b) {84, 86, 88, 90}
(c) {79, 81, 83, 85}
(d) {84, 87, 90, 93}



$\Rightarrow \quad 89 - K \geq 0 \Rightarrow K \leq 89$

$98 - K \geq 0 \Rightarrow K \leq 98$

$K \geq 0 \Rightarrow K \geq 0$

$\Downarrow$

$0 \leq x \leq 89$ — (i)

$n(H \cup I) \leq 100$

$89 + 98 - K \leq 100$

$187 - K \leq 100$

$K \geq 87$ — (ii)

$87 \leq x \leq 89$

(c) {79, 81, 83, 85}, K can not belong to the set

Shreya Sahu from Madhya Pradesh

QUESTION [JEE Mains 2019]

In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics, course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is.

(A) 42

(B) 1

(C) 102

(D) 38

Date :

Tab3

$M = \{2, 4, 6, --- 140\} = 70$
$P = \{3, 6, 9, --- 138\} = 46$
$C = \{5, 10, 15, --- 140\} = 28$

$n(M) = 70$
$n(P) = 46$
$n(C) = 28$

Even no. divisible by $3 = \{6, 12, -- 138\} = 23$
" $5 = \{10, 20 - 140\} = 14$
" $3 \& 5 = \{30, 60, 90, 120\} = 4$
no. divisible by $(3 \& 5) = \{15, 30, --- 135\} = 9$

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - (n(M \cup P) + n(P \cap C) + n(C \cap M)) + n(M \cap P \cap C)$

$\Rightarrow 70 + 46 + 28 - 23 - 14 - 9 + 4 = 102$

= total Student $- n(M \cup P \cup C)$
$140 - 102$
38 Ans

Sumit kumar

$n(M) \rightarrow$ those numbered students which are even i.e divisible by 2 & selected Mathematics course.

$P \rightarrow$ those numbered students which are divisible by 3 & opted physics course.

$C \rightarrow$ those numbered students which are divisible by 5 & opted Chemistry course.

$n(U) = 140$
$n(M) = 70$
$n(P) = 46$
$n(C) = 28$
$n(M \cap P) = 23$
$n(M \cap C) = 14$
$n(P \cap C) = 9$
$n(M \cap P \cap C) = 4$

$\because M = \{2, 4, 6, \ldots, 140\}$
$P = \{3, 6, 9, \ldots, 138\}$
$C = \{5, 10, 15, \ldots, 140\}$
$M \cap P = \{6, 12, \ldots, 138\}$
$M \cap C = \{10, 20, \ldots, 140\}$
$P \cap C = \{15, 30, \ldots, 135\}$
$M \cap P \cap C = \{30, 60, \ldots, 120\}$

$\Rightarrow n(M \cup P \cup C) = n(M) + n(P) + n(C) - (n(M \cap P) + n(M \cap C) + n(P \cap C)) + n(M \cap P \cap C)$.

$= 70 + 46 + 28 - (23 + 14 + 9) + 4$
$n(M \cup P \cup C) = 148 - (46)$
$= 102$

$\Rightarrow n(M \cup P \cup C)' = 140 - 102$
$= 38$ Ans

Vishal Yadav
Tah 3

In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted physics and those whose number is divisible by 5 opted chemistry course. Then the number of students who did not opt for any of the three course = ?

TAH 3

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - (n(M \cap P) + n(P \cap C) + n(C \cap M)) + n(M \cap P \cap C)$

$M = \{2, 4, 6, \ldots, 140\} \Rightarrow n(M) = 70$

$P = \{3, 6, 9, \ldots, 138\} \Rightarrow n(P) = 46$

$C = \{5, 10, 15, \ldots 140\} \Rightarrow n(C) = 28$

$M \cap P = \{6, 12, 18 \ldots 138\} \Rightarrow n(M \cap P) = 23$

$P \cap C = \{15, 30, 45 \ldots 135\} \Rightarrow n(P \cap C) = 9$

$C \cap M = \{10, 20, 30 \ldots 140\} \Rightarrow n(C \cap M) = 14$

$M \cap P \cap C = \{30, 60, 90, 120\} \longrightarrow n(M \cap P \cap C) = 4$

$n(M \cup P \cup C) = 70 + 46 + 28 - (23 + 9 + 14) + 4$

$= 144 - 46 + 4 = 102$

No. of students who did not opt for any of the three cour[ses]

