

ARITHMETIC MEAN**(i) Arithmetic Mean for Unclassified (Ungrouped or Raw)**

Data: If there are n observations, $x_1, x_2, x_3, \dots, x_n$, then their arithmetic mean

$$A \text{ or } \bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_{i=1}^n x_i}{n}$$

(ii) Arithmetic Mean for Discrete Frequency Distribution or Ungrouped Frequency Distribution:

Let f_1, f_2, \dots, f_n be corresponding frequencies of x_1, x_2, \dots, x_n . Then, arithmetic mean

$$A = \frac{x_1 f_1 + x_2 f_2 + \dots + x_n f_n}{f_1 + f_2 + \dots + f_n} = \frac{\sum_{i=1}^n x_i f_i}{\sum_{i=1}^n f_i}$$

(iii) Arithmetic Mean for Classified (Grouped) Data or Grouped Frequency Distribution:

For a classified data, we take the class marks x_1, x_2, \dots, x_n of the classes, then arithmetic mean by

$$A = \frac{\sum_{i=1}^n x_i f_i}{\sum_{i=1}^n f_i}$$

Combined Mean: If A_1, A_2, \dots, A_r are means of n_1, n_2, \dots, n_r observations respectively, then arithmetic mean of the combined group is called the combined mean of the observation.

$$A = \frac{n_1 A_1 + n_2 A_2 + \dots + n_r A_r}{n_1 + n_2 + \dots + n_r} = \frac{\sum_{i=1}^r n_i A_i}{\sum_{i=1}^r n_i}$$

MEDIAN**Median for Simple Distribution or Raw Data**

Firstly, arrange the data in ascending or descending order and then find the number of observations n .

(a) If n is odd, then $\left(\frac{n+1}{2}\right)$ th term is the median.

(b) If n is even, then there are two middle terms namely $\left(\frac{n}{2}\right)$ th

and $\left(\frac{n}{2} + 1\right)$ th terms, median is mean of these terms.

Median for Classified (Grouped) Data or Grouped Frequency Distribution

For a continuous distribution, median

$$M_d = l + \frac{\frac{N}{2} - C}{f} \times h$$

where, l = lower limit of the median class

f = frequency of the median class

N = total frequency = $\sum_{i=1}^n f_i$

C = cumulative frequency of the class just before the median class

h = length of the median class

Mode for Classified (Grouped) Distribution or Grouped Frequency Distribution

$$M_o = l + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

where, l = lower limit of the modal class

f_0 = frequency of the modal class

f = frequency of the pre-modal class

f = frequency of the post-modal class

h = length of the class interval

Relation Between Mean, Median and Mode

(i) Mean – Mode = 3 (Mean – Median)

(ii) Mode = 3 Median – 2 Mean

MEAN DEVIATION (MD)

(i) For simple (raw) distribution, $\delta = \frac{\sum_{i=1}^n |x_i - \bar{x}|}{n}$

where, n = number of terms, $\bar{x} = A$ or M_d or M_o

$$(ii) \text{ For unclassified frequency distribution, } \delta = \frac{\sum_{i=1}^n f_i |x_i - \bar{x}|}{\sum_{i=1}^n f_i}$$

$$(iii) \text{ For classified distribution, } \delta = \frac{\sum_{i=1}^n f_i |x_i - \bar{x}|}{\sum_{i=1}^n f_i}$$

where, x_i is the class mark of the interval.

STANDARD DEVIATION AND VARIANCE

(i) For simple distribution

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}} = \frac{1}{n} \sqrt{n \sum_{i=1}^n x_i^2 - \left(\sum_{i=1}^n x_i \right)^2}$$

where, n is a number of observations and \bar{x} is mean.

(ii) For discrete frequency distribution

$$\sigma = \sqrt{\frac{\sum_{i=1}^n f(x_i - \bar{x})^2}{N}} = \frac{1}{N} \sqrt{N \sum_{i=1}^n f_i x_i^2 - \left(\sum_{i=1}^n f_i x_i \right)^2}$$

(iii) For continuous frequency distribution

$$\sigma = \sqrt{\frac{\sum_{i=1}^n f_i (x_i - \bar{x})^2}{N}}$$

where, x_i is class mark of the interval

Standard Deviation of the Combined Series

If n_1, n_2 are the sizes, \bar{X}_1, \bar{X}_2 are the means and σ_1, σ_2 are the standard deviation of the series, then the standard deviation of the combined series is

$$\sigma = \sqrt{\frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}}$$

where,

$$d_1 = \bar{X}_1 - \bar{X}, d_2 = \bar{X}_2 - \bar{X}$$

and

$$\bar{X} = \frac{n_1 \bar{X}_1 + n_2 \bar{X}_2}{n_1 + n_2}$$

IMPORTANT POINTS TO BE REMEMBERED

- (i) The ratio of SD (σ) and the AM (\bar{x}) is called the coefficient of standard deviation $\left(\frac{\sigma}{\bar{x}} \right)$
- (ii) The percentage form of coefficient of SD i.e. $\left(\frac{\sigma}{\bar{x}} \right) \times 100$ is called coefficient of variation.
- (iii) The distribution for which the coefficient of variation is less is more consistent.
- (iv) Standard deviation of first n natural numbers is $\sqrt{\frac{n^2 - 1}{12}}$.
- (v) Standard deviation is independent of change of origin, but it depends on change of scale.