

CHAPTER

6



Principle of Mathematical Induction

- (i) $1 + 2 + 3 + \dots + n = \Sigma n = \frac{n(n+1)}{2}$
- (ii) $1^2 + 2^2 + 3^2 + \dots + n^2 = \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$
- (iii) $1^3 + 2^3 + 3^3 + \dots + n^3 = \Sigma n^3 = (\Sigma n)^2 = \left\{ \frac{n(n+1)}{2} \right\}^2$
- (iv) $1^4 + 2^4 + 3^4 + \dots + n^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- (v) $2 + 4 + 6 + \dots + 2n = \Sigma 2n = n(n+1)$
- (vi) $1 + 3 + 5 + \dots + (2n-1) = \Sigma(2n-1) = n^2$
- (vii) $x^n - y^n = (x-y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \dots + xy^{n-2} + y^{n-1})$,
where $n \in \mathbb{N}$
- (viii) $x^n + y^n = (x+y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 - \dots - xy^{n-2} + y^{n-1})$
where n is odd positive integer
- (ix) $a + (a+d) + (a+2d) + \dots + (a+(n-1)d)$
 $= \frac{n}{2} [2a + (n-1)d]$
- (x) $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^n-1)}{r-1}$, where $r \neq 1$
- (xi) $(a+b)^n = \sum_{r=0}^n {}^n C_r a^{n-r} b^r$
- (xii) $\cos(a) \cdot \cos(2a) \cdot \cos(4a) \dots \cos(2^{n-1}a) = \frac{\sin(2^n a)}{2^n \sin a}$
- (xiii) $\sin(a) + \sin(a+b) + \sin(a+2b) + \dots + \sin(a+(n-1)b)$
 $= \frac{\sin(nb/2)}{\sin(b/2)} \sin\left(a + (n-1)\frac{b}{2}\right)$
- (xiv) $\cos(a) + \cos(a+b) + \cos(a+2b) + \dots + \cos(a+(n-1)b)$
 $= \frac{\sin(nb/2)}{\sin(b/2)} \cos\left(a + (n-1)\frac{b}{2}\right)$