PRAYAS

JEE 2025

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Advance Session | Physics

Oscillations

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Topics *to be covered*

1. Oscillations
2.
3.
4.

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3. Two beads, each with charge q and mass m, are on a horizontal, frictionless, non-conducting, circular hoop of radius R. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by

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[ε_0 is the permittivity of free space.]

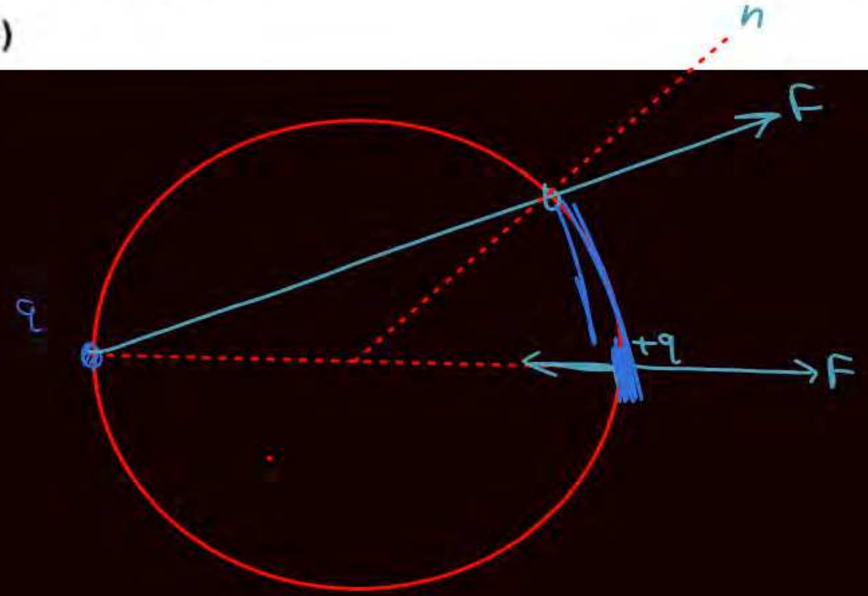
(A) $q^2 / (4\pi\varepsilon_0 R^3 m)$

(B) $q^2 / (32\pi\varepsilon_0 R^3 m)$

(C) $q^2 / (8\pi\varepsilon_0 R^3 m)$

(D) $q^2 / (16\pi\varepsilon_0 R^3 m)$

Answer (B)



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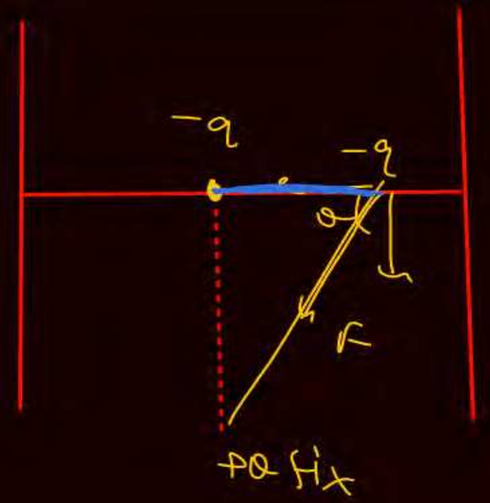

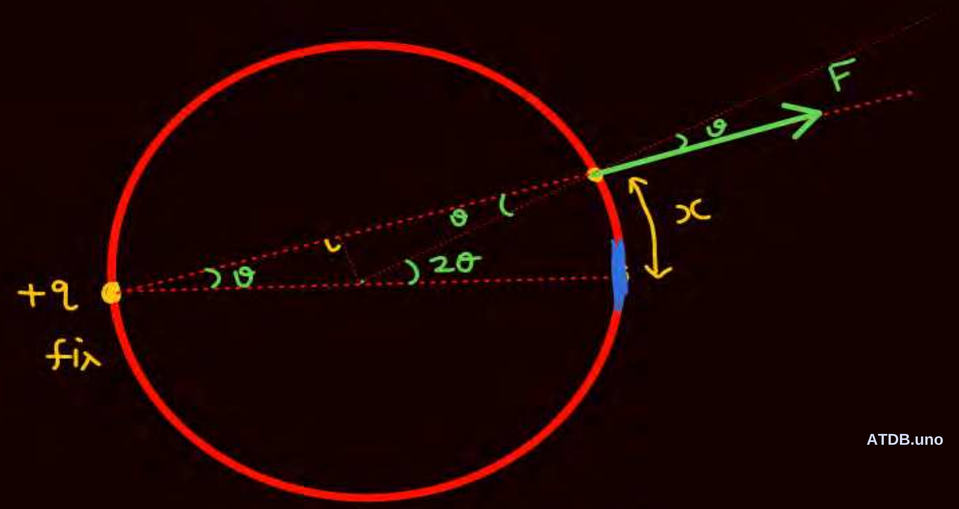

$$2\theta = \frac{x}{R}$$

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$$F_{net} = F\sin\theta$$

$$F_{net} = \frac{kqq}{(2R\cos\theta)^2}\left(\frac{2\theta}{2}\right)$$

$$F_{net} = \frac{kq^2}{4R^2}\,\frac{x}{R}\cdot 2 = \frac{k}{8R^3}q^2 x$$

$$a = \frac{\frac{1}{4\pi\epsilon_0}q^2}{R^3 m 8}\,x$$

4. A block of mass 5 kg moves along the x-direction subject to the force $F = (-20x + 10)$ N, with the value of x in metre. At time $t = 0$ s, it is at rest at position $x = 1$ m. The position and momentum of the block at $t = (\pi/4)$ s are

(A) -0.5 m, 5 kg m/s

(B) 0.5 m, 0 kg m/s

(C) 0.5 m, -5 kg m/s

(D) -1 m, 5 kg m/s

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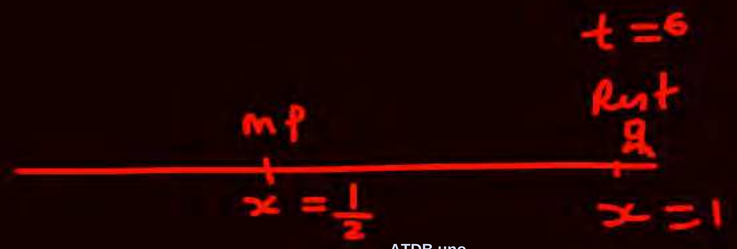
Answer (C)



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16. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. They system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10m from O. If the pebble hits the block at t = 1s, the value of v is:- (take g = 10 m/s^2) **[IIT-JEE 2012]**

E

$T = 1$

$\frac{2u\sin 45^\circ}{10} = 1$

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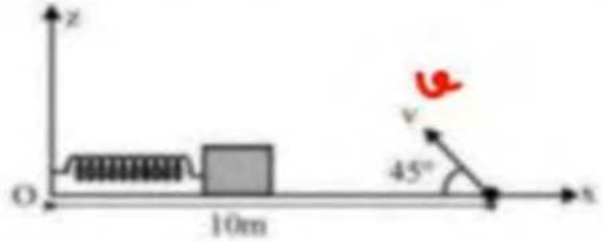


(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

Ans. (A)

. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is **[IIT JEE 2009]**

सरल आवर्त गति कर रहे एक कण का x-t ग्राफ दर्शाया गया है। t = 4 / 3 सैकण्ड पर कण का त्वरण होगा

