

CHAPTER

1



Relations and Functions

Relations

If A and B are two non-empty sets, then a relation R from A to B is a subset of $A \times B$.

Representation of a Relation

Roster form: In this form, we represent the relation by the set of all ordered pairs belongs to R .

Set-builder form: In this form, we represent the relation R from set A to set B as

$R = \{(a, b) : a \in A, b \in B \text{ and the rule which relate the elements of } A \text{ and } B\}$.

Domain, Codomain and Range of a Relation

Let R be a relation from a non-empty set A to a non-empty set B . Then, set of all first components or coordinates of the ordered pairs belonging to R is called the domain of R , while the set of all second components or coordinates of the ordered pairs belonging to R is called the range of R . Also, the set B is called the codomain of relation R .

Thus, domain of $R = \{a : (a, b) \in R\}$ and range of $R = \{b : (a, b) \in R\}$

Types of Relations

Empty or Void Relation: As $\phi \subset A \times A$, for any set A , so ϕ is a relation on A , called the empty or void relation.

Universal Relation: Since, $A \times A \subseteq A \times A$, so $A \times A$ is a relation on A , called the universal relation.

Identity Relation: The relation $I_A = \{(a, a) : a \in A\}$ is called the identity relation on A .

Reflexive Relation: A relation R on a set A is said to be reflexive relation, if every element of A is related to itself.

Thus, $(a, a) \in R, \forall a \in A \Rightarrow R$ is reflexive.

Symmetric Relation: A relation R on a set A is said to be symmetric relation iff $(a, b) \in R \Rightarrow (b, a) \in R, \forall a, b \in A$

i.e. $a R b \Rightarrow b R a, \forall a, b \in A$

Transitive Relation: A relation R on a set A is said to be transitive relation, iff $(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow (a, c) \in R, \forall a, b, c \in A$

Equivalence Relation

A relation R on a set A is said to be an equivalence relation, if it is simultaneously reflexive, symmetric and transitive on A .

Functions

Let A and B be two non-empty sets, then a function f from set A to set B is a rule which associates each element of A to a unique element of B .

Domain, Codomain and Range of a Function

If $f : A \rightarrow B$ is a function from A to B , then

- (i) the set A is called the domain of $f(x)$.
- (ii) the set B is called the codomain of $f(x)$.
- (iii) the subset of B containing only the images of elements of A is called the range of $f(x)$.

Number of Functions

Let X and Y be two finite sets having m and n elements respectively. Then each element of set X can be associated to any one of n elements of set Y . So, total number of functions from set X to set Y is n^m .

Number of One-One Functions

Let A and B are finite sets having m and n elements respectively, then the number of one-one functions from A to B is

$$= \begin{cases} n(n-1)(n-2)\dots(n-(m-1)), & n \geq m \\ 0, & n < m \end{cases}$$

Number of Onto (or Surjective) Functions

Let A and B are finite sets having m and n elements respectively, then number of onto (or surjective) functions from A to B is

$$= \begin{cases} n^m - {}^n C_1(n-1)^m + {}^n C_2(n-2)^m - {}^n C_3(n-3)^m + \dots, & n < m \\ n!, & n = m \\ 0, & n > m \end{cases}$$

Number of Bijective Functions

Let A and B are finite sets having m and n elements respectively, then number of bijective functions from A to B is

$$= \begin{cases} n!, & \text{if } n = m \\ 0, & \text{if } n > m \text{ or } n < m \end{cases}$$

Properties of Greatest Integer Function

- (i) $[x + n] = n + [x], n \in I$
- (ii) $[-x] = -[x], x \in I$
- (iii) $[-x] = -[x] - 1, x \notin I$
- (iv) $[x] \geq n \Rightarrow x \geq n, n \in I$
- (v) $[x] > n \Rightarrow x \geq n + 1, n \in I$
- (vi) $[x] \leq n \Rightarrow x < n + 1, n \in I$
- (vii) $[x] < n \Rightarrow x < n, n \in I$
- (viii) $[x + y] = [x] + [y + x - [x]]$ for all $x, y \in R$
- (ix) $[x + y] \geq [x] + [y]$

Important Points To Be Remembered

- (i) Constant function is periodic with no fundamental period.
- (ii) If $f(x)$ is periodic with period T , then $\frac{1}{f(x)}$ and $\sqrt{f(x)}$ are also periodic with same period T .
- (iii) If $f(x)$ is periodic with period T , then $kf(ax + b)$ is periodic with period $\frac{T}{|a|}$, where $a, b, k \in R$ and $a, k \neq 0$.

Properties of Even and Odd Functions

- (i) gof or fog is even, if both f and g are even or if f is odd and g is even or if f is even and g is odd.
- (ii) gof or fog is odd, if both of f and g are odd.

- (iii) If $f(x)$ is an even function, then $\frac{d}{dx}f(x)$ is an odd function and if $f(x)$ is an odd function, then $\frac{d}{dx}f(x)$ is an even function.
- (iv) The graph of an even function is symmetrical about Y -axis.
- (v) The graph of an odd function is symmetrical about origin or symmetrical in opposite quadrants.
- (vi) An even function can never be one-one, however an odd function may or may not be one-one.

Properties of Inverse Function

- (a) The inverse of a bijection is unique.
- (b) If $f: A \rightarrow B$ is a bijection and $g: B \rightarrow A$ is the inverse of f , then $fog = I_B$ and $gof = I_A$, where I_A & I_B are identity functions on the sets A & B respectively. If $fog = I$, then f is inverse of itself.
- (c) The inverse of a bijection is also a bijection.
- (d) If f & g are two bijections $f: A \rightarrow B, g: B \rightarrow C$ & gof exist, then the inverse of gof also exists and $(gof)^{-1} = f^{-1}og^{-1}$.
- (e) The graph of f^{-1} obtained by reflecting the graph of f about the line $y = x$.

General

If x, y are independent variables, then :

- (a) $f(xy) = f(x) + f(y) \Rightarrow f(x) = k \ln x$
- (b) $f(xy) = f(x) \cdot f(y) \Rightarrow f(x) = x^n, n \in R$ or $f(x) = 0$
- (c) $f(x + y) = f(x) \cdot f(y) \Rightarrow f(x) = a^{kx}$ or $f(x) = 0$
- (d) $f(x + y) = f(x) + f(y) \Rightarrow f(x) = kx$, where k is a constant.