$=$ Total students $- \; n(M \cup P \cup C)$

$= 140 - 102 = 38$ Ans

Shweta from UP

QUESTION

From 51 students taking examinations in Mathematics, Physics and Chemistry, 37 passed, Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. The largest possible number that could have passed all three examination is-

(A) 11

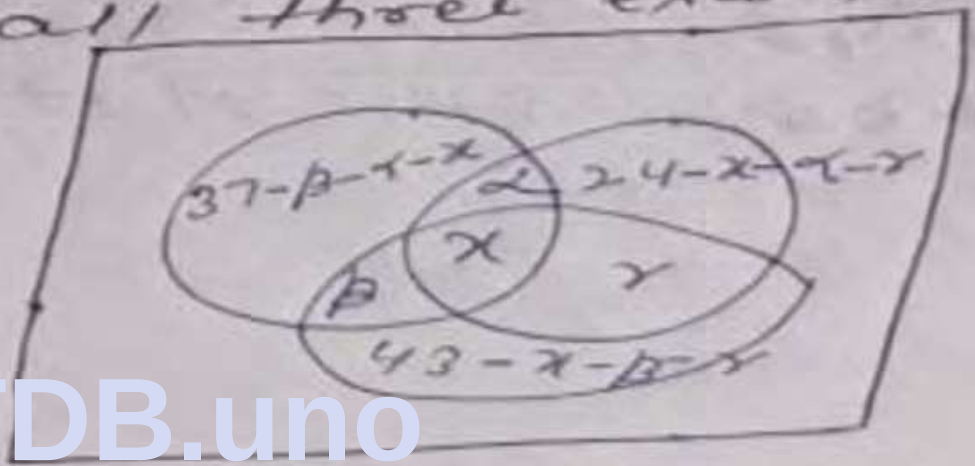
(B) 15

(C) 16

(D) 14

Q37. From 51 students taking examinations in mathematics, physics and chemistry, 37 passed mathematics, 24 physics & 43 chemistry. At most 19 passed mathematics & physics, at most 29 passed maths & chemistry & at most 20 passed physics & chemistry. The largest ~~...~~ possible number that could have passed all three examination is

(a) 11
(b) 15
(c) 16
(d) 14



$M = 37$

$P = 24$

$C = 43$

$n(M \cap P) \le 19$

$\Rightarrow \alpha + x \le 19$ — (I)

$n(M \cap C) \le 29 \rightarrow \beta + x \le 29$ — (II)

$n(P \cap C) \le 20 \rightarrow \gamma + x \le 20$ — (III)

$\alpha + \beta + \gamma \le 68 - 3x$ — (III)

$37 - \cancel{\beta} - \cancel{\alpha} - \cancel{x} + x + \cancel{\alpha} + \cancel{\beta} + \cancel{\gamma} + 24 - x - \alpha - \gamma + 43 - x - \beta - \gamma \le 51$

$61 + 43 - 2x - (\gamma + \beta + \gamma) \le 51$

$104 - 2x - 68 - 3x \le 51$

$x \le 51 - 36$

$x \le 15$

$\boxed{x_{max} = 15}$

Tah-4

Gautam from muzaffarpur bihar

QUESTION

In a class of 200 students, 70 played cricket, 60 played hockey and 80 played football. 30 played cricket and football, 30 played hockey and football, 40 played cricket and hockey. Find the maximum number of people playing all the three games and also the minimum number of people playing at least one game?

(A) 200, 100

(B) 30, 110

(C) 30, 120

(D) 20, 110

QUESTION

If $n(A) = 7$, $n(B) = 8$ and $n(A \cap B) = 4$, then which of the following columns.

(i)	$n(A \cup B)$	(a)	56
(ii)	$n(A \times B)$	(b)	16
(iii)	$n((B \times A) \times A)$	(c)	392
(iv)	$n(A \times B) \cap (B \times A))$	(d)	96
(v)	$n((A \times B) \cup (B \times A)$		

Tah 7 :- If $n(A) = 7$, $n(B) = 8$, & $n(A \cap B) = 4$, then which of the following columns.

