

CHAPTER

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Limits and Derivatives

Limit

- ❖ The expected value of the function as dictated by the points to the left of a point defines the left hand limit of the function at that point. Similarly the right hand limit.
- ❖ Limit of a function at a point is the common value of the left and right hand limits, if they coincide.
- ❖ For a function f and a real number a , $\lim_{x \rightarrow a} f(x)$ and $f(a)$ may not be same (In fact, one may be defined and not the other one).

Fundamental Theorems on Limits

- ❖ For functions f and g if $\lim_{x \rightarrow a} f(x)$ & $\lim_{x \rightarrow a} g(x)$ exists, then the following holds:

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$$

$$\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} \quad g(x) \neq 0 \text{ \& } \lim_{x \rightarrow a} g(x) \neq 0$$

Theorem

- ❖ Following are some of the standard limits

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}, \text{ for any +ve integer}$$

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x} \right)^x = e$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log_e a$$

L'hospital's Rule

If $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ is of form $\frac{0}{0}$

Then $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$

Derivatives

- ❖ Derivative of a function f at any point x is defined by

$$f'(x) = \frac{df(x)}{dx} = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- ❖ For functions u and v the following holds:

$$(u \pm v)' = u' \pm v'$$

$$(uv)' = u'v + uv'$$

$$\left(\frac{u}{v} \right)' = \frac{u'v - uv'}{v^2} \text{ provided all are defined.}$$

- ❖ Following are some of the standard derivatives.

$$\frac{d}{dx} (x^n) = nx^{n-1}$$

$$\frac{d}{dx} (\sin x) = \cos x$$

$$\frac{d}{dx} (\cos x) = -\sin x$$

$$\frac{d}{dx} (e^x) = e^x$$

$$\frac{d}{dx} (a^x) = a^x \log a$$

$$\frac{d}{dx} (\ln x) = \frac{1}{x}$$

$$\frac{d}{dx} (\tan x) = \sec^2 x$$

$$\frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$$

$$\frac{d}{dx} (\sec x) = \sec x \tan x$$