Laws of Motion.

Inertia

The property of an object by virtue of which it cannot change its state of rest or of uniform motion along a straight line on its own, is called **Inertia**. Greater the mass of a body greater will be its inertia and vice-versa. Inertia is of three types.

(i) **Inertia of rest** :- It is defined as the tendency of a body to remain in its position of rest.

(ii) **Inertia of motion** :- It is defined as the tendency of a body to remain in its state of uniform motion along a straight line.

(iii) **Inertia of direction** :- It is defined as inability of a body to change by itself its direction of motion.

Force

Force is a push or pull which changes or tries to change the state of rest, the state of uniform motion, size or shape of a body.
Its SI unit is newton(N) and its dimensional formula is $[MLT^{-2}]$.

Force can be categorised into two types :-

(i) Contact Forces :- Frictional force, tensional force spring force, normal force etc are the contact forces.

(ii) Distant Forces :- Electrostatic forces, gravitational forces, magnetic force etc are action at a distance forces.

Impulsive Force

A force which acts on a body for a short interval of time and produce a large change in momentum is called an impulsive force

Linear Momentum.

The total amount of motion present in a body. Linear momentum of a body is equal to the product of its mass and velocity. It is denoted by P

Linear momentum $p = mv$

Its SI unit is kg-m/s and dimensional formula is $[MLT^{-1}]$

It is a vector quantity and its direction is in the direction of velocity of the body.

Law of Conservation of Linear Momentum.

If no external forces acts on a system, then its total linear momentum remain conserved

Linear momentum depends on the frame of reference but law of conservation of linear momentum is independent from frame of reference.

Newton laws of motion are valid only in inertial frame of reference.

What is a system?

Two or more than two objects that interact with each other form a system.

Internal and external forces.

If the action-reaction pair exists in the considered system, then it is known as internal force otherwise it is known as external force

Free Body Diagram (FBD)

The diagrammatic representation of a body that is

isolated from it surrounding, showing all the external force acting on it, is known as the free-body diagram (FBD).

Steps for drawing the FBD

1 Isolate the free body
2 Draw the external forces
3 Choose the axes and resolve the forces

Newton's Law of Motion

1. Newton First Law of Motion

A body continues to be in its state of rest or in uniform motion along a straight line unless an external force is applied on it.
This law is also called law of Inertia.

Example :- when a carpet or a blanket is beaten with a stick, then the dust particles separate out from it.

2 Newton's Second Law of Motion :-

The rate of change of linear momentum is proportional to the applied force and change in momentum takes place in the direction of applied force

Mathematically

$$F \propto \frac{dp}{dt} \Rightarrow F = K\frac{d(mv)}{dt}$$

where K is a constant of proportionality and its value is one in SI and CGS system

$$F = \frac{mdv}{dt} = ma$$

The second law of motion is a vector law It is equivalent to three equations. One for each component of the vectors.

$$F_x = \frac{dp_x}{dt} = m \cdot a_x$$

$$F_y = \frac{dp_y}{dt} = m \cdot a_y$$

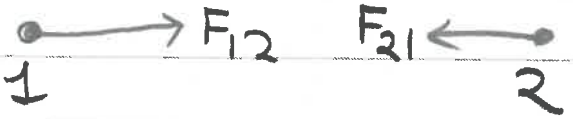
$$F_z = \frac{dp_z}{dt} = ma_x$$

Example

It is easier for a person to push an empty Shopping cart than a full one (this is depending on the mass of the object)

Newton's Third Law of motion :—

For every action there is an equal and opposite reaction and both acts on two different bodies



Mathematically $F_{12} = -F_{21}$

Examples :—

(i) Swimming becomes possible because of third law of motion.
(ii) Jumping of a man from a boat onto the bank of river.

Rocket

Rocket is an example of variable mass following law of conservation of momentum.

Thrust on the rocket at any instant.

$$F = -u\frac{dM}{dt}$$

where u = exhaust speed of the burnt gases

and $\frac{dM}{dt}$ = rate of combustion of fuel.

Velocity of rocket at any instant is given by

$$V = V_0 + u \log_e \frac{M_0}{M}$$

where

V_0 = initial velocity of the rocket
M_0 = initial mass of the rocket
and M = present Mass of the rocket

If effect of gravity is taken into account, then Speed of rocket

$$V = V_0 + u \log_e \left(\frac{M_0}{M}\right) - gt.$$

Equilibrium of a particle.

