

CHAPTER

12



# Sequences and Series

An arithmetic progression (A.P.) :  $a, a + d, a + 2d, \dots, a + (n - 1)d$  is an A.P.

Let  $a$  be the first term and  $d$  be the common difference of an A.P., then  $n^{\text{th}}$  term =  $t_n = a + (n - 1)d$

**The sum of first  $n$  terms of an A.P.**

$$S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} [a + \ell]$$

**$r^{\text{th}}$  term of an A.P. when sum of first  $r$  terms is given is**  
 $t_r = S_r - S_{r-1}$

**Properties of A.P.**

- (i) If  $a, b, c$  are in A.P.  
 $\Rightarrow 2b = a + c$  & if  $a, b, c, d$  are in A.P.  
 $\Rightarrow a + d = b + c$ .
- (ii) Three numbers in A.P. can be taken as  $a - d, a, a + d$ ; four numbers in A.P. can be taken as  $a - 3d, a - d, a + d, a + 3d$ ; five numbers in A.P. are  $a - 2d, a - d, a, a + d, a + 2d$  & six terms in A.P. are  $a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d$  etc.
- (iii) Sum of the terms of an A.P. equidistant from the beginning & end = sum of first & last term.

**Arithmetic Mean (Mean or Average) (A.M.):**

If three terms are in A.P. then the middle term is called the A.M. between the other two, so if  $a, b, c$  are in A.P.,  $b$  is A.M. of  $a$  &  $c$ .

**$n$ -Arithmetic Means Between Two Numbers:**

If  $a, b$  are any two given numbers &  $a, A_1, A_2, \dots, A_n, b$  are in A.P. then  $A_1, A_2, \dots, A_n$  are the A.M.

$$A_1 = a + \frac{b-a}{n+1}, A_2 = a + \frac{2(b-a)}{n+1}, \dots, A_n = a + \frac{n(b-a)}{n+1}$$

$$\sum_{r=1}^n A_r = nA \text{ where } A \text{ is the single A.M. between } a \text{ \& } b.$$

**Geometric Progression:**  $a, ar, ar^2, ar^3, ar^4, \dots$  with  $a$  as the first term &  $r$  as common ratio.

- (i)  $n^{\text{th}}$  terms =  $ar^{n-1}$
- (ii) Sum of the first  $n$  terms i.e.  $S_n = \begin{cases} a(r^n - 1) & r \neq 1 \\ na & r = 1 \end{cases}$

**Geometric Means (Mean Proportional) (G.M.):**

If  $a, b, c > 0$  are in G.P.  $b$  is the G.M. between  $a$  &  $c$ , then  $b^2 = ac$

**$n$  Geometric Means Between Positive Number  $a, b$ :** If  $a, b$  are two given numbers &  $a, G_1, G_2, \dots, G_n, b$  are in G.P. Then  $G_1, G_2, G_3, \dots, G_n$  are  $n$  G.M.s between  $a$  &  $b$ .  $G_2 = a(b/a)^{2/n+1}$ .  $G_n = a(b/a)^{n/n+1}$

**Harmonic Mean (H.M.):**

If  $a, b, c$  are in H.P.,  $b$  is the H.M. between  $a$  &  $c$ , then  $b = \frac{2ac}{a+c}$

H.M,  $H$  of  $a_1, a_2, \dots, a_n$  is given by  $\frac{1}{H} = \frac{1}{n} \left[ \frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} \right]$

**Relation between Means:**

$$G^2 = H \cdot A, \quad \text{C.M.} \geq \text{H.M. and A.M.} = \text{G.M.} = \text{H.M.}$$

If  $a_1, a_2, a_3, \dots, a_n$

**Important Results**

- (i)  $\sum_{r=1}^n (a_r \pm b_r) = \sum_{r=1}^n a_r \pm \sum_{r=1}^n b_r$
- (ii)  $\sum_{r=1}^n ka_r = k \sum_{r=1}^n a_r$
- (iii)  $\sum_{r=1}^n k = nk$  where  $k$  is a constant,
- (iv)  $\sum_{r=1}^n r = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$
- (v)  $\sum_{r=1}^n r^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$
- (vi)  $\sum_{r=1}^n r^3 = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$
- (vii)  $\sum_{i < j=1}^n a_i a_j = (a_1 + a_2 + \dots + a_n)^2 - (a_1^2 + a_2^2 + \dots + a_n^2)$