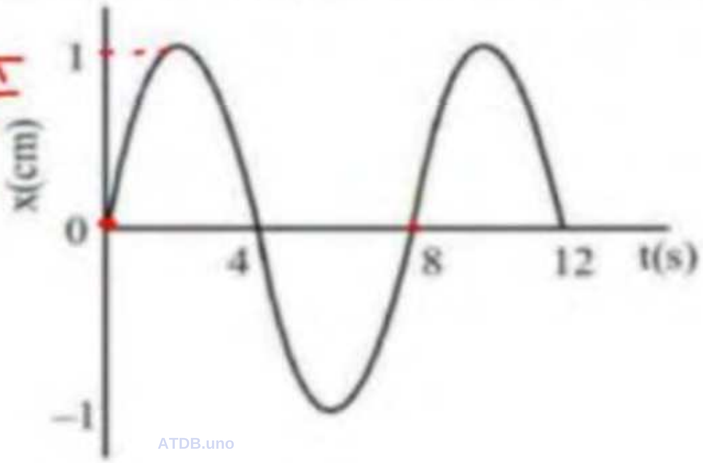
$A = 1$
$T = 8 \quad \omega = \frac{2\pi}{8}$

$x = A \sin(\omega t + 0)$

$v =$

$a =$

E



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(A) $\frac{\sqrt{3}}{32}\pi^2$ cm/s^2 (B) $\frac{-\pi^2}{32}$ cm/s^2 (C) $\frac{\pi^2}{32}$ cm/s^2 (D) $-\frac{\sqrt{3}}{32}\pi^2$ cm/s^2

ns. (D)

5. On a frictionless horizontal plane, a bob of mass $m = 0.1$ kg is attached to a spring with natural length $\ell_0 = 0.1$ m. The spring constant is $k_1 = 0.009\ \text{Nm}^{-1}$ when the length of the spring $\ell > \ell_0$ and is $k_2 = 0.016\ \text{Nm}^{-1}$ when $\ell < \ell_0$. Initially the bob is released from $\ell = 0.15$ m. Assume that Hooke's law remains valid throughout the motion. If the time period of the full oscillation is $T = (n\pi)$ s, then the integer closest to n is _____.

एक घर्षणरहित (frictionless) क्षैतिज समतल पर एक गोला (bob), जिसका द्रव्यमान $m = 0.1$ kg है, एक स्प्रिंग, जिसकी प्राकृतिक लम्बाई $\ell_0 = 0.1$ m है, से जुड़ा है। स्प्रिंग का नियतांक $k_1 = 0.009\ \text{Nm}^{-1}$ है जब स्प्रिंग की लम्बाई $\ell > \ell_0$ है, तथा $k_2 = 0.016\ \text{Nm}^{-1}$ है जब $\ell < \ell_0$ है। प्रारंभ में गोले को $\ell = 0.15$ m से छोड़ा जाता है। मान लें कि हुक का नियम (Hooke's law) पूरी गति के दौरान मान्य (valid) है। यदि एक पूरे दोलन का आवर्तकाल $T = (n\pi)$ s है, तो n का निकटतम पूर्णांक (nearest integer) _____ है। **[JEE-Advanced-2022]**

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Ans. 6

खिंचा है $\equiv k_1$

दबा है $\equiv k_2$

E

.05m खींचकर छोड़ दिया।

$$\frac{T_1}{4} + \frac{T_2}{4} + \frac{T_2}{4} + \frac{T_1}{4}$$

Ans

$$T_1 = 2\pi\sqrt{\frac{m}{K_1}}$$

$$T_2 = 2\pi\sqrt{\frac{m}{K_2}}$$

$\equiv$

1 oscillation

half

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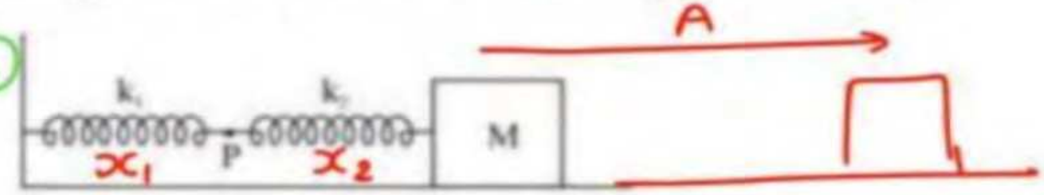
$$\frac{2\pi\sqrt{\frac{m}{K_1}}}{2} + \frac{2\pi\sqrt{\frac{m}{K_2}}}{2}$$

9. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is **[IIT JEE 2009]**

सरल आवर्त गति में द्रव्यमान M चित्रानुसार आयाम A से दोलन करता है। बिन्दु P का आयाम है:-



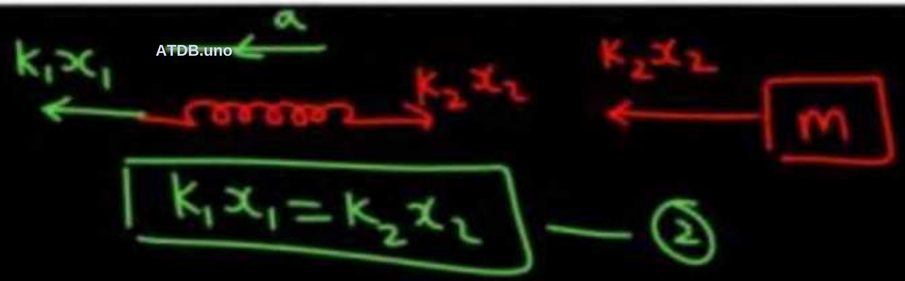
(A) $\frac{k_1 A}{k_2}$ (B) $\frac{k_2 A}{k_1}$ (C) $\frac{k_1 A}{k_1 + k_2}$ (D) $\frac{k_2 A}{k_1 + k_2}$

Ans. (D)

$x_1 + x_2 = A$ — (1)

$x_1 + \frac{k_1}{k_2} x_1 = A$

$x_1 = \frac{A k_2}{k_1 + k_2}$



$k_1 x_1 = k_2 x_2$ — (2)

find x_1

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A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M? **[JEE-Advanced-2016]**