(i)	$n(A \cup B)$	(a) 56
(ii)	$n(A \times B)$	(b) 16
(iii)	$n((B \times A) \times A)$	(c) 392
(iv)	$n((A \times B) \cap (B \times A))$	(d) 96.
(v)	$n((A \times B) \cup (B \times A))$	(e) 11

Vishal Yadav

Tah 7

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$= 7 + 8 - 4$
$= 11$

(ii) $n(A \times B) = n(A) \cdot n(B)$
$= 7 \cdot 8$
$= 56.$

(iii)
$n(B \times A) = 56$
$\Rightarrow n((B \times A) \times A) = 56 \times 7$
$= 392$

(iv) $n((A \times B) \cap (B \times A)) = (n(A \cap B))^2$
$= 16$

Date:
Page:

(v)

$$n((A\times B)\cup(B\times A)) = n(A\times B) + n(B\times A) - n((A\times B)\cap(B\times A))$$

$$= 56 + 56 - 16$$

$$= 96$$

Tah 7

7) A/Q $n(A) = 7$, $n(B) = 8$, $n(A \cap B) = 4$

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= \underline{11}$

(ii) $n(A \times B) = n(A) \cdot n(B) = 56$

(iii) $n((B \times A) \times A) = n(B \times A) \cdot n(A)$

$= n(B) \cdot n(A) \cdot n(A)$

$= 392$

(iv) $n((A \times B) \cap (B \times A)) = (n(A \cap B))^2 = 4^2 = 16$

(v) $n((A \times B) \cup (B \times A))$

$= n(A \times B) + n(B \times A) - n((A \times B) \cap (B \times A))$

$= 56 + 56 - 16$

$= 96$

$\boxed{\text{By: Mayank Raj}}$ From: Jharkhand

TAH7 If $n(A) = 7$, $n(B) = 8$ and $n(A \cap B) = 4$, then which of the following columns.

(i) $n(A \cup B)$ - None a) 56
(ii) $n(A \times B)$ - (a) b) 16
(iii) $n((B \times A) \times A)$ - (c) c) 392
(iv) $n((A \times B) \cap (B \times A))$ - (b) d) 96
(v) $n((A \times B) \cup (B \times A))$ - (d)

(i) $7 + 8 - 4 = 11$

(ii) $n(A) \cdot n(B) = 7 \times 8 = 56$

(iii) $n(B \times A) \cdot n(A)$
$56 \times 7 = 392$

(iv) $(n(A \cap B))^2 = 4^2 = 16$

(v) $n(A \times B) + n(B \times A) - n((A \times B) \cap (B \times A))$
$56 + 56 - 16$
$= 96$

QUESTION

If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and $C = \{1, 5, 4, 3\}$ then find the number of elements in

(i) $A \times B \times C$

(ii) $(A \times B) \cap (B \times C)$

(iii) $(A \times B \times C) \cap (B \times C \times A)$

Th:- 8:→ If $A=\{1,2,3\}$, $B=\{2,3,4\}$ and $C=\{1,5,4,3\}$ then ... the no. of element in

(I) $A\times B\times C$

$n(A)=3$ $\quad n(A\cap B)=2$
$n(B)=3$ $\quad n(B\cap C)=2$
$n(C)=4$ $\quad n(A\cap C)=2$

→ $n(A\times B\times C)=3\times3\times4=36$

(ii) $(A\times B)\cap(B\times C)=n(A\cap B)\cdot n(B\cap C)$
$=2\cdot2=4$

Vishal Yadav
Tah 8

(iii) $(A\times B\times C)\cap(B\times C\times A)$

if $(\alpha,\beta,\gamma)\in(A\times B\times C)\cap(B\times C\times A)$

$\Rightarrow(\alpha,\beta,\gamma)\in(A\times B\times C)$ & $(\alpha,\beta,\gamma)\in(B\times C\times A)$.

$\Rightarrow\alpha\in A$ & B $\Rightarrow\alpha\in A\cap B$
$\beta\in B$ & C $\quad\beta\in B\cap C$
$\gamma\in C$ & A $\quad c\in C\cap A$

$\Rightarrow n((A\times B\times C)\cap(B\times C\times A))=n(A\cap B)\cdot n(B\cap C)\cdot n(A\cap C)$

Tah-8

If $A=\{1,2,3\}$, $B=\{2,3,4\}$
$C=\{1,5,4,3\}$
Find no. of elements in ⟶
$n(A)=3$, $n(B)=3$, $n(C)=4$

i) $A\times B\times C$.
$n(A\times B\times C)=n(A).n(B).n(C)$
$=3\times3\times4$
$n(A\times B\times C)=36$

ii) $(A\times B)\cap(B\times C)$

$n((A\times B)\cap(B\times C))=2\times2=4$

2 2

iii) $(A\times B\times C)\cap(B\times C\times A)$

$n[(A\times B\times C)\cap(B\times C\times A)]=2^3\times2\times2=8$

2×2 2

(Solution to KTK)