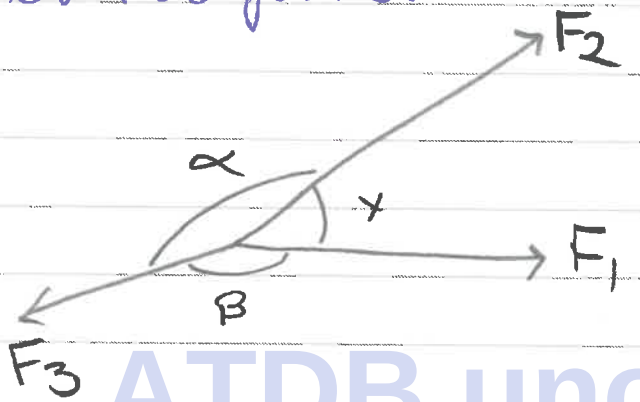
When the vector sum of the forces acting on a body is zero. then the body is said to be in equilibrium.

If two forces F_1 and F_2 act on a particle,

then they will be in equilibrium if $F_1 + F_2 = 0$

Lami's theorem.

It states that, if three forces acting on a particle are in equilibrium, then each force is proportional to the sine of the angle between the other two forces.



$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Common Forces in Mechanics.

Some of the comman forces that we come across in mechanics are given as

Weight

It is a field force. It is the force with which a body is pulled towards the centre of the earth due to gravity. It has the magnitude

mg where m is the mass of the body, and g is the acceleration due to gravity.

Normal Reaction :-

It is contact force. It is the force between two surfaces in contact, which is always perpendicular to the surface of contact.

Tension

→ Tension force always pulls a body
→ Tension is a reactive force
→ Tension across a massless pulley or frictionless pulley remain Constant.

Ideal spring.

→ An Ideal spring is made of an ideal string (massless and inextensible) wound in the form of a helix

→ Due to the design of spring, its length can vary on the application of the force

→ **Restoring nature** : Spring has a tendency to resist any kind of deformation.

Spring force : It is a restoring force developed in a string/spring. It is given by Hooke's law $F = -kx$ where x is the change in length of spring. Spring constant (k) denotes the stiffness of spring which signifies how difficult it is to deform the spring.

Friction

A force acting on the point of contact of the objects, which opposes the relative motion is called friction.

It acts parallel to the contact surfaces

Frictional forces are produced due to intermolecular interactions acting between the molecules of the bodies in contact.

Friction is of three types :-

1 Static Friction

It is an opposing force which comes into play when one body tends to move over the surface of the other body but actual motion is not taking place.

Static friction is a self-adjusting force which increases as the applied force is increased. Static friction opposes impending motion.

Limiting friction

It is the maximum value of static friction when body is at the verge of starting motion

$$\text{Limiting friction } f_{s(max)} = \mu_l R$$

where μ_l = coefficient of Limiting friction and R = normal reaction.

Limiting friction do not depend on area of contact surfaces but depends on their nature, ie smoothness or roughness.

Angle of friction It is the angle which the resultant of the force of limiting friction and the normal reaction (N) makes with the direction of R

$$\mu_l = \tan\theta$$

Angle of Repose or Angle of Sliding :- It is the minimum angle of inclination of a plane with the horizontal, such that a body placed on it, just begins to slide down.

If angle of repose in α and coefficient of limiting friction is μ_l then

$$\mu_l = \tan\alpha$$

Kinetic friction

It is an opposing force that comes into existence when one object is actually moving over the surface of other object.

$$\text{Kinetic friction}(f_K) = \mu_K R$$

where μ_K = coefficient of kinetic friction and R = normal reaction.

Kinetic friction is of two types.

(a) Sliding friction
(b) Rolling friction

As rolling friction < Sliding friction, therefore it is easier to roll a body than to slide.

Motion on a Rough Inclined plane

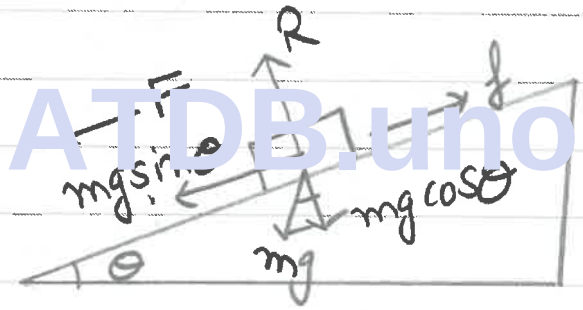
When an object moves along an inclined plane then different forces act on it like

normal reaction of plane, friction force acting in opposite direction of motion etc.

Different relations for the motion are given below

Normal reaction of plane $R = mg\cos\theta$

and net force acting downwald on the block, $F = mg\sin\theta - f$.