E

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

Clear

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

ABD

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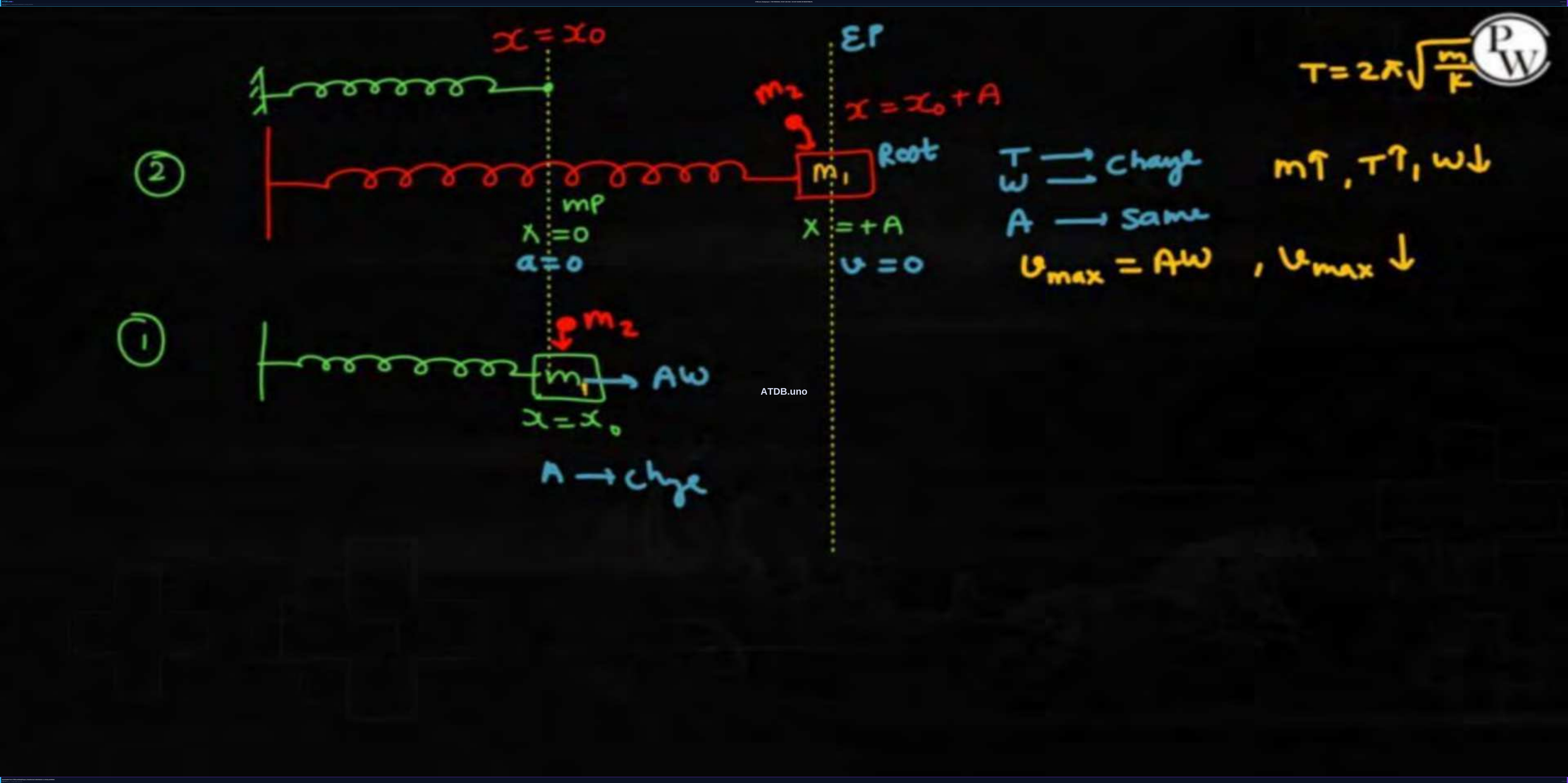
x = x0
EP
m2
x = x0 + A
Root
m1
mP
x = 0
a ≠ 0
x = +A
v = 0
2
1
m2
m1
Aω
x = x0
A → chnge
T = 2π√(m/k)
T/ω ⟶ chnge
A ⟶ same
vmax = Aω
mT, T↑, ω↓
vmax ↓
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1. A simple pendulum has time period T_1. When the point of suspension moves vertically up accordin to the equation $y = kt^2$ where $k = 1$ m/s^2 and t is time then the time period of the pendulum is T_2 the

$\left(\frac{T_1}{T_2}\right)^2$ is :-

$y = kt^2 = t^2$

$v_y = 2t$

$a_y = 2$

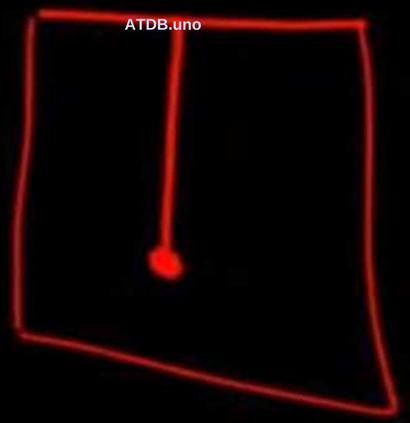
[IIT JEE (Scr) 2005]

(A) $\frac{5}{6}$ (B) $\frac{11}{10}$ (C) $\frac{6}{5}$ (D) $\frac{5}{4}$

Ans. (C)

E

$T = 2\pi\sqrt{\frac{l}{g+a}}$



$+a = 2$

A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are :- **[IIT-JEE 2011]**

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

Ans. (B)

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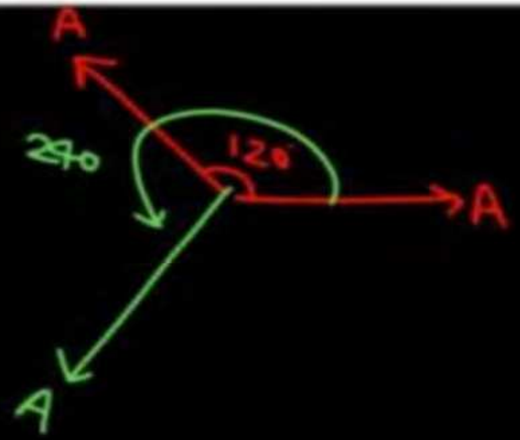

3. Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM :- **[IIT JEE 2006]**

(A) for any value of A, B and C (except $C = 0$) (B) if $A = -B$; $C = 2B$, amplitude = $|B\sqrt{2}|$

(C) if $A = B$; $C = 0$ (D) if $A = B$; $C = 2B$, amplitude = $|B|$

फलन : $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ सरल आवर्त गति को निरूपित करता है :-

(A) A, B तथा C के किसी भी मान के लिये ($C = 0$ के अतिरिक्त)

(B) यदि $A = -B$, $C = 2B$, आयाम = $|B\sqrt{2}|$

(C) यदि $A = B$; $C = 0$

(D) यदि $A = B$; $C = 2B$, आयाम = $|B|$

Ans. (A,B,D)

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$x = A\sin^2\omega t + B\cos^2\omega t + \frac{C}{2}\sin 2\omega t$

$= -B\sin^2\omega t + B\cos^2\omega t + \frac{C}{2}\sin 2\omega t$

$= B(\cos^2\omega t - \sin^2\omega t) + \frac{C}{2}\sin 2\omega t$

$x = B\cos 2\omega t + \frac{C}{2}\sin 2\omega t$

$\sqrt{B^2 + \left(\frac{C}{2}\right)^2}$

1. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is $0.5\ u_0$, it collides elastically with a rigid wall. After this collision :- **[JEE-Advanced-2013]**