QUESTION

Paragraph

There exists a matrix Q such that $PQP^T = N$, where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given N is a diagonal matrix of form $N = \text{diag.}(n_1, n_2, n_3)$ where n_1, n_2, n_3 are satisfying the equation $\det.(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

The value of det. (adj N) is equal to

[Note : adj M denotes the adjoint of a square matrix M]

(A) 4

(B) $\frac{1}{4}$

(C) $\frac{1}{9}$

(D) 9

Ans. D

KTK01. $PQP^T = N$, $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $N = \text{diag}\begin{bmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & 0 & n_3 \end{bmatrix}$

$Q = N$ $\quad \det(P - nI) = 0$, $n_1 < n_2 < n_3$

$$P - nI = \begin{bmatrix} 1-n & 2 & 0 \\ 2 & 1-n & 0 \\ 0 & 0 & 1-n \end{bmatrix}$$

$$|P - nI| = (1-n)(1-n)^2 - 2(2(1-n)) = 0$$

$$= (1-n)\{(1-n)^2 - 4\} = 0$$

$$= 1-n = 0 \qquad (1-n-2)(1-n+2) = 0$$

$n = 1 = n_2$ $\qquad n = -1 = n_1$ $\qquad n = 3 = n_3$

(a) $\det(\text{adj } N)$ value

$$= N = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$= \text{adj } N = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix} = -3I$$

$$= |\text{adj } N| = |-3I|^{3-1}$$

$$= |-3|^2 = 9 \text{ Ans}$$

Shivani
From bihar

KTK-1

$P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad PQP^T = N \quad , (P - nI) = 0$

$P^T = P$

$$PP^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$(P - nI) = 0$ taking det in bot sides

$|P - nI| = 0$

$$\begin{vmatrix} 1-n & 2 & 0 \\ 2 & (1-n) & 0 \\ 0 & 0 & (1-n) \end{vmatrix} = 0 \Rightarrow (1-n)\{(1-n)^2 - 4\} = 0$$

$(1-n) = 0 \quad , (1-n-2)(1-n+2) = 0$

$n = 1 \quad , (-1-n)(3-n) = 0$

$n = 1, \quad n = -1 \quad , n = 3$

$\therefore n_1 = -1, n_2 = 1, n_3 = 3$

$$N = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, |(\text{adj } N)| = |N|^{3-1} = |N|^2$$

$$= (-1+4)^2 = 3^2 = 9$$

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Paragraph-1

Given N is diagonal matrix of form $N = \text{diag}(n_1, n_2, n_3)$, where n_1, n_2, n_3 are satisfying the eq^n $\det(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

KTK I A

The value of $\det(\text{adj } N)$ is equal to—

[Note: adj M denotes the adjoint of a square matrix M]

$\Rightarrow \quad P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad$ N is diagonal matrix of form, $N = \text{diag}(n_1, n_2, n_3)$ where, n_1, n_2, n_3 are satisfying the eq^n $\det(P - nI) = 0$

$$\det(P - nI) = \begin{vmatrix} 1-n & 2 & 0 \\ 2 & 1-n & 0 \\ 0 & 0 & 1-n \end{vmatrix} = 0$$

$\Rightarrow (1-n)[n^2 - 2n + 1 - 4] = 0$

$\Rightarrow (n-1)(n^2 - 2n - 3) = 0$

$\Rightarrow (n-1)(n+1)(n-3) = 0$

$\therefore n = -1, 1, 3$

$n_1 = -1, n_2 = 1, n_3 = 3$ (as $n_1 < n_2 < n_3$)

$$N = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\det(N) = -3$.

Now, $\det(\text{adj } N) = |N|^{3-1}$

$= (-3)^2 = (9)$ Ans

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QUESTION

Paragraph

There exists a matrix Q such that $PQP^T = N$, where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given N is a diagonal matrix of form $N = \text{diag.}(n_1, n_2, n_3)$ where n_1, n_2, n_3 are satisfying the equation $\det.(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

If $Q^T = Q + \alpha I$, then the value of α is equal to

(A) -1

(B) 0

(C) 1

(D) $\frac{-1}{3}$

Ans. B

QUESTION

Paragraph

There exists a matrix Q such that $PQP^T = N$, where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given N is a diagonal matrix of form $N = \text{diag.}(n_1, n_2, n_3)$ where n_1, n_2, n_3 are satisfying the equation $\det.(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