Acceleration on inclined plane

$$a = g(\sin\theta - \mu\cos\theta)$$

Note*

when angle of inclination (θ) of the plane from horizontal is less than the angle of repose (α) then

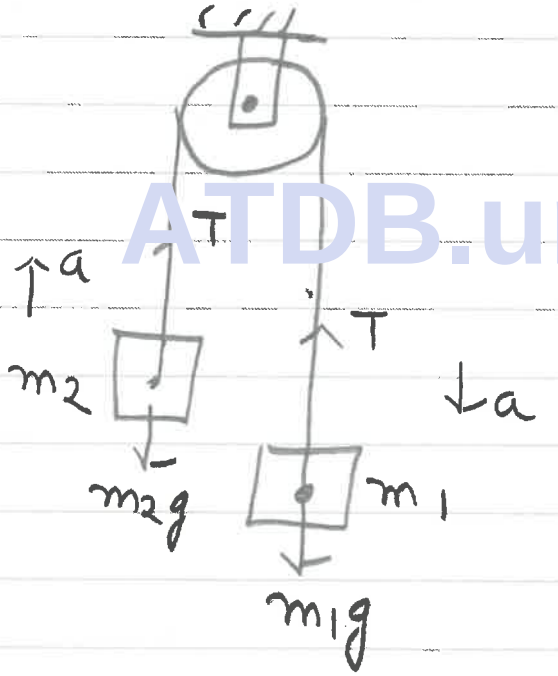
(i) minimum force required to move the body up the inclined plane

$$F_1 = mg(\sin\theta + \mu\cos\theta)$$

(ii) minimum force required to push the body down the inclined plane

$$F_2 = mg(\mu\cos\theta - \sin\theta)$$

Pulley Mass System.



When unequal masses m_1 and m_2 are suspended from a pulley ($m_1 > m_2$)

$$m_1g - T = m_1a \text{ and } T - m_2g = m_2a$$

On solving equation we get.

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

$$T = \frac{2 m_1 m_2}{(m_1 + m_2)} g.$$

Constrained Motion.

Constrained motion results when an object is forced to move in a restricted way.

There a two primary constraints.

1. String / Rod constraints.
2. Wedge constraints.

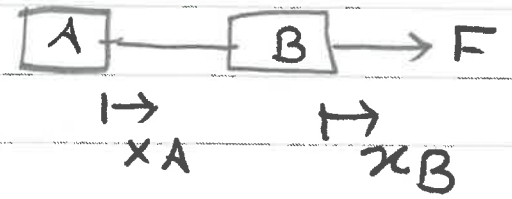
String constraints.

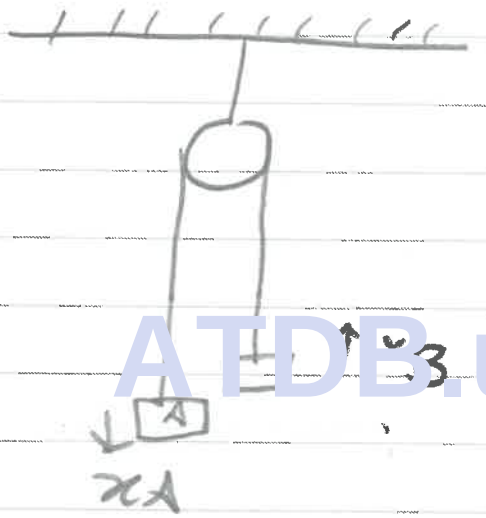
Consider object connected through a string that has the following properties.

→ The length of the string remains constant, ie string is inextensible

which means that if one block moves x distance the other block also moves the same distance
It can be mathematically written as
$x_A = x_B$.

Where x_A is displacement of block A and x_B is displacement of ~~block~~ block B





→ It always remain tight and does not slack

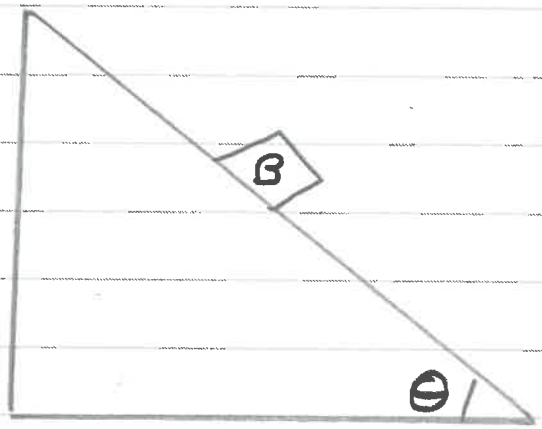
Parameters of the motion of such object along the length of the string and in the direction of extension have a definite relation between them.

Here, the parameters of motion means the displacement, velocity and acceleration.

Object mean the bodies that are directly connected to string.

Definite relation means the constraint relation such as $x_A = x_B$

Wedge constraints.



Wedge constrained motion: When a body moves over a wedge. It follows certain sliding constr-aint known as wedge constrains. Wedge Constraint is the component of velocity and acceleration perpendicular to the contact surface (interface) of two object. It is always equal if there is no deformation and the objects remain in contact.