(A) the speed of the particle when it returns to its equilibrium position is u_0

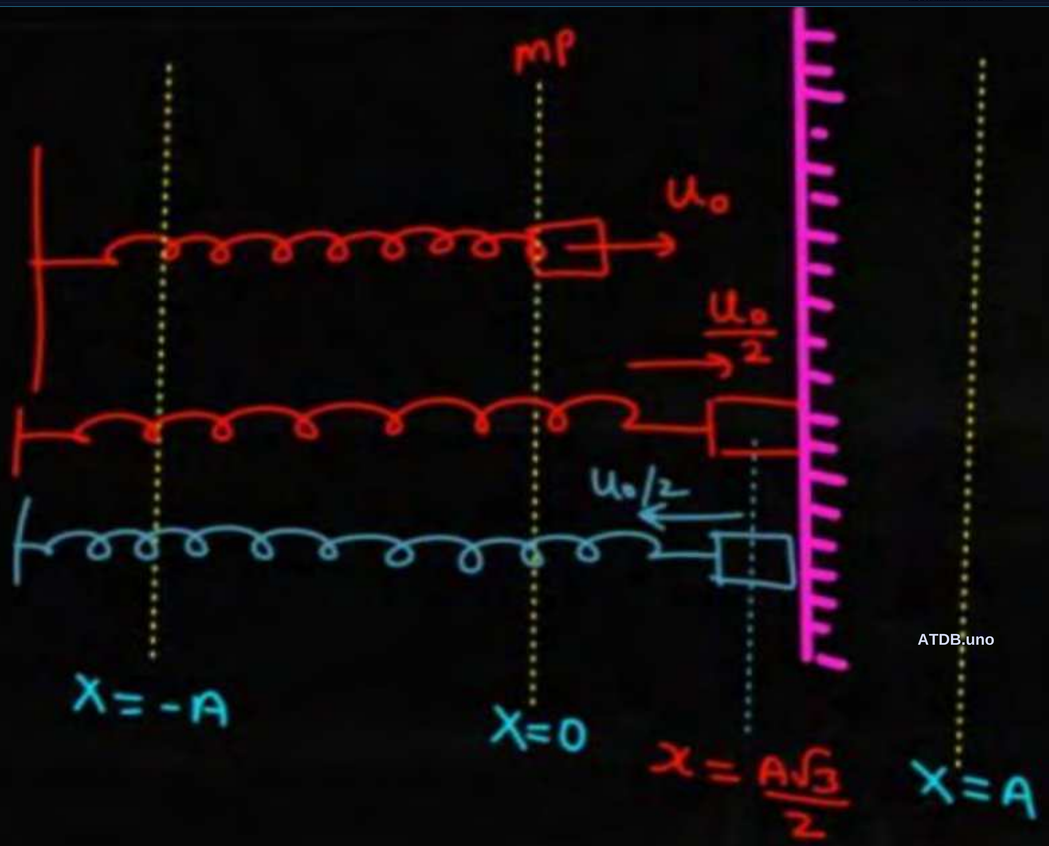
(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(D) the time at which the particle passes through the equilibrium position for the second time is

$$t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$$

Ans. (A,D)



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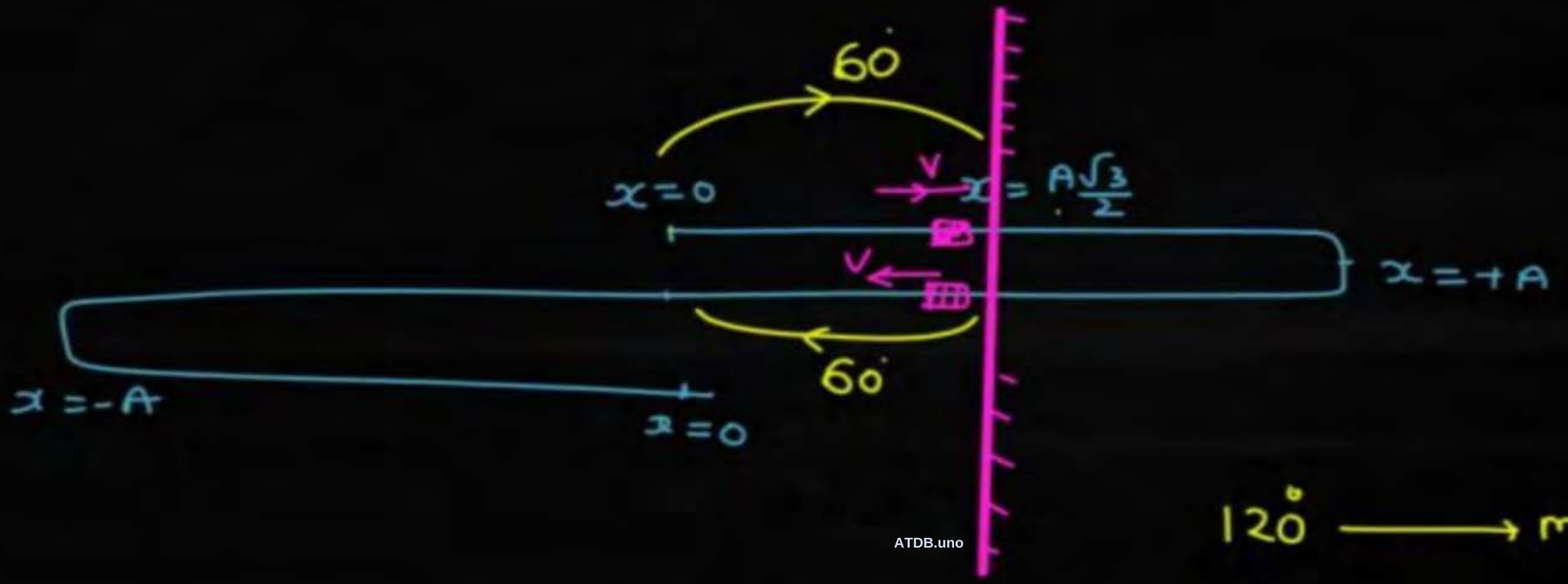
$$x = A\sin\omega t$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$\frac{A\omega}{2} = \omega\sqrt{A^2 - x^2}$$

$$x = \frac{A\sqrt{3}}{2}$$



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$120^\circ \longrightarrow$ mp वापसी

$120 + 90 + 90 \equiv 300$

$360 \longrightarrow 2\pi\sqrt{\frac{m}{k}}$

300

$\frac{2\pi\sqrt{\frac{m}{k}}}{360} \times 300$

30. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 Nm^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 ms^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _____. **[Adv. 2018]**

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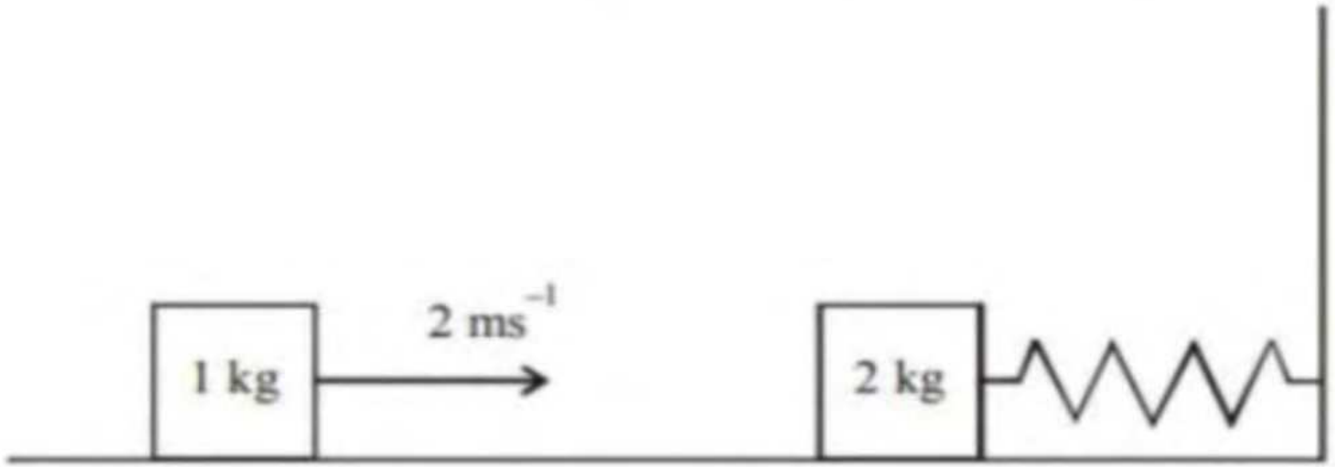


30. (2.09)

T

Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation (s) is (are)