The trace of matrix P^{2012} is equal to

[Note: The trace of a matrix is the sum of its diagonal entries]

(A) $3^{2011} + 2$

(B) 3^{2012}

(C) $3^{2012} + 2$

(D) 3^{2011}

Ans. C

Paragraph - 3 The trace of matrix P^{2012} is equal to —

[Note: The trace of a matrix is the sum of its diagonal entries]

KTK I C

$\rightarrow$ $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore tr(P^2) = 11 = (3^2 + 2) \cdot$$

$$P^4 = P^2 \cdot P^2 = \begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 41 & 40 & 0 \\ 40 & 41 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$tr(P^4) = 41 + 41 + 1 = 83 = (3^4 + 2).$$

$$P^6 = P^4 \cdot P^2 = \begin{bmatrix} 41 & 40 & 0 \\ 40 & 41 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 365 & 364 & 0 \\ 364 & 365 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$tr(P^6) = 365 + 365 + 1 = 731 = (3^6 + 2).$$

$\vdots$

General form for P^{2n} matrix, where $n \in N$

$$tr(P^{2n}) = 3^{2n} + 2.$$

Now, $tr(P^{2012}) = (3^{2012} + 2)$ Ans

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(Solution to RPP)

QUESTION

(RPP 1)

Consider the equation $3x^4 - 18x^3 + px^2 - 8qx + 3q = 0$. The equation has only positive real roots then the value of $\frac{p}{q}$ is

(A) $\frac{1}{8}$

(B) 4

(C) $\frac{1}{4}$

(D) 8

Ans. D

RPP-1

$3x^4 - 18x^3 + px^2 - 8qx + 3q = 0 \; \begin{cases}\alpha\\ \beta\\ \gamma\\ \delta\end{cases}$, $\alpha, \beta, \gamma, \delta \in R^+$

$S_1 = \alpha + \beta + \gamma + \delta = \frac{-(-18)}{3} = 6.$

$S_2 = \Sigma\alpha\beta = \alpha\beta + \alpha\gamma + \alpha\gamma + \beta\gamma + \beta\gamma + \gamma\delta = \frac{p}{3}$

$S_3 = \Sigma\alpha\beta\gamma = \alpha\beta\gamma + \alpha\gamma\delta + \beta\gamma\delta + \beta\delta\alpha = \frac{-(8q)}{3} = \frac{8q}{3}$

$S_4 = \alpha\beta\gamma\delta = \frac{3q}{3} = q.$

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Now $\quad \frac{S_3}{S_4} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{8}{3}$

AM $\overset{HM}{\cancel{GM}}$ ineq,

$$\frac{4}{\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta}} = \frac{\alpha + \beta + \gamma + \delta}{4}$$

$$\frac{4}{8/3} \leq \frac{6}{4} \Rightarrow 3/2 \leq 3/2 \Rightarrow \text{HM} = \text{AM satisfy}$$

$$\therefore \alpha = \beta = \gamma = \delta = 3/2$$

Now $\quad S_2 = \frac{p}{3} = 6 \cdot \left(\frac{3}{2}\right)^2 = \frac{6 \times 9}{4} \rightarrow p = \frac{81}{2}$

$S_4 = \cdot q = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$

$\therefore \frac{p}{\ } = \frac{81/2}{\ } = 16/2 = 8.$

RPP-1 Consider the eq^n $3x^4 - 18x^3 + px^2 - 8qx + 3q = 0$. The eq^n has only positive real roots then the value of p/q is—

RPP 1

$\rightarrow$ $3x^4 - 18x^3 + px^2 - 8qx + 3q = 0$ $\alpha, \beta, \gamma, \delta \rightarrow R^+$

$S_1 = \alpha+\beta+\gamma+\delta = 18/3 = 6$.

$S_2 = \sum \alpha\beta = \alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{p}{3}$.

$S_3 = \sum \alpha\beta\gamma = \alpha\beta\gamma + \alpha\gamma\delta + \alpha\beta\delta + \beta\gamma\delta = \frac{8q}{3}$.

$S_4 = \alpha\beta\gamma\delta = \frac{3q}{3} = q$.

Now, $\frac{S_3}{S_4} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} + \frac{1}{\delta} = \frac{8}{3}$

from AM-HM inequality,

$$\frac{4}{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}} \le \frac{\alpha+\beta+\gamma+\delta}{4}$$

$\Rightarrow \frac{4}{8/3} \le \frac{6}{4}$

$\Rightarrow \frac{3}{2} \le \frac{3}{2} \Rightarrow$ HM = AM.