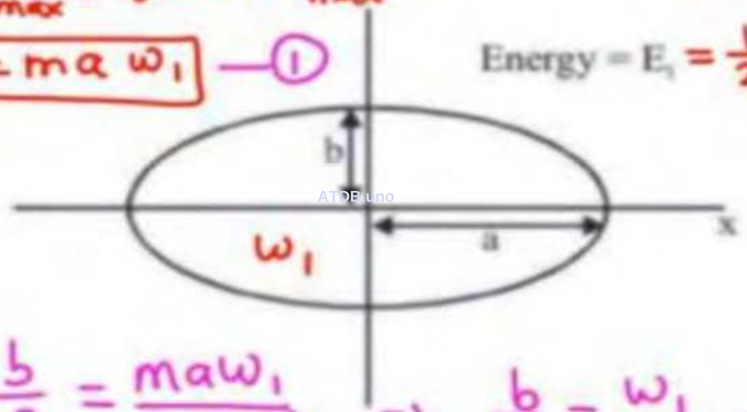
[JEE-Advanced-2015]

$T.E. = \frac{1}{2} m\omega^2 A^2$

$A_1 = a$

$P_{max} = b = mv_{max}$

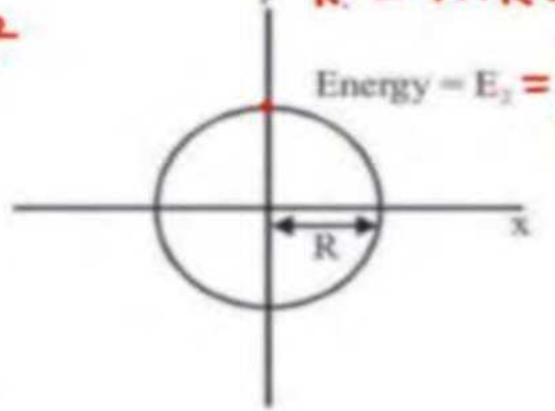
$b = ma\omega_1$ — (1)



Energy $= E_1 = \frac{1}{2}\omega_1^2 a^2$

$P_{max} = mv_{max}$

$R = mR\omega_2$ — (2)

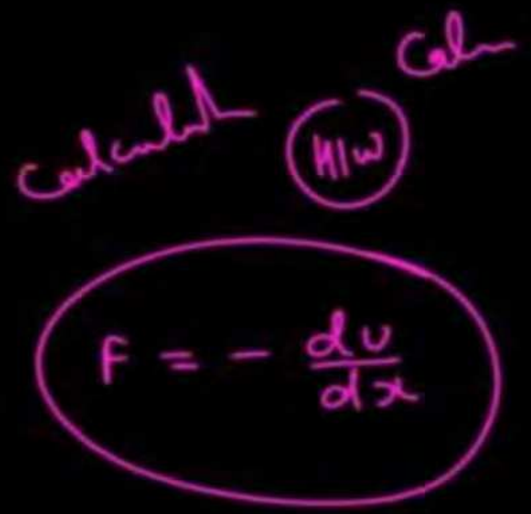


Energy $= E_2 = \frac{m}{2}\omega_2^2 R^2$

$\frac{(1)}{(2)}$ ⇒ $\frac{b}{R} = \frac{ma\omega_1}{mR\omega^2}$ ⇒ $\frac{b}{a} = \frac{\omega_1}{\omega_2} = \frac{1}{n^2}$

(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

s. (B,D)



3. A particle of unit mass is moving along the x-axis under the influence of a force and its total enegy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column-II **[JEE-Advanced-2015]**

Column-I	**Column-II**
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P) The force acting on the particle is zero at $x = a$.
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q) The force acting on the particle is zero at $x = 0$.
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R) The force acting on the particle is zero at $x = -a$.
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a}-\frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S) The particle experiences an attractive force towards $x = 0$ in the region $\lvert x\rvert < a$.
	(T) The particle with total enegy $\frac{U_0}{4}$ can oscillate about the point $x = -a$.

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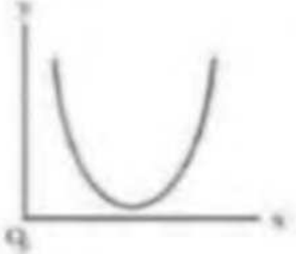
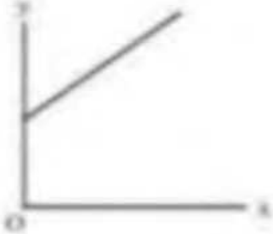
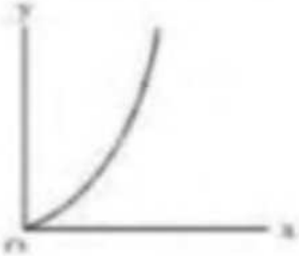
Ans. (A)-P,Q,R,T; (B)-Q,S; (C)-P,Q,R,S; (D)-P,R,T

Column-I		Column-II	
(A)	Potential energy of a simple pendulum (y axis) as a function of displacement (x axis)	(P)	
(B)	Displacement (y axis) as a function of time (x axis) for an one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.	(Q)	
(C)	Range of a projectile (y axis) as a function of its velocity (x axis) when projected at a fixed angle.	(R)	
(D)	The square of the time period (y axis) of a simple pendulum as a function of its length (x axis)	(S)	

Ans. (A) P,S (B) Q,R,S (C) S (D) Q

खण्ड -I खण्ड -II [IIT JEE 2008]

Two simple harmonic motions are represented by the equations

$y_1 = 10\sin(3\pi t + \pi/4)$ and $y_2 = 5\left(\frac{\sin 3\pi t}{2} + \frac{\sqrt{3}\cos 3\pi t}{2}\right)$

Their amplitudes are in the ratio of ..... **(1986 - 2 Marks)**

10

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10. (1)

12. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then. **(1999S - 3marks)**

(a) the resultant amplitude is $(1+\sqrt{2})a$

(b) the phase of the resultant motion relative to the first is 90°

(c) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion

(d) the resulting motion is not simple harmonic.

$x_1 = A\sin\omega t$

$x_2 = A\sin(\omega t + 45)$

$x_2 = A\sin(\omega t + \phi_0)$

E

$(1^2 + 2 + 2\sqrt{2})$

$A_{net} = A\sqrt{2} + A$

$\frac{1}{2}kA^2$

12. (a, c)

7. A particle free to move along the x-axis has potential energy given by $U(x) = k\,[1-\exp(-x^2)]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then

[1999S - 2marks]

(a) at points away from the origin, the particle is in unstable equilibrium

(b) for any finite nonzero value of x, there is a force directed away from the origin

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(c) if its total mechanical energy is $k/2$, it has its minimum kinetic energy at the origin.

(d) for small displacements from $x = 0$, the motion is simple harmonic

H/W

E (m)

$U = K - k e^{-x^2}$

$F = -\frac{dU}{dx} = -[0 - k e^{-x^2}(-2x)]$

$F = -2Kx e^{-x^2}$

If $x \ll 1$

SHM

$F \longrightarrow -2Kx$

7. (d)

8. A particle executes simple harmonic motion with a frequency. f. The frequency with which its kinetic energy oscillates is **[1987 - 2marks]**

(a) $f/2$ (b) f (c) $2f$ (d) $4f$

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8. (c)

Hlw
J

9. An object of mass 0.2 kg executes simple harmonic oscillation along the x-axis with a frequency of $(25/\pi)$ Hz. At the position $x = 0.04$, the object has Kinetic energy of 0.5 J and potential energy 0.4 J. The amplitude of oscillations is .....m. **[1994 - 2marks]**

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9. (0.06)

10. A linear harmonic oscillator of force constant 2×10^6 N/m and amplitude 0.01 m has a total mechanical energy of 160 J. Its **[1989 - 2 Mark]**

(a) maximum potential energy is 100 J

(b) maximum kinetic energy is 100 J

(c) maximum potential energy is 160 J

(d) maximum potential energy is zero

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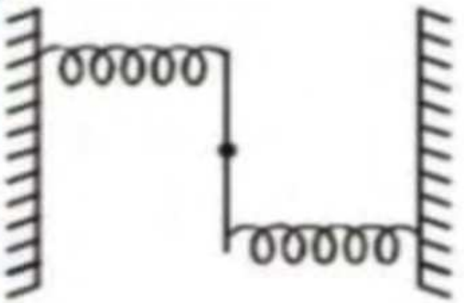
10. (b, c)

21. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane.



The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is

[2009]

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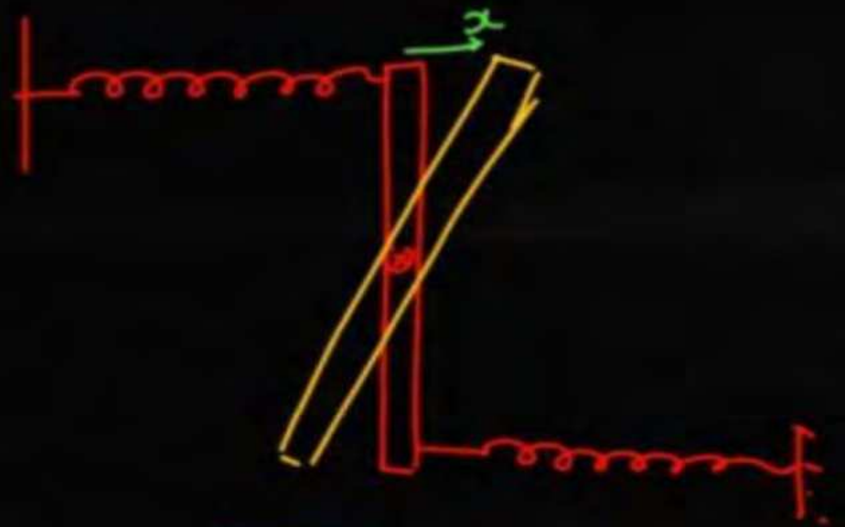
(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$

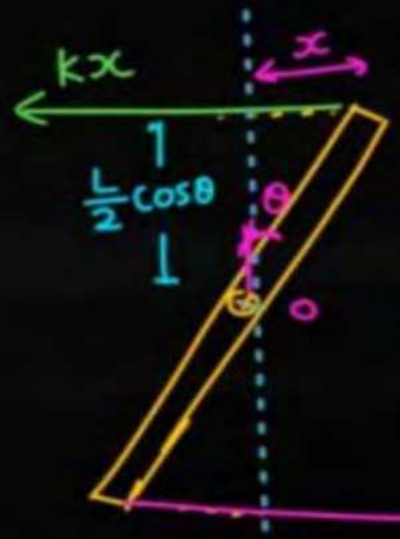
(b) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$

(c) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

(d) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

21. (c)





$\tau = kx\frac{L}{2}\cos\theta \times 2$

$\tau = k\cos\theta \cdot L \cdot x$

$\tau = k\cos\theta\, L\, \frac{L\theta}{2}$

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$T = 2\pi\sqrt{\frac{I}{\frac{kL^2}{2}}} = 2\pi\sqrt{\frac{mL^2}{12 \cdot \frac{kL^2}{2}}}$

$T = 2\pi\sqrt{\frac{m}{6k}}$

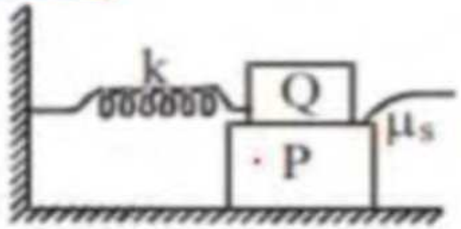
$x = \frac{L}{2}\sin\theta$

$x \approx \frac{L}{2}\theta$

$\vec{\tau} = -\left(\frac{kL^2}{2}\right)\vec{\theta}$

$f = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$

23. A block P of mass m is placed on a horizontal frictionless plane. A second block of same mass m is placed on it and is connected to a spring of spring constant k, the two blocks are pulled by distance A. Block Q oscillates without slipping. What is the maximum value of frictional force between the two blocks. **[2004S]**



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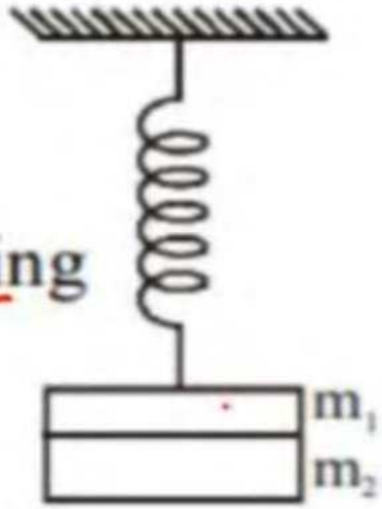
(a) $kA/2$ (b) kA

(c) $\mu_s mg$ (d) zero

23. (a)

38. Two masses m_1 and m_2 are suspended together by a massless spring of spring constant k. When the masses are in equilibrium, m_1 is removed without disturbing the system. Find the angular frequency and amplitude of oscillation of m_2.



E / C

$\frac{m_1 g}{k}$

$T = 2\pi\sqrt{\frac{m_2}{k}}$

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SHM

$$\vec{F}_{net} = -k\vec{x}$$

$$m\vec{a} = -k\vec{x} \qquad \vec{a} = -\frac{k}{m}\vec{x}$$

Solve $\Rightarrow$ $\dfrac{d^2\vec{x}}{dt^2} = -\dfrac{k}{m}\vec{x}$

$$x = A\sin(\omega_0 t + \phi)$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$x = A\sin(\omega_0 t + \phi)$

$\phi = 0$ (let)

$$TE = \frac{1}{2}kA^2$$

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$$\vec{F}_{net} = -k\vec{x} - b\vec{v}$$

$$\vec{a} = -\frac{k\vec{x}}{m} - \frac{b\vec{v}}{m}$$

$$\frac{d^2\vec{x}}{dt^2} = -\frac{k}{m}\vec{x} - \frac{b}{m}\frac{d\vec{x}}{dt}$$

$$x = A\sin(\omega t + \phi)$$

$$x = A_0 e^{-rt}\sin(\omega t + \phi)$$

A = Amplitude $= A_0 e^{-rt}$

$$\vec{F}_v \propto -\vec{v}$$

$$\vec{F}_v = -b\vec{v}$$

damping Const

$$r = \frac{b}{2m}$$

पुराना SHM

$\vec{F} = -K\vec{x}$

$x = A_0 \sin(\omega_0 t + \phi)$

$A = A_0 e^{-\gamma t}$

$\gamma = \frac{b}{2m}$

Const $\omega = \sqrt{\omega_0^2 - \gamma^2}$

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$\omega < \omega_0$

$T > T_0$

$T \longrightarrow$ Const

$T > T_0$

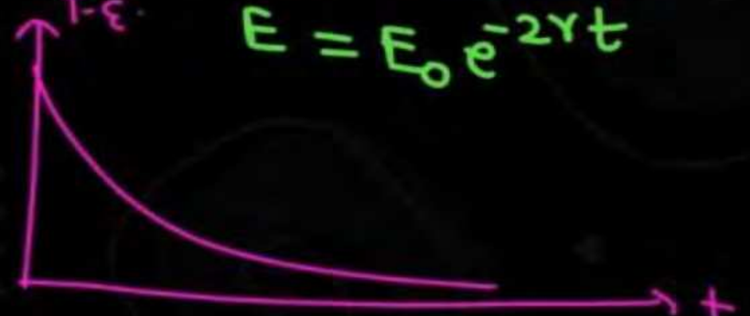
If $b = 0$, Undamped oscillation

$\gamma = 0$, $A = A_0 =$ Const

Total Energy $= \frac{1}{2} K A^2$

$T.E. = \frac{1}{2} K (A_0 e^{-\gamma t})^2$

$T.E. = \frac{1}{2} K A_0^2 e^{-2\gamma t}$



$E = E_0 e^{-2\gamma t}$

Q If at $t=0$ amplitude of damp oscillation is 10 m. After 2 sec amplitude become 6m. find amplitude after 2 more sec.