$\boxed{\alpha = \beta = \gamma = \delta}$

$\alpha+\beta+\gamma+\delta = 6$

$\boxed{\alpha = \beta = \gamma = \delta = \frac{3}{2}}$

from S_2,

$\frac{p}{3} = 6 \cdot \left(\frac{3}{2}\right)^2$

$\Rightarrow \boxed{p = 18 \times \frac{9}{4} = \frac{81}{2}}$

from S_4, $q = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$

Now, $\frac{p}{q} = \frac{81/2}{81/16} = (8)$ Ans.

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QUESTION

(RPP 2)

If both the roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceed 3 then

(A) $a < \frac{1}{2}$

(B) $a > \frac{1}{2}$

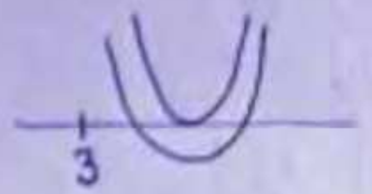
(C) $a < 1$

(D) $a > \frac{11}{9}$

Ans. D

RPP 02

$x^2 - 6ax + 2 - 2a + 9a^2 = 0$



$f(3) > 0$ & $-\frac{b}{2a} > 3$ & $D \geq 0$

$f(3) > 0$

$9 - 18a + 2 - 2a + 9a^2 > 0$

$9a^2 - 20a + 11 > 0$

$(a-1)(a - \frac{11}{9}) > 0$

$\boxed{a \in (-\infty, 1) \cup (\frac{11}{9}, \infty)}$

$\frac{6a}{2} > 3$

$\boxed{a > 1}$

$36a^2 - 4(2 - 2a + 9a^2) > 0$

$\cancel{36a^2} - 8 + 8a - \cancel{36a^2} > 0$

$(a-1) > 0$

$\boxed{a > 1}$

'∩'

$a \in (\frac{11}{9}, \infty)$

$\boxed{a > \frac{11}{9}}$ Ans

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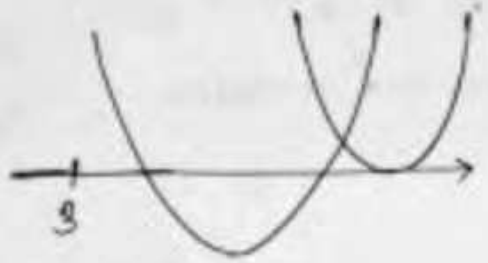
RPP-2 If both the roots of the eq^n $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceed 3 then –

RPP 2



(1) $f(3) > 0$
(2) $-\frac{b}{2a} > 3$
(3) $D \geq 0$

(1) $f(3) > 0$

$\Rightarrow 9 - 18a + 2 - 2a + 9a^2 > 0$

$\Rightarrow 9a^2 - 20a + 11 > 0$

$\Rightarrow 9a^2 - 11a - 9a + 11 > 0$

$\Rightarrow (9a - 11)(a - 1) > 0$

$a \in (-\infty, 1) \cup (\frac{11}{9}, \infty)$ (i).

(2) $-\frac{B}{2A} > 3$

$\Rightarrow \frac{6a}{2} > 3$

$\Rightarrow 3a > 3$

$\boxed{\therefore a > 1}$

$\therefore a \in (1, \infty)$ (ii).

(3) $D \geq 0$

$\Rightarrow 36a^2 - 4(2 - 2a + 9a^2) \geq 0$

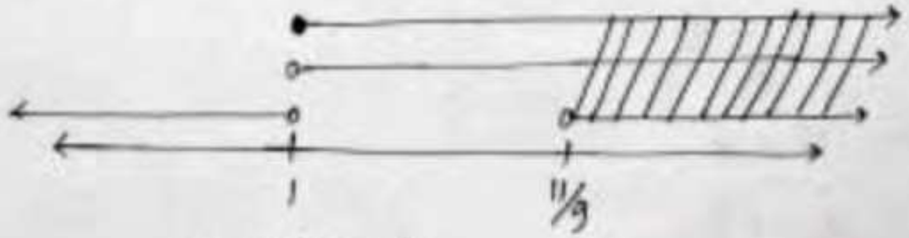
$\Rightarrow \cancel{36a^2} - 8 + 8a - \cancel{36a^2} \geq 0$

$\Rightarrow 8a - 8 \geq 0$

$\Rightarrow a - 1 \geq 0$

$\therefore a \in [1, \infty)$ (iii).