$t=0$, $A_0=10$

$t=2$ $A=6$

$t=4$ $A=?$

$$A = A_0 e^{-\gamma t}$$

$$6 = 10\, e^{-\gamma \cdot 2} \Rightarrow e^{-2\gamma} = \cdot 6$$

$$A = 10\, e^{-4\gamma}$$

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$A = A_0 e^{-\gamma t}$

$t = 0$	$t = 5$	$t = 15$
A_0	$\cdot 9 A_0$	A

Q Amplitude of a damped oscillator decreases to $\cdot 9$ times to its original magnitude in 5 sec. In another 10 sec it will decrease to 'a' times to its original magnitude where 'a' equal to?

JEE mains 2013

$\cdot 9 A_0 = A_0 e^{-5\gamma}$

$\cdot 9 = e^{-5\gamma}$

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$A = A_0 e^{-15\gamma}$

$= A_0 (e^{-5\gamma})^3 = A_0 (\cdot 9)^3 = \cdot 729\, A_0$

(A) $\cdot 81$

(B) $\cdot 729$

(C) $\cdot 6$

(D) $\cdot 7$

Q Displacement of damped harmonic oscillator is given by $x = e^{-\cdot 1t}\cos(10\pi t + \phi)$

time taken for its amplitude of vibration to drop to half of its initial value is close to

JEE main 2019

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$\frac{A_0}{2} = A_0 e^{-\gamma t}$ $\quad (\gamma = \cdot 1)$

$2^{-1} = e^{-\gamma t}$

$\ln 2 = \gamma t \ln e$

$2 = e^{\gamma t}$

$2.3 \times \log 2 = \cdot 1 \times t$

$\cdot 69 = \cdot 1 t$

$t = 6.9$

Ans (7)

5 oscillation → 1
10 " → 2

Q A damped harmonic oscillator has a frequency of 5 oscillation per second. Amplitude drop to half of its value for every 10 oscillation. The time it will takes to drop to $\frac{1}{1000}$ of the original amplitude is closed to

JEE main 2019

(a) 50 sec (b) 100 sec (c) 20 sec (d) 10 sec

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A_0 → $A_0/2$
$t = 0$ → $t = 2$

$$\frac{A_0}{2} = A_0 e^{-\gamma \times 2}$$

$$\frac{A_0}{1000} = A_0 e^{-\gamma t}$$

$$\frac{A_0}{2} = A_0 e^{-2r}$$

$$2 = e^{2r}$$

$$\ln 2 = 2r \ln e$$

$$\boxed{r = \frac{1}{2}\ln 2}$$

$$\frac{A_0}{1000} = A_0 e^{-rt}$$

$$1000^{-1} = e^{-rt}$$

$$\ln 1000 = rt \ln e$$

$$3 \ln 10 = rt$$

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$$3 \ln 10 = \frac{\ln 2}{2} t$$

$$t = \frac{6 \ln 10}{\ln 2} = \frac{6 \times 2.3 \log 10}{2.3 \log 2} = \frac{6 \times 1}{.3} = \frac{60}{3}$$

$$= 20 \text{ sec}$$

Q A pendulum with time period of 1sec is loosing energy due to damping. At certain time its energy is 45J. If after completing 15 oscillation, its energy become 15J find damping constant

JEE mains

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$$E = E_0 e^{-2\gamma t}$$

$$15 = 45\, e^{-2\times\gamma\times 15}$$

$$\ln 3 = 2\gamma \times 15$$

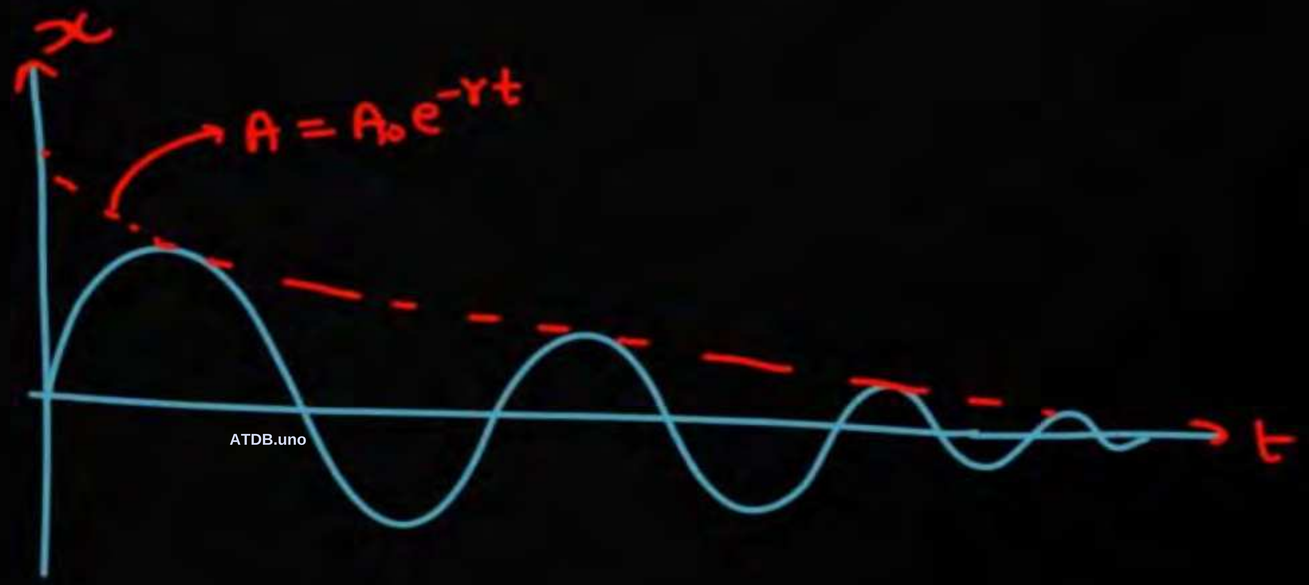
$$\gamma = \frac{\ln 3}{30}$$

∴

$$A_d = A = A_0 e^{-\gamma t}$$

$$\gamma = \frac{b}{2m}$$

$$\omega = \sqrt{\omega_0^2 - \gamma^2}$$



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$t_1 = T/4$

$t_2 = T/4$

$t_2 > t_1$

$t_1 + t_2 = T/2$

$$\omega = \sqrt{\omega_0^2 - \gamma^2}$$

$$\omega = \omega_0 \sqrt{1 - \frac{\gamma^2}{\omega_0^2}}$$

$$\omega = \omega_0 \sqrt{1 - \lambda^2}$$

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$\gamma < \omega_0 \Rightarrow$ Underdamping
$\gamma = \omega_0 \Rightarrow$ Critical damping
$\gamma > \omega_0 \Rightarrow$ Overdamping.

$$\lambda = \frac{\gamma}{\omega_0} = \text{damping ratio}$$

$\lambda < 1 \Rightarrow$ Underdamping
$\lambda = 1 \Rightarrow$ Critical damping
$\lambda > 1$ Overdamping.