(i) ∩ (ii) ∩ (iii) $\Rightarrow$



$a \in (\frac{11}{9}, \infty)$

$\boxed{\therefore a > \frac{11}{9}}$ Ans

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QUESTION

(RPP 3)

If $f(x) = ax^2 + 6x - a$ has maximum value 10 then sum of all possible value(s) of 'a' is

(A) –20

(B) –10

(C) 10

(D) 20

Ans. B

RPP 03

$f(x) = ax^2 + 6x - a$

Max Value exist if $a<0$

$f(x) = ax^2 + 6x - a$

$f\left(\frac{-b}{2a}\right) = 10$

$f\left(\frac{-6}{2a}\right) = a\left(\frac{-6}{2a}\right)^2 + 6\left(\frac{-6}{2a}\right) - a = 10$

$\frac{36}{4a} - \frac{36}{2a} - a = 10$

$\frac{-36}{4a} - a = 10$

$36 + 4a^2 = -40a$

$4a^2 + 40a + 36 = 0$

$a^2 + 10a + 9 = 0$

$(a+1)(a+9) = 0$

$a = -1, -9$

Sum $= -1 - 9 = -10$

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RPP3

If $f(x) = ax^2 + 6x - a$ has maximum value 10 then Sum of all possible value(s) of 'a' is—

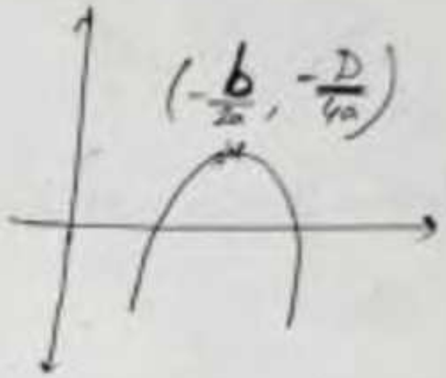
RPP 3

→ for a quadratic equation maximum and minimum value occurs at vertex.

$f(x) = ax^2 + 6x - a$.

↳ for occuring maximum value

$a<0 \rightarrow$ downward parabola



Now, $-\frac{b}{2a} = -\frac{3}{a}$.

$f\left(-\frac{3}{a}\right) = a\left(-\frac{3}{a}\right)^2 + 6\left(-\frac{3}{a}\right) - a$

$\Rightarrow 10 = \frac{9}{a} - \frac{18}{a} - a$

$\Rightarrow 10a = -9 - a^2$

$\Rightarrow a^2 + 10a + 9 = 0$

$a = \frac{-10 \pm \sqrt{100 - 36}}{2}$

$\Rightarrow a = \frac{-10 \pm 8}{2}$

$\Rightarrow a = -5 \pm 4$

$\therefore a = -9, -1$

$\therefore$ The sum of all possible values of 'a' $= (-9-1) = (-10)$.

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QUESTION

(RPP 4)

Let α and β are roots of equation $7x^2 - 5x - 1 = 0$, then

$$\lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\alpha - 5)^r} + \frac{1}{(7\beta - 5)^r} \right) \text{ is}$$

(A) 9

(B) -3

(C) 3

(D) $\frac{19}{13}$

Ans. A

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RPP-4

$7x^2-5x-1=0 \;\big\langle^{\alpha}_{\beta}$

$\lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\alpha-5)^r}+\frac{1}{(7\beta-5)^r}\right)$

$7\alpha^2-5\alpha-1=0$

$7\alpha^2-5\alpha=1$

$\alpha=\frac{1}{7\alpha-5}$

lly

$\beta=\frac{1}{7\beta-5}$

$\Rightarrow \lim_{n\to\infty} \sum_{r=0}^{n} (\alpha^r+\beta^r)$

$\Rightarrow \frac{1}{1-\alpha}+\frac{1}{1-\beta}=\frac{1-\beta+1-\alpha}{(1-\alpha)(1-\beta)}=\frac{2-(\alpha+\beta)}{1-(\alpha+\beta)+\alpha\beta}$

$\Rightarrow \frac{2-5/7}{1-5/7-1/7}=\frac{\frac{14-5}{7}}{\frac{7-5-1}{7}}=9$

$$\lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\alpha-5)^r} + \frac{1}{(7\beta-5)^r} \right) \text{ is } -$$