Relaxation time → it is the time interval in which amplitude decreases by $\frac{1}{e}$ times.

(time const)

$$A = A_0 e^{-\gamma t}$$

$$\frac{A_0}{e} = A_0 e^{-\gamma \tau}$$

$$e^{-1} = e^{-\gamma \tau}$$

$$\tau = \frac{1}{\gamma} = \frac{2m}{b}$$

$$\gamma = \frac{b}{2m}$$

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Damping Decrement → factor by which amplitude decreases in one time period.

$$= e^{\gamma \cdot T}$$

$t \Rightarrow$

$$A_1 = A_0 e^{-\gamma t}$$

$$A_2 = A_0 e^{-\gamma(t+T)}$$

$$\frac{A_1}{A_2} = \frac{e^{-\gamma t}}{e^{-\gamma(t+T)}} = e^{-\gamma t + \gamma(t+T)} = e^{\gamma T}$$

$$\frac{A_1}{A_2} = e^{\gamma T}$$

$$\ln \frac{A_1}{A_2} = \ln e^{\gamma T}$$

$$= \gamma T$$

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Logarithm decrement (λ), $\lambda = \gamma \cdot T$

$$\ln \frac{A_1}{A_2} = \ln e^{\gamma T}$$

$$\frac{A_1}{A_2} = e^{\gamma T}$$

$$\omega = \sqrt{\omega_0^2 - \gamma^2}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \gamma^2}}$$

$$\ln \frac{A_1}{A_2} = \gamma T = \frac{b}{2m} T$$

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$A_1 \longrightarrow$ Amplitude at time t

$A_2 \longrightarrow$ " " " $t+T$

Quality factor $Q = \frac{\pi}{\lambda} = \frac{\pi}{\gamma T}$

$\gamma \uparrow$, A जल्दी खत्म $\equiv$ $Q \downarrow$

$\gamma \downarrow$, A हॉल्फ-लाइफ कम होगा, $Q \uparrow$

$\lambda \downarrow$

$E = E_0 e^{-2\gamma t}$

$A = A_0 e^{-\gamma t}$

$\gamma \uparrow$, A जल्दी खत्म E होगी

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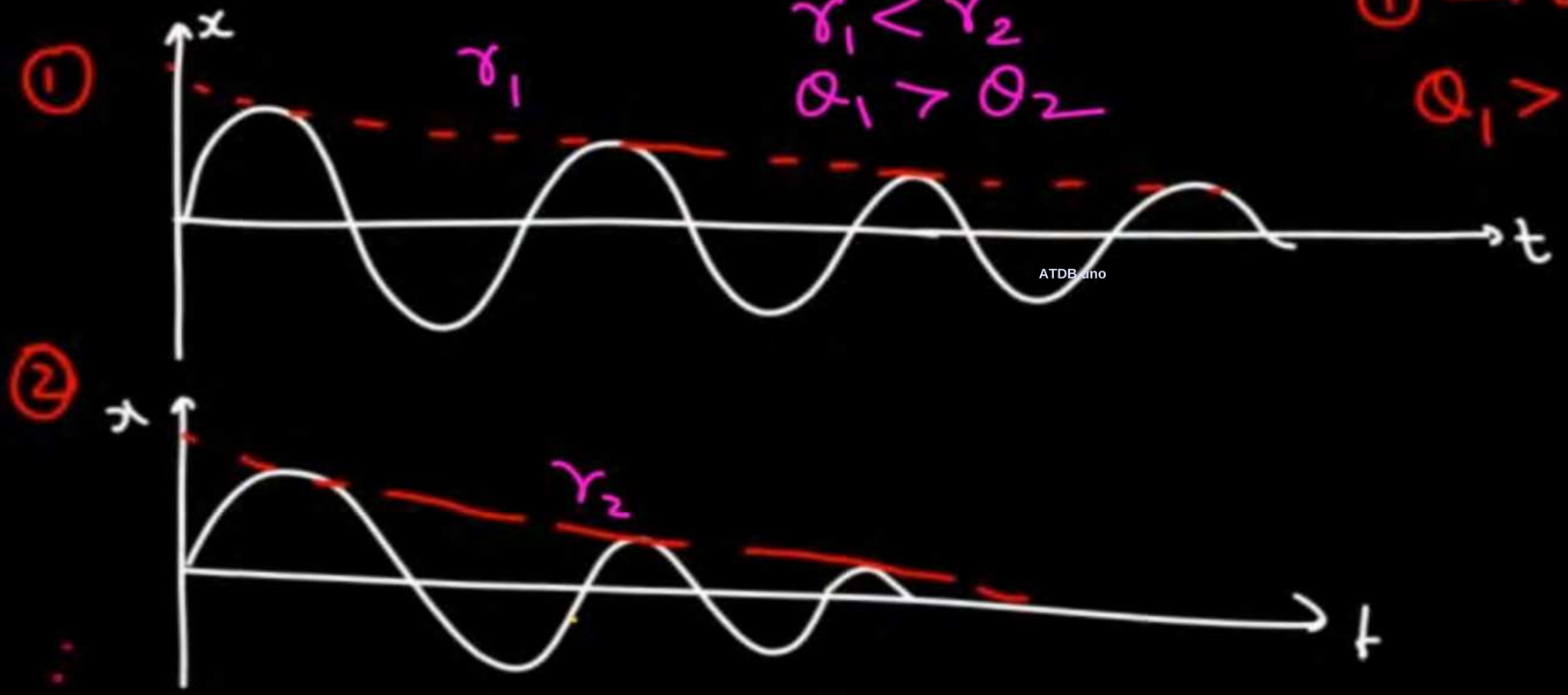
Q which graph has heighest Q factor (m → same)



$\gamma_1 < \gamma_2$

$Q_1 > Q_2$

① → ε̸t̸e̸t̸ - ∃

$Q_1 > Q_2$

$\gamma_1 < \gamma_2$

$b_1 < b_2$

$\gamma = \frac{b}{2m}$

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Summary

$\vec{F} = -K\vec{x} - b\vec{v}$ $\quad \gamma = \frac{b}{2m}$

$A = A_0 e^{-\gamma t}$

$x = A_0 e^{-\gamma t} \sin(\omega t + \phi)$

$\omega = \sqrt{\omega_0^2 - \gamma^2}$

$\gamma = \frac{b}{2m}$

ω → damped angular freq

ω_0 → Natural angular freq.

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Case 1

$\omega_0 > \gamma$ (Underdamping) oscillation

$T, \omega \rightarrow$ const

$\omega < \omega_0$

$T > T_0$

ω_0 → natural ang. freq

Case-2 Critical Damping

$\omega_0 = \gamma$ $\quad T \longrightarrow \infty$

(No oscillation)

Ex. → door,

Case 3 Overdamping

$\omega_0 < \gamma$

No oscillation

$\omega \longrightarrow$ imaginary (maths)

Home Work

- Solve JA SHM again if you have solved once

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