RPP 4

$\rightarrow$

$$7x^2 - 5x - 1 = 0 \quad \alpha, \beta$$

$$7\alpha^2 - 5\alpha - 1 = 0$$

$$\rightarrow \boxed{\frac{1}{7\alpha - 5} = \alpha}$$

$$\text{lly} \; \boxed{\frac{1}{7\beta - 5} = \beta}$$

Now, $\displaystyle \lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\alpha-5)^r} + \frac{1}{(7\beta-5)^r} \right)$

$$= \lim_{n\to\infty} \sum_{r=0}^{n} (\alpha^r + \beta^r)$$

$$= \frac{1}{1-\alpha} + \frac{1}{1-\beta} = \frac{1-\beta+1-\alpha}{(1-\alpha)(1-\beta)}$$

$$= \frac{2-(\alpha+\beta)}{1-(\alpha+\beta)+\alpha\beta}$$

$$= \frac{2-\frac{5}{7}}{1-\frac{5}{7}-\frac{1}{7}} = \frac{\frac{14-5}{7}}{\frac{7-5-1}{7}}$$

$$= (9) \text{ Ans}$$

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RPP-4

$7x^2 - 5x - 1 = 0 <^{\alpha}_{\beta}$ then $\lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\alpha-5)^r} + \frac{1}{(7\beta-5)^r} \right)$

Soln:- $S = \alpha + \beta = \frac{5}{7}$, $\alpha\beta = \frac{-1}{7}$

$$S = \underbrace{\lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\alpha-5)^r} \right)}_{\text{infinite GP}} + \underbrace{\lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\beta-5)^r} \right)}_{\text{infinite GP}}$$

$$= \left(1 + \frac{1}{7\alpha-5} + \left(\frac{1}{7\alpha-5}\right)^2 + \ldots \infty \right) + \left(1 + \frac{1}{7\beta-5} + \left(\frac{1}{7\beta-5}\right)^2 + \ldots \infty \right)$$

$$= \frac{1}{1 - \frac{1}{7\alpha-5}} + \frac{1}{1 - \frac{1}{7\beta-5}} = \frac{7\alpha-5}{7\alpha-6} + \frac{7\beta-5}{7\beta-6}$$

$$= \frac{2\times 49(\alpha\beta) - 42(\alpha+\beta) - 35(\alpha+\beta) + 60}{49\alpha\beta - 42(\alpha+\beta) + 36} = \frac{-14-30-25+60}{-37+36}$$

$$= \frac{-9}{-1} = \underline{9}$$

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QUESTION

(RPP 5)

If α, β are the roots of the equation $x^2 - 2x + 3$, then the value of

$$\sum_{r=1}^{10}(r+\alpha)(r+\beta) \text{ is equal to}$$

(A) -525

(B) -305

(C) 305

(D) 525

Ans. D

RPP-5.

If α, β are the roots of the eq^n x^2-2x+3, then the value of $\sum_{r=1}^{10}(r+\alpha)(r+\beta)$ is equal to —

RPP 5

$\Rightarrow$ $x^2-2x+3=0 <^{\alpha}_{\beta}$

$\alpha+\beta=2, \ \alpha\beta=3.$

$(r+\alpha)(r+\beta) = r^2+r(\alpha+\beta)+\alpha\beta$

$= (r^2+2r+3)$

Now, $\sum_{r=1}^{10}(r+\alpha)(r+\beta) = \sum_{r=1}^{10}(r^2+2r+3)$

$= \sum_{r=1}^{10}(r^2) + 2\sum_{r=1}^{10}(r) + 3\times10$

$= \frac{10(10+1)(20+1)}{6} + 2\cdot\frac{10\times11}{2} + 30$

$= \frac{10\times11\times21}{6} + 110 + 30$

$= (525)$ - Ans

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Date ______
Page ______

RPP-5

if α, β are the roots of the equation
$x^2 - 2x + 3.$

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$\alpha + \beta = 2$

$\alpha\beta = 3$

$$\sum_{r=1}^{10} (r+\alpha)(r+\beta)$$

$$= r^2 + r(\alpha+\beta) + \alpha\beta$$

$$= r^2 + r\cdot 2 + 3$$

$$= \frac{\cancel{10}^{5}\cdot 11\cdot \cancel{21}^{7}}{\cancel{6}\ \cancel{3}} + \frac{\cancel{2}\cdot 10\cdot 11}{\cancel{2}} + 30$$

$$= 35 \times 11 + 110 + 30$$

$$= 385 + 140$$

$$= \underline{525} \quad \text{Ans.}$$


THANK
YOU